CS 736 : Assignment - MRI Reconstruction, Model Fitting

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Maximum Marks: 100
Due Date : 24 Mar 2016, Fri, 11:55 pm

Please read, carefully, the instructions for submission at `http://www.cse.iitb.ac.in/~suyash/cs736/submissionStyle.pdf`

5 marks are reserved for submission in the described format.

1. (45 marks) Reconstructing a Phantom Magnetic Resonance Image.

   Download the 2D noiseless image and the sparsely-acquired k-space data (i.e., frequency-domain data) image available at
   `http://www.cse.iitb.ac.in/~suyash/cs736/assignmentImageReconstructionPhantom.mat.zip`

   In the kspace-data image, the center of kspace (i.e., low frequencies) lies at the corners of the image. Pixels where the data isn't acquired have value zero in the kspace-data image. The kspace-mask image indicates the pixels where the data wasn't acquired.

   Implement a maximum-a-posteriori Bayesian image-reconstruction algorithm that uses a sparse-acquisition transformation model, a noise model, and a MRF-prior model that uses a 4-neighbor neighborhood system (each pixel has 4 neighbors: left, right, up, down; the neighborhood wraps around at image boundaries) that has cliques of size *no* more than 2.

   Use gradient ascent (or descent) optimization with dynamic step size. Ensure that the value of the objective function (i.e., the log posterior or its negative) at each iteration increases (or decreases if using gradient descent). Use the inverse Fourier transform of the kspace data as the initial solution.

   Use 3 different MRF priors where the potential functions $V(x_i, x_j) := g(x_i - x_j)$ underlying the MRF penalize the difference between the neighboring pixel values x_i, x_j as follows (see class Notes at `http://www.cse.iitb.ac.in/~suyash/cs736/Notes_AlgoMIP_ImagePrior.pdf` for details). You may rely on the *circshift()* function in Matlab when computing differences between every pixel in the image and its neighbor.

   Introduce a parameter $\alpha \in [0, 1]$ to control the weighting between the prior (weight α) and the likelihood (weight $1 - \alpha$).

   Specifically, implement the following functionality as part of the reconstruction algorithm:

   (a) (1 marks) A Complex-Gaussian noise model. You don't need the noise level because that parameter can be absorbed in $1 - \alpha$ that you'll tune manually (Tuning α essentially manipulates the noise level, in case of the likelihood. So we can ignore the noise level σ when tuning α manually. Use $\sigma = 1$).

(b) (1 marks) MRF prior: Quadratic function: $g_1(u) := |u|^2$.

(c) (1 marks) MRF prior: Discontinuity-adaptive Huber function: $g_2(u) := 0.5|u|^2$, when $|u| \leq \gamma$ and $g(u) := \gamma|u| - 0.5\gamma^2$, when $|u| > \gamma$, where $0 < \gamma < \infty$ is a constant.

(d) (1 marks) MRF prior: Discontinuity-adaptive function: $g_3(u) := \gamma|u| - \gamma^2 \log(1 + |u|/\gamma)$, where $0 < \gamma < \infty$ is a constant.

(e) (16 marks) A sparse-acquisition transformation model that is able to simulate a general sparse MRI acquisition strategy that acquires any subset of pixels in the Fourier domain.

For each MRF prior, manually tune the parameters α and γ (where applicable) to reconstruct the noisy image in order to achieve the least possible relative root-mean-squared error (RRMSE). The RRMSE for 2 complex images A and B is defined as :
RRMSE$(A, B) = \sqrt{\sum_p (|A(p)| - |B(p)|)^2} / \sqrt{\sum_p |A(p)|^2}$, where the summation is over all pixels p. Always use the noiseless image as A.

Report the following:

(a) (0 mark) Report the RRMSE between the inverseFourier-kspaceData and noiseless images.

(b) (10 marks) Report the optimal values of the parameters and the corresponding RRMSEs for each of the 3 reconstruction algorithms. For each optimal parameter value reported (for each of the 3 reconstruction algorithms), give evidence of the optimality of the reported values by reporting the RRMSE values for two nearby parameter values (around the optimal) at plus/minus, say, 20% of the optimal value. That is, if a^*, b^* are the optimal parameter values, then report:
a^*, b^*, RRMSE(a^*, b^*),
RRMSE$(1.2a^*, b^*)$, RRMSE$(0.8a^*, b^*)$,
RRMSE$(a^*, 1.2b^*)$, RRMSE$(a^*, 0.8b^*)$.
(Tip: the optimal values for α might be very close to extreme marks of the allowed range. Be aware of that possibility.) .

(c) (10 marks) Show the following 5 images (at each pixel, show the magnitude of the pixel value) in the report using exactly the same colormap (i) Noiseless image, (ii) inverseFourier-kspaceData image, (iii) Image reconstructed using quadratic prior $g_1(\cdot)$ and optimal parameter tuning, (iv) Image reconstructed using Huber prior $g_1(\cdot)$ and optimal parameter tuning, (v) Image reconstructed using discontinuity-adaptive prior $g_3(\cdot)$ and optimal parameter tuning.

(d) (5 marks) Show the plots of the objective-function values (vertical axis) versus iteration (horizontal axis) corresponding to each of the 3 reconstructed results in (iii), (iv), and (v) above.

2. (25 marks) Reconstructing a Magnetic Resonance Image of the Brain.

Download the kspace-data image available at `http://www.cse.iitb.ac.in/~suyash/cs736/assignmentImageReconstructionBrain.mat.zip`

In the kspace-data image, the center of kspace (i.e., low frequencies) lies at the corners of the image. Pixels where the data isn't acquired have value zero in the kspace-data image. The kspace-mask image indicates the pixels where the data wasn't acquired.

Use all 3 maximum-a-posteriori Bayesian reconstruction algorithms implemented to reconstruct the noisy brain image.

Manually tune the parameters to give the best reconstructed image that, based on your judgment, gives the right tradeoff between noise/artifact removal and edge preservation.

Report the following:

(a) (16 marks) Show the following 4 images (at each pixel, show the magnitude of the pixel value) in the report using exactly the same colormap (i) InverseFourier-KspaceData image, (ii) Image reconstructed using quadratic prior $g_1(\cdot)$ and manual parameter tuning, (iii) Image reconstructed using Huber prior $g_1(\cdot)$ and manual parameter tuning, and (iv) Image reconstructed using discontinuity-adaptive prior $g_3(\cdot)$ and manual parameter tuning.

(b) (9 marks) Show the plots of the objective-function values (vertical axis) versus iteration (horizontal axis) corresponding to each of the 3 reconstructed results in (ii), (iii), and (iv) above.

3. (25 marks) Diffusion Tensor Magnetic Resonance Imaging.

Consider a diffusion-MRI experiment (in 2D) that performs diffusion imaging for a chosen set of N gradient directions $\{g_i\}_{i=1}^N$ and provides the values $\{S(g_i)\}_{i=1}^N$ corresponding to each direction.

The direction vectors are:

$\{g_i\}_{i=1}^6 = \{[1,0],[0.866,0.5],[0.5,0.866],[0,1],[-0.5,0.866],[-0.866,0.5]\}$.

For a particular pixel in the image, the acquired data for each direction vector (in the same sequence as above) are:

$\{S(g_i)\}_{i=1}^6 = \{0.5045 - i0.0217, 0.6874 + i0.0171, 0.3632 + i0.1789, 0.3483 + i0.1385, 0.2606 - i0.0675, 0.2407 + i0.1517\}$.

Use a diffusion-tensor model that represents diffusion using a 2×2 symmetric positive-definite matrix D.

Assume $S_0 = 1$ and $b_0 = 0.1$.

(a) (15 marks) Given this data, estimate D using a suitable optimization algorithm, report D, and plot the sequences of the logarithm of the objective function and the 4 entries in D over iteration.

(b) (5 marks) Report the (principal) direction (unit vector) along which the diffusion in the 2D plane is the strongest.

(c) (5 marks) How much more (by what multiplicative factor) is the diffusion in the principal direction as compared to the diffusion in the direction orthogonal to it ?