ECO111: Economy, Society & Public Policy

Work, wellbeing, and scarcity

Labour and Production

- Benarasi sarees are considered among the finest sarees in India.
- A community of artisans have been traditional weavers of the saree for generations.
- Consider the example of Sakina, who belongs to one such household.
- Sakina spends her time between doing household work and doing embroidery work on completed sarees.
- Contractor pays her a daily wage.

Labour and Production

- Labour can be thought of as in input in the production of goods.
- Labour is work.
- Work activity is often difficult to measure.
- Hard to measure the effort required by different activities in a comparable way.
- Economists often measure labour simply as the number of hours worked by individuals engaged in production.

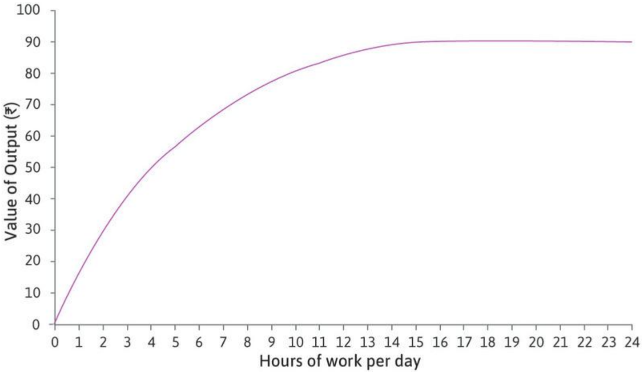
Labour and Production

- Sakina can vary the hours she spends embroidering.
- Assume that the hours she spends per day will increase the amount of embroidery that she will produce and hence the amount of income she will receive at the end, ceteris paribus.
- Work refers to all of the time that Sakina spends working on embroidering per day.

Labour and Production

Hours of work time	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15 or more
Units of Cloth (Rs.)	0	20	33	42	50	57	63	69	74	78	81	84	86	88	89	90
Value (Rs.)	0	20	33	42	50	57	63	69	74	78	81	84	86	88	89	90

Labour and Production



Labour and Production

- Sakina's **production function**: it translates the number of hours per day spent working (her input of labour) into output.
- Assume that every unit of embroidered cloth is worth one rupee, so that the number of units and the value of these units are the same.
- Final output might also be affected by unpredictable events.
- Think of the production function as telling us what Sakina will get under normal conditions (if she is neither lucky nor unlucky).

Labour and Production

- Sakina can obtain more output by working more, so the curve slopes upward.
- At 15 hours of work per day she gets the highest income she is capable of, which is Rs. 90.
- Any time spent working beyond that does not affect her output (she will be so tired that even working more she is not able to finish any more embroidering), and the curve becomes flat.

Labour and Production

- Sakina's **average product of labour** equals average output per hour of work.
- If she works for 4 hours per day she produces output worth Rs. 50.
- Hence average product of labour at 4 hours of work equals Rs. $50/4$ $=$ Rs. 12.50.
- It is the slope of a ray from the origin to the curve at 4 hours per day.

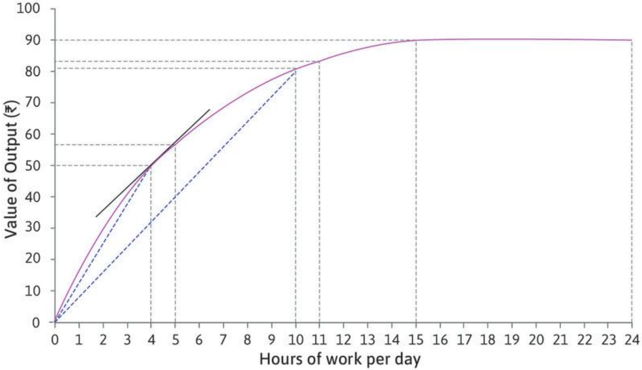
Labour and Production

- Sakina's **marginal product** is the increase in her output from increasing work by one hour.
- At each point on the production function, the marginal product is the increase in the value of output from working one more hour.
- The marginal product corresponds to the slope of the production function.

Labour and Production

- Sakina's production function in becomes flatter, the more hours she works so the marginal product of an additional hour falls as we move along the curve.
- The marginal product is **diminishing**.
- This captures the idea that an extra hour of work helps a lot if you are not working much, but if you are already working a lot, then working even more does not help very much.

Labour and Production



Labour and Production

- If we compare the marginal and average products at any point on Sakina's production function, we find that the marginal product is below the average product.
- For example, when she works for four hours her average product is $50/4=12.5$ rupees per hour,
- An extra hour's work raises the value of output from 50 to 57, so the marginal product is Rs. 7.
- This happens because the marginal product is diminishing.
- The average product is also diminishing.

Preferences

- If Sakina has the production function shown above, how many hours per day will she choose to work?
- The decision depends on her preferences—the things that she cares about.
- It is here that people and cultures differ.

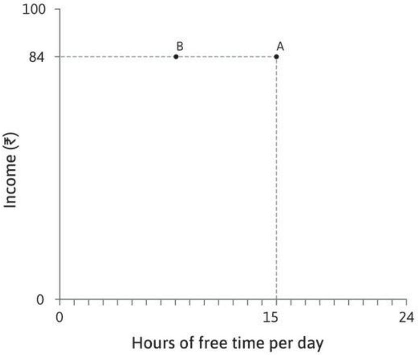
Preferences

- Let us assume that Sakina sells the output for its value.
- For example, if she produces Rs. 90 worth of cloth, she is able to sell it all, and that is her income.
- If Sakina cared only about the things she could buy with her income, she should work for 15 hours a day.
- Sakina also cares about her time and her relationships.
- She faces a **trade-off**: how much income is she willing to give up in order to spend time on things other than embroidery work?

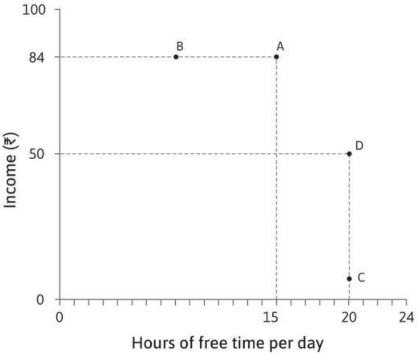
Preferences

- One can illustrate Sakina's preferences over free time and income in a diagram.
- Diagram has time spent on other activities on the horizontal axis and final income on the vertical axis.
- Free time is defined as all the time that she does not spend working.
- Every point in the diagram represents a different combination of free time and final income.
- Note that some of these combinations may not be possible for Sakina to achieve (depends on the production function).

Preferences



Preferences



Preferences

- For a given income, she prefers a combination with more time for other things than paid work to one with less time.
- Similarly, if two combinations both have 20 hours of time outside paid work, she prefers the one with a higher income.
- Compare points A and D in the table. Would Sakina prefer D (low income, plenty of free time) or A (higher income, less free time)? One way to find out would be to ask her.

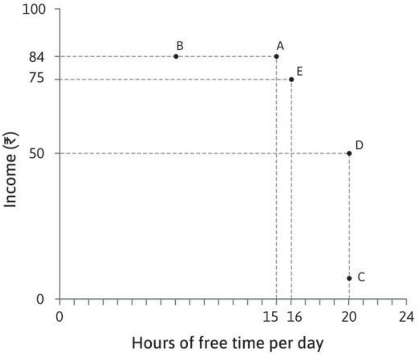
Preferences

- Suppose she says she is indifferent between A and D, meaning she would feel equally satisfied with either outcome.
- We say that these two outcomes would give Sakina the same **utility**.
- And we know that she prefers A to B, so B provides lower utility than A or D.

Preferences

- A systematic way to graph Sakina's preferences would be to start by looking for all of the combinations that give her the same utility as A and D.
- One could ask Sakina another question: 'Imagine that you could have the combination at A (15 hours of free time, Rs. 84). How many rupees would you be willing to sacrifice for an extra hour of free time?'
- Suppose that after due consideration, she answers 'nine'. Then we know that she is indifferent between A and E (16 hours, Rs. 75).

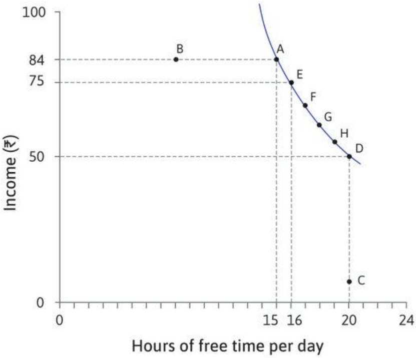
Preferences



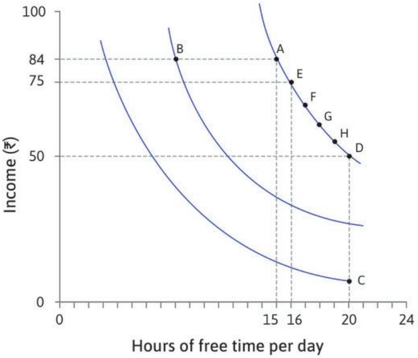
Preferences

- We could ask the same question about combination E, and so on until point D.
- Sakina is indifferent between A and E, between E and F, and so on, which means she is indifferent between all of the combinations from A to D.
- The combinations can be joined together to form a downward-sloping curve, called an **indifference curve**.
- Indifference curve joins together all of the combinations that provide equal utility or 'satisfaction'.

Preferences



Preferences



Preferences

- If one looks at the three curves drawn, one can see that the one through A gives higher utility than the one through B.
- The curve through C gives the lowest utility of the three.
- To describe preferences we don't need to know the exact utility of each option; we only need to know which combinations provide more or less utility than others.

Preferences

- The curves we have drawn capture our typical assumptions about people's preferences between two goods.
- In other models, these will often be consumption goods such as food or clothing, and we refer to the person as a consumer.
- In our model of a textile worker's preferences, the goods are 'final income' and 'free time'.

Preferences

- **Usual** (note that there may be exceptions depending on nature of goods) properties of indifference curves:

1. Indifference curves slope downward due to trade-offs.
2. Higher indifference curves correspond to higher utility levels.
3. Indifference curves are usually smooth.
4. Indifference curves do not cross.
5. As you move to the right along an indifference curve, it becomes flatter.

Preferences

- If Sakina is at A, with 15 hours of free time and an income of 84, she would be willing to sacrifice 9 rupees for an extra hour of free time, taking her to E.
- We say that her **marginal rate of substitution (MRS)** between income and free time at A is 9.
- MRS is the reduction in her income that would keep Sakina's utility constant following a one-hour increase of free time.

Preferences

- Indifference curves are becoming gradually flatter because it seems reasonable to assume that the more free time and the lower the income she has, the less willing she will be to sacrifice further income in return for free time, so her MRS will be lower.
- The MRS is the slope of the indifference curve, and it falls as we move to the right along the curve.

Opportunity Costs

- Sakina wants both her income and her free time to be as high as possible.
- But given her production function, she cannot increase her free time without getting a lower income.
- Free time has an **opportunity cost**: to get more free time, Sakina has to forgo the opportunity of getting a higher income.

Opportunity Costs

- In economics, opportunity costs are relevant whenever we study individuals choosing between alternative and mutually exclusive courses of action.
- Suppose you are considering between choosing action A and action B.
- When we consider the cost of taking action A we include the fact that if we do A, we cannot do B.
- So ‘not doing B’ becomes part of the cost of doing A.
- This is called an opportunity cost because doing A means forgoing the opportunity to do B.

Feasible Set

- Consider Sakina's problem of choosing between income and free time.
- Free time has an opportunity cost in the form of lost income
- Need a way to describe which alternatives are available to her.
- Use the notion of production function discussed previously.

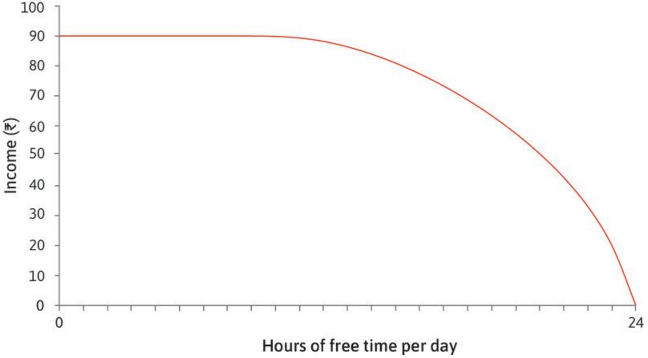
Feasible Set

- Recall that the production function is a relation which translates Sakina's hours of work into output (units of cloth).
- We also assumed that she is able to sell all the cloth at a cost of 1 rupee per unit of cloth.
- Her value of output and final income are thus the same.
- How does her final income depends on the amount of her free time?

Feasible Set

- There are 24 hours in a day.
- Sakina must divide this time between working (all the hours devoted to embroidering) and free time (all the rest of her time).
- If Sakina works solidly for 24 hours, that means zero hours of free time and a final income of 90.
- If she chooses 24 hours of free time per day (i.e., she does not work for pay), we assume she will get an income of zero.

Feasible Set



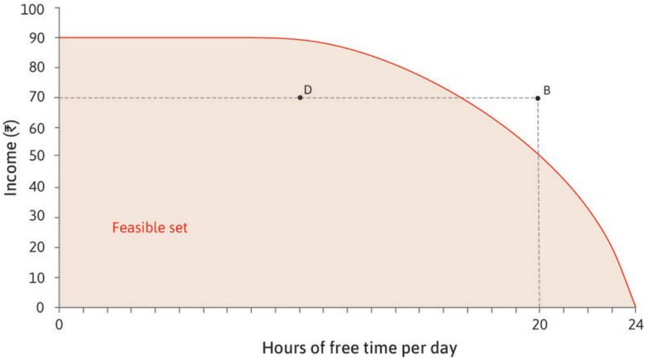
Feasible Set

- The axes are income and free time, the two goods that give Sakina utility.
- If we think of her choosing to consume a combination of these two goods, the curved line shows what is feasible.
- It represents her **feasible frontier:** the highest income she can achieve given the amount of free time she wants.

Feasible Set

- Any combination of free time and income that is on or inside the frontier is feasible.
- Combinations outside the feasible frontier are said to be **infeasible** given Sakina's abilities and conditions of work.
- On the other hand, even though a combination lying inside the frontier is feasible, choosing it would imply Sakina has effectively thrown away something that she values.

Feasible Set



Feasible Set

- The feasible frontier is a constraint on Sakina's choices.
- It represents the trade-off she must make between income and free time.
- At any point on the frontier, taking more free time has an opportunity cost in terms of income points foregone, corresponding to the slope of the frontier.

Feasible Set

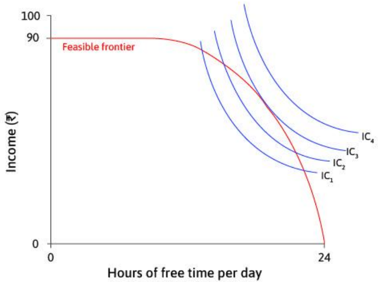
- The feasible frontier shows the **marginal rate of transformation** (MRT): the rate at which Sakina can transform free time into income points.
- Note that we have now identified two trade-offs:

1. The marginal rate of substitution (MRS): It measures the trade-off that Sakina is willing to make between income and free time.
2. The marginal rate of transformation (MRT): This measures the trade-off that Sakina is constrained to make by the feasible frontier.

Decision Making

- The final step in this decision-making process is to determine the combination of income and free time that Sakina will choose.
- Need to bring together her her feasible frontier and indifference curves.
- Indifference curves indicate what Sakina prefers, and their slopes shows the trade-offs that she is willing to make.
- Feasible frontier is the constraint on her choice, and its slope shows the trade-off she is constrained to make.

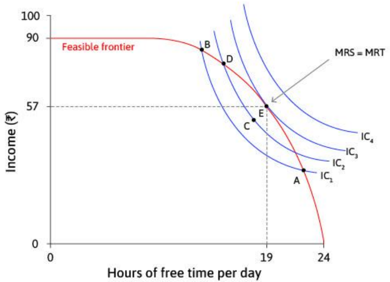
Decision Making



Decision Making

- IC_4 represents the highest level of utility because it is the furthest away from the origin. However, no combination of income and free time on IC_4 is feasible, however, because the whole indifference curve lies outside the feasible set.
- Suppose that Sakina considers choosing a combination somewhere in the feasible set, on IC_1. She can increase her utility by moving to points on higher indifference curves until she reaches a feasible choice that maximizes her utility.

Decision Making



Decision Making

- Sakina maximises her utility at point E, at which her indifference curve is tangent to the feasible frontier.
- Her optimal combination of income and free time is at the point where the marginal rate of transformation (MRT) is equal to the marginal rate of substitution (MRS).
- The model predicts that Sakina will choose to spend 5 hours each day working, and 19 hours on other activities.

Decision Making

- Modelled the worker's decision on work hours as a **constrained choice** problem: a decision-maker (Sakina) pursues an objective (utility maximization in this case) subject to a constraint (her feasible frontier).
- Both free time and income are scarce for Sakina because she values both of them.
- Each has an opportunity cost: More of one good means less of the other.

Decision Making

- In constrained choice problems, the solution is the individual's optimal choice.
- If we assume that utility maximization is Sakina's goal, the optimal combination of income and free time is a point on the feasible frontier at which:

$$MRS = MRT$$

- **Query:** What happens if both the feasible frontier and the indifference curves are linear?

Mathematical Summary

- Sakina's production function describes how her daily hours of work translate into her final value of output.
- We have seen that her marginal product at each point is the slope of the function, and her average product is the slope of the ray to the origin.
- A general mathematical representation of the production function is

$$y = f(h)$$

  where y is the rupee value of the final amount of cloth (her output), h is hours of work per day (the input) and $f(h)$ is the production function.

Mathematical Summary

- When Sakina is working for h hours per day, her average product of labour (APL) is calculated by dividing the rupee value of the final amount of cloth produced by the number of hours worked:

$$APL = \frac{y}{h} = \frac{f(h)}{h}$$

  This is the rupee value of average number of cloth produced per hour of work.

- In the diagram, it is the slope of the ray to the origin.

Mathematical Summary

- Sakina's marginal product of labour (MPL) is the increase in her cloth produced from increasing work time by one hour.
- More precisely, it is the rate at which her cloth production increases as work time increases, which corresponds to the slope of the production function.
- Sakina's marginal product when she works for hours is given by the derivative of the production function:

$$MPL = \frac{dy}{dh} = f'(h)$$

Mathematical Summary

- Suppose she works for h hours a day.
- To find her marginal product, we consider how her cloth production would change if she increased her work time by Δh hours.
- If the production increases by Δy , then the change in production per unit change of work time is:

$$\frac{\triangle y}{\Delta h} = \frac{f(h + \Delta h) - f(h)}{\Delta h}$$

- As Δh tends towards zero, this fraction tends towards the derivative of the function.

Mathematical Summary

- Consider the production function

$$y = Ah^{\alpha}$$

where A and α are constants such that $A > 0$ and $0 < \alpha < 1$.

- The average product of labour is then:

$$APL = \frac{y}{h} = \frac{f(h)}{h} = Ah^{\alpha-1}$$

Mathematical Summary

- The marginal product of labour is then:

$$MPL = \frac{dy}{dh} = f'(h) = A\alpha h^{\alpha-1} > 0$$

- The assumption $\alpha < 1$ implies that $MPL < APL$.

Mathematical Summary

- Sakina's production function has the property of diminishing marginal productivity.
- What does this mean for the mathematical properties of the production function?
- If the production function is $y = f(h)$, the marginal product of labour is $dy/dh = f'(h)$.
- The marginal product of labour diminishes is

$$\frac{d}{dh}\left(\frac{dy}{dh}\right) = \frac{d^2 y}{dh^2} < 0$$

that is if $f''(h) < 0$.

Mathematical Summary

- For the production function $y = Ah^{\alpha}$, we had found that $f'(h) = A\alpha h^{\alpha-1}$.
- The derivative of the marginal product is given by

$$f''(h) = A\alpha(\alpha - 1)h^{\alpha-2} < 0$$

since $0 < \alpha < 1$.
- The marginal product of labour is diminishing.

Mathematical Summary

- What's the relation between marginal product and average product of labour?

$$
\begin{aligned}
\frac{d}{dh}\left(\frac{f(h)}{h}\right) &= \frac{f'(h)}{h} - \frac{f(h)}{h^2} \\
&= \frac{1}{h}\left[f'(h) - \frac{f(h)}{h}\right] \\
&= \frac{1}{h}\left[MPL - APL\right]
\end{aligned}
$$

Mathematical Summary

- Preferences can be represented mathematically by writing down a utility function, which tells us how a person's 'units of utility' depend on the goods available.
- Sakina only cares about two goods: her hours of free time and her income. If she has t units of free time and y income, her utility is given by a function:

$$U(t, y)$$

- Since both income and free time are goods—Sakina would like to have as much of each as possible—the utility function must have the property that increasing either t or y would increase U .
- Marginal utility of a variable is given by the partial derivative of utility function with respect to that variable

Mathematical Summary

- Indifference curves are the contours of the utility surface, joining points of equal utility.
- In Sakina's case, an indifference curve shows all the combinations of free time and income that give her the same level of utility. The equation of a typical indifference curve is:

  $$U(t, y) = c$$

  where the constant c stands for the utility level achieved on the curve.
- Different values of c correspond to different indifference curves.

Mathematical Summary

- Given any combination (t, y) of free time and income, Sakina's marginal rate of substitution (MRS) (that is, her willingness to trade income for an extra hour of free time) is given by the slope of the indifference curve $U(t, y) = c$ through that point.
- The slope of the indifference curve through any point (t, y) is given by the formula:

$$\frac{dy}{dt} = -\frac{\partial U}{\partial t} / \frac{\partial U}{\partial y}$$

- The marginal rate of substitution is defined (in the text) as the absolute value of the slope.

Mathematical Summary

- Example: Suppose Sakina's utility function is given by

$$U = t^{\alpha} y^{\beta}$$

  where α and β are positive constants.

- Marginal utility of free time is given by

$$\frac{\partial U}{\partial t} = \alpha t^{\alpha-1} y^{\beta}.$$

- The marginal rate of substitution is given by

$$|\frac{dy}{dt}| = | - \frac{\partial U}{\partial t} / \frac{\partial U}{\partial y}| = \frac{\alpha}{\beta}\frac{y}{t}.$$

Mathematical Summary

- Sakina's decision of how much to work is constrained by the feasible set of combinations of free time and income.
- So she faces a trade-off: to get a good income via cloth production, she has to give up some free time.
- The marginal rate of transformation (MRT) measures the size of the trade-off.

Mathematical Summary

- Sakina's production function is given by

$$y = f(h)$$

  where y is the value of output (her income) and h her hours of work, and f is an increasing function.

- The feasible frontier is the relationship between her free time and income.
- If Sakina takes t hours of free time, her hours of work are:

$$h = 24 - t$$

Mathematical Summary

- Substituting into the production function, we obtain the equation of the feasible frontier:

$$y = f(24 - t)$$

- The marginal rate of transformation (MRT) is the rate at which the income changes as free time changes:

$$\frac{dy}{dt} = -f'(24 - t)$$

- The marginal rate of transformation is defined (in the text) as the absolute value of the slope.

Mathematical Summary

- Sakina wants to get as high an income as possible whilst sacrificing the least possible amount of free time.
- Sakina's utility function is

$$U(t, y)$$

  where utility depends positively on hours of free time t and the income y.
- If the production function is $y = f(h)$, where h is hours of work, the equation of the feasible frontier is:

$$y = f(24 - t)$$

Mathematical Summary

- Sakina's problem is to choose $t$and y to maximize $U(t, y)$ subject to the constraint $y = f(24 - t)$.
- This is an example of what is known in mathematics as a problem of constrained optimization.

Mathematical Summary

- One way to solve Sakina's problem is to use the constraint to substitute for y in terms of t in the utility function.
- Utility is expressed as a function of the single variable t:

$$U(t, f(24 - t))$$

which may be maximized with respect to t by equating its derivative to zero.

Mathematical Summary

- The derivative can be obtained via the chain rule:

$$\frac{dU}{dt} = \frac{\partial U}{\partial t} + \frac{\partial U}{\partial y}\frac{dy}{dt}$$

- The term dy/dt equals $-f'\left(24 - t\right)$ so that

$$\frac{dU}{dt} = \frac{\partial U}{\partial t} - f'\left(24 - t\right)\frac{\partial U}{\partial y}$$

Mathematical Summary

- At the point that solves Sakina's maximization problem, $dU/dt = 0$. So at this point:

$$\frac{\partial U}{\partial t} / \frac{\partial U}{\partial y} = f'(24 - t)$$

which gives us

$$MRS = MRT$$

Mathematical Summary

- So far we have shown that the values of t and y we are looking for must satisfy the first-order condition.
- To solve the problem fully and find the optimal values, we need to note that they must also lie on the feasible frontier.
- We have a pair of simultaneous equations:

$$\begin{aligned} \frac{\partial U}{\partial t} \Big/ \frac{\partial U}{\partial y} &= f'(24 - t) \\ y &= f(24 - t) \end{aligned}$$

which must be satisfied by t and y.

Mathematical Summary

- Let's find out the optimal allocation for the case where $U(t,y) = t^a y^b$ and $f(h) = h^\alpha$.
- Sakina's problem is to choose t and y to maximize $U(t,y) = t^a y^b$, subject to the constraint $y = (24 - t)^\alpha$.
- The first order condition $MRS = MRT$ gives us

$$\frac{ay}{bt} = \alpha\,(24 - t)^{\alpha - 1} = \frac{\alpha y}{24 - t}$$

which gives us

$$t = \frac{24}{1 + (\alpha b/a)}.$$

Mathematical Summary

- Substituting the value of t into the production function we get

$$y = \left(\frac{24\alpha b}{a + \alpha b}\right)^{\alpha}$$

- These are the values of t and y that give Sakina the highest utility she can achieve within the feasible set.