EE 798I: NANOPHOTONICS **Assignment 1** **2025-26 First Semester**

Q.1) **Vector calculus** **(50 marks)**

a) Prove that $\nabla.(\nabla \times \mathbf{A}) = \mathbf{0}$ and $\nabla \times (\nabla f) = \mathbf{0}$. Also, illustrate the two equalities through an example for each case.

b) Show that $\mathbf{E_0} \times (\boldsymbol{k} \times \mathbf{E_0})$ is parallel to $\boldsymbol{k}$ for any two vectors $\mathbf{E_0}$ and $\boldsymbol{k}$. Relate it to the plane wave solutions discussed in class.

c) Expand $\nabla.(\nabla f)$ (divergence of gradient) and $\nabla(\nabla.\boldsymbol{F})$ (gradient of divergence)

d) Recall the expression for the electric field due to a uniformly charged sphere of radius r and charge density ρ in electrostatics. Show that it satisfies Maxwell's equations.

Q.2) **Plane waves in a uniform, isotropic, linear, time-invariant medium** $(\boldsymbol{\epsilon}, \boldsymbol{\mu})$ **(120 marks)**

a) Write the general expression for a plane wave as discussed in class and show that it satisfies Maxwell's equations.

b) Find out the magnetic field strength and Poynting vector for a circularly polarized plane wave as well as for a standing wave. Compare the Poynting vector with the case of a linearly polarized plane wave. Provide a simple explanation for your observation.

c) Give an example of an elliptically polarized plane wave propagating along z-direction with the major and minor axes along x and y respectively.

d) Write an expression for an equal superposition of two counter-propagating plane waves with orthogonal polarizations. Is it a standing wave?

e) Write an expression for an equal superposition of two counter-propagating plane waves with same polarization but a relative phase difference of $\pi/2$. Is it a standing wave?

f) Consider a general superposition of two plane waves. Does the Poynting vector also follow a similar superposition principle? Prove it. (Hint: Consider an example)

g) Write down the time domain expressions for the electric and magnetic fields for a plane wave of angular frequency ω propagating in the $(\hat{x} + \hat{z})/\sqrt{2}$ direction and polarized along the $\hat{y}$ direction. Also write down the expressions for the Poynting vector, electric polarization, and magnetization density.

h) As in part (g) write down the expressions for the electric and magnetic fields for a plane wave propagating in the same direction but polarized in the orthogonal direction. Also verify if it satisfies the energy conservation relation.

Q.3) **Linearity of Maxwell's equations: Superposition principle** **(30 marks)**

Prove the superposition principle as discussed in the class, i.e., show that if $(\boldsymbol{E_1}(\boldsymbol{r}, t), \boldsymbol{H_1}(\boldsymbol{r}, t))$ and $(\boldsymbol{E_2}(\boldsymbol{r}, t), \boldsymbol{H_2}(\boldsymbol{r}, t))$ are the solutions of Maxwell's equations in a source free medium, the fields $(a_1\boldsymbol{E_1} + a_2\boldsymbol{E_2}, a_1\boldsymbol{H_1} + a_2\boldsymbol{H_2})$ also satisfy the Maxwell's equations for arbitrary real numbers a_1 and a_2.

Reformulate the superposition principle when there are sources present. Have you encountered superposition property anywhere else? Give examples.