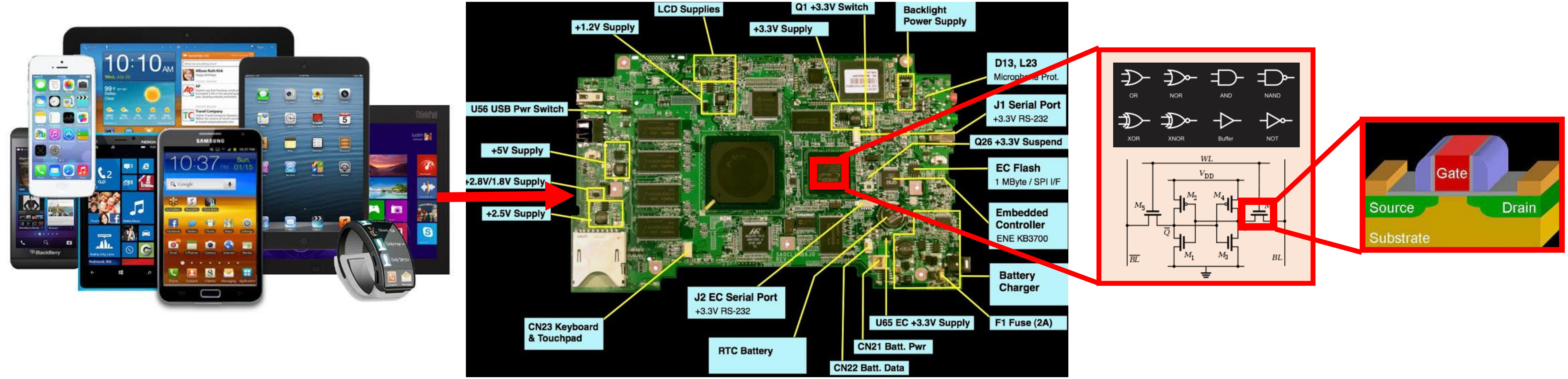


# ESC201: Introduction to Electronics

## Module 5: Amplifiers

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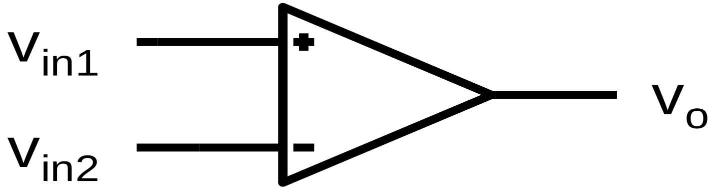
IIT Kanpur

# Difference Amplifier

An amplifier that is:

- sensitive to difference in input voltages; and
- insensitive to what is common.




$$v_{id} = v_{in1} - v_{in2}$$

$$v_{ic} = \frac{v_{in1} + v_{in2}}{2}$$

$$v_o = A_d v_{id} + A_{cm} v_{ic}$$

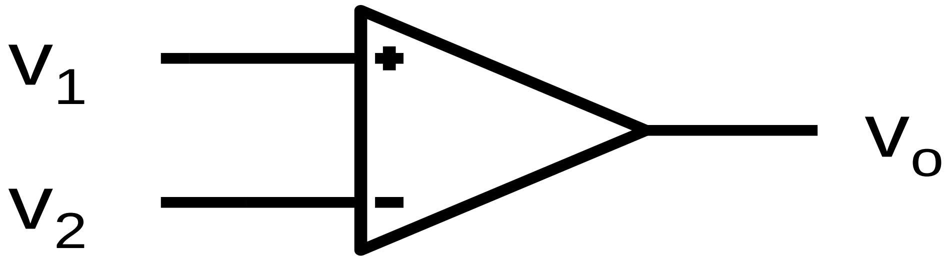
$A_d$ : Differential mode gain

$A_{cm}$ : Common mode gain

$$A_d >> A_{cm}$$

Common Mode Rejection Ratio: $CMRR = \dfrac{A_d}{A_{cm}}$

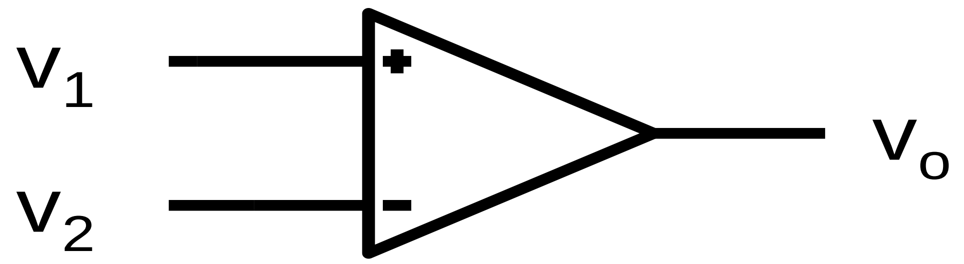
# Ideal Operational Amplifier

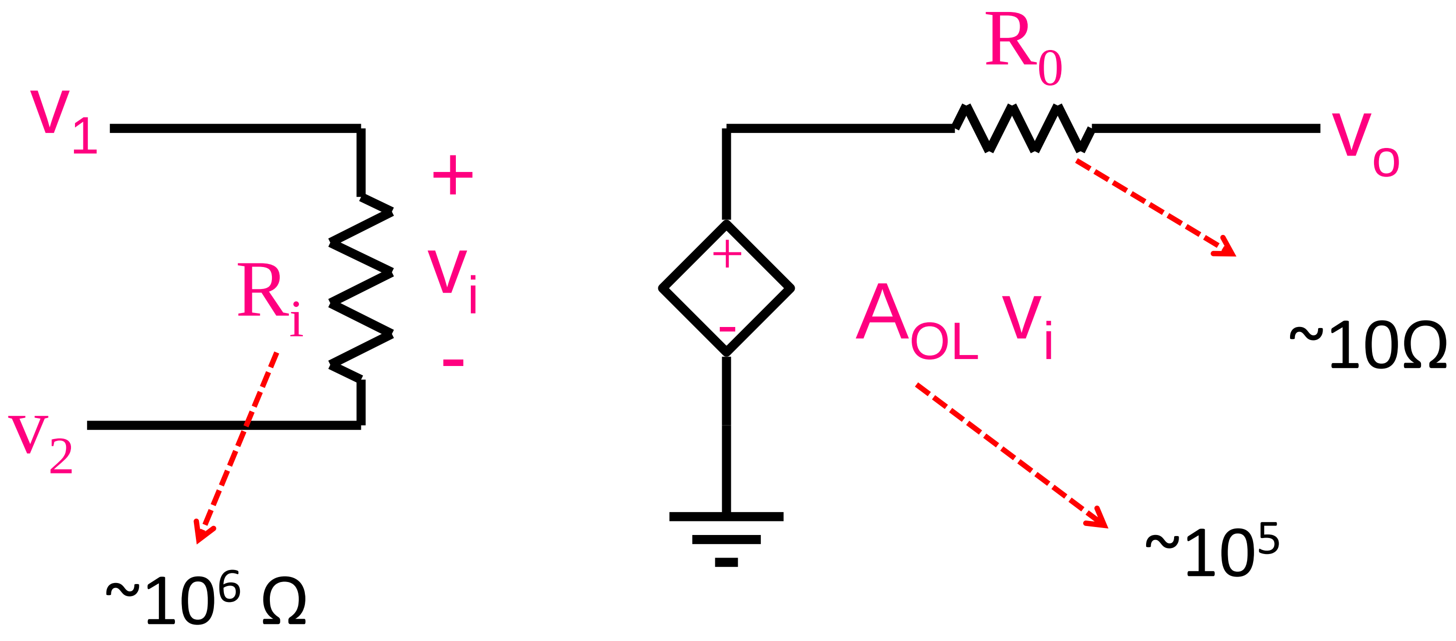



1. Infinite Differential-mode voltage gain
2. Infinite Common mode Rejection ratio
3. Infinite Input Resistance
4. Zero output Resistance
5. ….

# Equivalent Circuit Model

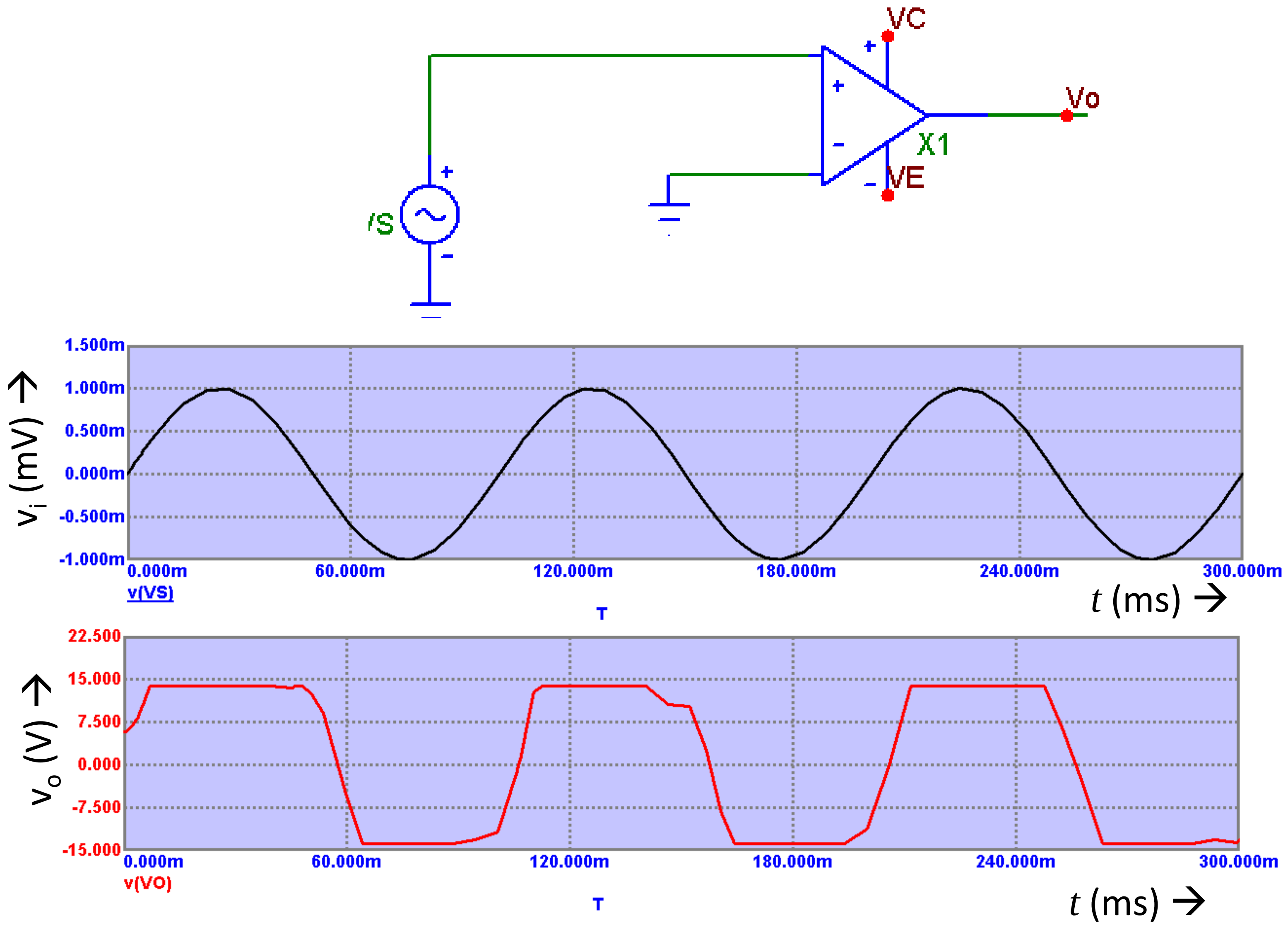
**A simple equivalent circuit model of an op-amp**



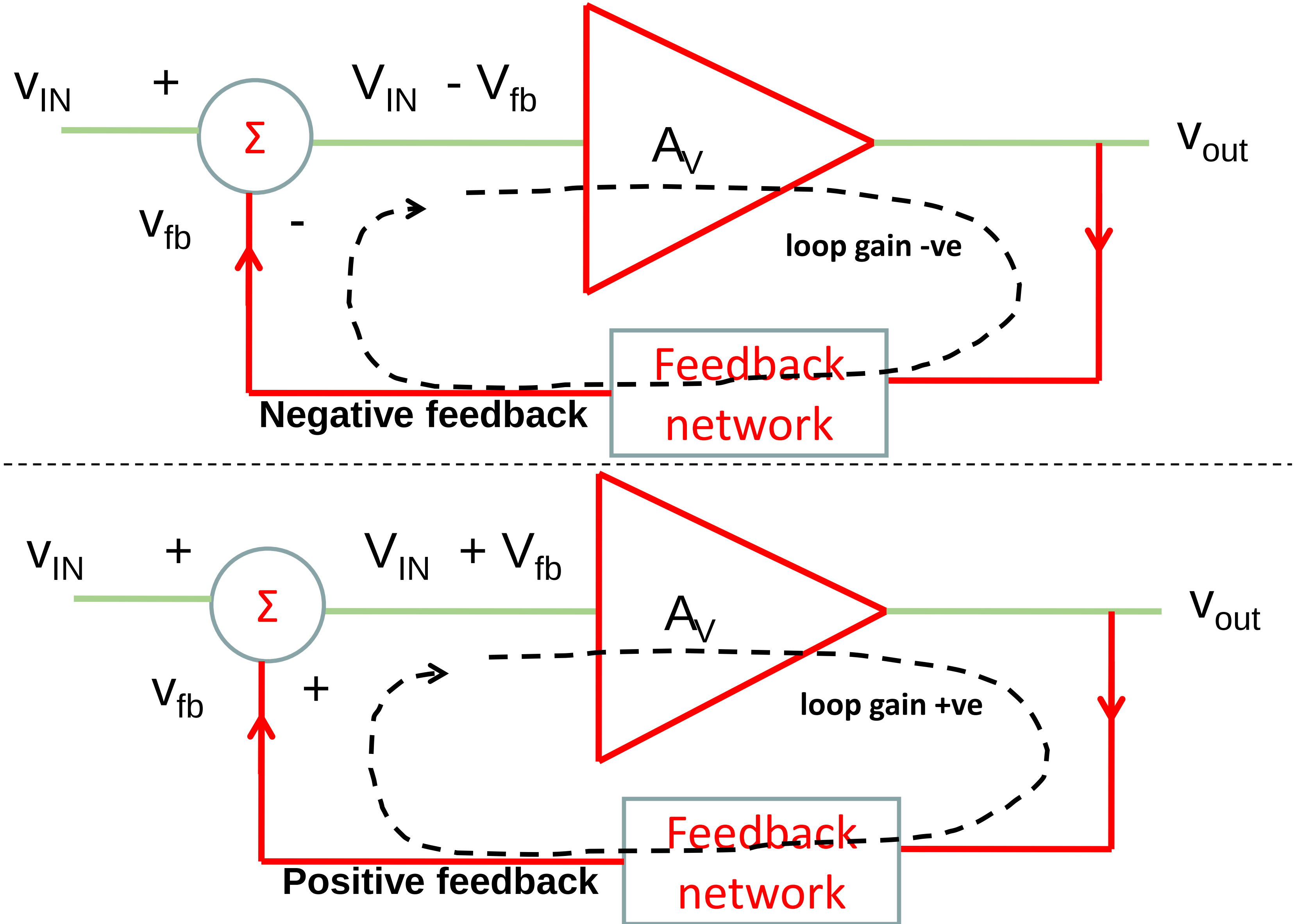




This assumes very high CMRR

# Simulation Results



# Negative and Positive Feedback



# Op-amp Circuit Classification by Feedback

Op-amp circuits

No control on the process at all

Open loop

Closed loop
Or feedback

Will result in running away of process to an extreme

Positive feedback

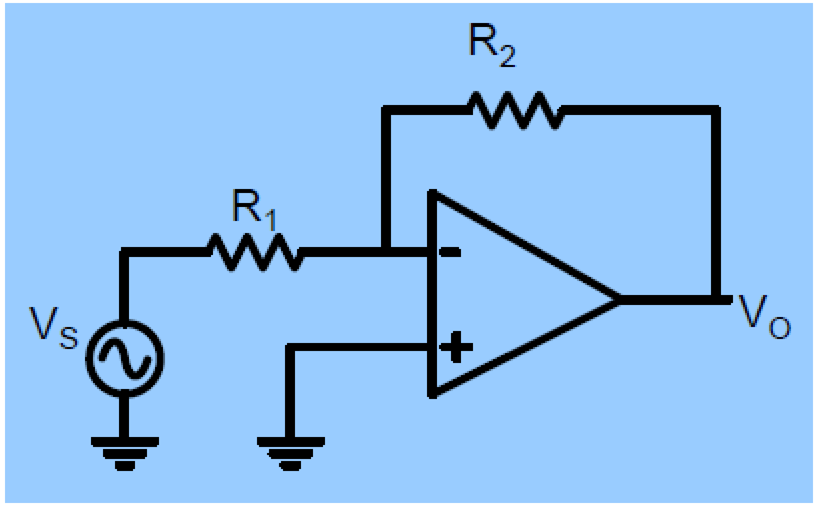
Needed to keep a process under control

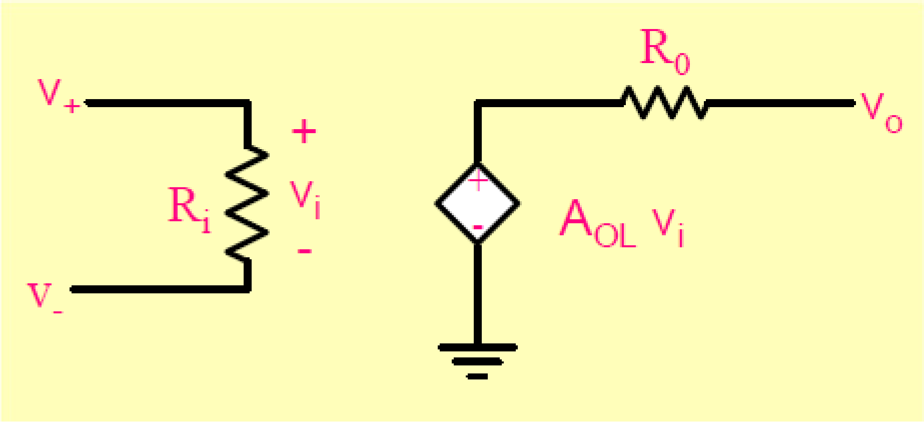
Negative feedback

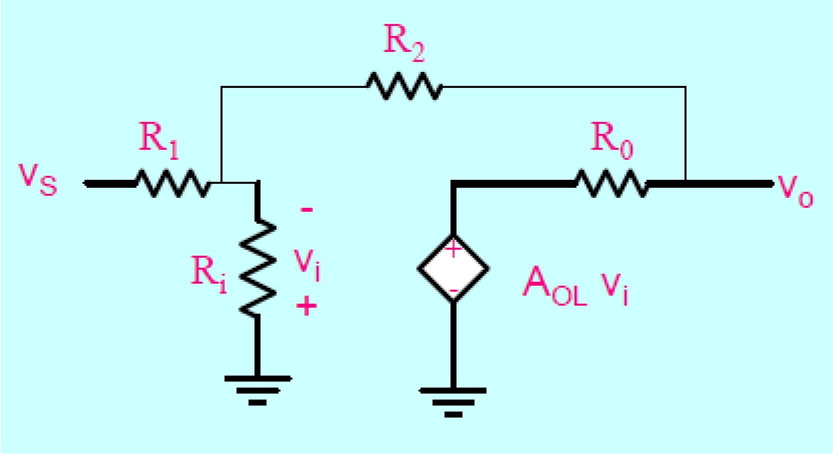
**Most Op-amp Circuits employ <u>negative feedback</u>**

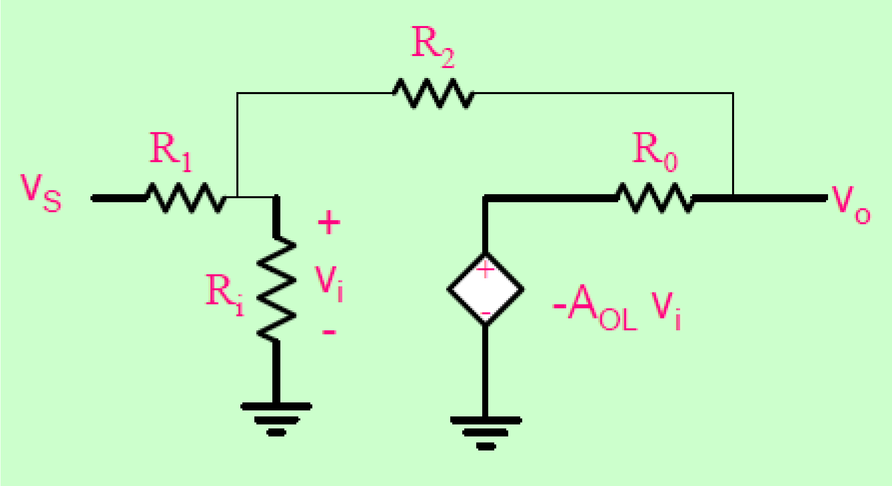
**Note**: The classification and behaviour is true for all systems in this universe!

# Inverting amplifier

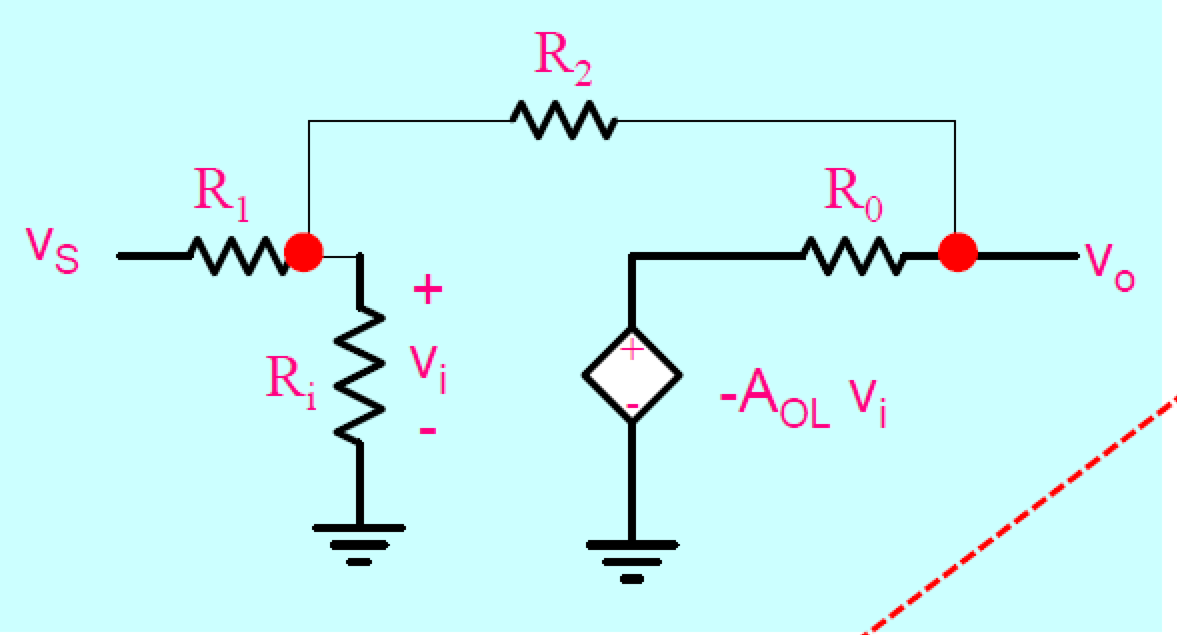


# Nodal Analysis




$$\frac{v_S - v_i}{R_1} = \frac{v_i}{R_i} + \frac{v_i - v_o}{R_2} \quad (1)$$

$$\frac{-A_{OL} v_i - v_o}{R_o} = \frac{v_o - v_i}{R_2} \quad (2)$$

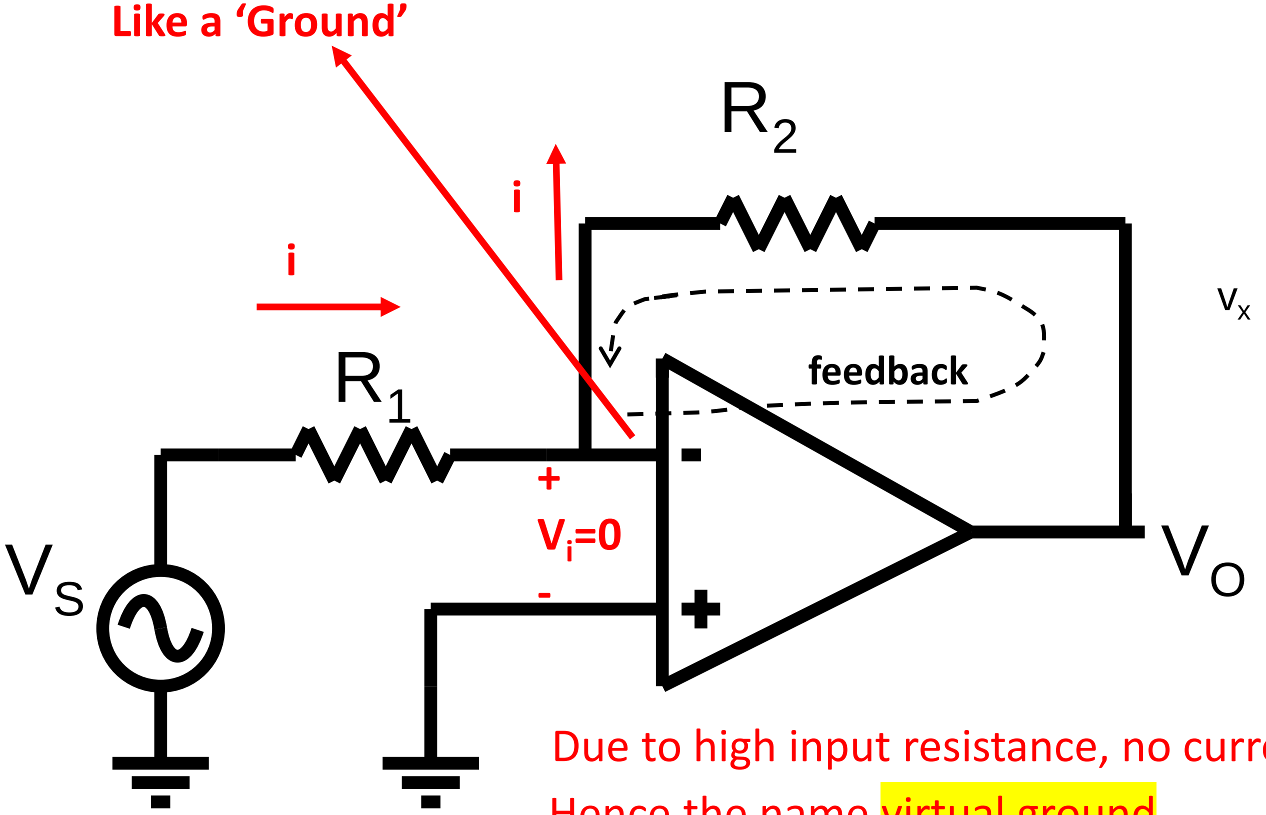
$$\frac{v_S}{R_1} = -\frac{v_o}{R_2} + v_i \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_i}\right) \quad (3)$$

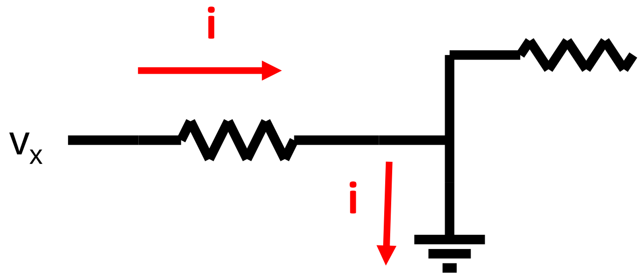
$$v_i = v_o \frac{\frac{1}{R_o} + \frac{1}{R_2}}{\frac{-A_{OL}}{R_o} + \frac{1}{R_2}} \quad (4)$$

$$\boxed{\text{As } A_{OL} \to \infty \quad v_i \to 0}$$

This is called the Virtual Ground property

# Inverting Amplifier




A real ground will sink all current

Due to high input resistance, no current is sunk in op-amp

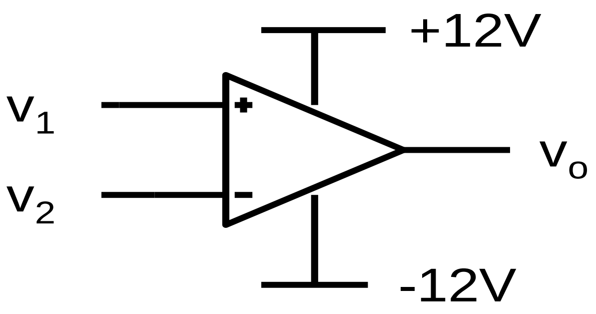
Hence the name virtual ground

Under negative feedback with high gain

Op-amp input is a nullator: no current/no voltage: open circuit and short circuit at the same time!

Op-amp output is a norator: can supply any value of current and voltage

# Virtual Ground Property




for op-amp connected in negative feedback

$$v_1 \cong v_2$$

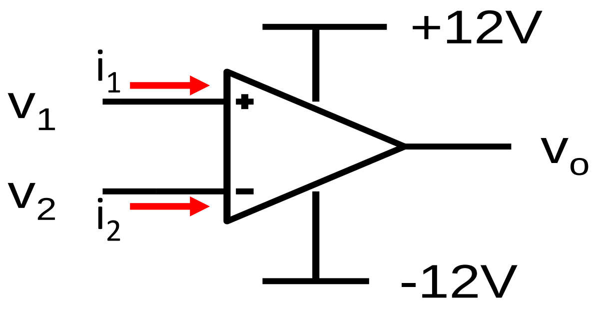
In an op-amp with negative feedback, the voltage of the inverting terminal is equal to the voltage of the non-inverting terminal if the gain of the op-amp is sufficiently high

This property does not hold under certain conditions such as

- open loop,
- positive feedback
- or if the op-amp is saturated.

# Input of an Op-amp is a Nullator

**For an ideal op-amp circuits under negative feedback**




With negative feedback:

1. $v_1 \approx v_2$

(as in a short circuit)

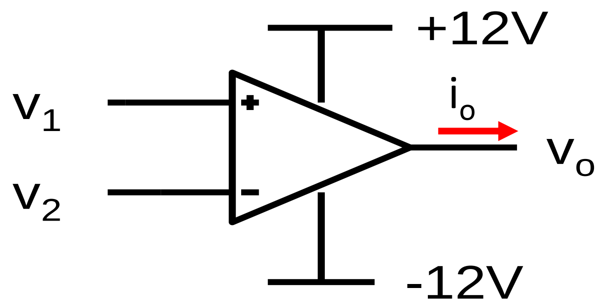
2. $i_1 \approx 0$; $i_2 \approx 0$

(as in an open circuit)

The op-amp input appears to have properties of short and open circuit simultaneously !
Its behaviour approaches that of a nullator!

# Output of an Op-amp is a Norator

**For an ideal op-amp circuits under negative feedback**




With negative feedback:

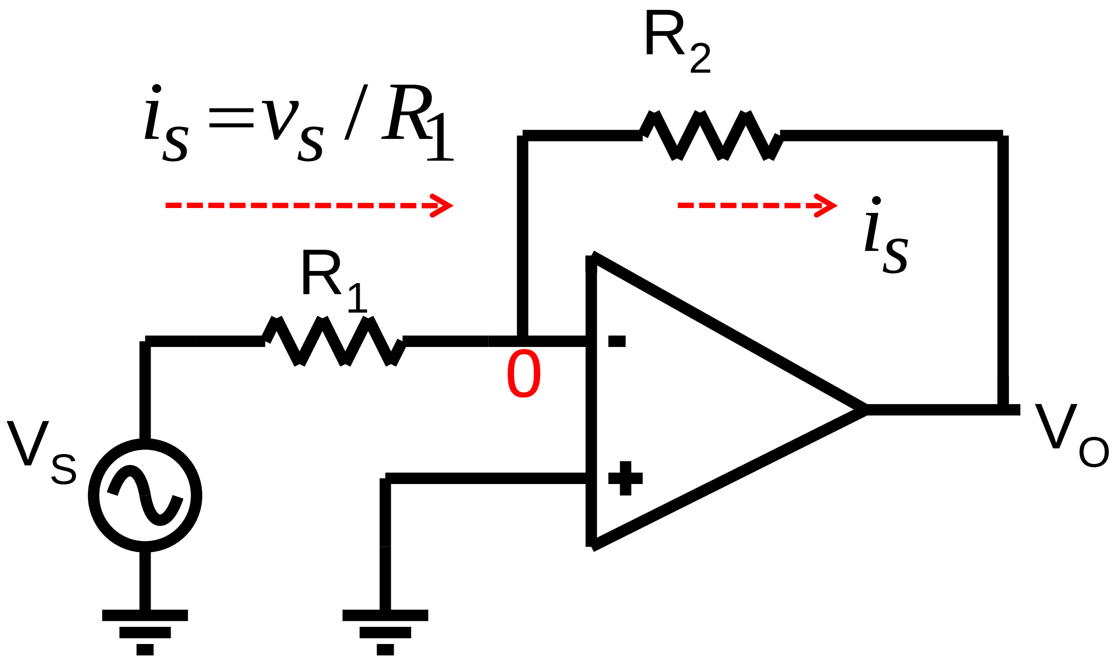
1. $v_o$ can take any voltage between the supply voltages (as in an open circuit)

2. $i_o$ can supply any current needed - up to a value. (as in a short circuit)

At the output side op-amp appears to be like a short and an open circuit simultaneously !
Its behaviour approaches that of a norator!

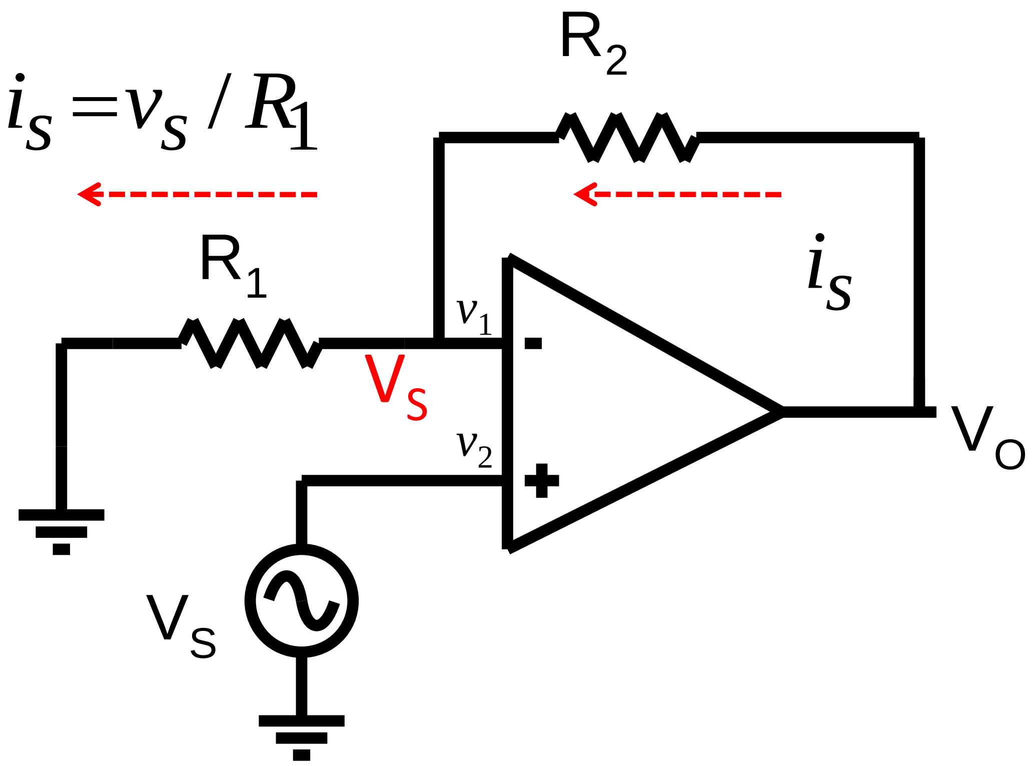
# Inverting Amplifier

Re-analyze inverting amplifier with the concepts discussed




$$\frac{0 - v_O}{R_2} = i_S = \frac{v_S}{R_1} \qquad \frac{v_O}{v_S} = -\frac{R_2}{R_1}$$

# Non-Inverting Amplifier



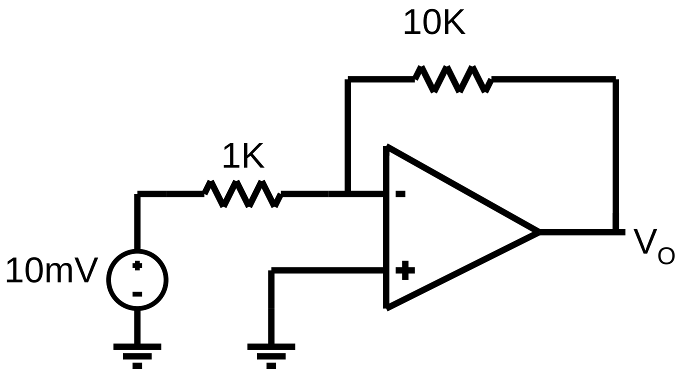

1. $v_1 = v_2$

2. $i_1 = i_2 = 0$

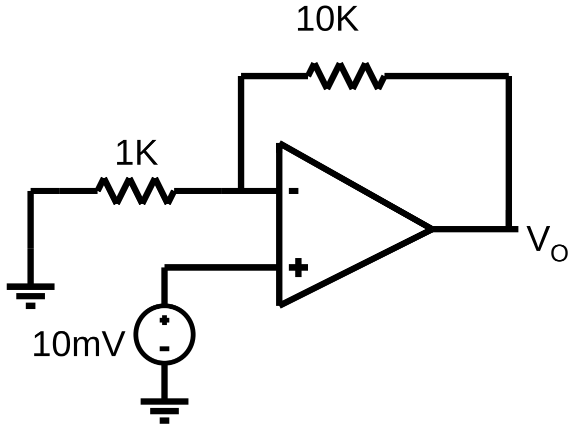
$$\frac{v_o - v_S}{R_2} = i_s = \frac{v_s}{R_1}$$

$$\frac{v_o}{v_S} = 1 + \frac{R_2}{R_1}$$

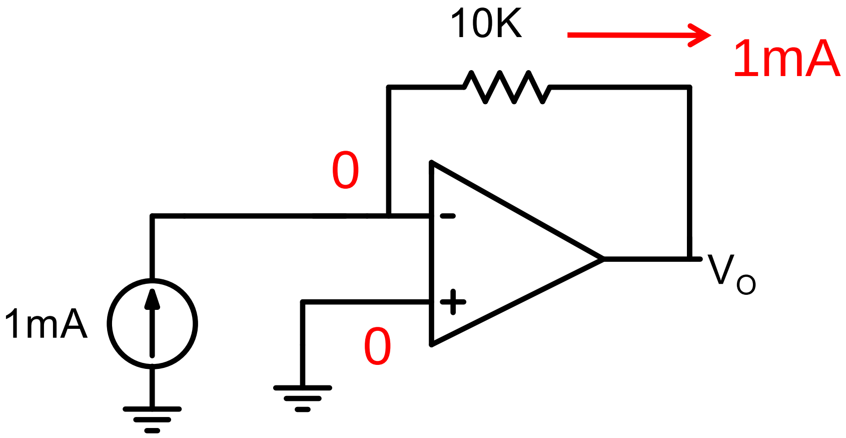
## Examples




$$\frac{v_o}{v_S} = -\frac{R_2}{R_1} \Rightarrow v_o = -100mV$$




$$\frac{v_o}{v_S} = 1 + \frac{R_2}{R_1} \Rightarrow v_o = 110mV$$

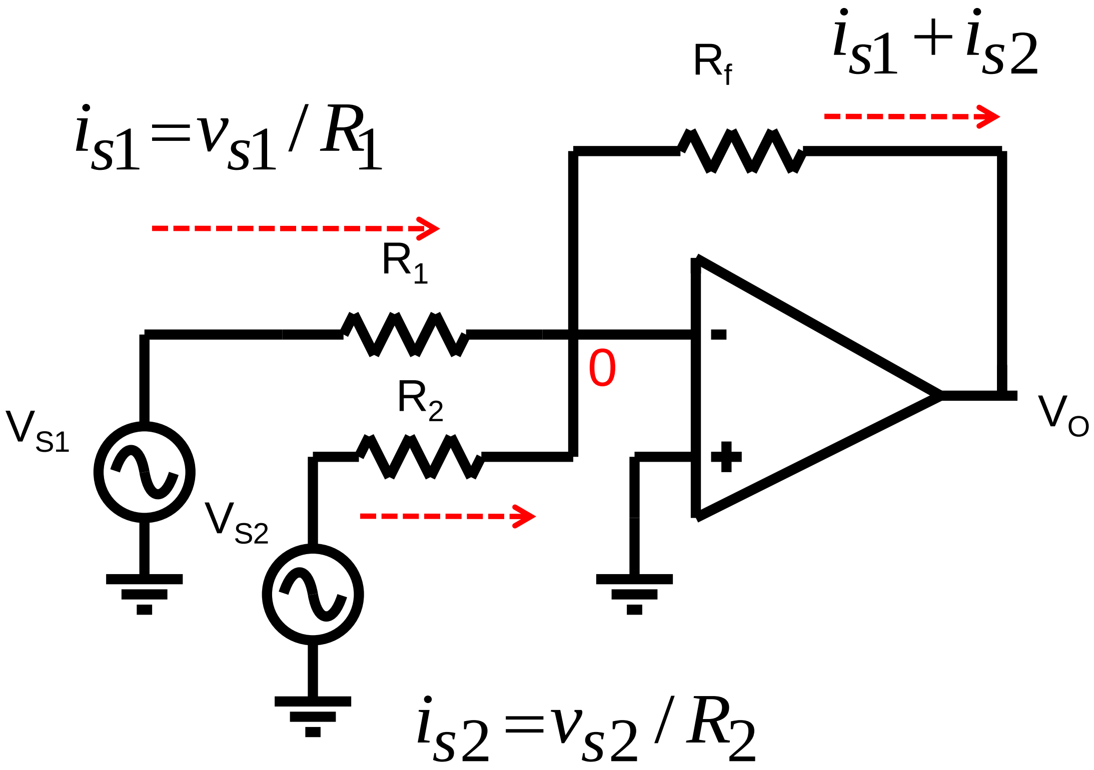



$$\frac{0 - v_o}{10K} = 1mA \qquad v_o = -10V$$

# Op-amp with Negative Feedback

- We have seen we can build
  - Inverting amplifiers
  - Noninverting amplifiers
- We will see that we can build
  - Adders
  - Subtractors
  - Weighted adders and subtractors
- And further
  - Integrator, differentiator
  - Log amplifier, antilog amplifier
  - Multiplier
  - Temperature sensor, ….
  - And much more!

# Adder

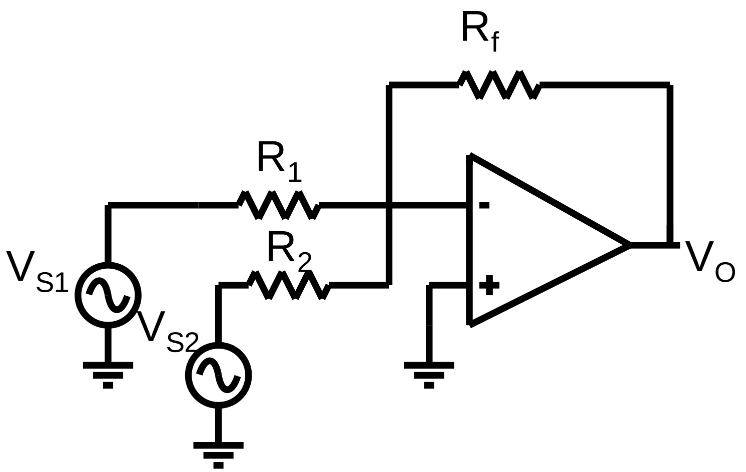


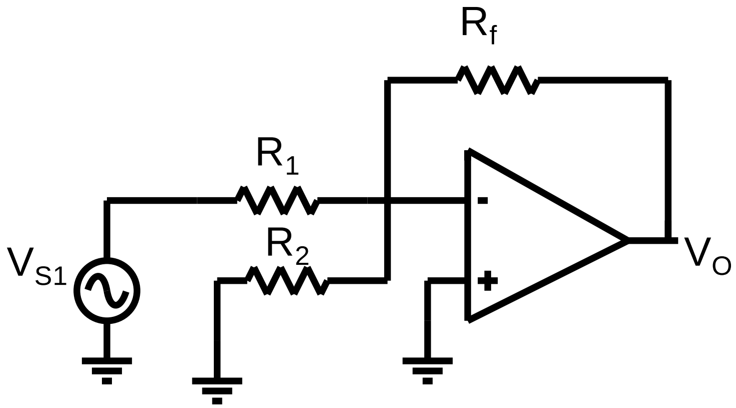

$$\frac{0-v_o}{R_f}=i_{s1}+i_{s2}=\frac{v_{s1}}{R_1}+\frac{v_{s2}}{R_2}$$

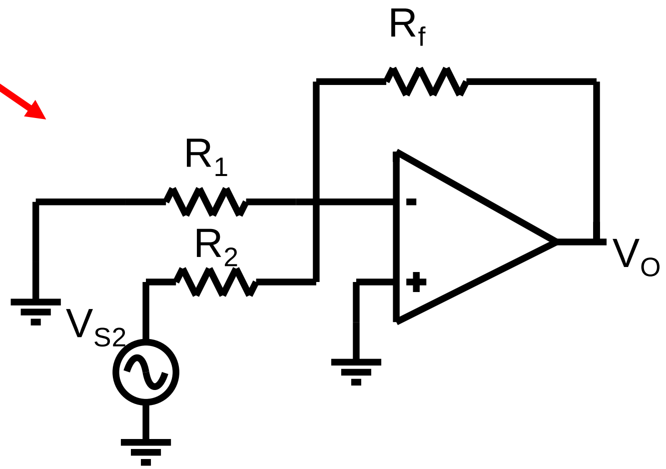
$$v_o=-\left(\frac{R_f}{R_1}v_{s1}+\frac{R_f}{R_2}v_{s2}\right)$$

For $R_1=R_2=R$ $\quad v_o=-\frac{R_f}{R}(v_{s1}+v_{s2})$

# Adder: Alternative Analysis



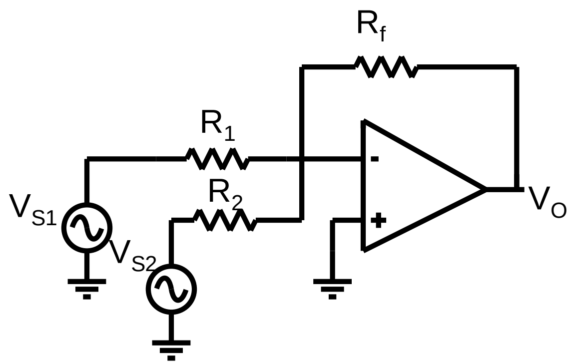




$$v_o = \left\{-\left(\frac{R_f}{R_1}\right)v_{s1}\right\} + \left\{-\left(\frac{R_f}{R_2}\right)v_{s1}\right\}$$

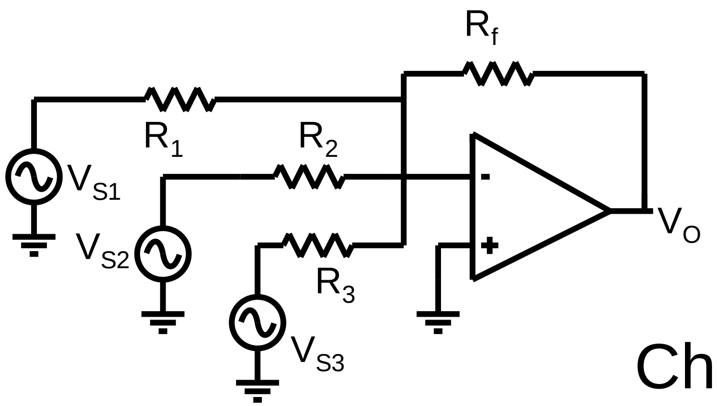
## Design Example 1

Design a circuit that would generate the following output given three input voltages $v_{s1}$, $v_{s2}$ and $v_{s3}$.

$$v_o = -10v_{s1} - 4v_{s2} - 5v_{s3}$$




$$v_o = -\frac{R_f}{R_1}v_{s1} - \frac{R_f}{R_2}v_{s2}$$




$$v_o = -\frac{R_f}{R_1}v_{s1} - \frac{R_f}{R_2}v_{s2} - \frac{R_f}{R_3}v_{s3}$$

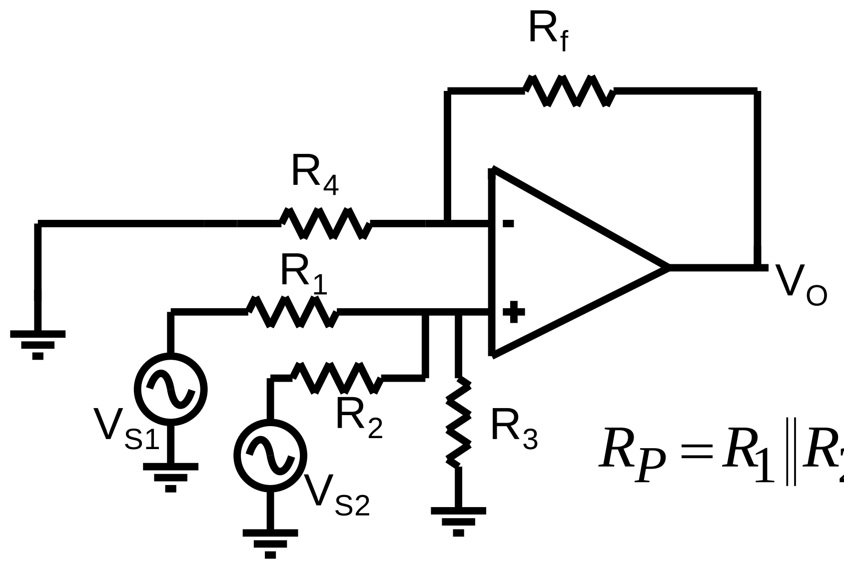
Choose : $R_f = 10\text{ k}\Omega$ $\Rightarrow R_1 = 1\text{ k}\Omega$

$\Rightarrow R_2 = 2.5\text{ k}\Omega$ $\Rightarrow R_3 = 2\text{ k}\Omega$

## Design Example 2

Design a circuit that would generate the following output given two input voltages $v_{s1}$ and $v_{s2}$.

$$v_o = 10v_{s1} + 4v_{s2}$$




$$v_o = (\frac{R_p}{R_1}v_{s1} + \frac{R_p}{R_2}v_{s2}) \times (1 + \frac{R_f}{R_4})$$

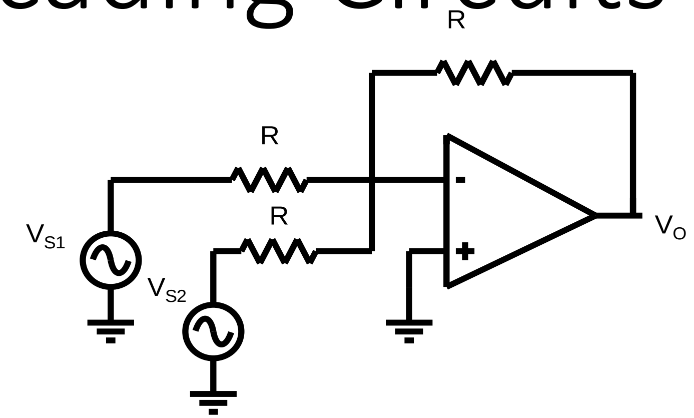
$$R_P = R_1 \| R_2 \| R_3$$

The voltage at the non-inverting terminal due to each signal has to be evaluated - it is a voltage divider configuration for each of the supply considered separately.

Voltage contribution by $i^{th}$ source to input node: $v_i \cdot (R_p/R_i)$

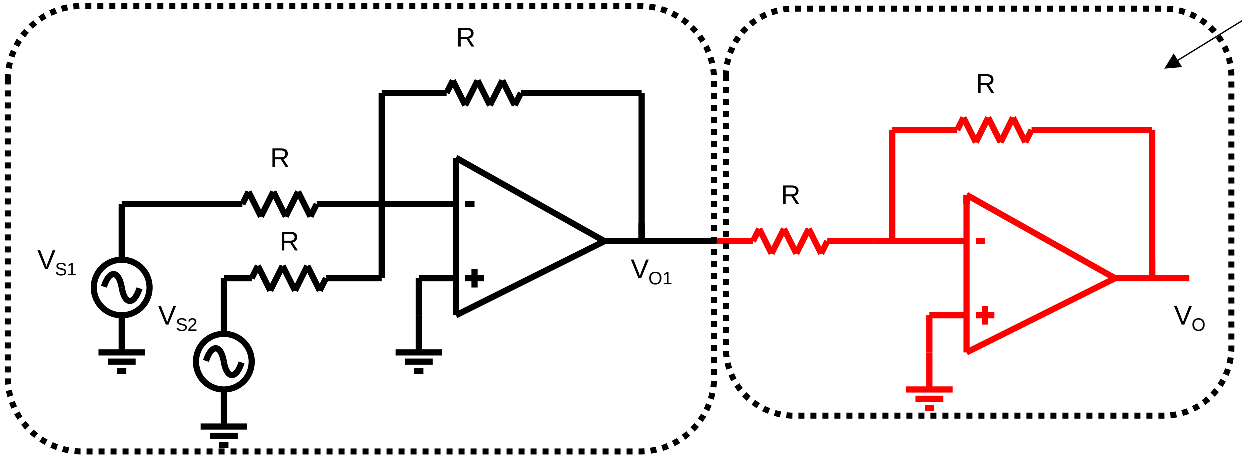
**Why don't you give it a try and design the circuit?**

# Cascading Circuits




$$v_o = -(v_{s1} + v_{s2})$$

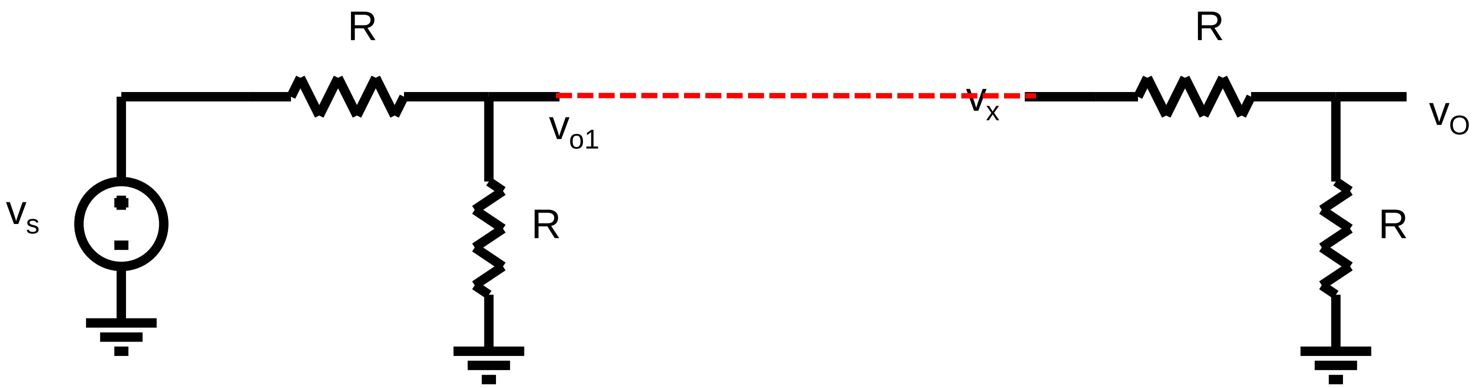
Unity gain inverting circuit




$$v_{o1} = -(v_{s1} + v_{s2}) \qquad v_o = -v_{o1} \quad \Rightarrow \quad v_o = (v_{s1} + v_{s2})$$

Have we made some assumption here ? When does the assumption hold?

## Example 1




$$\frac{v_{o1}}{v_s} = 0.5$$

$$\frac{v_o}{v_x} = 0.5$$

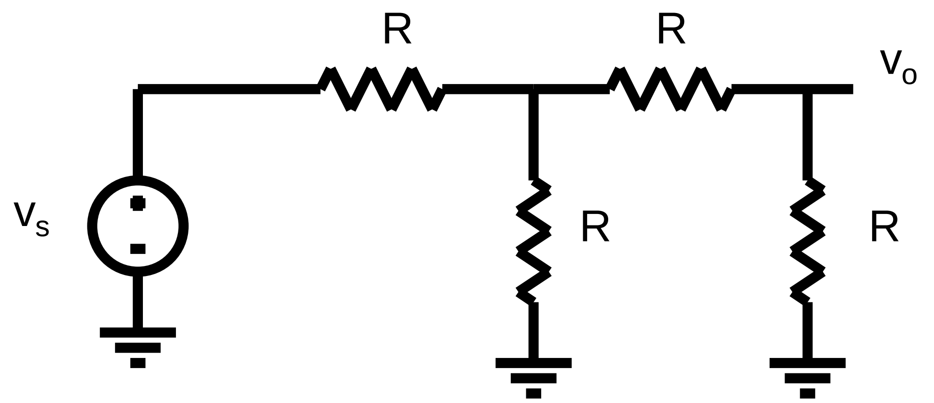
Connect the two circuits. $v_{o1} = v_x$

$$\frac{v_o}{v_x} = \frac{v_o}{v_{o1}} = 0.5$$

$$\frac{v_o}{v_s} = \frac{v_o}{v_{o1}} \times \frac{v_{o1}}{v_s} = 0.5 \times 0.5 = 0.25$$

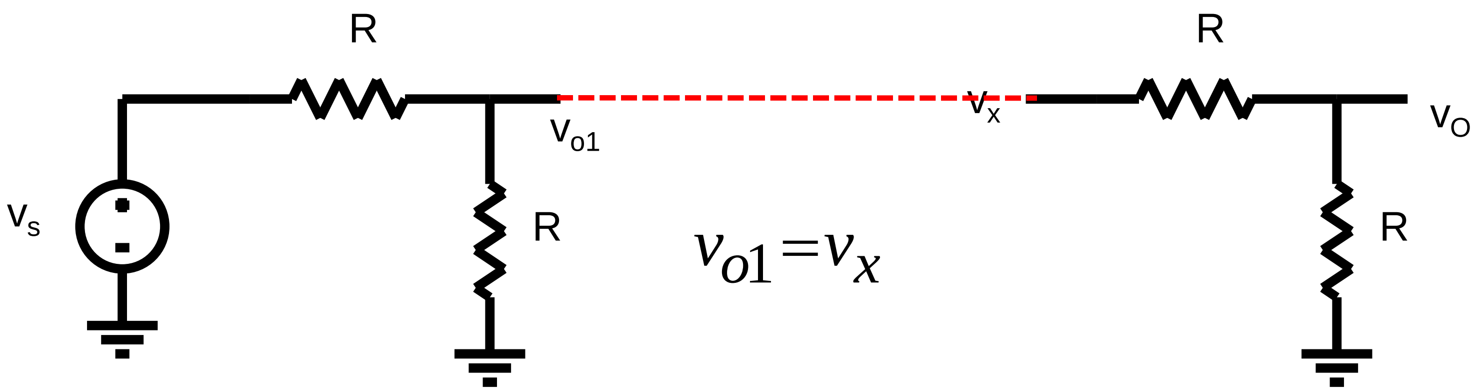
Is this indeed true? When can this be valid even as an approximation?

BUT




$$\frac{v_o}{v_s} = 0.2$$

Where is the error ?



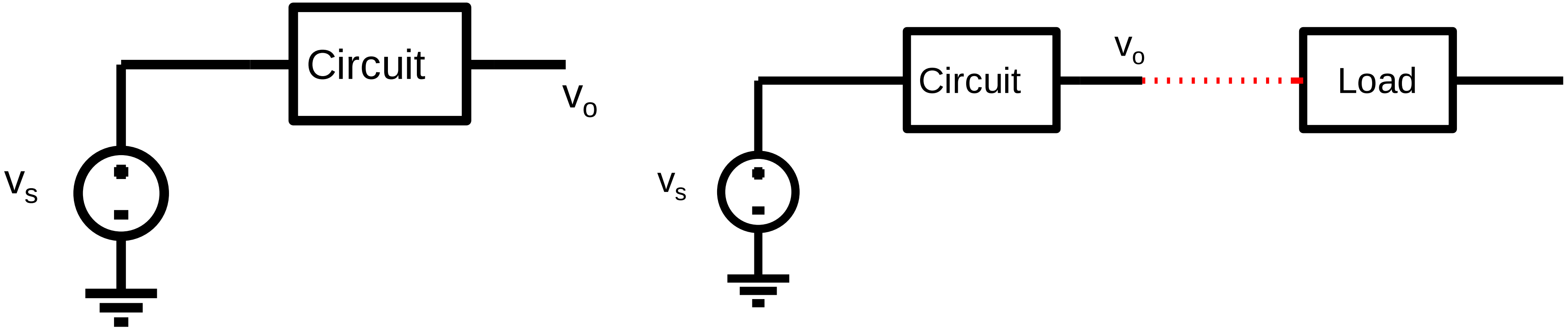

$$v_{o1} = v_x$$

$$\frac{v_{o1}}{v_s} = 0.5 \qquad \frac{v_{o1}}{v_s} \neq 0.5 \qquad \frac{v_o}{v_x} = 0.5$$

Left-side circuit gets 'loaded' by the right-side circuit.

Left-side circuit's output vs. input characteristics get modified due to the loading.

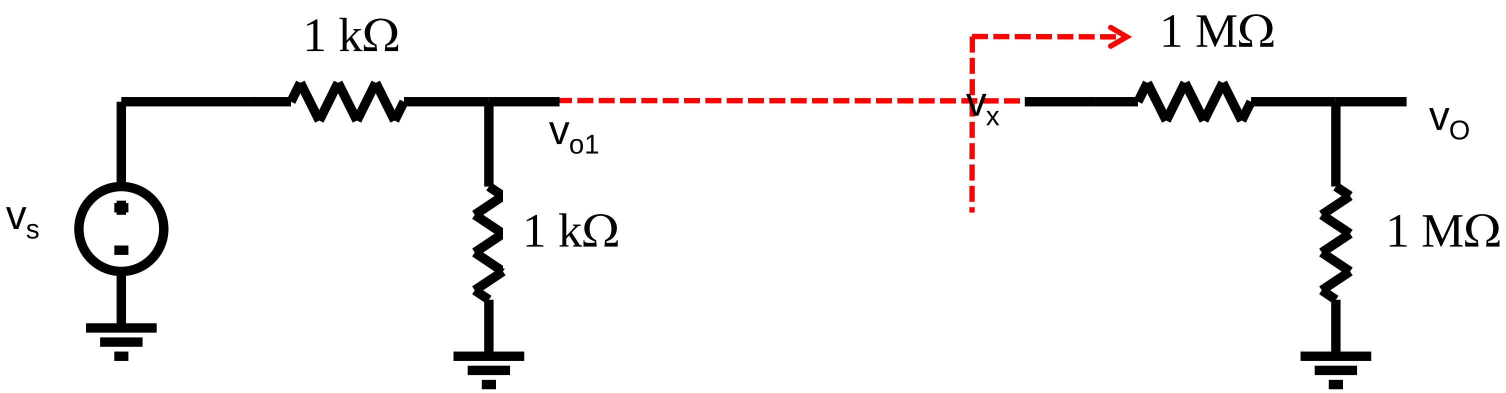
# Loading Effect




$V_o$ in general gets altered when we connect a load to it

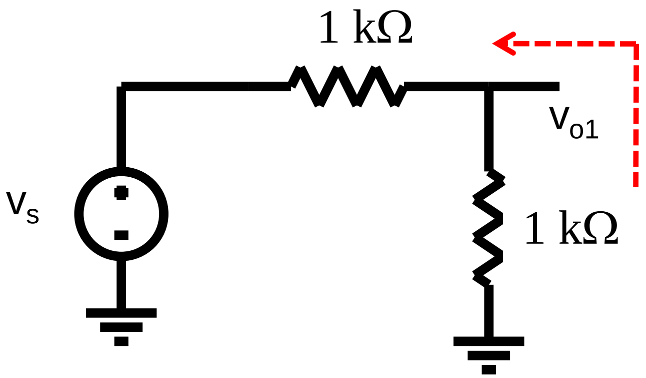
Under what conditions is change in $V_O$ small upon connection of a load ?

# Example 2




$$\frac{v_{o1}}{v_s} = 0.5 \qquad \frac{v_{o1}}{v_s} \cong 0.5$$

We can describe this effect in terms of output resistance



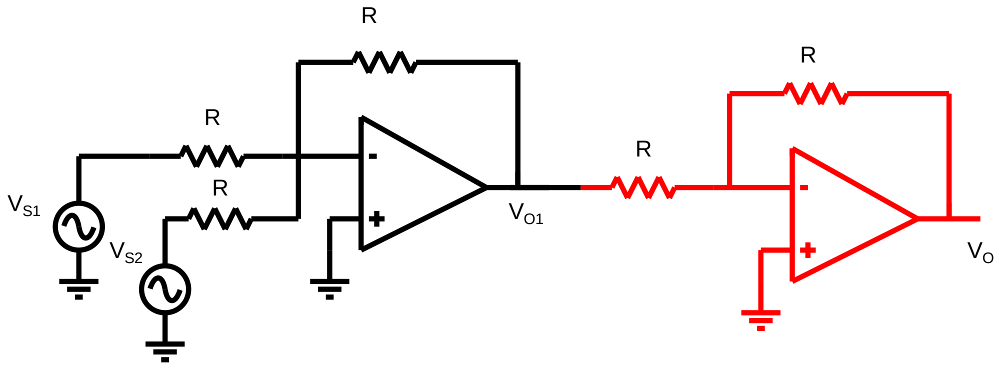

$$R_o = 0.5\ \text{k}\Omega \qquad R_L = 2\ \text{M}\Omega$$

# Loading Effect

Whenever output resistance of a circuit is much smaller than the load resistance, the loading effect is minimal.

$$R_O << R_L$$

# Cascading Circuits (contd.)




$$v_{o1} = -(v_{s1} + v_{s2}) \qquad v_o = -v_{o1} \qquad v_o = (v_{s1} + v_{s2})$$

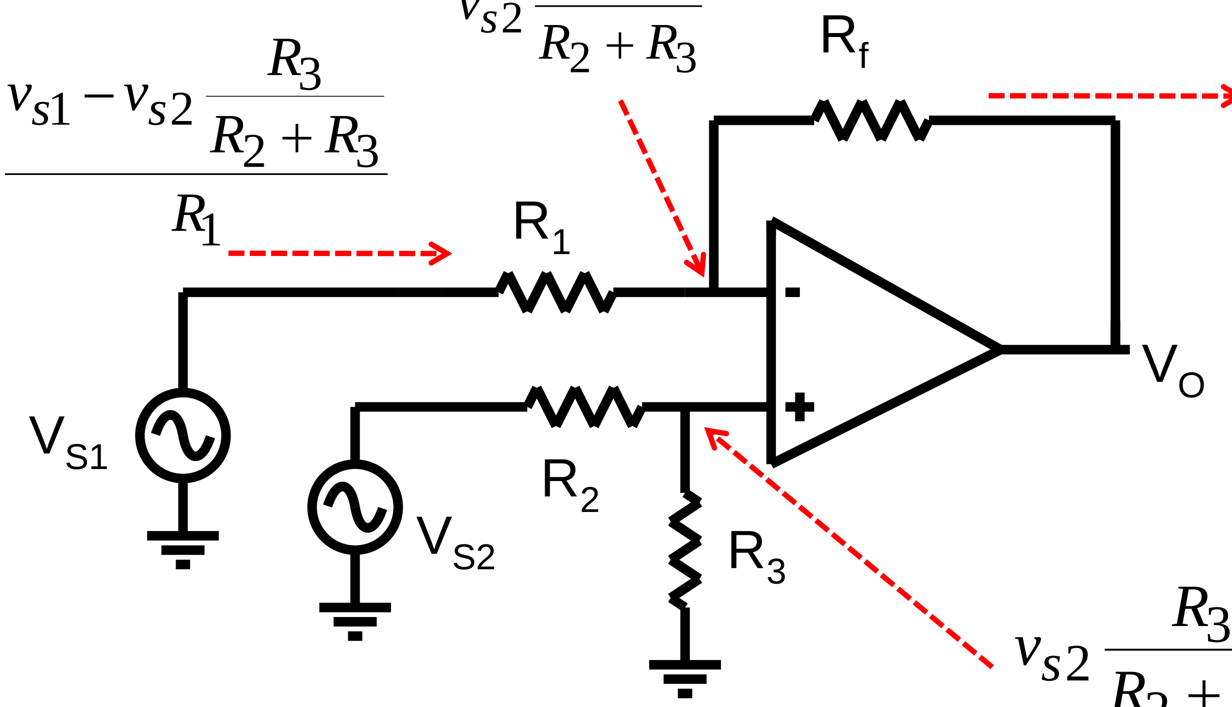
The assumption made here is that there is no loading

A reasonable because op-amps output has very low resistance

# Subtractor

$$v_{s1} - v_{s2}\frac{R_3}{R_2+R_3} \over R_1$$

$$v_{s2}\frac{R_3}{R_2+R_3}$$



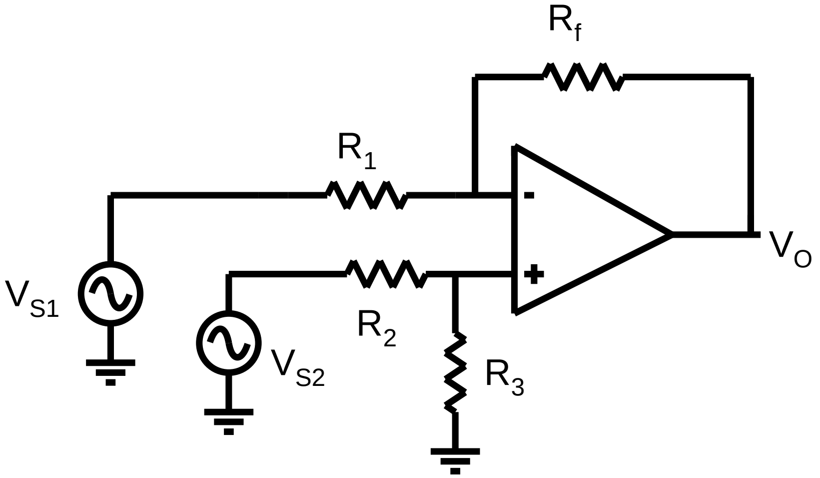

$$\frac{v_{s1} - v_{s2}\frac{R_3}{R_2+R_3}}{R_1}$$

$$v_{s2}\frac{R_3}{R_2+R_3}$$

$$\frac{v_{s2}\frac{R_3}{R_2+R_3} - v_o}{R_f} = \frac{v_{s1} - v_{s2}\frac{R_3}{R_2+R_3}}{R_1}$$

$$v_o = v_{s2}\frac{\frac{R_3}{R_2}}{(1+\frac{R_3}{R_2})}(1+\frac{R_f}{R_1}) - (\frac{R_f}{R_1})v_{s1}$$

# Subtractor Output

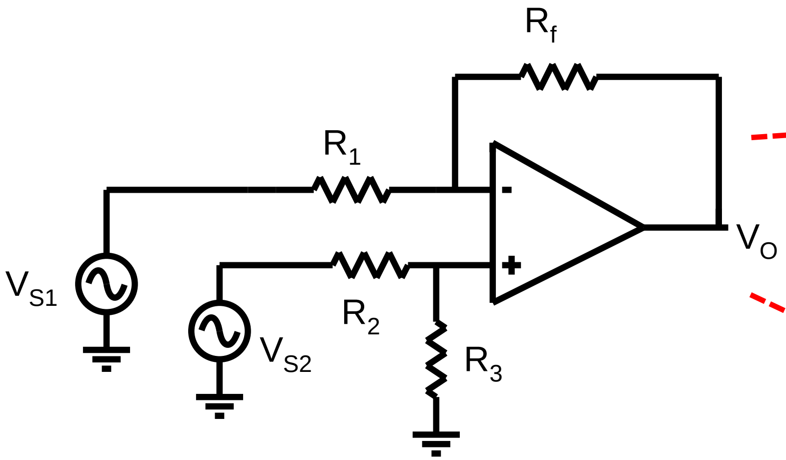



$$v_o = v_{s2}\frac{\frac{R_3}{R_2}}{(1+\frac{R_3}{R_2})}(1+\frac{R_f}{R_1}) - (\frac{R_f}{R_1})v_{s1}$$

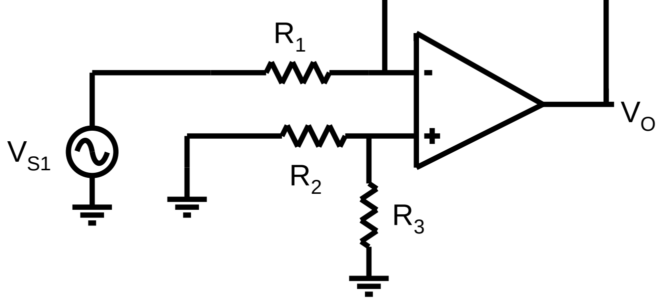
Choose $\frac{R_3}{R_2} = \frac{R_f}{R_1}$

$$v_o = \frac{R_f}{R_1}(v_{s2} - v_{s1})$$

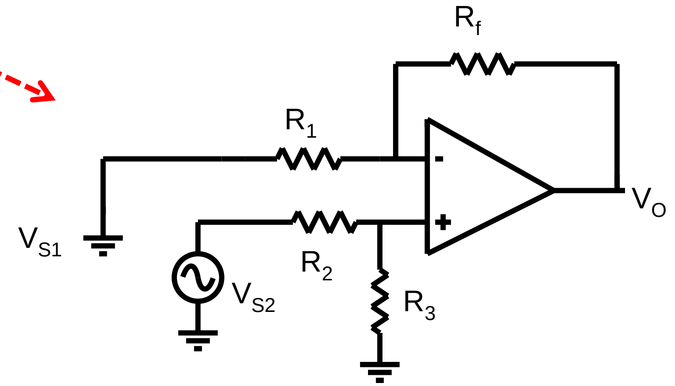
# Subtractor: Alternative Analysis




Inverting amplifier




Non-inverting amplifier

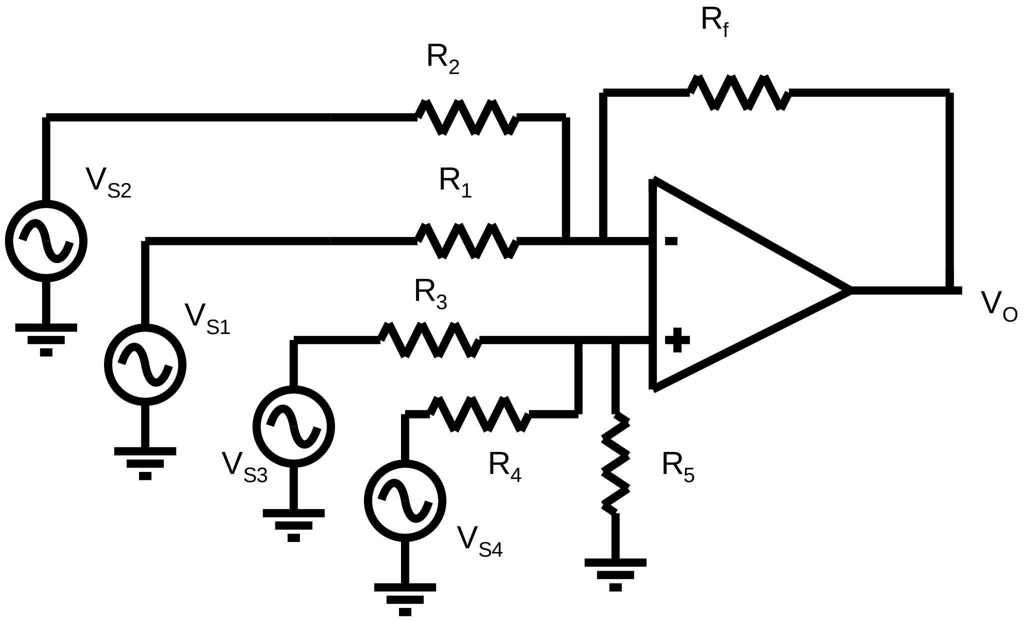



Use superposition theorem

$$v_o = -(\frac{R_f}{R_1})v_{s1} + \left\{ v_{s2}\frac{R_3}{(R_3+R_2)} \times (1+\frac{R_f}{R_1}) \right\}$$

Analysis is made simpler by **Re-Using** results derived earlier

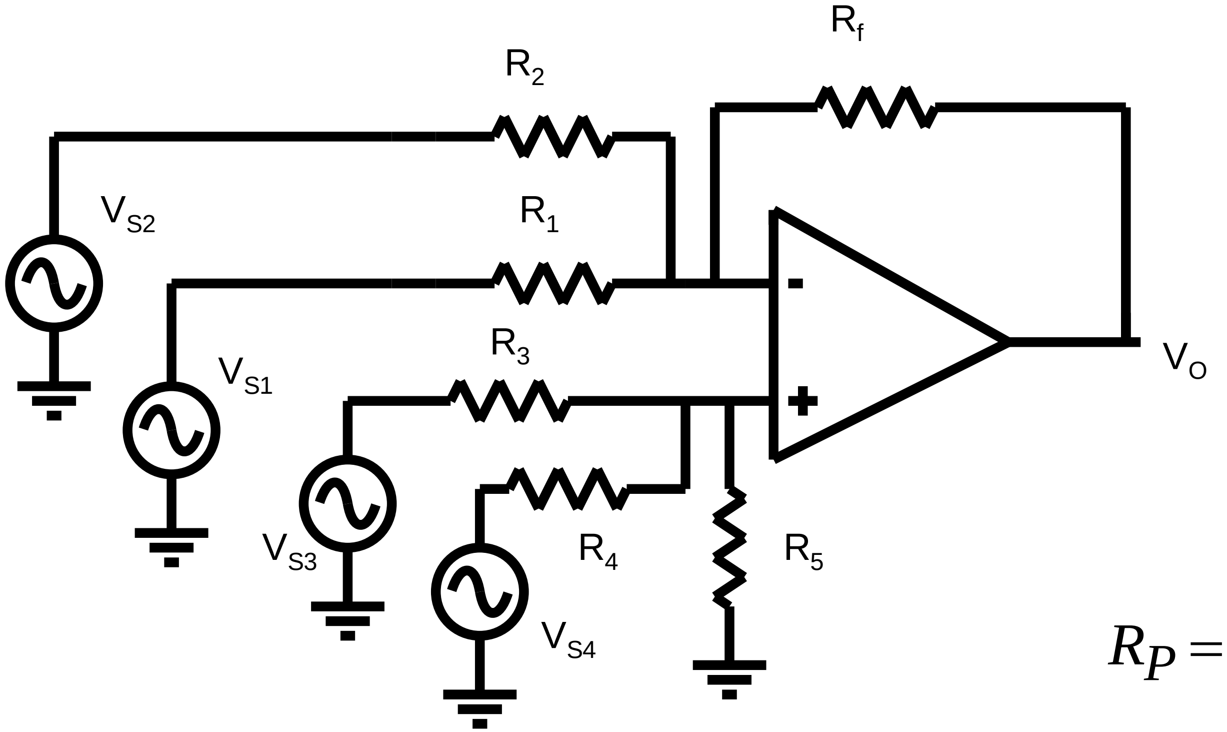
# Adder/Subtractor




$$v_o = -(\frac{R_f}{R_1})v_{s1} + \left\{-(\frac{R_f}{R_2})v_{s2}\right\} + \left\{v_{s3}\ \frac{R_5 \| R_4}{R_5 \| R_4 + R_3} \times (1 + \frac{R_f}{R_1 \| R_2})\right\}$$

$$+ \left\{v_{s4}\ \frac{R_5 \| R_3}{R_5 \| R_3 + R_4} \times (1 + \frac{R_f}{R_1 \| R_2})\right\}$$

# Adder/Subtractor – contd.

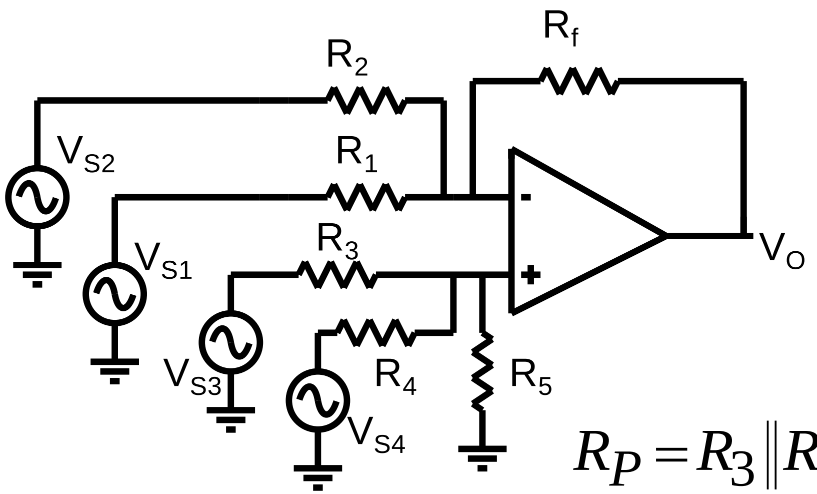



$$R_P = R_3 \| R_4 \| R_5$$

$$v_o = -\left(\frac{R_f}{R_1}\right)v_{s1} \quad + \quad -\left(\frac{R_f}{R_2}\right)v_{s2} \quad + \quad v_{s3}\,\frac{R_P}{R_3} \quad \times\left(1+\frac{R_f}{R_1 \| R_2}\right)$$

$$+ \qquad v_{s4}\,\frac{R_P}{R_4} \quad \times\left(1+\frac{R_f}{R_1 \| R_2}\right)$$

## Example




$$v_o = -10v_{s1} - 4v_{s2} + 5v_{s3} + 2v_{s4}$$

$$R_P = R_3 \| R_4 \| R_5$$

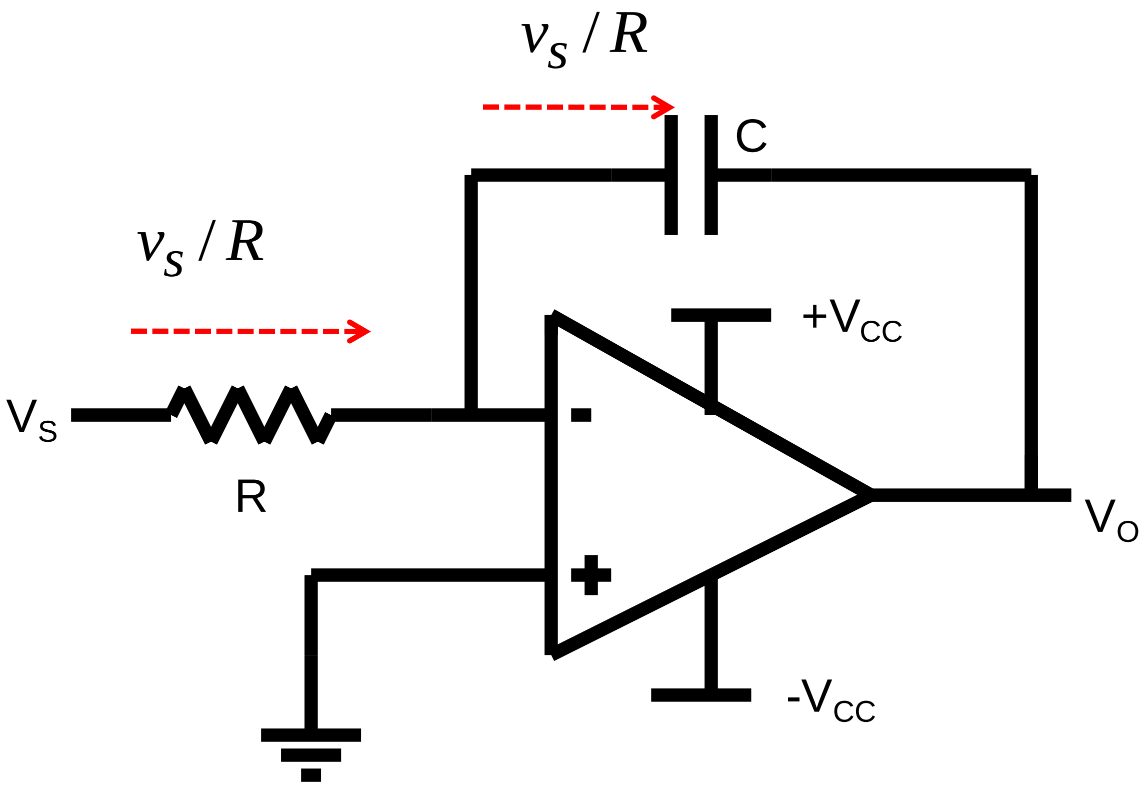
$$v_o = -(\frac{R_f}{R_1})v_{s1} - -(\frac{R_f}{R_2})v_{s2} + (1+\frac{R_f}{R_1\|R_2})\times\frac{R_P}{R_3}v_{s3} + (1+\frac{R_f}{R_1\|R_2})\times\frac{R_P}{R_4}v_{s4}$$

Choose : $R_f = 10\ \mathrm{k\Omega}$ $\Rightarrow R_1 = 1\ \mathrm{k\Omega}$ $\Rightarrow R_2 = 2.5\ \mathrm{k\Omega}$

$$\Rightarrow \frac{R_P}{R_3} = 0.33 \qquad \Rightarrow \frac{R_P}{R_4} = 0.133 \qquad \Rightarrow \frac{R_4}{R_3} = 2.5$$

Choose : $R_3 = 1\ \mathrm{k\Omega}$ $\Rightarrow R_4 = 2.5\ \mathrm{k\Omega}$ $\Rightarrow R_P = 0.33\ \mathrm{k\Omega}$ $\Rightarrow R_5 = 0.625\ \mathrm{k\Omega}$

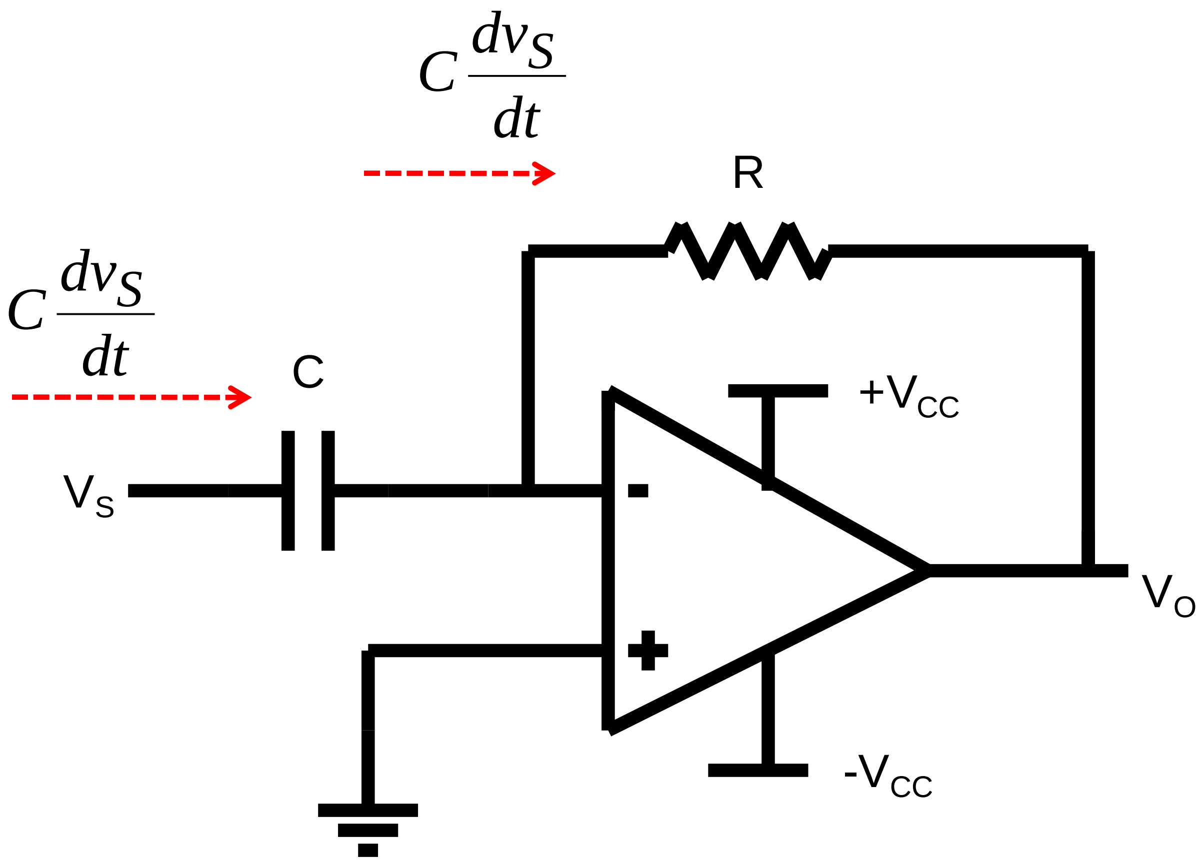
# Integrator




$$\frac{v_S}{R} = -C\frac{dv_O}{dt} \Rightarrow v_O(t) = -\frac{1}{RC}\int v_S\,dt$$

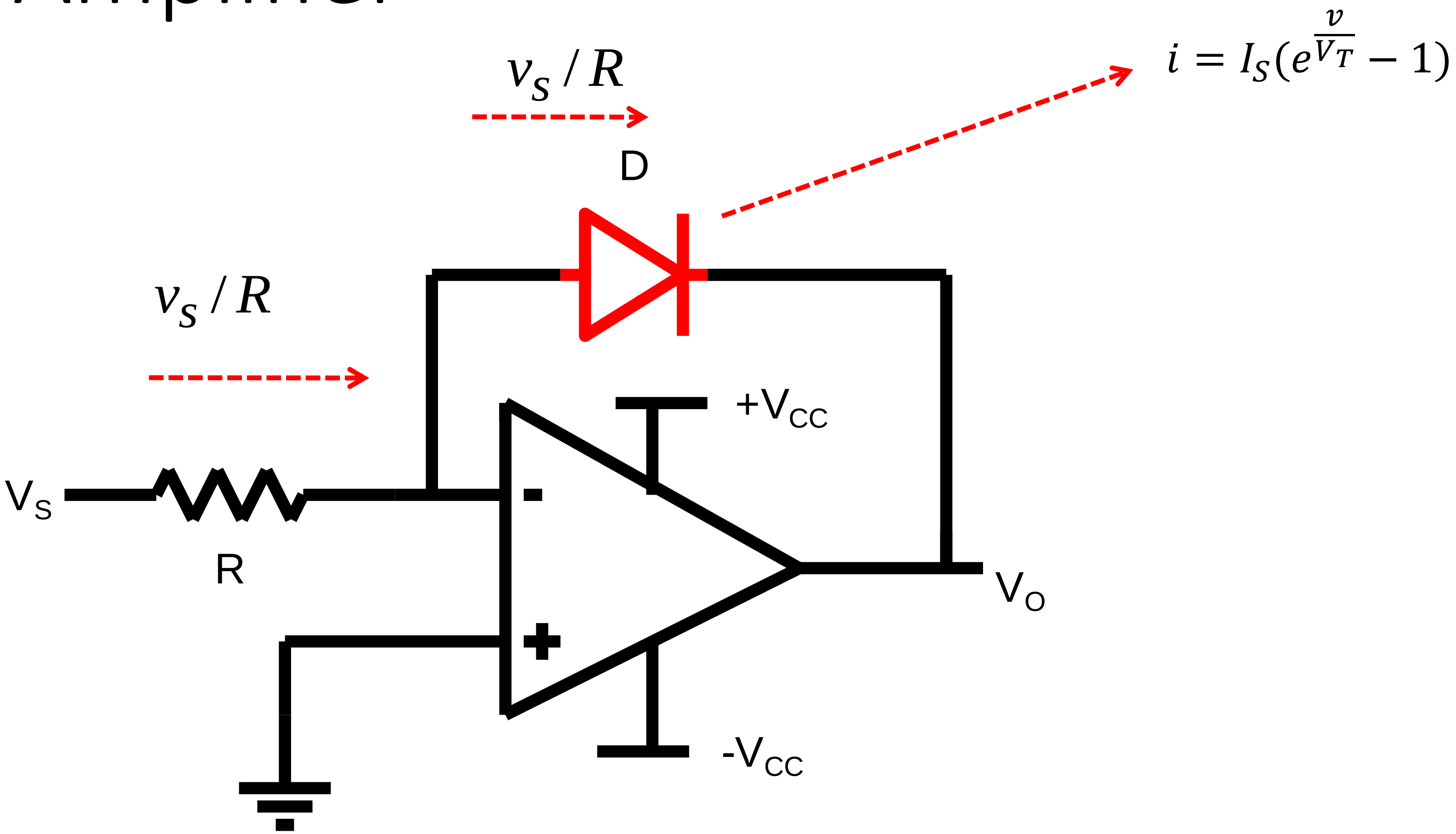
Caution: To ensure negative feedback for dc component in Vs, a large resistor is often connected in parallel to C. Else, the dc component of input might saturate the output voltage

# Differentiator



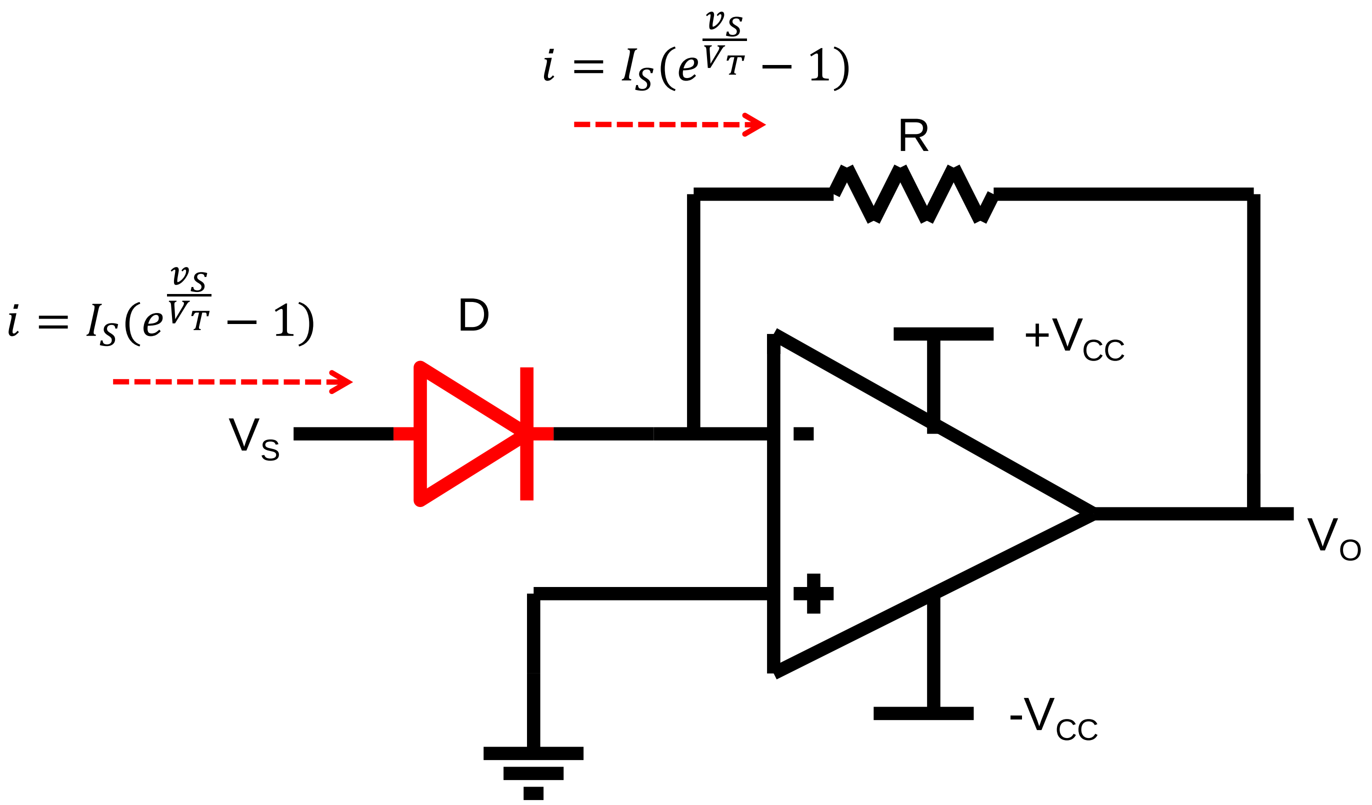

$$-\frac{v_O}{R} = C\frac{dv_S}{dt} \Rightarrow v_O(t) = -RC\frac{dv_S}{dt}$$

# Log Amplifier




$$\frac{v_S}{R} = I_S\left(e^{-\frac{v_O}{V_T}} - 1\right) \Rightarrow -v_O = V_T \times \ln(1 + \frac{v_S}{Ri_S}) \cong V_T \times \ln(\frac{v_S}{RI_S})$$

# Antilog Amplifier




$$-\frac{v_O}{R} = I_S\left(e^{\frac{v_S}{V_T}} - 1\right) \Rightarrow v_O = -RI_S(e^{\frac{v_S}{V_T}} - 1) \cong -RI_S \times e^{\frac{v_S}{V_T}}$$

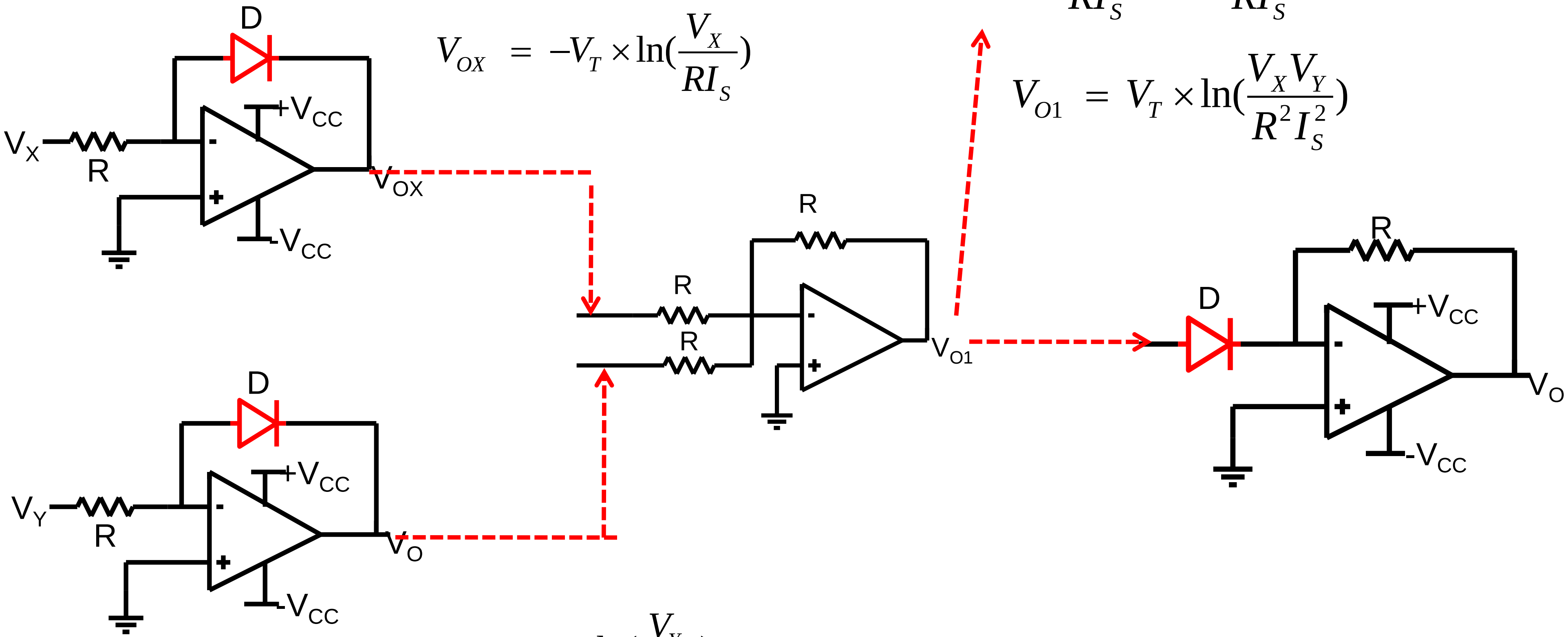
# Multiplier

Use log and anti-log functions

$$V_{O1} = V_T \times (\ln(\frac{V_X}{RI_S}) + \ln(\frac{V_Y}{RI_S}))$$

$$V_{OX} = -V_T \times \ln(\frac{V_X}{RI_S})$$

$$V_{O1} = V_T \times \ln(\frac{V_X V_Y}{R^2 I_S^2})$$

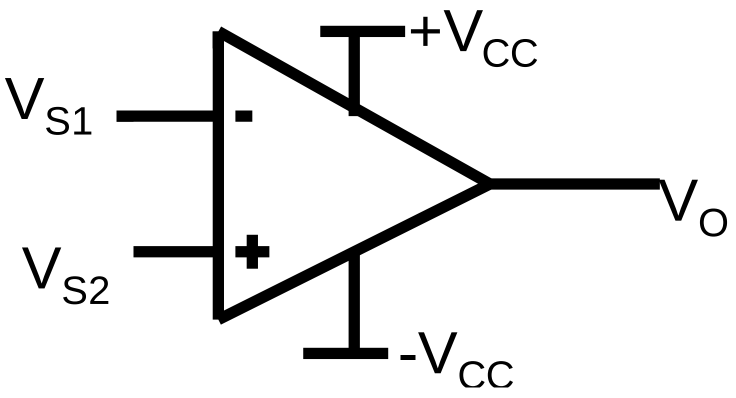


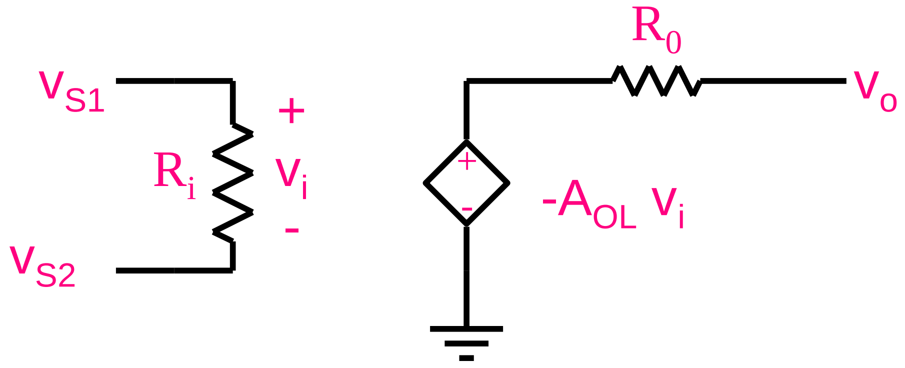

$$V_{OY} = -V_T \times \ln(\frac{V_Y}{RI_S})$$

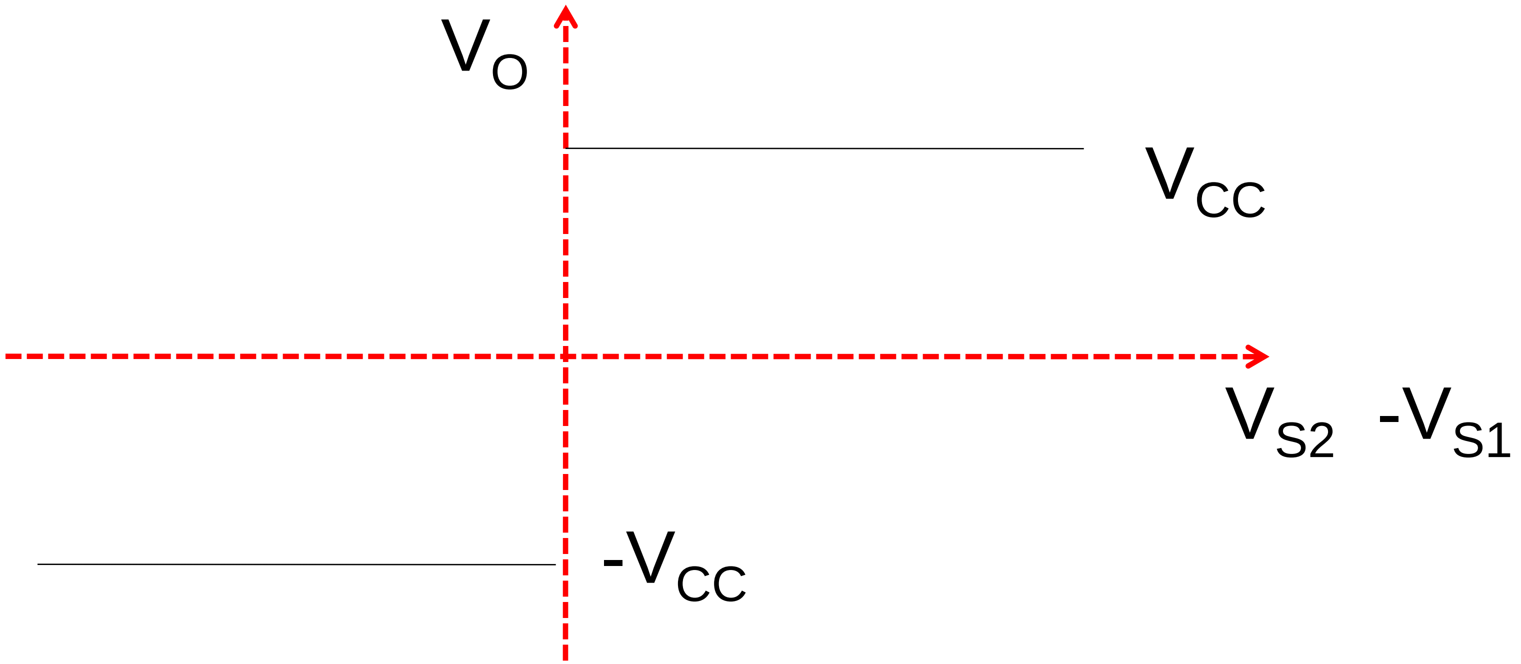
$$V_O \cong -RI_S \times e^{\frac{V_{O1}}{V_T}} = -\frac{V_X V_Y}{RI_S}$$

# Comparator

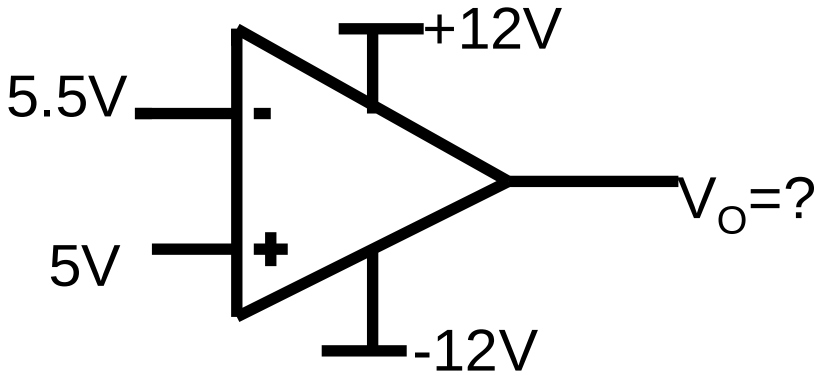
Open Loop condition



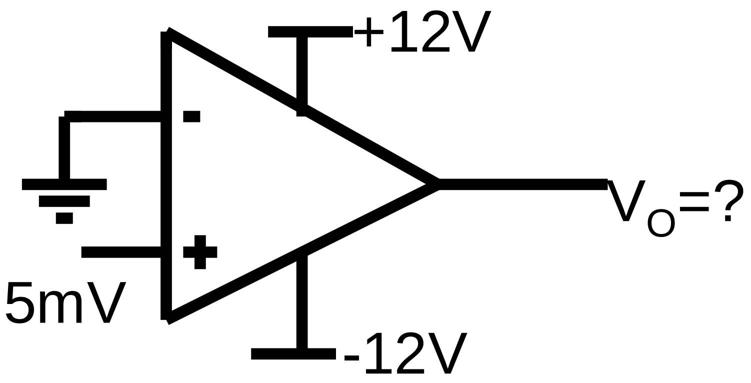



Example 1



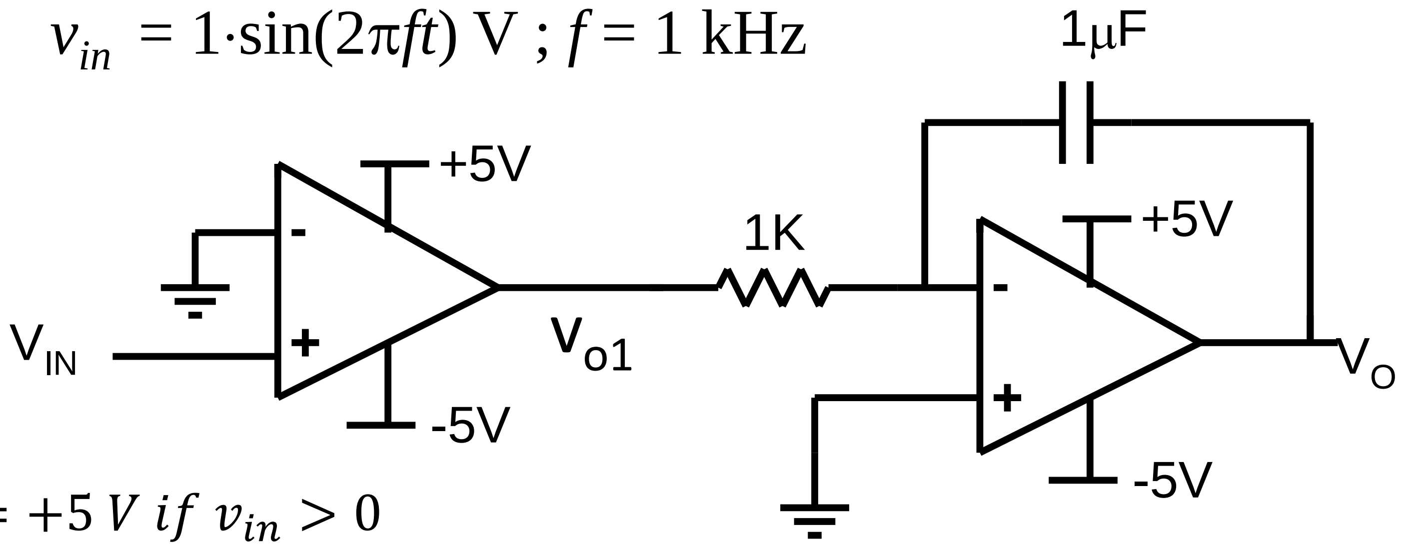

~ -12V




~ +12V

# Example 2

$$v_{in} = 1\cdot\sin(2\pi ft)\ \text{V}\ ;\ f = 1\ \text{kHz}$$




$$V_{O1} = +5\ V\ if\ v_{in} > 0$$
$$= -5\ V\ if\ v_{in} < 0$$