Sub Code:KAS303

Paper Id: 199353 **Roll No:**

B. TECH
(SEM III) THEORY EXAMINATION 2019-20
MATHEMATICS III

Time: 3 Hours ***Total Marks: 100***

Note: **1.** Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x 10 = 20

Qno.	Question	Marks	CO
a.	Write $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 5 & 7 & 4 & 2 & 8 & 1 & 6 \end{pmatrix}$ as a product of disjoint cycles.	2	3
b.	Prove that $(p \vee r) \wedge q$ is a valid argument from the premises p, q.	2	3
c.	Give one example of the relation on the set $A = \{0, 1, 2\}$ which is reflexive and not symmetric, but transitive	2	4
d.	Negate the proposition: $\left(\exists x\, p(x) \vee \exists\, y\, q(y)\right)$.	2	3
e.	Consider the set $D_{50} = \{1, 2, 5, 10, 25, 50\}$ and the relation is '|' i.e."divides" be a partial ordering relation on D_{50}. Then (i) Draw the Hasse diagram of D_{50} with relation divides. (ii) Determine all the upper bounds of 5 and 10. (i) Determine all the lower bounds of 5 and 10.	2	5
f.	The integer n is odd if and only if n^2 is odd.	2	4
g.	What is the change of scale property of Z −Transform.	2	2
h.	Evaluate $L\{\, te^{-t} \sin 2t\}$.	2	1
i.	What is bounded Lattice?	2	5
j.	What is Pigeonhole Principle?	2	4

SECTION B

2. Attempt any *three* of the following: 3 x 10 = 30

Qno.	Question	Marks	CO
a.	State and prove Lagrange's theorem.	10	3
b.	Find the Laplace transform of the following periodic function: $f(t) = \sin\left(\frac{\pi t}{a}\right)\, for\ 0 < t < a$. (Rectified sine wave of period a.)	10	1
c.	$f: A \to B$ and $g : B \to C$ be one –to-one and onto functions, then gof is also one-to-one and onto function. Show that $(gof\,)^{-1} = f^{-1}og^{-1}$.	10	4
d.	Convert the Boolean function: $f(x, y, z) = (x + y + z)(xy + x'z)'$ into conjunctive and disjunctive normal form.	10	5
e.	Find the Fourier sine transform of $e^{-\|x\|}$. Hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2}\, dx$.	10	2

SECTION C

3. Attempt any *one* part of the following: 1 x 10 = 10

Qno.	Question	Marks	CO
a.	Among 100 students, 32 study Mathematics, 20 study Physics, 45 study Biology, 15 study Mathematics and Biology, 7 study Mathematics and Physics, 10 study Physics and Biology and 30 do not study any of three subjects. (i) Find the number of students studying all three subjects. (ii) Find the number of students studying exactly one of the three subjects.	10	4
b.	Prove by Mathematical Induction $1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$.	10	4

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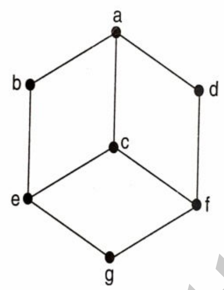
4. Attempt any *one* part of the following: 1 x 10 = 10

Qno.	Question	Marks	CO
a.	(i) Design a circuit corresponding to the Boolean expressions $(x \wedge \bar{y}) \vee (\bar{x} \wedge y)$, $(x_1 \vee x_2) \wedge x_1$. (ii) Minimize the following Boolean expression using k-map: $f(x,y,z) = (x\bar{y}z + \bar{x}yz + xy + \bar{x}\bar{y}z)$.	10	5
b.	(i) Solve by Z –Transform: $y_{k+2} - 3y_{k+1} + 2y_k = 0$, with $y_0 = 0, y_1 = 1$. (ii) Find $Z\left\{\cosh\left(\frac{k\pi}{2} + \alpha\right)\right\}$.	10	2

5. Attempt any *one* part of the following: 1 x 10 = 10

Qno.	Question	Marks	CO
a.	Construct the meet and join table of the Lattice $(L, \vee, \wedge)$ as shown in the following figure:  	10	5
b.	Prove that a poset has at most one greatest element and one least element.	10	5

6. Attempt any *one* part of the following: 1 x 10 = 10

Qno.	Question	Marks	CO
a.	Check whether the statement: $(p \leftrightarrow q) \wedge (q \leftrightarrow r) \rightarrow (p \leftrightarrow r)$ is a tautology or not.	10	3
b.	Solve the recurrence relation $a_{r+2} - 2a_{r+1} + a_r = 2^r$ by the method of generating function with the initial conditions $a_0 = 2, a_1 = 1$.	10	4

7. Attempt any *one* part of the following: 1 x 10 = 10

Qno.	Question	Marks	CO
a.	The temperature u in the semi-infinite rod $0 \le x < \infty$ is determined by the differential equation $\frac{\partial u}{\partial t} = k\,\frac{\partial^2 u}{\partial x^2}$ subject to the conditions, (i) $u = 0$ when $t = 0, x > 0$ (ii) $\frac{\partial u}{\partial x} = -\alpha$ (a constant), when $x = 0, t > 0$. By using cosine transform find the distribution of the temperature u.	10	2
b.	Solve the following simultaneous equations using Laplace transform: $\frac{dx}{dt} - y = e^t$, $\frac{dy}{dt} + x = \sin t$. with $x(0) = 1, y(0) = 0$	10	1