CS 736 : Assignment Image Denoising with MRFs

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Maximum Marks: 80
Due Date : 5 Mar 2017, Sunday, 11:55 pm

Please read, carefully, the instructions for submission at `http://www.cse.iitb.ac.in/~suyash/cs736/submissionStyle.pdf`

1. (50 marks) Denoising a Phantom Magnetic Resonance Image.

   Download the 2D noiseless image and the 2D noisy image available at
   `http://www.cse.iitb.ac.in/~suyash/cs736/assignmentImageDenoisingPhantom.mat.zip`

   Implement a maximum-a-posteriori Bayesian image-denoising algorithm that uses a noise model coupled with a MRF prior that uses a 4-neighbor neighborhood system (each pixel has 4 neighbors: left, right, up, down; the neighborhood wraps around at image boundaries) that has cliques of size *no* more than 2.

   Use gradient ascent (or descent) optimization with dynamic step size. Ensure that the value of the objective function (i.e., the log posterior or its negative) at each iteration increases (or decreases if using gradient descent). Use the noisy image as the initial solution.

   Use 3 different MRF priors where the potential functions $V(x_i, x_j) := g(x_i - x_j)$ underlying the MRF penalize the difference between the neighboring voxel values x_i, x_j as follows (see class Notes at `http://www.cse.iitb.ac.in/\%7Esuyash/cs736/Slides_AlgoMIP_ImagePrior.pdf` for details). You may rely on the *circshift()* function in Matlab when computing differences between every pixel in the image and its neighbors.

   Introduce a parameter $\alpha \in [0, 1]$ to control the weighting between the prior (weight α) and the likelihood (weight $1 - \alpha$).

   Specifically, implement the following functionality as part of the denoising algorithm:

   (a) (3 marks) A Complex-Gaussian noise model. You don't need the noise level because that parameter can be absorbed in $1 - \alpha$ that you'll tune manually (Tuning α essentially manipulates the noise level, in case of the likelihood. So we can ignore the noise level σ when tuning α manually. Use $\sigma = 1$).

   (b) (3 marks) MRF prior: Quadratic function: $g_1(u) := |u|^2$.

   (c) (3 marks) MRF prior: Discontinuity-adaptive Huber function: $g_2(u) := 0.5|u|^2$, when $|u| \leq \gamma$ and $g(u) := \gamma|u| - 0.5\gamma^2$, when $|u| > \gamma$, where $0 < \gamma < \infty$ is a constant.

   (d) (3 marks) MRF prior: Discontinuity-adaptive function: $g_3(u) := \gamma|u| - \gamma^2 \log(1 + |u|/\gamma)$, where $0 < \gamma < \infty$ is a constant.

For each MRF prior, manually tune the parameters α and γ (where applicable) to denoising the noisy image in order to achieve the least possible relative root-mean-squared error (RRMSE). The RRMSE for 2 complex images A and B is defined as :
RRMSE$(A, B) = \sqrt{\sum_p (|A(p)| - |B(p)|)^2} / \sqrt{\sum_p |A(p)|^2}$, where the summation is over all pixels p. Always use the noiseless image as A.

Report the following:

(a) (0 point) Report the RRMSE between the noisy and noiseless images.

(b) (15 marks) Report the optimal values of the parameters and the corresponding RRMSEs for each of the 3 denoising algorithms. For each optimal parameter value reported (for each of the 3 denoising algorithms), give evidence of the optimality of the reported values by reporting the RRMSE values for two nearby parameter values (around the optimal) at plus/minus 20% of the optimal value. That is, if a^*, b^* are the optimal parameter values, then report:
$a^*, b^*,$ RRMSE(a^*, b^*),
RRMSE$(1.2a^*, b^*)$, RRMSE$(0.8a^*, b^*)$,
RRMSE$(a^*, 1.2b^*)$, RRMSE$(a^*, 0.8b^*)$.
(Tip: the optimal values for α might be very close to extreme limits of the allowed range. Be aware of that possibility.) .

(c) (15 marks) Show the following 5 images (at each pixel, show the magnitude of the pixel value) in the report using exactly the same colormap (i) Noiseless image, (ii) Noisy image, (iii) Image denoised using quadratic prior $g_1(\cdot)$ and optimal parameter tuning, (iv) Image denoised using Huber prior $g_1(\cdot)$ and optimal parameter tuning, (v) Image denoised using discontinuity-adaptive prior $g_3(\cdot)$ and optimal parameter tuning.

(d) (8 marks) Show the plots of the objective-function values (vertical axis) versus iteration (horizontal axis) corresponding to each of the 3 denoised results in (iii), (iv), and (v) above.

2. (30 marks) Denoising a Magnetic Resonance Image of the Brain.

Download the 2D noisy image available at
`http://www.cse.iitb.ac.in/~suyash/cs736/assignmentImageDenoisingBrainNoisy.mat.zip`

Use all 3 maximum-a-posteriori Bayesian denoising algorithms implemented to denoise the noisy brain image.

Manually tune the parameters to give the best denoised image that, based on your judgment, gives the right tradeoff between noise removal and edge preservation.

Report the following:

(a) (3 marks) Using the (complex) intensities in the background (i.e., air), estimate the noise level, i.e., the standard deviation of the i.i.d. Gaussian noise in the real or imaginary components of the complex MR image.

(b) (18 marks) Show the following 4 images (at each pixel, show the magnitude of the pixel value) in the report using exactly the same colormap (i) Noisy image, (ii) Image denoised using quadratic prior $g_1(\cdot)$ and manual parameter tuning, (iii) Image denoised using Huber prior $g_1(\cdot)$ and manual parameter tuning, and (iv) Image denoised using discontinuity-adaptive prior $g_3(\cdot)$ and manual parameter tuning.

(c) (9 marks) Show the plots of the objective-function values (vertical axis) versus iteration (horizontal axis) corresponding to each of the 3 denoised results in (ii), (iii), and (iv) above.