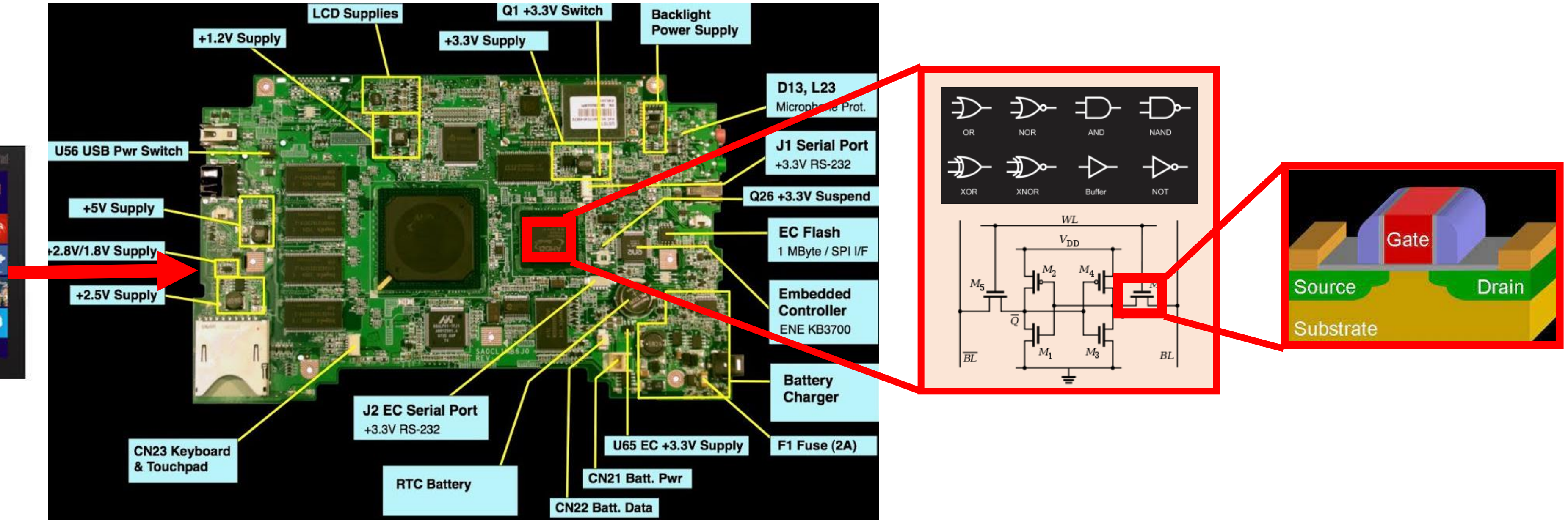

ESC201: Introduction to Electronics

Module 2: Elements with Memory

Dr. Shubham Sahay,
Assistant Professor,
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IIT Kanpur

Capacitor and Inductor

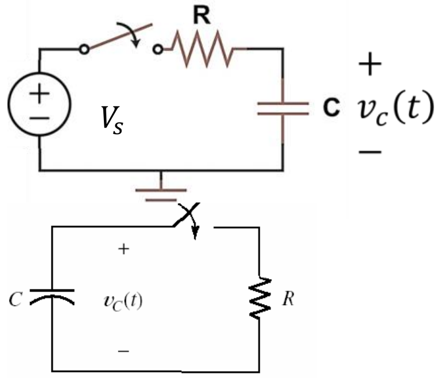
$$v_C(t) = v_C(\infty) + \left(v_C(0^+) - v_C(\infty)\right)e^{-\frac{t}{RC}}$$

Final Voltage (steady state) — Initial Voltage — Final Voltage

$$\underbrace{v_C(0^+) - v_C(\infty)}_{\text{Change in voltage}}$$

$$v_C(t) = V_s\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$v_C(t) = V_i e^{-\frac{t}{\tau}}$$

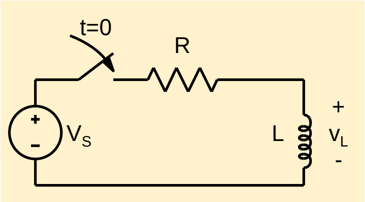


$$\tau = RC$$

$$i(t) = i(\infty) + \left(i(0^+) - i(\infty)\right)e^{-\frac{R}{L}t}$$

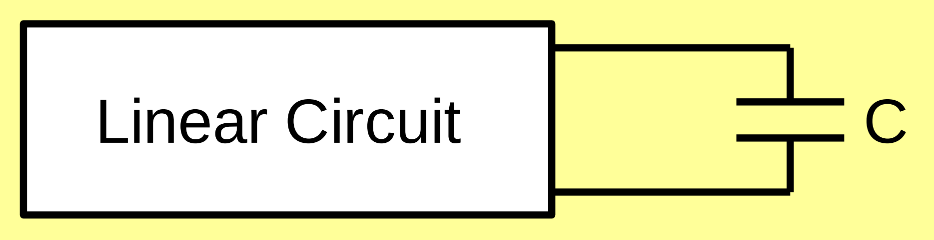
$$i(t) = \frac{V_S}{R} - \frac{V_S}{R}e^{-\frac{R}{L}t}$$

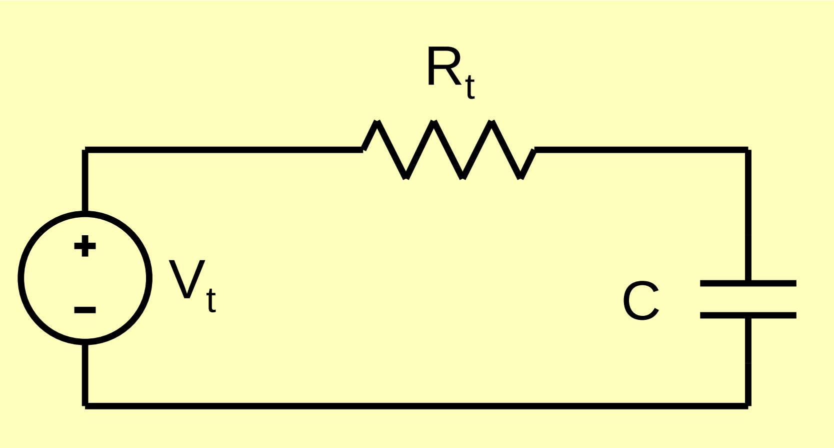
$$i(t) = i_0 e^{-\frac{R}{L}t}$$



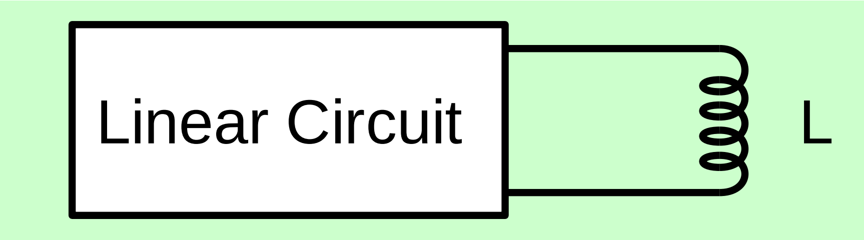
Circuit analysis with Inductor/Capacitor

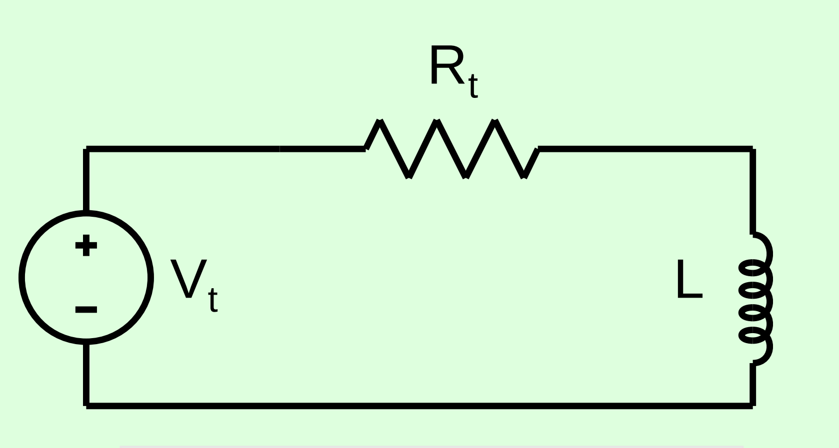
Easy if the circuit contains a single L or C: Thevenin equivalent





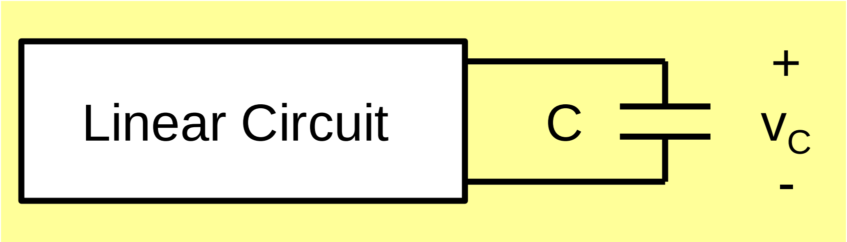
$$x(t) = x(\infty) + \{x(0^+) - x(\infty)\}e^{-\frac{t}{\tau}}$$

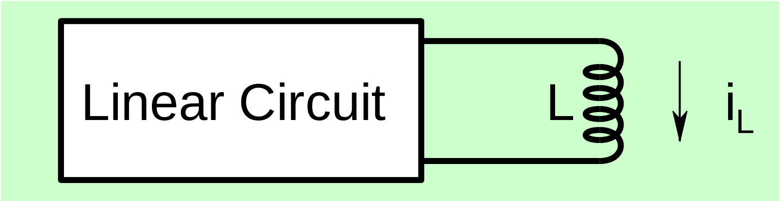


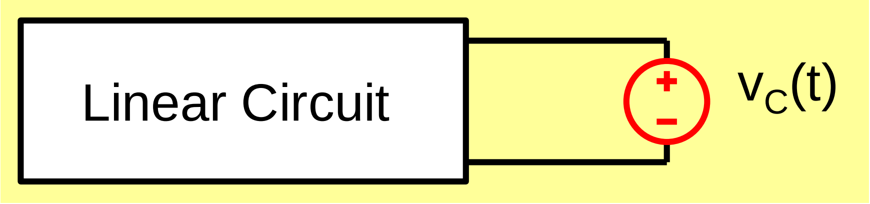


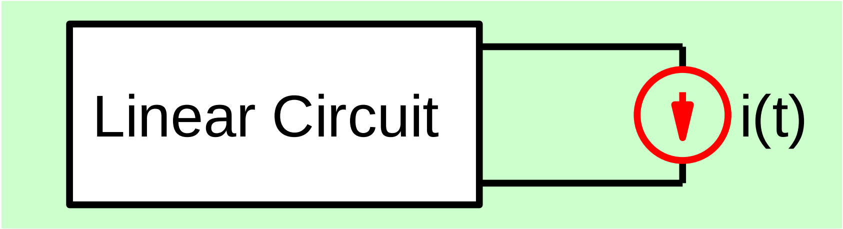
$$\tau = \frac{L}{R_{eq}} \; or \;\; R_{eq}C$$

Voltages & currents inside the circuit?





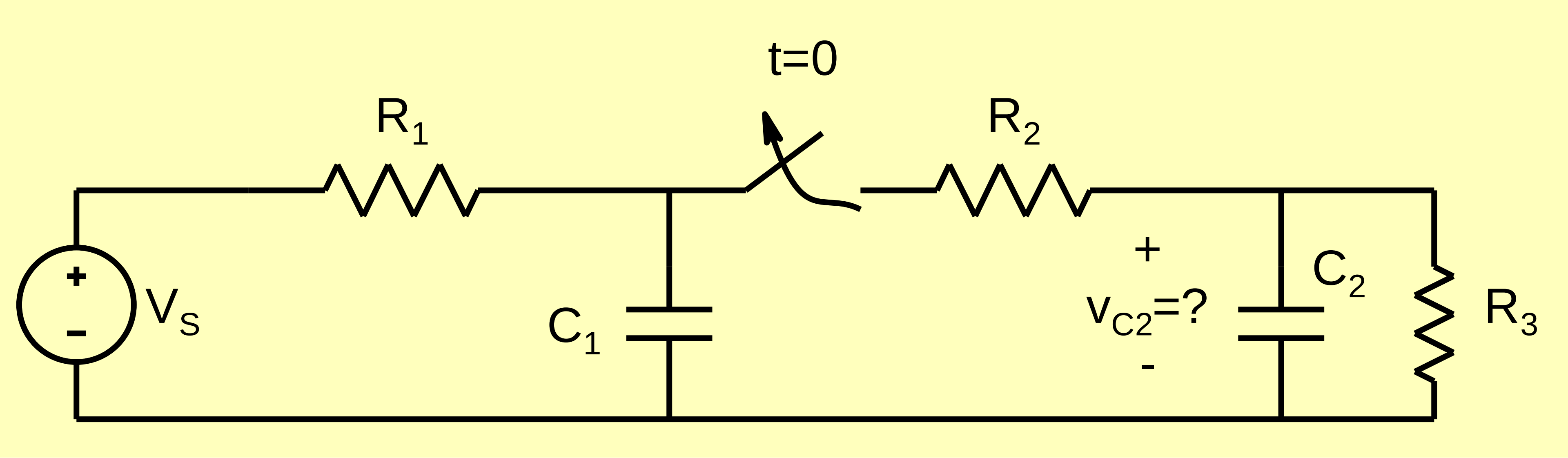


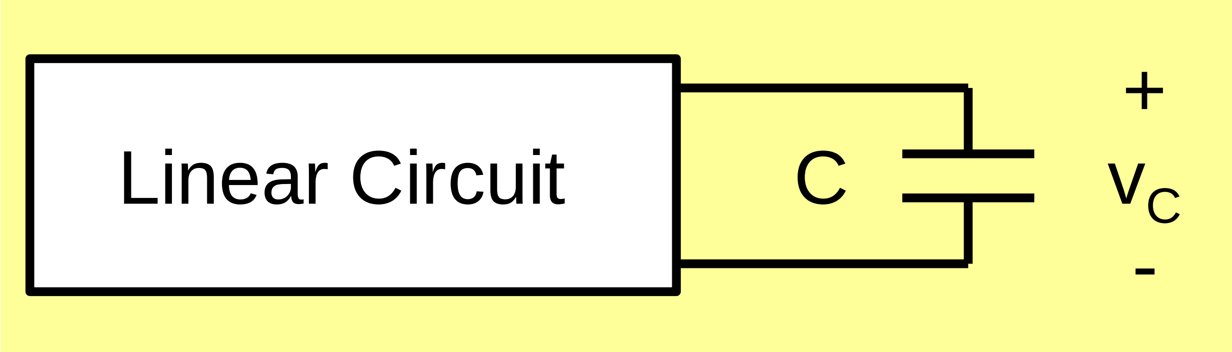


Can Capacitance be Negative?

Why Intel is crazy about Negative Capacitance?

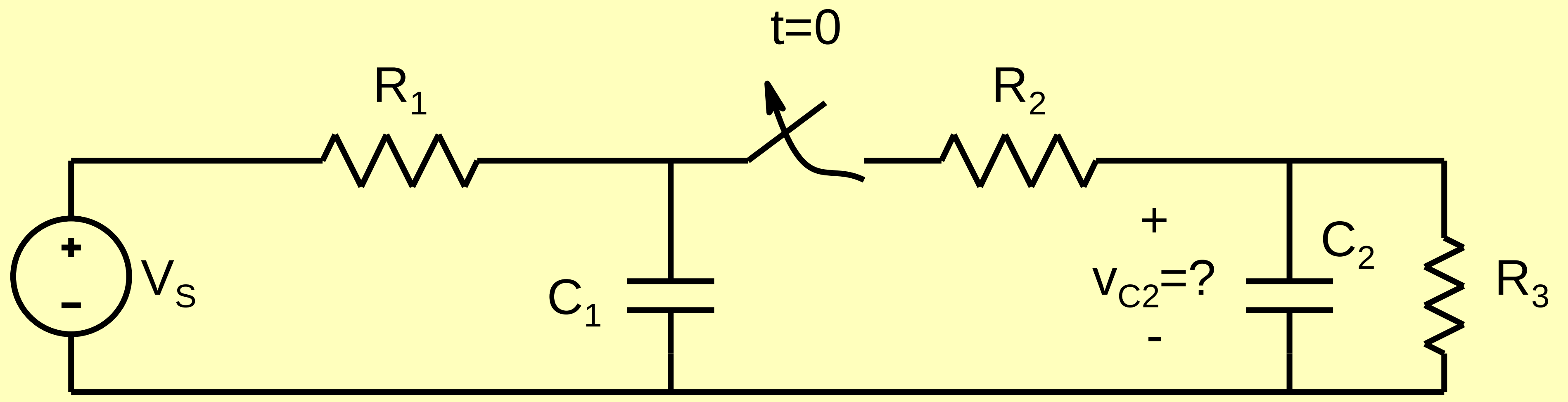
Can we solve this 2 capacitor problem using our present approach?

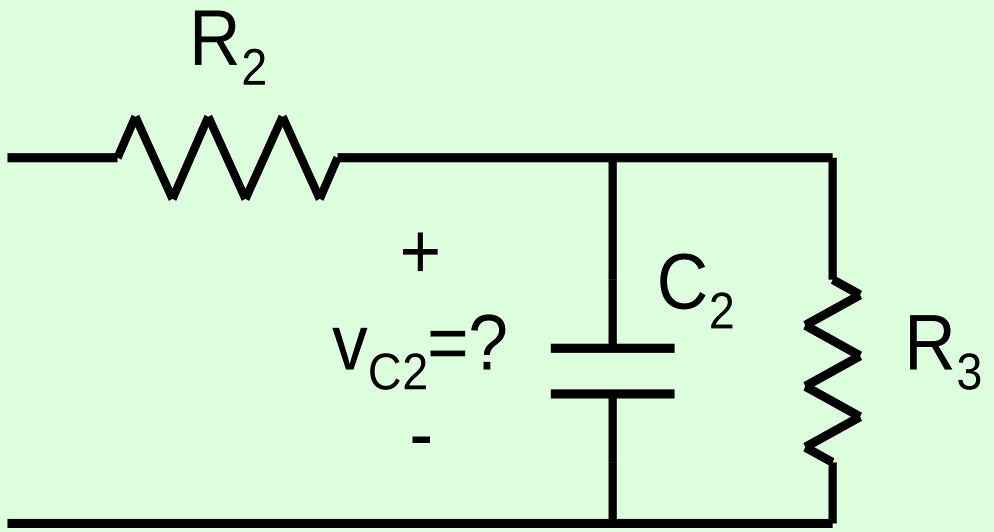




Circuit for t > 0

Can we solve this 2 capacitor problem using our present approach?



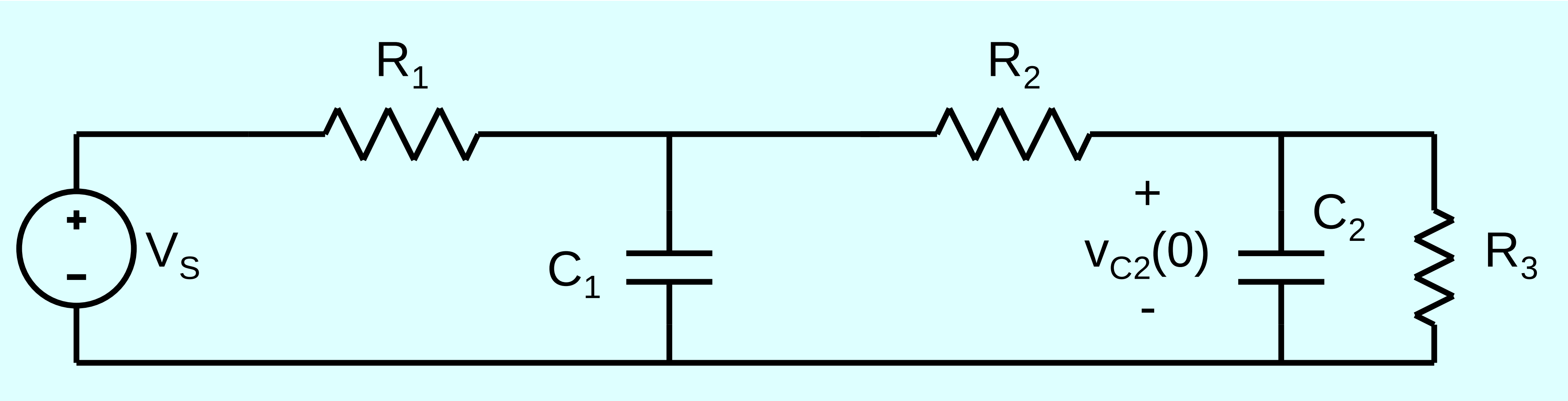


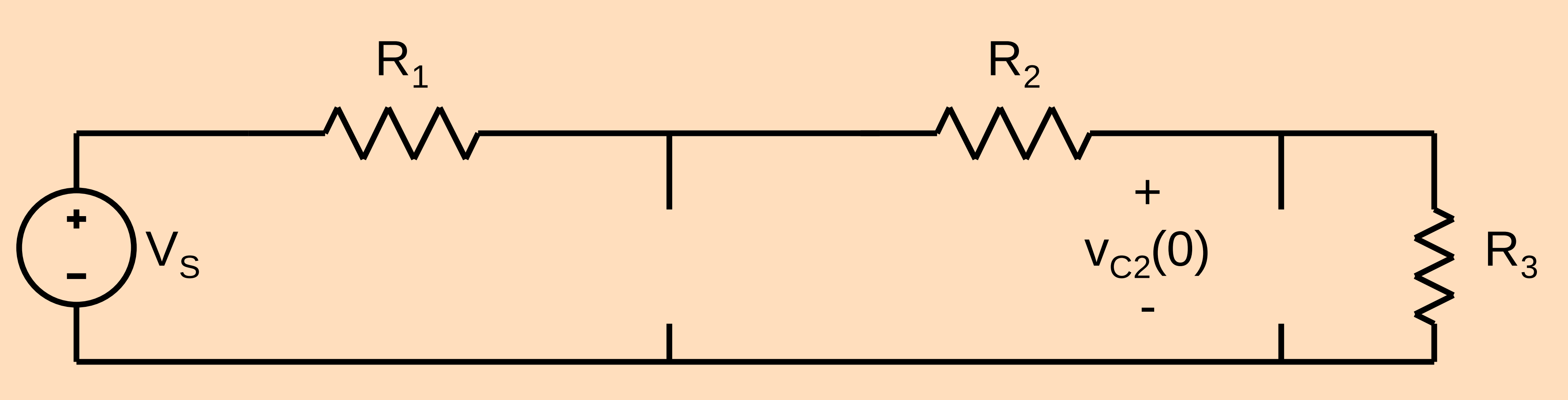
Circuit for t > 0

$$v_{c2}(0^+) = v_{c2}(0^-)$$

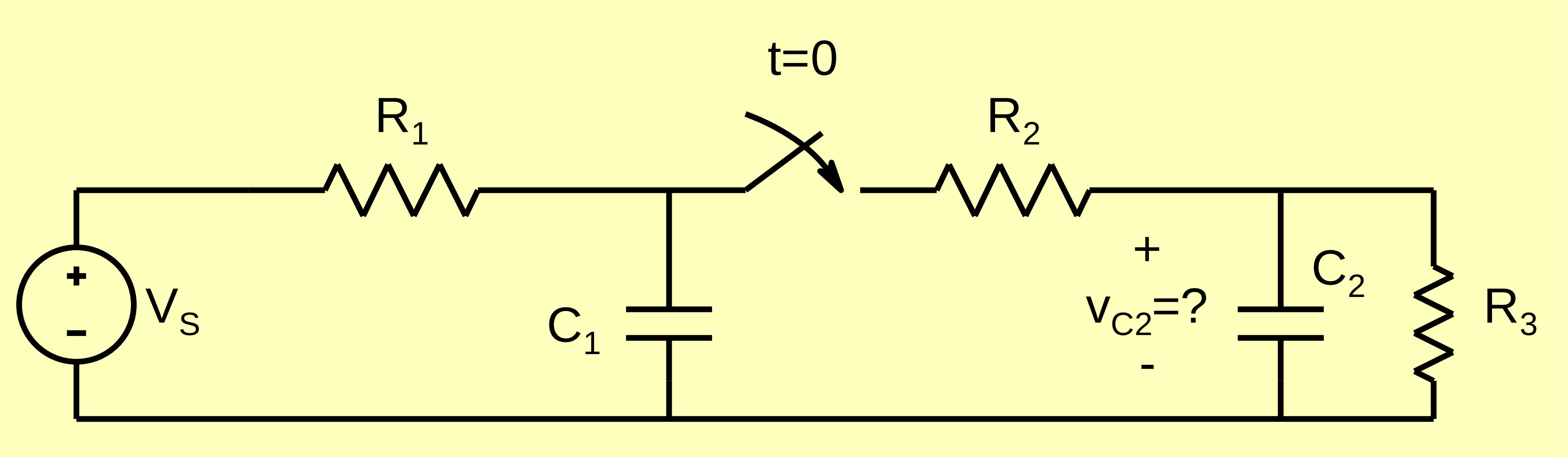
$$v_{c2}(t) = v_{c2}(\infty) + \{v_{c2}(0^+) - v_{c2}(\infty)\}e^{-\frac{t}{\tau}}$$

$$v_{c2}(0^+) = v_{c2}(0^-)$$





Will our approach work here?



No, because circuit for $t > 0$ has two capacitances

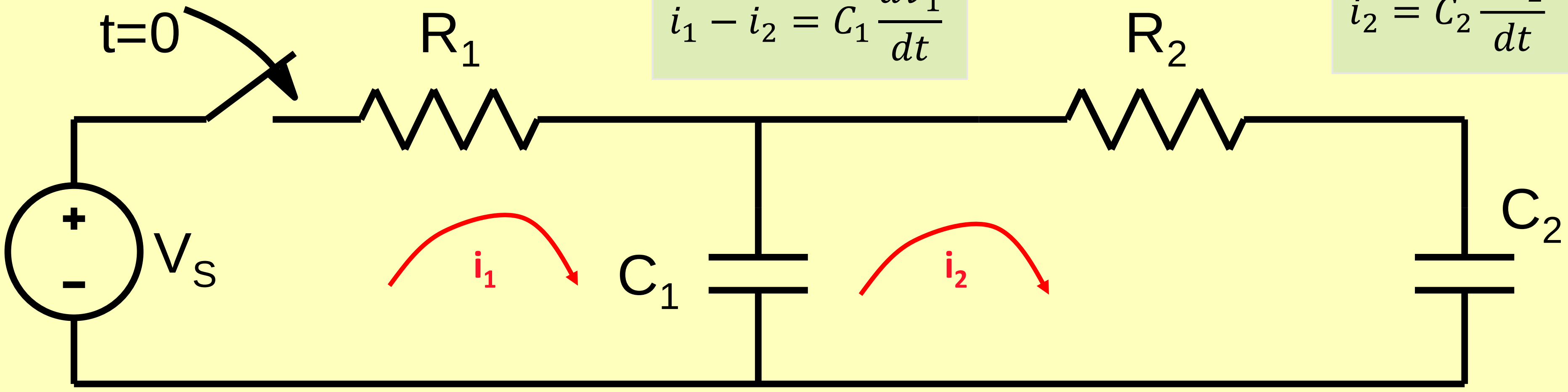
As long as the circuit has single capacitor or inductor for the time interval for which the analysis is being carried out, the stated approach will work fine.

Circuits with Multiple Capacitors

What happens when there is more than one storage element?

$$i_1 - i_2 = C_1 \frac{dv_1}{dt}$$

$$i_2 = C_2 \frac{dv_2}{dt}$$

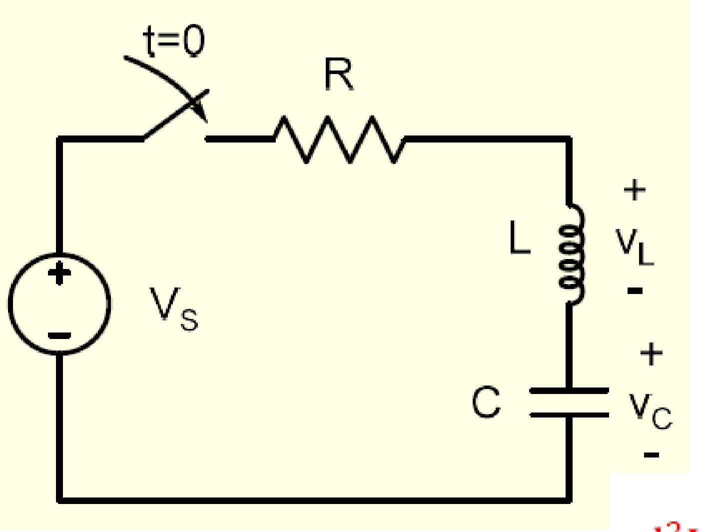


$$V_S = i_1 R_1 + v_{C1}$$

$$v_{C1} = i_2 R_2 + v_{c2}$$

$$R_1 R_2 C_1 C_2 \frac{d^2 v_{c2}}{dt^2} + (R_1 C_1 + R_1 C_2 + R_2 C_2) \frac{d v_{c2}}{dt} + v_{c2} = V_S$$

Series RLC circuit



$$V_S = I \times R + L\frac{dI}{dt} + V_C \qquad I = C\frac{dV_C}{dt}$$

$$V_S = C\frac{dV_C}{dt} \times R + LC\frac{d^2V_C}{dt} + V_C$$

$$\frac{d^2V_C}{dt^2} + \frac{R}{L} \times \frac{dV_C}{dt} + \frac{V_C}{LC} = \frac{V_S}{LC} \qquad V_C(t) = A \times e^{st} \qquad As^2e^{st} + \frac{R}{L}Ase^{st} + \frac{A}{LC}e^{st} = 0$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \qquad s = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

$$\omega_O = \frac{1}{\sqrt{L \times C}} \qquad Q = \frac{\omega_O L}{R} \qquad \frac{s}{\omega_O} = -\frac{1}{2Q} \pm \sqrt{\frac{1}{4Q^2} - 1}$$

How to find constants?

$$V_C(t) = V_S + A \times e^{s_1 t} + B \times e^{s_2 t}$$

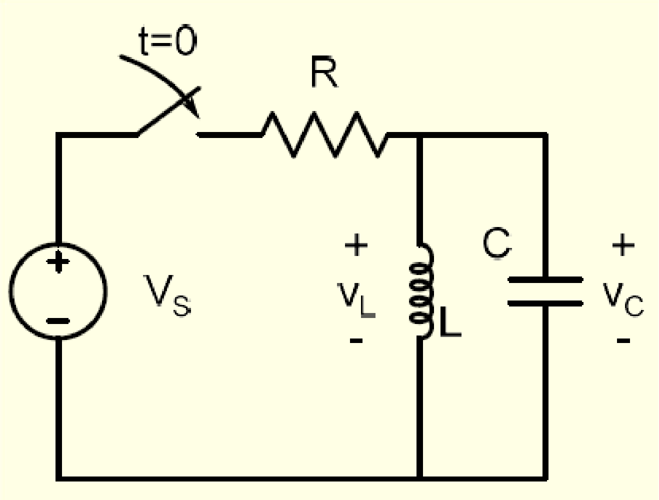
$$\omega_O = \frac{1}{\sqrt{L \times C}} \qquad Q = \frac{\omega_O L}{R} \qquad \frac{s}{\omega_O} = -\frac{1}{2Q} \pm \sqrt{\frac{1}{4Q^2} - 1}$$

$$V_C(0) = V_S + A + B = 0 \qquad (1)$$

$$I_C(t) = C\frac{dV_C(t)}{dt} = C\{V_S + As_1 \times e^{s_1 t} + Bs_2 \times e^{s_2 t}\}$$

$$I_C(0) = 0 = C\{V_S + As_1 + Bs_2\} \qquad (2)$$

Parallel RLC Circuit



$$V_S = I \times R + V_L \qquad I = C\frac{dV_C}{dt} + I_L \qquad V_L = L\frac{dI_L}{dt}$$

$$\frac{V_S}{R} = LC\frac{d^2 I_L}{dt^2} + \frac{L}{R} \times \frac{dV_L}{dt} + I_L$$

$$\frac{d^2 I_L}{dt^2} + \frac{1}{RC} \times \frac{dI_L}{dt} + \frac{I_L}{LC} = \frac{V_S}{RLC}$$

$$I_L(t) = A \times e^{st} \qquad As^2 e^{st} + \frac{1}{RC}Ase^{st} + \frac{A}{LC}e^{st} = 0$$

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \qquad s = -\frac{1}{2RC} \pm \sqrt{\frac{1}{4R^2C^2} - \frac{1}{LC}}$$

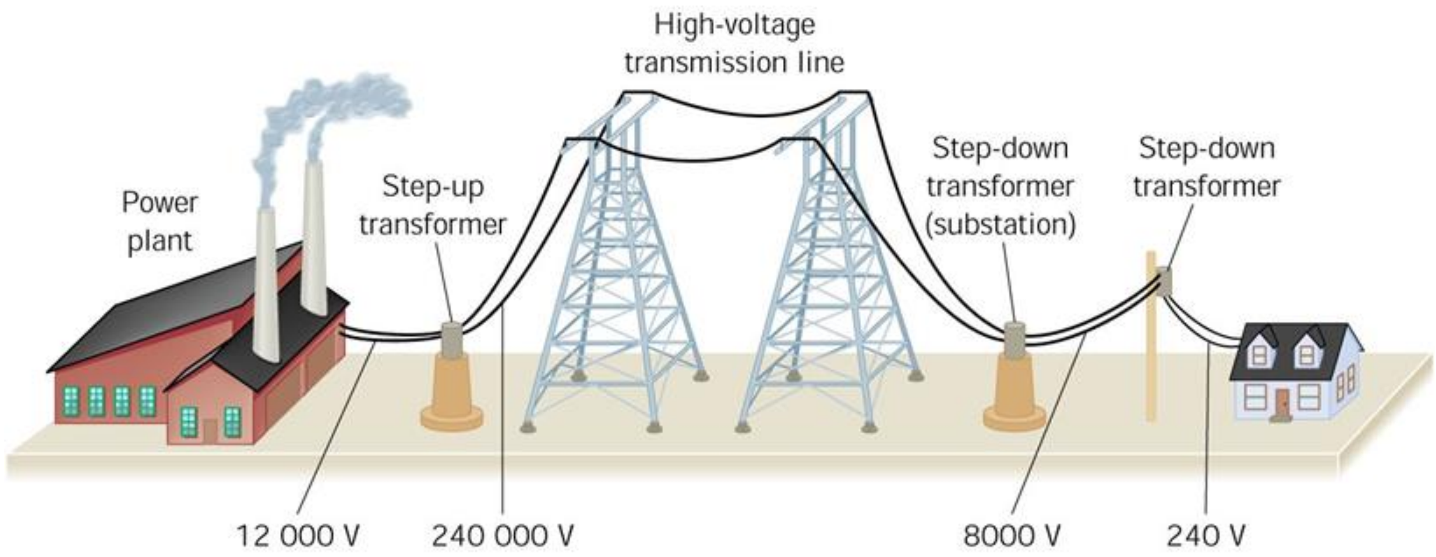
$$\omega_O = \frac{1}{\sqrt{L \times C}} \qquad Q = \frac{R}{\omega_O \times L} \qquad \frac{s}{\omega_O} = -\frac{1}{2Q} \pm \sqrt{\frac{1}{4Q^2} - 1}$$

Sinusoidal Signals

- Appear in many practical applications
  - Electric power is distributed by sinusoidal currents and voltages

$p = v \times i$	$2.2KW = 2.2KV \times 1A$ $2.2KW = 220V \times 10A$

$Loss = i^2 R_{wire}$



Communication

20 Hz -20KHz

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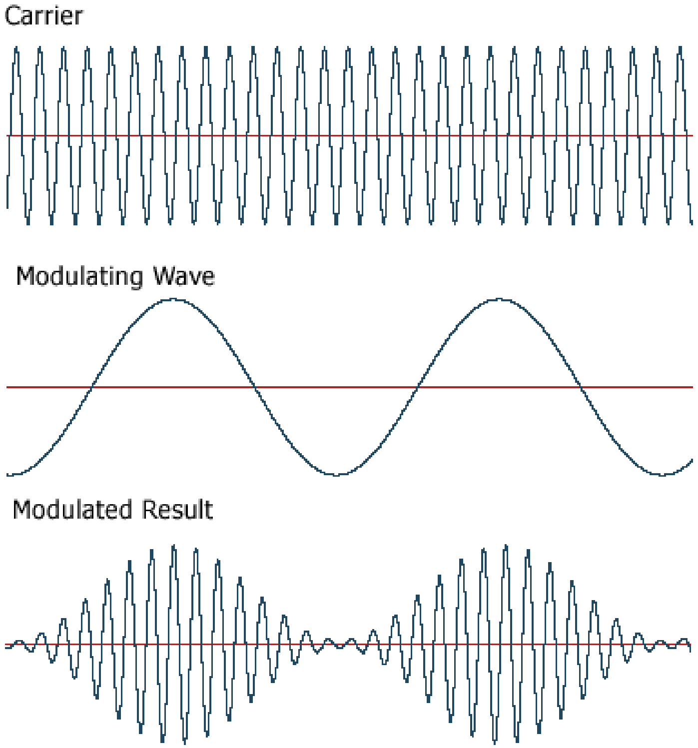
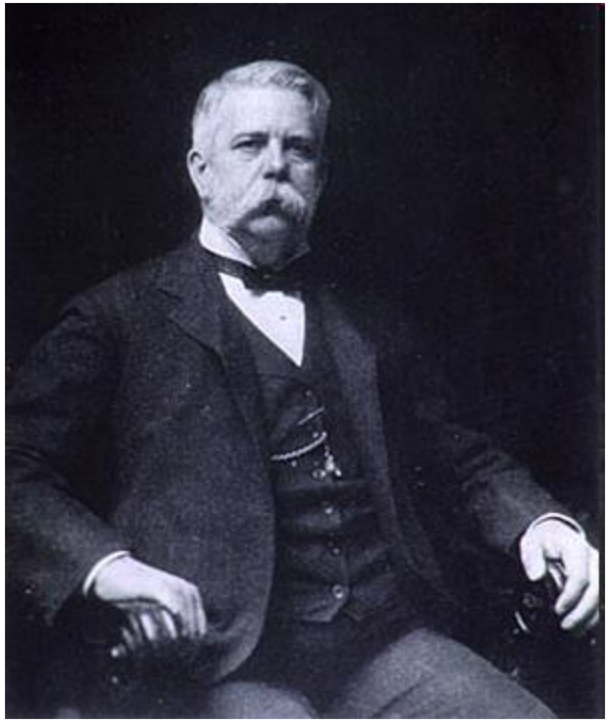


War of currents: AC vs DC

Tell Westinghouse to stick to air brakes. He knows all about them.

-Thomas Edison

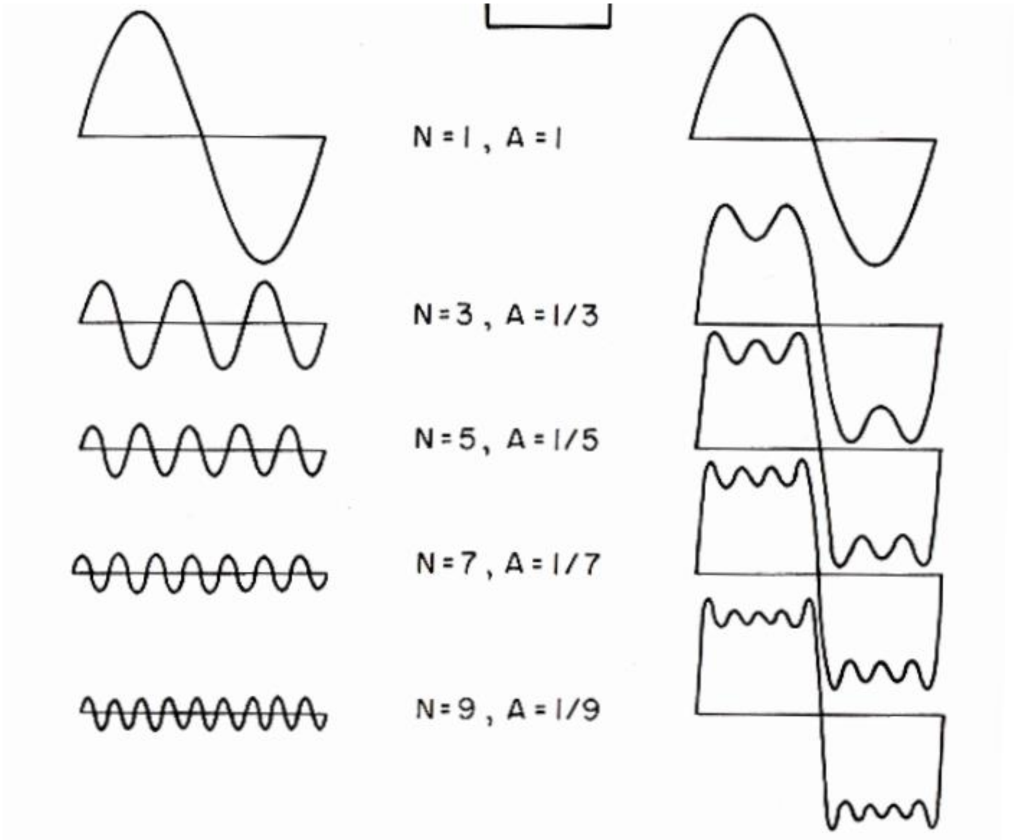


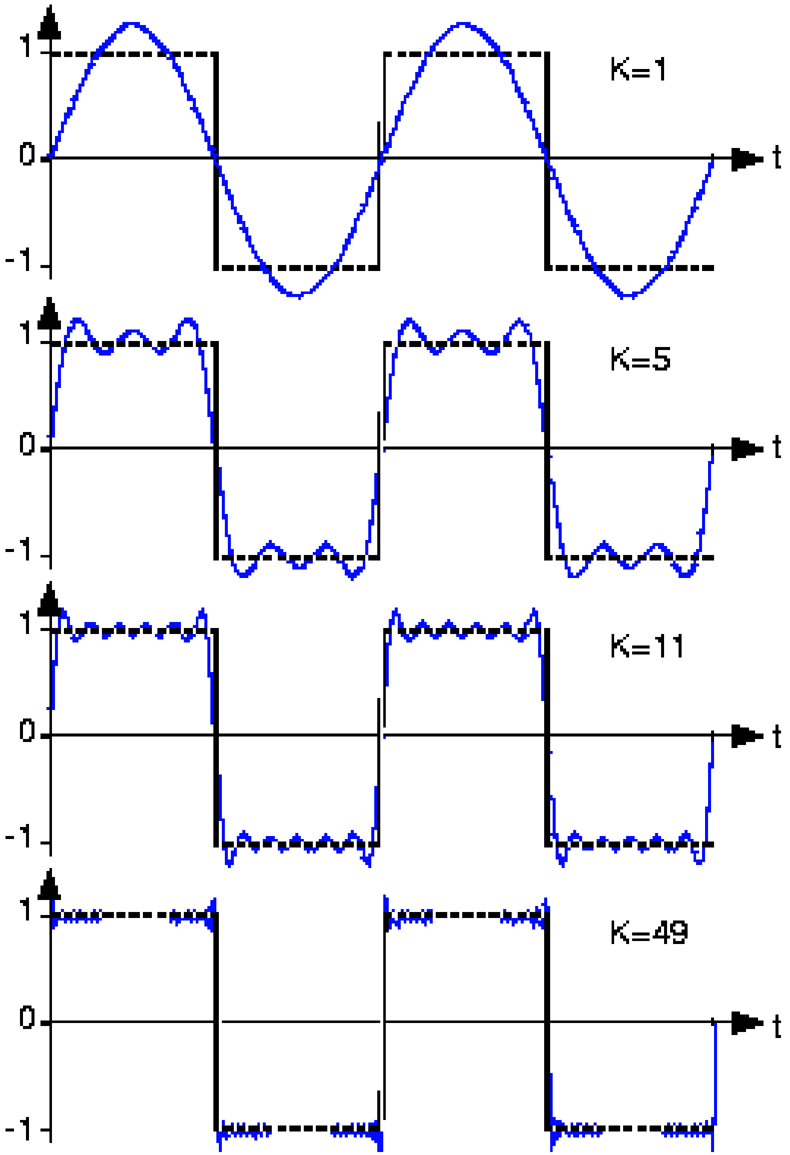
George Westinghouse formed an alliance with Nikola Tesla

https://en.wikipedia.org/wiki/War_of_Currents

Fourier Analysis

$$A\sin\left(\frac{N\pi t}{T}\right) \quad \rightarrow \frac{1}{N}\sin\left(\frac{N\pi t}{T}\right)$$





$$f(t) = \frac{4}{\pi}\sum_{1,3,5}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi t}{T}\right)$$

Sinusoids through Circuits

Sinusoids have following interesting property

- Derivative is a sinusoid
- Integral is a sinusoid

$$\frac{\mathrm{d}}{\mathrm{d}t}\sin\omega t = \cos\omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i_c = C\frac{dv_c}{dt}$$

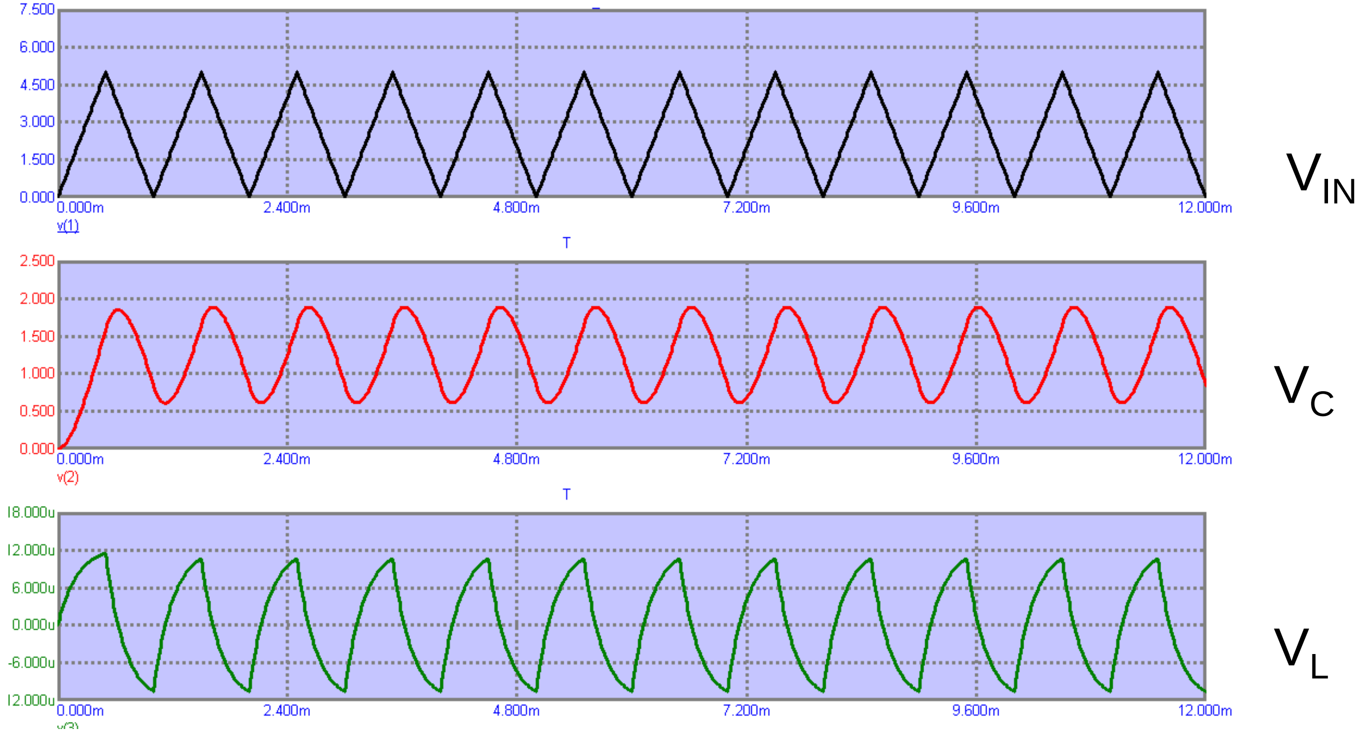
$$\int \sin\omega t\ \mathrm{d}t = -\cos\omega t = \sin\left(\omega t - \frac{\pi}{2}\right)$$

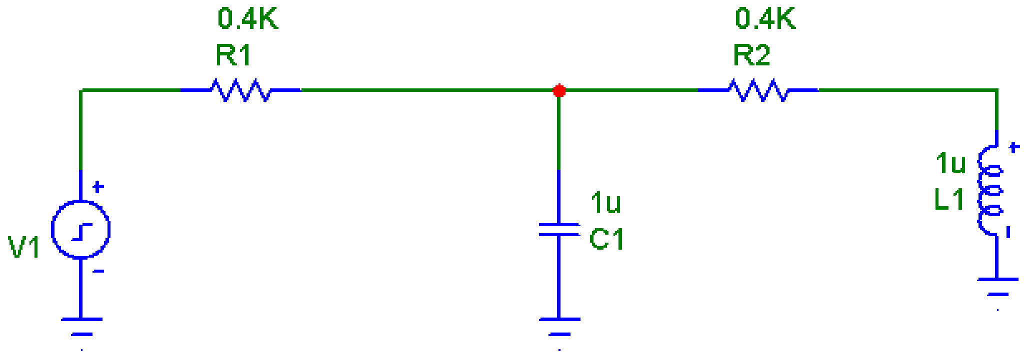
$$v = L\frac{di}{dt}$$

So as a sinusoidal signal goes through a circuit, it remains a sinusoid of the same frequency.

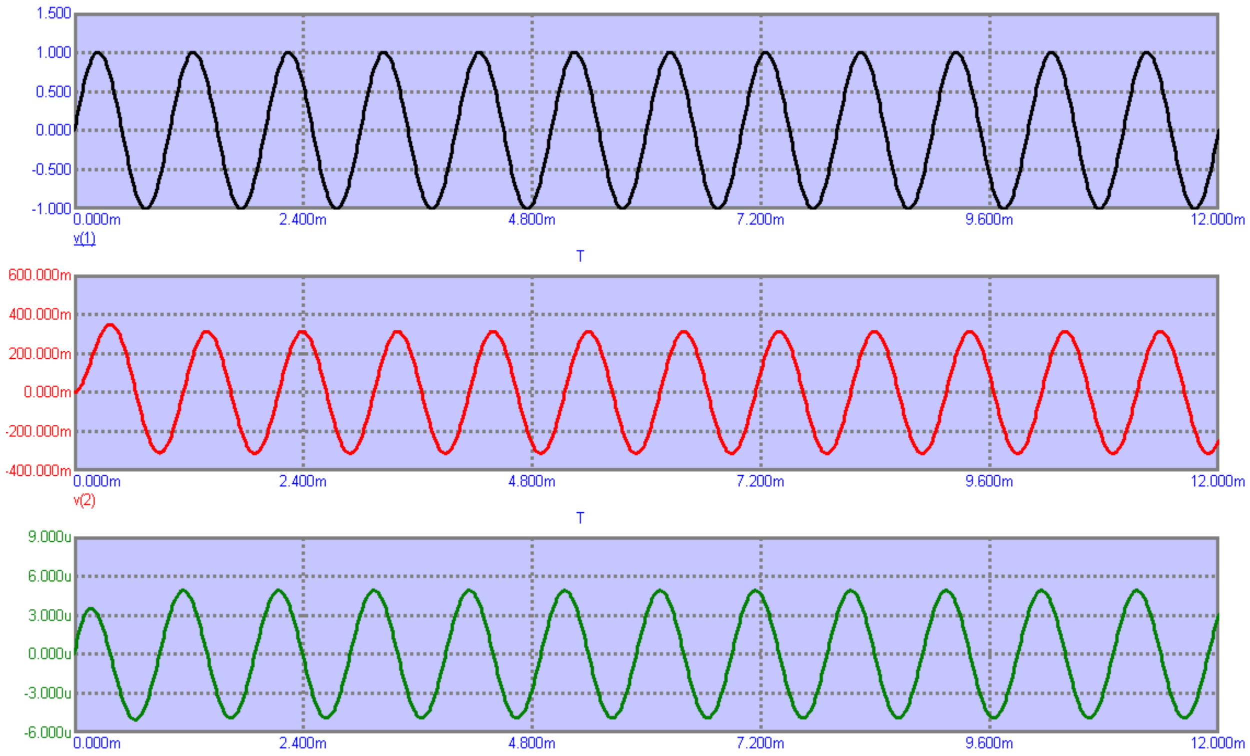
True for any Linear time-invariant system.

Triangular signal





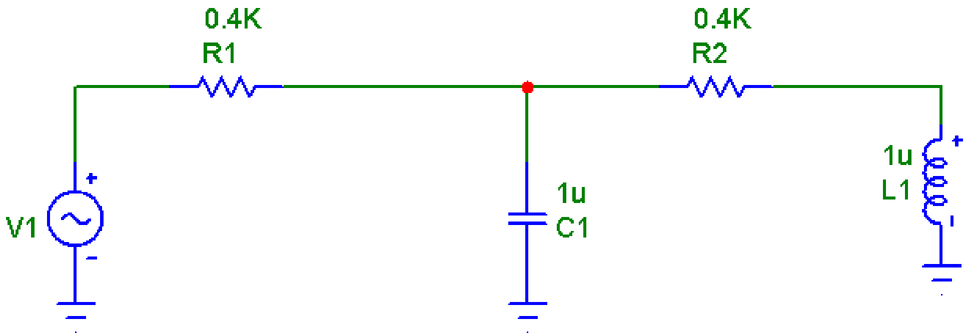
Sinusoidal signal



V_{IN}

V_C

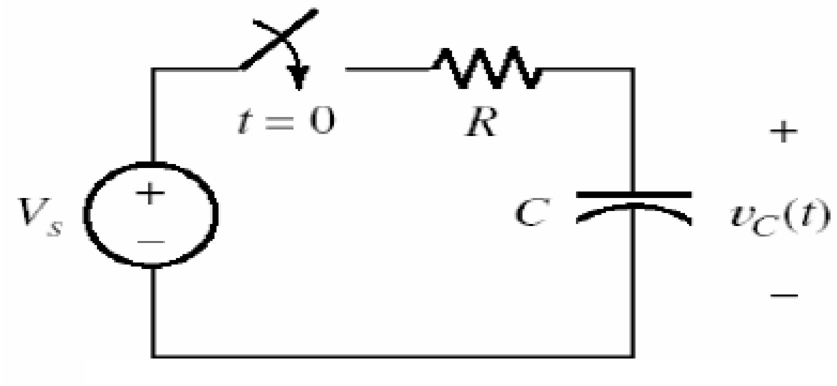
V_L

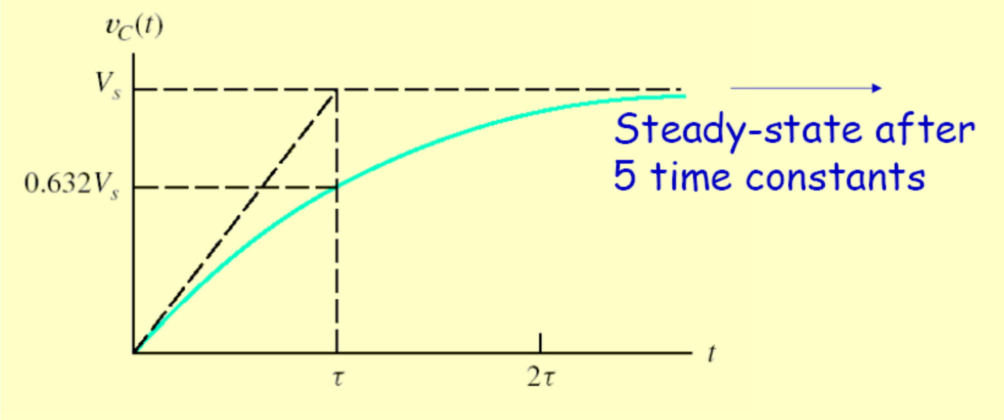


Voltage everywhere in the circuit is sinusoidal

Transient & forced response

- Split solution into two components:
  - Transient (time-dependent component)
  - Forced (steady-state)



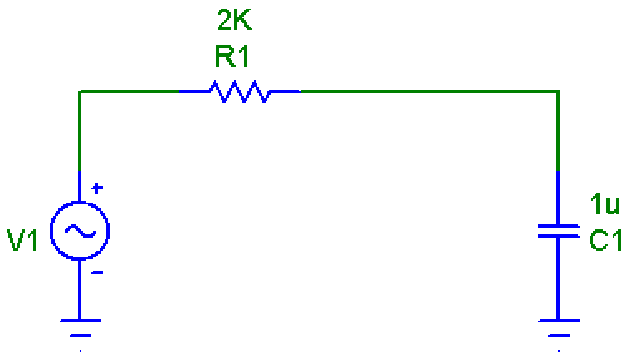


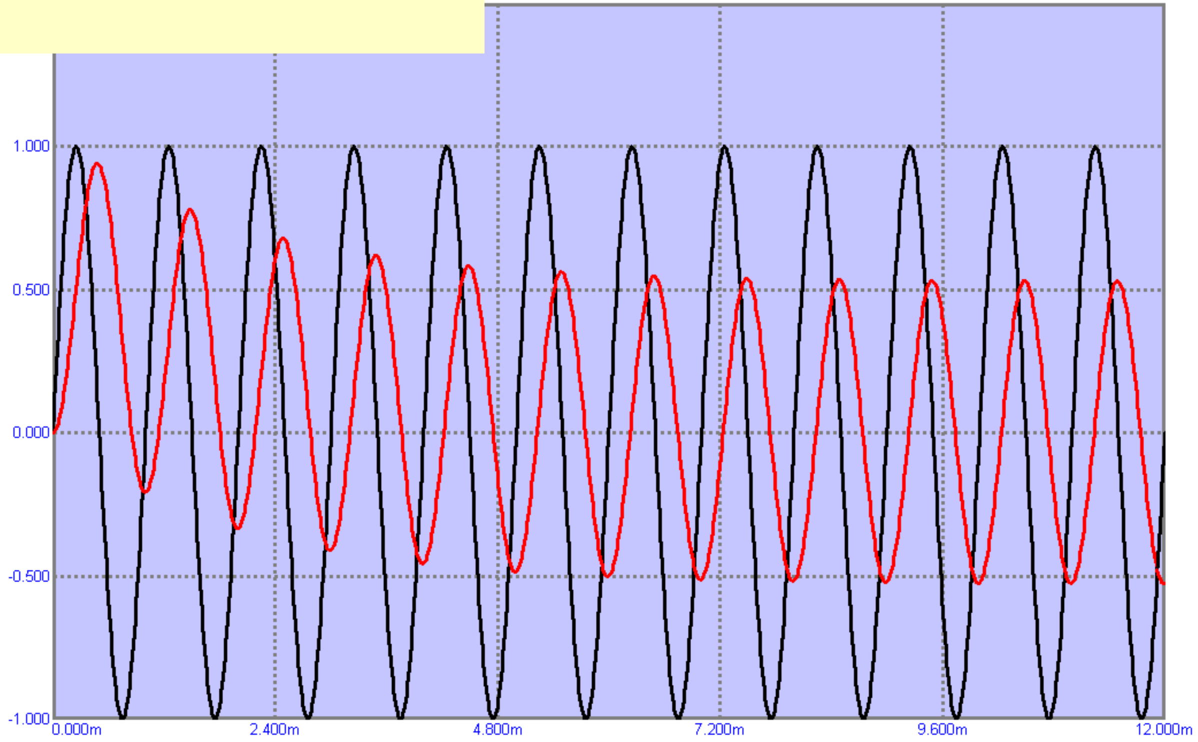
$$v_C(t) = \underbrace{V_s}_{\text{Steady-state or forced response}} - \underbrace{V_s e^{-t/\tau}}_{\text{Transient response}}$$

Sinusoidal steady state

Sinusoidal Steady-State

- Whenever the forced input to the circuit is sinusoidal the response will be sinusoidal
- If the input persists, the response will persist and we call it steady-state response





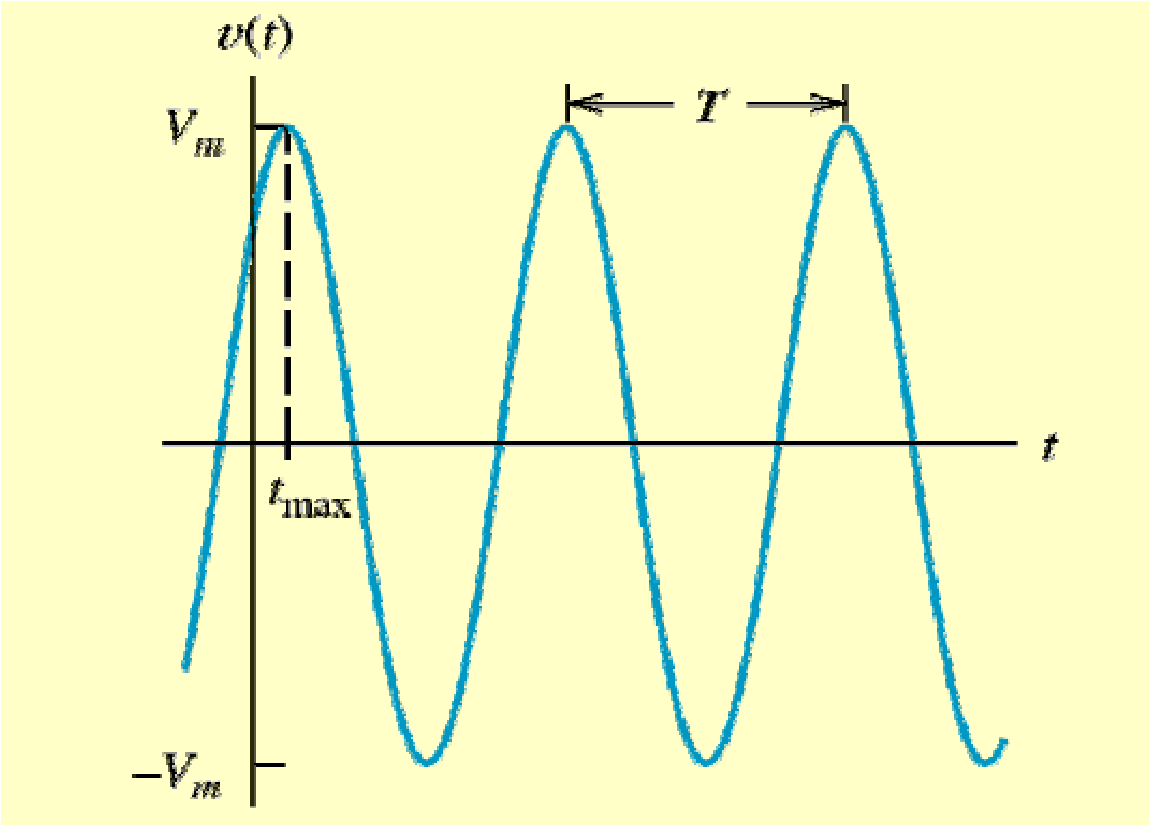
Preliminaries: Representation of Sinusoidal Signals

Canonical Form

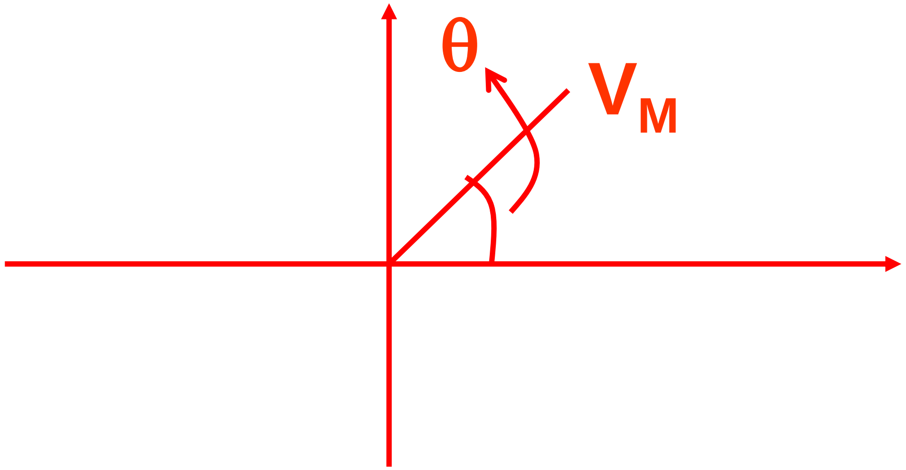
$$v(t) = V_m \cos(\omega t + \theta)$$

peak value

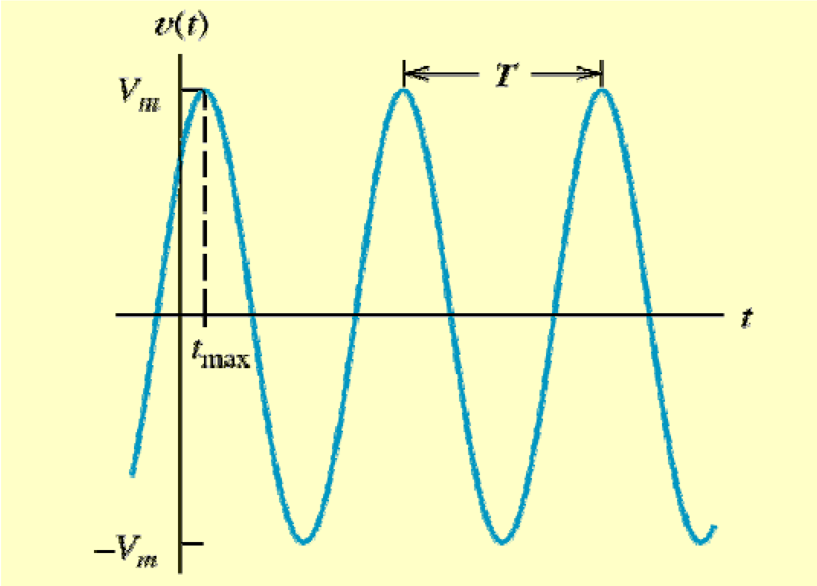
phase



Representation of Sinusoidal Signals



$v(t) = V_m \cos(\omega t + \theta)$



ω is the angular frequency in radians per second

T is the period , where $f = \dfrac{1}{T}$ is the frequency

$\omega = \dfrac{2\pi}{T}$ $\quad\omega = 2\pi f$ $\quad\theta$ is the phase angle

Representation of Sinusoidal Signals…

$$5\sin(4\pi t - 60^o)$$

$$= 5\cos(4\pi t - 60^o - 90^o)$$

$$v(t) = V_m \cos(\omega t + \theta)$$

$$\sin(z) = \cos(z - 90^\circ)$$

Amplitude = 5 ;
Phase = -150°

Phase in radians: $360^o = 2\pi$ $\qquad \theta = \frac{-150}{360} \times 2\pi = -2.618$ radians

$\omega = 4\pi\ rad/s$

$\omega = \frac{2\pi}{T} = 4\pi \Rightarrow T = 0.5s$

$f = \frac{1}{T} = 2Hz$

Preliminaries:

Find the phase difference between the two currents

$$i_1 = 4\sin(377t + 25^o)$$

$$i_2 = -5\cos(377t - 40^o)$$

Canonical Form $x(t) = x_m \cos(\omega t + \theta)$

$$i_1 = 4\cos(377t + 25^o - 90^o)$$

$$\theta_1 = -65^o$$

$$i_2 = 5\cos(377t - 40^o + 180^o)$$

$$\theta_2 = 140^o$$

$$\theta_1 - \theta_2 = -205^o$$

$$\sin(\omega t \pm 180^o) = -\sin \omega t$$

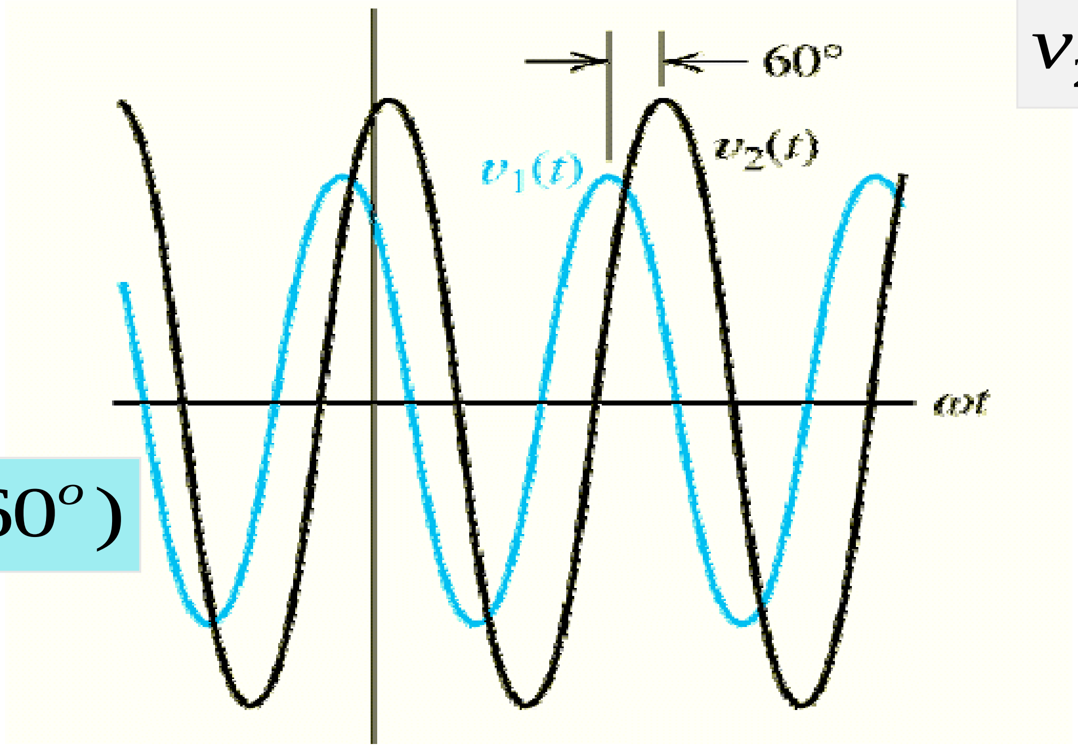
$$\cos(\omega t \pm 180^o) = -\cos \omega t$$

$$\sin(\omega t \pm 90^o) = \pm\cos \omega t$$

$$\cos(\omega t \pm 90^o) = \mp\sin \omega t$$

Phase relationship

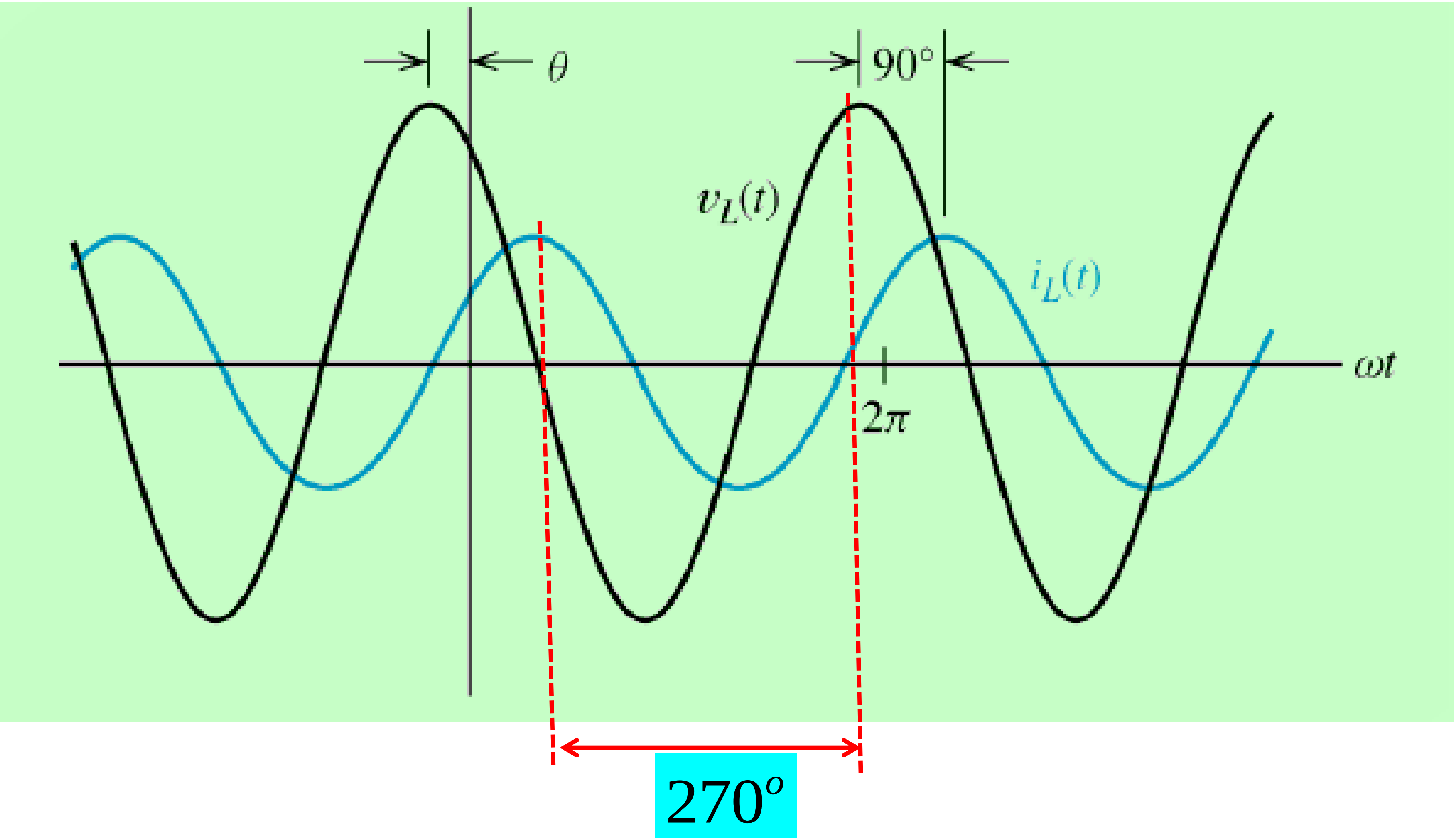
$$v_2(t) = v_{2m} \cos(\omega t)$$



$$v_1(t) = v_{1m} \cos(\omega t + 60^o)$$

The peaks of $v_1(t)$ occur 60^{o} before the peaks of $v_2(t)$.

In other words, $v_1(t)$ leads $v_2(t)$ by 60^{o} .



Voltage leads current by 90° or lags current by 270° ?

Phase difference is usually considered between -180 to 180°

Add or subtract 360° to bring the phase between -180 to 180°

$$i_1 = 4\cos(377t - 65^o) \qquad i_2 = 5\cos(377t + 140^o)$$

Does i_2 lead i_1 ?

$$\theta_1 - \theta_2 = -205^o$$

$$\theta_1 - \theta_2 = -205^o + 360^o = 155^o$$

i_1 leads i_2 by 155º