LECTURE 7

19 August 2024

Stochastic steady state

- Stochastic steady state modeled by mixed strategy NE
- Consider the example of matching pennies

	Head	*Tail*
Head	\$1, −\$1	−\$1, \$1
Tail	−\$1, \$1	\$1, −\$1

- Is there a pure strategy NE?
- What is a mixed strategy NE?
- If a player prefers the deterministic outcome *'a'* to the deterministic outcome *'b'*, it is very plausible that if $p>q$, then she prefers the lottery in which *a* occurs with probability *p* and b with probability *1-p*, to the situation where *a* occurs with probability *q and b* with probability *1-q*.

- In a case of more than two outcomes for some players, we cannot extrapolate or generalize in this way from preferences regarding deterministic outcomes to preferences regarding lotteries over outcomes.
- Example: A game has three possible outcomes *a*, *b*, and *c* and a player prefers *a* to *b* to *c*. Then do they prefer the deterministic outcome *b* to the lottery in which *a* and *c* each occur with probability ½?

vNM preferences

- For every player *i* there is a payoff function u_i with the property that player *i* prefers one lottery over outcomes to another if and only if, according to u_i , the expected value of the first lottery exceeds the expected value of the second lottery.
- Such preferences are called vNM preferences named after von Neumann and Morgenstern.
- The assumption that a player's preferences are represented by the expected value of a payoff function does not place restrictions on their attitudes to risk.
- If the players are allowed to randomize then the numbers are payoffs whose expected values represent the players' preferences regarding lotteries over outcomes

- A strategic game with vNM preferences consists of:
  i. a set of players
  ii. for each player, a set of actions
  iii. for each player, preferences regarding lotteries over action profiles that may be represented by the expected value of a payoff function over action profiles

- By expected payoff functions with vNM preferences the following two games are different. But with deterministic payoffs do they resemble any of the games we have done so far?

	Q	F
Q	$2,2$	$0,3$
F	$3,0$	$1,1$

	Q	F
Q	$3,3$	$0,4$
F	$4,0$	$1,1$

- A mixed strategy is when each player with vNM preferences, are allowed to choose a probability distribution over their set of actions than restricting to choose a single deterministic action.
- The players are not restricted from choosing a deterministic strategy.

Mixed Strategy Nash Equilibrium

- A mixed strategy NE is a mixed strategy profile α^* with the property that no player *i* has a mixed strategy α_i such that they prefer the lottery over the outcomes generated by the strategy profile $(\alpha_i, \alpha^*_{-i})$ to the lottery over the outcomes generated by the strategy profile α^* .
- Find the mixed strategy NE for the matching pennies game.

- Find all the equilibria (pure and mixed strategy) in the BoS game.

	B	S
B	$2, 1$	$0, 0$
S	$0, 0$	$1, 2$