Mixed Strategy NE

(Source: An Introduction to Game Theory by Martin J. Osborne)

Suppose player 1 plays T with probability p and player 2 plays L with probability q. The probability pertaining to each of the action pairs for the players are given in the payoff matrix below

	$L\ (q)$	$R\ (1-q)$
$T\ (p)$	pq	$p(1-q)$
$B\ (1-p)$	$(1-p)q$	$(1-p)(1-q)$

i.e, (T, L) occurs with a probability equal to pq, and so on for each action pair. Now the preferences here are vNM preferences. Payoffs are determined by expected payoffs. Suppose $u_1(\ )$ represents the payoff function for player 1and $u_2(\ )$. Therefore, the expected payoff for player 1 for each pair of actions are:

(T, L) : $pq.u_1(T, L)$
(T, R) : $p(1-q).u_1(T, R)$
(B, L) : $(1-p)q.u_1(B, L)$
(B, R) : $(1-p)(1-q).u_1(B, R)$

Let us consider the example of the BoS game. vNM preferences of the game is given in the following payoff matrix

	B	S
B	$2, 1$	$0, 0$
S	$0, 0$	$1, 2$

Suppose player 1 attaches probability p to action B and (1 – p) to action S, and player 2 attaches probability q to action B and probability (1 – q) to action S. Player 1 would play both B and S with positive probabilities if and only if

If, $E_1(B) = E_1(S)$
- ➔ $q.2 + (1-q).0 = q.0 + (1-q).1$
- ➔ $2q = 1-q$
- ➔ $q = 1/3$

Then player 1 has equal expected payoffs for playing B or S, under which situation player 1 shall play both with a positive probability.

If, $E_1(B) > E_1(S)$
- ➔ $2q > 1-q$
- ➔ $q > 1/3$

Then player 1 has higher expected payoff for B than S, hence player 1 chooses p = 1 or plays B with certainty

If, $E_1(B) < E_1(S)$

➔ q < 1/3

Then player 1 has higher expected payoff for S than B, hence player 1 chooses p = 0 or plays S with certainty

Similarly for player 2, when player 1 mixes its strategy as (p, 1-p)

If, $E_2(B) = E_2(S)$

➔ p.1 + (1-p).0 = p.0 + (1-p).2

➔ p = 2 – 2p

➔ p = 2/3

Then, player 2 mixes its strategies and plays both B and S with positive probabilities q and (1 – q) respectively

If, $E_2(B) > E_2(S)$

➔ p > 2/3

Then, q =1

If, $E_2(B) < E_2(S)$

➔ p < 2/3

Then, q = 0

The best response function of player 1 (identifies the best responses to each of player 2's actions) is

$$B_1(q) = \begin{cases} \{0\} & if\ q < 1/3 \\ \{p: 0 \leq p \leq 1\} & if\ q = 1/3 \\ \{1\} & if\ q > 1/3 \end{cases}$$

The best response of player 2 to player 1's actions are

$$B_2(p) = \begin{cases} \{0\} & if\ p < 2/3 \\ \{q: 0 \leq q \leq 1\} & if\ p = 2/3 \\ \{1\} & if\ q > 2/3 \end{cases}$$

In NE both players have best responses to the other players' actions or best responses intersect at NE. Plotting the two best response functions in the figure below. It is clear from the figure that there are points where mutual responses of the players intersect : (p=0, q=0), (p=2/3, q=1/3), (p=1, q=1). The first and the last constitute the psNE (B, B) and (S, S) respectively. The second one is the msNE (p=2/3, q=1/3)