Questions

1. Convert the following numbers into the number system indicated
   - (a) $(1010.011)_2$ to decimal
   - (b) $(FA)_{16}$ to decimal
   - (c) $(101110101101)_2$ into hexadecimal
   - (d) $(FA)_{16}$ to binary

Ans 1. (a) $(1010.011)_2$

$= (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) + (0 \times 2^{-1}) + (1 \times 2^{-2}) + (1 \times 2^{-3})$

$= 8 + 0 + 2 + 0 + 0\cancel{.5} + 0.25 + 0.125$

$= 10.375$

(b) $(FA)_{16}$

$= (15 \times 16^1) + (10 \times 16^0)$

$= 240 + 10$

$= 250$

(c) $(101110101101)_2$

$= (1011)(1010)(1101)$

$= (BAD)_{Hex}$

(d) $(FA)_{16}$

$= (1111)(1010)$

$= 11111010$

2. Convert the decimal number 27.25 into a binary number.

Ans 2.

	27	remainder
2	13	1
2	6	1
2	3	0
2	1	1
2	0	1

$\therefore 27 = 11011$

0.	25	
0.	5	$\times 2$
1.	0	$\times 2$

$\therefore 0.25 = (.01)_2$

$\therefore (27.25)_{10} = (11011.01)_2$

3. What is the largest decimal number that you can represent using 8bits? How many bits are required to represent decimal numbers less than or equal to 10^6?

Ans 3. Largest binary number that can be represented using 8 bits is $(11111111)_2 = 255$

Let the number of bits be n. The largest binary number using these 8 bits is

$$2^{n-1} + 2^{n-2} + 2^{n-3} + \dots + 2^1 + 1$$

$$= 2^n - 1 \text{ (Sum of a GP)}$$

From the given condition,

$$2^n - 1 \geq 10^6$$

$$\therefore n \geq \log_2(10^6 + 1)$$

$$\therefore n = 20.$$

4. Determine the number system in which the following arithmetic operations have been carried out. Give justifications for your answer.

(a) 24+17=40

$\therefore$ (a) $24 + 17 = 40$

Let the number system be x.

$$\therefore 2x^1 + 4x^0 + 1 \times x^1 + 7x^0 = 4x^1 + 0$$

$$\Rightarrow 2x + 4 + x + 7 = 4x$$

$$\Rightarrow 11 = 4x - 3x$$

or $x = 11$

(b) $22 \times 5 = 132$

(b) $22 \times 5 = 132$

Let the number system be x.

$$\therefore (2x^1 + 2x^0)(5x^0) = (1x^2 + 3x^1 + 2x^0)$$

$$\Rightarrow (2x+2)(5) = (x^2 + 3x + 2)$$

$$\Rightarrow 10x + 10 = x^2 + 3x + 2$$

$$\Rightarrow x^2 - 7x - 8 = 0$$

$$\Rightarrow x^2 - 8x + x - 8 = 0$$

$$\Rightarrow x(x-8) + 1(x-8) = 0$$

$$\Rightarrow (x+1)(x-8) = 0$$

$x = -1$ is inadmissible.

$$\therefore x = 8$$

5. Obtain 1's and 2's complement of the following binary numbers:
   (a) 10000000
   (b) 10101010
   (c) 01110101
   (d) 10011100

(a) 10000000

1's complement : 01111111

2's complement : 10000000

(b) 10101010

1's complement : 01010101

2's complement : 01010110

(c) 01110101

1's complement : 10001010

2's complement : 10001011

(d) 10011100

1's complement : 01100011

2's complement : 01100100

6. (a) What is the minimum number of bits required to represent -32 in 2's complement form?

<u>Ans6</u> · (a) $(32)_{10} = (100000)_2 = (0100000)_2$ —
↳ for sign.

For -32, take 2's complement of $+32$

$(-32)_{10} = (1100000)_2$ — Six bits

(b) 11011111 is a number in 2's complement. Is it positive or negative? What is its magnitude?

(b) Since the sign bit is 1, the number is negative. 2's complement of 11011111 is 00100001

$(00100001)_2 = (33)_{10}$

7. Carry out the following four operations using 8bit 2's complement representation:

$$\pm 24 \pm 32$$

Verify that operations have been properly carried out.

Ans 7. $(+32)_{10} = (00100000)_2$

$(-32)_{10} = (11100000)_2$

$(+24)_{10} = (00011000)_2$

$(-24)_{10} = (11101000)_2$

+32 +24:

$$\begin{array}{r} 00100000 \\ +\ 00011000 \\ \hline 00111000 \end{array}$$

= 56

-32 + 24

$$\begin{array}{r} 11100000 \\ +\ 00011000 \\ \hline 11111000 \end{array}$$

This is a negative number.
2's complement of 11111000
= 00001000 = 8

+32 - 24

$$\begin{array}{r} 00100000 \\ +\ 11101000 \\ \hline 100001000 \end{array}$$

= 8

-32 - 24

$$\begin{array}{r} 11100000 \\ +\ 11101000 \\ \hline 111001000 \end{array}$$

This is a negative number.
2's complement of 111001000 = 00111000 = 56

8. Show that the Boolean expression $x + \bar{x}.y$ is equivalent to $x + y$ using basic postulates and theorems of Boolean algebra.

$$x + \bar{x}y$$
$$= (x+\bar{x}).(x+y)$$
$$= 1\cdot(x+y)$$
$$= x+y$$

9. Reduce the following expressions to a minimum number of literals using basic postulates and theorems of Boolean algebra.

   (a) $f = (x+y).(\bar{y}+\bar{x})$

   (b) $f = ABCD + \bar{A}BD + AB\bar{C}D$

(a) $f=(x+y).(\cancel{x}\bar{y}+\bar{x})$

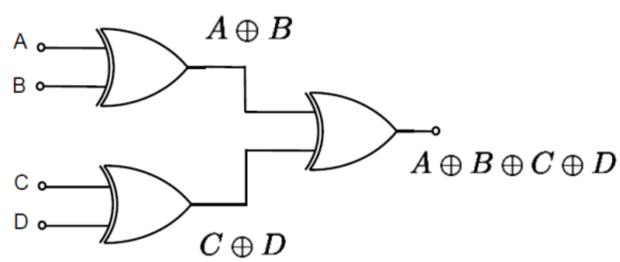
$$\Rightarrow f = x\bar{y} + x.\bar{x} + y.\bar{y} + y.\bar{x}$$
$$\Rightarrow f = x\bar{y} + y\bar{x}$$

(b) $f = ABCD + \bar{A}BD + AB\bar{C}D$

$$\Rightarrow f = ABD(C+\bar{C}) + \bar{A}BD$$
$$\Rightarrow f = ABD + \bar{A}BD$$
$$\Rightarrow f = BD(A+\bar{A})$$
$$\Rightarrow f = BD$$

10. Consider four-input function F(A, B, C, D) that outputs 1 whenever an odd number of its inputs are 1, (a) construct the truth table (b) write down the Boolean expressions, present an implementation of the function using two-input XOR gate

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0



11. Four switches operate a lamp as follows: the lamp lights up if switches 1,3 and 4 are closed and switch 2 is open, or if 2, 4 are closed and 3 is open, or if all the switches are kept closed. Express this as a boolean function in a standard sum of product form and solve it using k- map. (Use bit '1' when switch is closed and bit '0' when switch is open).

Let four switches are represented by w, x, y, z
For closed switch variable will have value 1 and for open switch variable will have value 0.

1, 3, 4 closed ; 2 open $\rightarrow w\bar{x}yz$
2, 4 closed ; 3 open $\rightarrow x\bar{y}z$
All closed $\rightarrow wxyz$

wx \ yz	00	01	11	10
00	0	0	0	0
01	0	1	0	0
11	0	1	1	0
10	0	0	1	0

$f = x\bar{y}z + wyz$

12. Obtain the truth table for the following function: $(x.y + z)(y + x.z)$ and write it as sum of products (SOP) and product of sums (POS).

$f = (x \cdot y + z) \cdot (y + x \cdot z)$

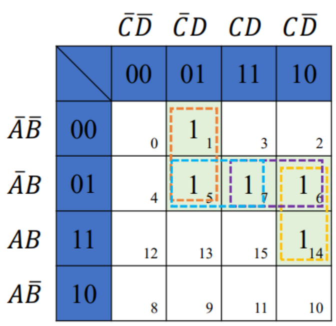
x	y	z	$(x \cdot y + z)$	$(y + x \cdot z)$	f
0	0	0	0	0	0
0	0	1	1	0	0
0	1	0	0	1	0
0	1	1	1	1	1
1	0	0	0	0	0
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

SOP: $f = \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$

POS: $f = (x+y+z)(x+y+\bar{z})(x+\bar{y}+z)(\bar{x}+y+z)$

13. Simplify the following 4-variable functions into sum-of-products form using K-map.
    a. $\sum(1,5,6,7,14)$

$\sum(1,5,6,7,14)$

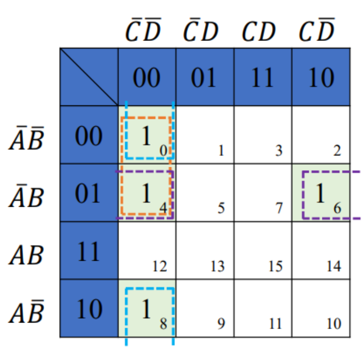


$\boxed{\bar{A}\bar{C}D + BC\bar{D} + \bar{A}BD}$

$\boxed{\bar{A}\bar{C}D + BC\bar{D} + \bar{A}BC}$ there are two answers possible.

b. $\sum(0,4,6,8)$

$\sum(0,4,6,8)$



$\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{D}$

c. $\sum(0,1,4,6,8,9,14)$

$\sum(0,1,4,6,8,9,14)$

		$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
		00	01	11	10
$\bar{A}\bar{B}$	00	1_{0}	1_{1}	3	2
$\bar{A}B$	01	1_{4}	5	7	1_{6}
AB	11	12	13	15	1_{14}
$A\bar{B}$	10	1_{8}	1_{9}	11	10

$\boxed{\bar{B}\bar{C} + \bar{A}B\bar{D} + BC\bar{D}}$

$\boxed{\bar{B}\bar{C} + \bar{A}\bar{C}\bar{D} + BC\bar{D}}$

d. $\sum(1,4,7,11,13,14)$

		$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
		00	01	11	10
$\bar{A}\bar{B}$	00	0	1_{1}	3	2
$\bar{A}B$	01	1_{4}	5	1_{7}	6
AB	11	12	1_{13}	15	1_{14}
$A\bar{B}$	10	8	9	1_{11}	10

$\bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}\bar{D} + \bar{A}BCD$
$+ A\bar{B}CD + AB\bar{C}D + ABC\bar{D}$

This cannot be minimized any further.

14. Simplify the following 4-variable functions into product-of-sums form using K-map

a. $\Pi(1,3,5,7,13,15)$

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	1	1	1

$F = (x_1+\bar{x}_4)(\bar{x}_2+\bar{x}_4)$

b. $\Pi(1,3,6,9,11,12,14)$

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00		0	0	
01				0
11	0			0
10		0	0	

$F = (x_2+\bar{x}_4)(\bar{x}_1+\bar{x}_2+x_4)\cdot$
$(\bar{x}_2+\bar{x}_3+x_4)$

c. $\Pi(1,3,5,7,9,11,12,13,14,15,)$

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00		0	0	
01		0	0	
11	0	0	0	0
10		0	0	

$F = (\bar{x}_1 + \bar{x}_2) . (\bar{x}_4)$

d. $\Pi(0,1,3,4,5,7,12,13,15)$

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	0	0	0	
01	0	0	0	
11	0	0	0	
10				

$F = (x_1 + x_3)(x_1 + \bar{x}_4)(\bar{x}_2 + x_3)(\bar{x}_2 + \bar{x}_4)$

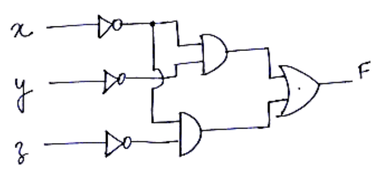
15. Design a combinational circuit with 3 inputs and 1 output

(a) The output is 1 when the binary value of the inputs is less than 3. The output is 0 otherwise

x	y	z	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

$x \backslash yz$	00	01	11	10
0	1	1		1
1				

$F = \bar{x}\bar{y} + \bar{x}\bar{z}$



(b) The output is 1 when the binary value of inputs is an odd number.

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$x \backslash yz$	00	01	11	10
0		1	1	
1		1	1	

$F = z$