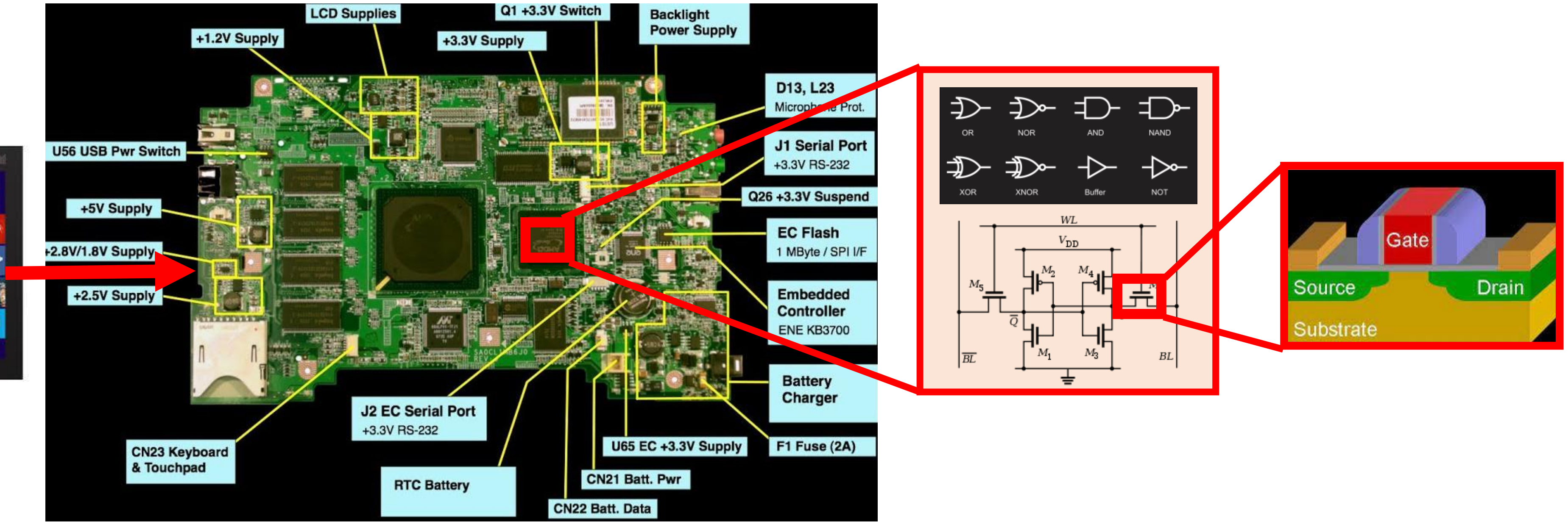

ESC201: Introduction to Electronics

Module 3: Frequency Domain Analysis

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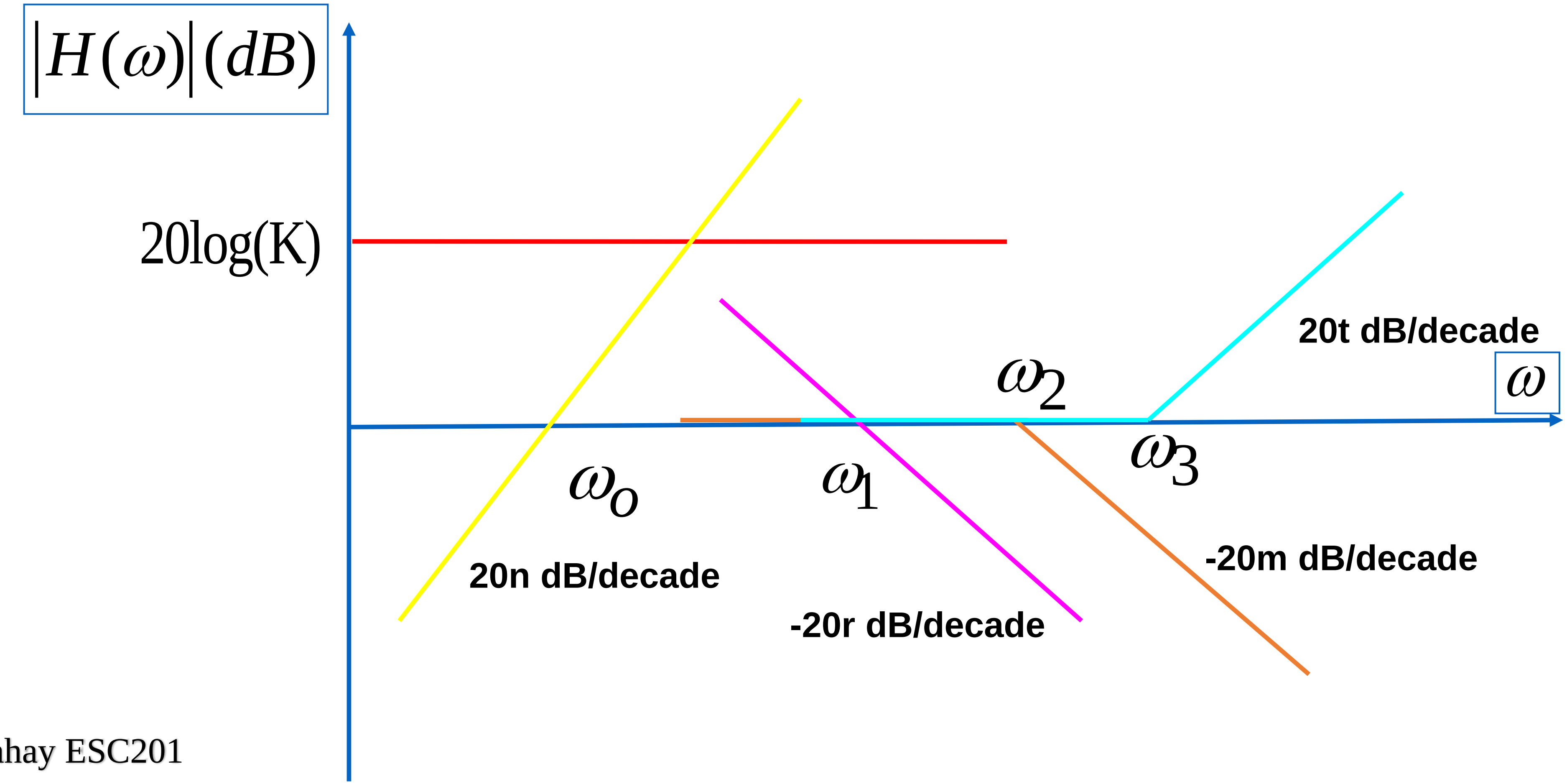
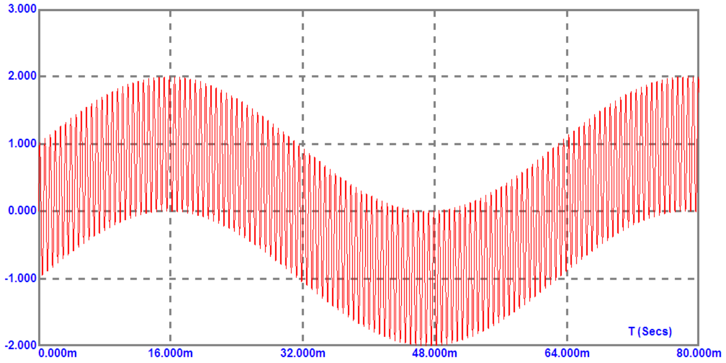
Department of Electrical Engineering,
IIT Kanpur

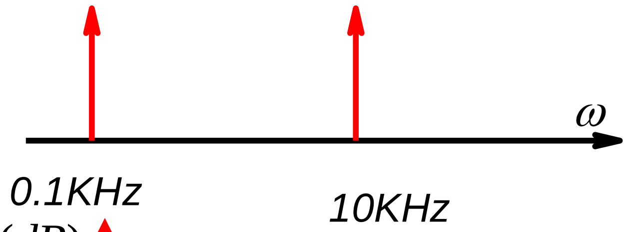
Bode Plot

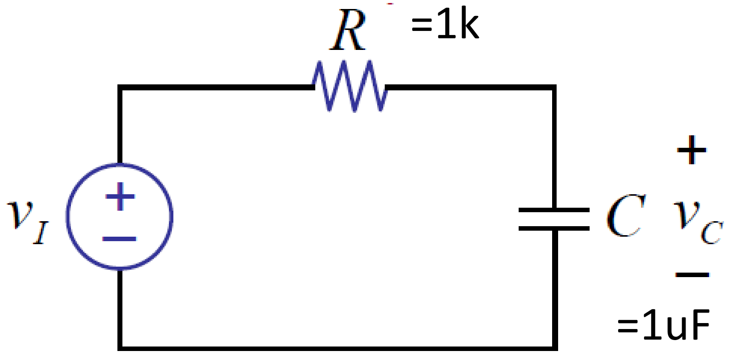
$$H(\omega) = K \times j(\omega/\omega_o)^n \times \frac{1}{j(\omega/\omega_1)^r} \frac{1}{\{1+j(\omega/\omega_2)\}^m} \times \{1+j(\omega/\omega_3)\}^t$$



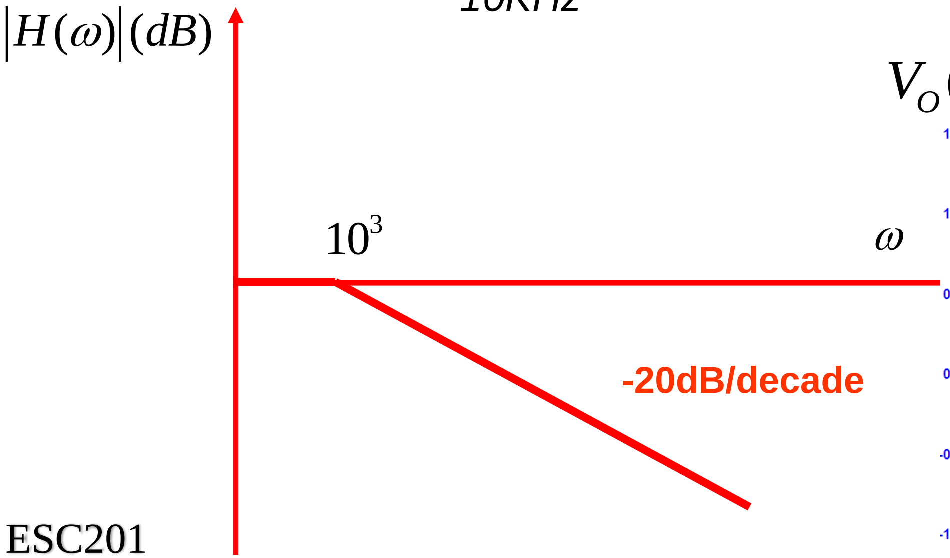
Recall Example



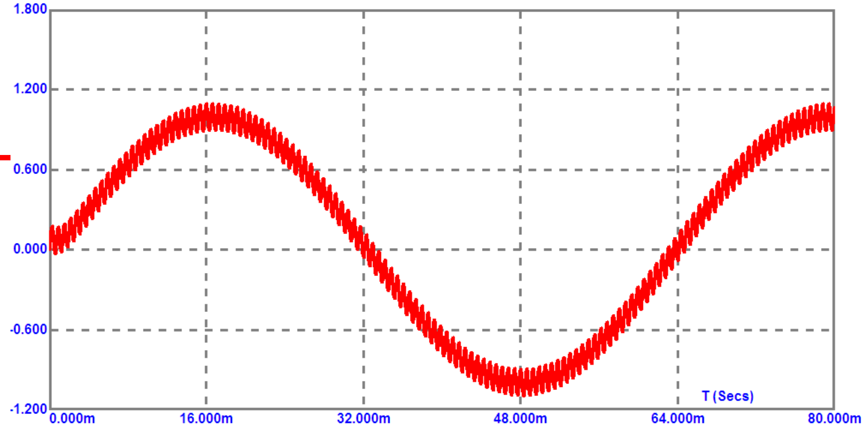




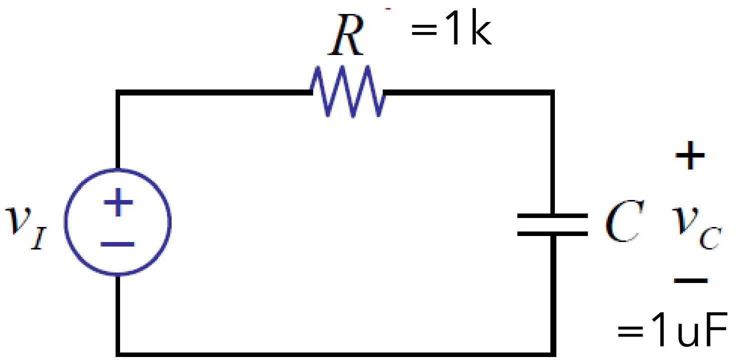
$$H(\omega) = \frac{1}{1 + j\omega 10^{-3}} = \frac{1}{1 + j\dfrac{\omega}{10^3}}$$

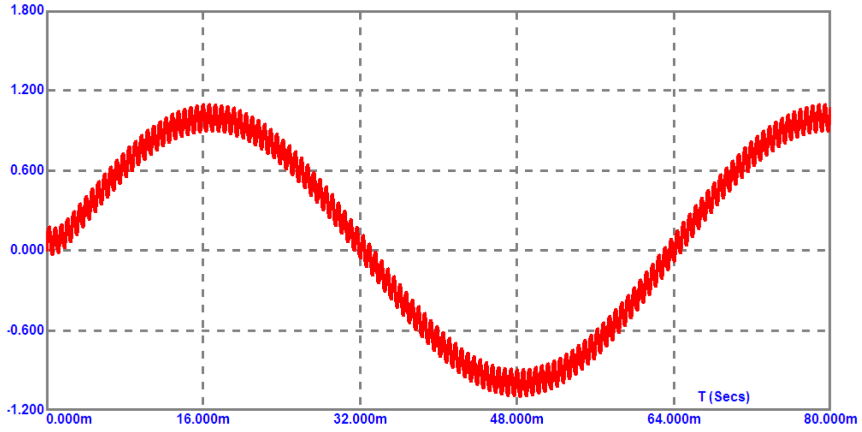


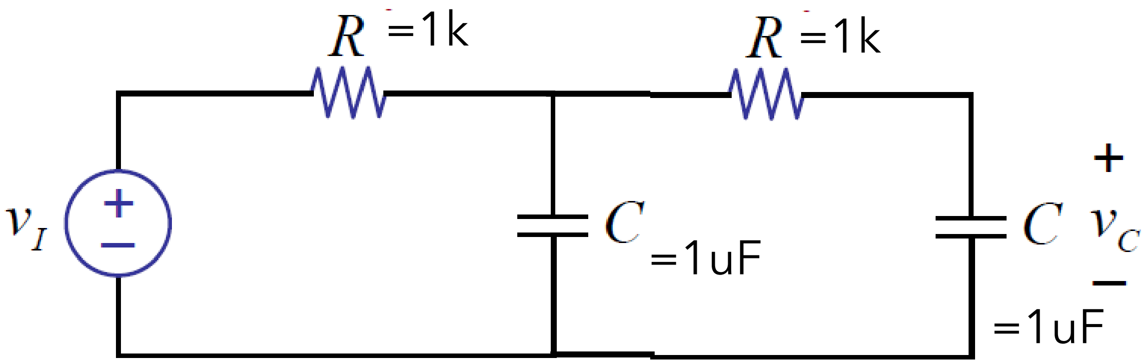
$$V_O(t) = 1Sin(100t) + 0.1Sin(10^4 t)$$

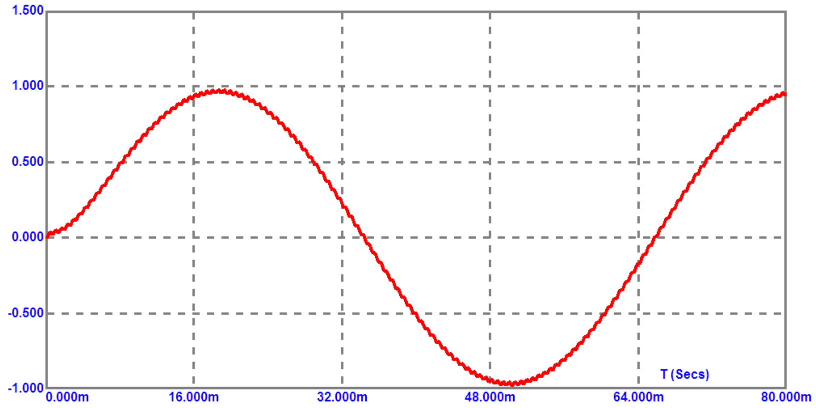


Example: low pass filter…

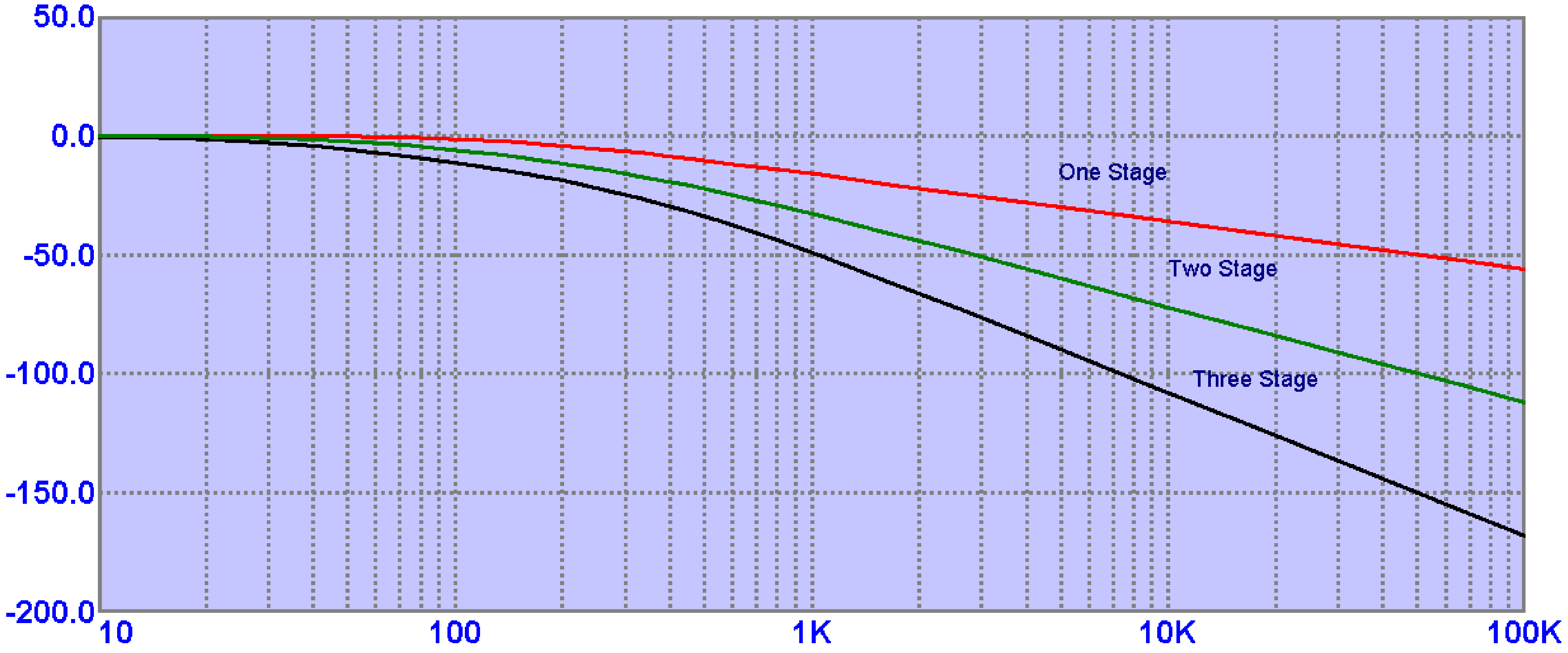




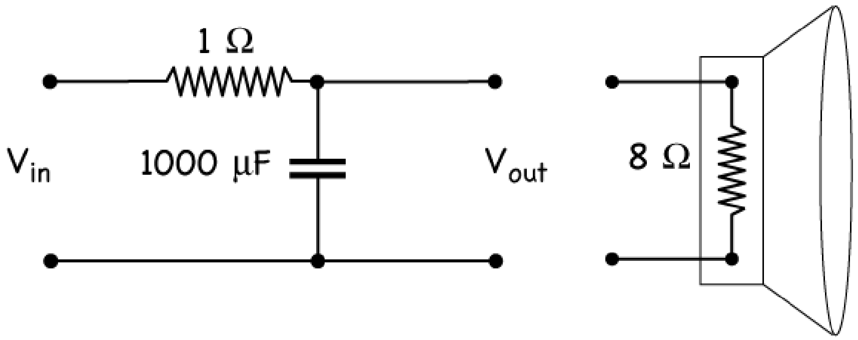




Adding more RC stages, makes the characteristics sharper

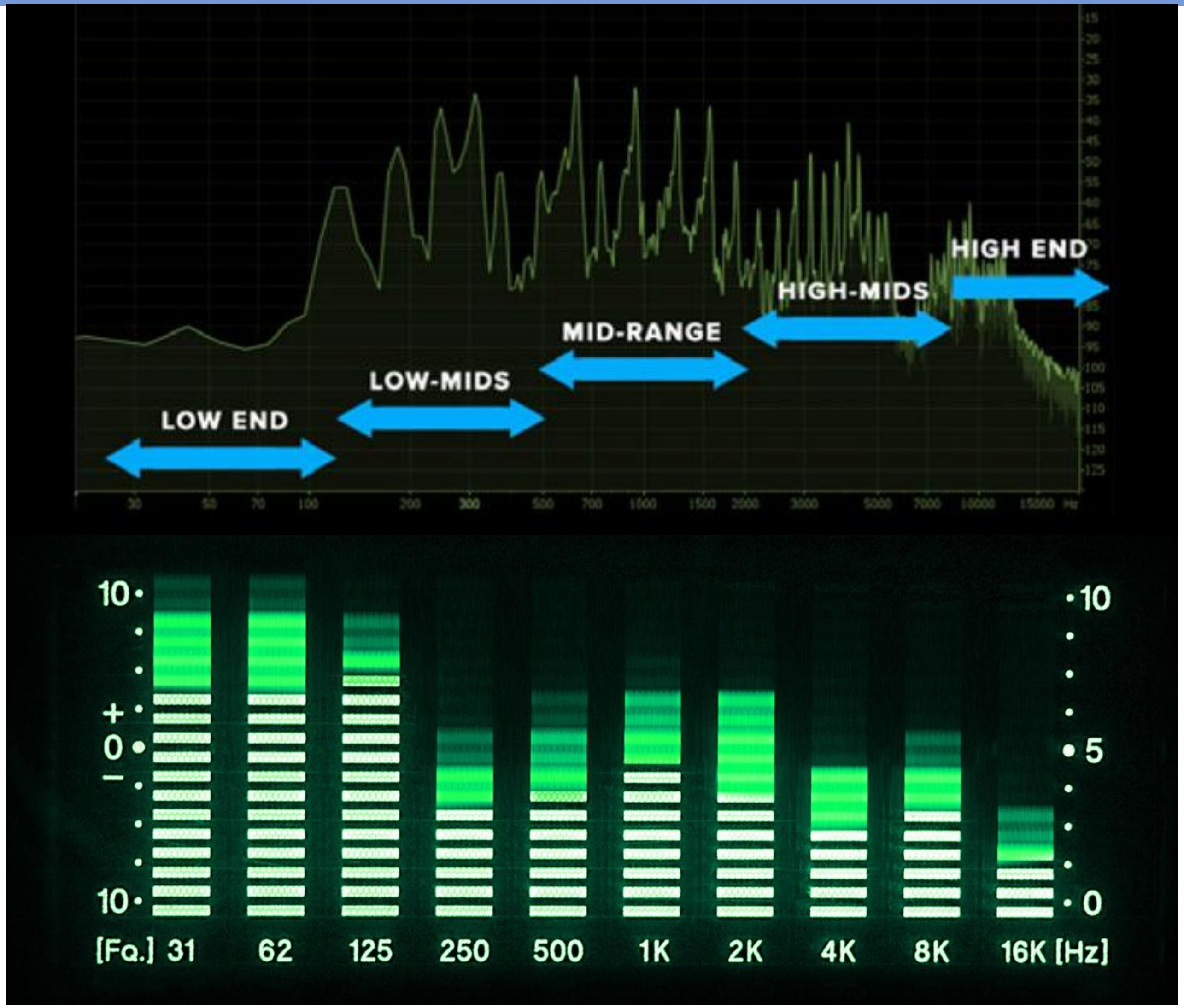


Filters



Low-Pass Audio Filter

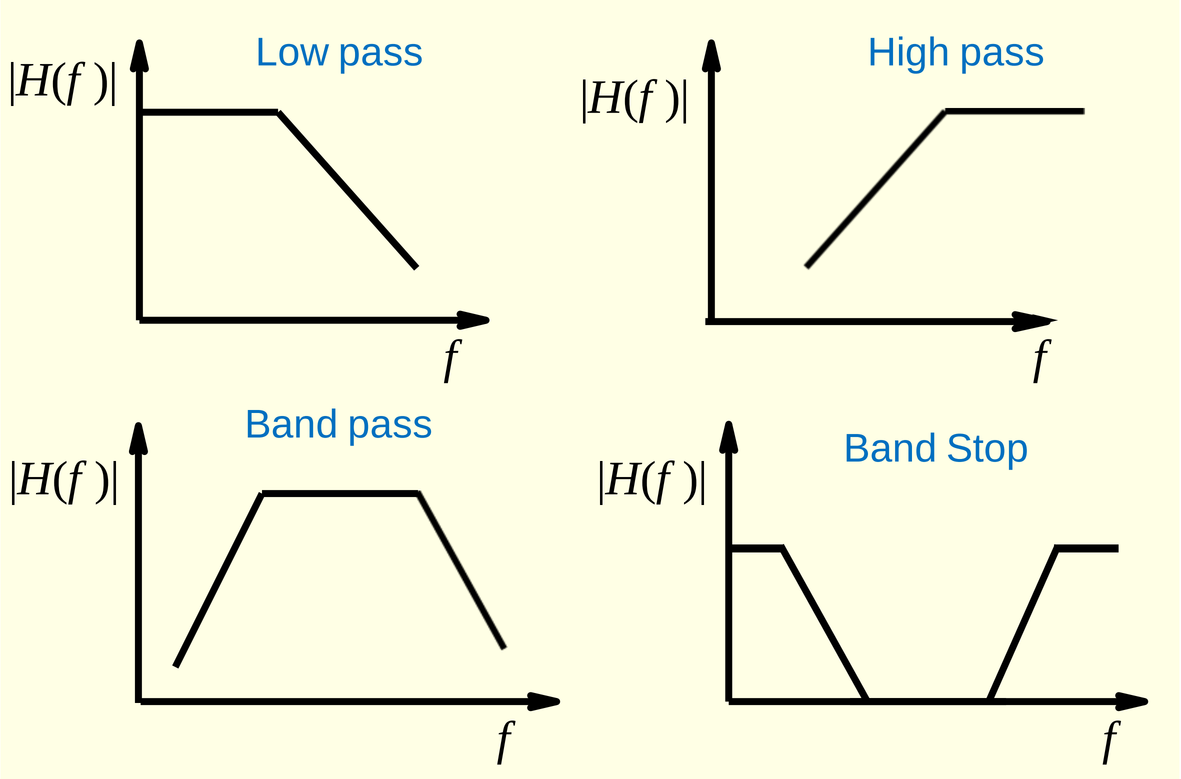
Removing Noise from Images



Equalization of Sound

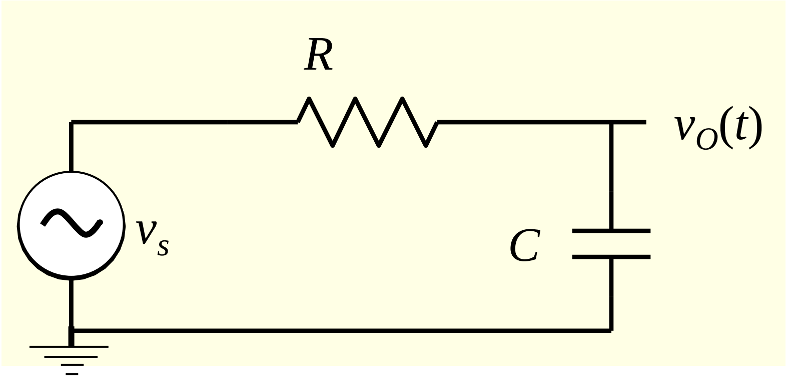
Filters for Electrical Signals

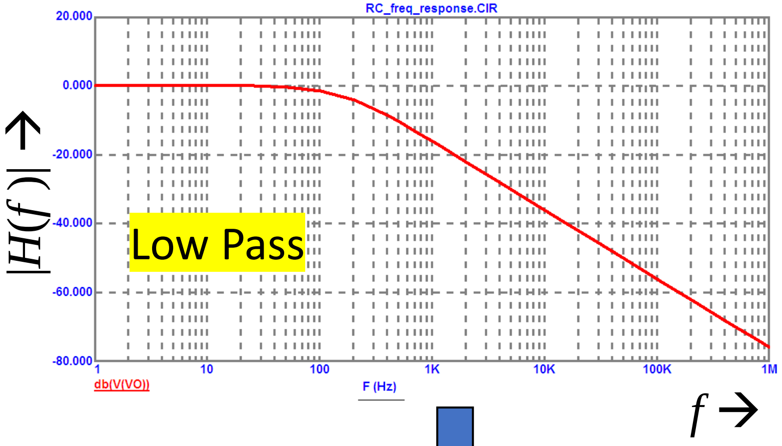
Filter – pass / amplify signal in a band of frequency and reject / attenuate the remaining

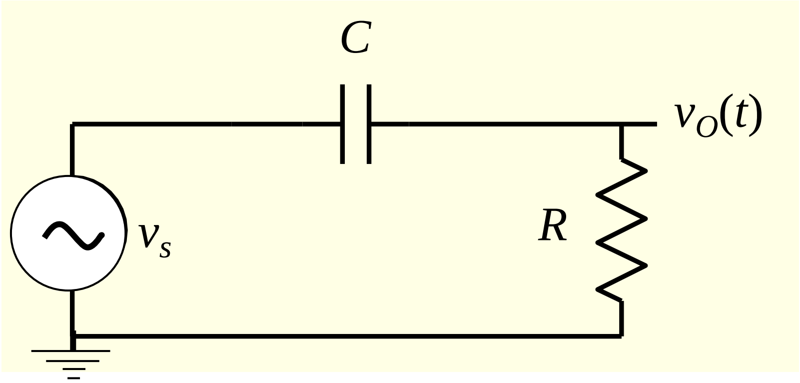


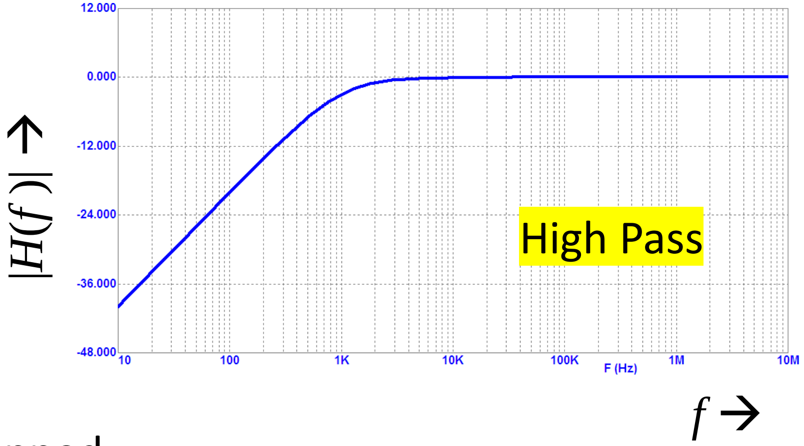
- Many practical applications in electronics and electrical systems
- Tuning radios, cleaning up communication signals, removing higher frequencies from power systems, … + many other areas

RC Filters



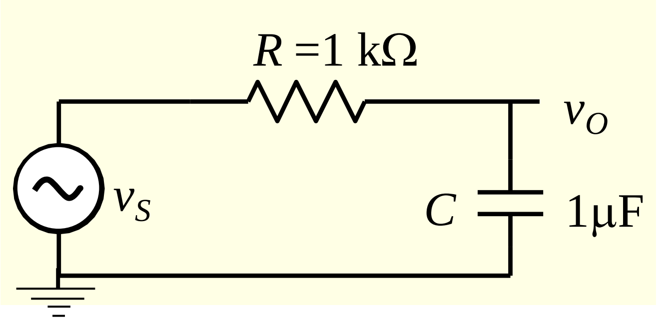




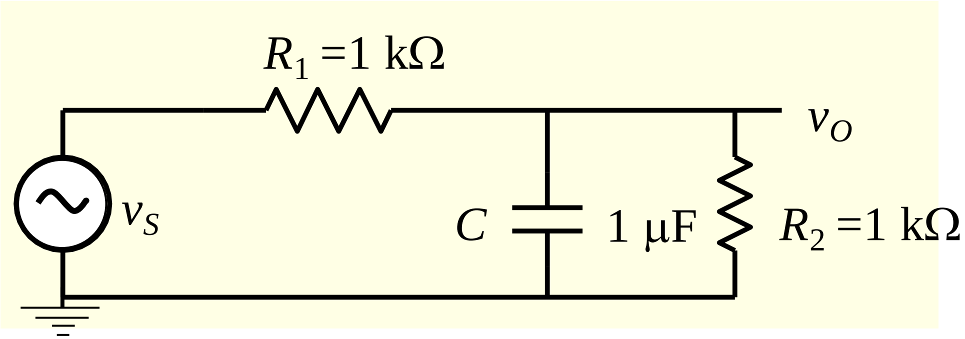


- Capacitor and resistor positions are swapped
- Low pass becomes high pass filter!
- In both cases $\omega_{3dB} = (R \cdot C)^{-1} \rightarrow f_{3dB} = (2\pi \cdot R \cdot C)^{-1}$

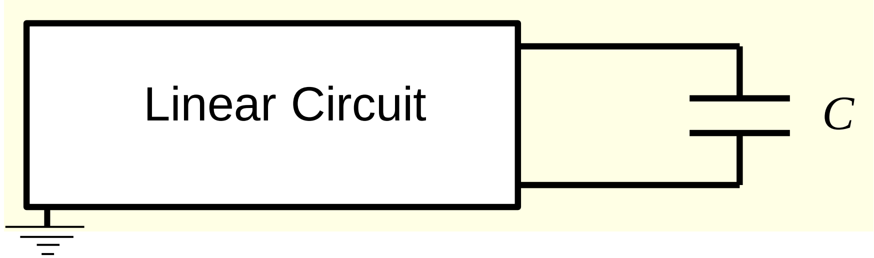
Single Capacitor Circuit $\omega_{3\mathrm{dB}}$



$$\omega_{3\mathrm{dB}} = \frac{1}{RC} = 10^3\, rad/s$$



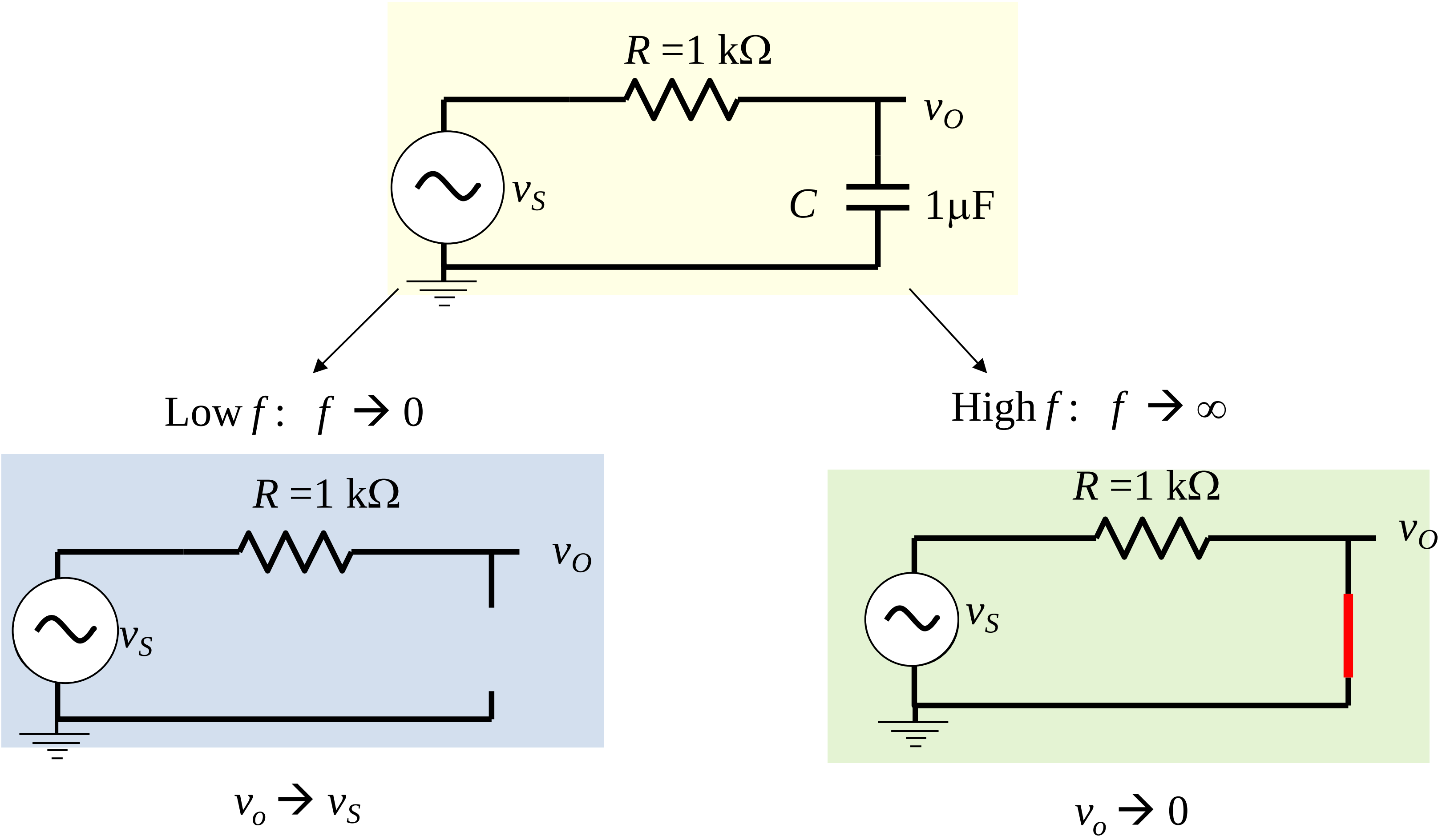
$$\omega_{3\mathrm{dB}} = \frac{1}{R_1 \| R_2\, C}$$



$$\omega_{3\mathrm{dB}} = \frac{1}{\tau} = \frac{1}{R_{\mathrm{eq}} C}$$

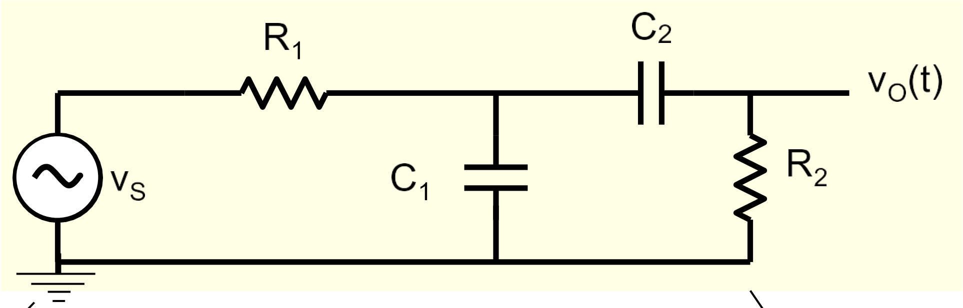
Find Thévenin resistance across the capacitance to find R_{eq} in time constant

Asymptotic Behaviour

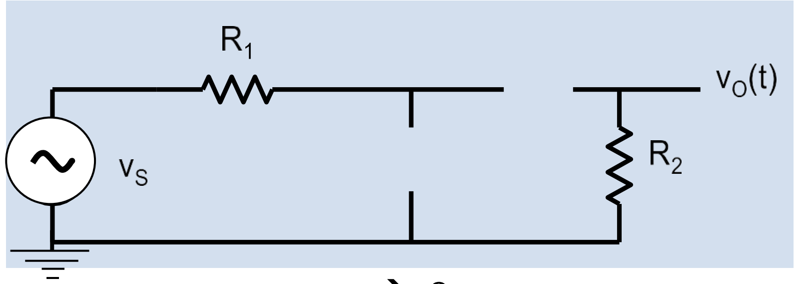


Clear hints of a low-pass filter behaviour!

Circuit Attenuating High and Low Frequency

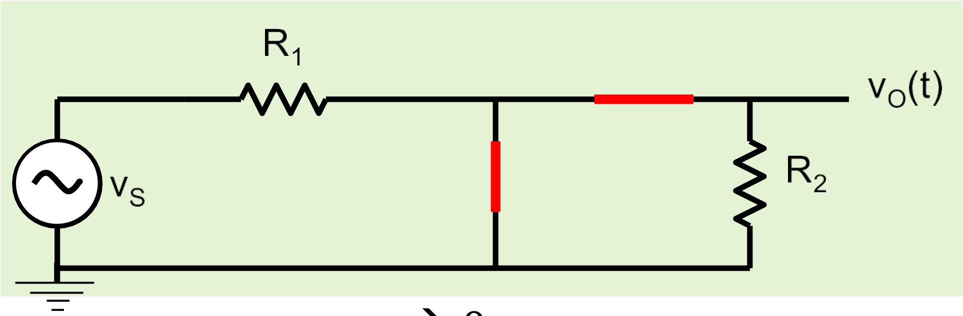


Low f: $f \rightarrow 0$



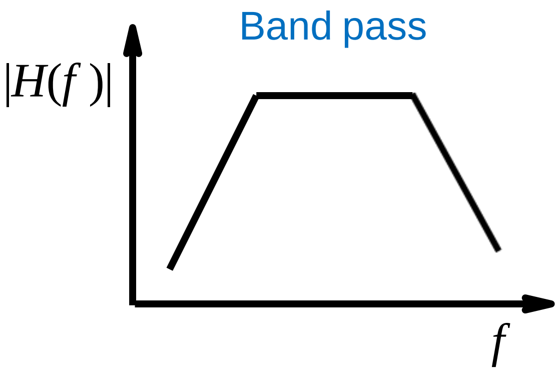
$v_o \rightarrow 0$

High f: $f \rightarrow \infty$

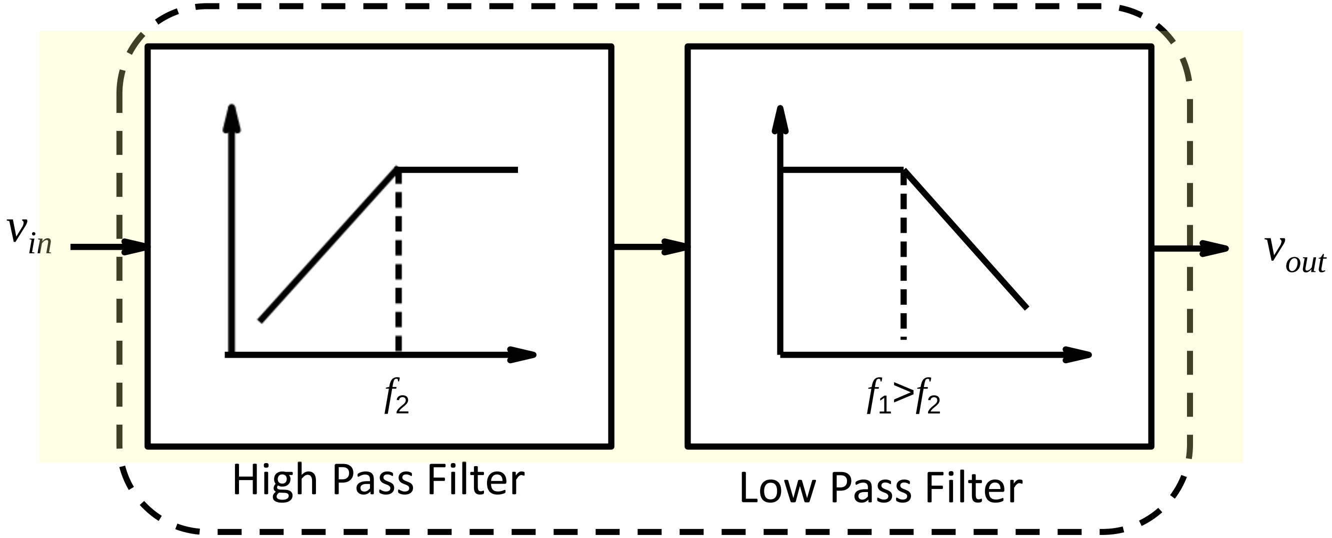


$v_o \rightarrow 0$

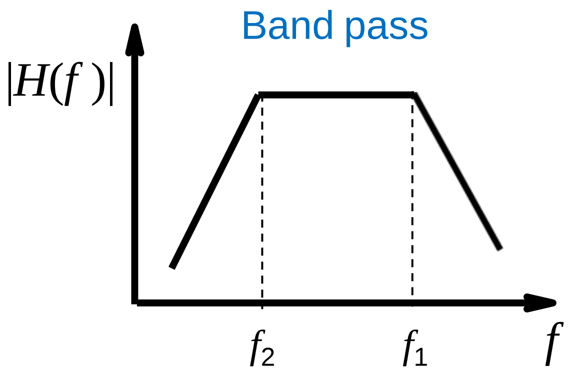
Could this be a bandpass filter?

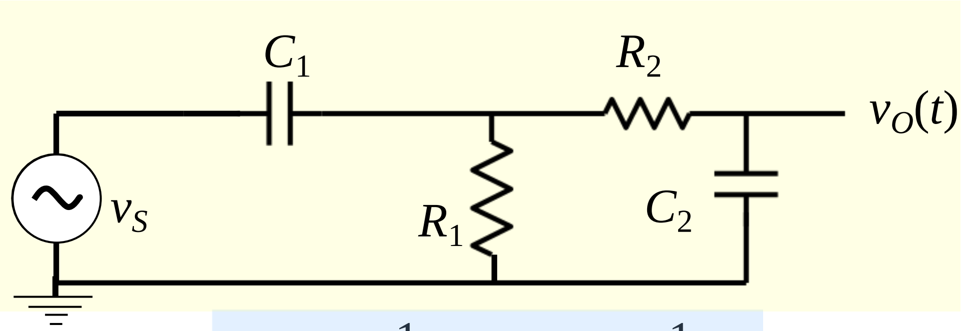


A Bandpass Filter



For $f_1 > f_2$, it is a band pass filter

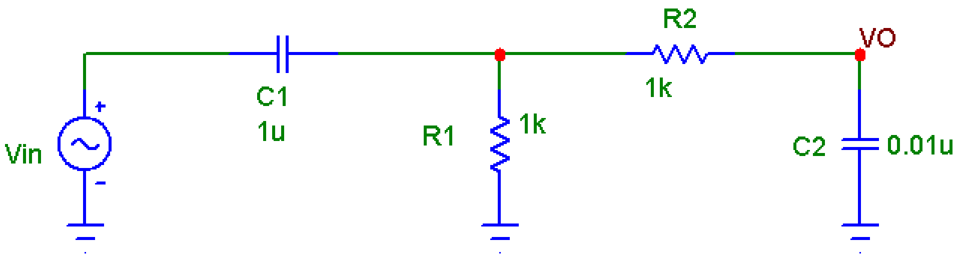


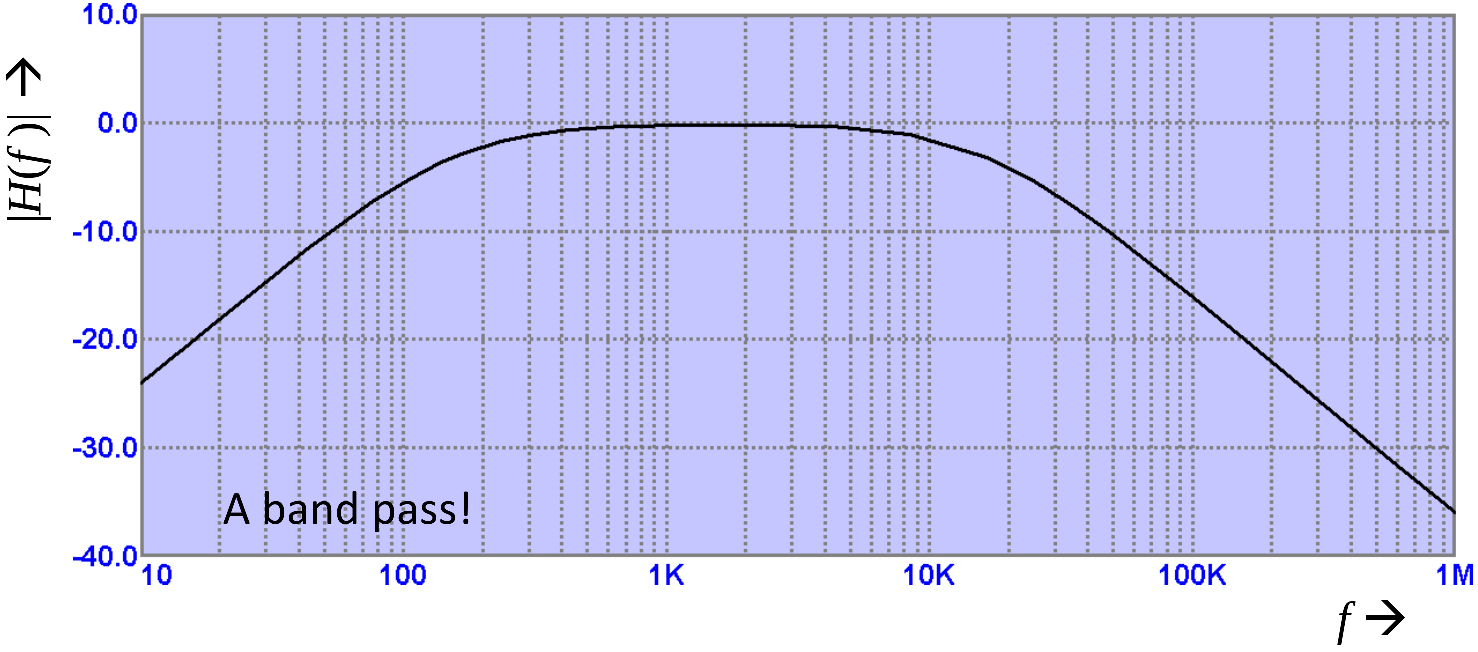


$$f_2 \cong \frac{1}{2\pi R_1 C_1} \; ; \; f_1 \cong \frac{1}{2\pi R_2 C_2}$$

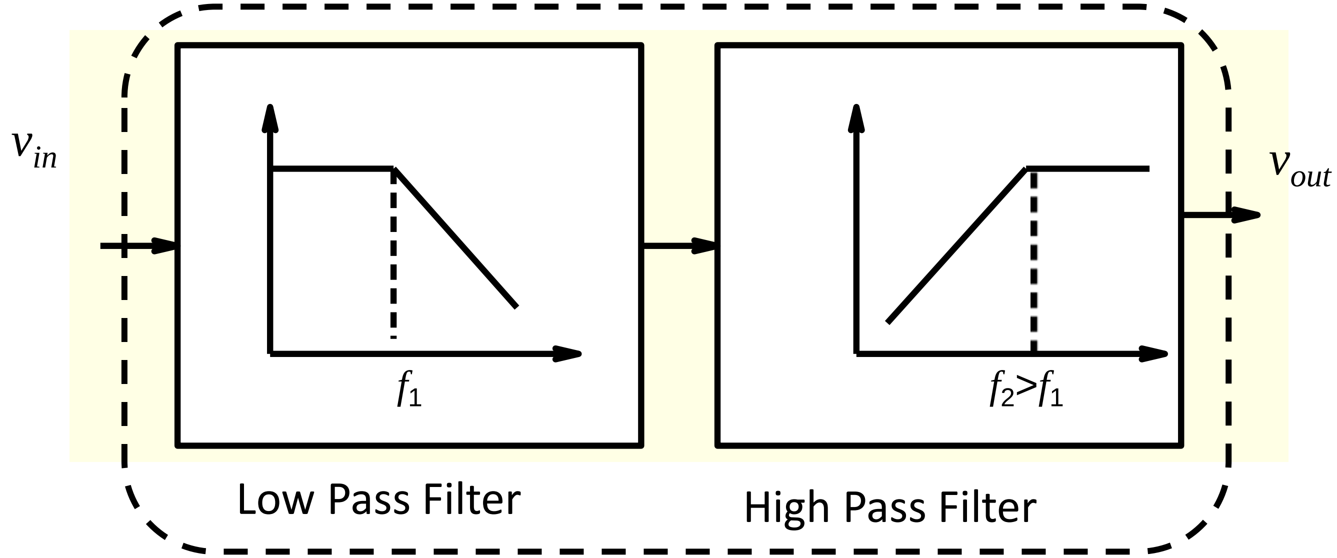
Example: Band Pass filter

$$f_2 \cong \frac{1}{2\pi R_1 C_1} \; ; f_1 \cong \frac{1}{2\pi R_2 C_2}$$

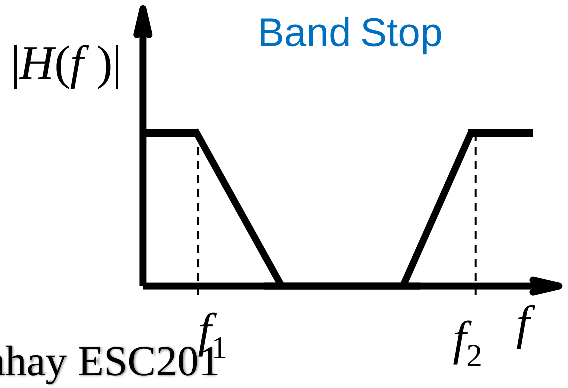




Band Stop Filter



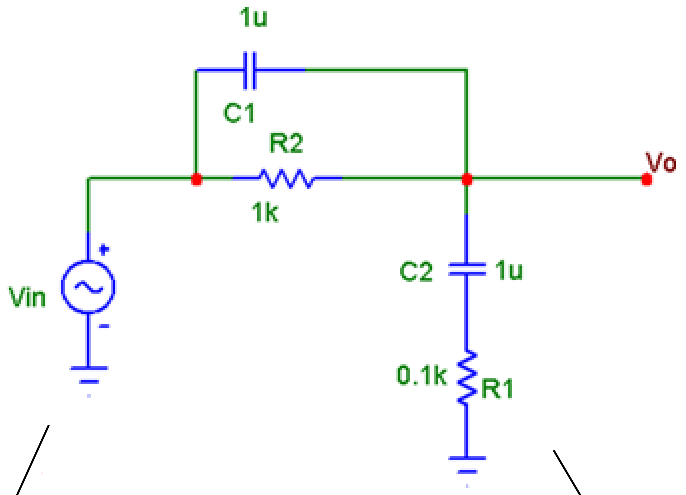
For $f_1 < f_2$, it is a band pass filter

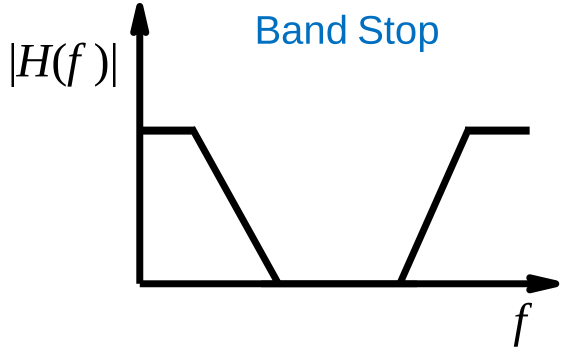


Will this work?

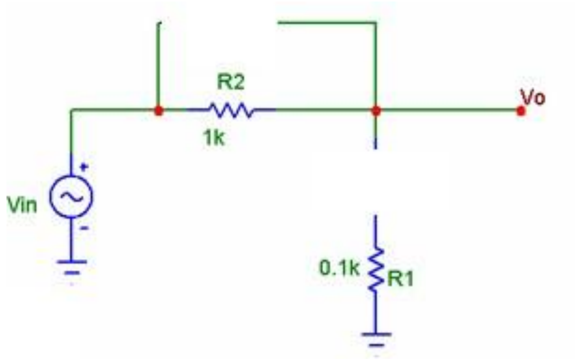
Well, it does work if designed properly!

Example: Band Stop filter



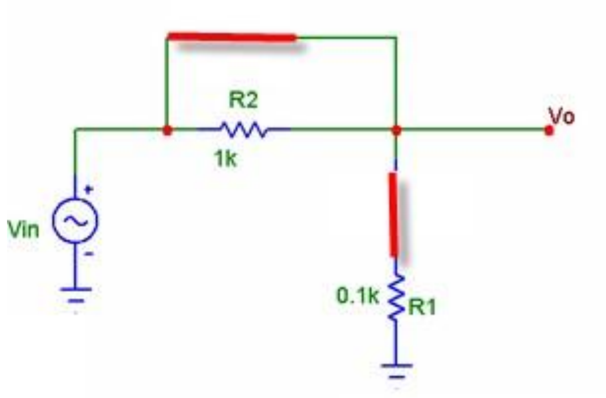


Low f: $f \rightarrow 0$



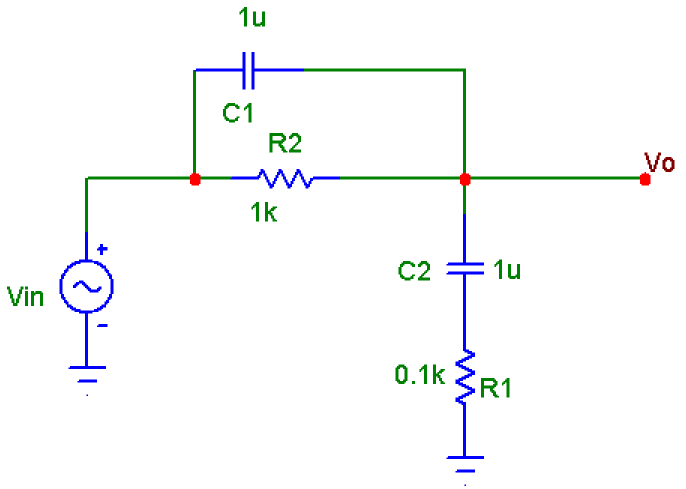
$v_o \rightarrow v_{in}$

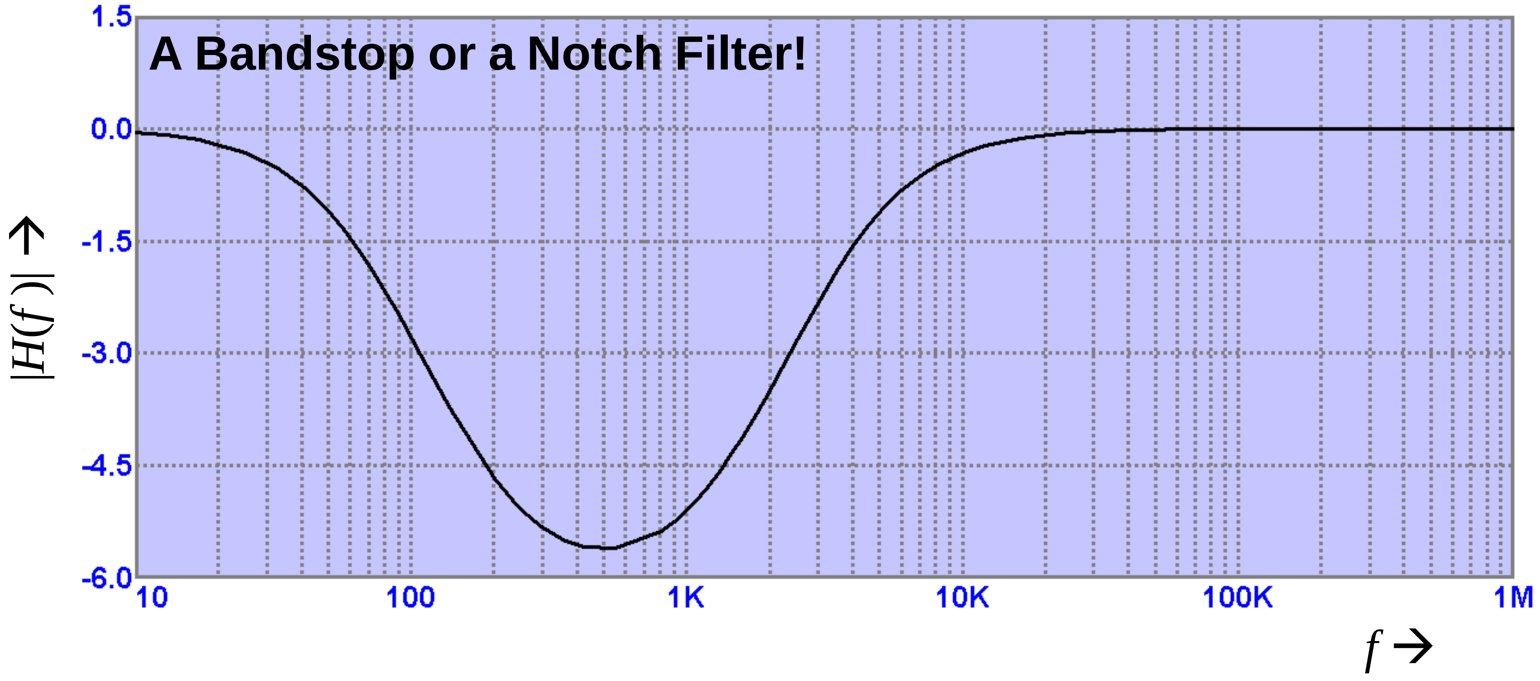
High f: $f \rightarrow \infty$



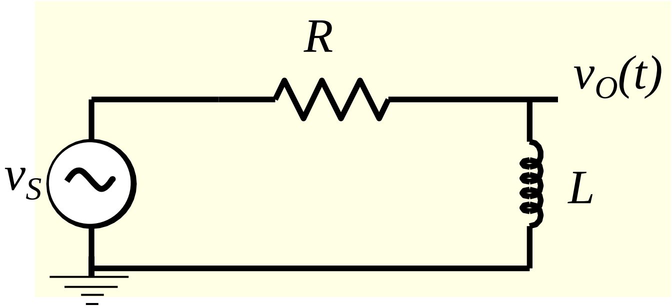
$v_o \rightarrow v_{in}$

Example: Band Stop filter (continued)





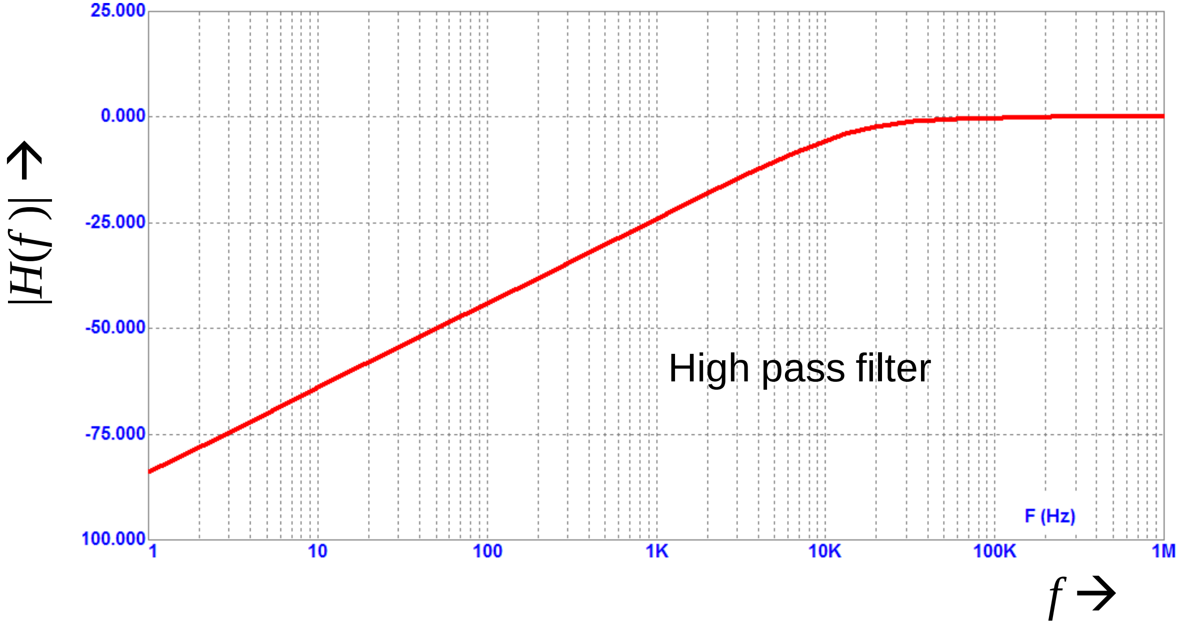
RL High Pass Filters



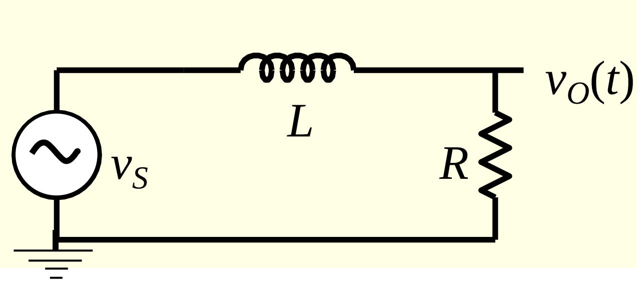
$$H(\omega) = \frac{V_O(\omega)}{V_S(\omega)}$$

$$H(\omega) = \frac{j\omega L}{R + j\omega L} = \frac{j(\omega/\omega_{3dB})}{1 + j(\omega/\omega_{3dB})}$$

$$\omega_{3dB} = \frac{R}{L}$$



RL Low Pass Filters

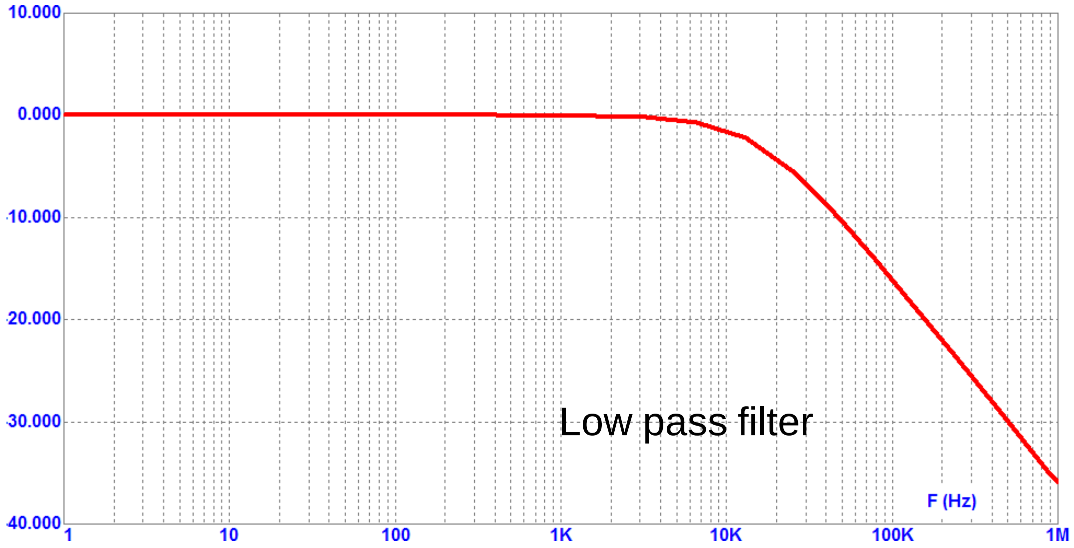


$$H(\omega) = \frac{V_O(\omega)}{V_S(\omega)}$$

$$H(\omega) = \frac{R}{R + j\omega L} = \frac{1}{1 + j(\omega / \omega_{3dB})}$$

$$\omega_{3dB} = \frac{R}{L}$$

$|H(f)| \rightarrow$



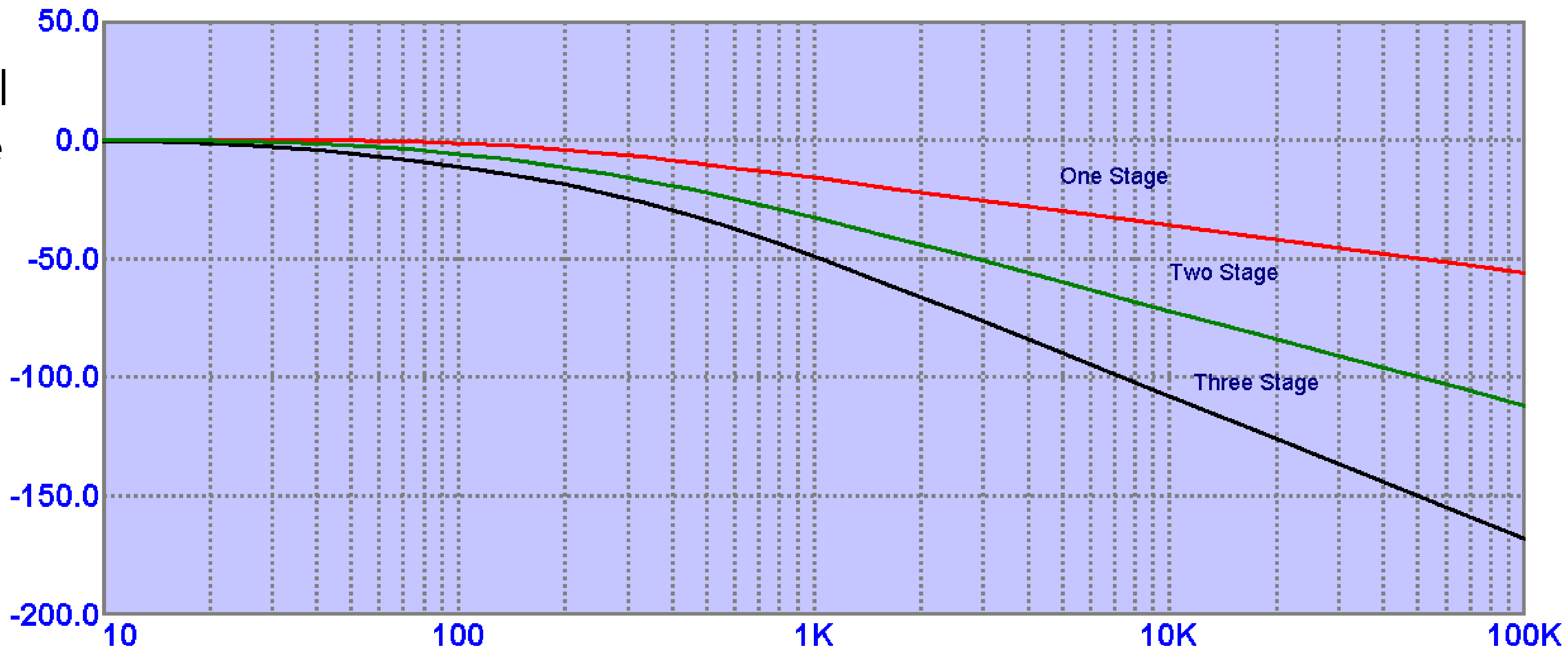
$f \rightarrow$

FM Radio

Different radio channels are separated by very narrow frequency interval.

For exampl

MHz) or Re



log(98.3) = 10 decades

This implies an attenuation of -6000 dB/decade !!

Resonance

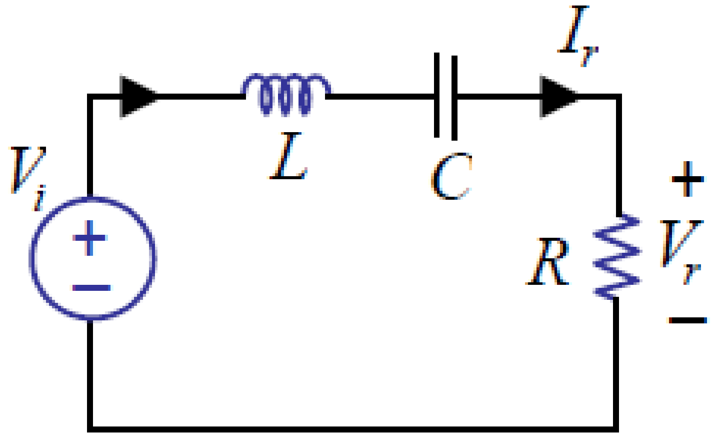
- Every system has its own natural frequency of oscillation.

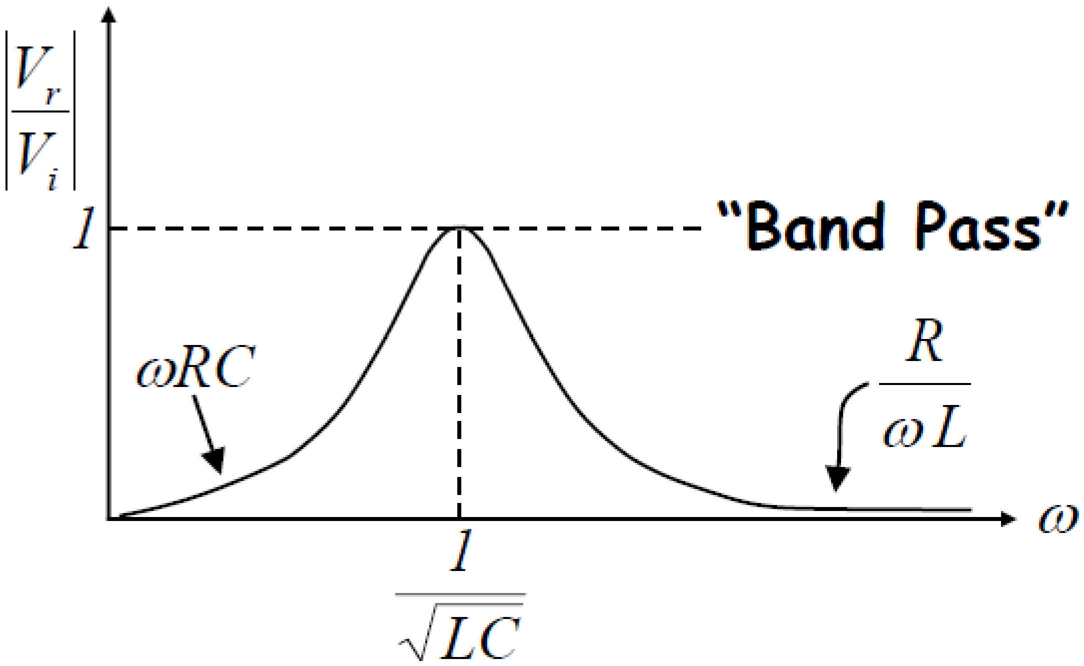
- If you apply an external stimulus at natural frequency: System exhibits an extremely large response

RLC Circuit

$$\frac{V_r}{V_i} = \frac{R}{j\omega L + \frac{1}{j\omega C} + R}$$

$$\left|\frac{V_r}{V_i}\right| = \frac{\omega RC}{\sqrt{\left(1-\omega^2 LC\right)^2 + \left(\omega RC\right)^2}}$$





Observe:

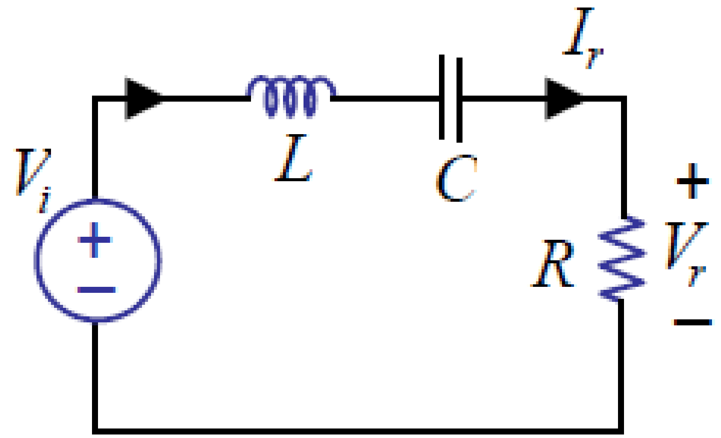
$$\text{Low } \omega: \quad \approx \omega RC$$

$$\text{High } \omega: \quad \approx \frac{R}{\omega L}$$

$$\omega\sqrt{LC} = 1: \quad \approx 1$$

Series Resonant Circuit

$$I_r = \frac{V_i}{Z_{eq}} \qquad Z_{eq} = R + j\omega L - j\frac{1}{\omega C}$$



Resonance is a condition in which capacitive and inductive reactance cancel each other to give rise to a purely resistive circuit

$$|I_r(\omega)| = \frac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

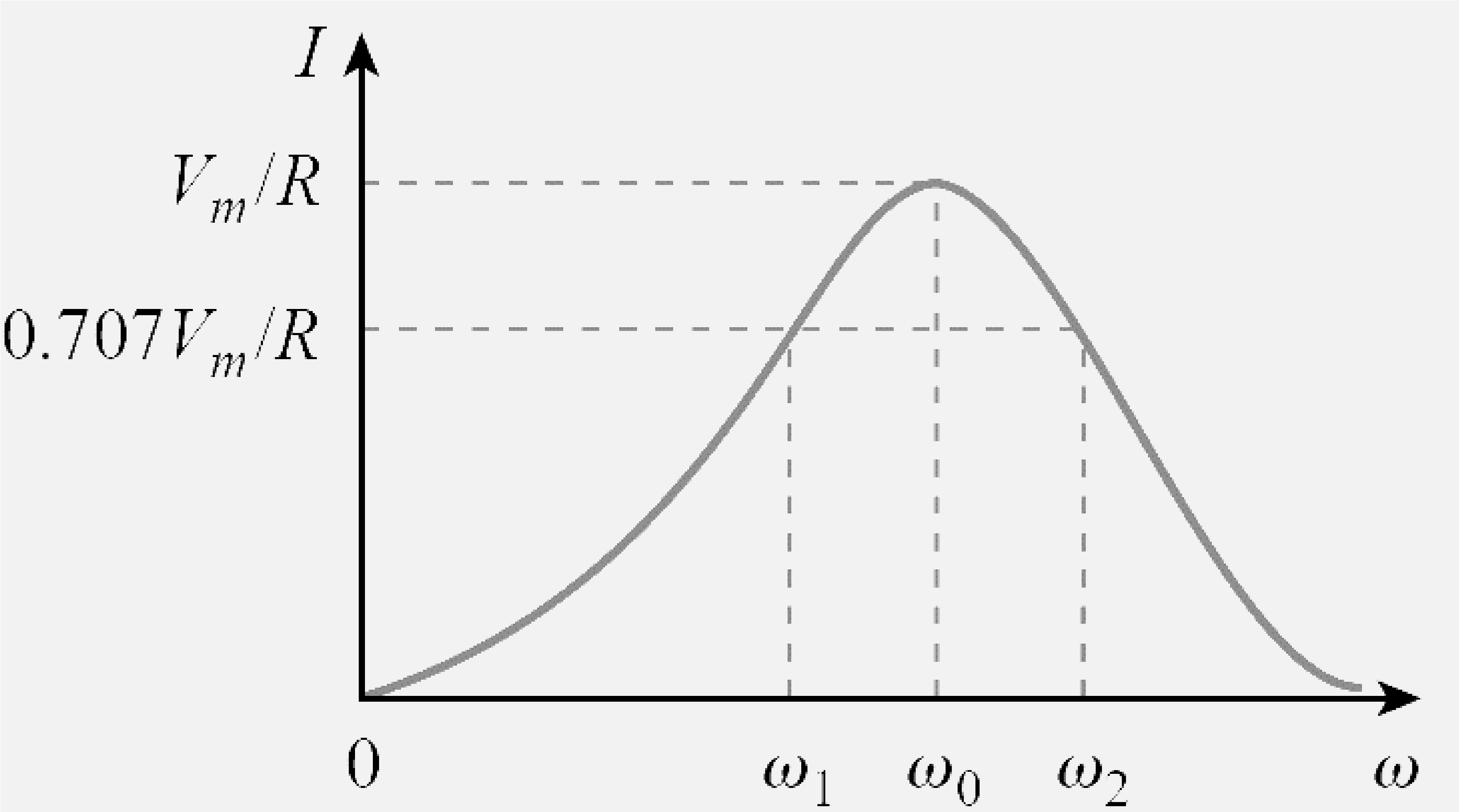
$$j\omega_O L - j\frac{1}{\omega_O C} = 0$$

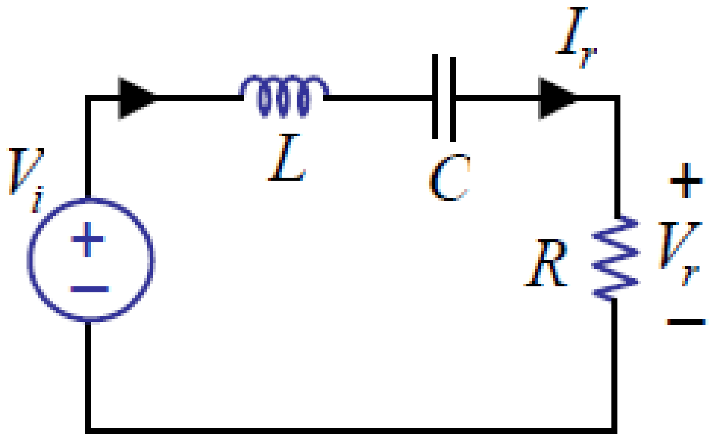
$$\Rightarrow \omega_O = \frac{1}{\sqrt{LC}}$$ Resonant frequency $$f_O = \frac{1}{2\pi\sqrt{LC}}$$

$$Z_{eq} = R$$

Current and voltage are in phase (power factor is unity)!

Series Resonant Circuit





ω_1 and ω_2 are called half-power frequencies

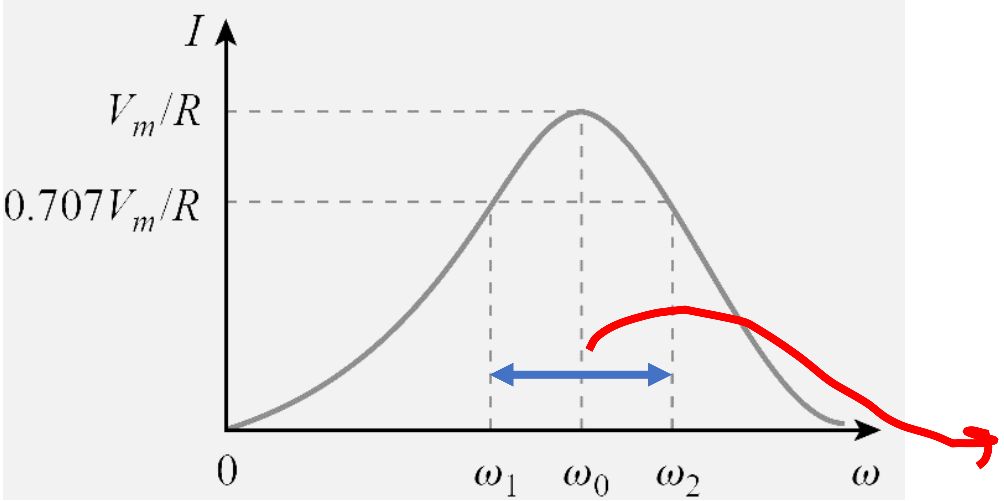
$$\omega_O = \frac{1}{\sqrt{LC}}$$

Half-power Frequency

$$|I(\omega)| = \frac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$$\omega_O = \frac{1}{\sqrt{LC}}$$

ω_1 and ω_2
half-power frequencies



$$\omega_O = \sqrt{\omega_1 \omega_2}$$

$$\Delta\omega = \omega_2 - \omega_1 = \frac{R}{L}$$

Bandwidth

$$|I(\omega_1)| = \frac{V_m}{\sqrt{R^2 + (\omega_1 L - \frac{1}{\omega_1 C})^2}} = \frac{V_m}{\sqrt{2}R}$$

$$|I(\omega_2)| = \frac{V_m}{\sqrt{R^2 + (\omega_2 L - \frac{1}{\omega_2 C})^2}} = \frac{V_m}{\sqrt{2}R}$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

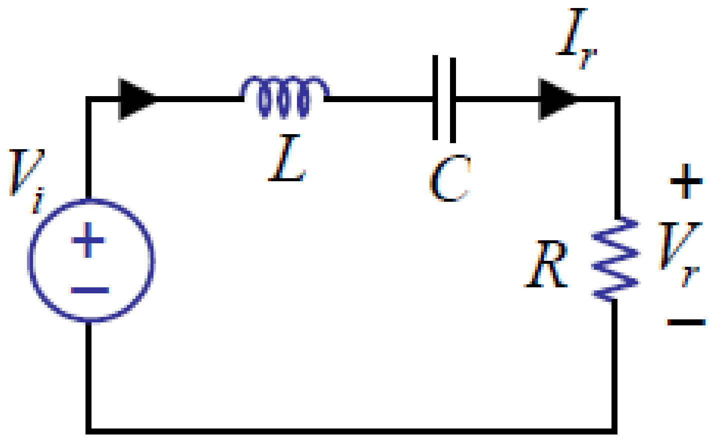
$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$Q = \frac{\omega_0}{\Delta\omega}$$

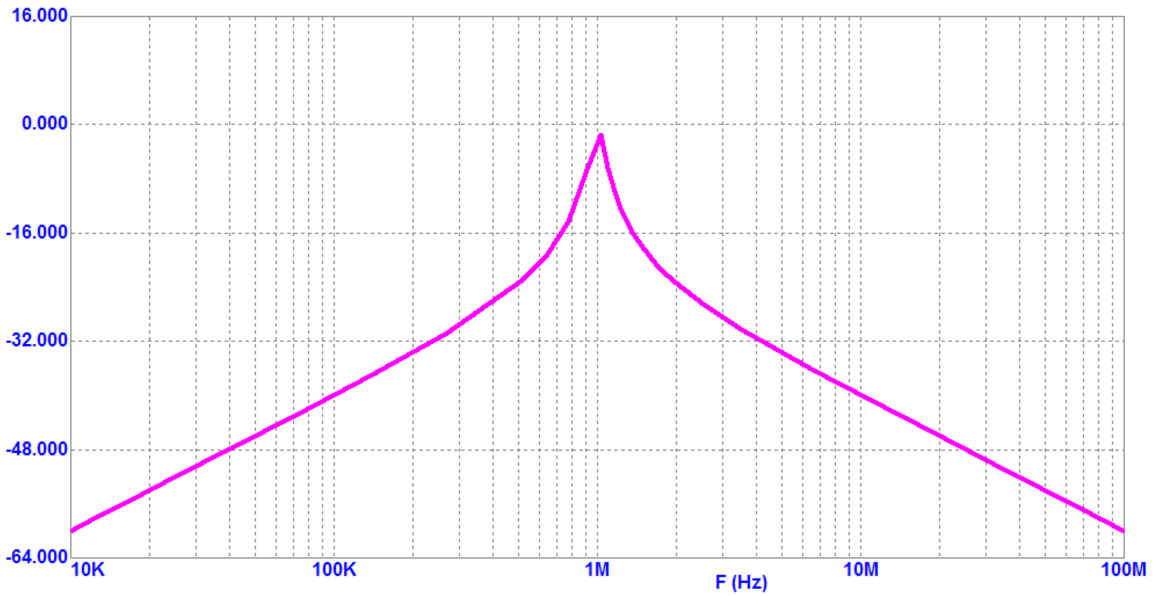
Quality Factor

$$Q = \frac{\sqrt{L}}{\sqrt{C}R}$$

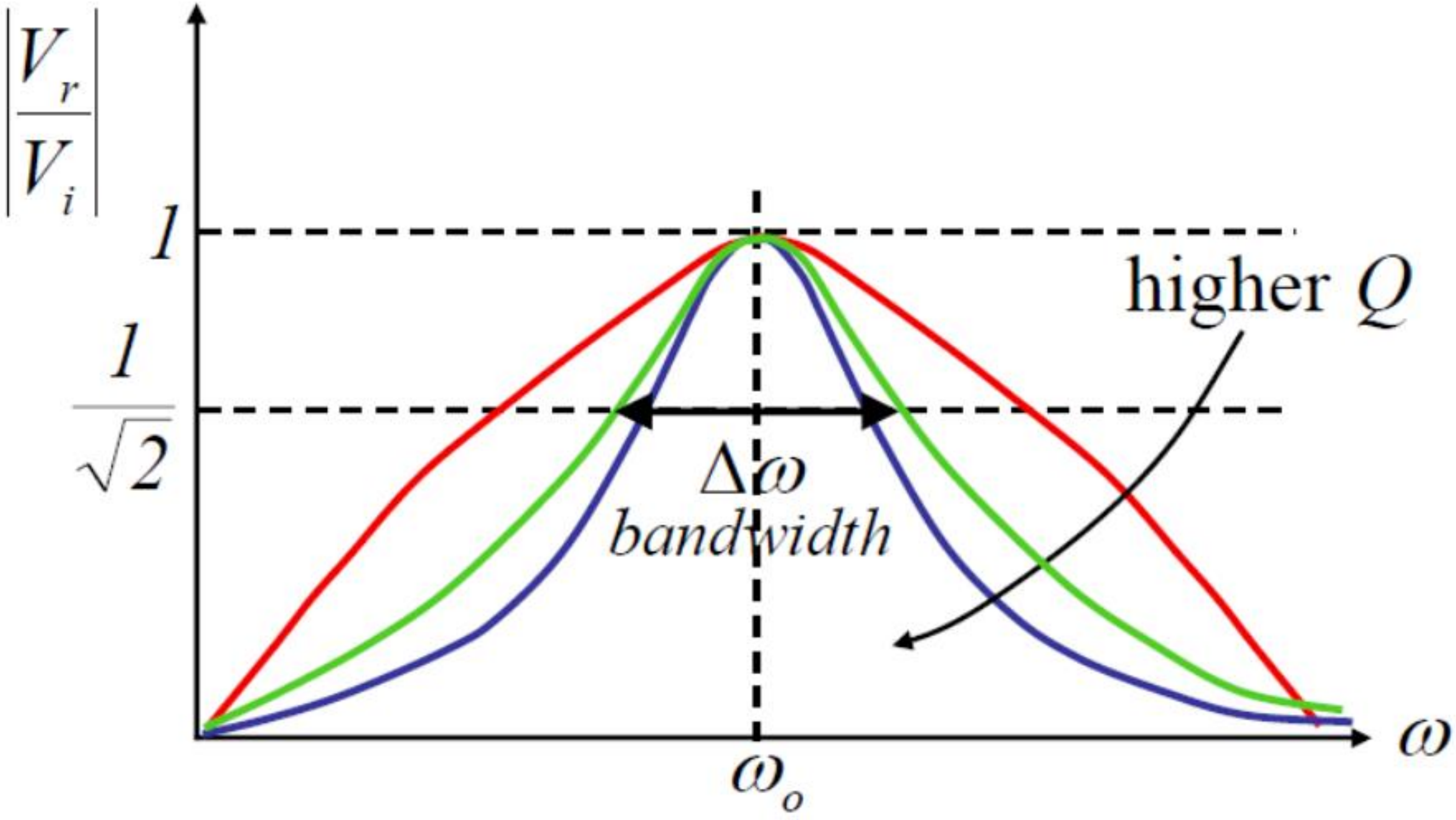
RLC Circuit



$$|V_o(\omega)| = |V_I| \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$



Bandwidth & Quality Factor



$$Q = \frac{\omega_0}{\Delta\omega}$$

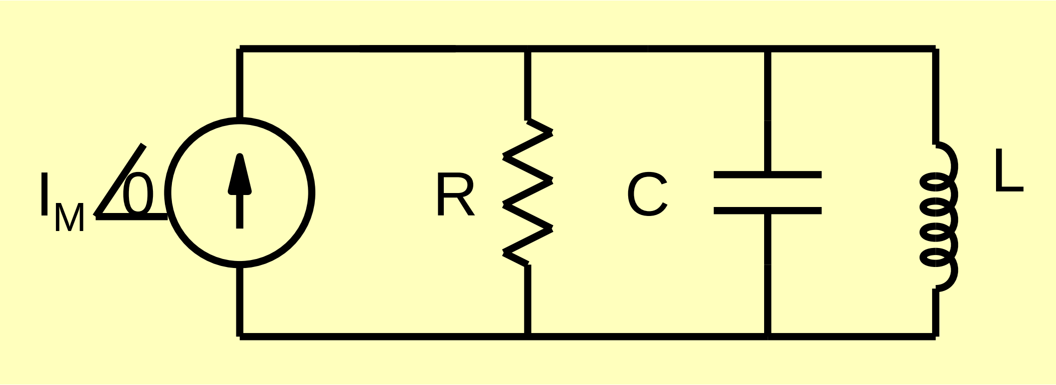
Quality Factor

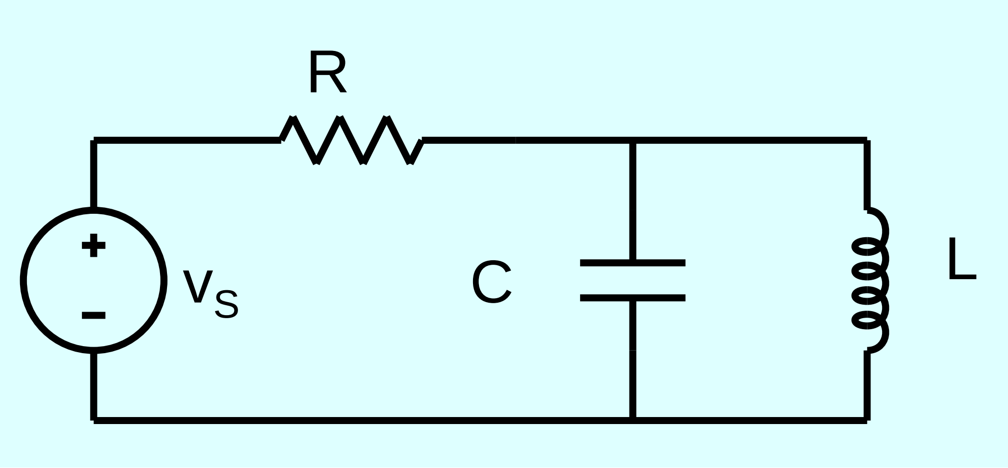
Q represents sharpness of resonance

For high Q circuits:

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2}$$

Parallel Resonance





$$Y_{eq} = \frac{1}{R} + j\omega C - j\frac{1}{\omega L}$$

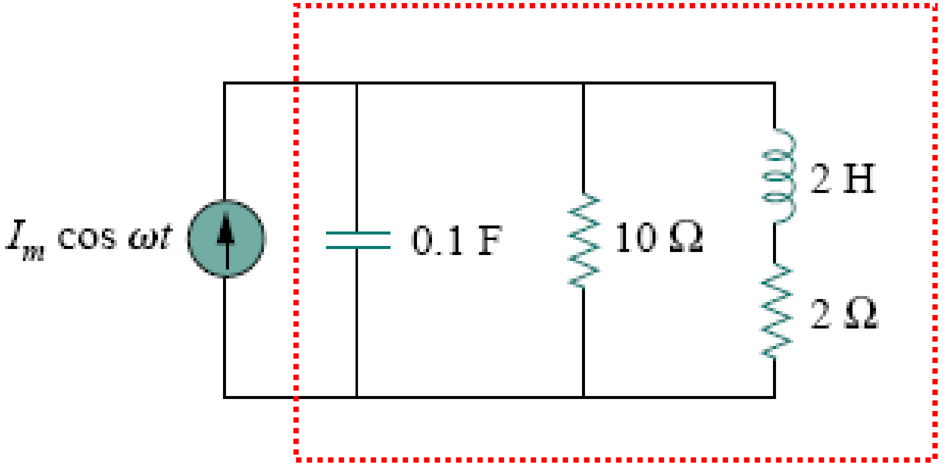
Resonant frequency:

$$j\omega_O C - j\frac{1}{\omega_O L} = 0 \Rightarrow \omega_O = \frac{1}{\sqrt{LC}}$$

$$f_O = \frac{1}{2\pi\sqrt{LC}}$$

$$Z_{eq} = R$$

Example



What is the resonant frequency ?

$$\mathbf{Y} = j\omega 0.1 + \frac{1}{10} + \frac{1}{2 + j\omega 2} = 0.1 + j\omega 0.1 + \frac{2 - j\omega 2}{4 + 4\omega^2}$$

At resonance, $\text{Im}(\mathbf{Y}) = 0$

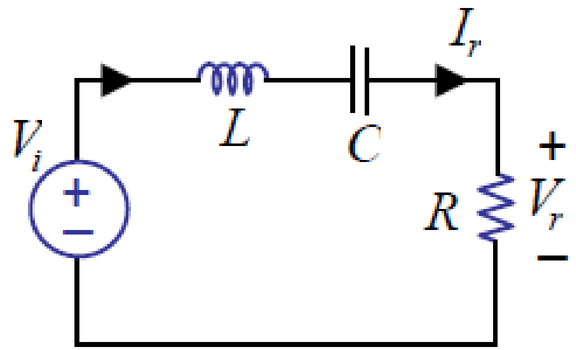
$$\omega_0 0.1 - \frac{2\omega_0}{4 + 4\omega_0^2} = 0 \qquad \Longrightarrow \qquad \omega_0 = 2 \text{ rad/s}$$

$$f_o = \frac{\omega_o}{2\pi}$$

RLC Circuit

$$|V_o(\omega)| = |V_I|\frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$$|V_O(\omega)| = \frac{1}{\sqrt{1 + Q^2\left(\frac{\omega^2}{\omega_O^2} - 1\right)^2}}$$



For $\omega=\omega_O$, V_O = 1 so the signal simply passes through !

How much Q do we need to pass 450KHz but reject 460KHz by 60dB?

$$\omega_0 = 2\pi \times 450 \times 10^3 = 2.827 \times 10^6\, rad/s$$

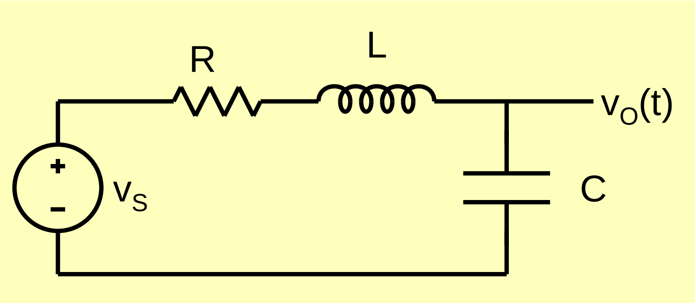
$$\omega = 2\pi \times 460 \times 10^3 = 2.89 \times 10^6\, rad/s$$

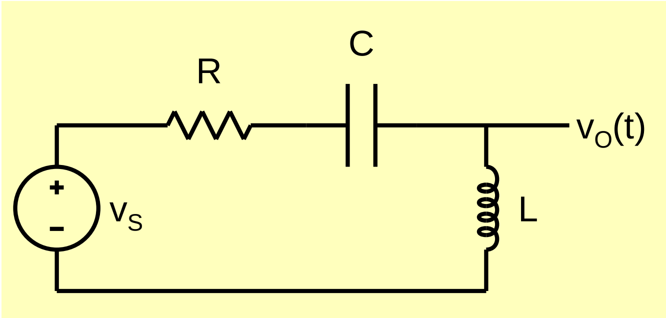
For an attenuation of -60dB or 10^{-3} at ω:

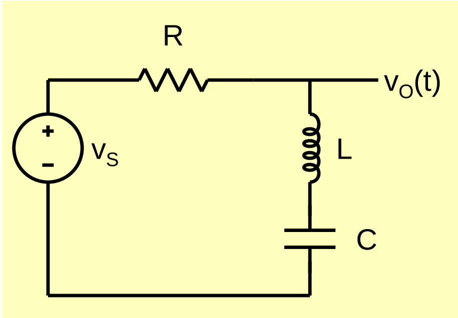
$$Q = \frac{1000}{(460/450)^2 - 1} = 23000$$

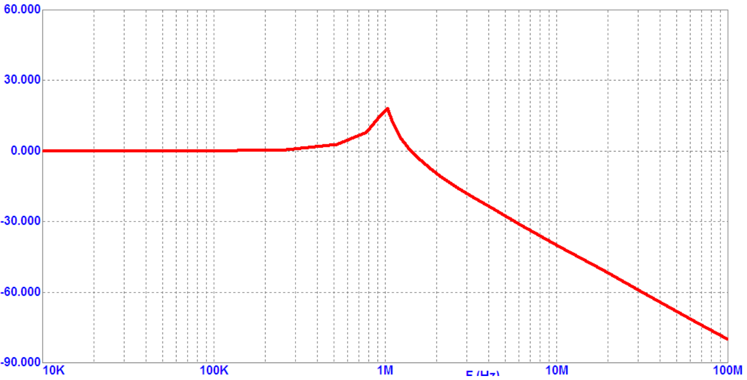
This is a large value of Q!

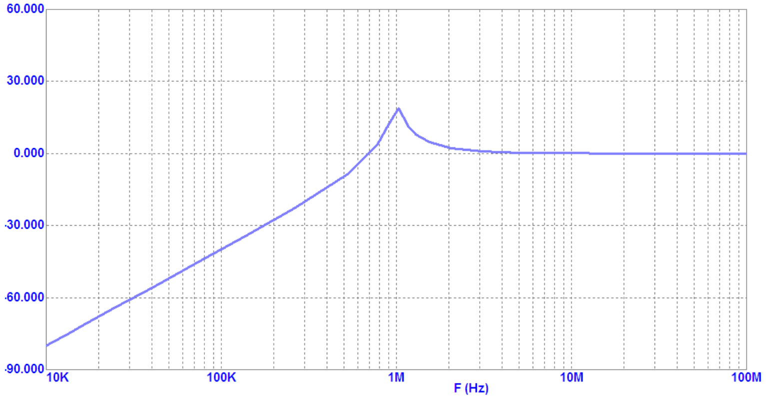
RLC Filters

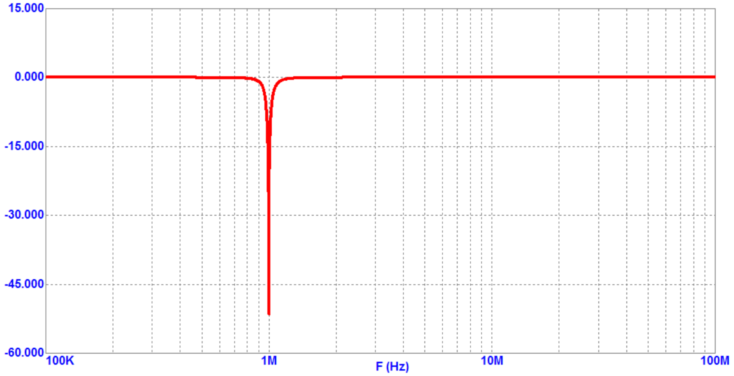

R
L
vO(t)
vS
C


R
C
vO(t)
vS
L


R
vO(t)
vS
L
C


60.000
30.000
0.000
-30.000
-60.000
-90.000
10K
100K
1M
10M
100M


60.000
30.000
0.000
-30.000
-60.000
-90.000
10K
100K
1M
F (Hz)
10M
100M


15.000
0.000
-15.000
-30.000
-45.000
-60.000
100K
1M
F (Hz)
10M
100M