Advanced Heat Transfer Assignment

Himanshu Sharma 21ME30030

April 7, 2024

1 Check for dominance of Natural Convection

$$\begin{aligned} \text{Ra}_{\text{H}} &= \frac{H^{3}g\beta_{\infty}(T_{0} - T_{\infty})}{\nu\alpha} \\ &= 2.34 \times 10^{13} \\ v_{c} &= \frac{\alpha}{H} \text{Ra}_{\text{H}}^{1/2} \\ &= 5.73 \text{ m/s} \\ \text{Ri} &= \frac{Hg\beta_{\infty}(T_{0} - T_{\infty})}{v_{c}^{2}} \\ &= 0.80 \end{aligned}$$

2 Results

2.1 θ

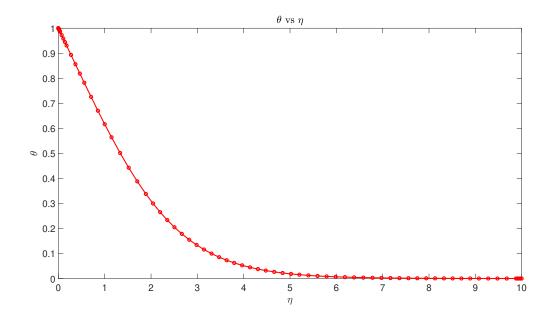


Figure 1: Non Dimensional Temperature, θ as a function of η

2.2 *G*

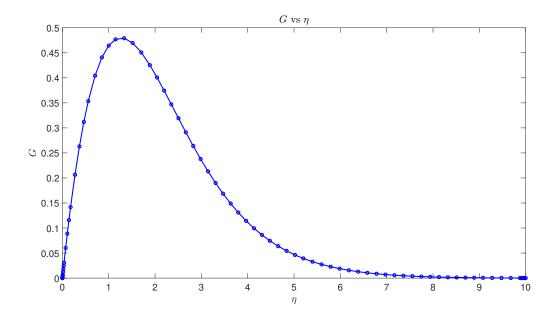


Figure 2: G as a function of η

2.3 Heat Transfer Coefficient

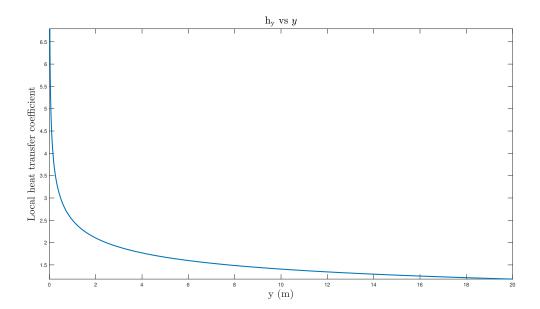


Figure 3: Heat Transfer Coefficient

$$\begin{aligned} {\rm Nu_y} &= 0.3920 \ {\rm Ra_y^{1/4}} \\ h_y &= k_{\rm air} {\rm Nu_y}/y = Cy^{-1/4} \\ h &= \frac{\int_0^H h {\rm dy}}{H} = 1.5765 \ {\rm W/m^2 K} \end{aligned}$$

2.4 Heat Flux

$$q_0^{"} = h(T_0 - T_) = 63.06 \text{ W/m}^2$$

2.5 Thermal Boundary Layer

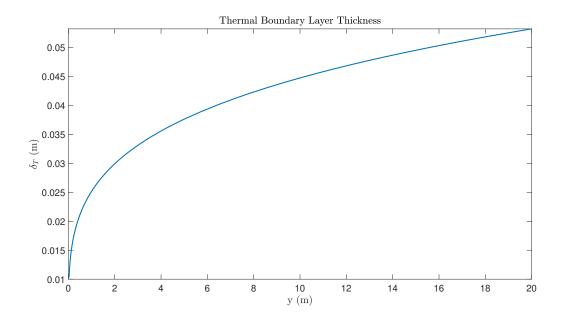


Figure 4: Thermal Boundary Layer

$$\delta_t(y) = 4.6727 \ y \text{Ra}_{y}^{-1/4}$$

2.6 Streamlines

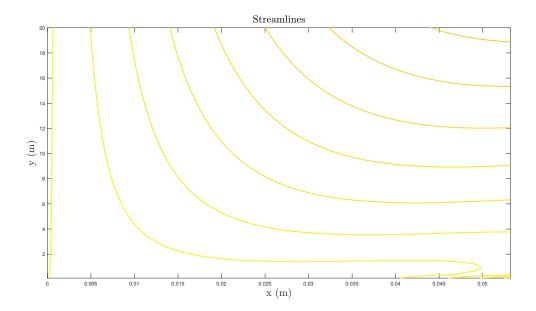


Figure 5: Streamlines

Heatlines 2.7

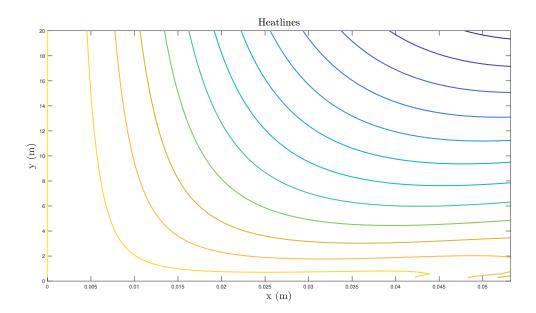


Figure 6: Heatlines

3 Comparison with Scaling Analysis

Thermal Boundary Layer Thickness

from scaling analysis:
$$\begin{split} \delta_t \sim H \mathrm{Ra_H^{-1/4}} \\ \delta_t \sim 0.0092 \mathrm{~m} \end{split}$$

from similarity solution:

$$\begin{split} & \delta_t(y) = 4.6727 \ y \text{Ra}_{\text{y}}^{-1/4} \\ & \delta_{t,\text{max}} = \delta_t(H) = 0.0532 \ \text{m} \end{split}$$

Characteristic Velocity

from scaling analysis: $\begin{array}{l} v_c \sim \frac{\alpha}{H} \mathrm{Ra}_H^{1/2} \\ v_c \sim 5.73 \mathrm{\ m/s} \end{array}$

$$v_c \sim \frac{\alpha}{H} \text{Ra}_H^{7/2}$$

 $v_z \sim 5.73 \text{ m/s}$

from similarity solution:
$$v_c(y) = \frac{\alpha}{y} \mathrm{Ra}_y^{1/2}$$

$$v_{c,\mathrm{max}} = v_c(H) = 5.73~\mathrm{m/s}$$

Nusselt Number

from scaling analysis: $\begin{aligned} \mathrm{Nu} \sim \mathrm{Ra}_\mathrm{H}^{1/4} \\ \mathrm{Nu} \sim 2180 \end{aligned}$

 $\begin{aligned} & \text{from similarity solution:} \\ & \text{Nu}_y = -\theta(0) \text{Ra}_y^{1/4} \\ & \text{Nu}_H = 853 \end{aligned}$