PRACTICE QUESTIONS
14 September 2024

Utility functions and their properties.

Utility Function	Completeness	Transitivity	Strict Monotonicity	Monotonicity	Nonsatiation
$u(x,y) = by$	√	√	X	√	√
$u(x,y) = ax$	√	√	X	√	√
$u(x,y) = ax - by$	√	√	X	X	√
$u(x,y) = ax + by$	√	√	√	√	√
$u(x,y) = A\min\{ax, by\}$	√	√	X	√	√
$u(x,y) = Ax^{\alpha}y^{\beta}$	√	√	X	√	√

Q1. For each of the utility functions in the table above

a. Find the marginal utility for good x *and* y
b. Are the marginal utilities positive? Are they strictly positive? Connect your results with the properties of monotonicity and strict monotonicity.
c. Find the marginal rate of substitution of x for y. Does it increase or decrease in the amount of good x?
d. Find the indifference curve reaching a utility level of u =10, and another IC of u = 20. Do the IC cross either axis?

{Hint: $MU_x = \frac{dU}{dx}, MU_y = \frac{dU}{dY}$
If both MU*s* are non-negative it implies addition of one unit of either goods gives at least as much utility as earlier, implying monotonicity, if it is strictly positive for both goods it implies strict monotonicity
$MRS_{xy} = \frac{MU_x}{MU_y}$ }

Q2. Consider an individual with utility function

$$u(x,y) = \min\{x + 2y, 2x + y\}$$

Find the indifference curve at a utility level of u = 10. Interpret the preference of this consumer.

{Hint: Use $10 = \min\{x + 2y, 2x + y\}$, there are multiple situations when u =10, a) x + 2y = 10 and 2x + y ≥ 10, b) x + 2y = 2x + y = 10, c) x + 2y ≥ 10 and 2x + y = 10}

Q3. Consider Chelsea's Cobb-Douglas utility function, $u(x,y) = xy$. What is the preference ordering of Chelsea for the bundles A = (1, 2) and B = (3, 8)? Consider the following transformation of the utility function, $v(x,y) = [u(x,y)]^2$, $z(x,y) = \ln[u(x,y)]$, and $l(x,y) = \frac{1}{u(x,y)}$. Which of these transformations continues to represent Chelsea's preferences as earlier?

{Hint: u(1, 2) = 2 < 24 = u(3, 8). Therefore, Chelsea prefers B to A. Now see if any of the transformations are increasing and it should represent these preference relations.}

Q4. Assume Ayush's preferences are represented with the linear utility function $u(x,y) = 3x + 4y$

a. What is Ayush's MU for good x and good y?
b. Are his preferences monotonic
c. Find Ayush's indifference curve for a given level of utility $\bar{u}$
d. Find MRS between x and y

{Hint: For monotonicity find dU/dy and dU/dx, if both are non-negative it satisfies monotonicity and if both are strictly positive it satisfies strict monotonicity.}