Department of Electrical Engineering,

Indian Institute of Technology, Kanpur

ESC201	**Mid-semester Examination**	**27/02/2025**
Total Marks: 30		**Time: 2 hours**

Instructions

- **Please read the questions carefully.**
- **The question paper carries 30 marks which will be scaled to 20 marks for final grading.**

1) Professor X (Charles Xavier) is a mutant who possesses telepathy powers and can tune into minds of fellow mutants (working at a frequency of 1 MHz) and even non-mutants (working at a frequency of 1 KHz) through his device Cerebro (which acts like an amplifier with a gain of 20 dB). Magneto (another powerful mutant) does not want Professor X to invade his mind and wants to design a helmet with a filter attenuating (rejecting) the brain waves corresponding to the mutant frequency (1 MHz) by -60 dB while passing the non-mutant frequency (1 KHz). Upon passing the brain waves of Professor X through spectrum analyser, Magneto gets to know that its time-domain representation can be given as:

$v(t) = 0.1\cos(2\pi 1000t + 30^{o}) + 5\cos(2\pi 10^{6}t + 45^{o})$ V

Can you please help Magneto design this filter to block telepathy by answering the following questions:

(a) What filter topology do you need? **(1 mark)**

(b) Do the specifications necessitate the use of a resonant filter? NO **(1 mark)**

(c) You are given a resistance of 50Ω, a variable capacitance and a variable inductance to design this filter. Draw the schematic of the filter and find out the values of the components needed. **(2 marks)**

(d) Draw the representative (not to exact scale) Bode magnitude and phase plot for the designed filter. **(3 marks)**

(e) What are the amplitudes of the individual sinusoidal components in $v(t)$ after passing through Cerebro and at the output of the designed filter? **(1 mark)**

→ We want to pass 1KHz while attenuating 1MHz signal by −60 dB. Difference b/w frequencies = $\log_{10}\frac{(1MHz)}{1KHz}$ decades

→ In 3 decades of frequency, we need to reduce the gain by −60 dB. Slope required = −20 dB/decade

⇒ We can use a simple low pass filter (a) 1 with corner frequency at 1KHz to introduce slope of −20dB/decade

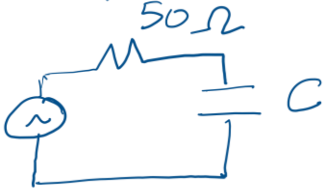
Please give marks

One can even use a band pass filter (central freq. = 1KHz) & appropriate credit must be given if design steps are correct.

Or even band stop filter with notch at 1MHz.

→ ∵ Required slope is −20 dB/decade, a resonant filter is not necessary. Simple RC filter can do it. (1)

→ I am using simple RC filter with corner frequency of 1 KHz



Using $f_c = \dfrac{1}{2\pi RC}$ (1)

$10^3 = \dfrac{1}{2\pi \times 50 \times C}$ or $C = \dfrac{1}{2\pi \times 50 \times 10^3} = 0.31 \times 10^{-5}$ F

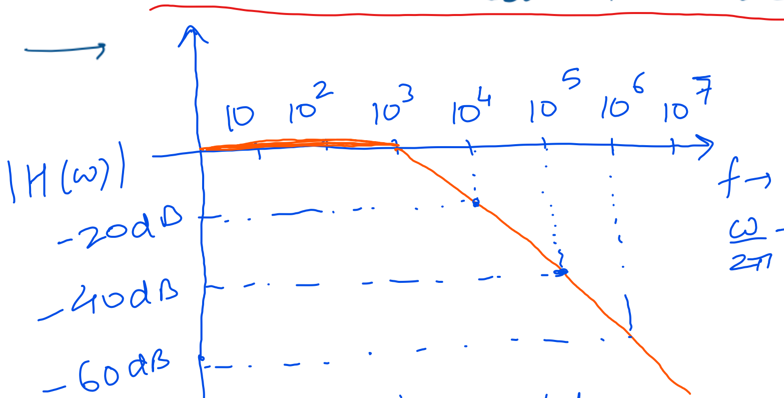
$C = 3.1\ \mu F$ (1)

One can even use low pass filter based on LR circuit. Give full credits for alternate designs.

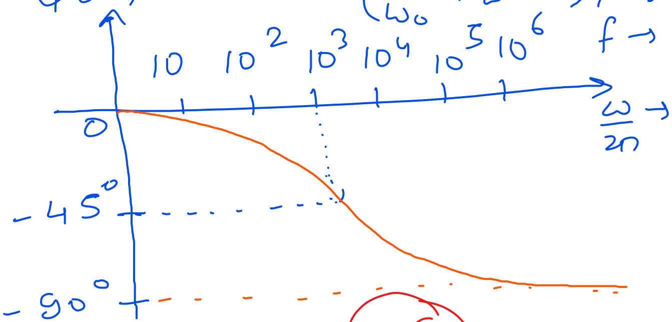
→

$\phi(\omega) = -\tan^{-1}\left(\dfrac{\omega}{\omega_0}\right)$

$\omega \to 0, \phi \to 0$
$\omega = \omega_0, \phi = -45°$
$\omega \to \infty, \phi = -90°$



Bode Magnitude plot (1.5)

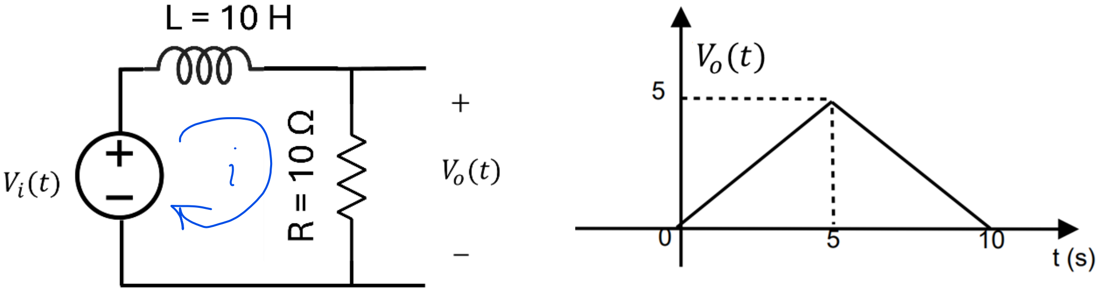


(1.5)

→ Since cerebro acts like an amplifier with gain of 20 dB, amplitudes get multiplied by 10X → (0.5)

→ After passing through the designed filter, amplitude of signal corresponding to 1 KHz is unchanged while the one corresponding to 1 MHz gets attenuated by 60 dB or 0.001X. (0.5)

3) Consider the circuit and the output $V_o(t)$ shown below. Compute and draw the input signal $V_i(t)$ as a function of t, that must be applied to produce this output signal. (**4 marks**)



→ Using KVL, we get,

$$L\frac{di}{dt} + iR - V_i(t) = 0 \quad 1 \quad \& \quad V_o(t) = iR$$

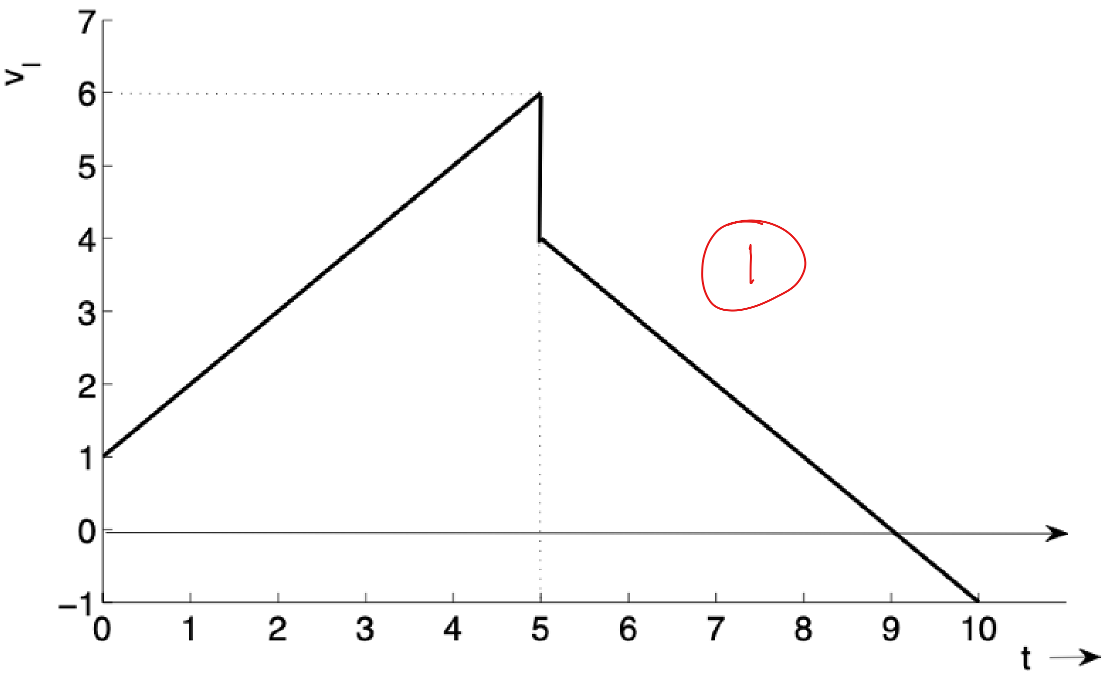
$$\therefore i = \frac{V_o(t)}{R}$$

substituting back

$$\rightarrow \frac{L}{R}\frac{dV_o(t)}{dt} + V_o(t) = V_i(t)$$

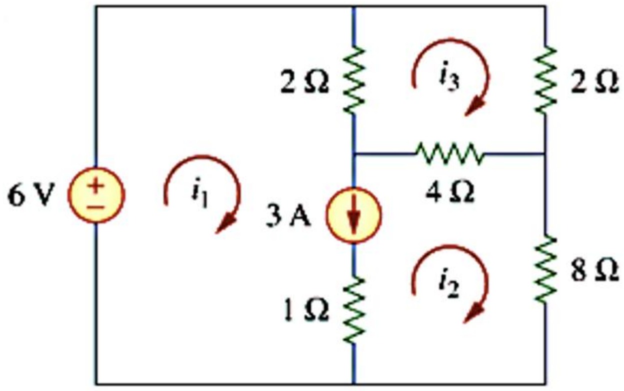
$$\because \frac{L}{R} = 1 \Rightarrow V_i(t) = V_o(t) + \frac{dV_o(t)}{dt} \quad 1$$

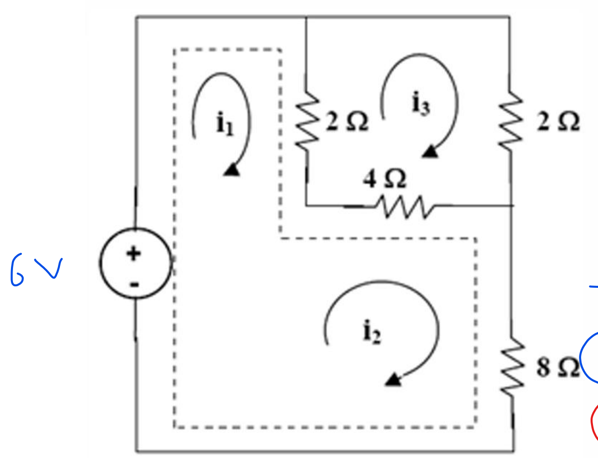
$$V_o(t) = \begin{cases} t, & 0 \le t \le 5 \\ 10-t, & 5 \le t \le 10 \end{cases} \quad 0.5 \quad \Rightarrow V_i(t) = \begin{cases} t+1, & 0 \le t \le 5 \\ 9-t, & 5 \le t \le 10 \end{cases} \quad 0.5$$



4) Using the mesh analysis, Find out the mesh currents i_1, i_2, and i_3 in the following circuit:

(5 marks)





∵ we have a current source in the loop, we need to (1) use super-mesh analysis as we do not know precisely voltage across 3A current source.

→ Applying super-mesh analysis:

$$(1) \rightarrow 2(i_1 - i_3) + 4(i_2 - i_3) + 8i_2 - 6 = 0 \quad (1)$$

$$i_1 - i_2 = 3 \quad (1) \Rightarrow i_1 = i_2 + 3$$ substi[tute] here

for the other loop, $(2) \rightarrow 2i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) = 0$ (1)

(1) & (2) can be re-written as:

$$(1) \rightarrow 2i_2 + 6 - 2i_3 + 4i_2 - 4i_3 + 8i_2 - 6 = 0 \Rightarrow 14i_2 - 6i_3 = 0$$

$$\Rightarrow i_3 = \frac{7}{3} i_2$$

$$(2) \rightarrow 2i_3 + 4i_3 - 4i_2 + 2i_3 - 2i_2 - 6 = 0 \Rightarrow 8i_3 - 6i_2 = 6$$

$$\Rightarrow 8 \times \frac{7}{3} i_2 - 6i_2 = 6$$

$$\Rightarrow (56 - 18) i_2 = 18$$

$$\Rightarrow i_2 = \frac{18}{38} = 0.47A$$

$$\Rightarrow i_3 = \frac{7}{3} i_2 = \frac{7}{3} \times \frac{9}{19} = 1.11 A$$

$$\& \ i_1 = i_2 + 3 = \frac{9}{19} + 3 = \frac{66}{19} = 3.47A$$

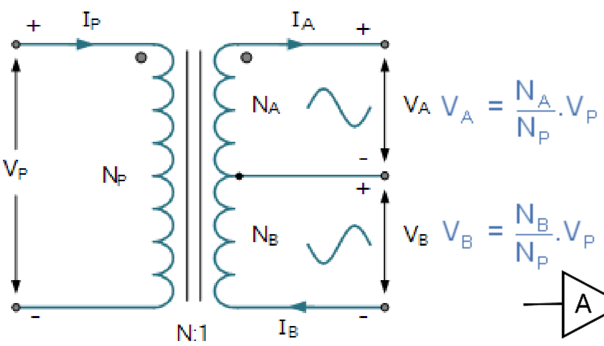
(1) cut 0.5 if values are incorrect.

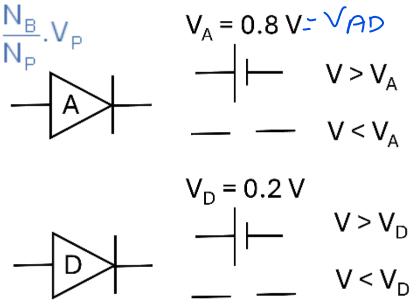
5) A transformer with a centre-tap along with the equivalent models for the adamantium diode (extracted from wolverine) and the vibranium diode (extracted from Wakanda) are shown below (left). Tony Stark wants to design a full-wave rectifier with these components to charge his ironman suite with a peak voltage $|V_o|$ of exactly 12 V when the household supply of 220 V RMS is fed at the input. The circuit he designs in a hurry is also shown below (right).

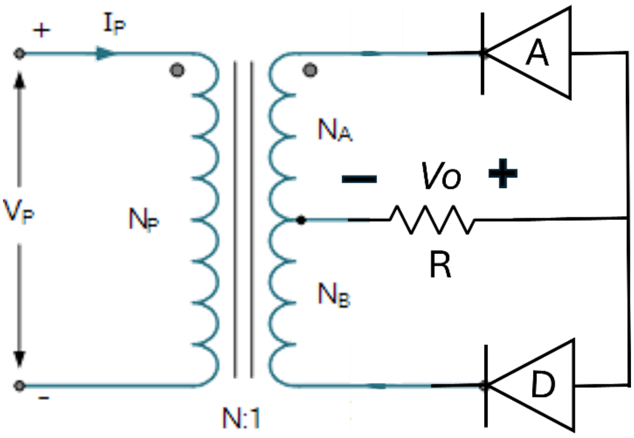
(a) Does the circuit even act like a rectifier? Is so, why? If not, why? Also plot the output waveform assuming a peak voltage $|V_o|$ of exactly 12 V. **(2 marks)**

(b) Find out the turn ratios N_A/N_P and N_B/N_P to achieve the desired peak voltage $|V_o|$ of exactly 12 V at the output. **(4 marks)**

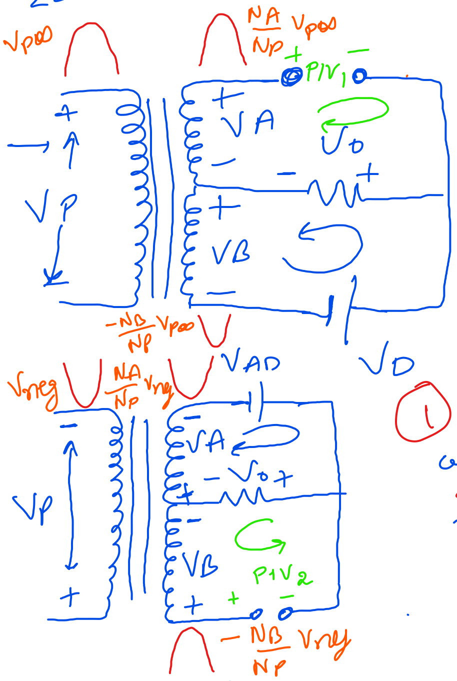
(c) Find out the peak inverse voltage across the two diodes. **(2 marks)**







220V RMS = 311.12 V ← Vp



(1)

for +ve cycle of Vp; adamantium diode is reverse-biased while vibranium diode is forward biased. Using KVL,

$\therefore \; V_o + V_B - V_D = 0 \Rightarrow V_D = V_D - V_B$

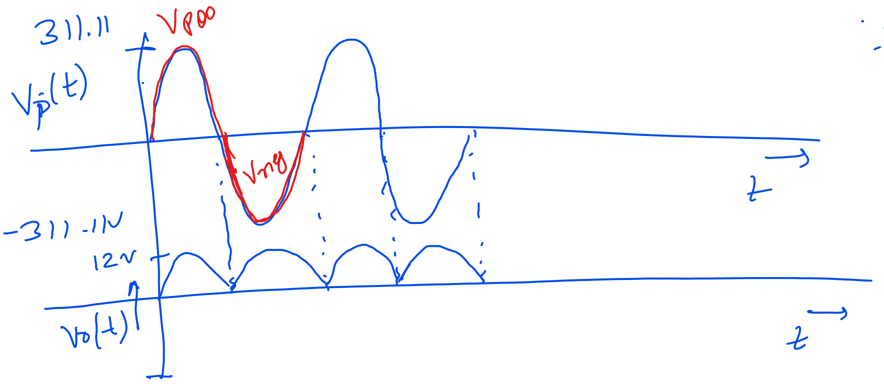
$\Rightarrow V_O = V_D - \left(-\frac{N_B}{N_P}V_{pos}\right) = V_D + \frac{N_B}{N_P}V_{pos}$ — (1)

(1) for −ve cycle of Vp, vibranium diode is reverse biased while adamantium diode is forward biased. Using KVL,

$V_O + V_A - V_{AD} = 0 \Rightarrow V_O = V_{AD} - V_A = V_{AD} - \frac{N_A}{N_P}V_{neg}$

$= V_{AD} + \frac{N_A}{N_P}V_{pos}$ — (2)

Given the peak of V_O $|V_O| = 12\,V \Rightarrow V_O = +12\,V$ (Vpeak).



∵ the circuit converts i/p voltage to only one polarity of o/p voltage, it can be considered as a rectifier.

(1)

→ To obtain exactly $|V_0| = 12V$ or $V_0 = +12\,V$ Vpeak,

from (1) → $V_0 = V_D + \frac{N_B}{N_P} \cdot V_P \Rightarrow 12 - V_D = \frac{N_B}{N_P} V_P \Rightarrow \frac{N_B}{N_P} = \frac{11.8}{311.11}$ (1)

$= 0.038$ (1)

from (2) → $V_0 = V_{AD} + \frac{N_A}{N_P} V_P \Rightarrow 12 - V_{AD} = \frac{N_A}{N_P} \cdot V_P \Rightarrow \frac{N_A}{N_P} = \frac{11.2}{311.11} = 0.036$ (1)

→ For PIV_1 → Applying KVL in the green loop for +ve cycle,
(Adamantium)

$PIV_1 + V_0 - V_A = 0 \Rightarrow PIV_1 = V_A - V_0 = \frac{N_A}{N_P} \times 311.11 - 12$ (0.5)

$= 11.2 - 12 = -0.8V$ (0.5)

ly PIV_2 → Apply KVL in the green loop for -ve cycle
(Vibranium)

$PIV_2 + V_0 - V_B = 0 \Rightarrow PIV_2 = V_B - V_0$ (0.5)

$= \frac{N_B}{N_P} \times 311.11 - 12$

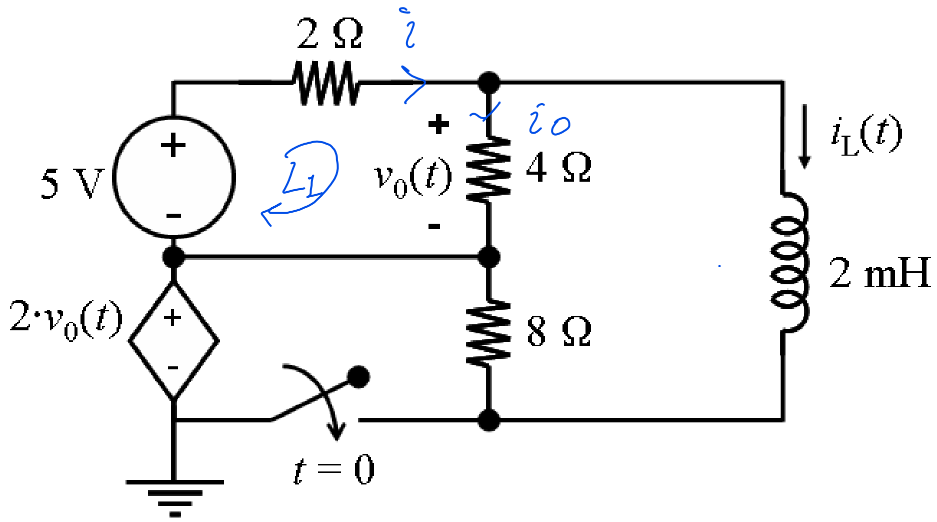
Deduct only 0.5 if answers are incorrect.

$\approx -0.2\,V.$ (0.5)

→ $v_0(t) = v_0(t\to\infty) + [v_0(t=0^+) - v_0(t\to\infty)]\, e^{-\frac{t}{\tau}}$

→ $i_L(t) = i_L(t\to\infty) + [i_L(t=0^+) - i_L(t\to\infty)]\, e^{-\frac{t}{\tau}}$ 0.5

2) The switch in the circuit given below is kept open for a long time and is closed at $t = 0$. Determine the expressions for the current $i_L(t)$ and the voltage $v_0(t)$ for $t > 0$ (**5 marks**)



→ $i_0 = \frac{v_0}{4}$ & $i = i_0 + i_L$

Applying KVL in L1:

$2(i_0 + i_L) + 4i_0 = 5$

⇒ $v_0 + \frac{v_0}{2} + 2i_L = 5$

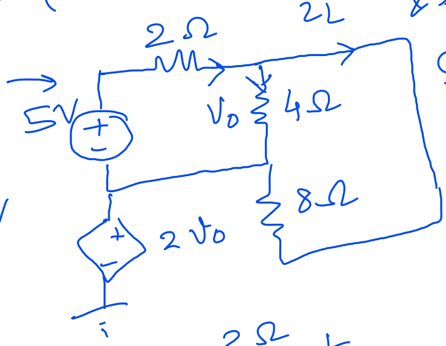
⇒ $\frac{3v_0}{2} = (5 - 2i_L)$

2) $v_0(t) = \frac{2}{3}[5 - 2i_L(t)]$

∵ $v_0(t)$ is a fn of $i_L(t)$, it cannot change instantaneously or $v_0(t=0^-) = v_0(t=0^+)$ 1

$\boxed{t = 0^-}$

$t<0$ & steady state $\frac{di}{dt} = 0$



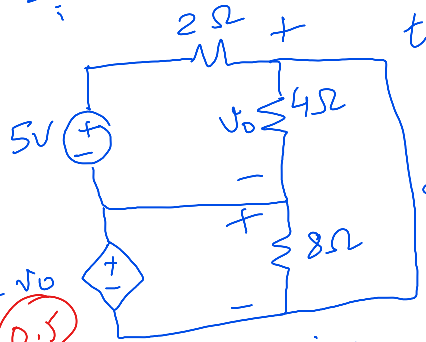
→ Using resistor divider configuration,

$v_0(t=0^-) = 5\left[\frac{4\|8}{2+4\|8}\right] = \frac{4}{7}\times 5 = \frac{20}{7}$ V

∴ $v_0(t=0^+) = \frac{20}{7}$ V 0.5

$\boxed{t\to\infty}$

$t\to\infty$ $\frac{di}{dt} = 0$ for steady state



→ $t\to\infty$, $v_0 + 2v_0 = 0$

⇒ $v_0(t\to\infty) = 0$ 0.5

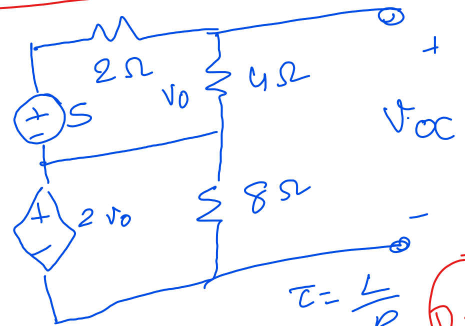
→ $i_L(t=0^+) = i_L(t=0^-)$

Using current division, $i_L(t=0^-)$ can be obtained as: $\frac{5}{(2+4\|8)}\cdot\left(\frac{4}{4+8}\right) = \frac{5}{14}$ A $= i_L(t=0^+) = i_L(t=0^-)$ 0.5

→ ∵ $v_0(t\to\infty) = 0$; $i_L(t\to\infty) = \frac{5}{2} = 2.5$ A 0.5

→ To find time const. τ, we need to find R_{th}.

→ Due to dependent source, find V_{oc} & I_{sc}. 0.5



$I_{sc} = \frac{5}{2} = 2.5$A from $(t\to\infty)$ ckt.

$V_{oc} = 3v_0(t) = 3\times 5\times \frac{4}{2+4} = 10$V 0.5

$R_{th} = \frac{V_{oc}}{I_{sc}} = 4\Omega$

$\tau = \frac{L}{R}$ 0.5

$\tau = 0.5$ ms