Applied Discrete Modelling

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Assignment- 2

**System Specification:**

A quality tester is fed by material flows from two different sources. Only one of the two sources can be active, either *source 0* or *source 1*. The average duration of an activity period of source 0 is 2 minutes. The average duration of source 1 being active is 3 minutes. At the beginning of the simulation source 0 is active.

Assuming, that in each time step, one item is produced, the probability for the item to test OK is 0.9 for source 0 and 0.95 for source 1.

**Implementation**

Extend your DTMC solution program to discretize any CTMC using a given time step. The program should import CTMC specifications in the format given in the exercise.

**Tasks and Questions**

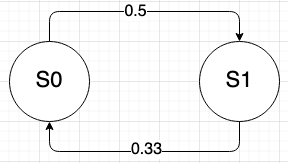
Specify and draw the CTMC representing the system.

Use your program to answer the following questions:

* What is the probability that source 0 is active after 8 minutes for different discretization time steps (e.g. 2, 1, 0.5, 0.25, 0.1)?
* What is the probability of source 0 being active in steady state for different discretization time steps (e.g. 2, 1, 0.5, 0.25, 0.1)?
* What is the average probability of testing an item OK in steady state?

**Task 01:** Specify and draw the CTMC representing the system.

Sol: The CTMC for the previously constructed DTMC, will be very similar except that CTMCs have **no self-transitions**.



Infinitesimal Generator Matrix (IGM) for the CTMC:

**Q** = The difference between Probability Transition Matrix (PTM) and IGM is that the sum along the rows is not 1 but 0.

For further evaluations we will be converting the IGM to PTM by using this formula:

**P** = **Q** + **I.Δ**

**I**: Identity matrix of size of IGM

**Δ**: It is the time-step for this conversion (we typically take the magnitude of the largest diagonal element and divide it by 1), so **Δ** > 1/max(|diag.|)

**Task 02:** Programming questions

**2.1** What is the probability that source 0 is active after 8 minutes for different discretization time steps (e.g. 2, 1, 0.5, 0.25, 0.1)?

Sol:

|  |  |
| --- | --- |
| Discretization Time Step | Probability P(0) |
| 2 | 0.6167245060324097 |
| 1 | 0.6024099159036089 |
| 0.5 | 0.6078632322148652 |
| 0.25 | 0.6642725014733947 |
| 0.1 | 0.8011978072495772 |

**2.2** What is the probability of source 0 being active in steady state for different discretization time steps (e.g. 2, 1, 0.5, 0.25, 0.1)?

Sol:

|  |  |
| --- | --- |
| Discretization Time Step | Probability P(0) |
| 2 | Cannot find with precision of over 15 decimal places |
| 1 | 0.6024096385542165 |
| 0.5 | 0.6024096385542161 |
| 0.25 | 0.6024096385542167 |
| 0.1 | 0.6024096385542097 |

**2.3** What is the average probability of testing an item OK in steady state?

Sol: The average probability of testing an item OK in steady state is 0.9198795180722776.