GAME THEORY

Lecture 2

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Source: An introduction to game theory by Martin J. Osborne

Theory of rational choice

- A decision maker chooses the best action according to their preferences, among all the actions available to them.
- “Rationality” lies within the consistency of their decisions when faced with different sets of available actions.
- **Actions :** A set consisting of all the available choices the player can choose from. Generally, a subset of actions are available to the player depending on the restrictions on the game.

- Players are aware of their preferences. ‘**Preferences’** are consistent.
- Preferences are represented by what is called **payoff functions.**
- **Payoff function** associates a number with each action in such a way that actions with higher numbers are preferred.

The payoff function *u* represents a decision maker’s preferences if, for any actions *a* in *A* and *b* in *A*

u(a) > *u(b)* if and only if the decision maker

prefers *a* to *b*

- The preferences convey **ordinal** information, and hence so does payoff functions too. It does not tell us by how much one action is preferred to the other.

Q. Person 1 cares about both her income and about person 2's income. The value she attaches to each unit of her own income is the same as the value she attaches to two units of person 2's income. Order the outcomes according to her preference, given the first component is person 1's income and the second component is person 2's income

(1,4), (2,1), (3,0)

- **Theory of rational choice** is, ‘the action chosen by a decision-maker is at least as good (based on their preferences) as every other available actions.’
- In consumption theory, the set of available actions is the set of all bundles of goods that the consumer can afford.
- In production theory, the set of available actions is the set of all input-output combinations.

- A **strategic game (with ordinal preferences)** is a model of interacting decision makers which consists of the following.
- Decision makers are called **players.**
- Each player has a set of possible **actions.**
- Each player is affected by the actions of all players, not only their own action. This leads to interaction between players.
- Each player has **preferences** about the action profile (list of all the players' actions.)

- **Simultaneous games:** Time is absent from the model.
- Each player chooses their action simultaneously, ie, no player while choosing their actions are informed of the actions chosen by the other players.

PRISONER'S DILEMMA

		Suspect 2	
		Quiet	*Fink*
Suspect 1	*Quiet*	2, 2	0, 3
	Fink	3, 0	1, 1

- Prisoner's Dilemma models a situation in which there are gains from cooperation but each player has an incentive to '*free ride*' whatever the other player does.
- Working on a joint project

	Work hard	*Goof off*
Work hard	2, 2	0, 3
Goof off	3, 0	1, 1

- Duopoly

	High	*Low*
High	1000, 1000	−200, 1200
Low	1200, −200	600, 600

BACH OR STRAVINSKY

	Bach	*Stravinsky*
Bach	2, 1	0, 0
Stravinsky	0, 0	1, 2

- Two official of a political party deciding the stand to take on an issue. Suppose, they disagree about the best stand, but are both better off if they take the same stand than if they take different stands.
- Two merging firms that currently use different computer technologies. They both will be better off if they both use the same technology. Each firm prefers that the common technology adopted be the ones the firm has been using.
- Aspects of both conflict and cooperation are present in both the Prisoner's Dilemma and Bach or Stravinsky games.

MATCHING PENNIES

	Head	*Tail*
Head	$1, -1$	$-1, \quad 1$
Tail	$-1, \quad 1$	$1, -1$

- The players' interests are diametrically opposed. Such games are **strictly competitive** games.
- Player 1 wants to take the same action as player 2. But player 2 wants to take a different action than player 1.

STAG HUNT

- Here, there is no conflict between interests.

	Stag	*Hare*
Stag	2, 2	0, 1
Hare	1, 0	1, 1

NASH EQUILIBRIUM

- In a game, the best action for any given player depends, in general, on the other players' actions.
- So players must have a *belief* about the other players' actions.

A Nash Equilibrium is an action profile a with the property that no player i can do better by choosing an action different from* a_i^* *, given that every other player j adheres to* a_j^*

- In some sense Nash Equilibrium corresponds to a steady state. It embodies a stable social norm, if everyone else adheres to it, no individual wishes to deviate from it.

- What is the Nash Equilibrium when multiple players play the Stag Hunt game?