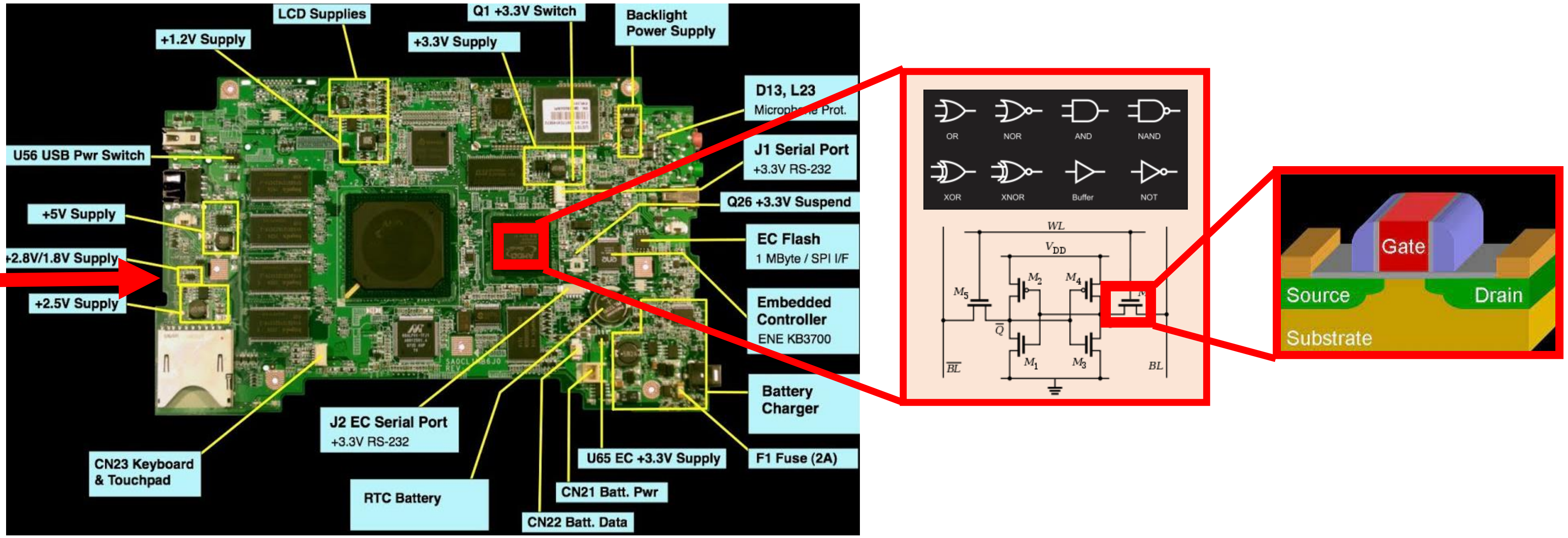

ESC201: Introduction to Electronics

Module 1: Circuit Analysis

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IIT Kanpur

Recap: Techniques of Circuit Analysis

Nodal Analysis

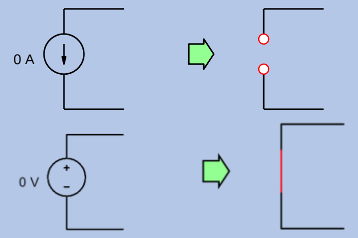
1. Identify and number the nodes
2. Pick Ground node/Reference node wisely, if it is not already specified
3. Writing KCL Equations in Terms of the Node Voltages

Mesh Analysis

1. Assign mesh currents i_1, i_2, …, in to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

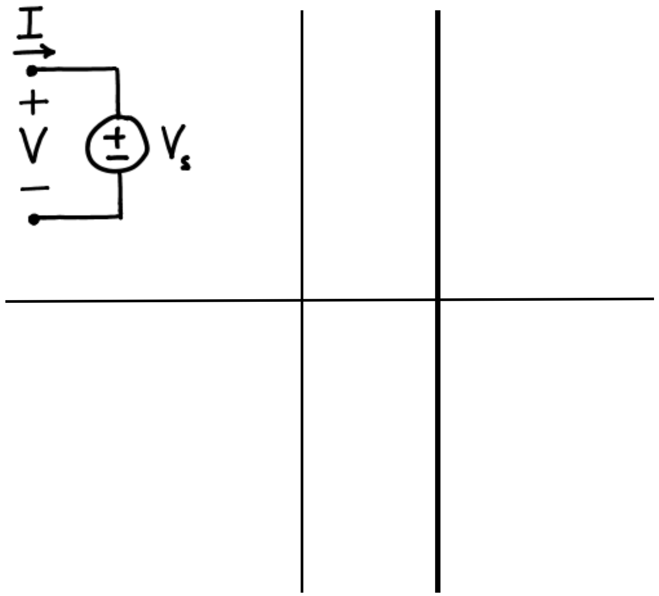
Superposition Method for Linear Circuits

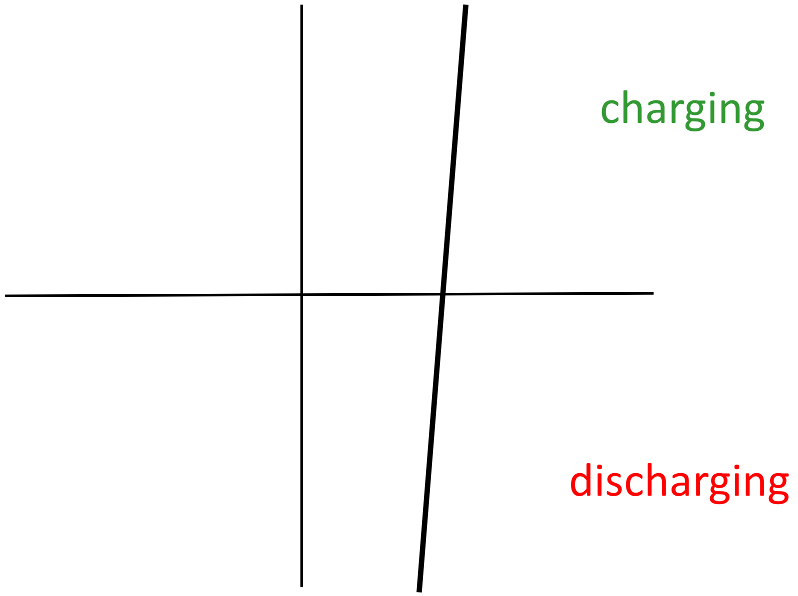
The superposition principle states that the total response is the sum of the responses to each of the independent sources acting individually.
Linear circuit: linear elements, independent voltage/current Sources, and linear dependent sources



Non-ideal sources

- How to model non-ideal batteries?
- Draw intuition from the VI characteristic



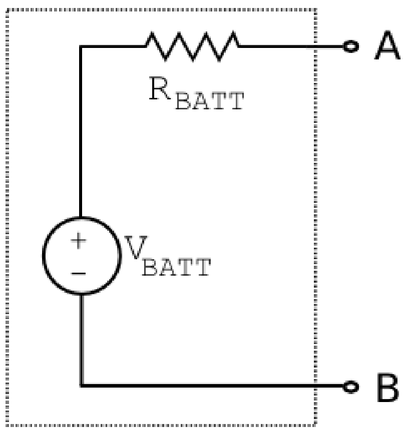


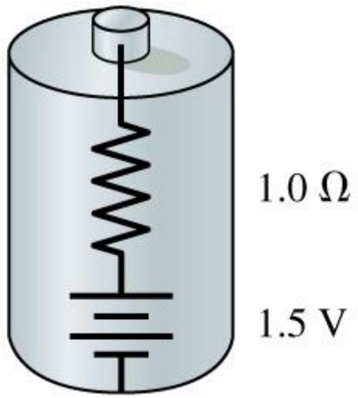
Ideal battery

Real battery

Non-ideal battery

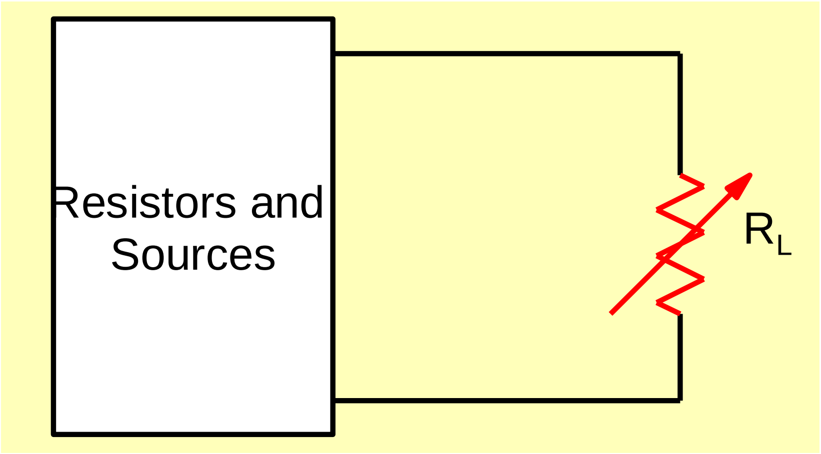
- The non-ideal battery consists of linear elements inside it
- Thevenin equivalent of a battery!

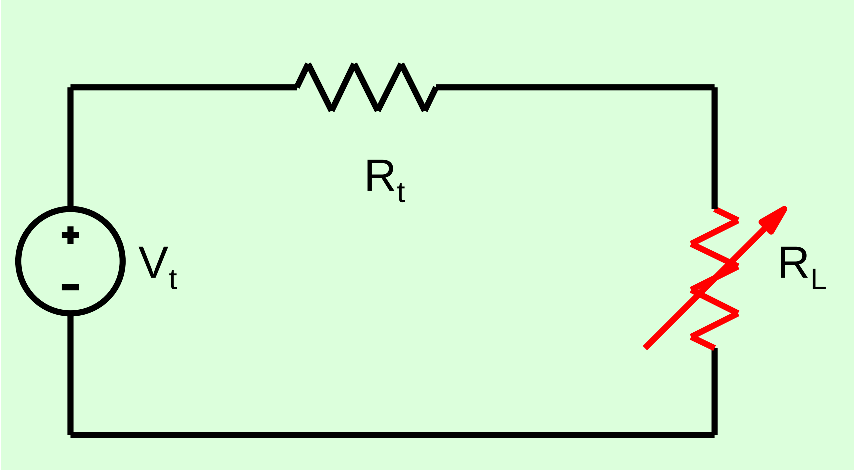




Validity: only when current magnitude is not too large and voltage is around V_{BATT}

General Case





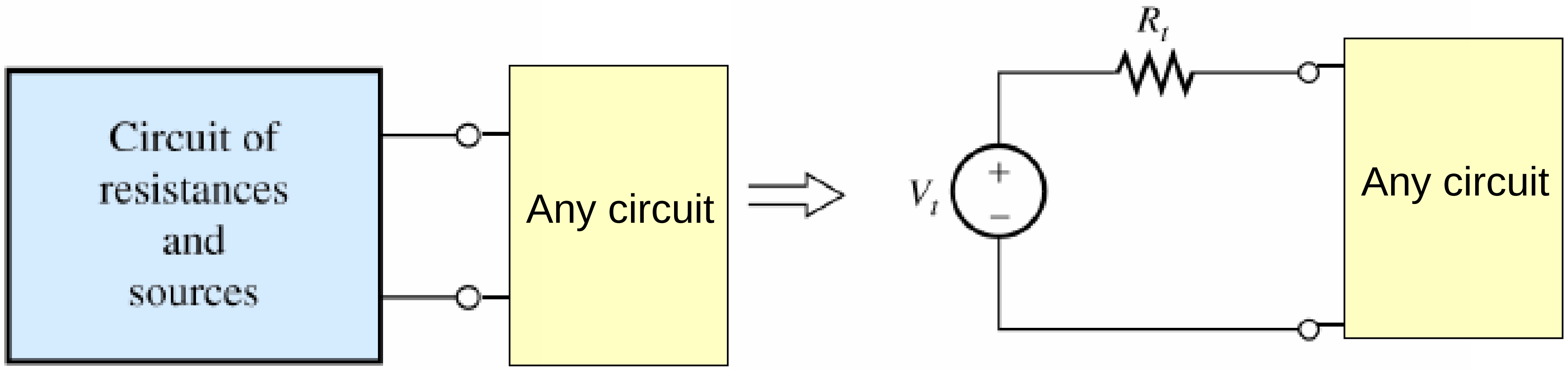
Maximum power is delivered to the load when $\mathbf{R_L = R_t}$

Thévenin’s Theorem

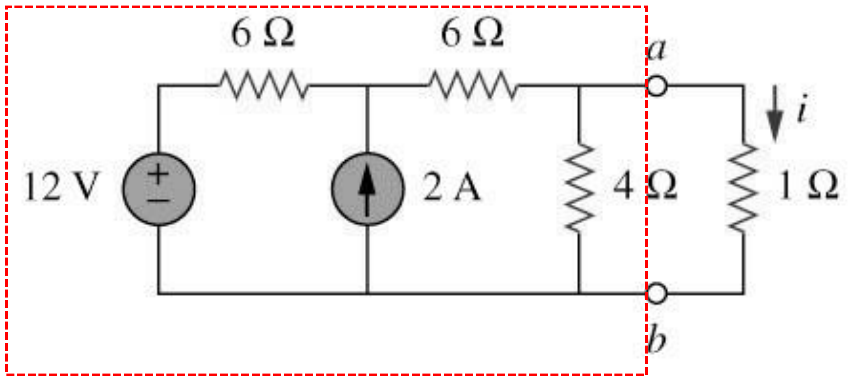
“Any linear circuit containing several voltages and resistances can be replaced by just one single voltage in series with a single resistance connected across the load”

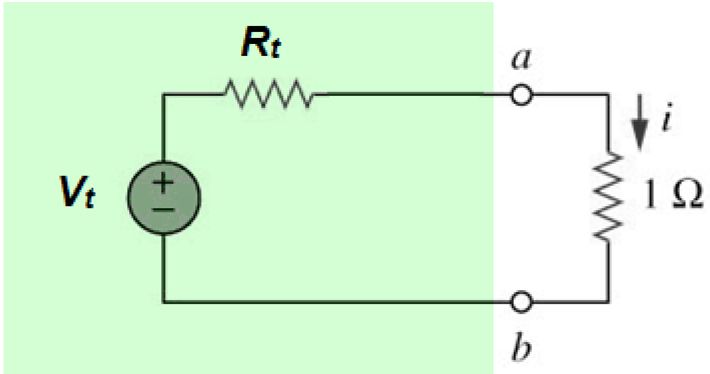
Any linear circuit with power supplies (voltage sources and/or current sources) and resistances can be replaced by a single resistance connected in series with a voltage source to make it equivalent to the original circuit.

Thévenin Equivalent Circuits

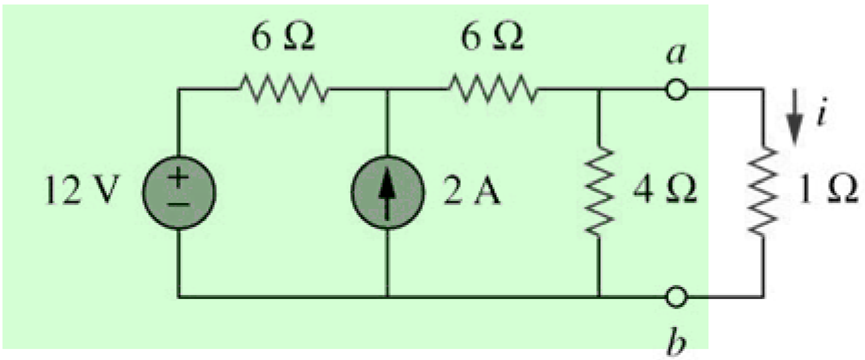


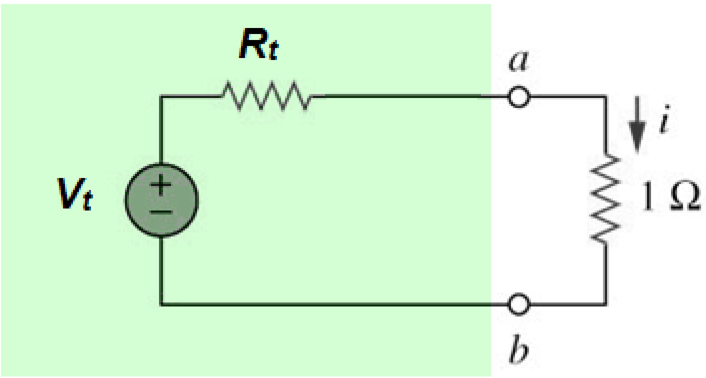
Thévenin equivalent circuit

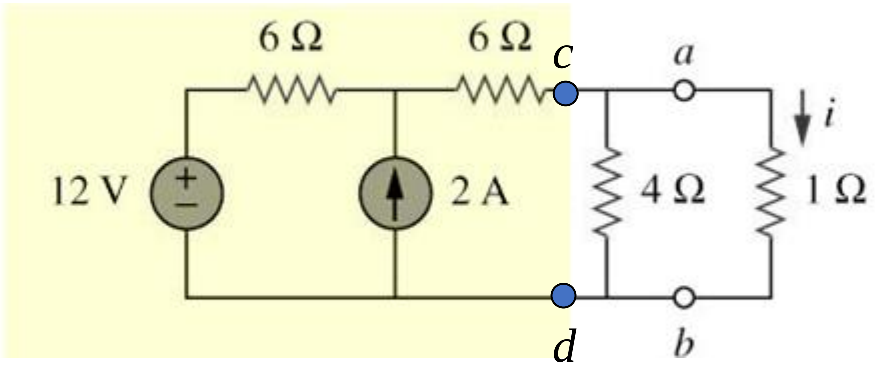


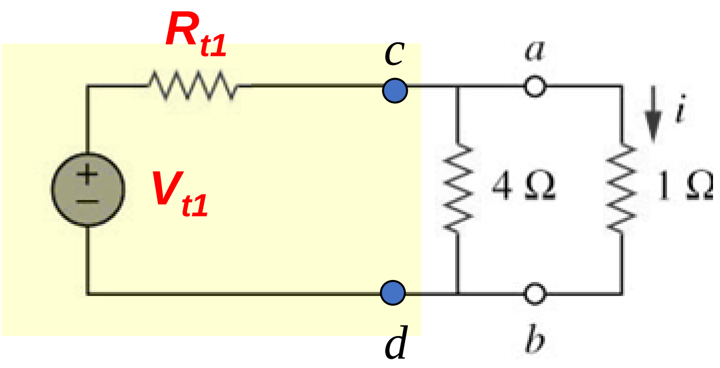


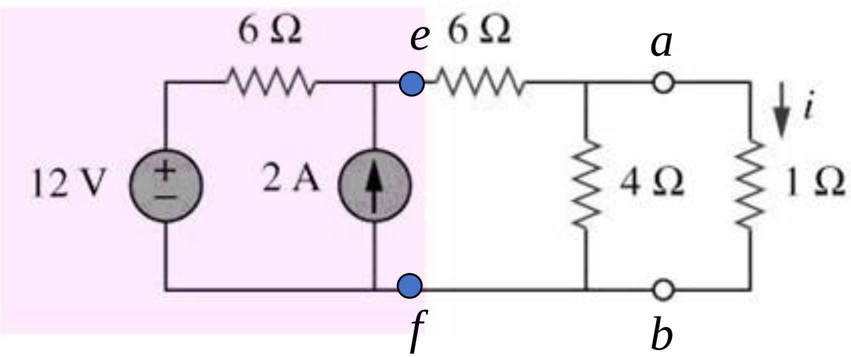
Thévenin's Theorem Applies to Any Part of the Circuit

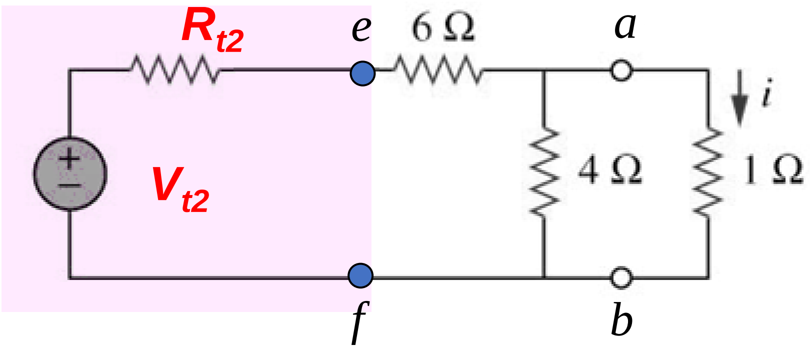




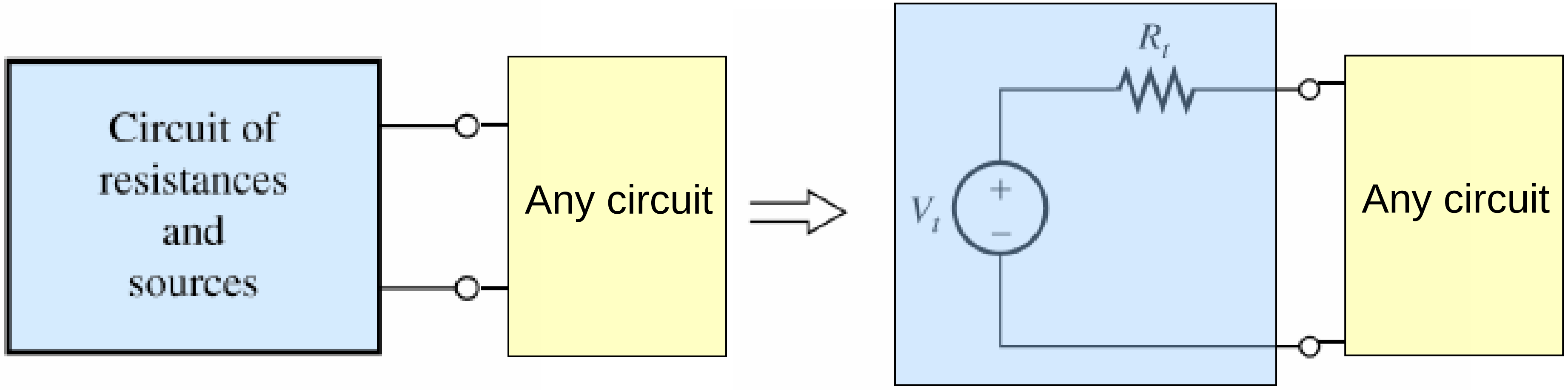






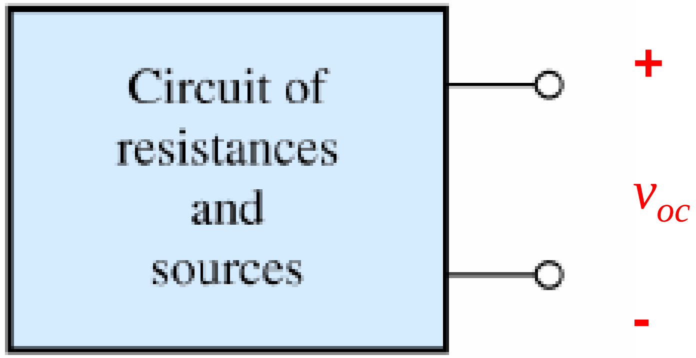


Thévenin Voltage

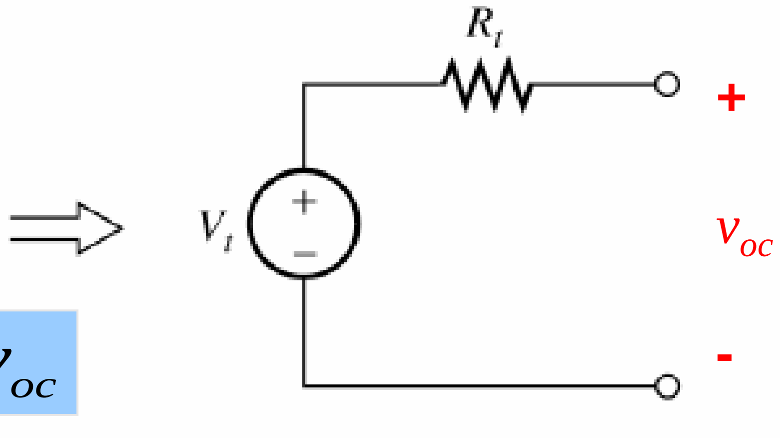


Thévenin equivalent

What is Thévenin Voltage V_t ?



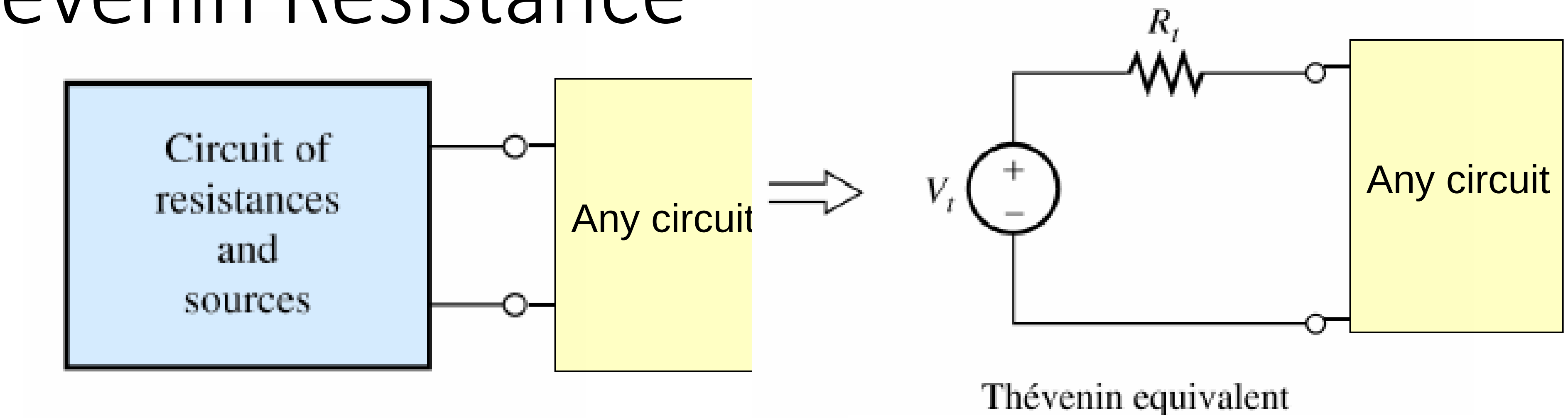
$$V_t = v_{oc}$$



Thévenin equivalent circuit

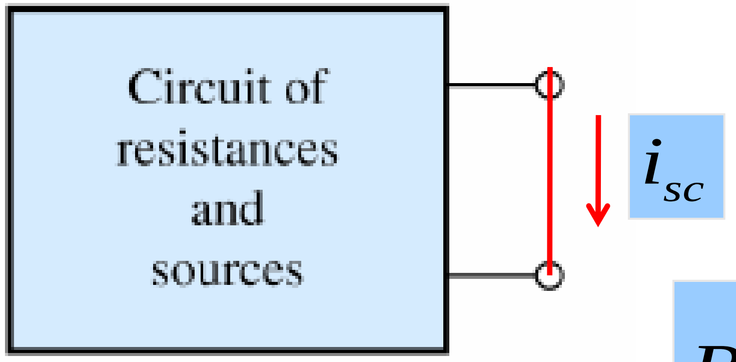
Since the circuits are equivalent their v_{oc} must be same!

Thévenin Resistance

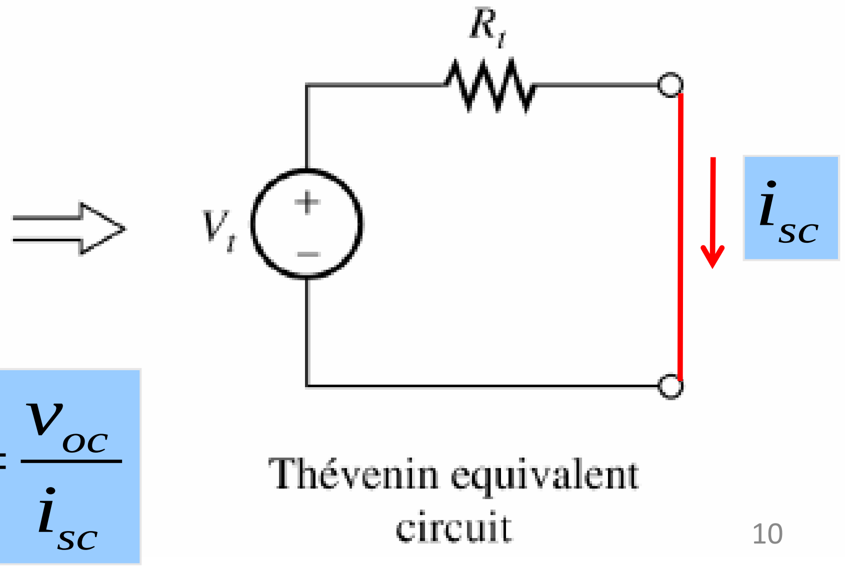


What is R_t ?

The first level solution:

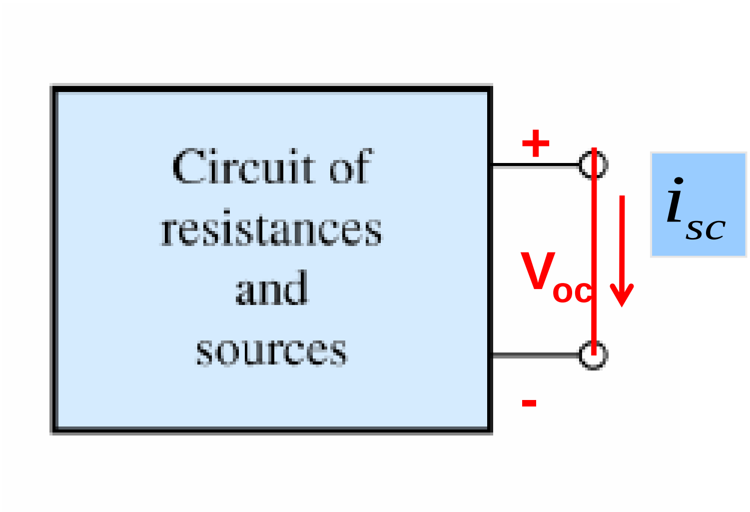


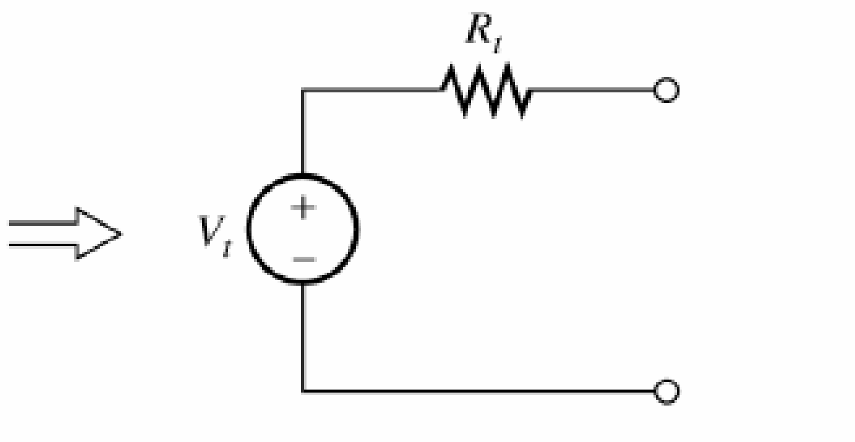
$$R_t = \frac{V_t}{i_{sc}} = \frac{v_{oc}}{i_{sc}}$$



Since the circuits are equivalent their i_{sc} must be same!

Thévenin Parameters



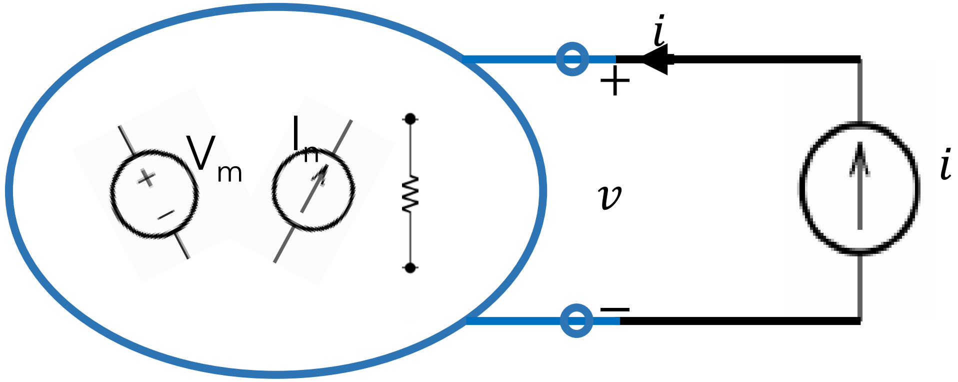


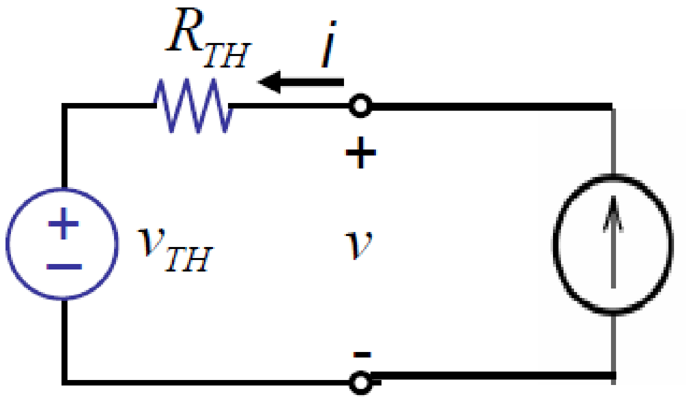
Thévenin equivalent circuit

$$V_t = v_{oc}$$

$$R_t = \frac{v_{oc}}{i_{sc}}$$

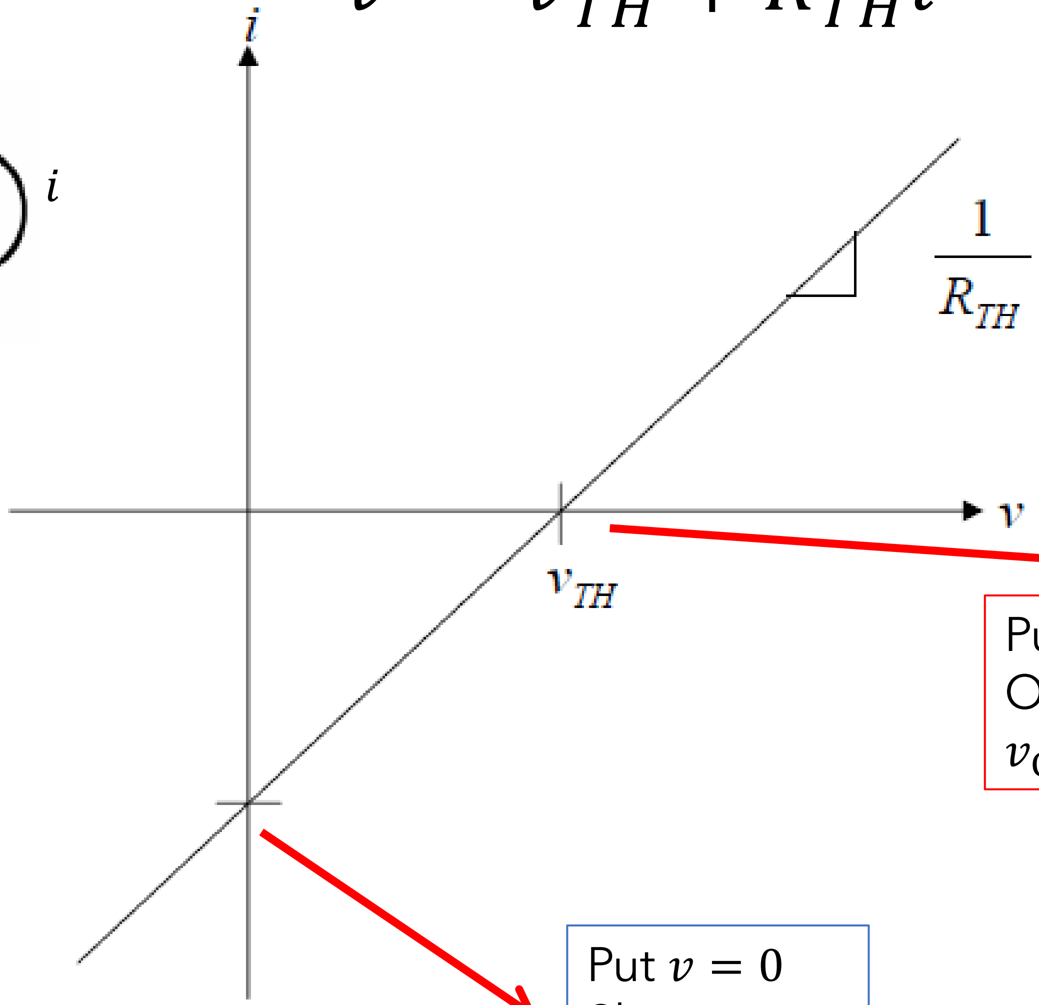
V_{TH} and R_{TH} from V-I Characteristics





Equal
Characteristics

$$v = v_{TH} + R_{TH} i$$



Put $i = 0$
Open circuit
v_{OC}

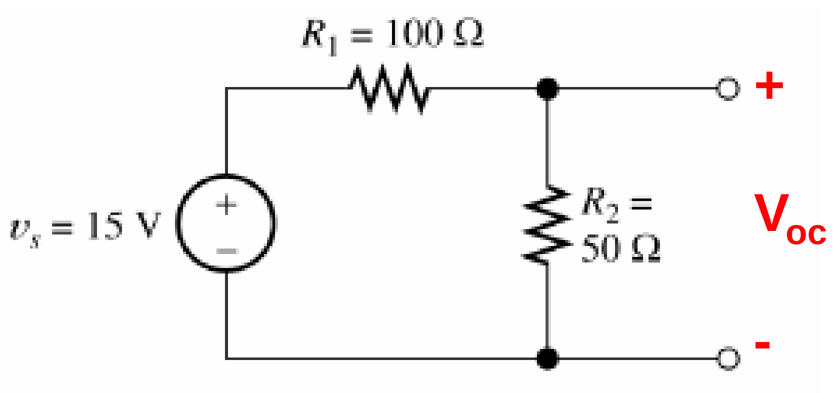
Put $v = 0$
Short circuit
i_{SC}

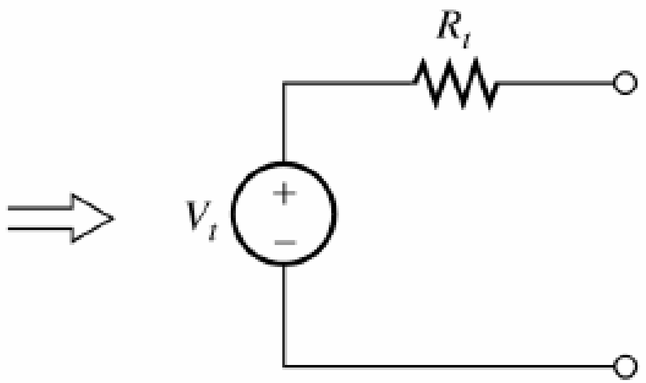
$$0 = v_{TH} + R_{TH} i_{SC}$$

$$-v_{TH} = R_{TH} i_{SC}$$

$$\boxed{R_{TH} = \frac{v_{TH}}{-i_{SC}} = \frac{v_{OC}}{-i_{SC}}}$$

Examples

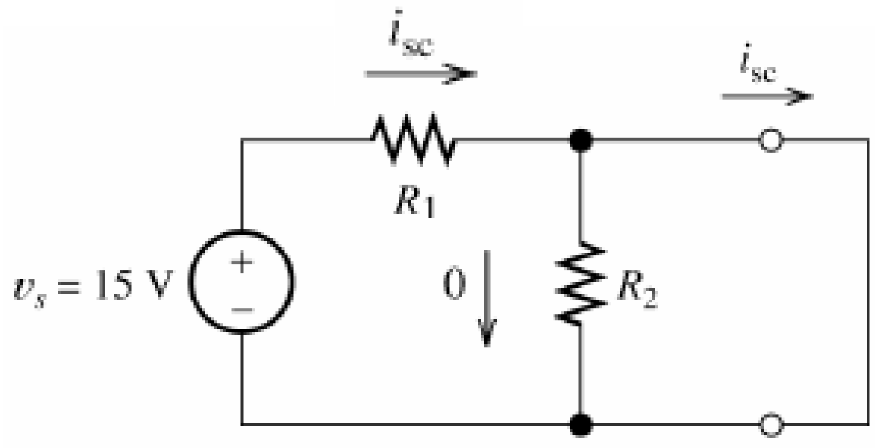




Thévenin equivalent circuit

$$V_t = v_{oc}$$

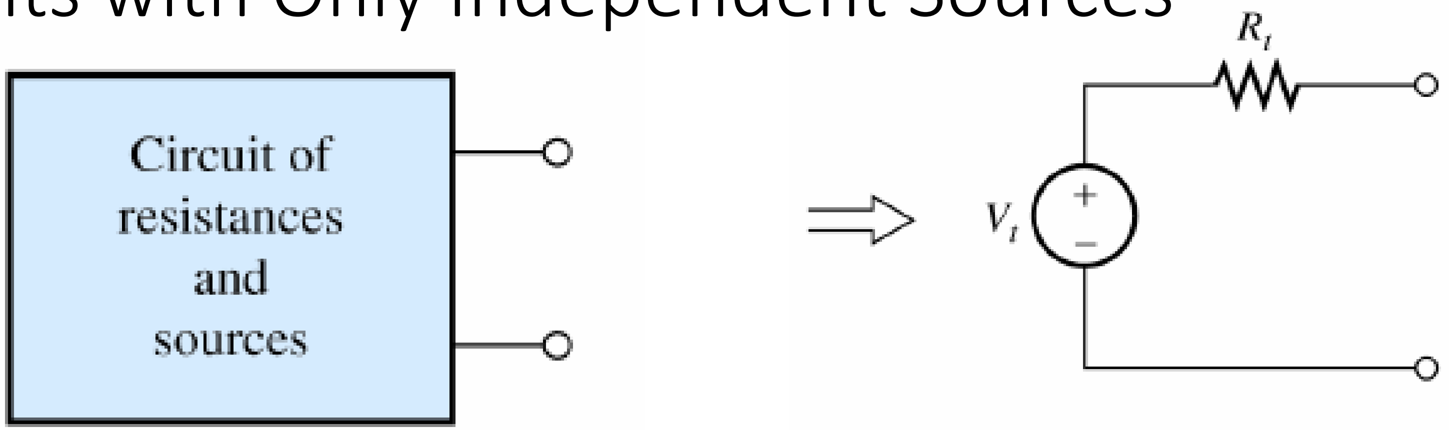
$$V_t = \frac{R_2}{R_2 + R_1} \times 15 = 5 \text{ V}$$



$$i_{sc} = \frac{v_s}{R_1} = 0.15A$$

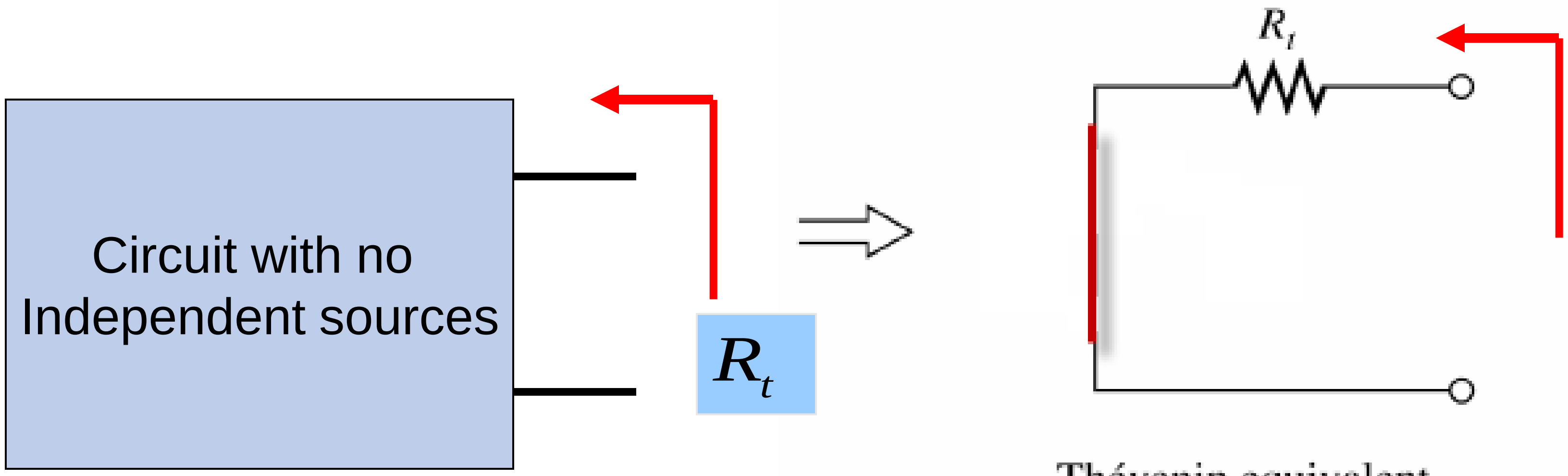
$$R_t = \frac{v_{oc}}{i_{sc}} = 33.3\Omega$$

For Circuits with Only Independent Sources



Thévenin equivalent circuit

Suppose we make all independent sources zero in the circuit



Thévenin equivalent circuit

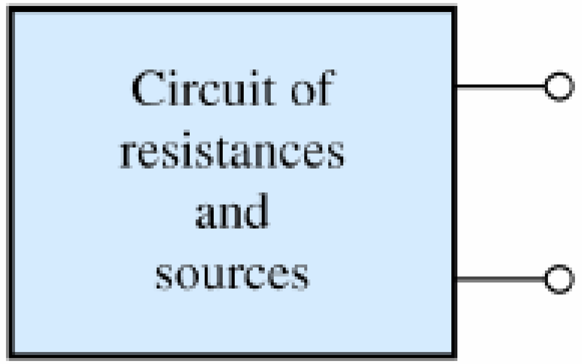
Reduces to resistive circuit evaluation we saw earlier!

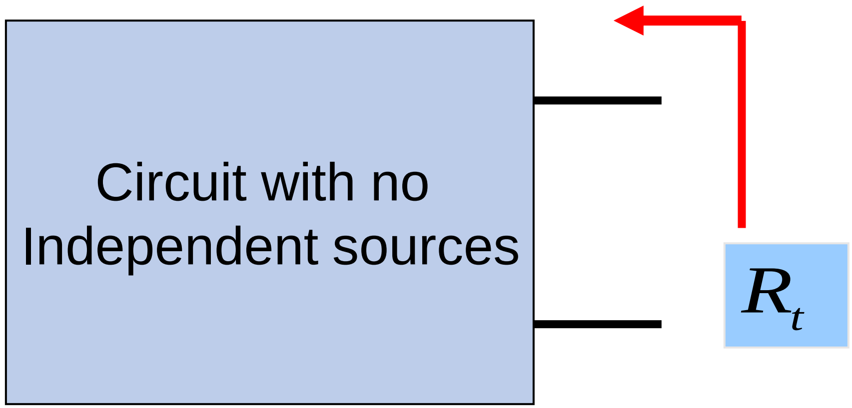
Evaluation of R_t

- Turn off independent sources in the original network:

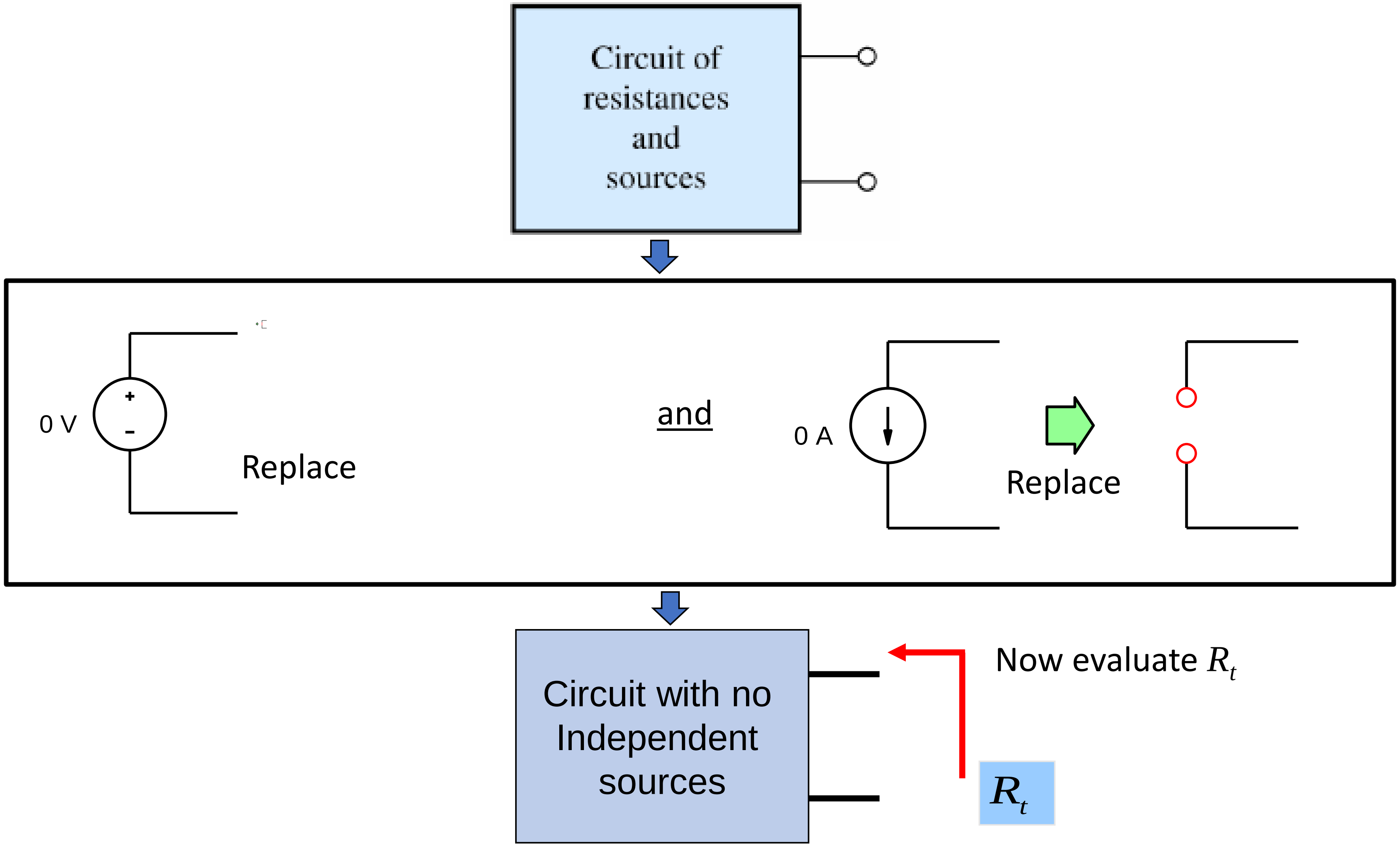
  - Replace a voltage source with a short circuit
  - Replace a current source with an open circuit

- 2. Compute the resistance between the terminals

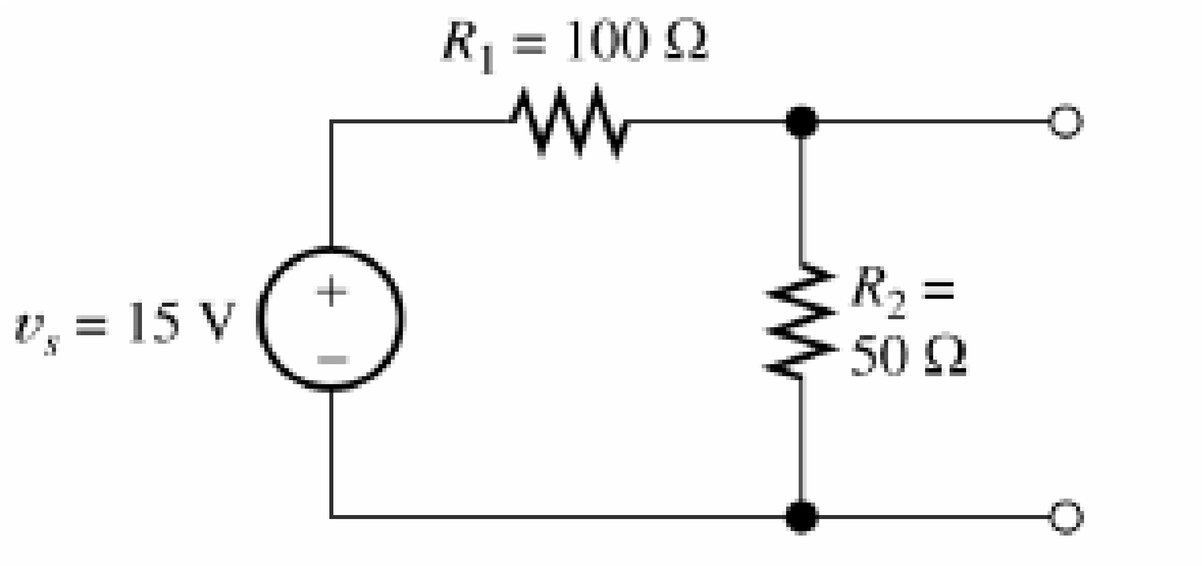


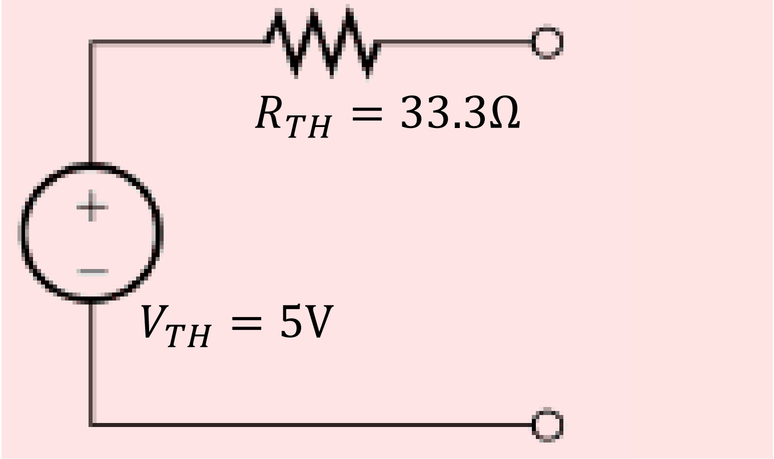


Procedure to Directly Evaluate R_t

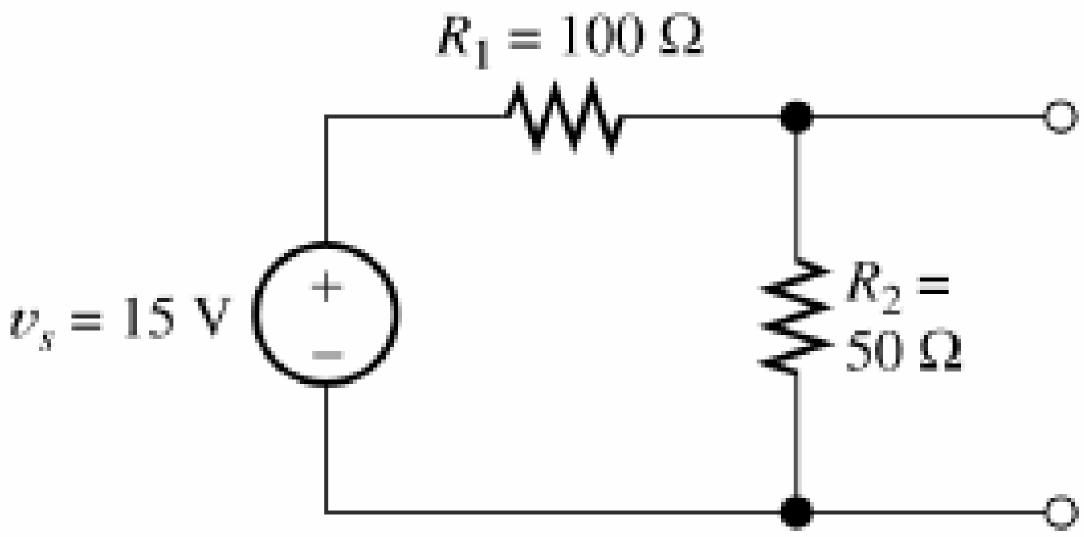


Thevenin Equivalent: Example

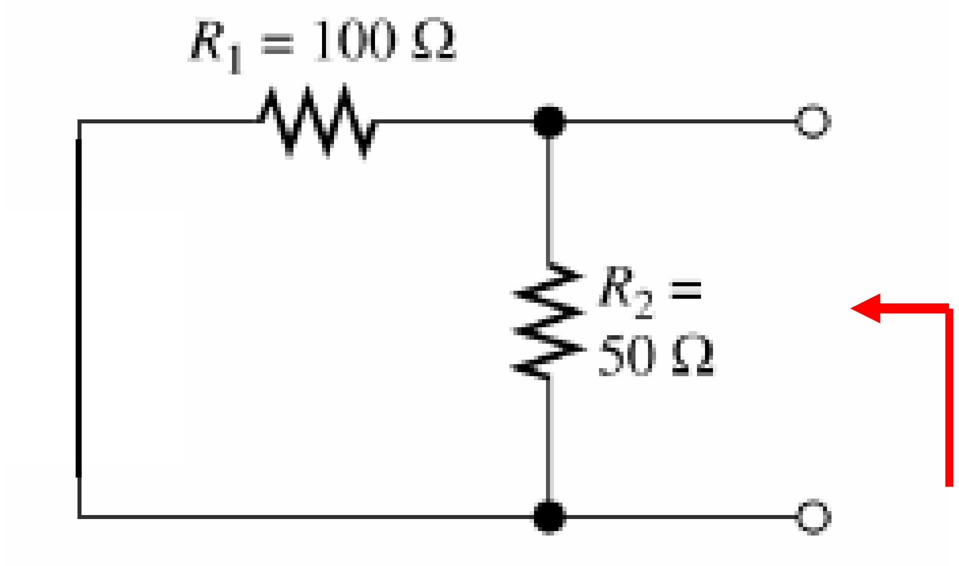




Thevenin Equivalent: Example: Direct R_{TH}



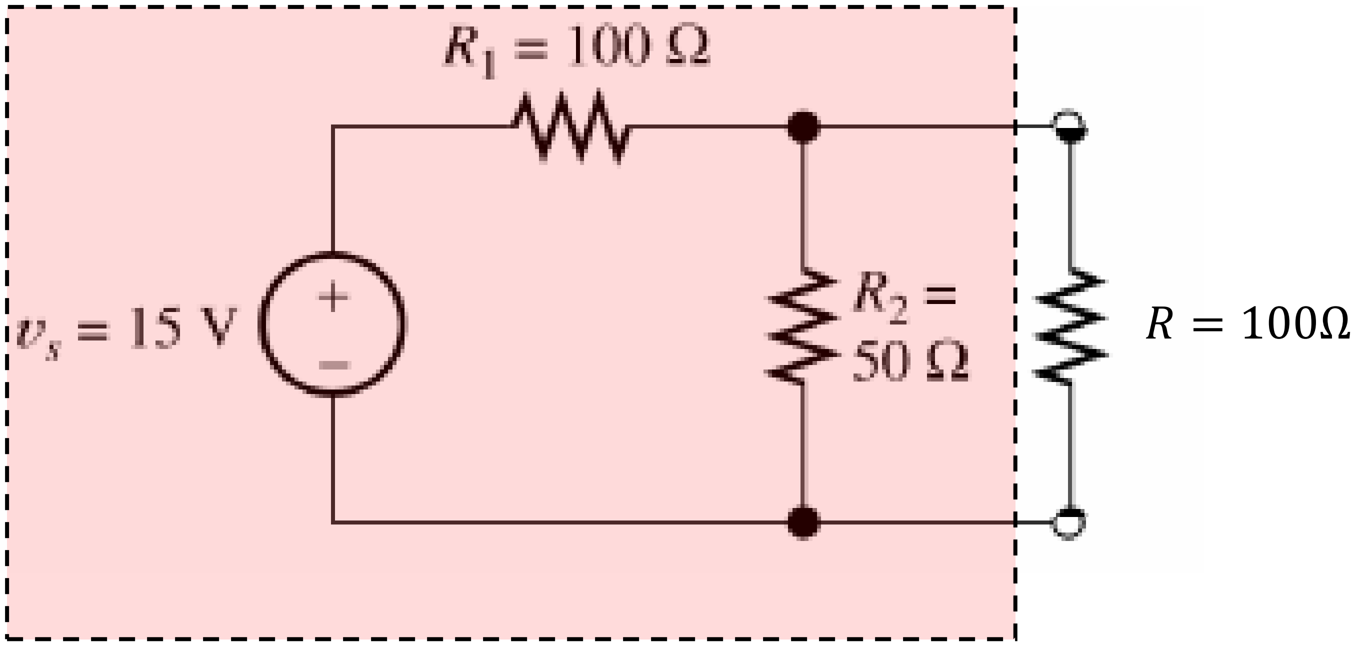
Thevenin Equivalent: Example : Direct R_{TH}

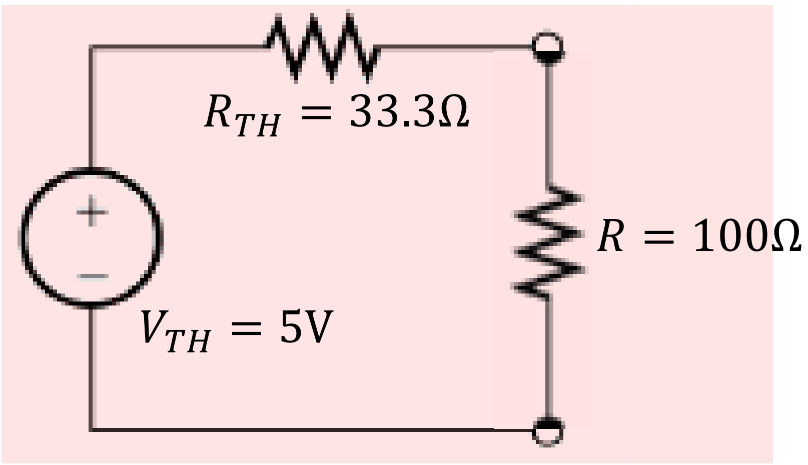


$$R_{TH} = 100||50 = 33.3\Omega$$

How to Use Thevenin Equivalent

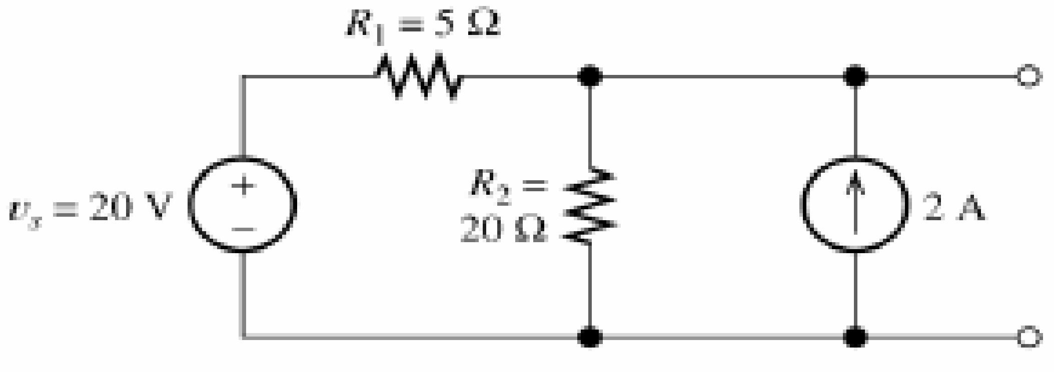
Compute Current in R

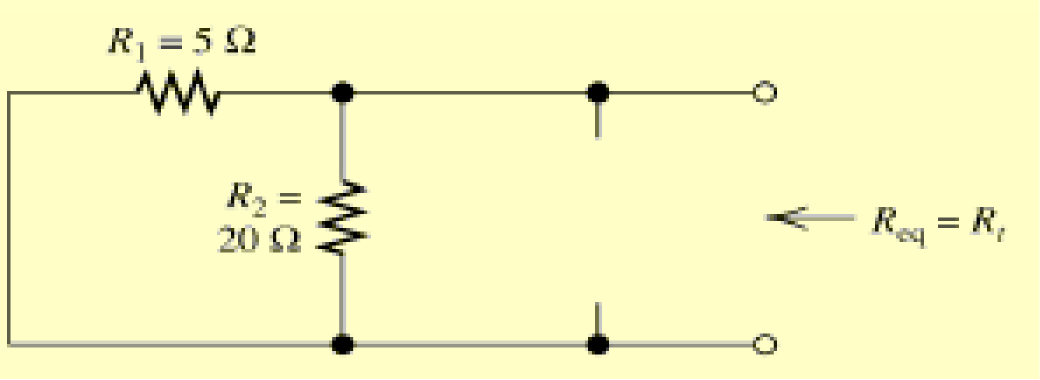




$$i = \frac{5}{33.3 + 100}\text{A}$$

Example

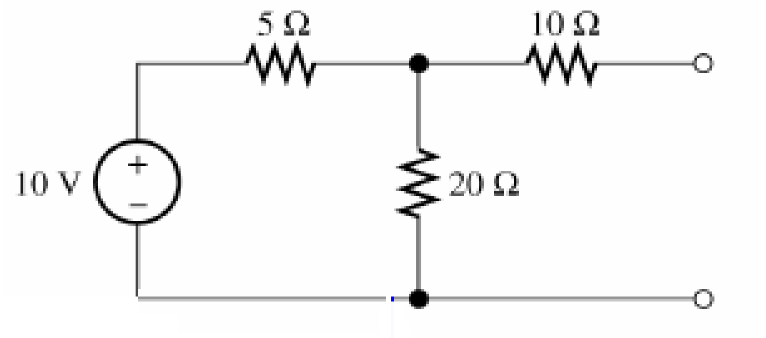




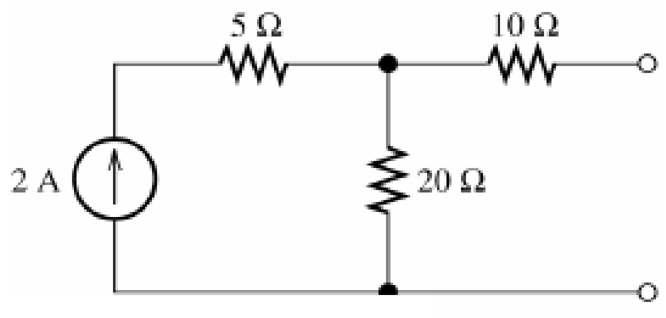
$$R_{eq} = \frac{5 \times 20}{5 + 20} = 4\Omega$$

Example

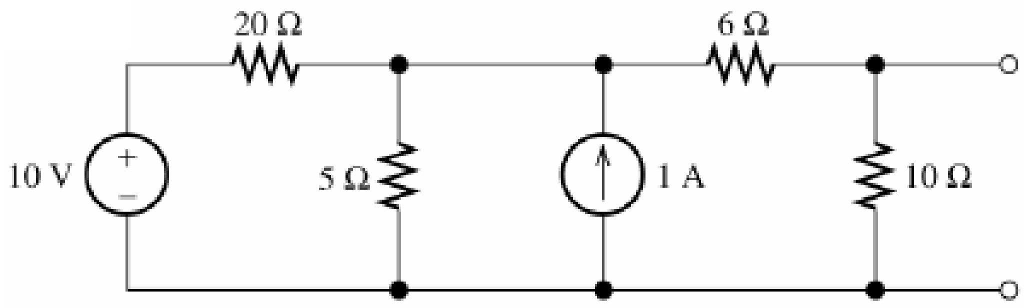
Find Thévenin resistance R_t for each of the circuits shown below



R_t=10+(5||20)=14 Ω



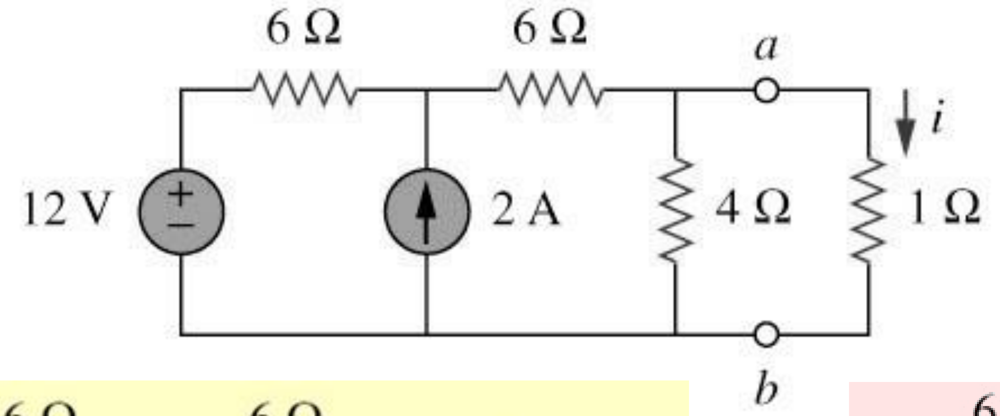
R_t=10+20 = 30 Ω

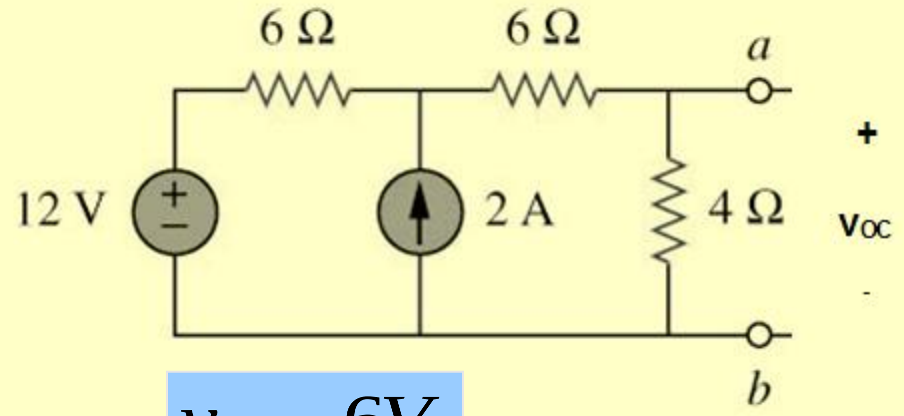


R_t = ((20||5)+6)||10) = 5 Ω

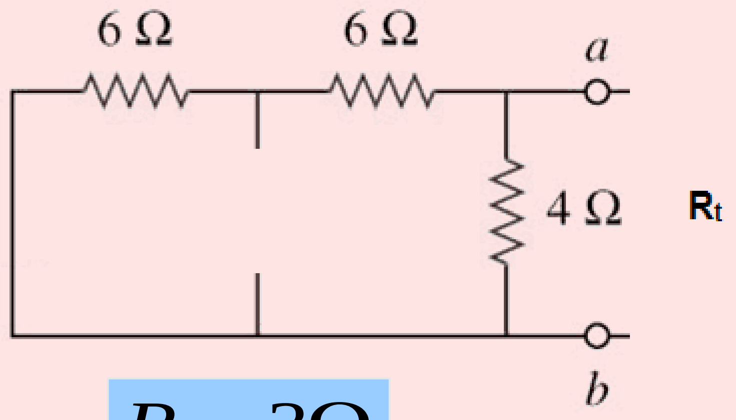
Example

Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit shown below. Hence find i.

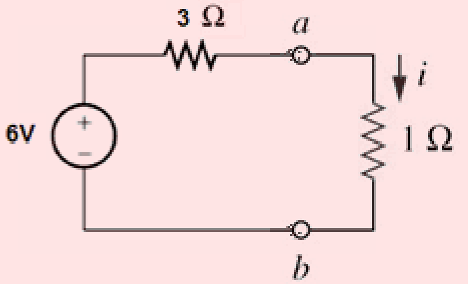




$$v_{oc} = 6V$$



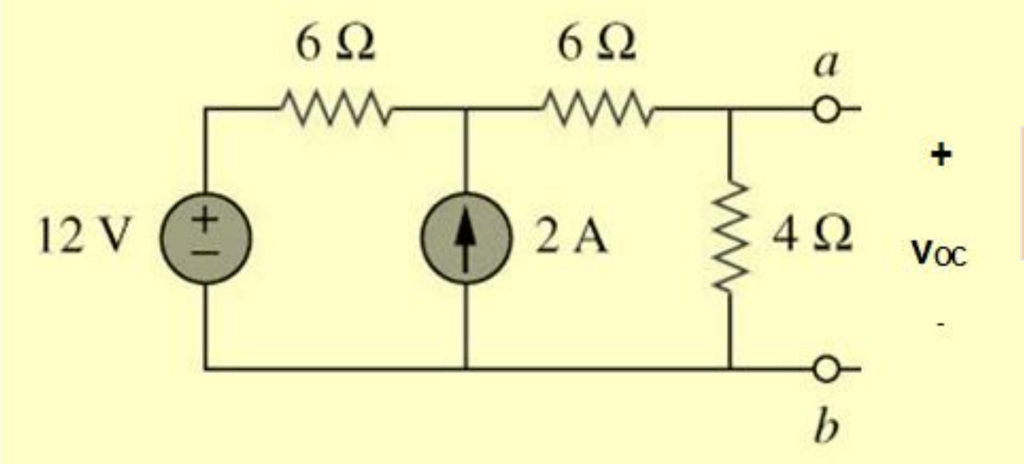
$$R_t = 3\Omega$$



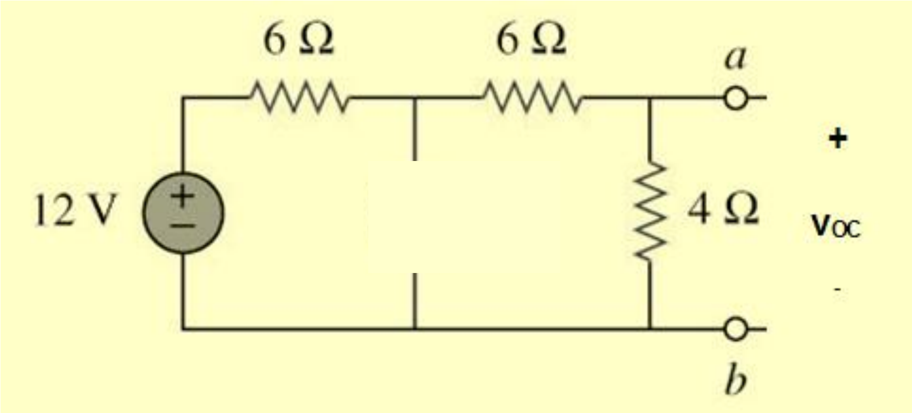
$$i = 1.5A$$

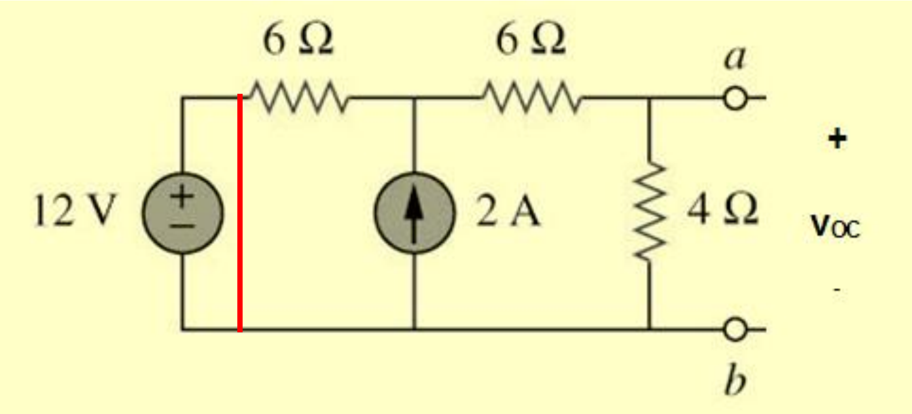
Use Superposition

$$v_{oc} = 6V$$



$$V_{oc} = V_{oc1} + V_{oc2} = 6$$

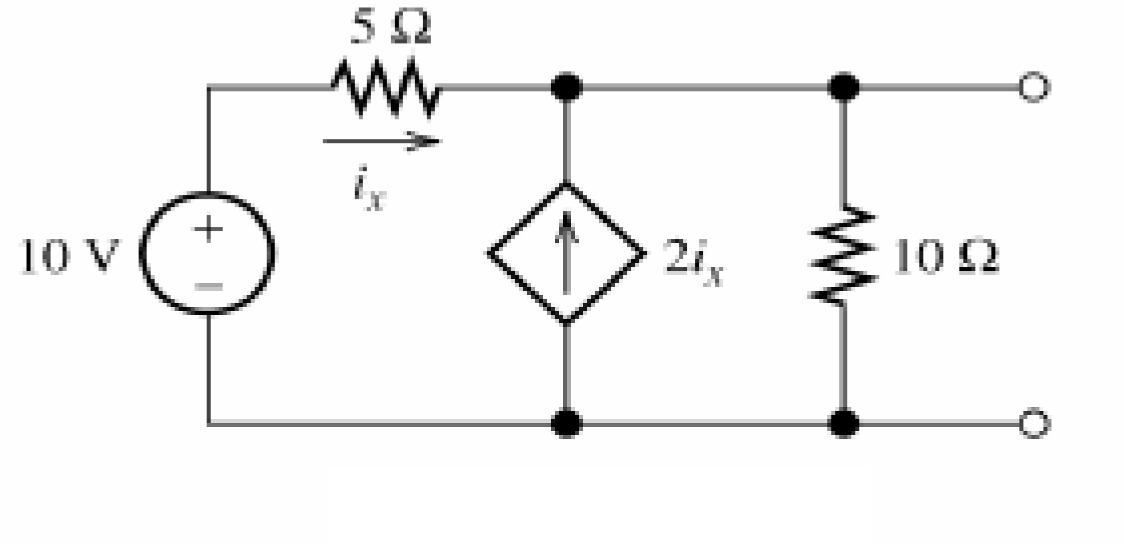


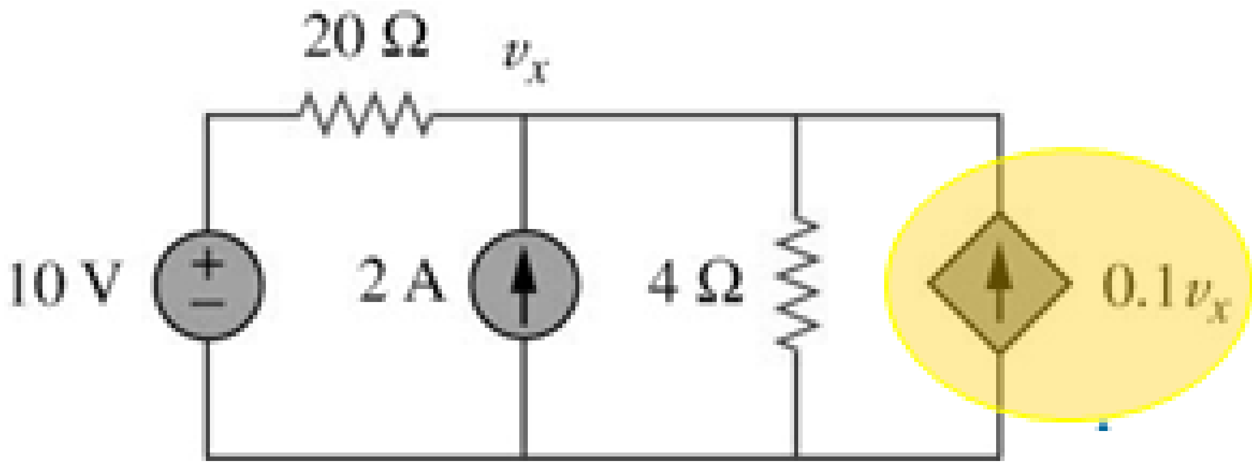


$$V_{oc1} = \frac{4}{4+12} \times 12 = 3$$

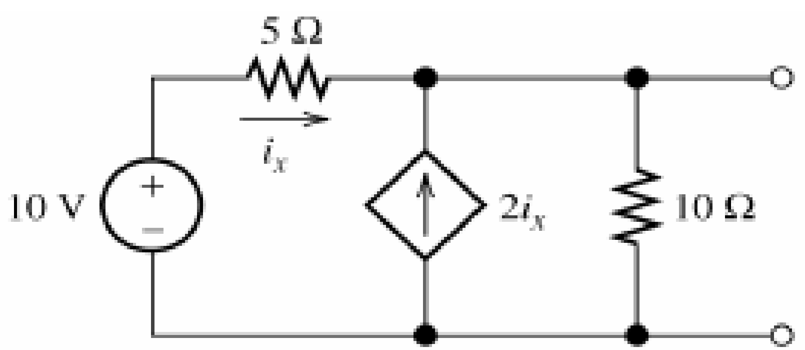
$$V_{oc2} = 4 \times \left(2 \times \frac{6}{6+10} \right) = 3$$

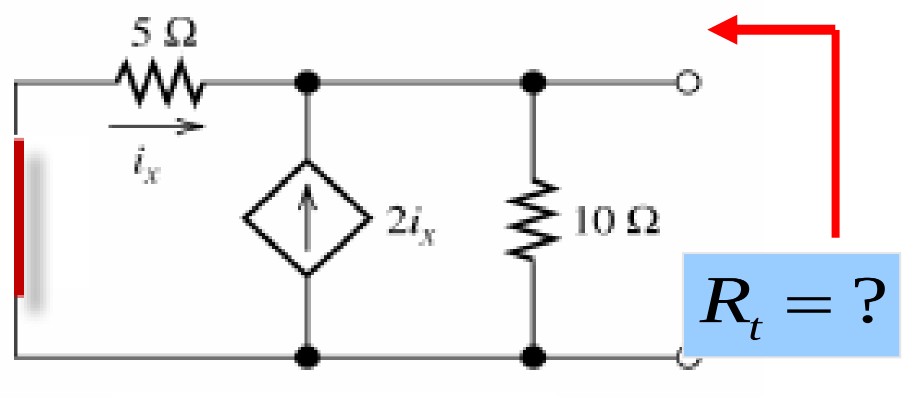
What If There Are Dependent Power Supplies?





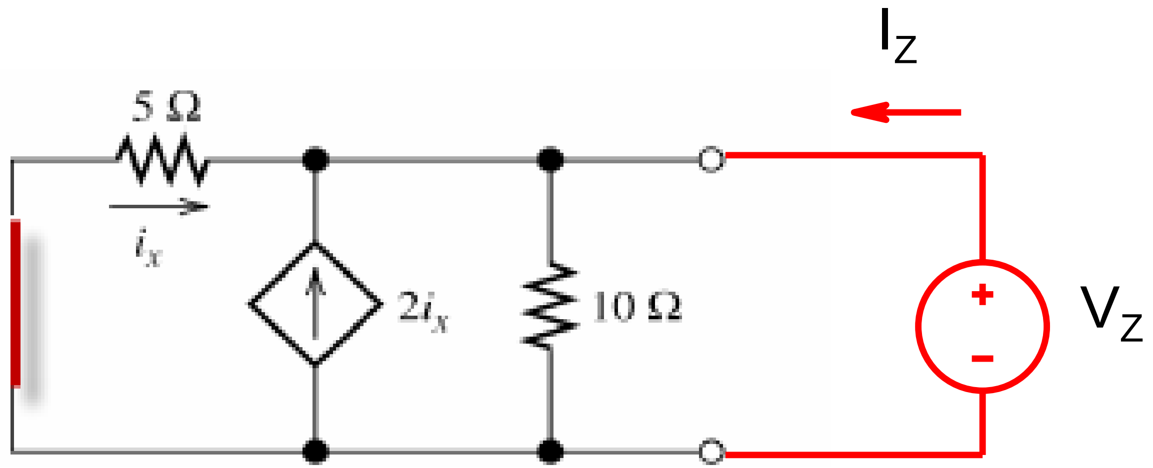
Thévenin Resistance for Circuit with Dependent Sources





Not easy to evaluate!

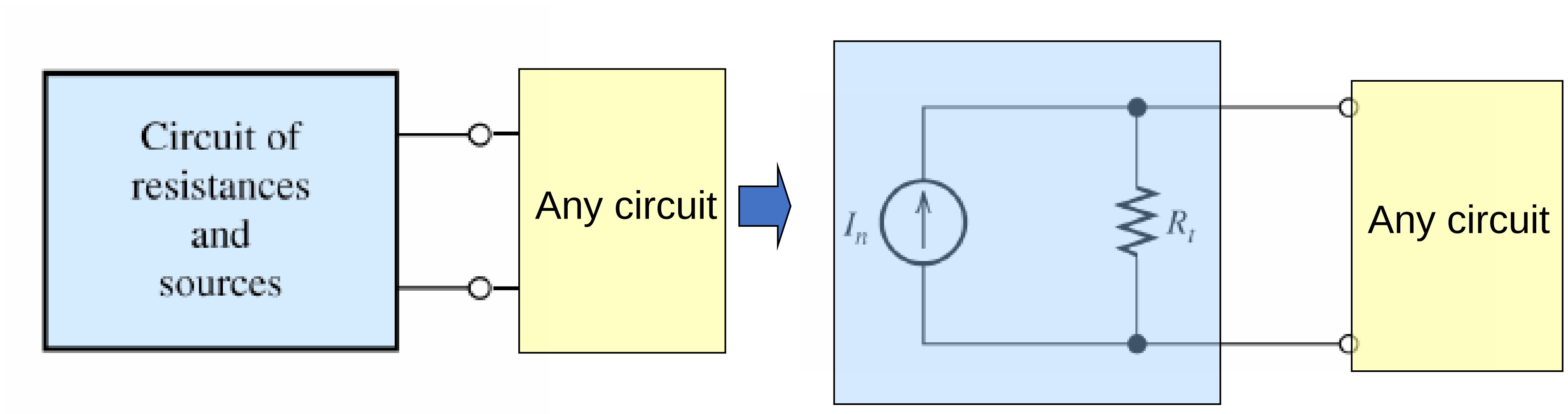
Procedure:
Add a power sources at evaluation nodes and then evaluate R_t



$$R_t = \frac{V_Z}{I_Z}$$

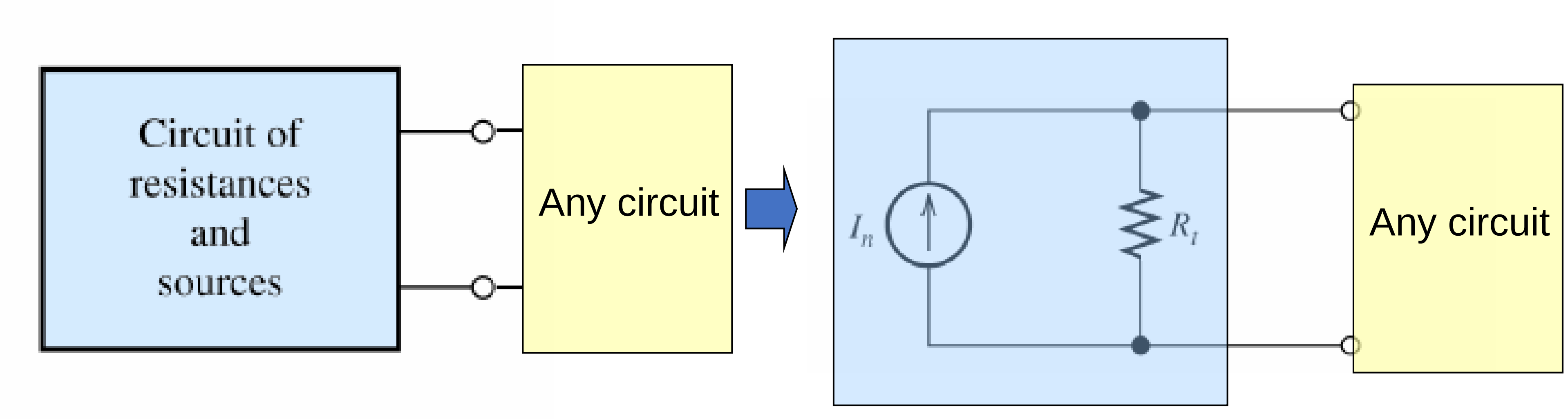
Norton's Theorem

"Any linear circuit containing several energy sources and resistances can be replaced by a single Constant Current generator in parallel with a Single Resistor"

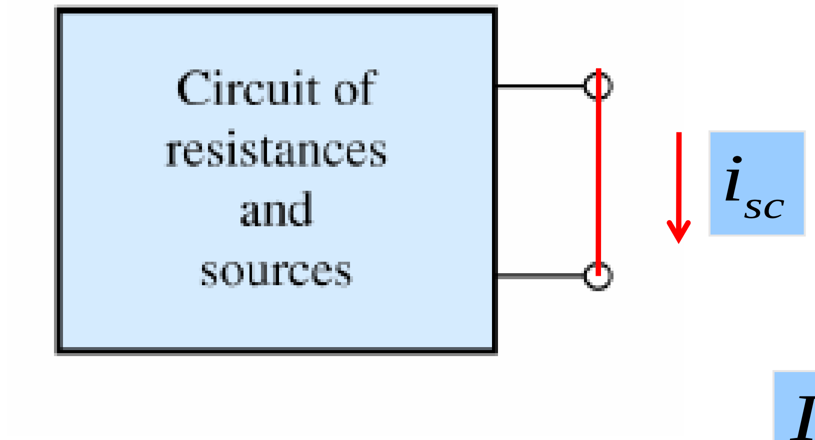


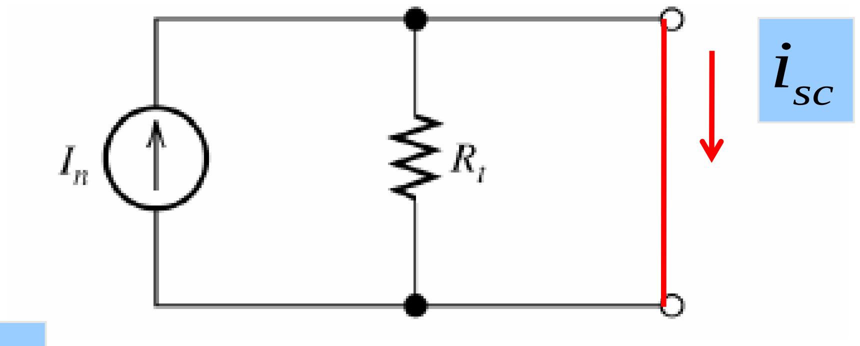
Parameters I_n and R_n = R_t

Norton Current



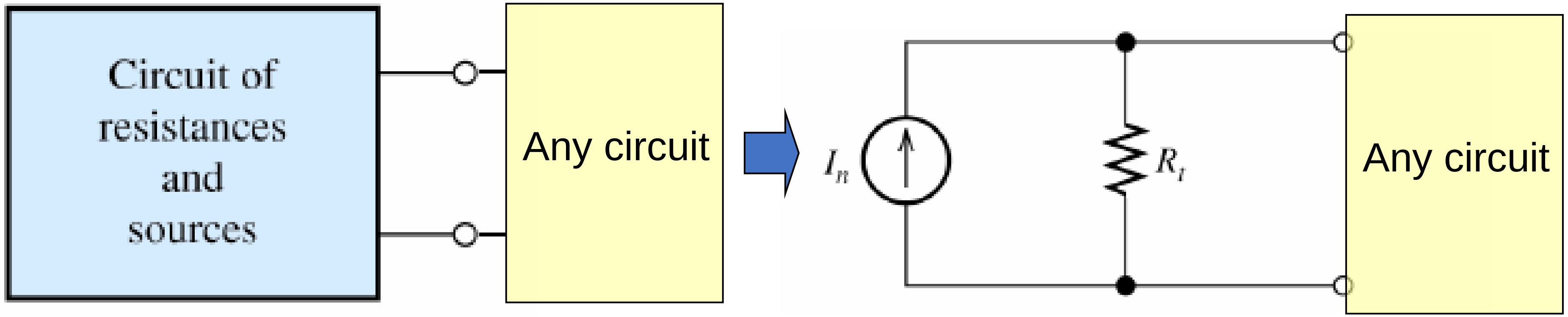
How do we find I_n ?



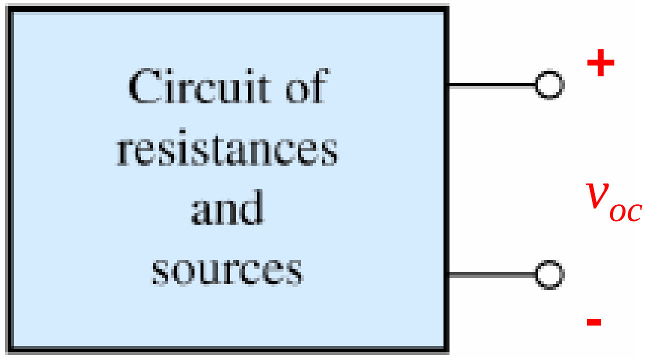


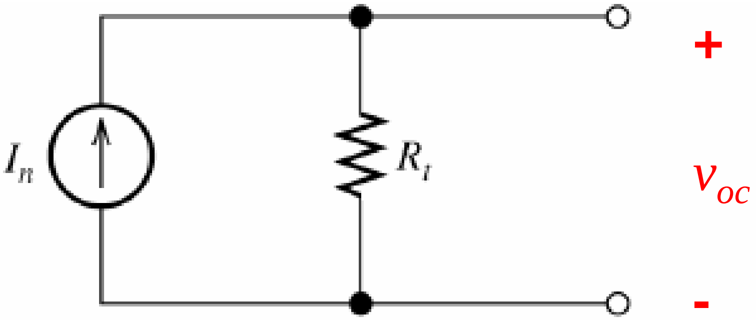
$$I_n = i_{sc}$$

Norton Resistance



How do we find R_t ?

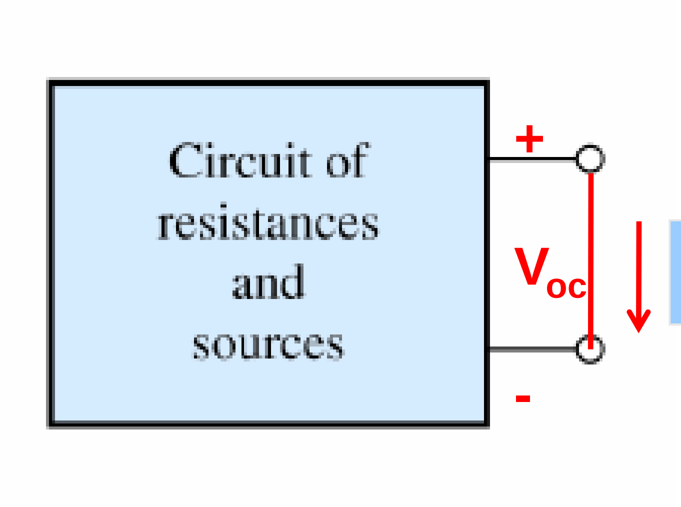




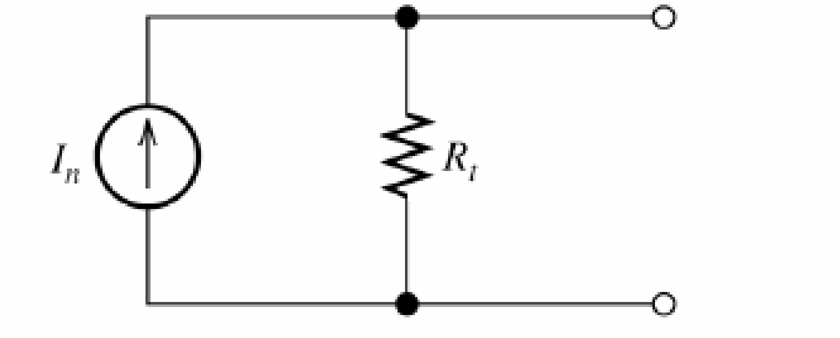
$$R_t = \frac{v_{oc}}{i_{sc}}$$

$$v_{oc} = I_n \times R_t$$

Norton Parameters



i_{sc}

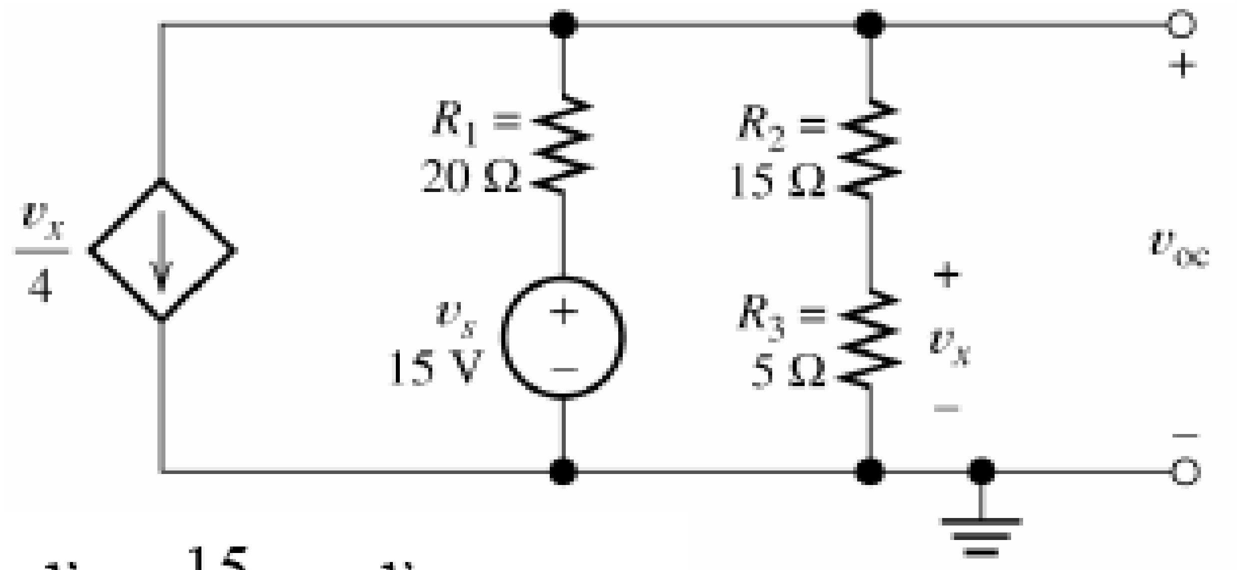


$$I_n = i_{sc}$$

$$R_t = \frac{v_{oc}}{i_{sc}}$$

Norton resistance is the same as Thévenin Resistance

Norton Equivalent: example

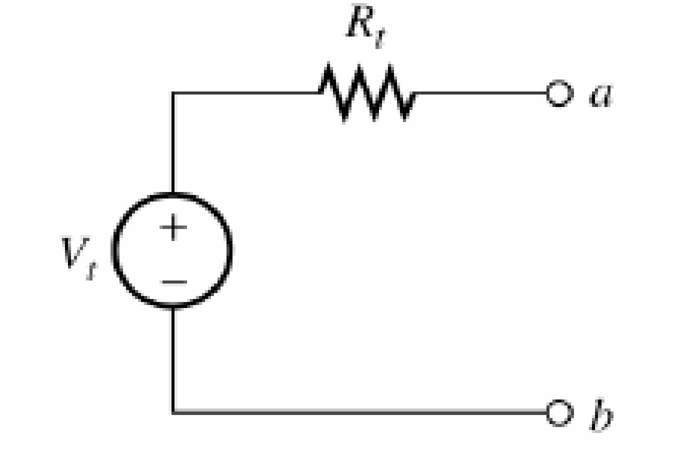


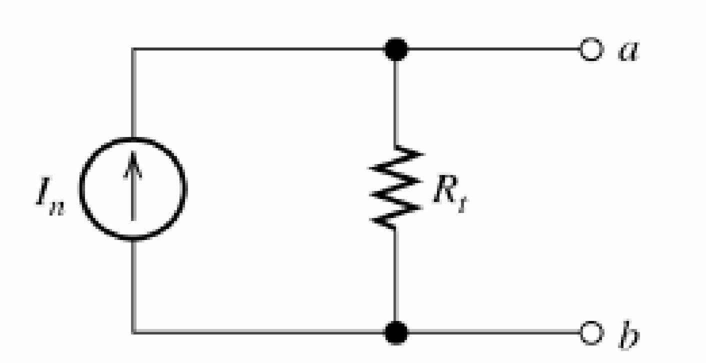
$$\frac{v_x}{4} + \frac{v_{oc} - 15}{R_1} + \frac{v_{oc}}{R_2 + R_3} = 0$$

$$v_x = \frac{R_3}{R_2 + R_3} v_{oc} = 0.25 v_{oc}$$

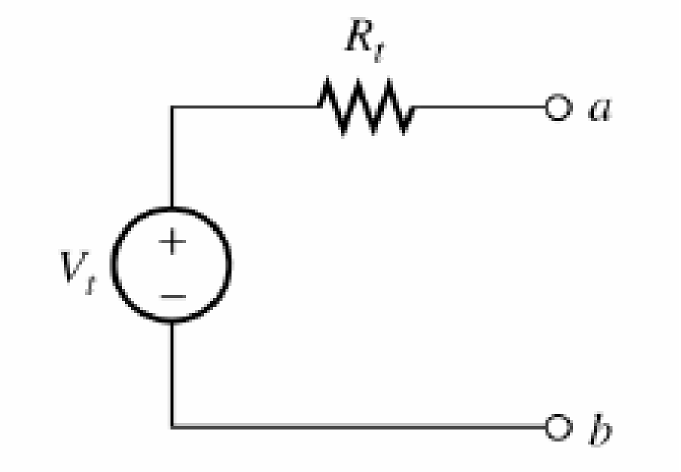
$$\boxed{v_{oc} = 4.62\text{V}}$$

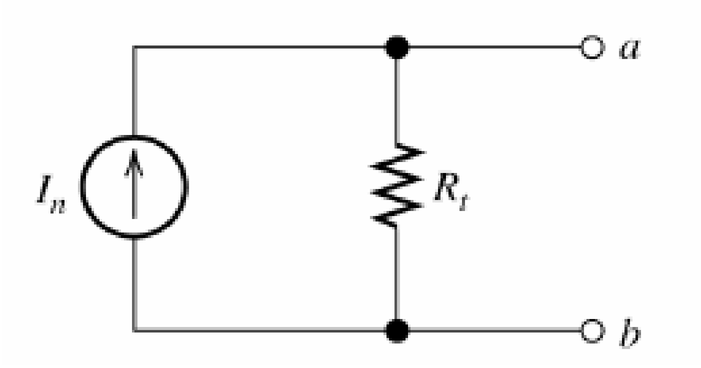
Source Transformation





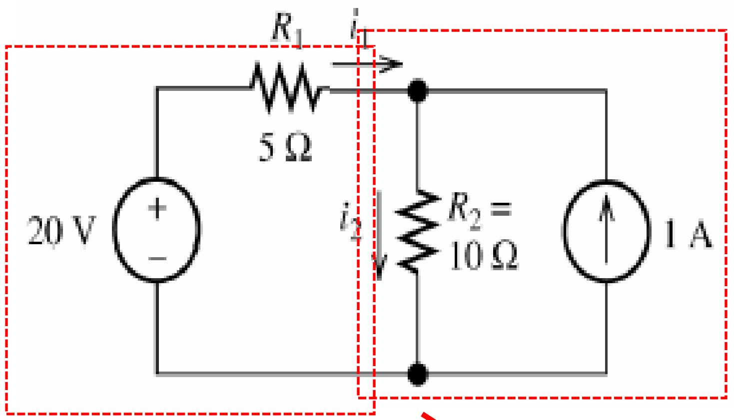
$$I_n = \frac{V_t}{R_t}$$



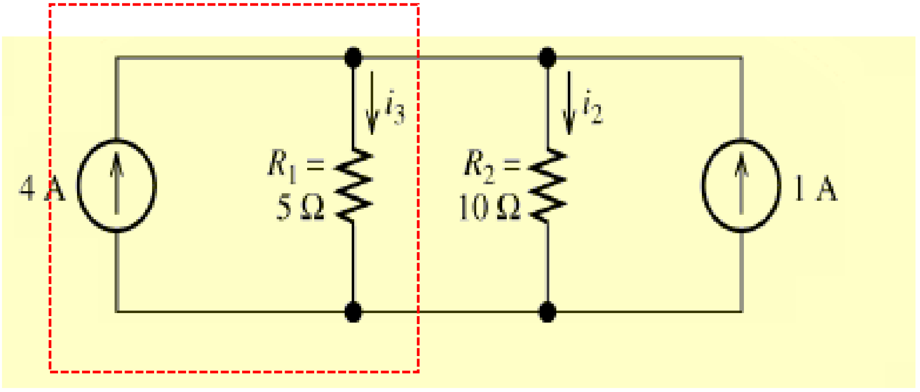


$$V_t = I_n \times R_t$$

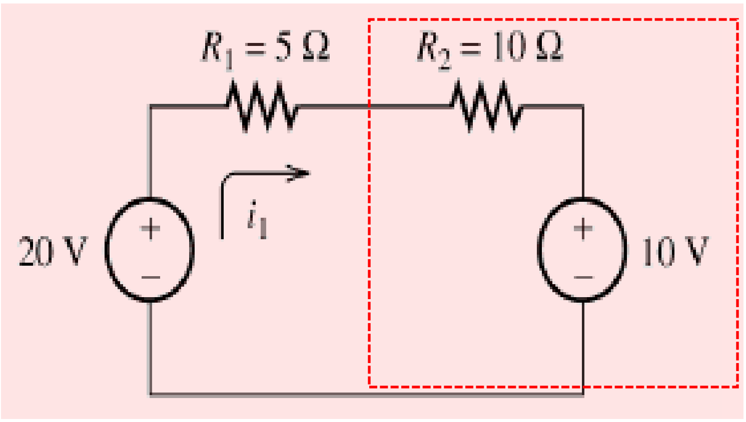
Example



R_1
i_1
5 Ω
20 V
i_2
R_2 =
10 Ω
1 A



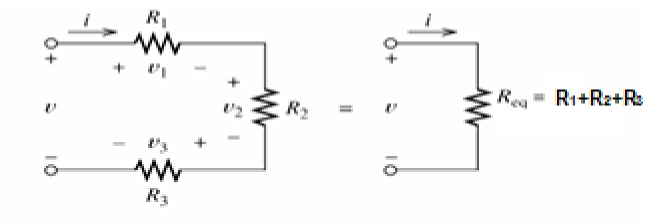
i_3
i_2
4 A
R_1 =
5 Ω
R_2 =
10 Ω
1 A



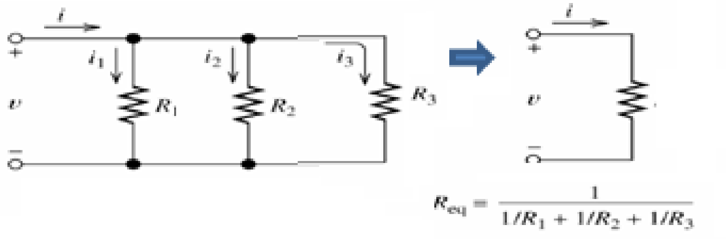
R_1 = 5 Ω
R_2 = 10 Ω
i_1
20 V
10 V

Summary

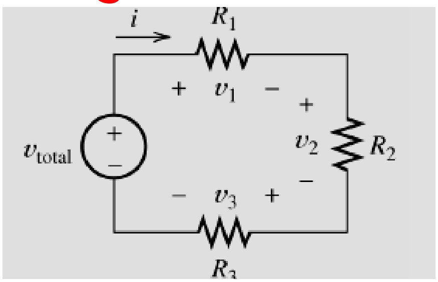
Series/Parallel resistances



(a) Three resistances (b) Equivalent



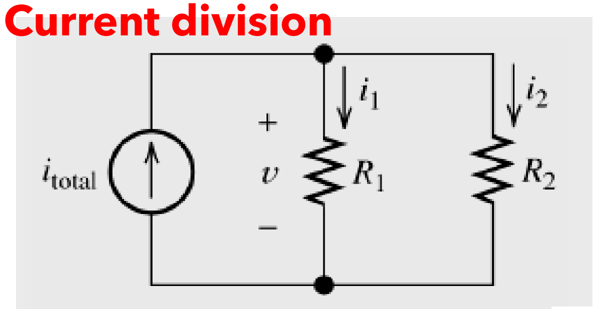
Voltage division



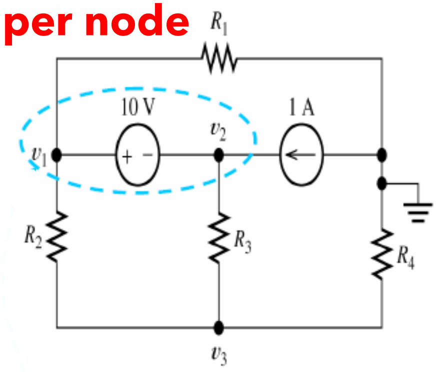
$$v_2 = R_2 i = \frac{R_2}{R_1 + R_2 + R_3} v_{total}$$

$$i_2 = \frac{v}{R_2} = \frac{R_1}{R_1 + R_2} i_{total}$$

Current division



Super node



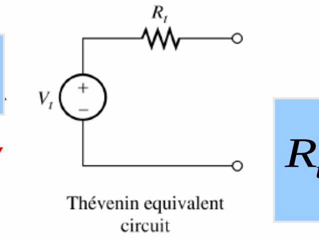
Nodal Analysis:

1. Identify and number the nodes
2. Choose a reference node
3. Write KCL for each node such that Sum of currents leaving a node is zero

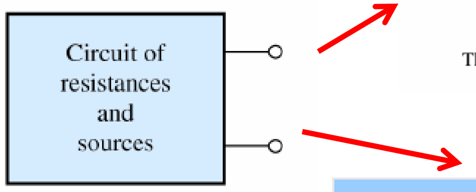
Mesh Analysis

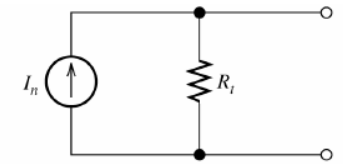
1. Assign mesh currents i_1, i_2, …, in to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

$$V_t = v_{oc}$$



$$R_t = \frac{v_{oc}}{i_{sc}}$$

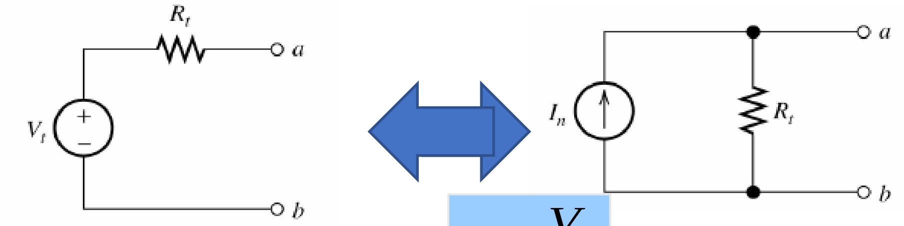




Thevenin & Norton

$$I_n = i_{sc}$$

Source Transformation



$$V_t = I_n \times R_t$$

$$I_n = \frac{V_t}{R_t}$$

The **superposition principle** states that the total response is the sum of the responses to each of the independent sources acting individually.