# ECO111 Quiz 1 Solutions

September 2024

## Q1

|   | A | B |   |   | A | B |
|---|---|---|---|---|---|---|
| A | 1, 1, 1 | 0, 0, 0 |   | A | 0, 0, 0 | 0, 0, 0 |
| B | 0, 0, 0 | 0, 0, 0 |   | B | 0, 0, 0 | 4, 4, 4 |
|   | A |   |   |   | B |   |

The best response of player 1 when player 2's and 3's actions are:

- ( ,A,A) is A

- ( ,A,B) is both A and B

- ( ,B,A) is both A and B

- ( ,B,B) is B

By this reasoning we find the PSNE are (A,A,A) and (B,B,B)

To find MSNE:
If player 2 plays both A and B with positive probabilities such that $P_2(A) = q$ and;
if player 3 plays both A and B with positive probabilities such that $P_3(A) = r$
Player 1 plays both A and B with positive probabilities equal to p and 1-p respectively iff

$$E_1(A) = E_1(B) \tag{1}$$

$$E_1(A) = 1.qr + 0.(1-q)r + 0.q(1-r) + 0.(1-q)(1-r) \tag{2}$$

$$E_1(B) = 0.qr + 0.(1-q)r + 0.q(1-r) + 4.(1-q)(1-r) \tag{3}$$

Plugging in 2 & 3 in 1 we get

$$qr = 4(1-q)(1-r) \tag{4}$$

Similarly play 2 plays both A and B with positive probability iff

$$pr = 4(1-p)(1-r) \tag{5}$$

Player 3 plays both A and B with positive probability iff

$$pq = 4(1-p)(1-q) \tag{6}$$

Solving 4, 5, and 6 we get

$$p = q = r = 2/3 \tag{7}$$

Therefore the only MSNE for this game is $\{(2/3,1/3), (2/3,1/3), (2/3,1/3)\}$

## Q2

We have

$$2GDP = GDP(1 + \frac{8}{100})^n \tag{8}$$

$$2 = 1.08^n$$
$$\text{Given,} \quad \sqrt{2} = 1.08^{4.5}$$

Thus,  n = 9 years

## Q3

|  | Brian | |
|---|---|---|
| | Football | Netflix |
| Esther | 5*,8 | 1,2 |
| | 2,6 | 4*,4 |

**a**

Esther's best response to Brian watching:

- Football is Football

- Netflix is Netflix

Thus, Esther has no dominated strategy as both actions are played.
Brian's best response to Esther watching:

- Football is Football

- Netflix is Football

Thus, Brian's dominated strategy is watching Netflix.

**b**

The PSNE as marked in the payoff matrix and explained using best responses in part (a) is (Football, Football).

**c**

Suppose Brian watch both Football and Netflix with positive probabilities equal to q and 1-q respectively.
And, suppose Esther watch both Football and Netflix with positive probabilities equal to p and 1-p respectively.

Brian will have a positive value for q iff

$$E_B(Football) = E_B(Netflix) \tag{9}$$

which implies  8p + 6(1-p) = 2p + 4(1-p)
4p = -2
p = -1/2  Which is not possible as probabilities are never negative. Thus Brian never mixes his strategy.
Given Brian never mixes his strategy and plays q = 1 (since Netflix is dominated strategy and never played), the best response of Esther is to have p = 1, ie, watch Football. Thus, there is no MSNE.

**d**

In the BoS game, although the players have difference preferences between the actions, both the players also have higher preference when together than apart.

Here Brian always prefers to watch Football irrespective of Esther's actions, while Esther prefers to watch something together with Brian. This game is not a coordination game like BoS.

### e

Yes, the PSNE here is Pareto Efficient. Deviation by any player to any other action leaves at least one player worse off.

# Q4

### a

Preferences of the individual based on the ordinal utility functions given are $c \succ a \succ d \succ b$ since $u(c) > u(a) > u(d) > u(b)$.
Any example that maintains this relationship.

### b

No, they do not because the new utilities are $u'(a) = 101 > u'(c) = 98 > u'(d) = 2 > u'(b) = 0$ which represents the preferences $a \succ c \succ d \succ b$

### c

Any $\alpha < 0$ represents the true preference of the individual as it is a positive linear transformation of the utility functions u(.)