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EE798I - NANOPHOTONICS

Assignment - 2

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Ans 1- **Boundary Conditions**

Consider medium 1 with properties $\varepsilon_1, \mu_1, \sigma_1$ and fields E_1, D_1, H_1, B_1 and medium 2 with properties $\varepsilon_2, \mu_2, \sigma_2$ and fields E_2, D_2, H_2, B_2. The interface may contain surface charge density ρ_s and surface current density J_s

Maxwells equations in phasor form are given by:

1. Faradays Law : $\oint_C E \cdot dl = -j\omega \int_S B \cdot ds$
2. Ampere's Law : $\oint_C H \cdot dl = \int_S (J + j\omega D) \cdot ds$
3. Gauss Law for E : $\oint_S D \cdot ds = \int_V \rho_v \, dV$
4. Gauss Law for B : $\oint_S B \cdot ds = 0$

A) Tangential component of E

Consider a small rectangular loop of width Δw and height Δh that straddles the boundary. The sides of length Δw are parallel to the interface Now, the line integral of E around the closed loop C :

$$\oint_C E \cdot dl = \int_{a\to b} E \cdot dl + \int_{b\to c} E \cdot dl + \int_{c\to d} E \cdot dl + \int_{d\to a} E \cdot dl$$

now, let $\Delta h \to 0 \Rightarrow$ integrals along $b\to c$ & $d \to a$ become 0

$$\rightarrow \boxed{\oint_C E \cdot dl = E_{t1} \Delta w - E_{t2} \Delta w}$$

now, RHS $= -j\omega \int_S B\cdot ds = -j\omega B_n(\Delta w \Delta h)$

where B_n is normal component of B passing through loop

as $\Delta h \to 0$, RHS $\to 0$

equating the LHS and RHS $E_{t1} = E_{t2}$

$\Rightarrow$ The tangential component of electric field E is continuous across the boundary. $\Rightarrow \hat{n} \times (E_1 - E_2) = 0$

B) Tangential Component of H

Using Ampere's Law in the same loop as (A)

$$\oint_C H\cdot dl = I_{enc} = \int_S (J + j\omega D)\cdot ds$$ now as $\Delta h \to 0$

$$\Rightarrow (H_{t1} - H_{t2})\Delta w = J_{sn}\Delta w$$

$$\Rightarrow \hat{n} \times (H_1 - H_2) = J_s$$

(c) Normal Component of D

Consider a small cylindrical surface of top/bottom area ΔS and height Δh that straddles the boundary.

Now, applying Gauss law: $\oint_S D\cdot ds = Q_{enc}$

also $$\oint_S D\cdot ds = \int_{top} D\cdot ds + \int_{bottom} D\cdot ds + \int_{side} D\cdot ds$$

shrink the pillbox to $\Delta h \to 0 \Rightarrow \int_{side} D\cdot ds = 0$

$$\Rightarrow \oint_S D \cdot ds = (D_1 \cdot \hat{n} - D_2 \cdot \hat{n}) \Delta S$$

as $h \to 0$, $Q_{enc} = \int_V \rho_v \, dV = \rho_s \Delta S$

$$\Rightarrow (D_1 - D_2) \cdot \hat{n} \, \Delta S = \rho_s \Delta S$$

$$\Rightarrow \hat{n} \cdot (D_1 - D_2) = \rho_s$$

(d) Normal component of B.

Consider the same geometry as (c)

$$\oint B \cdot ds = 0 \Rightarrow (B_{n1} - B_{n2}) \Delta S = 0$$

$$\Rightarrow \hat{n} \cdot (B_1 - B_2) = 0$$

To conclude, the boundary conditions are:

(i) $\hat{n} \times (E_1 - E_2) = 0$

(ii) $\hat{n} \cdot (B_1 - B_2) = 0$

(iii) $\hat{n} \times (H_1 - H_2) = J_s$

(iv) $\hat{n} \cdot (D_1 - D_2) = \rho_s$

Ans 2- Integral form of Maxwell's equations

The differential Maxwell equations in frequency domain are

$$\bar{\nabla} \cdot \bar{D} = \rho \quad (1)$$

$$\bar{\nabla} \cdot \bar{B} = 0 \quad (2)$$

$$\bar{\nabla} \times \bar{E} = -j\omega \bar{B} \quad (3) \text{ where } j = \sqrt{-1}$$

$$\nabla \times \bar{H} = \bar{J} + j\omega \bar{D} \quad (4)$$

A) Integrate (1) over arbitrary volume V:

$$\int_V \bar{\nabla}\cdot\bar{D}\, dV = \int_V \rho\, dV$$

Now by Gauss theorem:

$$\int_{\partial V} \bar{D}\cdot d\bar{S} = \int_V \rho\, dV$$

$$Q_{enc} = \int_V \rho\, dV \quad \Rightarrow \quad \boxed{\oint_{\partial V} \bar{D}\cdot d\bar{S} = Q_{enc}}$$

B) Integrate (2) over V, apply divergence theorem

$$\int \bar{\nabla}\cdot\bar{B}\, dV = \oint_{\partial V} B\cdot dS = 0$$

$$\Rightarrow \quad \boxed{\oint_{\partial V} \bar{B}\cdot d\bar{S} = 0}$$

c) Integrate (3) over any open surface S, apply stokes':

$$\int_S (\bar{\nabla}\times\bar{E})\cdot d\bar{S} = -j\omega \int_S \bar{B}\cdot d\bar{S}$$

now, by stokes $\oint_{\partial S} \bar{E}\cdot dl = -j\omega \int_S \bar{B}\cdot d\bar{S}$

Hence $\boxed{\oint_{\partial S} \bar{E}\cdot dl = -j\omega \int_S \bar{B}\cdot dS}$

d) Integrate (4) over an open surface, apply stokes'

$$\int_S (\nabla\times H)\cdot dS = \int_S \bar{J}\cdot dS + j\omega \int \bar{D}\cdot dS$$

Thus :
$$\boxed{\oint_{\partial S} \bar{H} \cdot dl = \int_S \bar{J} \cdot d\bar{S} + j\omega \int_S \bar{D} \cdot d\bar{S}}$$

The RHS contains both the conduction current through S and the ~~#~~ displacement current $j\omega \bar{D}$

(e) Charge Conservation (continuity eq^n)
Taking divergence of (4):
$$\bar{\nabla} \cdot (\bar{\nabla} \times \bar{H}) = \bar{\nabla} \cdot \bar{J} + j\omega \nabla \cdot \bar{D}$$

using $\bar{\nabla} \cdot \bar{D} = \rho$: $\bar{\nabla} \cdot \bar{J} + j\omega\rho = 0$

$$\Rightarrow \boxed{\oint_{\partial V} \bar{J} \cdot d\bar{S} = -j\omega \int_V \rho \, dV}$$

Ans 3- <u>A different scaling symmetry</u>

Maxwell's curl equations for first medium:

$$\bar{\nabla} \times \bar{E}_1 = -j\omega\mu_1(r) H_1 \qquad (1)$$

$$\bar{\nabla} \times \bar{H}_1 = j\omega\epsilon_1(r) E_1 \qquad (2)$$

Now for the second medium:

$$\bar{\nabla} \times \bar{E}_1 = -j\omega s \mu_1(r) H_2 \qquad (3)$$

$$\bar{\nabla} \times \bar{H}_1 = j\omega s s \epsilon_1(r) E_2 \qquad (4)$$

let $\omega' = s\omega$, substitute this in (3) & (4)

$$\bar{\nabla} \times \bar{E}_2(\bar{r}, \omega) = -j\omega' \mu_1(\bar{r}) \bar{H}_2(\bar{r}, \omega) \quad (5)$$

$$\bar{\nabla} \times \bar{H}_2(\bar{r}, \omega) = j\omega' \epsilon_1(\bar{r}) \bar{E}_2(\bar{r}, \omega) \quad (6)$$

This is exactly the same mathematical form as (1), (2)

Hence, if (E_1, H_1) is a solution in medium (ϵ_1, μ_1) at frequency ω, then the same fields are a solution in a scaled medium $(S\epsilon_1, S\mu_1)$ at the frequency ω/S, because:

$$E_2(\bar{r}, \omega) = E_1(\bar{r}, \omega s)$$
$$H_2(\bar{r}, \omega) = H_1(\bar{r}, \omega s)$$

Ans 4 **Symmetries in Time domain**

→ Time-domain Maxwell equations are:

(a) $\bar{\nabla} \cdot \bar{D}(\bar{r}, t) = \rho(\bar{r}, t)$

(b) $\bar{\nabla} \cdot \bar{B}(\bar{r}, t) = 0$

(c) $\bar{\nabla} \times E(\bar{r}, t) = -\dfrac{\partial \bar{B}(\bar{r}, t)}{\partial t}$

(d) $\bar{\nabla} \times \bar{H}(\bar{r}, t) = \bar{J}(\bar{r}, t) + \dfrac{\partial \bar{D}(\bar{r}, t)}{\partial t}$

→ For local, instantaneous, linear media:

$\bar{D} = \varepsilon \bar{E}$, $\bar{B} = \mu \bar{H}$

$\bar{J} = \sigma \bar{E} + \bar{J}_{ext}$

For dispersive linear media, we replace ϵ and μ by convolution kernels. The results remain same but with convolution scaling laws.

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(A) Scaling invariance - space / time scaling

let $\bar{r}' = a\bar{r}$ and $t' = bt$, $a > 0$, $b > 0$

let $\Phi'(\bar{r}', t') = \lambda_\Phi \Phi(\bar{r}, t)$

where $\Phi = \bar{E}, \bar{H}, \bar{D}, \bar{B}, \bar{J}, \rho$

we need to find relations b/w the λ's and the parameter transformations $\varepsilon', \mu', \sigma'$ so that the Maxwell eq^n in primed coordinates read exactly the same form.

$$\bar{\nabla}' \cdot \bar{D}' = \rho' \quad , \quad \bar{\nabla}' \cdot \bar{B}' = 0$$
$$\bar{\nabla}' \times \bar{E}' = -\partial_{t'} \bar{B}' \quad , \quad \bar{\nabla}' \times \bar{H}' = \bar{J}' + \partial_{t'} \bar{D}'$$

$$\nabla' = \frac{1}{a} \nabla \quad , \quad \partial_{t'} = \frac{1}{b} \partial_t$$

Now, apply Faraday's law:

$$\bar{\nabla}' \times \bar{E}' = \frac{\lambda_E}{a} (\bar{\nabla} \times \bar{E}) \quad , \quad -\partial_{t'} \bar{B}' = -\frac{\lambda_B}{b} \partial_t \bar{B}$$

$$\Rightarrow \boxed{\frac{\lambda_B}{\lambda_E} = \frac{b}{a}} \quad (1)$$

Applying Ampere-Maxwell law:

$$\bar{\nabla}' \times \bar{H}' = \frac{\lambda_H}{a} (\bar{\nabla} \times \bar{H}) = \frac{\lambda_H}{a} (\bar{J} + \partial_t D)$$

$$J' + \partial_{t'} D' = \lambda_J \bar{J} + \frac{\lambda_D}{b} \partial_t D$$

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$$\Rightarrow \boxed{\lambda_J = \frac{\lambda_H}{a}} \ (2) \quad , \quad \boxed{\frac{\lambda_D}{\lambda_H} = \frac{b}{a}} \ (3)$$

Applying Gauss's law:

$$\bar{\nabla}' D' = \frac{\lambda_D}{a}(\bar{\nabla}\cdot\bar{D}) = \frac{\lambda_D}{a}\rho \quad , \quad \rho' = \lambda_\rho \rho$$

$$\Rightarrow \boxed{\lambda_\rho = \frac{\lambda_D}{a}} \ (4)$$

Constitutive relations scaling:

we know, $\bar{D} = \varepsilon \bar{E}$, $\bar{B} = \mu \bar{H}$

$$\Rightarrow \lambda_D \bar{D} = \bar{D}' = \varepsilon' E' = \varepsilon' \lambda_E E \Rightarrow \boxed{\varepsilon' = \frac{\lambda_D}{\lambda_E}\varepsilon} \ (5)$$

$$\lambda_B \bar{B} = B' = \mu' \bar{H}' = \mu' \lambda_H H \Rightarrow \boxed{\mu' = \frac{\lambda_B}{\lambda_H}\mu} \ (6)$$

For ohmic conduction, $\bar{J}_{cond} = \sigma \bar{E}$

$$\lambda_j \bar{J}_{cond} = \lambda_j \sigma \bar{E} = \sigma' \lambda_E E \Rightarrow \boxed{\sigma' = \frac{\lambda_J}{\lambda_E}\sigma} \ (7)$$

let $\lambda_E = \lambda_H = 1$ (normalization scale)

From (1) - (4), we obtain:

$$\boxed{\lambda_B = \frac{b}{a}, \quad \lambda_D = \frac{b}{a}, \quad \lambda_J = \frac{1}{a}, \quad \lambda_\rho = \frac{b}{a^2}}$$

Hence the material parameters transform as:

$$\boxed{\varepsilon' = \frac{b}{a}\varepsilon \qquad \mu' = \frac{b}{a}\mu \qquad \sigma' = \frac{1}{a}\sigma}$$

Sources transform as:

$$\boxed{\bar{J}'(\bar{r}', t') = \frac{1}{a}\bar{J}(\bar{r}, t) \qquad \rho'(\bar{r}', t') = \frac{b}{a^2}\rho(\bar{r}, t)}$$

(B) Time-reversal symmetry in time-domain

Objective is to derive the time-reversal mapping that preserves the form of Maxwell's equations.

Define operator $T : t \mapsto -t$

let $\varphi'(\bar{r}, t) = S_\varphi \varphi(\bar{r}, -t)$

where $\varphi \in \{E, \bar{H}, \bar{D}, \bar{B}, \bar{J}, \rho\}$, $S_\varphi \in \{-1, 1\}$

Faraday's law:

$$\bar{\nabla} \times \bar{E}'(\bar{r}, t) = S_E \nabla \times \bar{E}(\bar{r}, -t) = S_E(-\partial_\tau \bar{B}(\bar{r}, \tau))\Big|_{\tau=-t}$$

$$= -S_E \partial_\tau \bar{B}(\bar{r}, \tau)\Big|_{\tau=-t}$$

$$-\partial_t B'(\bar{r}, t) = -\partial_t[S_B \bar{B}(\bar{r}, -t)] = -S_B(-\partial_\tau \bar{B}(\bar{r}, \tau))\Big|_{\tau=-t}$$

$$= S_B \partial_\tau \bar{B}(\bar{r}, \tau)\Big|_{\tau=-t}$$

equating left and right:

$$S_B = -S_E \qquad (1)$$

Now using Ampere-Maxwell :

LHS : $\bar{\nabla} \times \bar{H}' = S_H \nabla \times \bar{H}(\bar{r}, -t) = S_H (\bar{J}(\bar{r}, -t) + \partial_\tau \bar{D}(\bar{r}, \tau)|_{\tau=-t})$

RHS : $J' + \partial_t \bar{D}' = S_J \bar{J}(\bar{r}, -t) + \partial_t [S_D \bar{D}(\bar{r}, -t)]$

$= S_J \bar{J}(\bar{r}, -t) - S_D \partial_\tau \bar{D}(\bar{r}, \tau) |_{\tau=-t}$

comparing, we get : $S_H = S_J = -S_D$ — (2)

Now using Gauss's Law : $\bar{\nabla} \cdot \bar{D} = \rho$

$\bar{\nabla} \cdot \bar{D}' = S_D \bar{\nabla} \cdot D(\bar{r}, -t) = S_D \rho(\bar{r}, -t)$

$= \rho' = S_\rho \rho(r, -t)$

$\Rightarrow S_\rho = S_D$ — (3)

Solving and picking consistent assignments :

$E'(\bar{r}, t) = \bar{E}(\bar{r}, -t)$
$D'(\bar{r}, t) = \bar{D}(\bar{r}, -t)$
$B'(\bar{r}, t) = -\bar{B}(\bar{r}, -t)$
$H'(\bar{r}, t) = -\bar{H}(\bar{r}, -t)$
$J'(\bar{r}, t) = -\bar{J}(\bar{r}, -t)$
$\rho'(\bar{r}, t) = \rho(\bar{r}, -t)$

(c) Consistency b/w phasor and time-domain relations

(i) Sign flips in B and H are exactly what compensate the sign change in $j\omega$ when $\omega \mapsto -\omega$, so phasor version of Maxwell is consistent with the time domain parity

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(ii) Time reversal in phasors is implemented by $\omega \mapsto -\omega$ together with complex conjugation. The mappings $\vec{E} \mapsto \vec{E}$, $\bar{B} \mapsto -\bar{B}$ found in time domain translate to the same frequency-domain relations after $\omega \mapsto -\omega$ + conjugation is done.

Ans 5- Energy Conservation in frequency domain

Time domain Poynting's Theorem states that:

$$\nabla \cdot (\bar{E} \times \bar{H}^*) + \frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon |E|^2 + \frac{1}{2}\mu |H|^2\right) = -\bar{J} \cdot \bar{E}$$

divergence of poynting vector

rate of change of stored EM energy

power diss. per unit volume by currents

Frequency domain Maxwell's eq^{n}s:

$$\bar{\nabla} \times \bar{E} = -j\omega\mu \bar{H} \quad \text{--- (1)}$$
$$\bar{\nabla} \times \bar{H}^* = \bar{J}^* + j\omega\varepsilon \bar{E}^* \quad \text{--- (2)}$$

$$E \cdot (2) \equiv E \cdot (\bar{\nabla} \times \bar{H}^*) = \bar{E} \cdot \bar{J}^* + j\omega \epsilon |E|^2 \quad \text{--- (3)}$$
$$H^* \cdot (1) \equiv H^* \cdot (\bar{\nabla} \times \bar{E}) = -j\omega\mu |H|^2 \quad \text{--- (4)}$$

$$\Rightarrow -E \cdot (\bar{\nabla} \times \bar{H}^*) + H^* \cdot (\bar{\nabla} \times \bar{E}) = -\bar{E} \cdot \bar{J}^* - j\omega (\epsilon |E|^2 + \mu |H|^2)$$
$$\Rightarrow \bar{\nabla} \cdot (\bar{E} \times \bar{H}^*) = -\bar{E} \cdot \bar{J}^* - 2j\omega \left(\frac{\epsilon |E|^2 + \mu |H|^2}{2}\right)$$

$$\Rightarrow \boxed{\bar{\nabla} \cdot \bar{S} = -\frac{1}{2}(\bar{E} \cdot \bar{J}^*) + j\omega \frac{1}{2}(\bar{E} \cdot \bar{D}^* + \bar{H}^* \cdot \bar{B})}$$

The relations are consistent and the frequency domain theorem compactly encodes the time-averaged behaviour of the system.

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Ans 6 - **Light Propagation in lossy medium**

$n_{Si} + i K_{Si} = 5.57 + 0.387i$

$\lambda_0 = 400$ nm , $K_0 = 2\pi \backslash \lambda_0$, $\eta_0 = \sqrt{\mu_0 / \varepsilon_0}$

$E_0 = 100 Vm^{-1}$, $n_1 = 1$, $n_2 = n_{S_i} + i K_{S_i}$, $\eta_0 = 376.73\Omega$

For incident wave in air $(z<0)$:

$\tilde{E}_{inc}(z) = E_0 \exp(-jK_0 z)$

$\tilde{H}_{inc}(z) = (E_0 / \eta_0) \exp(-jK_0 z)$

$$\gamma = \frac{n_1 - N}{n_1 + N} , \quad N = n_{Si} - jK_{Si}$$

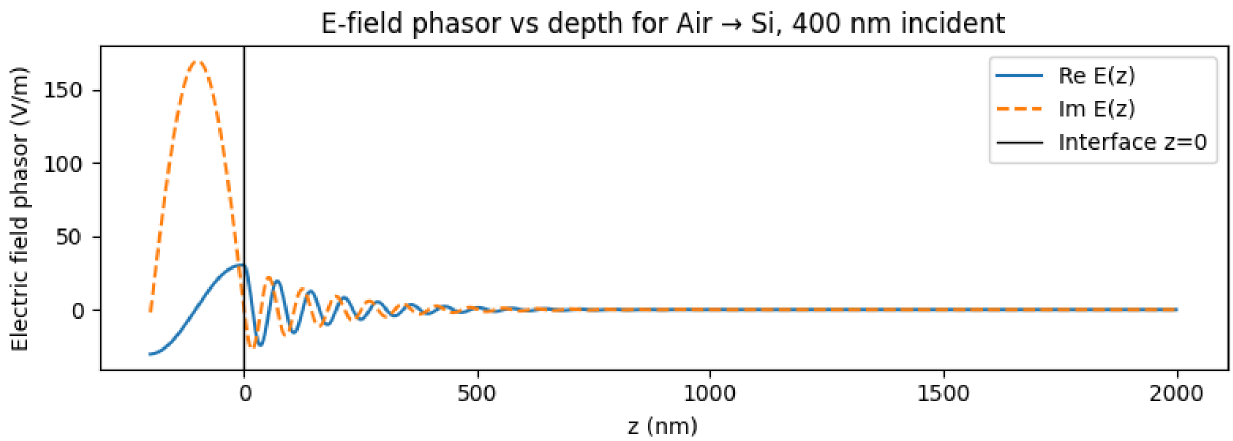
$$\tilde{E}(z) = \begin{cases} E_0 \exp(-jK_0 z) + \gamma E_0 \exp(+jK_0 z) , & z<0 \\ E_0 t \exp(-jK_0 N z) , & z>0 \end{cases}$$

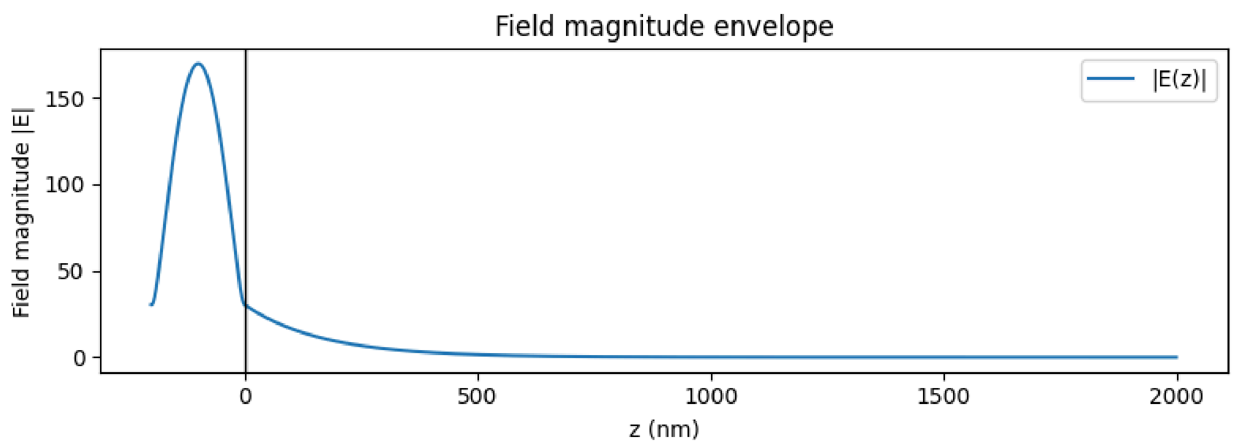
$$\tilde{H}(z) = \begin{cases} 1/\eta_0 \left(E_0 e^{-jK_0 z} - E_0 \gamma e^{+jK_0 z} \right) , & z<0 \\ 1/\eta_{Si} \; E_0 t \exp(-jK_0 N z) , & z>0 \end{cases}$$

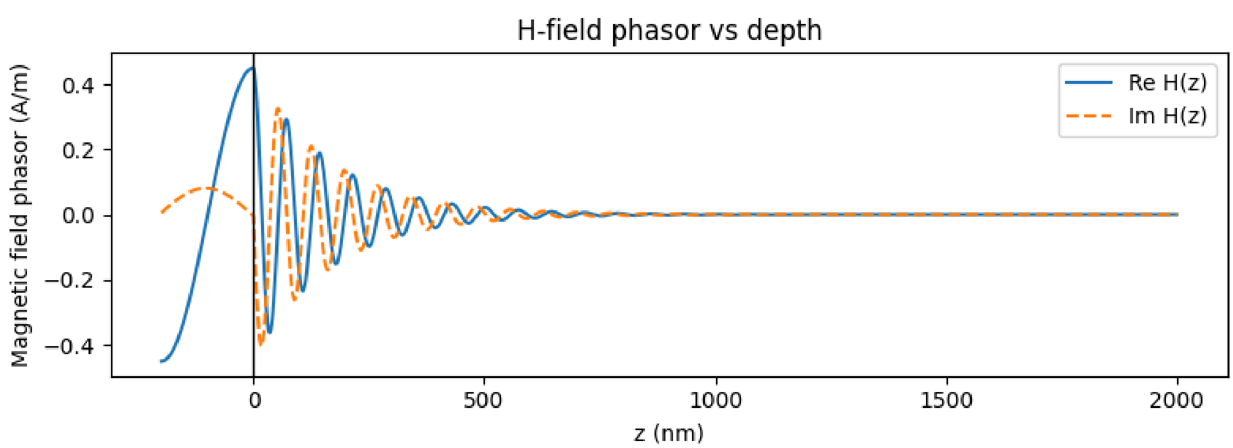
where $\eta_{Si} = \eta_0 / N$

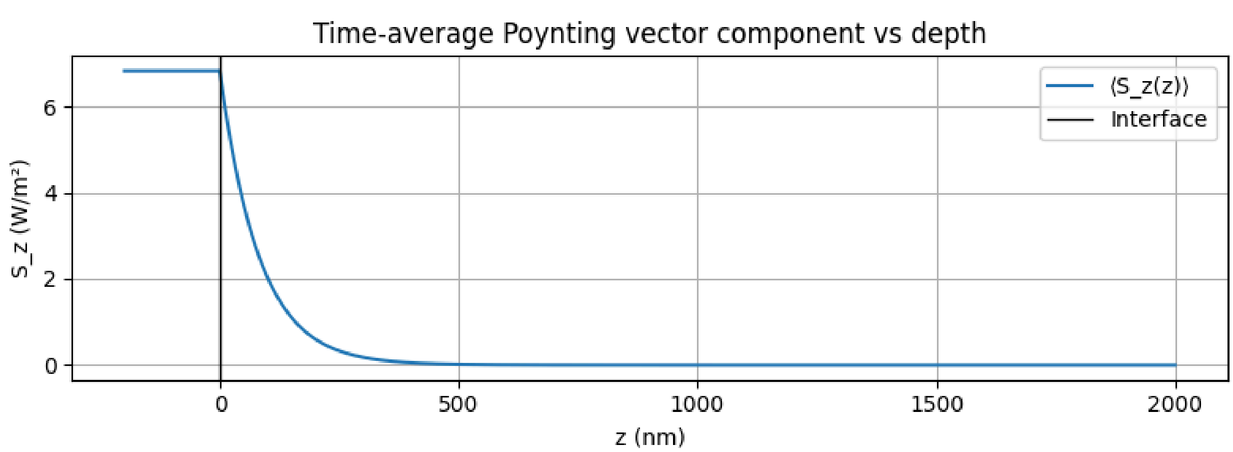
$$\langle S(z) \rangle = \frac{1}{2} \Re \{ \tilde{E}(z) \times \tilde{H}^*(z) \}$$

	Lossy Si	Lossless
1.	Exponential decay of field magnitude in Si	No decay of field magnitude, hence const magnitude
2.	Net poynting flux decreases with z (power is absorbed)	Net poynting flux is constant with z
3.	incident energy = (reflected + absorbed) energy	incident energy = (reflected + transmitted) energy

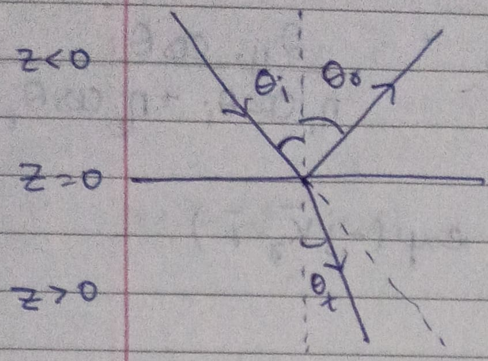

E-field phasor vs depth for Air → Si, 400 nm incident
Electric field phasor (V/m)
z (nm)
Re E(z)
Im E(z)
Interface z=0


Field magnitude envelope
Field magnitude |E|
z (nm)
|E(z)|


H-field phasor vs depth
Magnetic field phasor (A/m)
z (nm)
Re H(z)
Im H(z)


Time-average Poynting vector component vs depth
S_z (W/m²)
z (nm)
⟨S_z(z)⟩
Interface

Ans 7 - Reflection of circularly polarized light



ε_1, μ_1

ε_2, μ_2

x-z plane

$$\eta = \sqrt{\mu / \varepsilon}$$

$$k = \omega\sqrt{\mu\varepsilon}$$

Snell's law:

$$k_1 \sin\theta_i = k_2 \sin\theta_t$$

$\hat{y} \rightarrow$ TE direction

For any wave with propagation unit vector $\hat{k}$ in the x-z plane a convenient in-plane unit vector orthogonal to $\hat{k}$ is:

$$\hat{p} = \cos\theta\,\hat{x} - \sin\theta\,\hat{z}$$
$$\hat{p}_i = \cos\theta_i\,\hat{x} - \sin\theta_i\,\hat{z}$$
$$\hat{p}_r = \cos\theta_i\,\hat{x} + \sin\theta_i\,\hat{z}$$
$$\hat{p}_t = \cos\theta_t\,\hat{x} - \sin\theta_i\,\hat{z}$$

Right-circular polarized light:

$$\bar{E}_i(\bar{r}) = E_0 \frac{1}{\sqrt{2}}(\hat{s} + j\hat{p}_i)\, e^{-j\bar{k}_i \cdot \bar{r}}$$

here
$$\hat{s} = \hat{y}$$
$$\hat{k}_i = k_1(\sin\theta_i\,\hat{x} + \cos\theta_i\,\hat{z})$$
$$\hat{k}_r = k_1(\sin\theta_i\,\hat{x} - \cos\theta_i\,\hat{z})$$
$$\hat{k}_t = k_2(\sin\theta_t\,\hat{x} + \cos\theta_t\,\hat{z})$$

$$\Rightarrow \bar{E}_i(\bar{r}) = E_0 \frac{1}{\sqrt{2}}(\hat{y} + j\hat{p}_1)\exp(-j\bar{k}_1 \cdot \bar{r})$$

For TE (s) polarization:

$$r_s = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i - \eta_1\cos\theta_t} \qquad t_s = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

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For TM polarisation:

$$r_p = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} \qquad t_p = \frac{2\eta_2 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

$$E_r(\bar{r}) = E_0 \frac{1}{\sqrt{2}} (r_s \hat{y} + j r_p \hat{P}_r) \exp(-j\bar{K}_r \cdot \bar{r})$$

$$E_t(\bar{r}) = E_0 \frac{1}{\sqrt{2}} (t_s \hat{y} + j t_p \hat{P}_t) \exp(-j\bar{K}_t \cdot \bar{r})$$

Now, calculating the magnetic field phasors:

$$\bar{H} = (1/\eta)\hat{K} \times \bar{E}$$

$$\bar{H}_i(\bar{r}) = \frac{E_0}{\eta_1} \frac{1}{\sqrt{2}} (\hat{K}_i \times \hat{y} + j\hat{K}_i \times \hat{P}_i) \exp(-j\bar{K}_i \cdot \bar{r})$$

$$\bar{H}_r(\bar{r}) = \frac{E_0}{\eta_1} \frac{1}{\sqrt{2}} (r_s \hat{K}_r \times \hat{y} + j r_p \hat{K}_r \times \hat{P}_r) \exp(-j\bar{K}_r \cdot \bar{r})$$

$$\bar{H}_t(\bar{r}) = \frac{E_0}{\eta_2} \frac{1}{\sqrt{2}} (t_s \hat{K}_t \times \hat{y} + j t_p \hat{K}_t \times \hat{P}_t) \exp(-j\bar{K}_t \cdot \bar{r})$$

$$S = \frac{1}{2} \bar{E} \times \bar{H}^*, \qquad \langle S \rangle = \Re\left\{ \frac{1}{2} E \times H^* \right\}$$

For each plane wave region, the energy flow direction is along $\hat{K}$ and the magnitude for a single plane wave of E- amplitude ~~to~~ E_a is:

$$\langle S \rangle = \frac{|E_a|^2}{2\eta} \cos\phi \qquad \eta \to \text{wave impedance}$$

$$\langle S_{i,z} \rangle = \frac{|E_0|^2}{2\eta_1} \cos\theta_i$$

$$\langle S_{r,z} \rangle = -\frac{|E_0|^2}{2\eta_1} \cos\theta_i \, |r_{eff}|^2$$

reflected power, $P_r = \frac{|E_0|^2}{2\eta_1} \cos\theta_i \left(\frac{|r_s|^2 + |r_p|^2}{2} \right)$

reflected fraction of incident power, $R = \frac{|r_s|^2 + |r_p|^2}{2}$

transmitted intensity, $\langle S_{t,z} \rangle = \frac{|E_0|^2}{2\eta_2} \cos\theta_t \left(\frac{|t_s|^2 + |t_p|^2}{2} \right)$

$$\boxed{\langle S_{i,z} \rangle = \langle S_{r,z} \rangle = \langle S_{t,z} \rangle}$$

→ Since $r_s \neq r_p$, $t_s \neq t_p$, the reflected and transmitted fields are not pure right circular in general. They are elliptically polarized

→ For $\theta_i = 0$: $r_s = r_p$, $t_s = t_p$ hence polarization is preserved.

→ The handedness of the wave changes on reflection if there is a sign change b/w s and p components. Else it remains the same.

Ans 8 - Plane waves and their sources

There exists a surface current density sheet at $z = 0$

$\bar{J}_s(t) = J_0 \cos(\omega t) \hat{x}$ (time domain)

$\tilde{\bar{J}}_s = J_0 \hat{x}$ (phasor)

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Boundary cond: $\hat{z} \times (\bar{H}_{z=0^+} - \bar{H}_{z=0^-}) = \bar{J}_s$

$E_{z=0^+} = E_{z=0^-}$

Since the sheet is oscillating uniformly in x:

- Radiated fields will be plane waves propagating away from sheet: for $z>0$ wave travelling in $+z$
  for $z<0$ wave travelling in $-z$
- Waves are Transverse EM with:
  - $\bar{E} \parallel \hat{x}$ ($\bar{E}$ parallel to current)
  - $\bar{H}$ along $\hat{y}$
  - propagation along $\pm\hat{z}$

$$\boxed{\tilde{E}(z) = \begin{cases} \tilde{E}_0 \exp(-jK_0 z)\,\hat{x} & z>0 \\ \tilde{E}_0 \exp(jK_0 z)\,\hat{x} & z<0 \end{cases}}, \quad K_0 = \omega/C$$

now $\tilde{H} = \frac{1}{\eta_0}(\hat{k} \times \tilde{E})$, $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$

$$\Rightarrow \boxed{\tilde{H}_y(z) = \begin{cases} (+\tilde{E}_0/\eta_0)\exp(-jK_0 z), & z>0 \\ (-\tilde{E}_0/\eta_0)\exp(+jK_0 z), & z<0 \end{cases}}$$

Now applying boundary cond at $z=0$

$\hat{z} \times (H^+ - H^-) = \bar{J}_s$, $H^+ = E_0/\eta_0\,\hat{y}$

$H^- = -E_0/\eta_0\,\hat{y}$

$\Rightarrow H^+ - H^- = 2E_0/\eta_0\,\hat{y}$

now $\hat{z} \times (H^+ - H^-) = -2E_0/\eta_0\,\hat{x} = J_0\,\hat{x}$

$$\Rightarrow \boxed{E_0 = -J_0\eta_0/2}$$

Power flows away from sheet in both directions

$$\langle S \rangle = \frac{1}{2}\Re\{E \times H^*\} = \begin{cases} J_0^2\eta_0/8, & z>0 \\ -J_0^2\eta_0/8, & z<0 \end{cases}$$

$$P_{rad} = \frac{J_0^2\eta_0}{4}$$

The surface emits net power, drawn from the external source driving oscillations