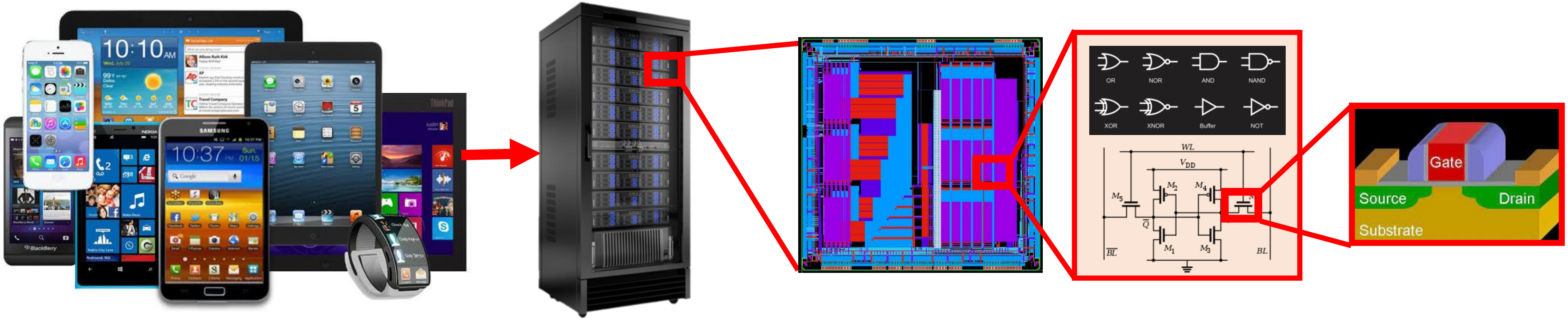

ESC201: INTRODUCTION TO ELECTRONICS

MODULE 6: DIGITAL CIRCUITS

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Representing Positive and Negative Numbers

Extra bit needed to carry sign information
"MSB" is often the sing bit

One option
Sign bit = 0 represents non-negative nos.
Sign bit = 1 represents negative numbers

decimal	Signed Magnitude
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
-0	1000
-1	1001
-2	1010
-3	1011
-4	1100
-5	1101
-6	1110
-7	1111

magnitude

magnitude

decimal	Signed 1's complement
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
-7	1000
-6	1001
-5	1010
-4	1011
-3	1100
-2	1101
-1	1110
0	1111

magnitude

magnitude

decimal	Signed 2's complement
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
-8	1000
-7	1001
-6	1010
-5	1011
-4	1100
-3	1101
-2	1110
-1	1111

magnitude

magnitude

Unique zero representation

$2^{n-1} - 1$ positive nos.

2's comp. of 0 and -8 are themselves

2^{n-1} negative nos.

Arithmetic with 2’s Complement

- The negative of a number A is represented by its 2’s complement
  - Negative of the negative of the number is the number itself
- To evaluate **A – B**, one can following the following algorithm
  - Find **–B** by taking 2’ complement of **B**
  - Then **A – B = A + (-B) = A +** (2’s complement of **B**)

Example

Adding or subtracting numbers with addition operation alone

To get a negative number, 2’s complement of positive number is taken

$$\begin{array}{r} +5 \\ +2 \\ \hline +7 \\ \hline \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ +\,0\,0\,1\,0 \\ \hline 0\,1\,1\,1 \\ \hline \end{array}$$

$$\begin{array}{r} +5 \\ -2 \\ \hline +3 \\ \hline \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ +\,1\,1\,1\,0 \\ \hline 0\,0\,1\,1 \\ \hline \end{array}$$

2’s comp. of +2

2’s comp. of +5

$$\begin{array}{r} -5 \\ +2 \\ \hline -3 \\ \hline \end{array} \qquad \begin{array}{r} 1\,0\,1\,1 \\ +\,0\,0\,1\,0 \\ \hline 1\,1\,0\,1 \\ \hline \end{array}$$

2’s complement is 0011 = 3

2’s comp. of +5

2’s comp. of +2

$$\begin{array}{r} -5 \\ -2 \\ \hline -7 \\ \hline \end{array} \qquad \begin{array}{r} 1\,0\,1\,1 \\ +\,1\,1\,1\,0 \\ \hline 1\,0\,0\,1 \\ \hline \end{array}$$

2’s complement is 0111 = 7

The Boolean Algebra

In the Boolean world, a variable can take just two values {0,1}

Positive Logic:

0 may represent → False, No, Low voltage, Low …(variable)

1 may represent → True, Yes, High voltage, High …(variable)

Negative Logic will be the inverse of the above, i.e., 0 being True and 1 being False

All interactions of Boolean variables can be represented by a combination of:

AND	➡	y is x_1 AND x_2	➡	$y = x_1 \cdot x_2$
OR	➡	y is x_1 OR x_2	➡	$y = x_1 + x_2$
NOT	➡	y is NOT x_1	➡	$y = \overline{x_1}$

In the examples above, y → response variable and x_1 and x_2 → independent variables

More About Basic Operations

Truth Table

AND: $y = x_1 . x_2$

y is 1 if and only if both x_1 and x_2 are 1, otherwise 0

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

OR: $y = x_1 + x_2$

y is 1 if either x_1 or x_2 is 1.
y is 0 if and only if both x_1 and x_2 are 0, otherwise 1

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

NOT: $y = \overline{x}$

y is the inverse of x
If y is 0, x is 1; and If y is 1, x is 0

x	y
0	1
1	0

Some Basic Postulates

P1: $x + 0 = x$	P1: $x . 1 = x$
commutativity P2: $x + y = y + x$	P2: $x . y = y . x$
distributivity P3: $x.(y+z) = x.y+x.z$	P3: $x+y.z = (x+y).(x+z)$ (please take note)
P4: $x + \overline{x} = 1$	P4: $x . \overline{x} = 0$

Some Basic Theorem

T1: $x + x = x$	T1: $x \cdot x = x$
T2: $x + 1 = 1$	T2: $x \cdot 0 = 0$
T3: $\overline{(\overline{x})} = x$	
T4: $x + (y+z) = (x+y)+z$	T4: $x \cdot (y.z) = (x.y).z$
(DeMorgan's theorem) T5: $\overline{(x+y)} = \overline{x} \cdot \overline{y}$	(DeMorgan's theorem) T5: $\overline{(x.y)} = \overline{x} + \overline{y}$
T6: $x + x.y = x$	T6: $x.(x+y) = x$

Proving Theorems

P1:	$x + 0 = x$
P2:	$x + y = y + x$
P3:	$x.(y+z) = x.y+x.z$
P4:	$x + \overline{x} = 1$

P1:	$x . 1 = x$
P2:	$x . y = y . x$
P3:	$x+y.z = (x+y).(x+z)$
P4:	$x . \overline{x} = 0$

Prove T1: $x + x = x$

$x + x = (x+x). 1$ (P1)

$= (x+x). (x+\overline{x})$ (P4)

$= x + x.\overline{x}$ (P3)

$= x + 0$ (P4)

$= x$ (P1)

Prove T1: $x . x = x$

$x . x = x.x+ 0$ (P1)

$= x.x + x.\overline{x}$ (P4)

$= x . (x+\overline{x})$ (P3)

$= x . 1$ (P4)

$= x$ (P1)

Proving More Theorems

P1:	$x + 0 = x$	P1:	$x \,.\, 1 = x$
P2:	$x + y = y + x$	P2:	$x \,.\, y = y \,.\, x$
P3:	$x.(y+z) = x.y+x.z$	P3:	$x+y.z = (x+y).(x+z)$
P4:	$x + \overline{x} = 1$	P4:	$x \,.\, \overline{x} = 0$

Prove : $x + 1 = 1$

$$x + 1 = x+(x+\overline{x})$$
$$= (x+x)+\overline{x}$$
$$= x + \overline{x}$$
$$= 1$$

$x + x\,.y = x$

$$= x \,.\, 1 + x.\, y$$
$$= x.\,(1+ y)$$
$$= x \,.\, 1$$
$$=x$$

$x + \overline{x}\,.y = x+y$

$$= (x + \overline{x}).\,(x+ y)$$
$$= 1.\,(x+ y)$$
$$= x + y$$

Exercise

De Morgan's theorem

$$\overline{(x_1 + x_2 + x_3+\ldots)} = \overline{x_1}\,.\,\overline{x_2}.\overline{x_3}\ \ldots \qquad \text{(A)}$$

$$\overline{(x_1.\, x_2.\, x_3.\ldots.)} = (\,\overline{x_1} + \overline{x_2} + \overline{x_3} + \ldots.) \qquad \text{(B)}$$

Prove this for both (A) and (B). You have to prove both forms P4 hold.

Start with only two variables x_1 and x_2, then extend.

Simplification of Boolean Expressions

De Morgan's Theorem

$$\overline{(x_1 + x_2 + x_3 + \ldots.)} = \overline{x_1} \, . \, \overline{x_2} \, . \, \overline{x_3} \, .$$

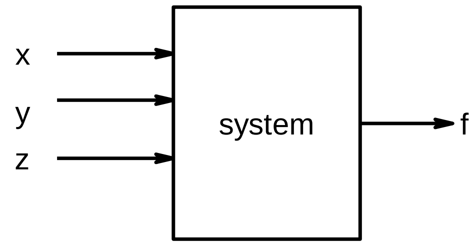
$$\overline{(x_1 . \, x_2 . \, x_3 \ldots..)} = (\overline{x_1} + \overline{x_2} + \overline{x_3} + \ldots..)$$

$$\overline{(\overline{x_1}.x_2 + \overline{x_2}.x_3)} = ?$$

$$= (x_1 + \overline{x_2}) \, . \, (x_2 + \overline{x_3})$$

$$= x_1 . \, x_2 + x_1 \, . \, \overline{x_3} + \overline{x_2} . \, \overline{x_3}$$

Design Flow



System Description

↓

Truth Table

↓

Boolean Expression

↓

Minimized Boolean Expression

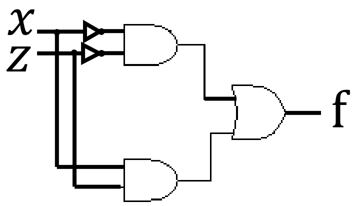
↓

Gate Netlist

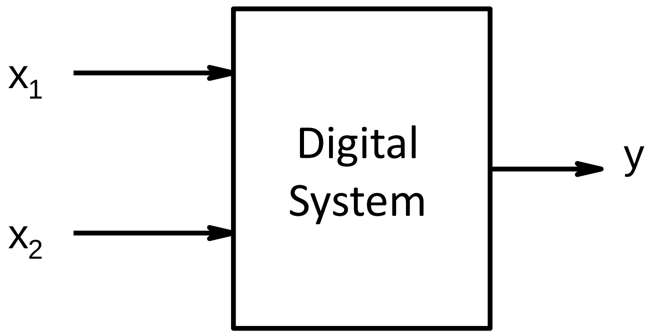
x	y	z	f
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$$\mathrm{f} = \overline{x}.\overline{y}.\overline{z} + \overline{x}.y.\overline{z} + x.\overline{y}.z + x.y.z$$

$$\Rightarrow \mathrm{f} = \overline{x}.\overline{z} + x.z$$



Representation of a Digital System



Description in words

y = 1 when x_1 is 0 and x_2 is 1

Truth Table

Indicates when response y is ‘true’

x_1	x_2	y
0	0	0
0	1	1
1	0	0
1	1	0

Boolean expression

$$y = \overline{x_1} . x_2$$

Boolean Function from Truth Tables

Example

x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	0

$$y = \overline{x_1} \,.\, \overline{x_2}$$

Example

x_1	x_2	y
0	0	0
0	1	0
1	0	1
1	1	0

$$y = x_1 \,.\, \overline{x_2}$$

Example

x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	1

$\overline{x_1} \,.\, \overline{x_2}$

$x_1 \,.\, x_2$

When more than one combination is ‘true’ combine them with OR operation

$$y = \overline{x_1} \,.\, \overline{x_2} + x_1 . x_2$$

Both True and False Can be Useful

Instead of writing expressions as sum of terms that make **y equal to 1**,
we can also write expressions using terms that make **y equal to 0**

Example

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

$$y = \overline{x_1}.x_2 + x_1.\overline{x_2}$$

By inspecting 'True' terms

$$\bar{y} = \overline{x_1}.\overline{x_2} + x_1.x_2$$

$$y = (x_1 + x_2).(\overline{x_1} + \overline{x_2})$$

By inspecting 'False' terms

Note
Inversion
In variables !

Example

x_1	x_2	y
0	0	1
0	1	1
1	0	1
1	1	0

$$y = \overline{x_1}.\overline{x_2} + \overline{x_1}.x_2 + x_1.\overline{x_2}$$

By inspecting 'True' terms

$$y = \overline{x_1} + \overline{x_2}$$

By inspecting 'False' terms

Here a simpler approach!

Expression Derived From False Terms

Example

x_1	x_2	y
0	0	1
0	1	0
1	0	1
1	1	1

$y = x_1 + \overline{x_2}$

Example

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

$y = x_1 + x_2$

Example

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

$x_1 + x_2$

$\overline{x_1} + \overline{x_2}$

$y = (x_1 + x_2).(\overline{x_1} + \overline{x_2})$

SoP Form With Min Terms for Two Inputs

A **min term** is a **product (AND)** that contains all the variables used in a function

The function is the **sum (OR) of min terms for which output function is 'True'**

x (MSB)	y (LSB)	min term	
0	0	$\overline{x}.\overline{y}$	m0
0	1	$\overline{x}.y$	m1
1	0	$x.\overline{y}$	m2
1	1	$x.y$	m3

Example

x	y	f_1
0	0	0
0	1	1
1	0	1
1	1	0

$$f_1 = \sum(1,2) = m_1 + m_2 = \overline{x}.y + x.\overline{y}$$

Example

$$f_2 = \sum(0,2,3) = ?$$

$$f_2 = \overline{x}.\overline{y} + x.\overline{y} + x.y$$

We are showing function in **Sum of Products (SoP)** form

Min Terms for Three Inputs

MSB x	y	LSB z	min terms	
0	0	0	$\bar{x} . \bar{y} . \bar{z}$	m0
0	0	1	$\bar{x} . \bar{y} . z$	m1
0	1	0	$\bar{x} . y . \bar{z}$	m2
0	1	1	$\bar{x} . y . z$	m3
1	0	0	$x . \bar{y} . \bar{z}$	m4
1	0	1	$x . \bar{y} . z$	m5
1	1	0	$x . y . \bar{z}$	m6
1	1	1	$x . y . z$	m7

$$\mathrm{f}_2 = \sum(1,4,7) = ?$$

$$\mathrm{f}_2 = \bar{x}.\,\bar{y}.z + x.\bar{y}.\bar{z} + x.y.z$$

Sum of Products form of function for three input variables