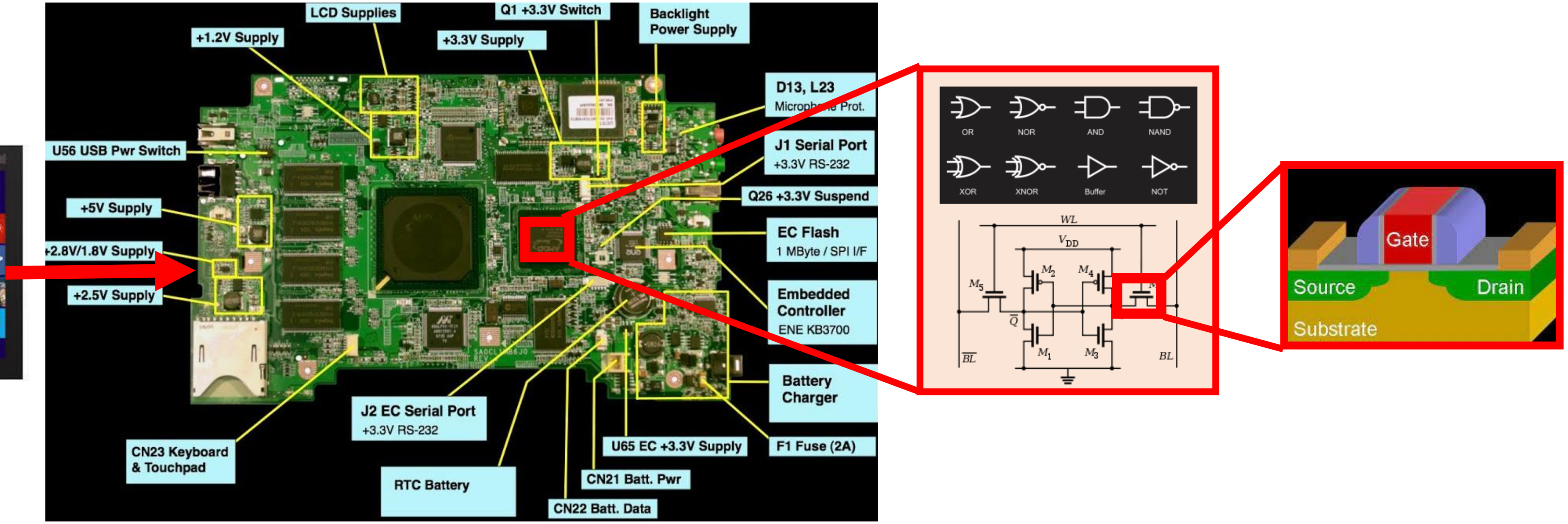

ESC201: Introduction to Electronics

Module 1: Circuit Analysis

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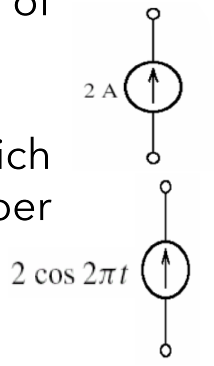
my GOV
मेरी सरकार
ONE INDIA, ONE FESTIVAL
MANY CELEBRATIONS
SHISHUR SAENKRAAT
MAGHI/LOHRI
MAGH BIHU
KHICHDI PARV
UTTARAYAN
MAKAR SAKRAAT
POUSH SÔNGKRĀNTI
MAKAR SANKRANTI
MAKARA SANKRANTI
PEDDA PANDUYA
MAKARA SANKRAMANA
PONGAL
MAKARA VILAKKU

Re-cap

Current: The time rate of flow of electrical charge $i(t) = \frac{dq(t)}{dt}$

– The units are amperes (A), which are equivalent to coulombs per second (*C/s*)

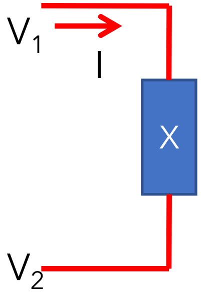
Direction of current flow is opposite to direction of electron flow



Voltage difference is the Source of current flow

Units of Voltage: Volts (V)

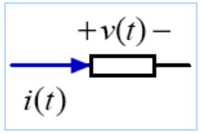
Power



$$P = (V_1 - V_2) \times I$$

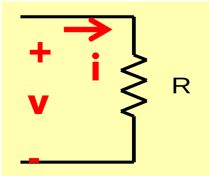
$$P(t) = \frac{dw}{dt} \Rightarrow \quad w = \int_{t1}^{t2} p(t)dt$$

Ohm's law



$$v(t) = R \times i(t)$$

$$i(t) = \frac{v(t)}{R} = G \times v(t)$$



$$P = i^2 \times R$$

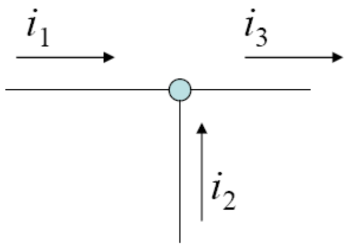
$$P = \frac{v^2}{R}$$

Two elements are connected in series if there is no other element connected to the node joining them. Same current flows

Two elements are connected in parallel if both ends of one element are connected directly to corresponding ends of the other. **Same voltage**

Kirchhoff's Current Law (KCL)

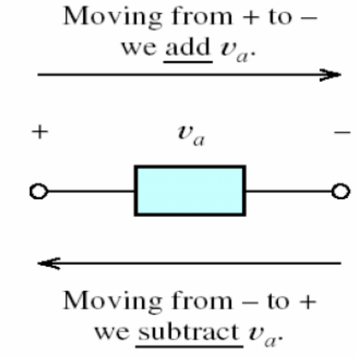
Sum of currents entering a node is equal to sum of currents leaving a node



$$i_1 + i_2 = i_3$$

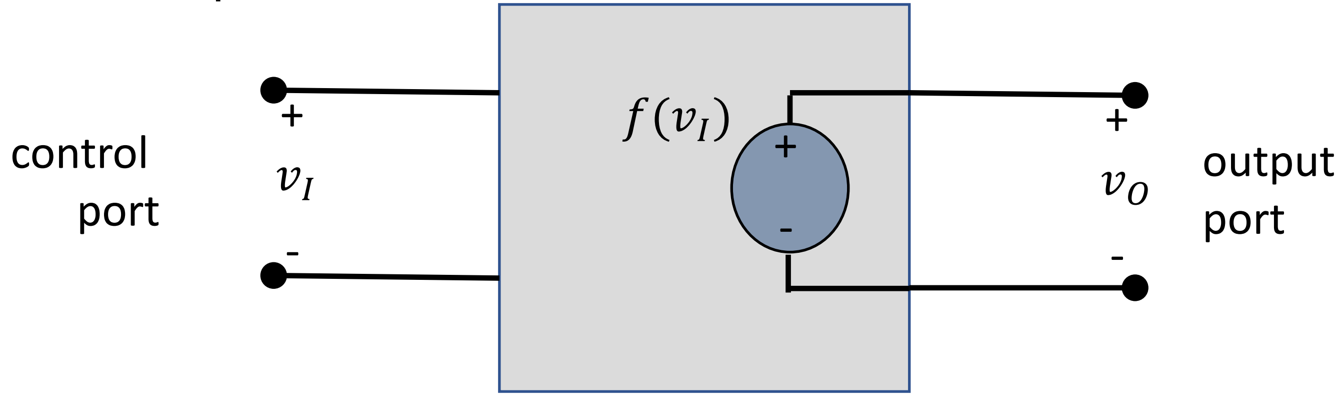
Kirchhoff's Voltage Law (KVL)

The algebraic sum of the voltages equals zero for any closed path (loop) in an electrical circuit

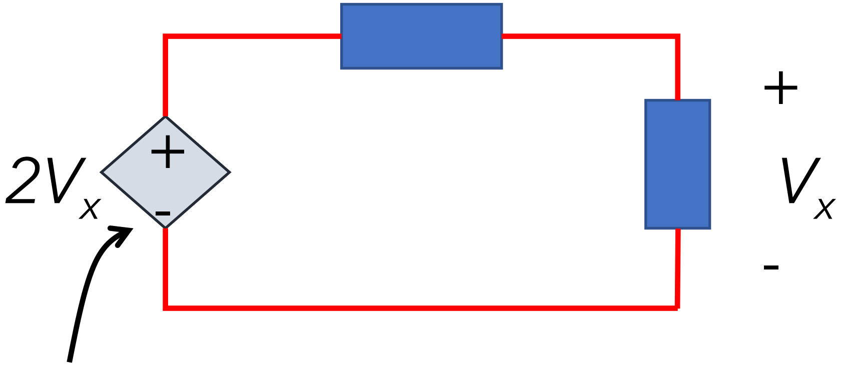


Dependent sources

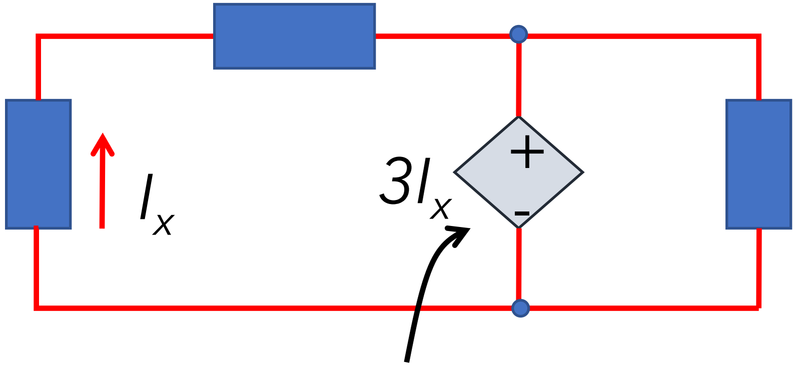
- A new lumped element
- Voltage/current source whose value depends on the voltage/current somewhere else in the circuit
- Can be viewed as a two-port device



Dependent (Controlled) Voltage Sources



Voltage-controlled Voltage source



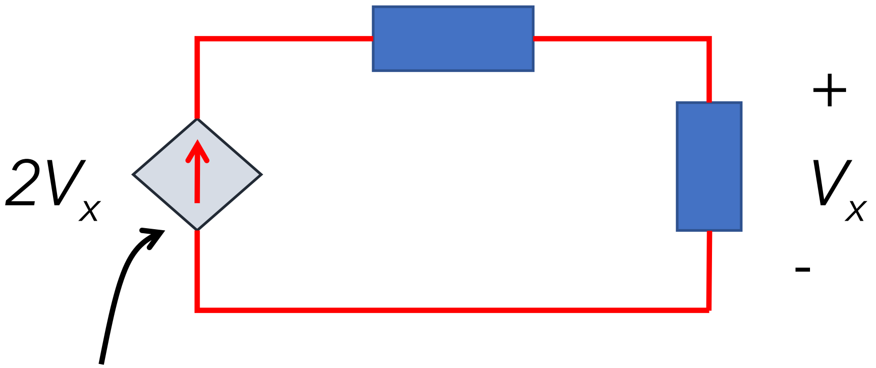
Current-controlled Voltage source

- Very useful in constructing circuit models for real-world devices such as transistors and amplifiers (we'll see examples in the course)

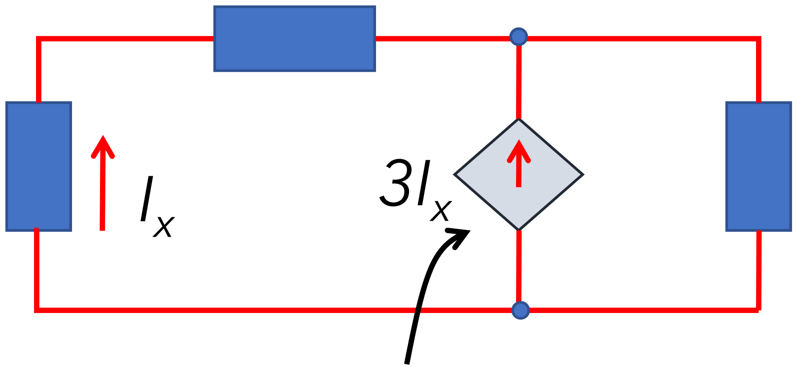
- For a voltage controlled source: $V = K_1 V_x$,
  K_1 is a gain parameter with no units

- For a current controlled source: $V = K_2 I_x$,
  K_2 is a gain parameter with units [V/A]

Dependent (Controlled) Current Sources



Voltage-controlled current source

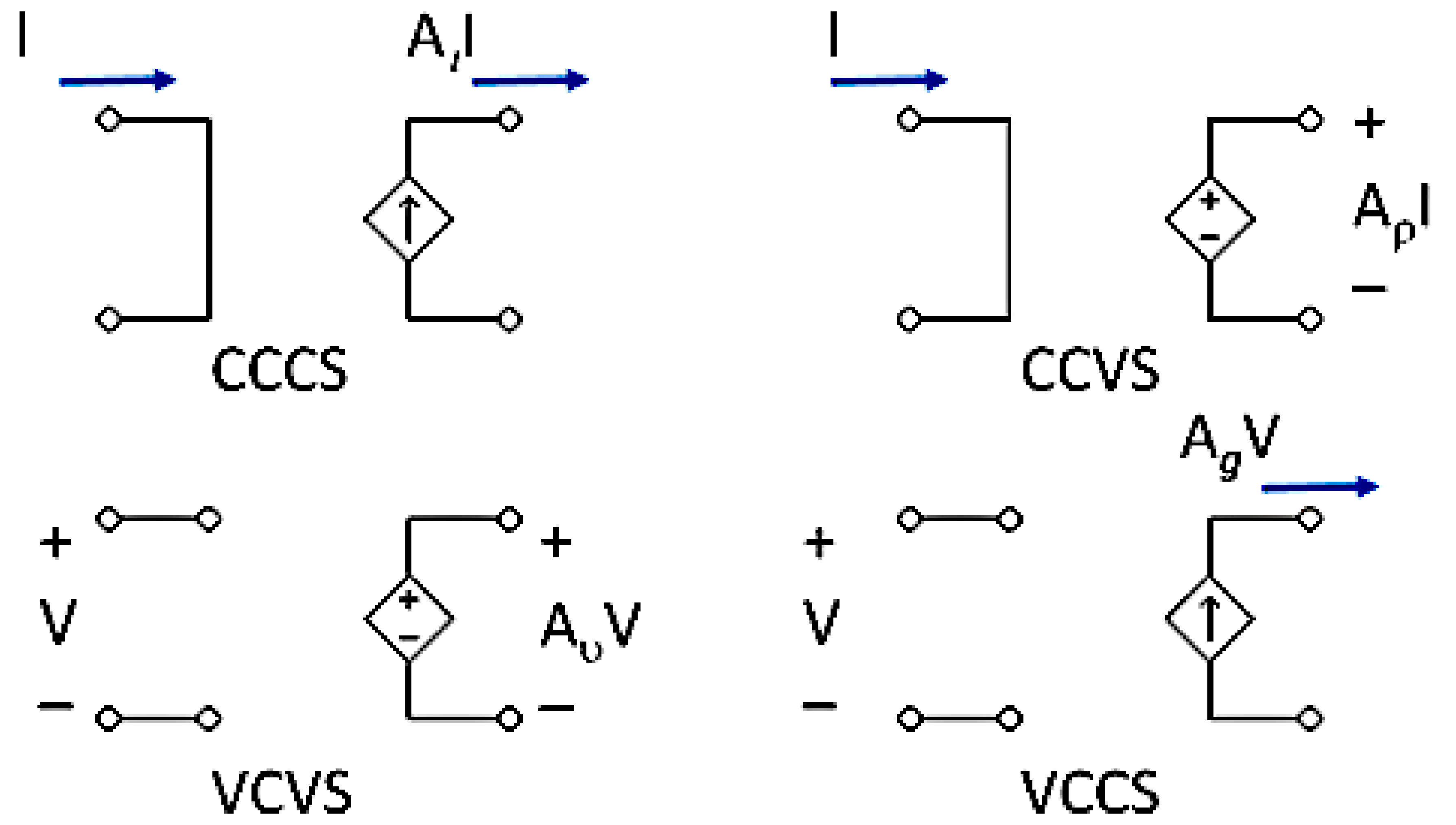


Current-controlled current source

- For a voltage controlled source: $I = K_3 V_x$
  K_3 *is a gain parameter with units* [A/V]

- For a current controlled source: $I = K_4 I_x$
  K_4 *is a gain parameter with no units*

Four combinations



Circuit Analysis

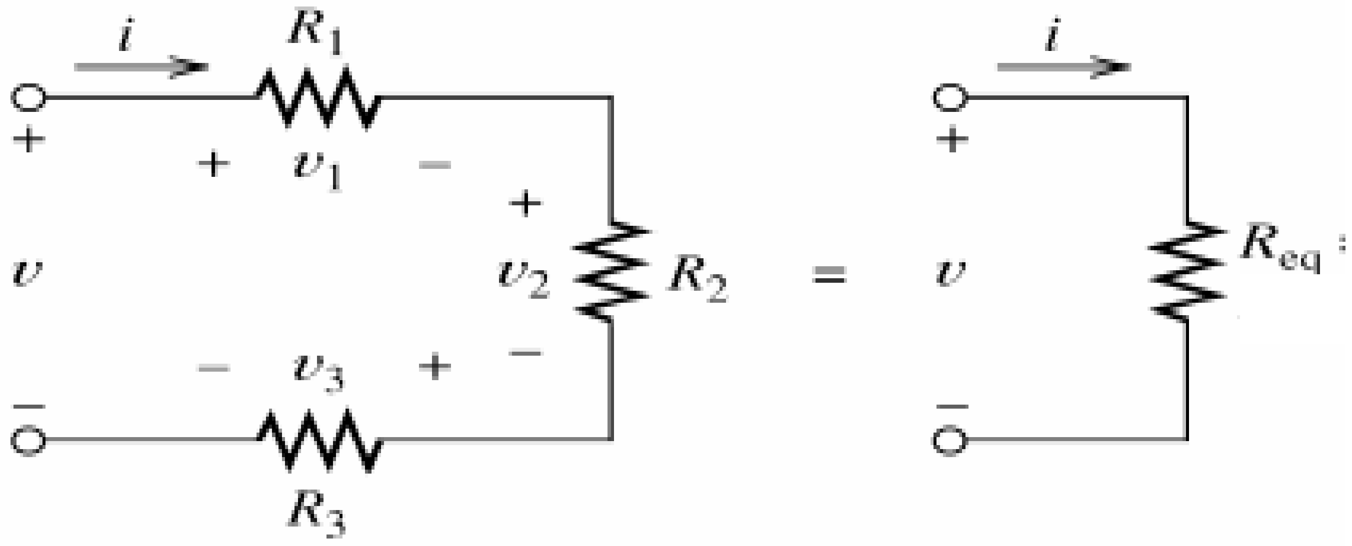
- Two important tasks:
  - Analysis
  - Synthesis

Approaches

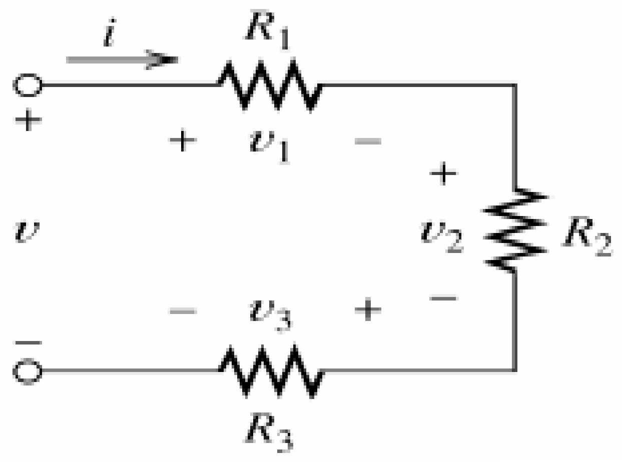
1. Solve circuits (i.e., find currents and voltages of interest) by combining resistances in series and parallel
2. Apply the voltage-division and current-division principles
3. Solve circuits by the node-voltage technique
4. Solve circuits by the mesh-current technique
5. Apply the superposition principle
6. Find Thévenin and Norton equivalents and apply source transformations

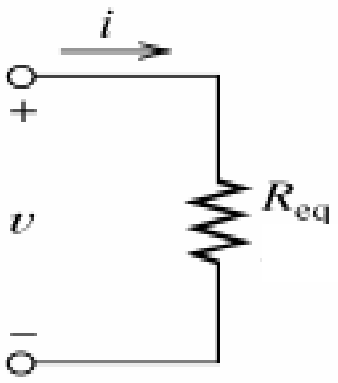
Concept of equivalent circuits

Two circuits are equivalent if they have the same current-voltage behavior.



Method 1: Combining Elements





$$v_1 = R_1 i$$
$$v_2 = R_2 i$$
$$v_3 = R_3 i$$

Using KVL :

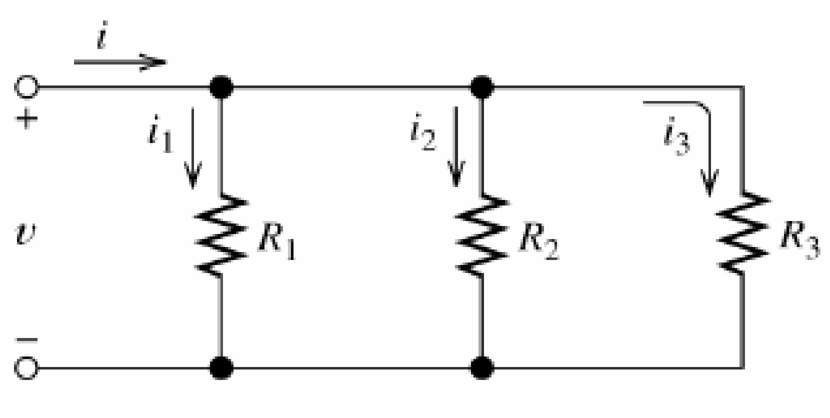
$$v = v_1 + v_2 + v_3$$
$$= (R_1 + R_2 + R_3)i$$

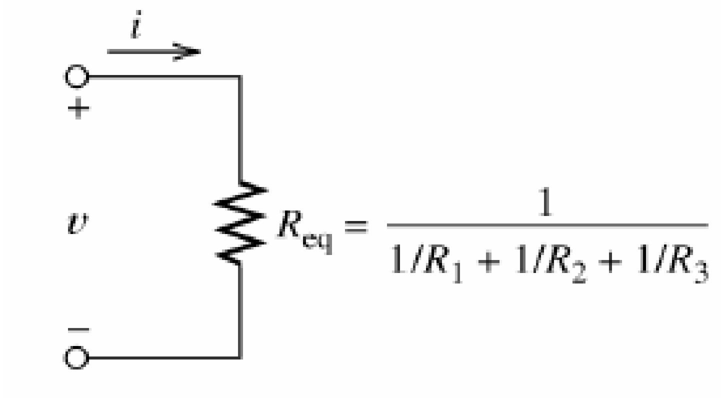
$$v = R_{eq} i$$

If we take $R_{eq} = R_1 + R_2 + R_3$

Both circuits are equivalent as far as ***v* vs *i*** relation is concerned.

Parallel Resistances





By KCL

$$i_1 = v / R_1$$
$$i_2 = v / R_2$$
$$i_3 = v / R_3$$

$$i = i_1 + i_2 + i_3$$
$$= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) v$$

If we take

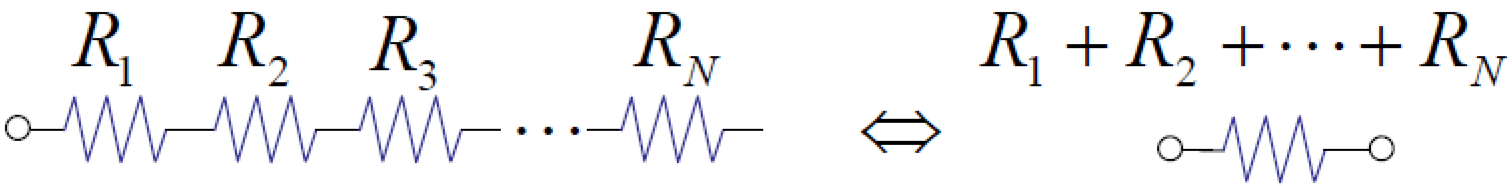
$$i = \left(\frac{1}{R_{eq}}\right) v$$

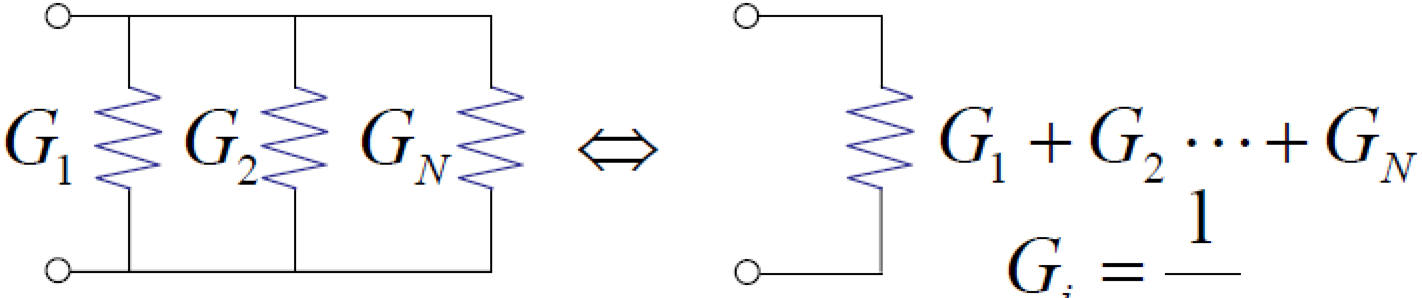
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$G_{eq} = G_1 + G_2 + G_3$$

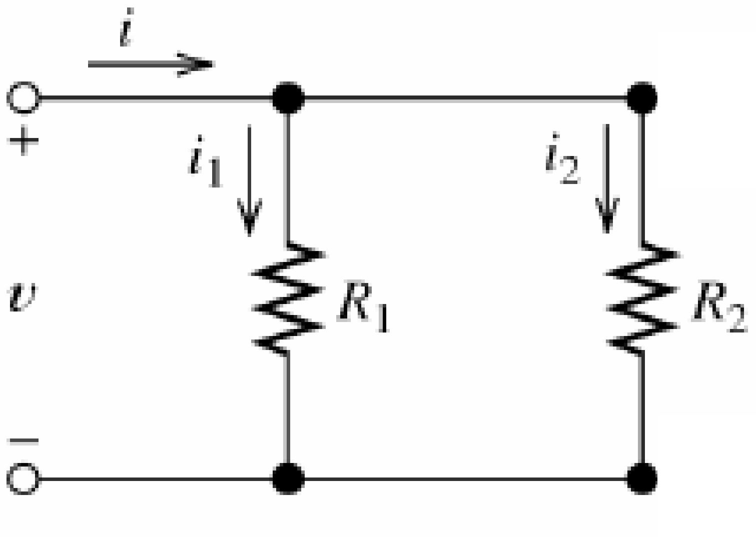
Element combination rules





$$G_i = \frac{1}{R_i}$$

Element combination rules

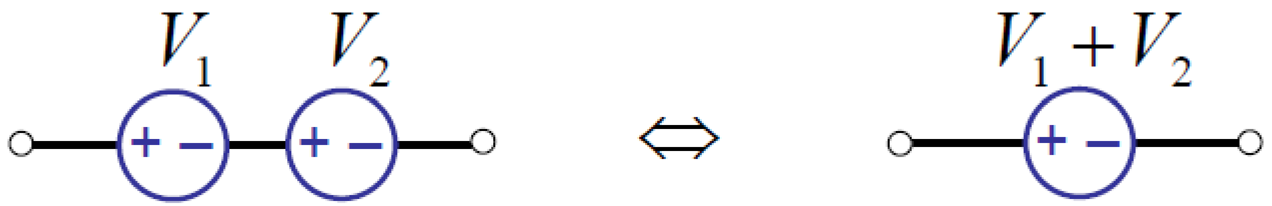


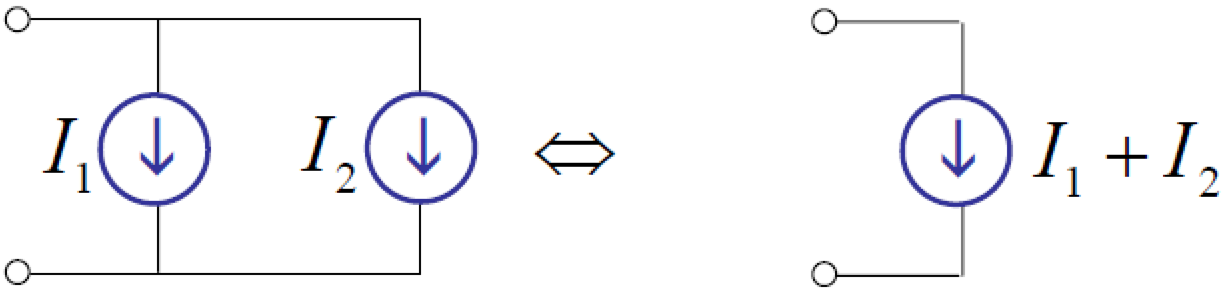
$$\frac{1}{R_{eq}} = \frac{1}{R_!} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

- Always R_{eq} is less than the smallest resistor
- If R_1 or R_2 is zero (short circuit), then $R_{eq} = 0$

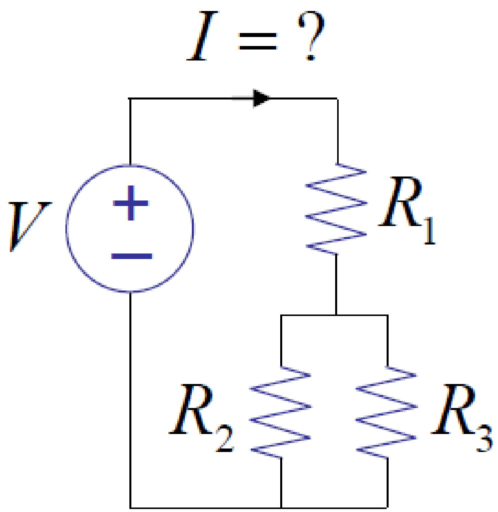
Element combination rules

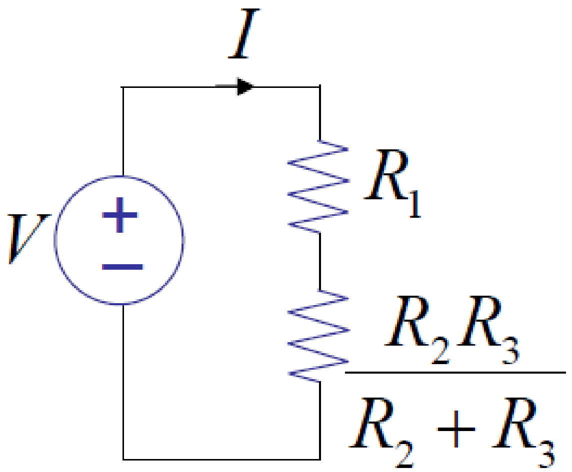


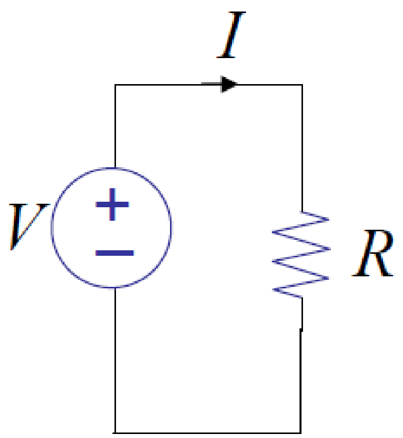


Example: repeatedly combine

$$I = \frac{V}{R}$$

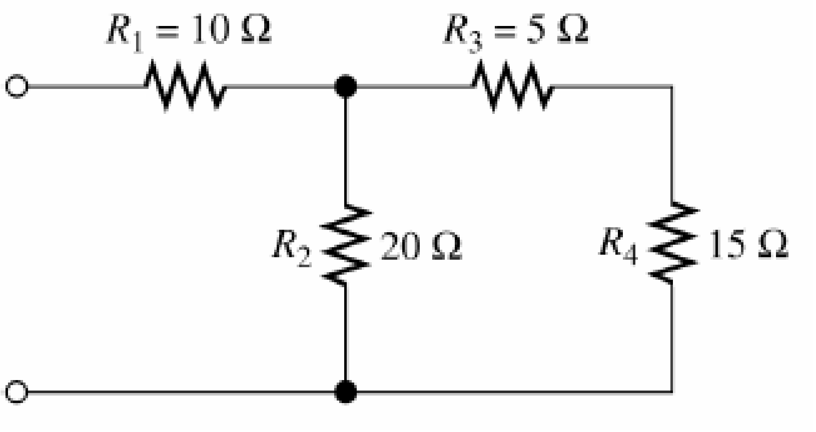


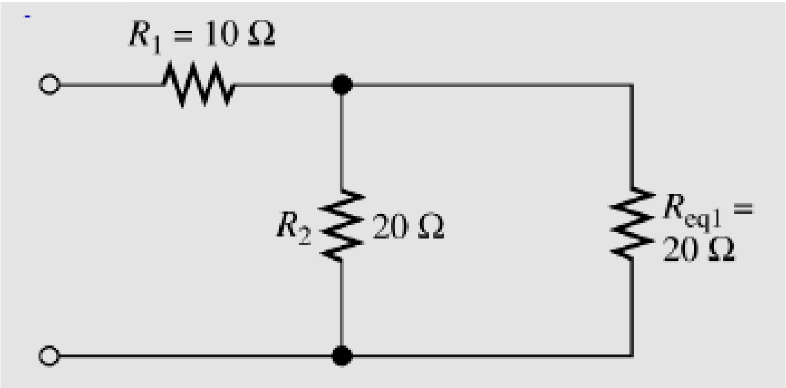


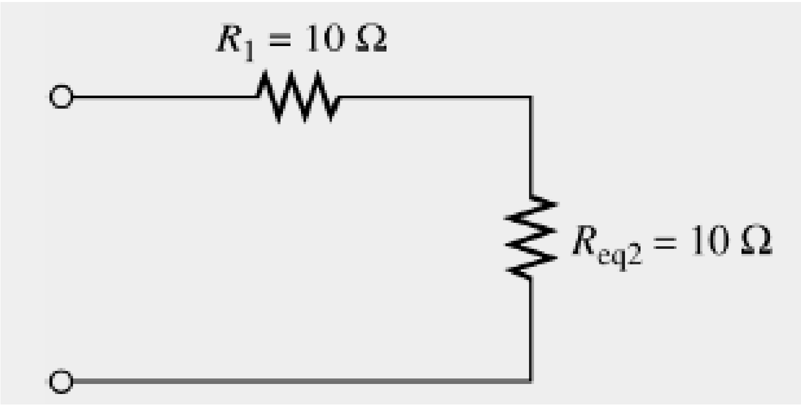


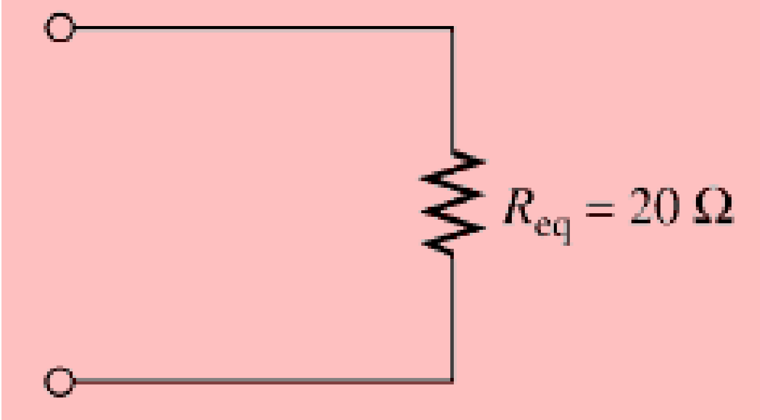
$$R = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

Example 2









$$\mathrm{R}_{eq} = \{(R_4 + R_3) \| R_2\} + R_1$$

Solving backwards: theory

1. Begin by locating a combination of resistances that are in series or parallel. Often the place to start with is the farthest from the source.

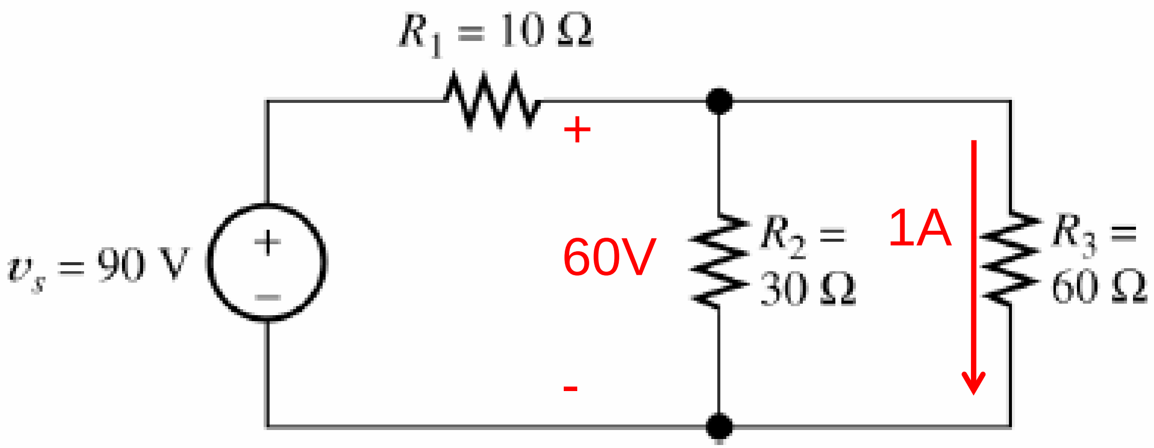
2. Redraw the circuit with the equivalent resistance for the combination found in step 1.

3. Repeat steps 1 and 2 until the circuit is reduced as far as possible. Often (but not always) we end up with a single source and a single resistance.

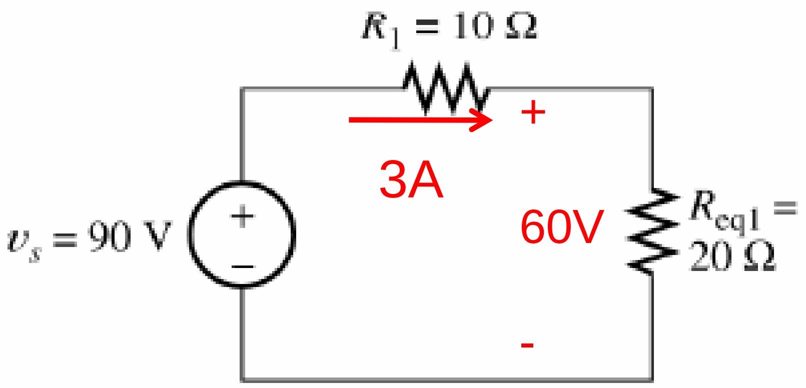
4. Solve for the currents and voltages in the final equivalent circuit. Then go back one step and solve for unknown voltages and current.

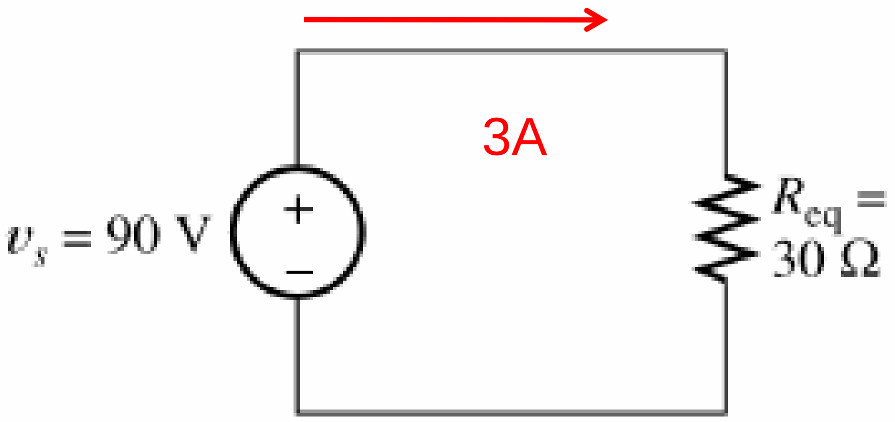
5. Repeat step 4 until the required current or voltage in the original circuit is found.

Solving backwards

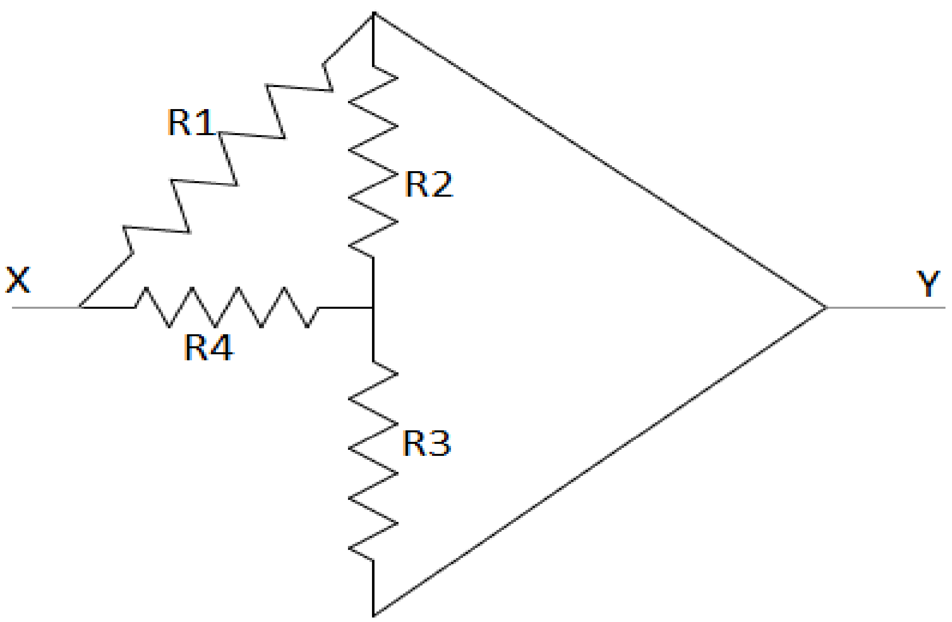


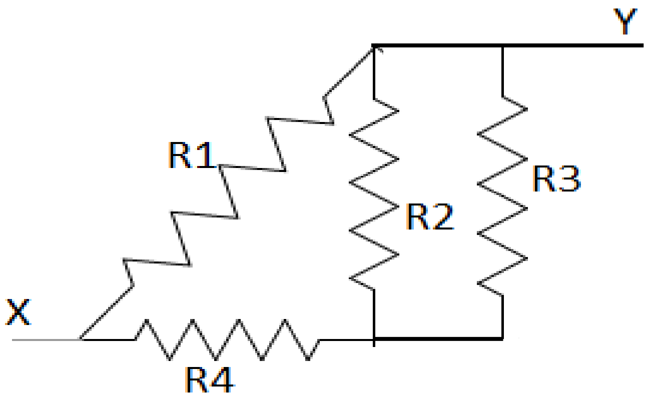
Find current in R_3





Don’t get confused!

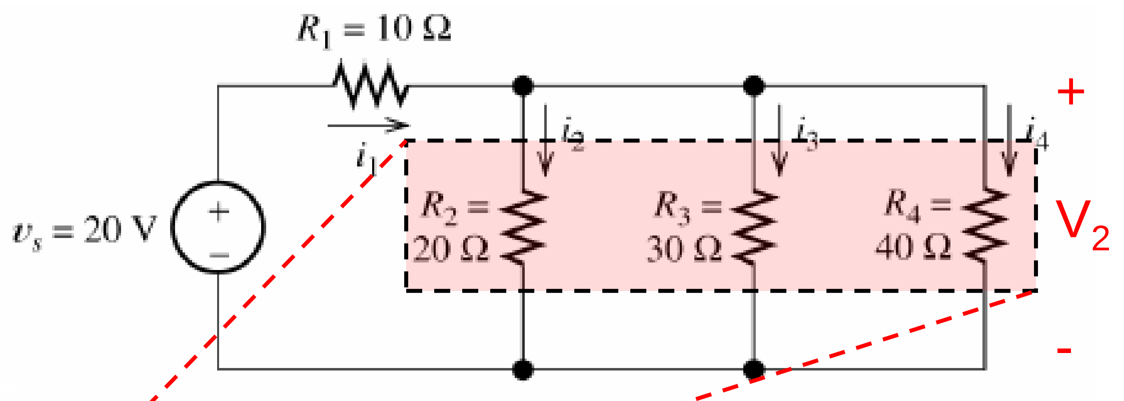


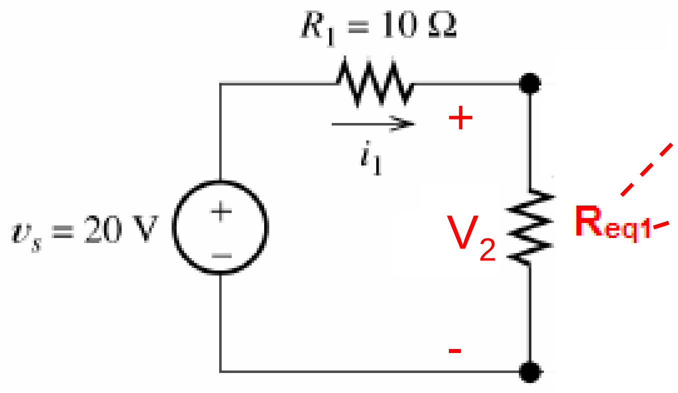


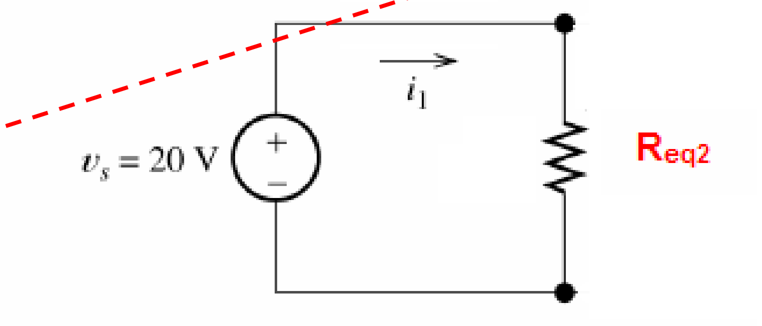
Example

Find all the currents

Compute i_2, i_3 and i_4







$$R_{eq1} = R_2 \| R_3 \| R_4$$

$$R_{eq2} = R_1 + R_{eq1}$$

Compute V_2

Solve for i_1

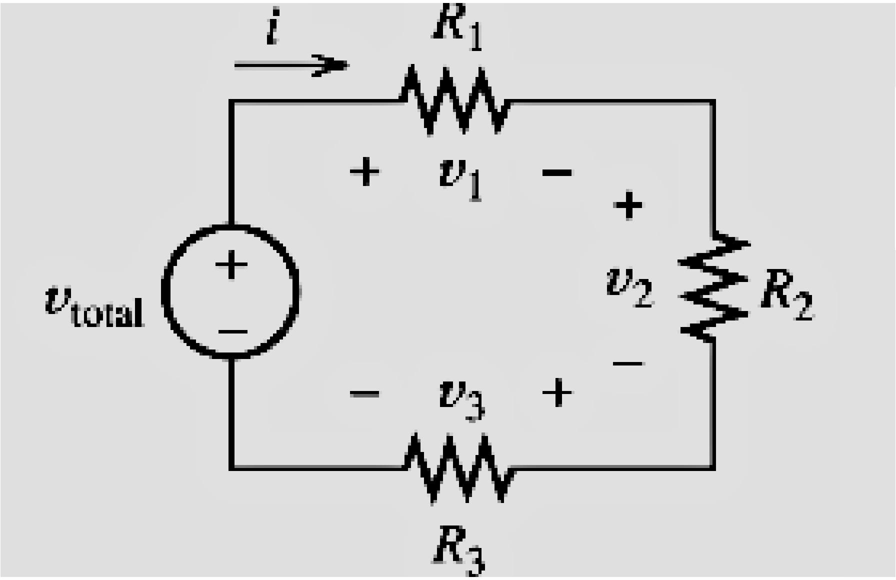
Ans. i_1=1.04 A, i_2=0.48 A, i_3=0.32, i_4=0.24 A

Voltage division

- A voltage applied to resistors connected in series will be divided among them in proportion to their resistance

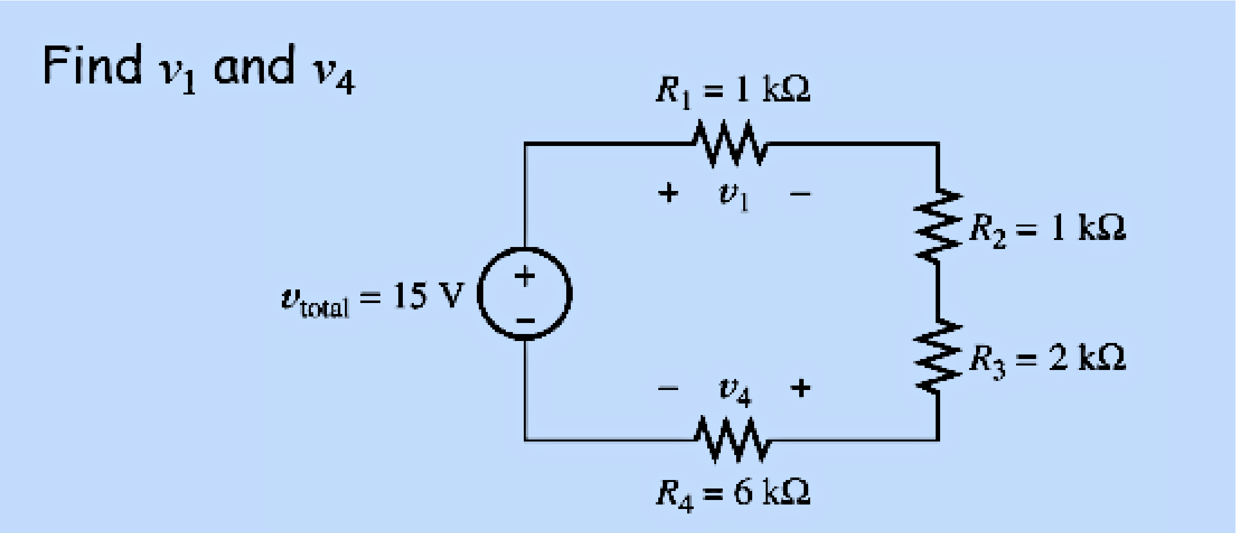
$$i = \frac{v_{total}}{R_1 + R_2 + R_3}$$

$$v_1 = R_1 i = \frac{R_1}{R_1 + R_2 + R_3} v_{\text{total}}$$



$$v_2 = R_2 i = \frac{R_2}{R_1 + R_2 + R_3} v_{\text{total}}$$

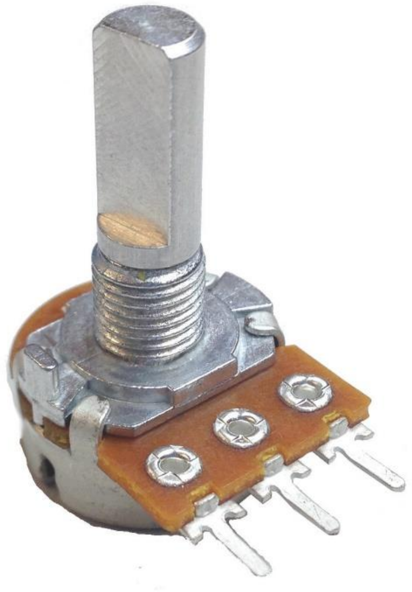
Voltage division: example

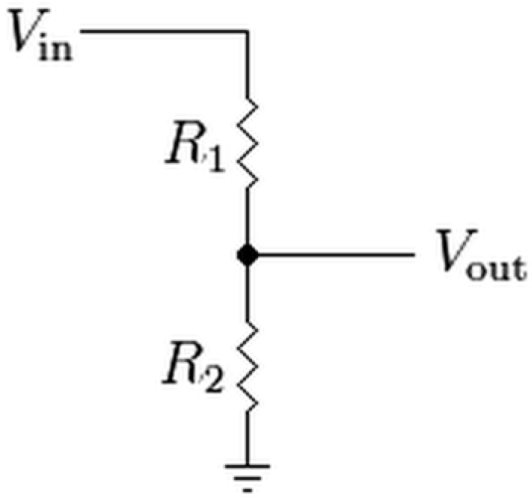


$$v_1 = \frac{1}{1+1+2+6} \times 15\ \text{V} = 1.5\ \text{V}$$

$$\text{v}_2 = \frac{6}{1+1+2+6} \times 15\ \text{V} = 9\ \text{V}$$

Potentiometer

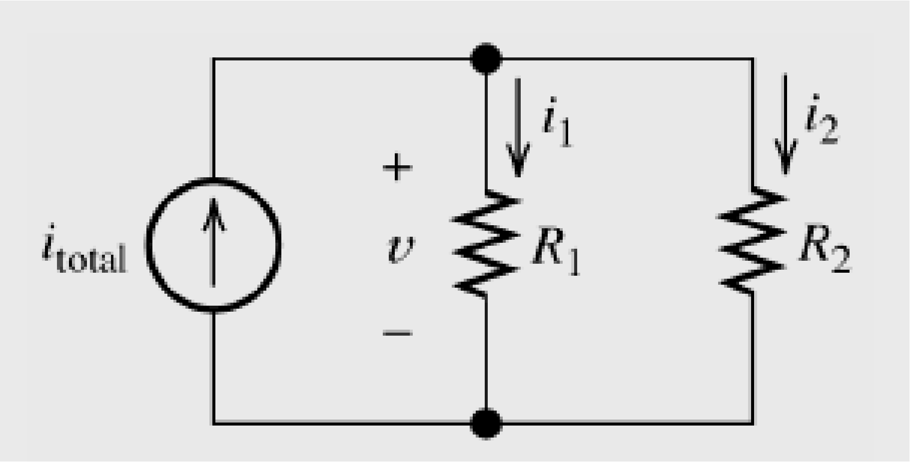




$$= V_{in} \frac{R_2}{R_1 + R_2}$$

Current division

- The total current flowing into a parallel combination of resistors will be divided among them in inverse proportion of their resistances.



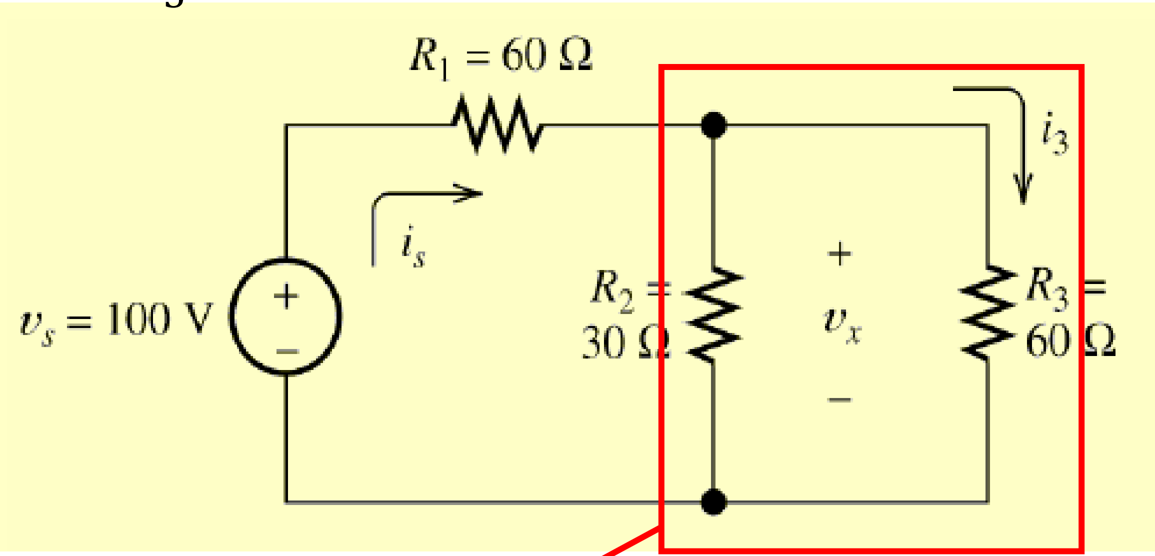
$$i_1 = \frac{v}{R_1} = \frac{R_2}{R_1 + R_2} i_{total}$$

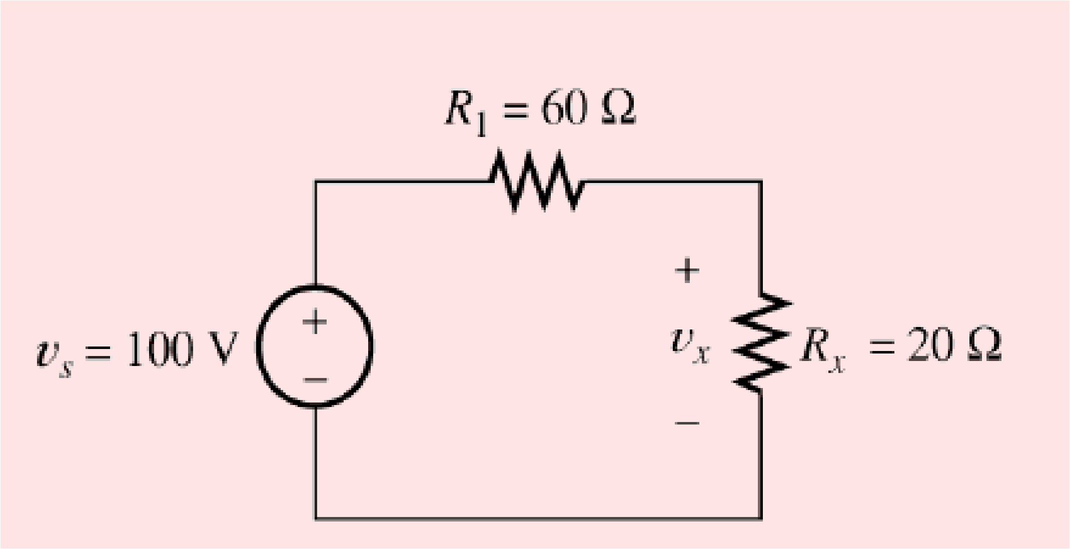
$$v = \frac{R_1 R_2}{R_1 + R_2} i_{total}$$

$$i_2 = \frac{v}{R_2} = \frac{R_1}{R_1 + R_2} i_{total}$$

Example

Find v_x and i_3





$$R_x = \frac{R_2 R_3}{R_2 + R_3} = \frac{30 \times 60}{30 + 60} = 20\Omega$$

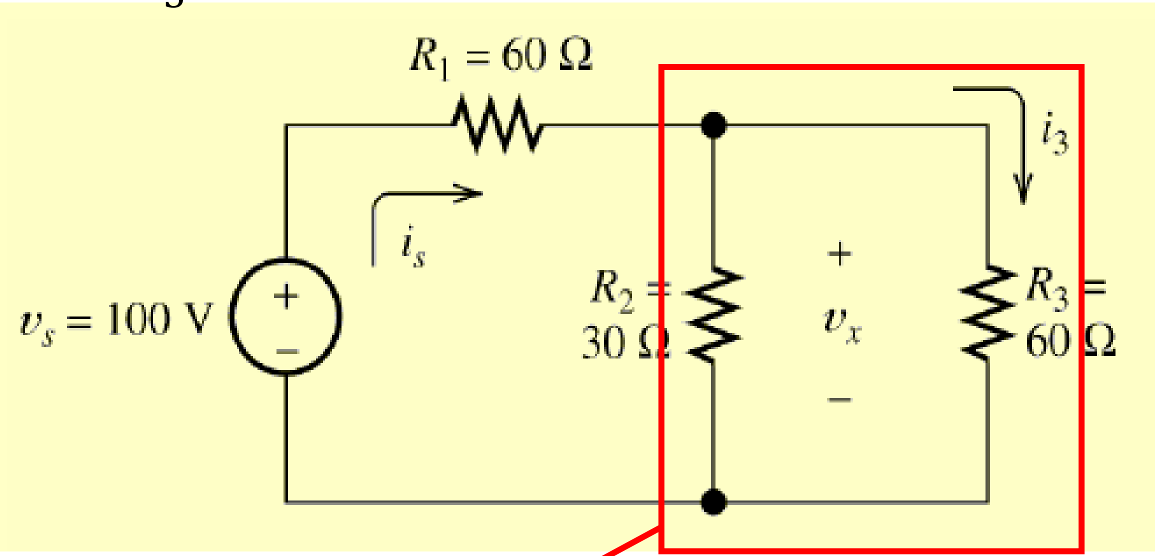
$$v_x = \frac{R_x}{R_1 + R_x} v_s = \frac{20}{60 + 20} 100 = 25V$$

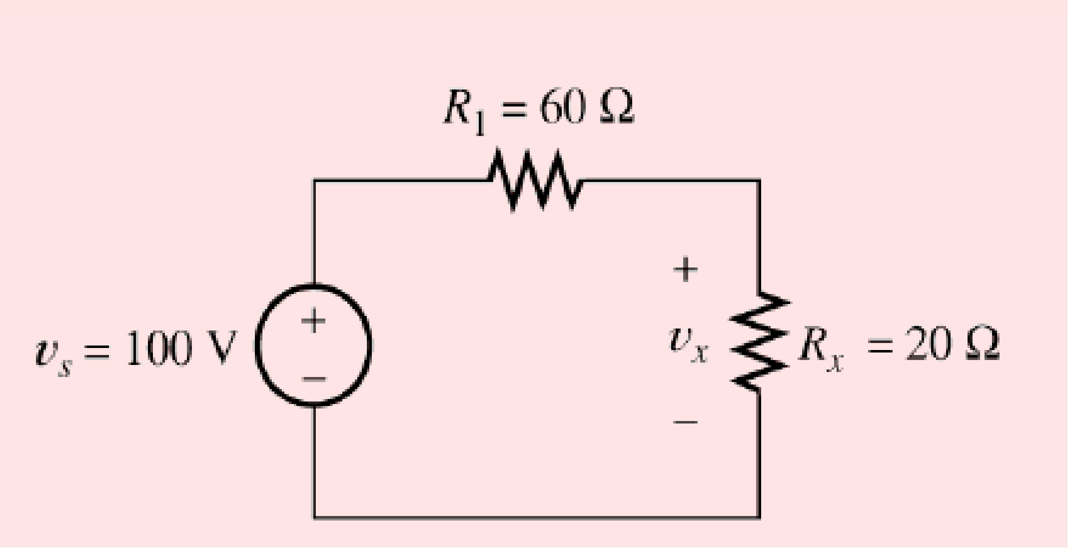
Find v_x using voltage division, and find i_3 using Ohm's law

$$i_3 = \frac{25\ V}{60\ \Omega} = 0.417\ \text{A}$$

Example

Find v_x and i_3





$$R_x = \frac{R_2 R_3}{R_2 + R_3} = \frac{30 \times 60}{30 + 60} = 20\Omega$$

$$i_s = \frac{v_s}{R_1 + R_x} = \frac{100}{60 + 20} = 1.25A$$

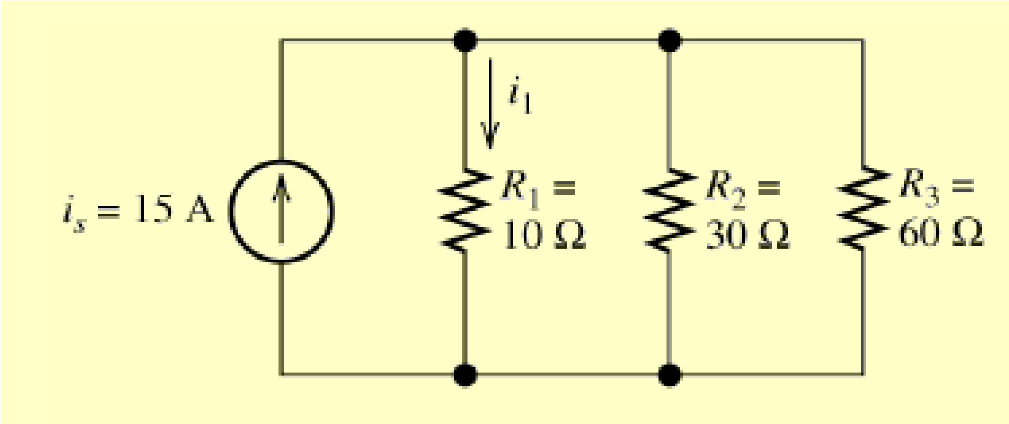
Find i_3 using voltage division, and v_x using Ohm's law

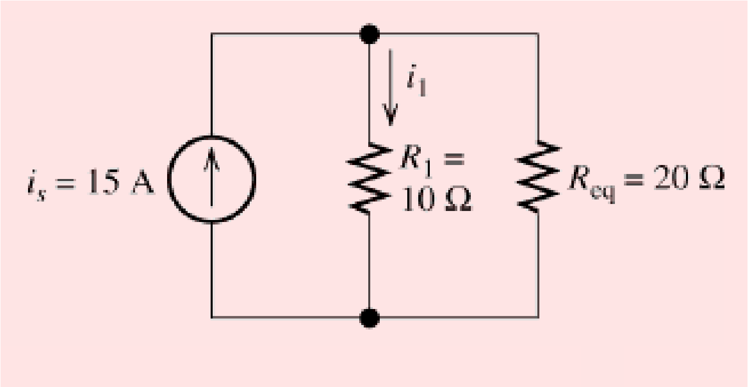
$$i_3 = \frac{R_2}{R_2 + R_3} i_s = \frac{30}{30+60} \times 1.25 = 0.417A$$

Example 2

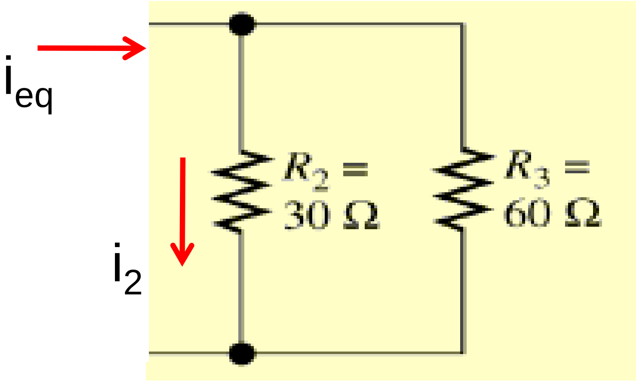
Use current division rule to find i_1

$$i_1 = \frac{R_{eq}}{R_1 + R_{eq}} i_s = \frac{20}{10+20} 15 = 10\text{A}$$





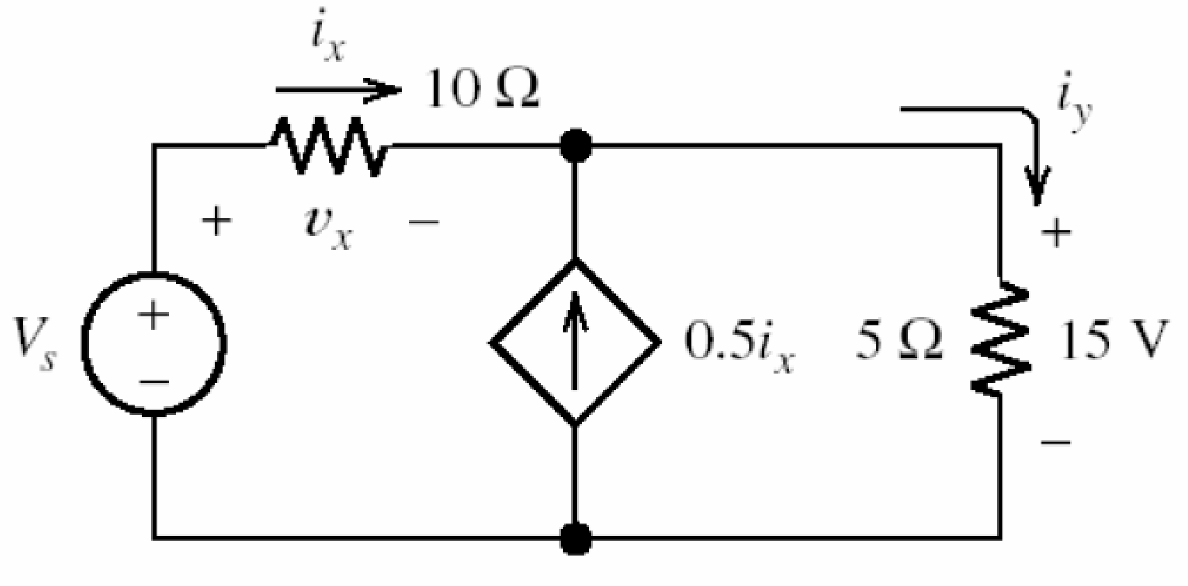
Suppose we want to find i_2 also



$$i_2 = \frac{R_3}{R_2 + R_3} i_{eq}$$

$$i_{eq} = \frac{R_1}{R_1 + R_{eq}} i_s$$

Example with Dependent Sources



Find source voltage.

$$i_y = \frac{15\ \text{V}}{5\ \Omega} = 3\ \text{A}$$

$$i_x + 0.5 i_x = i_y$$

$$i_x = 2\ \text{A}$$

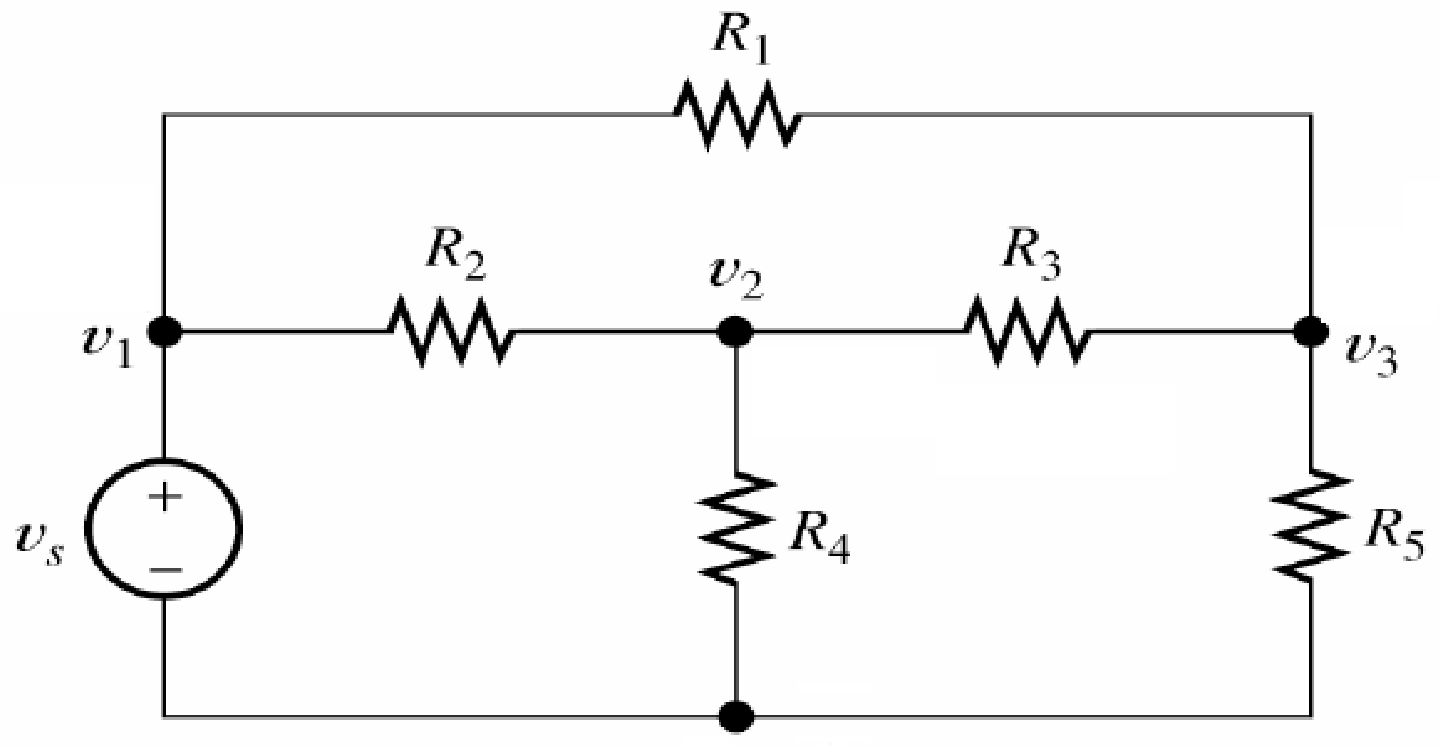
$$v_x = 10 i_x = 20\ \text{V}$$

$$V_s = v_x + 15$$

$$V_s = 35\ \text{V}$$

Limitations

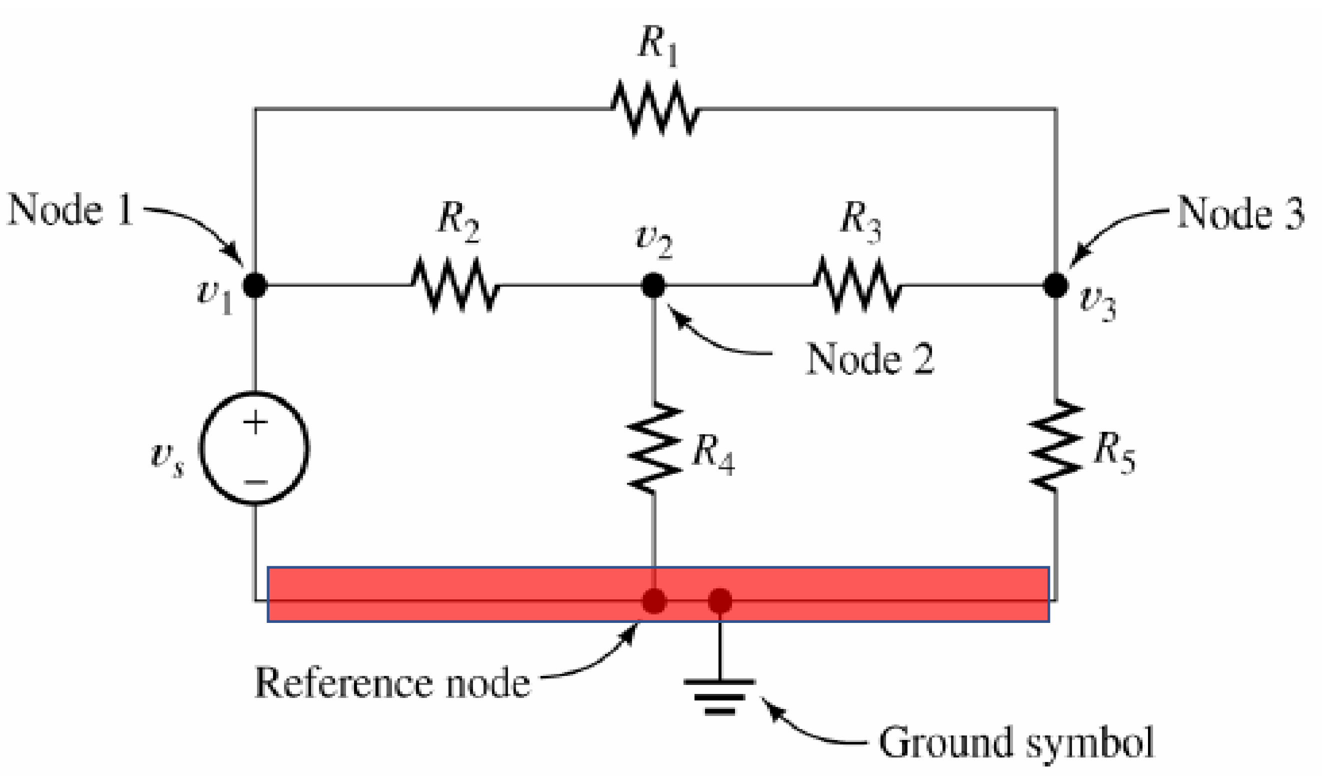
Although series/parallel equivalents and the current/voltage division principles are very important concepts, yet they are not sufficient to solve all circuits !!



Nodal Analysis

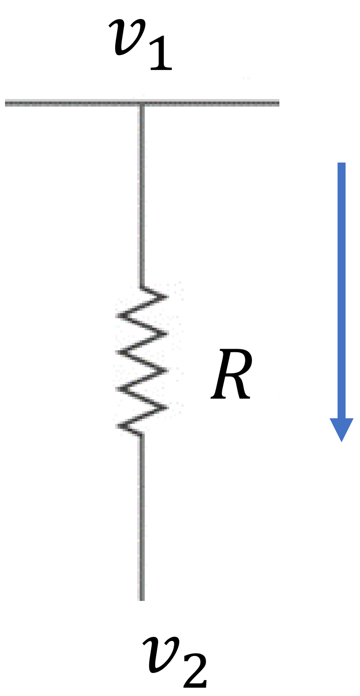
- In nodal analysis, the variables used to describe the circuit will be "Node Voltages" (Recall Nodes!)
  - Nodal voltage are the voltages of each node with respect to a pre-selected reference node
- Steps:
  - Designate a node as reference or ground
  - Label voltages of remaining nodes (unknown variables)
  - Use KCL for all nodes except ground,
    - Write currents in terms of node voltages (using I-V equations)
  - Solve for node voltages
  - Back solve for branch voltages, if required

Node method: Steps 1 & 2



Nodal Analysis will give values of node voltages v_1, v_2 and v_3 with respect to the reference node

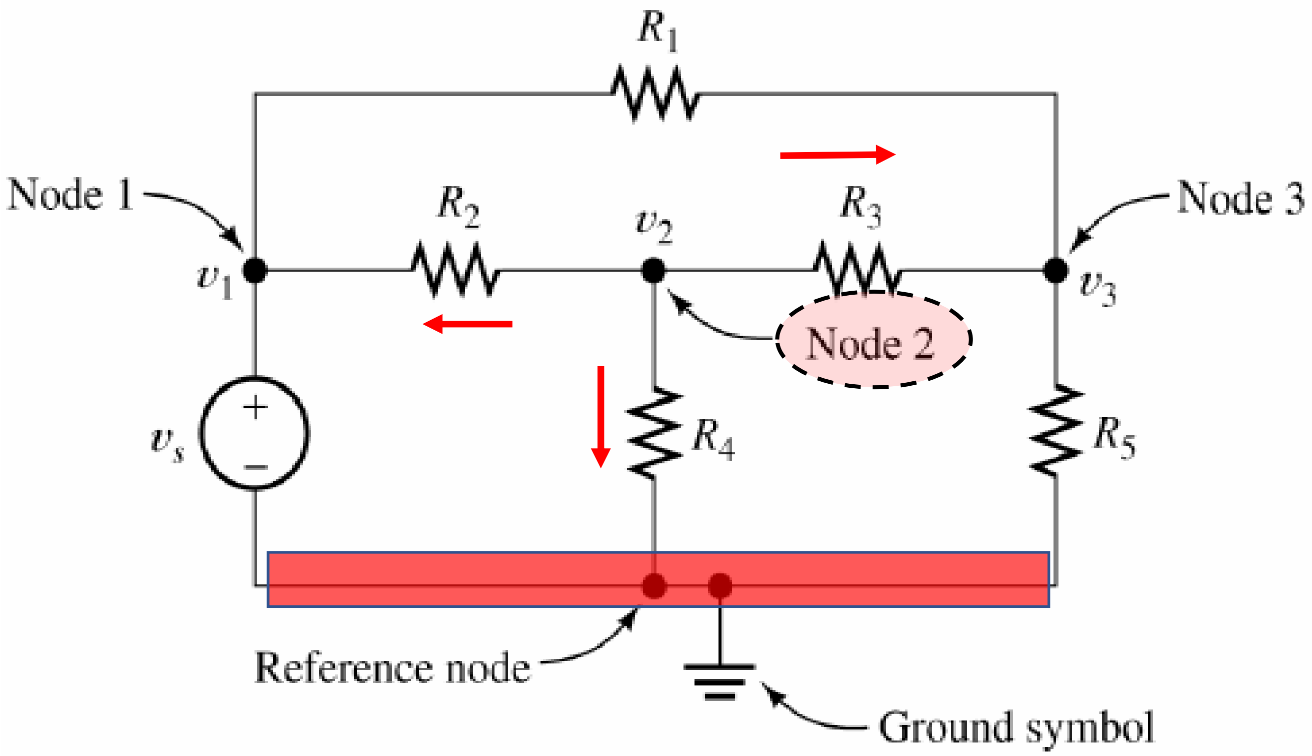
KCL in terms of voltages



$$i = \frac{v_1 - v_2}{R}$$

Node method: step 3

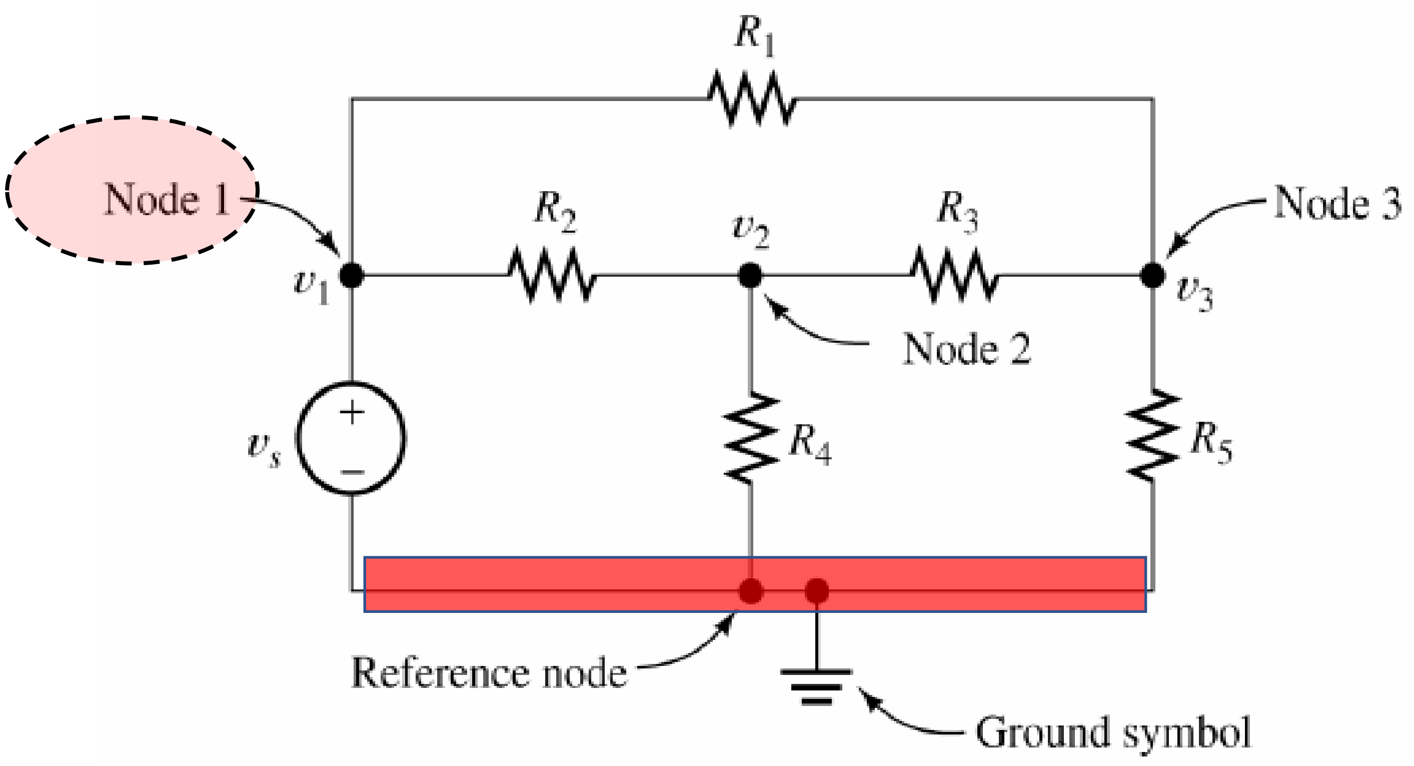
- Apply KCL using node voltages



- Sum of currents leaving node-2 is zero

$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_4} + \frac{v_2 - v_3}{R_3} = 0$$

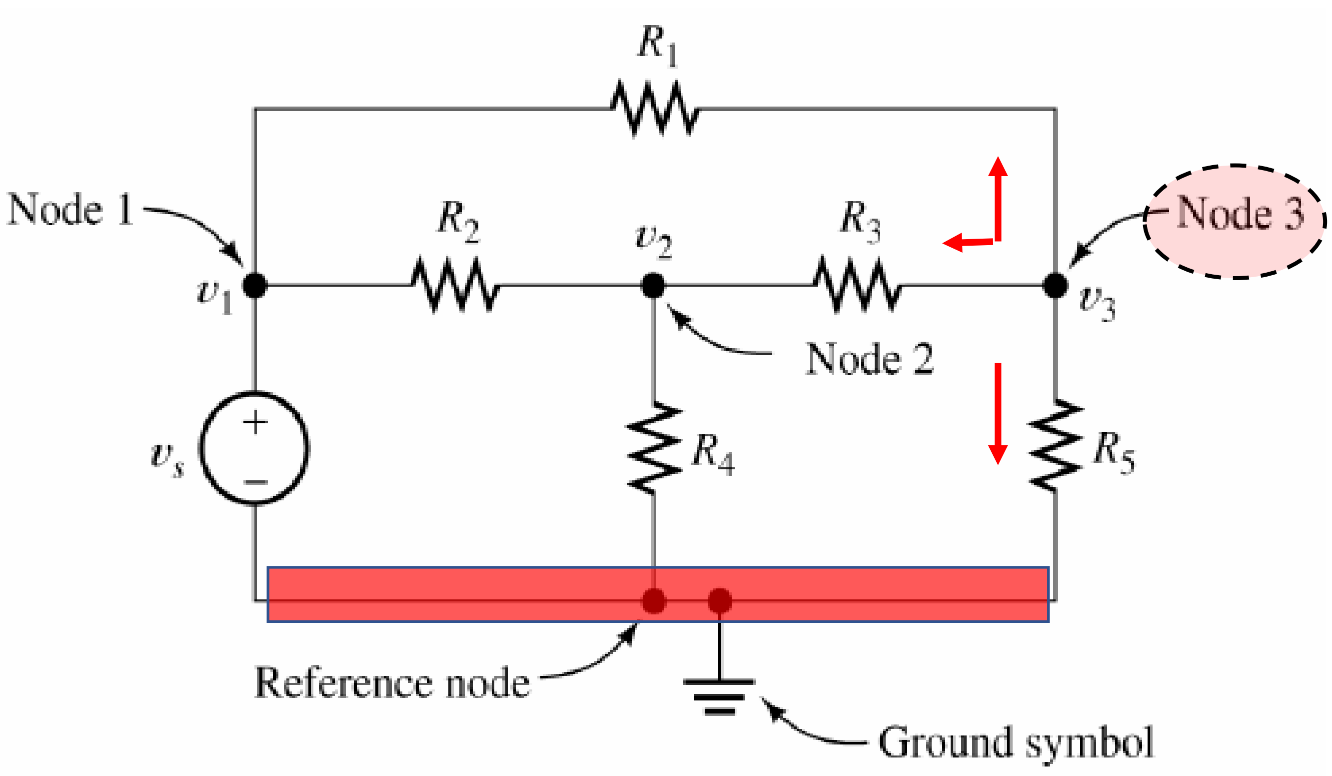
Node method: step 3



Voltage for node 1 is known

$$v_1 = v_s$$

Node method: step 3



$$\frac{v_3 - v_1}{R_1} + \frac{v_3}{R_5} + \frac{v_3 - v_2}{R_3} = 0$$

Node method: step 4

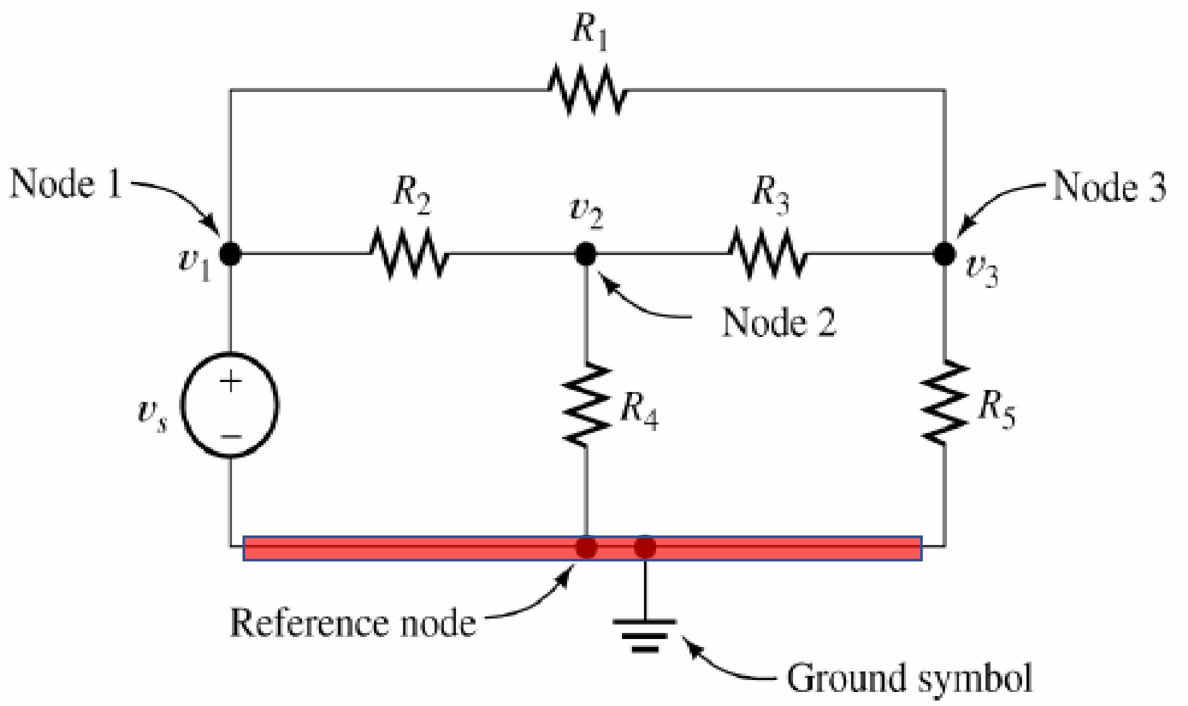
- Solve the three equations

N nodes
=>
N − 1 independent equations

Node 2 $$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_4} + \frac{v_2 - v_3}{R_3} = 0$$

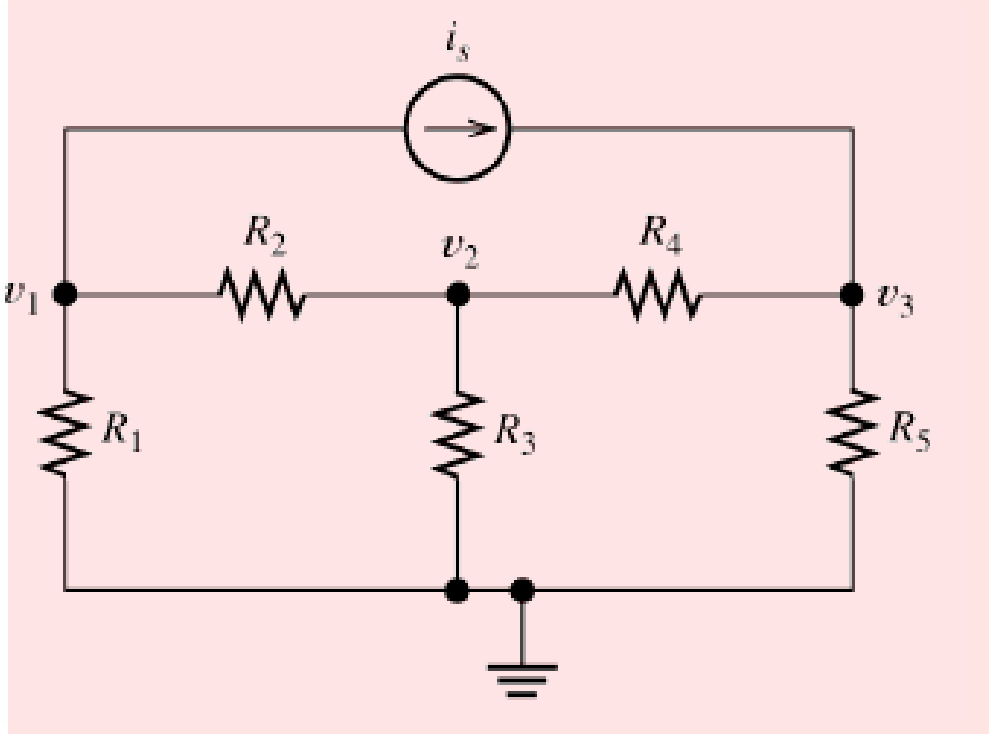
Node 3 $$\frac{v_3 - v_1}{R_1} + \frac{v_3}{R_5} + \frac{v_3 - v_2}{R_3} = 0$$

Node 1 $$v_1 = v_s$$



Node method: example 2

Circuits with Independent Current Sources



Node 1:

$$\frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} + i_s = 0$$

Node 2:

$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_3} + \frac{v_2 - v_3}{R_4} = 0$$

$$Node\,3:$$

$$\frac{v_3}{R_5} + \frac{v_3 - v_2}{R_4} - i_s = 0$$

Node method: example 3

Circuits with voltage sources connected with ground

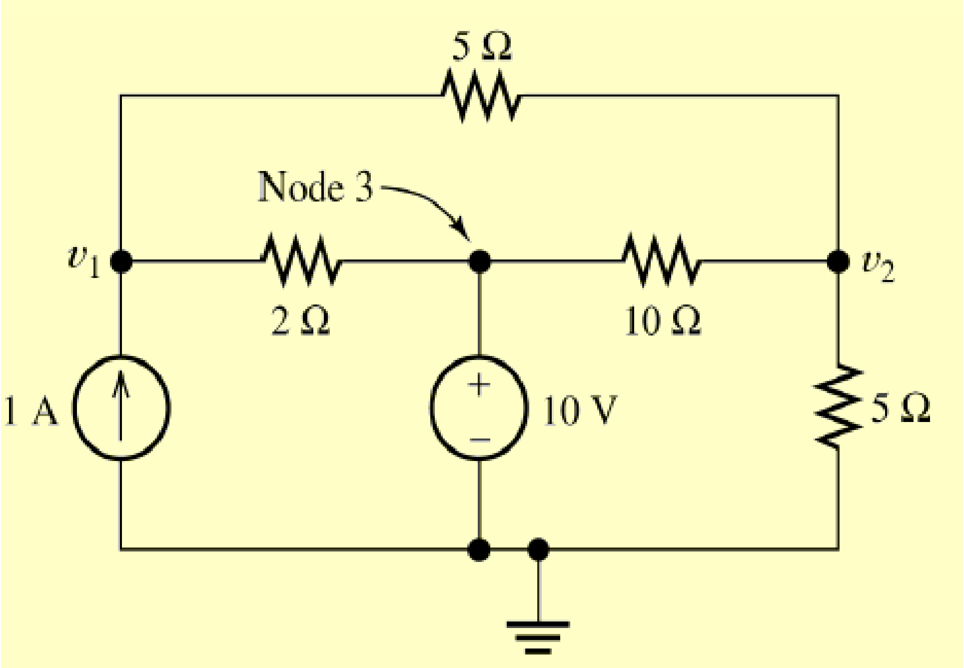
Number of unknowns = 2

$$\text{Node1}: \frac{v_1 - v_2}{5} + \frac{v_1 - 10}{2} = 1$$

$$0.7v_1 - 0.2v_2 = 6$$

$$\text{Node 2}: \frac{v_2}{5} + \frac{v_2 - 10}{10} + \frac{v_2 - v_1}{5} = 0$$

$$-0.2v_1 + 0.5v_2 = 1$$



$$v_1 = 10.32\,V$$
$$v_2 = 6.129\,V$$

Example 4: hanging nodes

Circuits with voltage sources that are not connected to ground

Super node

i

Node 1

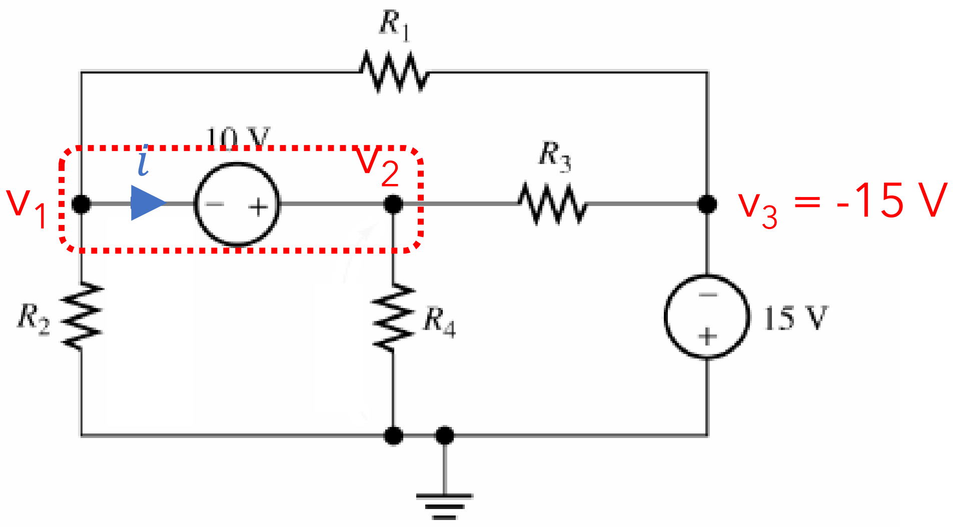
$$\frac{v_1}{R_2}+\frac{v_1-v_3}{R_1}+\boxed{\frac{v_1-v_2}{?}}=0$$

$-i$

Node 2

$$\frac{v_2}{R_4}+\frac{v_2-v_3}{R_3}+\boxed{\frac{v_2-v_1}{?}}=0$$

$$v_2-v_1=10$$



$$\frac{v_1}{R_2}+\frac{v_1-v_3}{R_1}+\frac{v_2}{R_4}+\frac{v_2-v_3}{R_3}=0$$