

Q.1) Boundary conditions (20 marks)

Derive the field continuity boundary conditions using the Maxwell's equations in the frequency domain. Clearly state all the assumptions if any.

Q.2) Integral form of Maxwell's equations (20 marks)

In the frequency domain derive the integral form of the Maxwell's equations from the differential form. Clearly state all the assumptions if any.

Q.3) A different scaling symmetry (10 marks)

Consider two different source-free media with permittivity and permeability distributions given by $(\epsilon_1(\mathbf{r}), \mu_1(\mathbf{r}))$ and $(s\epsilon_1(\mathbf{r}), s\mu_1(\mathbf{r}))$, where s is some constant. Relate the solutions of Maxwell's equations in the two media.

Q.4) Symmetries in Time domain (20 marks)

In the lecture we derived expressions for the scaling invariance and time reversal symmetry using the Maxwell's equations in the frequency domain. For time domain formulation, the analogous expressions were stated but not derived. Derive these analogous expressions in time domain. Are the relations in the phasor form consistent with the time domain relations?

Q.5) Energy conservation in Frequency domain (10 marks)

In the lecture we derived expression for the energy conservation using the Maxwell's equations in the time domain. Derive the analogous expressions in frequency domain. Are the relations in the phasor form consistent with the time domain relations?

Q.6) Light propagation in lossy medium (25 marks)

Consider a plane wave of free-space wavelength 400 nm and electric field amplitude 100 V/m normally incident from air onto a photodetector made of Silicon (very thick Si layer). You can find out the complex refractive index of Si on the link given below. Determine and plot the electric and magnetic field phasors throughout the structure. Also determine and plot the Poynting vector components. Compare the results with the case of lossless medium as discussed in class.

Link: <https://refractiveindex.info/?shelf=main&book=Si&page=Aspn>

Q.7) Reflection of circularly polarized light (25 marks)

Consider a right-circularly polarized plane wave incident from medium 1 (ϵ_1, μ_0) onto medium 2 (ϵ_2, μ_0) at an angle θ_1 . Find out the electric and magnetic field phasors throughout the structure and thus the reflection and transmission amplitudes. Determine the Poynting vector. Comment on the polarization of the reflected and transmitted wave.

Q.9) Planes waves and their sources

(20 marks)

Consider an oscillating surface current $\mathbf{J}_s = J_0 \cos(\omega t) \hat{x}$ in the $z = 0$ plane surrounded by air. Work out the solutions of Maxwell's equations in the presence of such a current source. Calculate the time averaged Poynting vector above and below the current source. Does the current source emit or absorb net power? If it does, where is the energy coming from or vanishing to?

Hint: Work in the frequency domain (phasors) and employ the field boundary conditions to figure out a solution.