

## PROBLEM SET 2

Q1. Each of two players has two possible actions, *Quiet* and *Fink*; each action pair results in the players' receiving amounts of money as shown in the matrix below. The players are not selfish, rather the preferences of each player  $i$  are represented by the payoff function  $m_i(a) + \alpha m_j(a)$ , where  $m_i(a)$  is the amount of money received by player  $i$ ,  $j$  is the other player, and  $\alpha$  is a given non-negative number

	<i>Quiet</i>	<i>Fink</i>
<i>Quiet</i>	2, 2	0, 3
<i>Fink</i>	3, 0	1, 1

- Formulate a strategic game that models this situation when  $\alpha=1$ . Is this game the Prisoners' Dilemma?
- Find the range of values of  $\alpha$  for which the resulting game is the Prisoners' Dilemma. For values of  $\alpha$  for which the game is not the Prisoners' Dilemma, find its Nash equilibria

Q2. Two people enter a bus. Two adjacent cramped seats are free. Each person must decide whether to sit or stand. Sitting alone is more comfortable than sitting next to the other person, which is more comfortable than standing

- Suppose that each person cares only about her own comfort. Model the situation as a strategic game. Is this game the Prisoners' Dilemma? Find its NE
- Suppose that each person is altruistic, ranking the outcomes according to the other person's comfort, and out of politeness, prefers to stand than to sit if the other person stands. Model this situation as a strategic game. Is this game the Prisoners' Dilemma? Find its NE
- Compare the peoples' comfort in the equilibria of the two games

Q3. Consider two variants of the  $n$ -hunter Stag Hunt in which only  $m$  hunters, with  $2 \leq m \leq n$ , need to pursue the stag in order to catch it. There is only one stag. Assume that a captured stag is shared only by the hunters that catch it.

- Assume as before, that each hunter prefers the fraction  $1/n$  of the stag to a hare. Find the NE of the strategic game that models this situation.
- Assume that each hunter prefers the fraction  $1/k$  of the stag to a hare, but prefers the hare to any smaller fraction of the stag, where  $k$  is an integer with  $m \leq k \leq n$ . Find the NE of the strategic game that models this situation

Q4. Find the NE in the following game. Give your comments about the NE

	<i>Bach</i>	<i>Stravinsky</i>
<i>Bach</i>	2, 2	0, 0
<i>Stravinsky</i>	0, 0	1, 1

Q5. Each of  $n$  people choose whether or not to contribute a fixed amount towards the provision of a public good. The good is provided if and only if at least  $k$  people contribute, where  $2 \leq k \leq n$ ; if it is not provided contributions are not refunded. Each person ranks outcomes from best to worst as follows: (i) any outcome in which the good is provided and they do not contribute, (ii) any outcome in which the good is not provided and they contribute, (iii) any outcome in which the good is not provided and they do not contribute, (iv) any outcome in which the good is not provided and they contribute. Formulate this situation as a strategic game and find its NE.

Is there a NE in which more than  $k$  people contribute?

One in which  $k$  people contribute

One in which less than  $k$  people contribute