NUMBER SYSTEMS

- KOUSTAV

CONCEPT – REMAINDERS

$\overset{1\ +\ 2}{11+12} \xrightarrow{R} 3$
10

$\overset{1\ \times\ 2}{11\times12} \xrightarrow{R} 2$
10

$\overset{2\ \times\ 4\ \times\ 6}{13\times15\times17} \xrightarrow{R} \frac{48}{11} \xrightarrow{R} 4$
11

$\overset{2\ \times\ 3\ +\ 4}{14\times15+16} \xrightarrow{R} 10$
12

$\overset{2\ \times\ 2\ +\ 0}{17\times27+55} \xrightarrow{R} 4$
5

1. On dividing a number by 5, we get 3 as remainder. What will be the remainder when the square of this number is divided by 5?

A. 0 B. 1 C. 2 ~~D. 4~~

$\frac{3^2}{5}$ R → 4

$\frac{8^2}{5}$ R → 4

$\frac{13^2}{5}$ R → 4

$$\begin{array}{r} 5\overline{)3\,(0} \\ -0 \\ \hline 3 \end{array}$$

2. On dividing a number by 774, we get 35 as remainder. What will be the remainder when the same number is divided by 18?

A. 14 B. 17 C. 18 D. 19

$\frac{35}{18}$ R ⇒ 17

774) N (Q

35

$N = 774Q + 35$

$0 \times q + 17$

R ⇒ $0 \times q + 17$

$= 17$

3. What is the remainder when 2^{25} is divided by 3?

A. 2 B. 1 C. 0 D. 3

$\frac{2^{25}}{3} \xrightarrow{R} \frac{2^{25}}{3} \xrightarrow{R} 2^{25}$ - - - - -

$\frac{2^{25}}{3} \xrightarrow{R} (-1)^{25} = -1$

$\Rightarrow 3 + (-1) = 2$ =

$3)2(0$ $3)2(1$

$\underline{-0}$ or $\underline{-3}$

2 -1

$(-1)^{odd} = -1$

$(-1)^{even} = 1$

$\frac{5^{44}}{3} \xrightarrow{R} \frac{2^{44}}{3} \xrightarrow{R} (-1)^{44} = 1$ =

$\frac{34^{34}}{7} \xrightarrow{R} \frac{6^{34}}{7} \xrightarrow{R} (-1)^{34} = 1$ =

$\frac{19^{19}}{5} \xrightarrow{R} \frac{4^{19}}{5} \xrightarrow{R} (-1)^{19} = -1$

$\Rightarrow 5 + (-1) = 4$ =

$\frac{19^{19}}{9} \xrightarrow{R} 1^{19} = 1$ =

4. What is the remainder when ($1^1 + 2^2 + 3^3 + \ldots + 100^{100}$) is divided by 4?

A. 3 B. 1 C. 2 ✓~~D. 0~~

$\frac{1^1 + 2^2 + 3^3 + 4^4}{4}$

$\Rightarrow 1 + 0 + (-1)^3 + 0^4$

$= 1 + 0 - 1 + 0$

$= 0$

$\frac{5^5 + 6^6 + 7^7 + 8^8}{4}$

$\Rightarrow \frac{1^5 + 2^6 + 3^7 + 0^8}{4}$

$\Rightarrow 1 + 0 + (-1)^7 + 0$

$= 0$

5. Find the remainder when 53^{12} is divided by 17.

A. 8 B. 0 C. 1 D. 16

$\frac{53^{12}}{17} \xrightarrow{R} \frac{2^{12}}{17} = \frac{(2^4)^3}{17}$

$= \frac{16^3}{17} \xrightarrow{R} (-1)^3 = -1$

$\Rightarrow 17 + (-1) = 16$

[Power of 2; nearest to 17]

$(a^x)^y = a^{xy}$

$\frac{32^{32}}{15} \xrightarrow{R} \frac{2^{32}}{15} = \frac{(2^4)^8}{15} = \frac{16^8}{15} \xrightarrow{R} 1^8 = 1$

$\frac{16^{24}}{9} \xrightarrow{R} (-2)^{24} = \frac{2^{24}}{9} = \frac{(2^3)^8}{9} = \frac{8^8}{9} \xrightarrow{R} (-1)^8 = 1$

$\frac{20^{21}}{9} \xrightarrow{R} \frac{2^{21}}{9} = \frac{(2^3)^7}{9} = \frac{8^7}{9} \xrightarrow{R} (-1)^7 = -1$

$\Rightarrow 9 + (-1) = 8$

$\frac{27^{22}}{8} \xrightarrow{R} \frac{3^{22}}{8} = \frac{(3^2)^{11}}{8} = \frac{9^{11}}{8} \xrightarrow{R} 1^{11} = 1$

$\frac{32^{33}}{15} \xrightarrow{R} \frac{2^{33}}{15} = \frac{2^{32} \times 2^1}{15} = \frac{(2^4)^8 \times 2^1}{15} = \frac{16^8 \times 2}{15}$

$\xrightarrow{R} 1^8 \times 2 = 2$

$\frac{20^{23}}{9} \xrightarrow{R} \frac{2^{23}}{9} = \frac{2^{21} \times 2^2}{9} = \frac{(2^3)^7 \times 2^2}{9} = \frac{8^7 \times 4}{9}$

$\xrightarrow{R} (-1)^7 \times 4 = -4 \Rightarrow 9 - 4 = 5$

6. The remainder when $(7^{21}+7^{22}+7^{23}+7^{24})$ is divided by 25:

A. 1 B. 24 C. 0 D. 12

$$\frac{7^{21}+7^{22}+7^{23}+7^{24}}{25}$$

$$= \frac{7^{20}\left(7+7^2+7^3+7^4\right)}{25}$$

$$= \frac{(7^2)^{10}\left(7+7^2+7^2\times 7+(7^2)^2\right)}{25}$$

$\Rightarrow (-1)^{10}\left(7+(-1)+(-1)\times 7+(-1)^2\right)$

$= 1(7-1-7+1)$

$= 0$

$25)\,49\,(2$

-50

-1

$\frac{7^2}{25} \Rightarrow R \to -1$

7. $P = (1!)^2 + (2!)^2 + (3!)^2 + \ldots + (100!)^2$.
The remainder when 5^{2P} is divided by 13 is:

A. 1 B. 12 C. 0 D. 2

$\frac{5^{2P}}{13} = \frac{25^{P}}{13}$ R $\Rightarrow (-1)^P$

[Ans $\Rightarrow$ 1 if P is even
Ans $\Rightarrow$ -1 or 12 if P is odd]

$P = (1!)^2 + (2!)^2 + (3!)^2 + \ldots + (100!)^2$

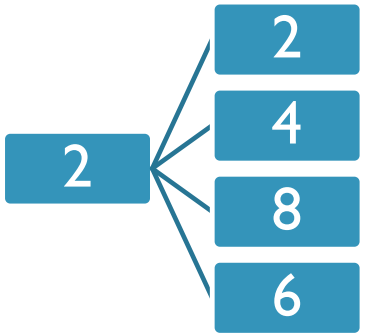
$(1!)^2$ ↓ odd + $(2!)^2 + (3!)^2 + \ldots + (100!)^2$ even

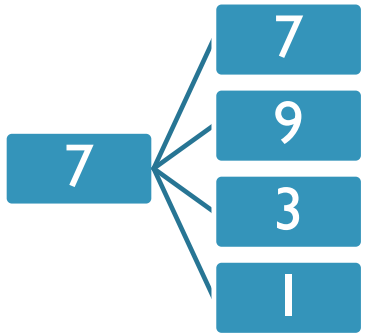
[If $n \geq 2$; $n!$ = even]

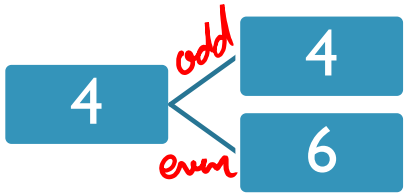
P = odd

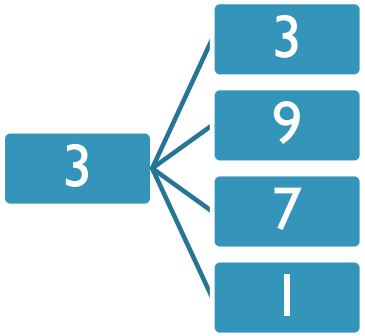
$(-1)^P = -1 \Rightarrow 13 - 1 = 12$

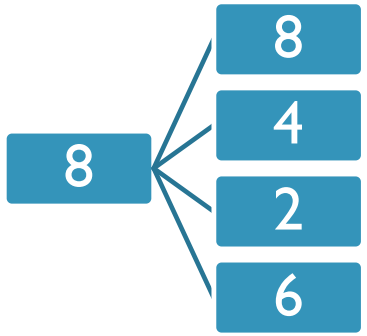
CONCEPT – CYCLICITY (UNIT'S PLACE)

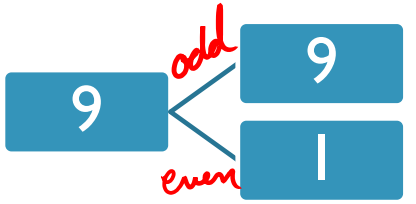


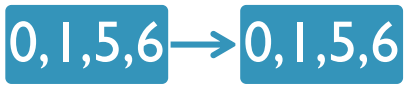












1. What is the last digit of the following expressions:

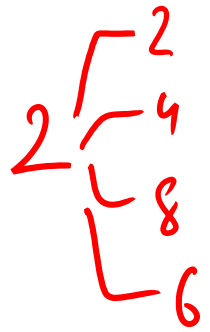
1.a) 2^5

2

1.b) 2^{25}

$\frac{25}{4} \xrightarrow{R} 1$

$2^1 = 2$



1.c) 2^{125}

$\frac{125}{4} \Rightarrow \frac{25}{4} \xrightarrow{R} 1$

$2^1 = 2$

1.d) 432^{1234}

$2^{1234} \Rightarrow 2^{34}$

$\frac{34}{4} \xrightarrow{R} 2$

$2^2 = 4$

2. What is the last digit of the expression 777^{777}?

A. 3 B. 1 C. 7 D. 9

777^{777}

$\Rightarrow 7^{777} \Rightarrow 7^{77}$

$\frac{77}{4}$ R $\rightarrow 1$

$7^1 = 7$

7 | 7
7 | 9
3
1

3. The unit's digit of the product $3^{1001} \times 7^{22002} \times 13^{333003}$ is:

A. 3 B. 1 C. 5 D. 9 ✓

$3^{01} \times 7^{02} \times 3^{03}$

$3 \times 9 \times 7$

9

3 → 9 → 7 → 1

7 → 9 → 3 → 1

4. The unit's digit of the sum $22^{222} + 33^{333} + 44^{444}$ is:

A. 3 B. 1 C. 5 D. 9

$2^{22} + 3^{33} + 4^{\text{even}}$

$\frac{22}{4} \Rightarrow 2$ $\frac{33}{4} \Rightarrow 1$

$2^2 + 3^1 + 6$

$4 + 3 + 6$

3

$2 \rightarrow 2, 4, 8, 6$

$3 \rightarrow 3, 9, 7, 1$

$4 \rightarrow 4, 6$

$4^{\text{odd}} \rightarrow 4$

$4^{\text{even}} \rightarrow 6$

5. N = 1! + 2! + 3! +……+ 2010!. What is the digit in the unit's place of N?

A. 3 B. 2 C. 1 D. 0

$1! + 2! + 3! + 4! + 5! + \cdots + 2010!$

$1 + 2 + 6 + 4 + \quad '0'$

3

[If $n \geq 2$; $n!$ = even

If $n \geq 5$; $n!$ ends with 0]

6. The unit's place of the product $34^{123!} \times 3456^{123456!}$ is:

A. 4 B. 8 C. 1 ✓ D. 6

$4^{123!} \times 6^{123456!}$

$4^{\text{even}} \times 6$

6×6

6

$4 \overline{)4}$
$\quad\ \ 6$

$6 \to 6$

7. $1234567890^{1234567890}$

ANSWER KEY – NUMBER SYSTEMS

REMAINDERS		CYCLICITY	
QUESTION	ANSWER	QUESTION	ANSWER
1	D	1	2, 2, 2, 4
2	B	2	C
3	A	3	D
4	D	4	A
5	D	5	A
6	C	6	D
7	B	7	-