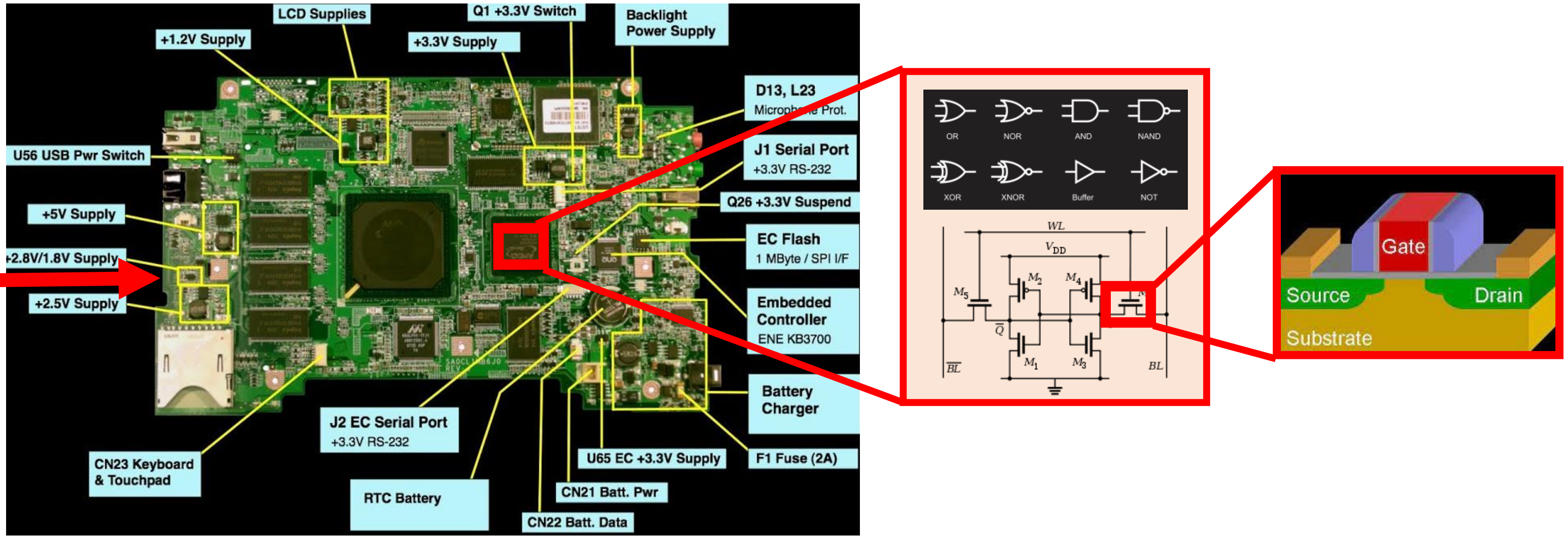
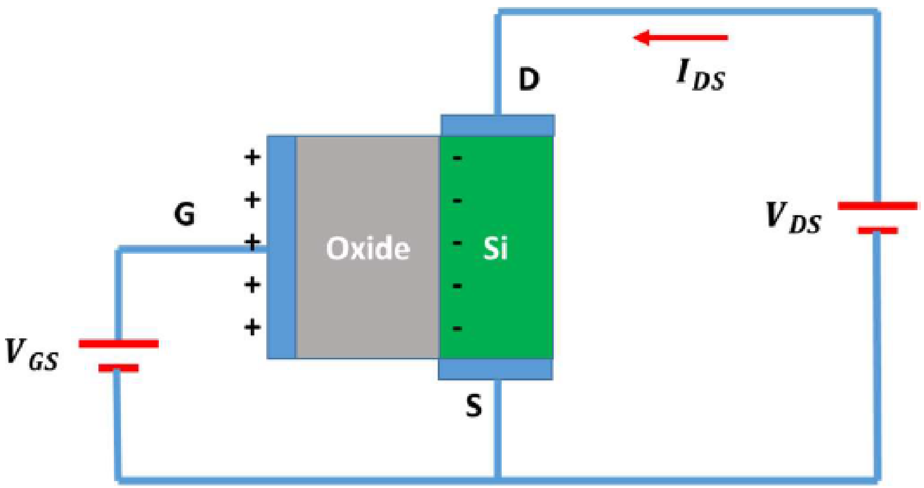


ESC201: Introduction to Electronics

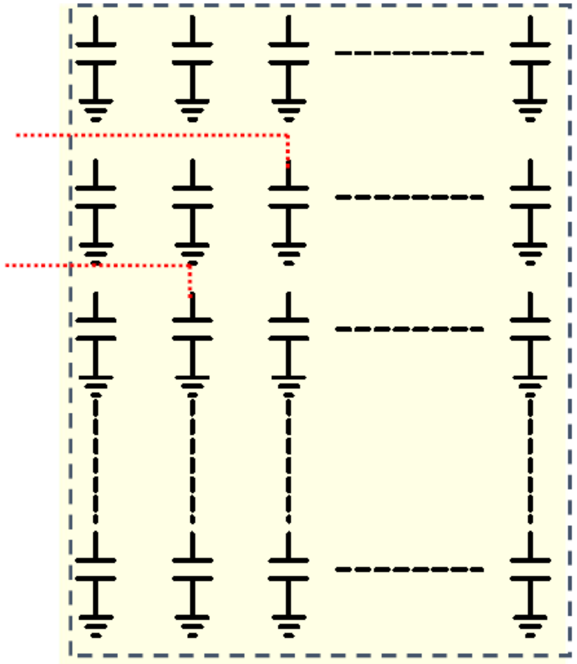
Module 2: Elements with Memory

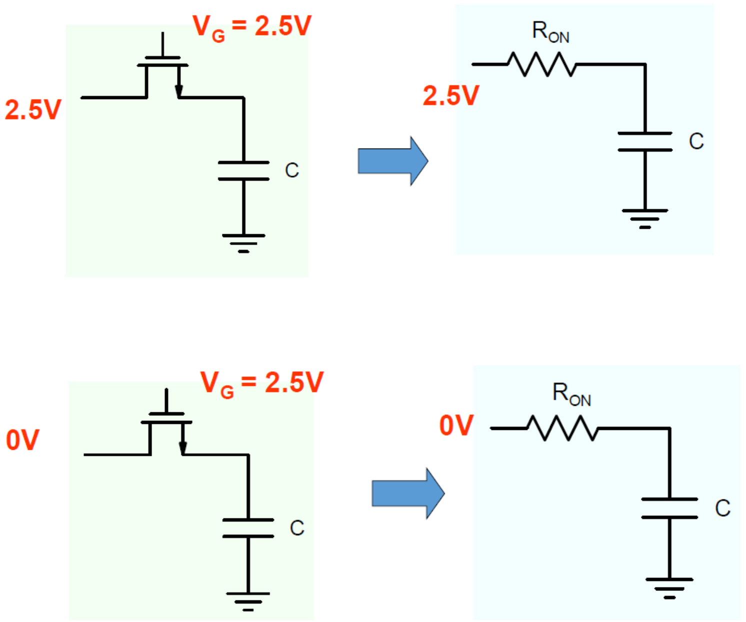
Dr. Shubham Sahay,
Assistant Professor,
Department of Electrical Engineering,
IIT Kanpur

Recap



Drain current is controlled by gate voltage





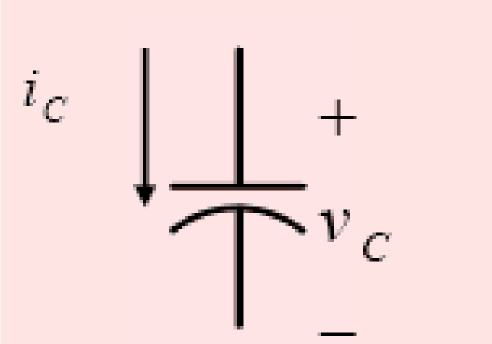
DRAM

Registers and SRAM

Hard disk

Two things to remember

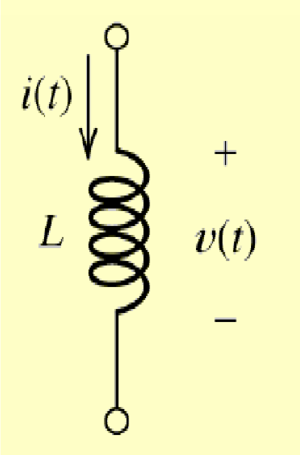
Voltage across a capacitor cannot change instantaneously



$$i_c = C\frac{dv_c}{dt}$$

Instantaneous change in voltage implies infinite current!

Current through an inductor cannot change instantaneously



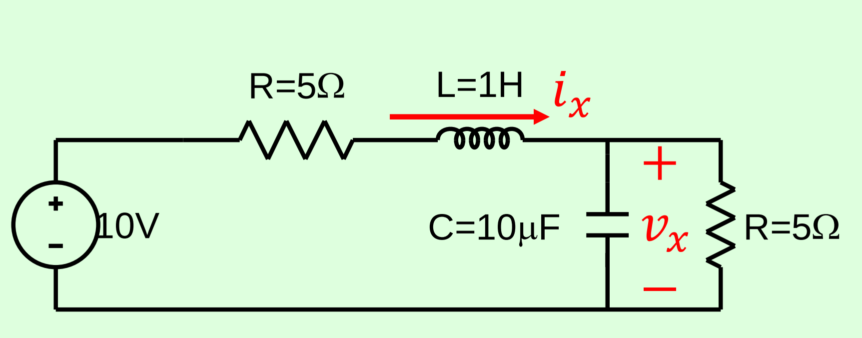
$$v = L\frac{di}{dt}$$

Instantaneous change in current implies infinite voltage!

Example 2: Steady state behavior after a change

Find voltage and current a long time after closing the switch

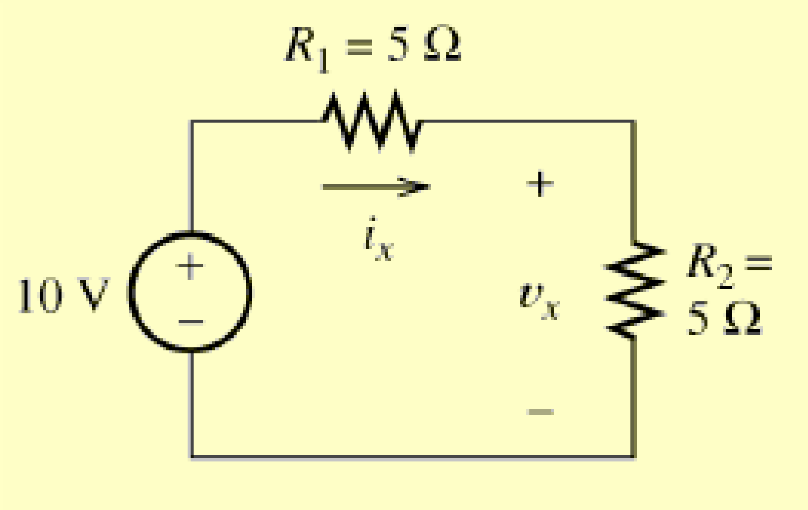
$t \to \infty$



Recall

A capacitor under dc or steady state acts like an open circuit.

An inductor under dc or steady state acts like a short circuit



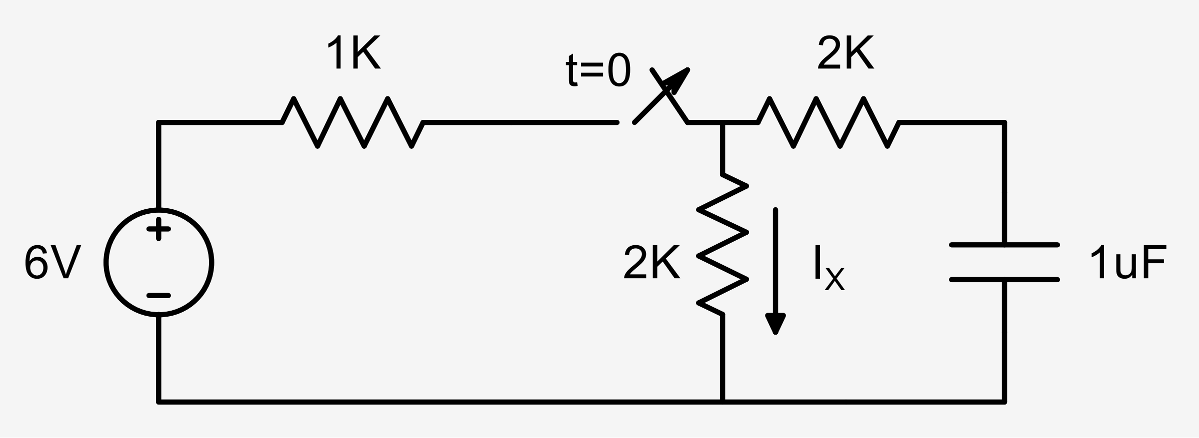
$$i_x = \frac{10}{R_1 + R_2} = 1\text{ A}$$

$$v_x = R_2 i_x = 5\text{ V}$$

$$i_L(t \to \infty) = 1 \quad \text{A}$$
$$v_c(t \to \infty) = 5 \quad \text{V}$$

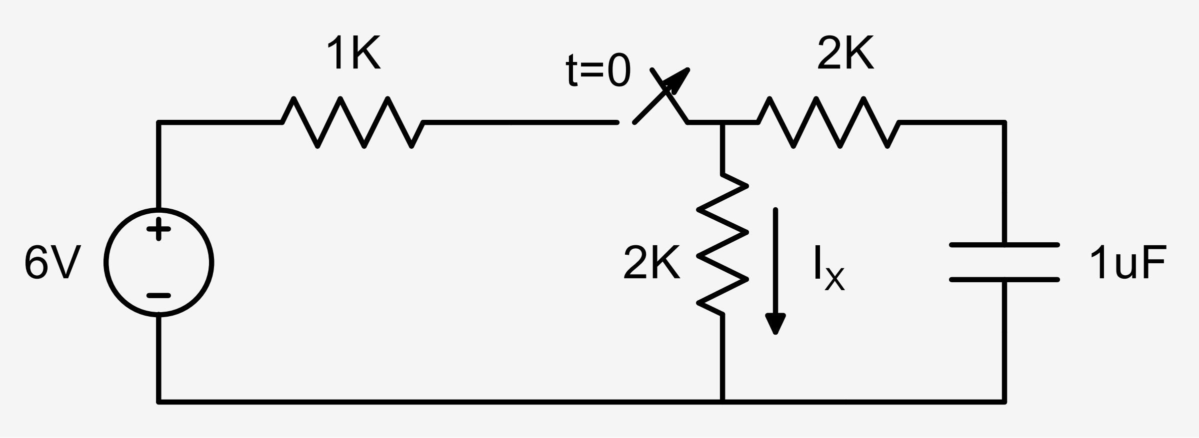
Example 3

Determine the current I_X immediately after switch is opened.



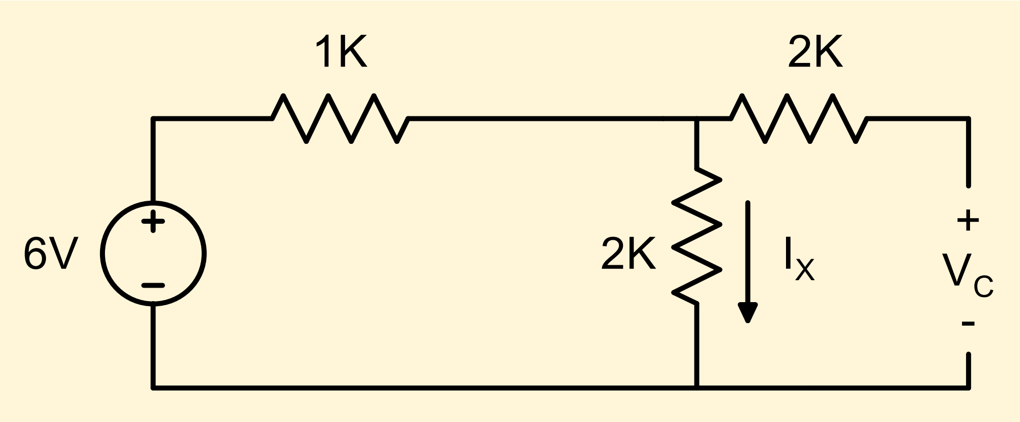
Example 3

Determine the current I_X immediately after switch is opened.



First analyze the circuit before circuit is opened.

Circuit for $t \leq 0$



$$v_C(0) = \frac{2}{3} \times 6 = 4V$$

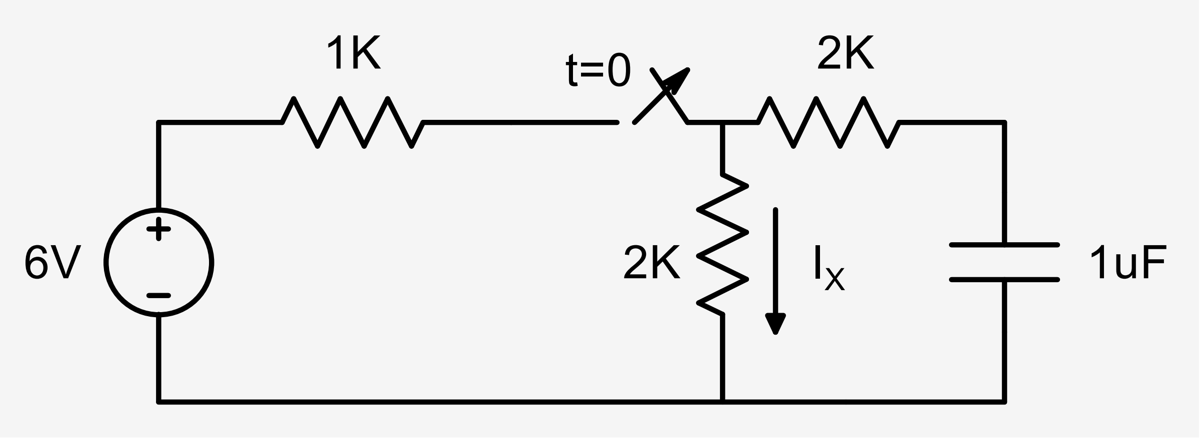
$$i_X(0) = \frac{v_C(0)}{2\text{k}\Omega} = 2mA$$

Voltage across a capacitor cannot change instantaneously

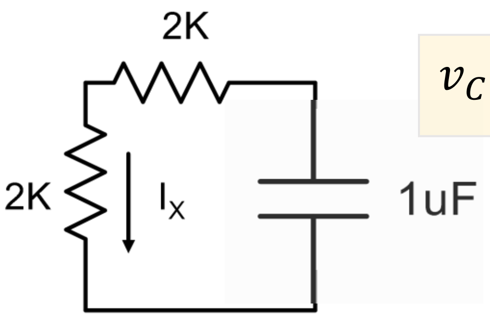
$$v_C(0^+) = 4V$$

Example 3:

Determine the current I_X immediately after switch is opened.



Circuit for $t > 0$

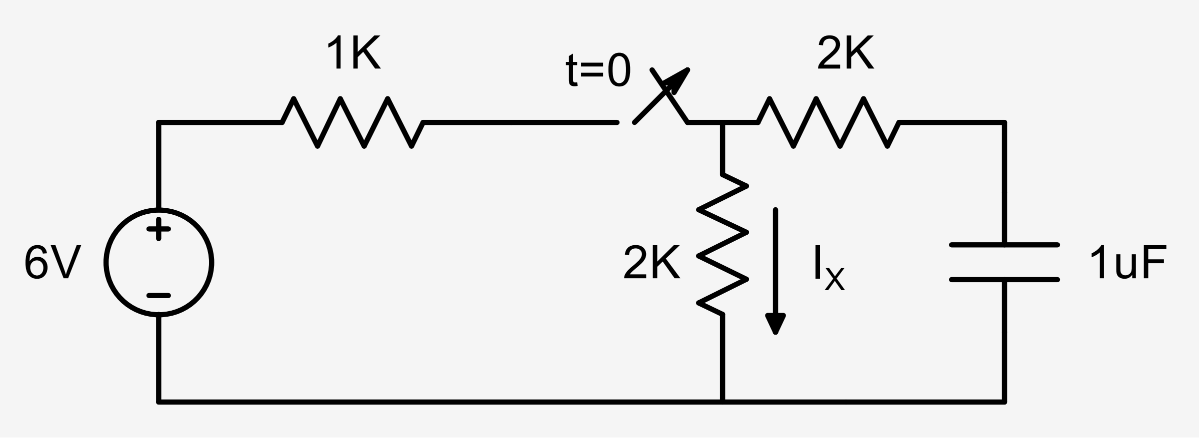


$$v_C(0^+) = 4V$$

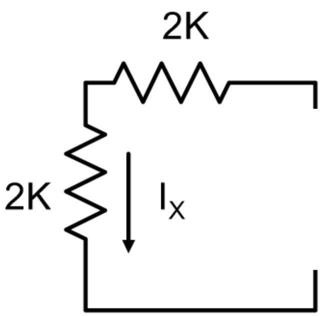
$$i_X(0^+) = \frac{v_C(0^+)}{4\text{k}\Omega} = 1mA$$

Example 3:

Determine the current I_X immediately after switch is opened.



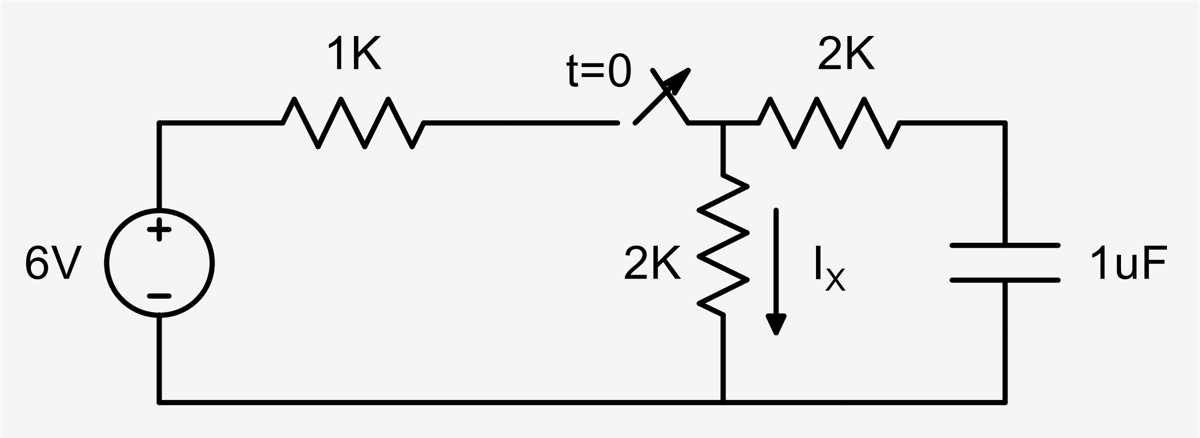
Circuit for $t \to \infty$



$i_X(\infty) = 0mA$

$v_C(\infty) = 0V$

Transient Analysis



$i_X(0) = 2mA$

$i_X(0^+) = 1mA$

$i_X(\infty) = 0mA$

$v_C(0) = 4V$

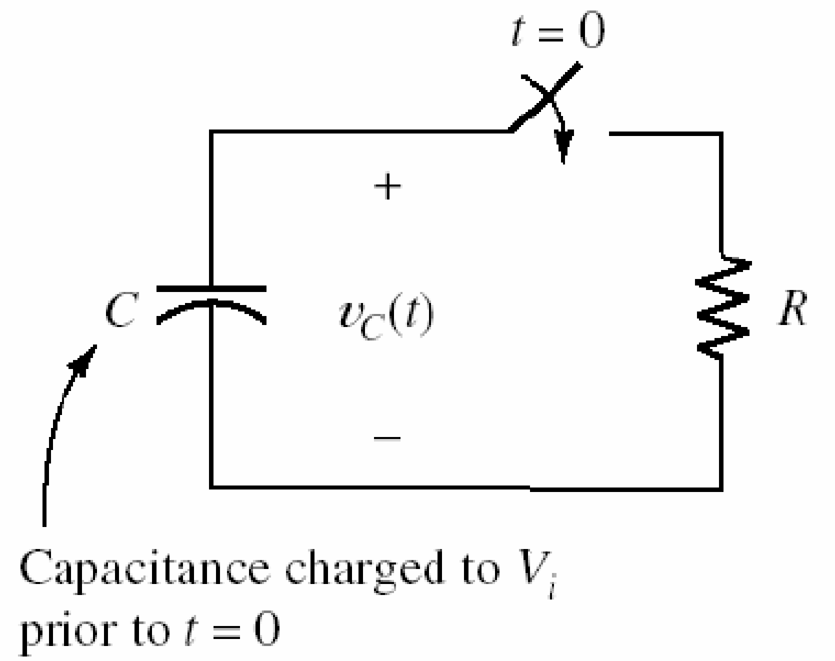
$v_C(0^+) = 4V$

Discharging

$v_C(\infty) = 0V$

- Behavior with time
- Voltage/current may change gradually
- Transient analysis

Capacitor: Discharging



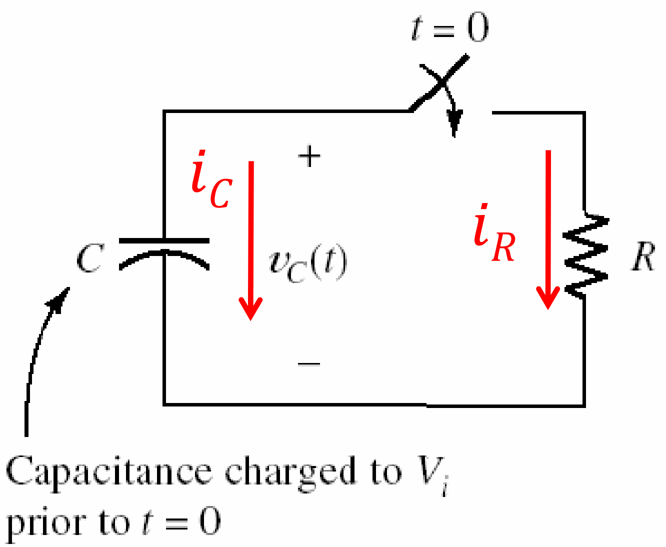
Initial voltage across capacitor is V_i.

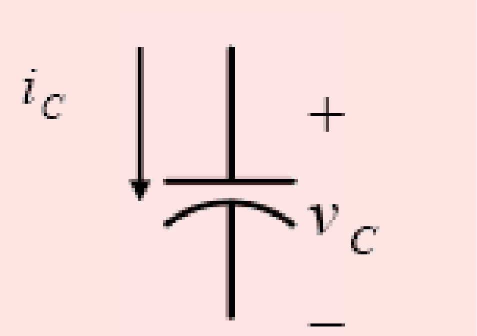
Steady state: voltage across capacitor is 0.

How does the capacitor voltage fall?

How long will it take for capacitor voltage to fall to half its initial value?

Capacitor: Discharging





$$i_c = C\frac{dv_c}{dt}$$

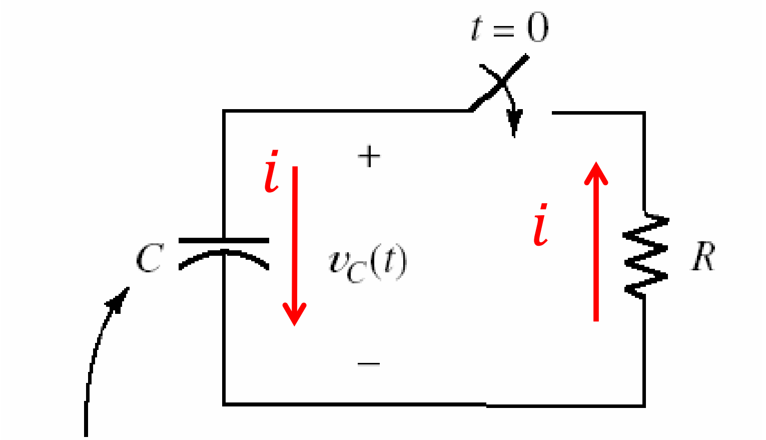
Write KCL at top node with switch closed:

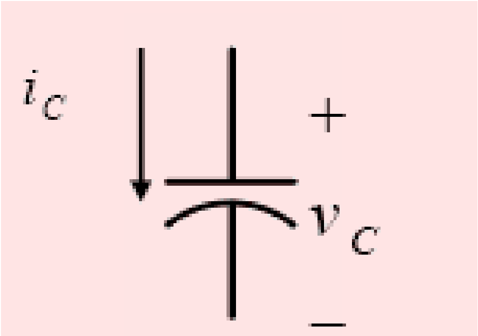
$$i_c(t) + i_R(t) = 0$$

$$C\frac{dv_C(t)}{dt} + \frac{v_C(t)}{R} = 0$$

$$\frac{dv_C(t)}{dt} = -\frac{1}{RC}v_C(t)$$

Capacitor: Discharging





$$i_c = C\frac{dv_c}{dt}$$

Applying KVL

$$v_C + iR = 0$$

$$v_C + C\frac{dv_c}{dt}R = 0$$

$$\frac{dv_c}{dt} = -\frac{1}{RC}v_C$$

$$\boxed{\frac{dv_c(t)}{dt} = -\frac{1}{RC}v_C(t)}$$

$$\boxed{\frac{dy}{dt} = -ay}$$

Differential Equation: First Order

$$\frac{dy}{dt} = -ay$$

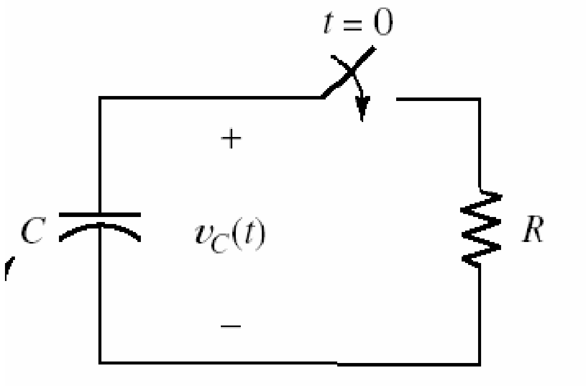
Solution: $y(t) = Ke^{-at}$

Constant K is often found from the initial condition

$$K = y(0)$$

$$y(t) = y(0)e^{-at}$$

Capacitor: Discharging



$$\frac{dv_c(t)}{dt} = -\frac{1}{RC}v_C(t)$$

$$v_C(t) = v_C(0)e^{-\frac{t}{RC}}$$

$$v_C(t) = v_C(0^+)e^{-\frac{t}{RC}}$$

$$\frac{dy}{dt} = -ay$$

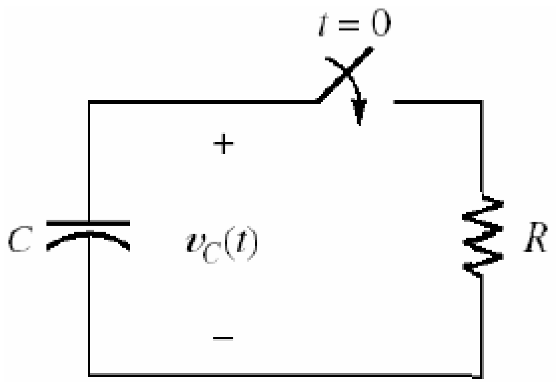
$$y(t) = y(0)e^{-at}$$

Voltage across a capacitor cannot change instantaneously

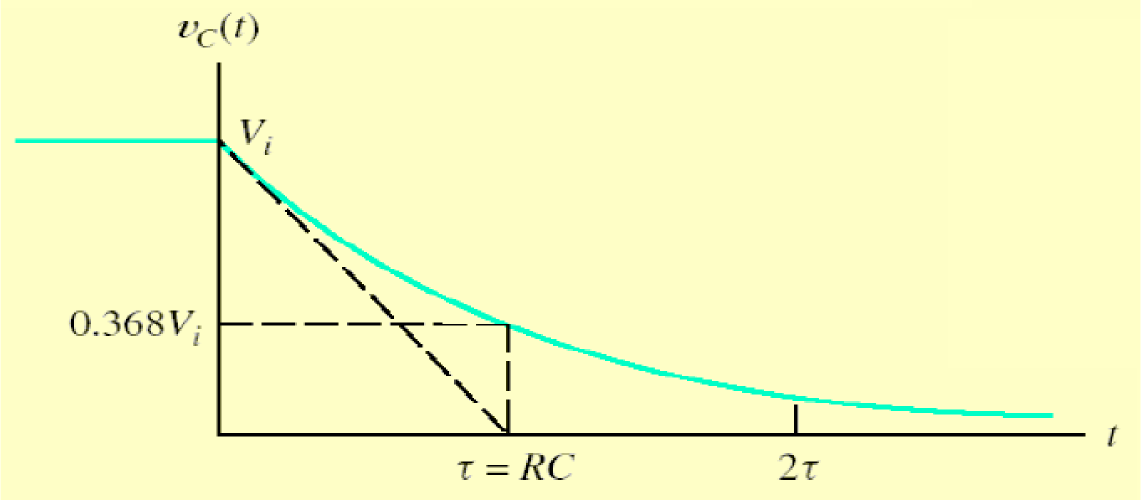
$$v_C(0^+) = v_C(0) = V_i$$

$$v_C(t) = V_i e^{-\frac{t}{RC}}$$

Capacitor: Rate of Discharge



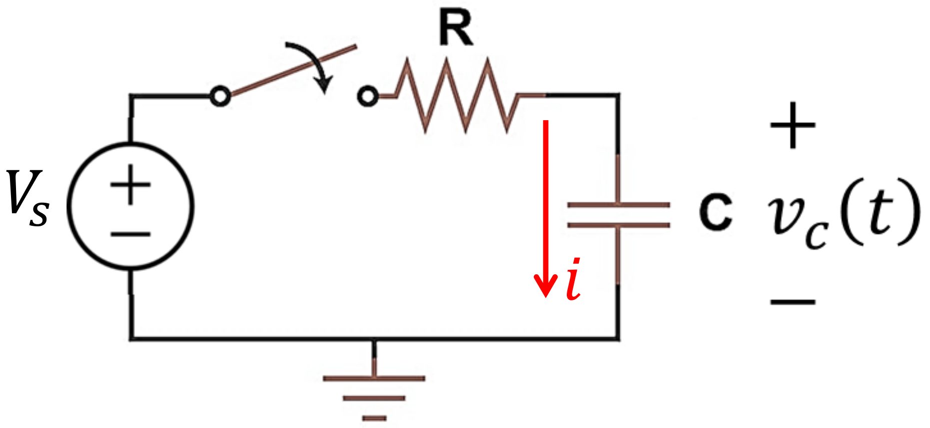
$$v_C(t) = V_i e^{-\frac{t}{RC}}$$



The time interval $\tau = RC$ is called the time constant of the circuit. After about **five time constants**, the voltage remaining on the capacitor will be negligible compared to the initial value

Time	τ	2τ	3τ	4τ	5τ
$V(t)/V_i$	0.368	0.135	.05	0.018	0.0067

Capacitor: Charging



$$i_c = C\frac{dv_c}{dt}$$

Initial Voltage 0

Applying KVL

$$-V_s + iR + v_C = 0$$

$$v_C + C\frac{dv_c}{dt}R = V_s$$

$$\frac{dv_c}{dt} = -\frac{v_c}{CR} + \frac{V_s}{CR}$$

$$\frac{dx}{dt} = -a_1 x + a_2$$

Differential Equation

$$\frac{dx}{dt} = -a_1 x + a_2$$

Solution: $x(t) = K_1 + K_2 e^{-a_1 t}$

Use $t \to \infty$:

$$x(\infty) = K_1$$

$$x(t) = x(\infty) + K_2 e^{-a_1 t}$$

Use $t = 0$:

$$x(0) = x(\infty) + K_2$$

$$x(t) = x(\infty) + (x(0) - x(\infty))e^{-a_1 t}$$

Differential Equation

$$\frac{dx}{dt} = -a_1 x + a_2$$

$$x(t) = x(\infty) + (x(0) - x(\infty))e^{-a_1 t}$$

Recall for discharging

$$\frac{dx}{dt} = -ax$$

$$x(t) = x(0)e^{-at}$$

A special case when $a_2 = 0$

Capacitor: Charging

$$\frac{dx}{dt} = -a_1 x + a_2$$

$$x(t) = x(\infty) + (x(0) - x(\infty))e^{-a_1 t}$$

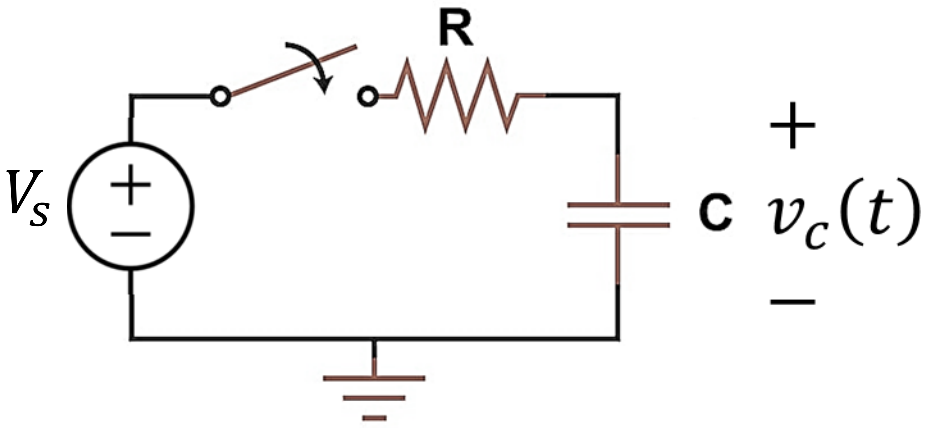
$$\frac{dv_c}{dt} = -\frac{v_c}{CR} + \frac{V_s}{CR}$$

$$v_C(t) = v_C(\infty) + (v_C(0^+) - v_C(\infty))e^{-\frac{t}{RC}}$$

Final Voltage (steady state)

Initial Voltage

Initial Voltage in Charging



Initial Voltage $v_C(0) = 0$

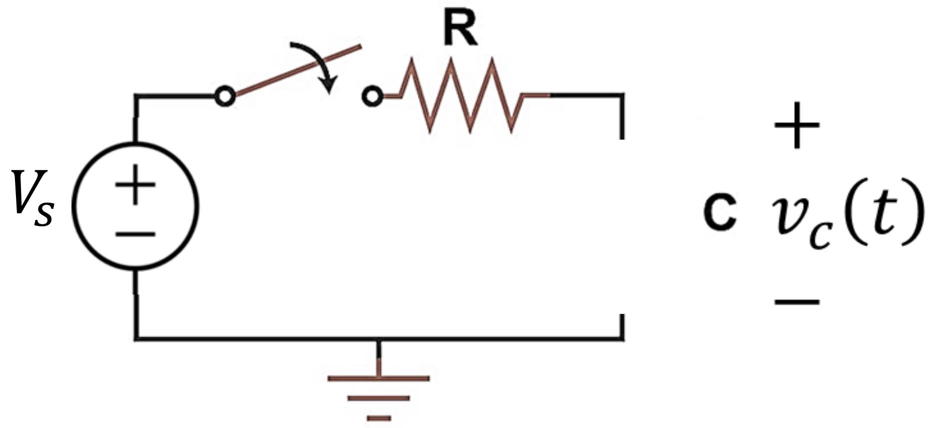
Recall
voltage across a capacitor cannot change instantly.

$$v_C(0^+) = v_C(0^-) = 0$$

$$v_C(t) = v_C(\infty) + (v_C(0^+) - v_C(\infty))e^{-\frac{t}{RC}}$$

$$v_C(t) = v_C(\infty) - v_C(\infty)e^{-\frac{t}{RC}}$$

Final Voltage in Charging



Recall

A capacitor under DC or steady state acts like an open circuit

$$v_C(\infty) = V_S$$

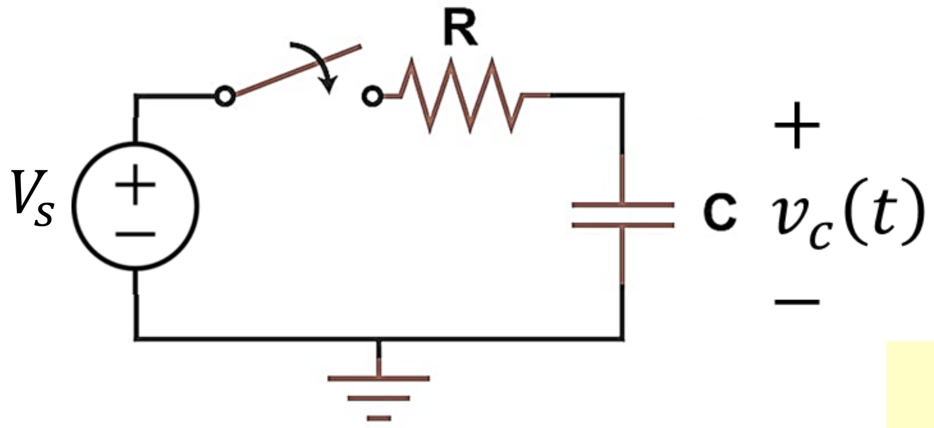
$$v_C(t) = v_C(\infty) + (v_C(0^+) - v_C(\infty))e^{-\frac{t}{RC}}$$

$$v_C(t) = v_C(\infty) - v_C(\infty)e^{-\frac{t}{RC}}$$

$$v_C(t) = V_s + (0 - V_s)e^{-\frac{t}{RC}}$$

$$v_C(t) = V_s - V_s e^{-\frac{t}{RC}}$$

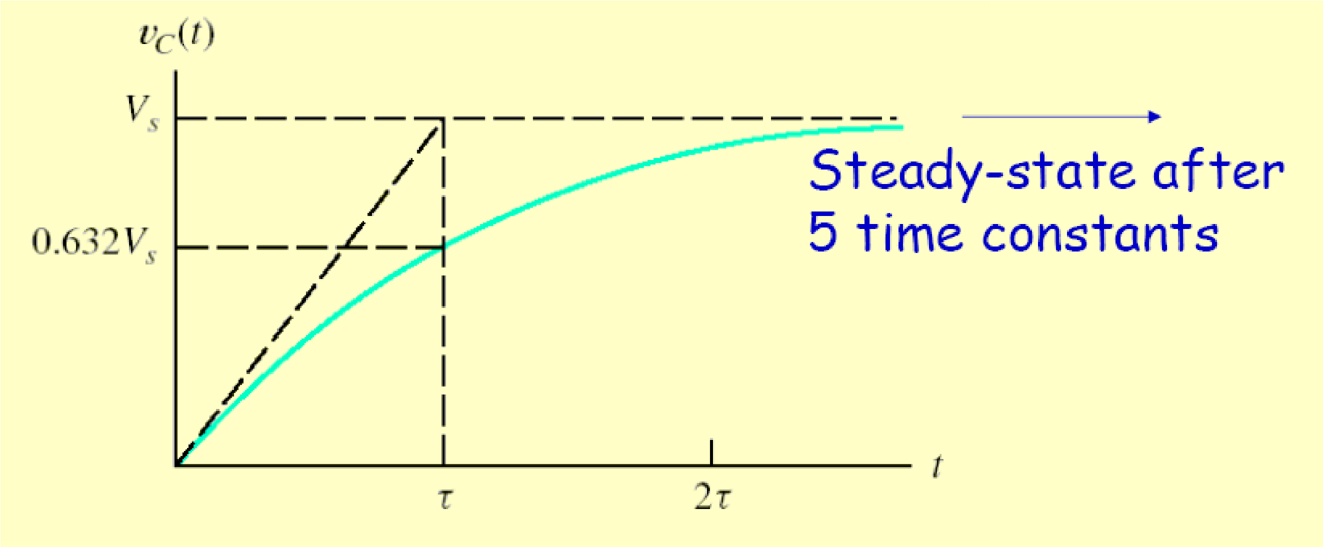
Capacitor: rate of charge



$$v_C(t) = V_s - V_s e^{-\frac{t}{RC}}$$

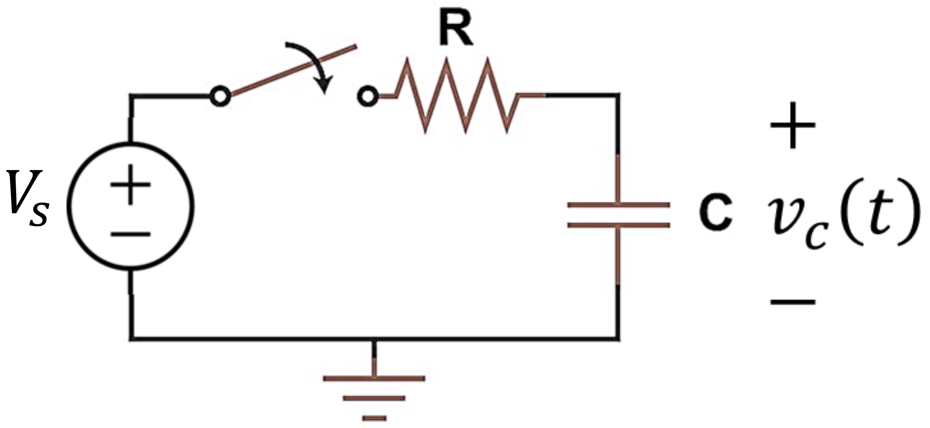
$$\boxed{v_C(t) = V_s\left(1 - e^{-\frac{t}{\tau}}\right)}$$

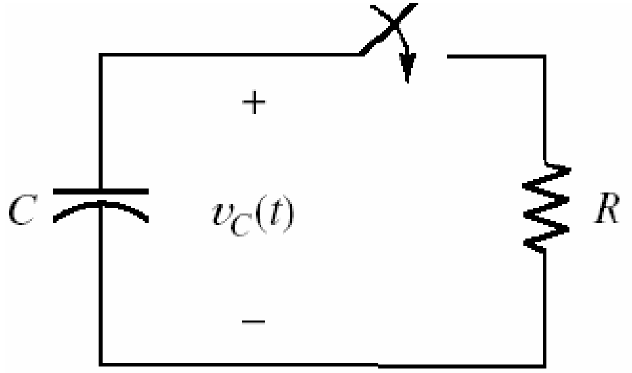
$$\tau = RC$$



Time	τ	2τ	3τ	4τ	5τ
$V(t)/V_i$	0.632	0.865	.95	0.982	0.993

Capacitor: discharging vs charging





$\tau = RC$

$$v_C(t) = V_s\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$v_C(t) = V_i e^{-\frac{t}{\tau}}$$

General result $$v_C(t) = v_C(\infty) + (v_C(0^+) - v_C(\infty))e^{-\frac{t}{RC}}$$

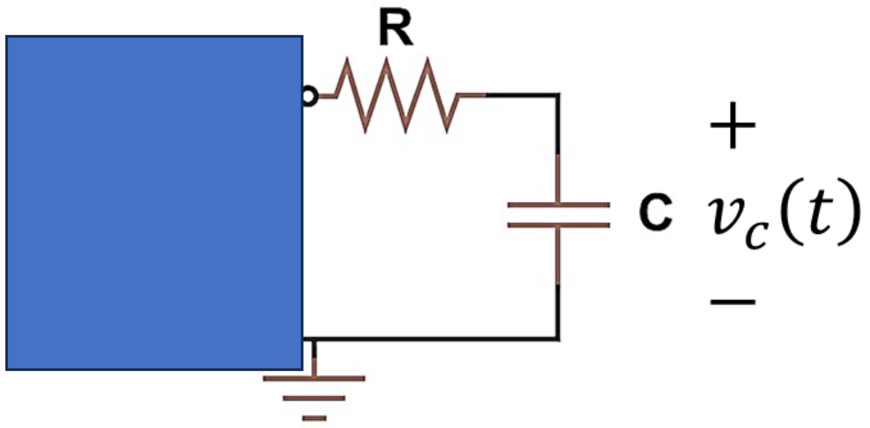
Final Voltage (steady state)

Initial Voltage

Final Voltage

Change in voltage

Example



If initial voltage across capacitor is V_m

Its final voltage is V_f

Equivalent resistance to this capacitor is R

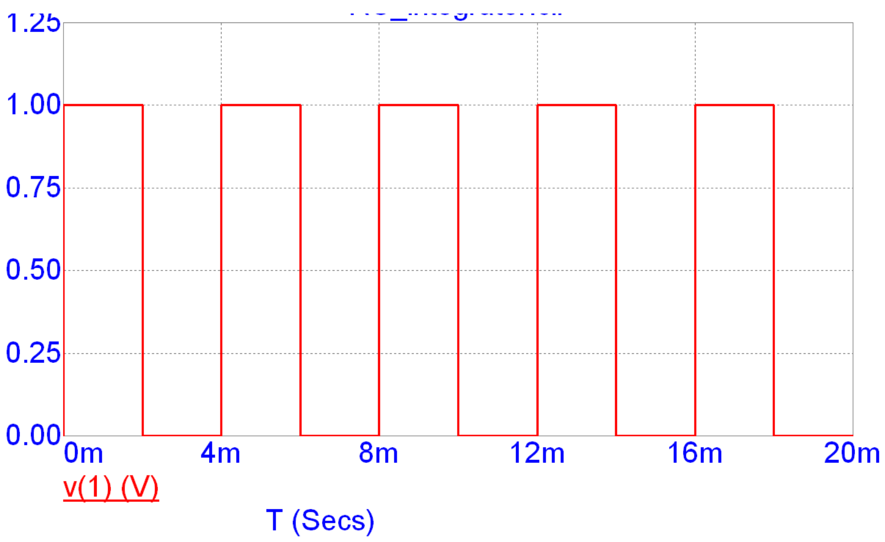
What will be the voltage after time t

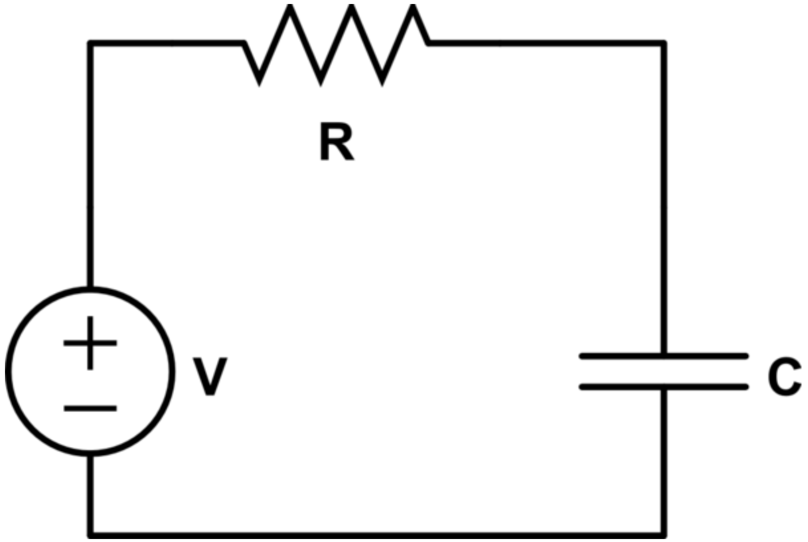
$$v_C(t) = v_C(\infty) + (v_C(0^+) - v_C(\infty))e^{-\frac{t}{RC}}$$

$$v_C(t) = V_f + (V_m - V_f)e^{-\frac{t}{RC}}$$

Example 4: Response under Square Pulse

Input voltage

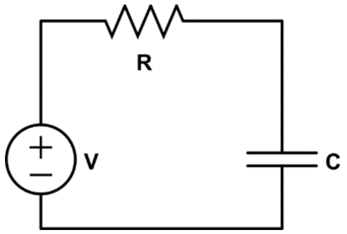


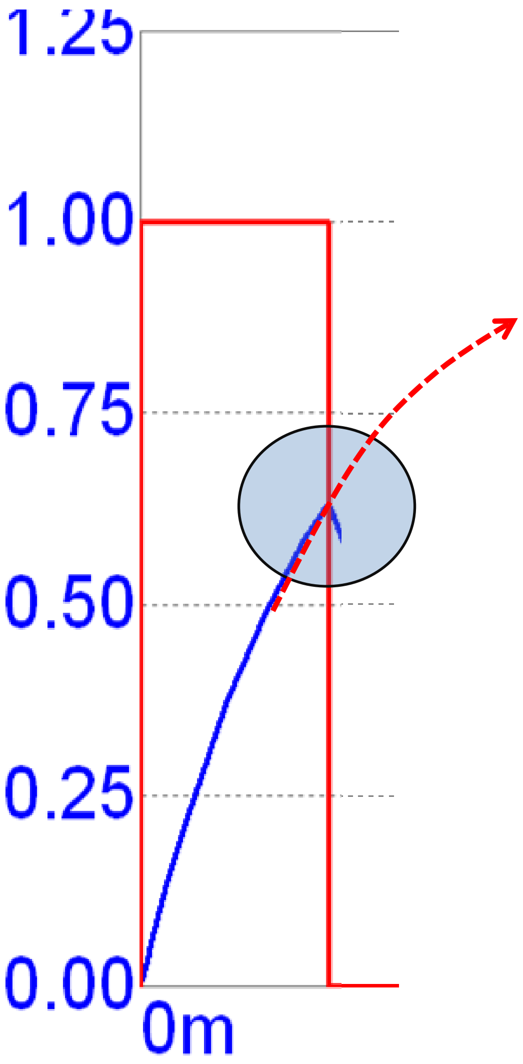


$$\tau = RC = 2 \text{ ms}$$

Capacitor tries to achieve voltage V at steady state

Example 4





$$v_c(\infty) = 1V; \quad v_c(0^+) = 0;$$

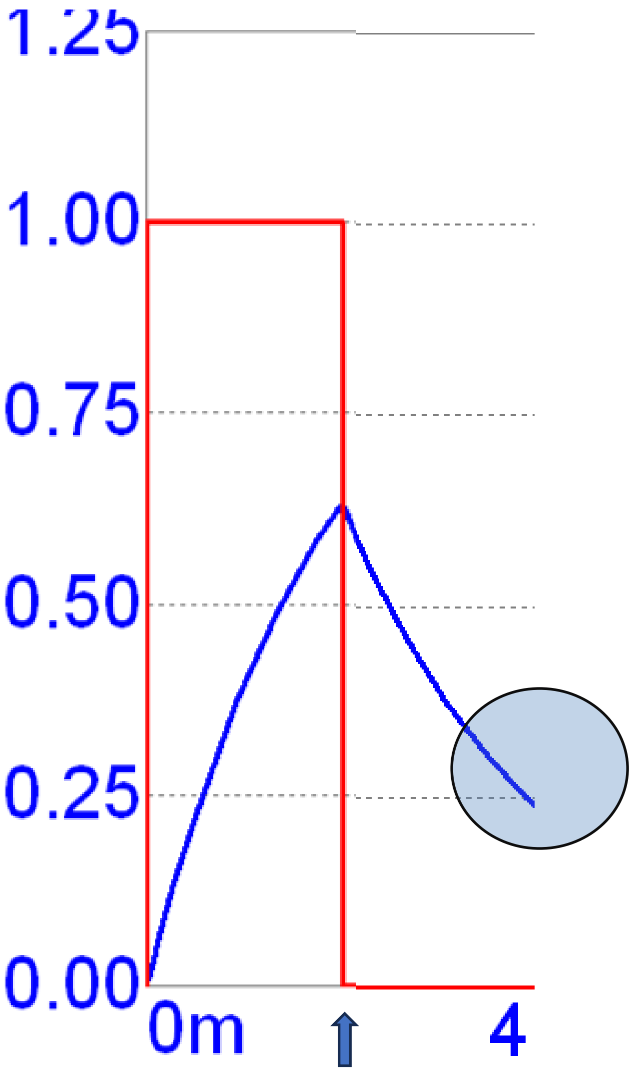
$$v_c(t) = v_c(\infty) + \left[v_c(0^+) - v_c(\infty)\right]e^{-\frac{t}{\tau}}$$

$$v_c(t) = 1 - e^{-\frac{t}{2}}$$

time t is in ms

$$v_c(2) = 1 - e^{-\frac{2}{2}} = 0.63V$$

Example 4



$$v_c(\infty) = 0; v_c(0^+) = 0.63V;$$

$$v_c(t) = v_c(\infty) + [v_c(0^+) - v_c(\infty)]\ e^{-\frac{t}{\tau}}$$

$$v_c(t) = 0.63e^{-\frac{t}{2}}$$

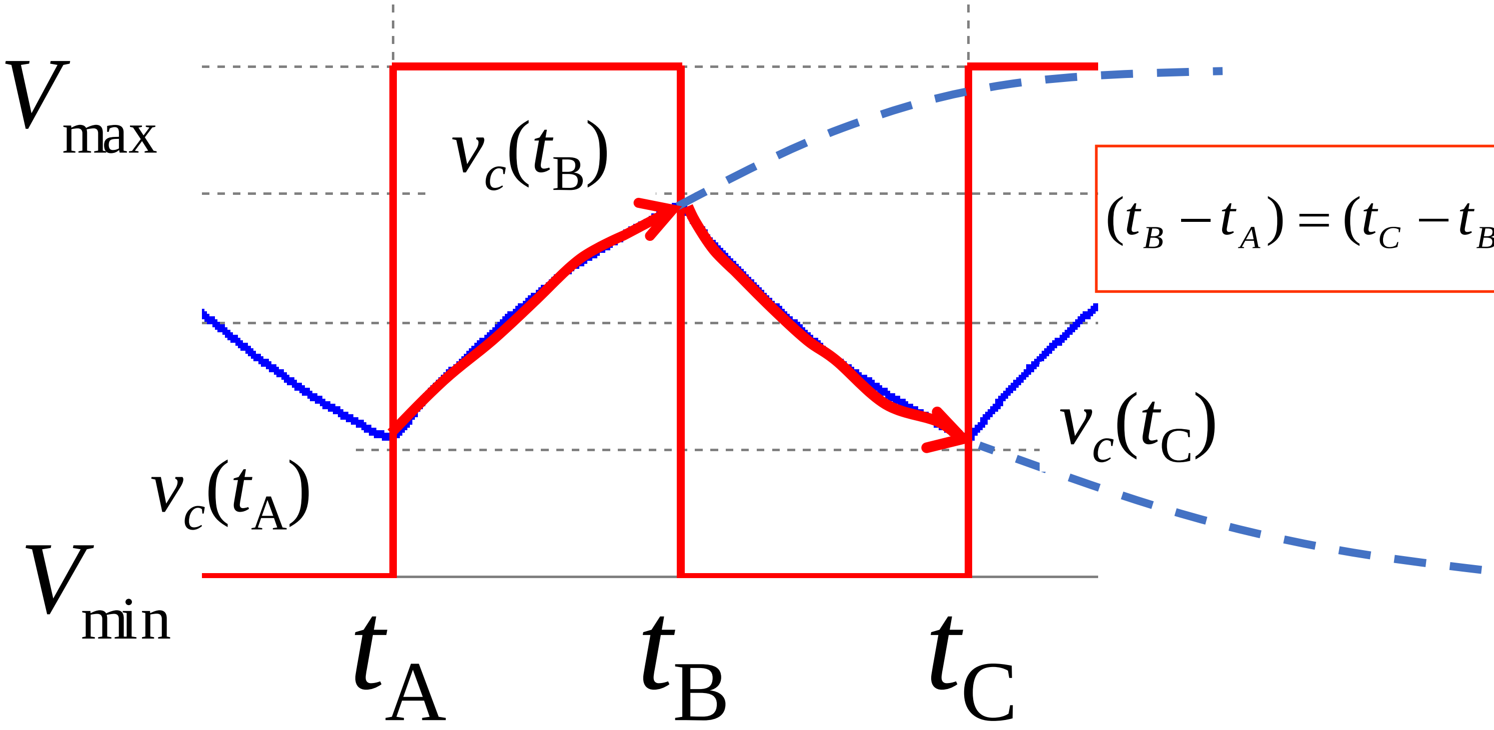
In the original time scale

$$v_c(t) = 0.63e^{-\frac{(t-2)}{2}}; 2 \leq t$$

$$v_c(4) = 0.63\ e^{-\frac{2}{2}} = 0.23V$$

Let us call this t as 0.

Example 3…

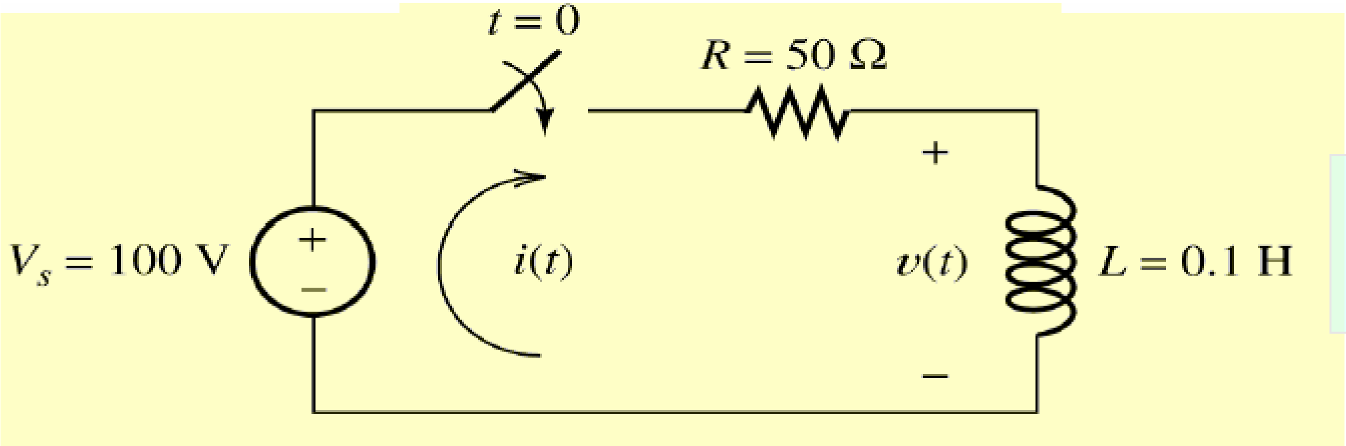


$$(t_B - t_A) = (t_C - t_B) = \frac{T}{2}$$

$$v_c(t) = V_{max} + \left[v_c(t_A) - V_{max}\right] e^{-\frac{(t - t_A)}{\tau}} \qquad t_A \le t \le t_B$$

$$v_c(t) = V_{min} + \left[v_c(t_B) - V_{min}\right] e^{-\frac{(t - t_B)}{\tau}} \qquad t_B \le t \le t_C$$

Circuits with Inductor



$$v = L\frac{di}{dt}$$

Let us write KVL

$$V_s = iR + v \qquad V_s = iR + L\frac{di}{dt} \qquad \frac{di}{dt} = -\frac{R}{L}i + \frac{V_s}{L}$$

$$\frac{dx}{dt} = -a_1 x + a_2$$

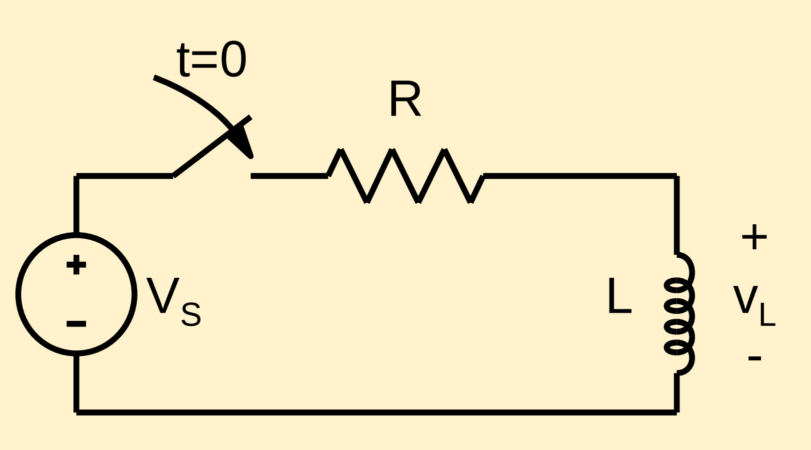
$$x(t) = x(\infty) + \{x(0^+) - x(\infty)\}e^{-a_1 t}$$

$$i(t) = i(\infty) + \left(i(0^+) - i(\infty)\right)e^{-\frac{R}{L}t}$$

Time Constant: $\tau = \frac{L}{R}$

$$e^{-\frac{t}{\tau}}$$

Example



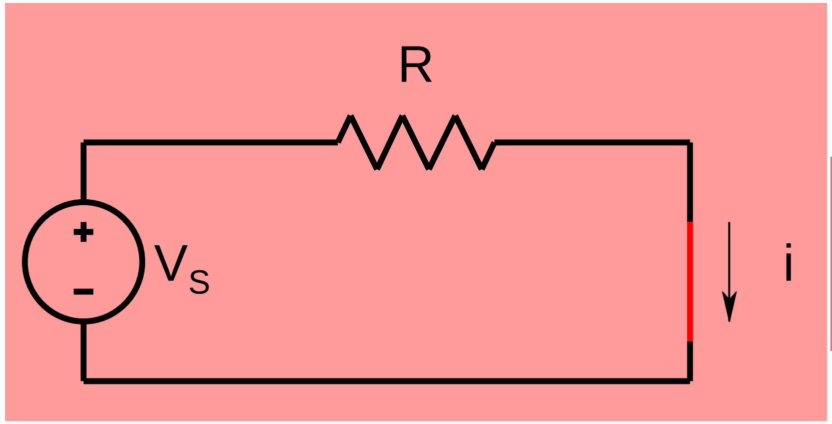
$$i(t) = i(\infty) + \left(i(0^+) - i(\infty)\right)e^{-\frac{R}{L}t}$$

What is $i(\infty)$?

We also note that inductor current cannot change instantly

$$i(0^+) = i(0^-) = 0$$

Inductor in steady state is like a short circuit

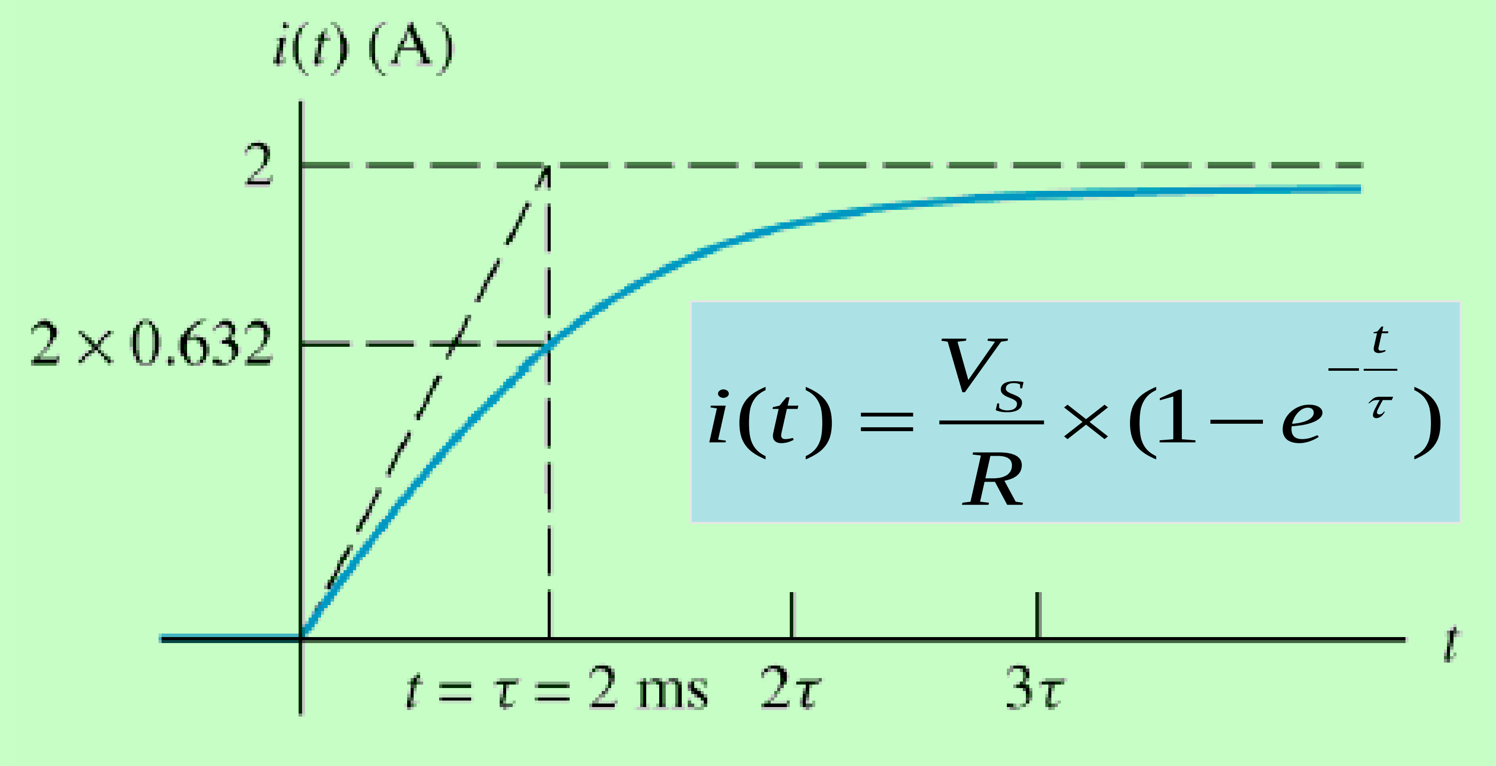


$$i(\infty) = \frac{V_S}{R}$$

$$i(t) = \frac{V_S}{R} + \left(i(0) - \frac{V_S}{R}\right)e^{-\frac{R}{L}t}$$

$$i(t) = \frac{V_S}{R} - \frac{V_S}{R}e^{-\frac{R}{L}t}$$

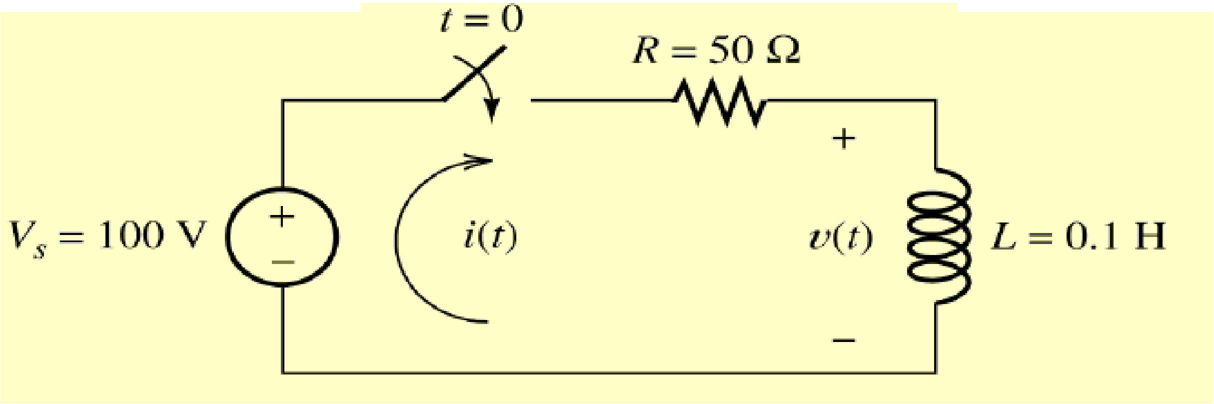
Inductor: current buildup rate



$$i(t) = \frac{V_S}{R} \times (1 - e^{-\frac{t}{\tau}})$$

$$\text{Time Constant} : \tau = \frac{L}{R}$$

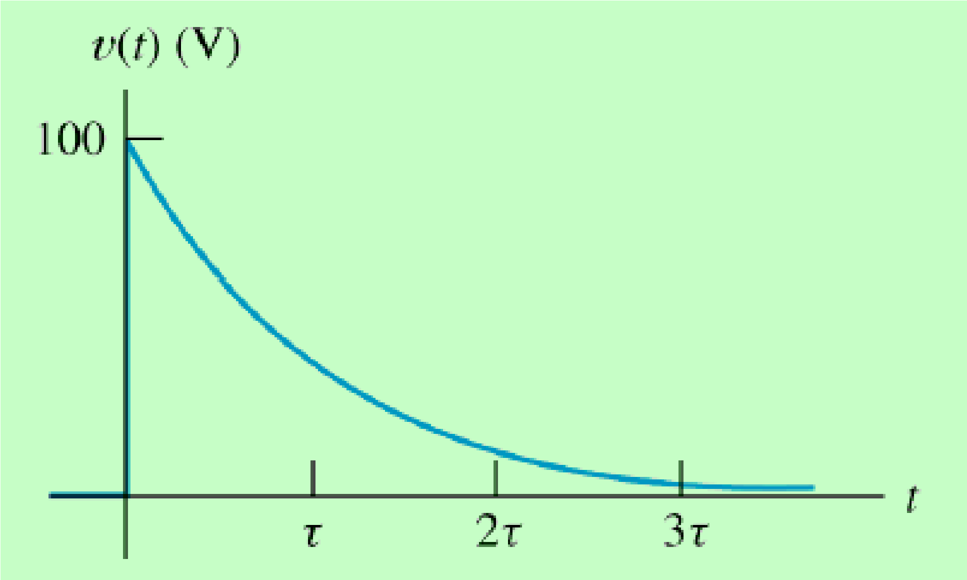
Inductor: voltage decay rate



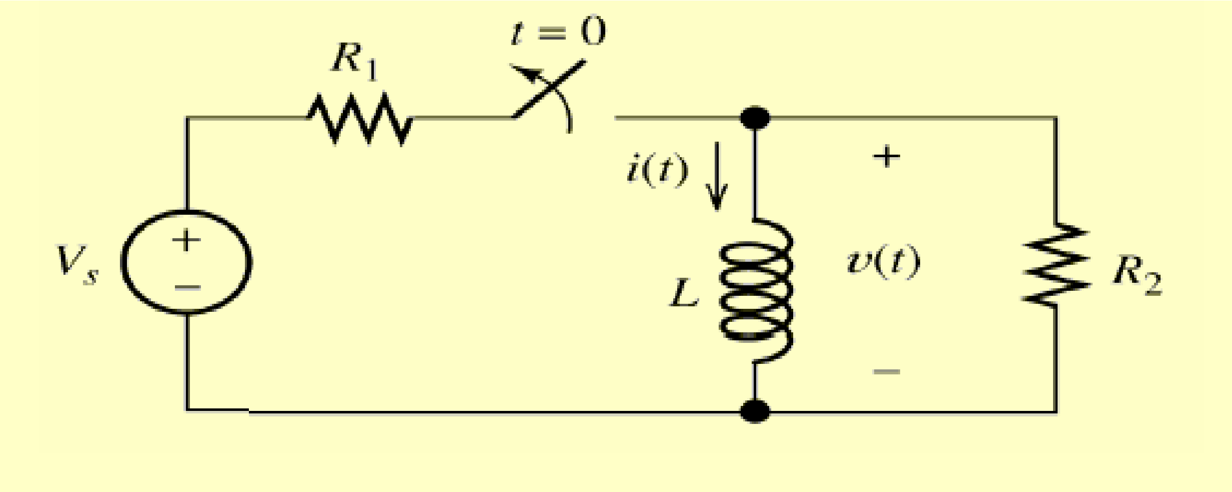
$$v = L\frac{di}{dt}$$

$$i(t) = \frac{V_S}{R} \times (1 - e^{-\frac{t}{\tau}})$$

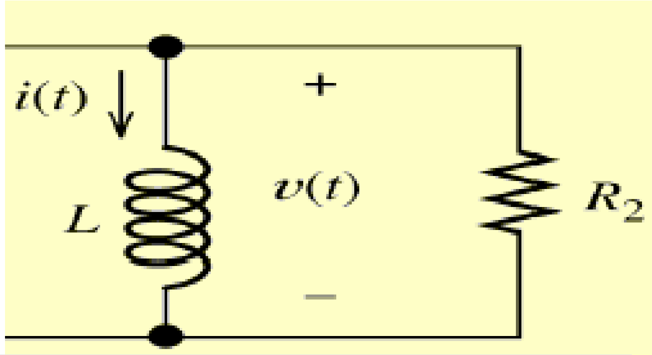
$$v(t) = V_s e^{-\frac{t}{\tau}}$$



Example 2



Circuit for t > 0



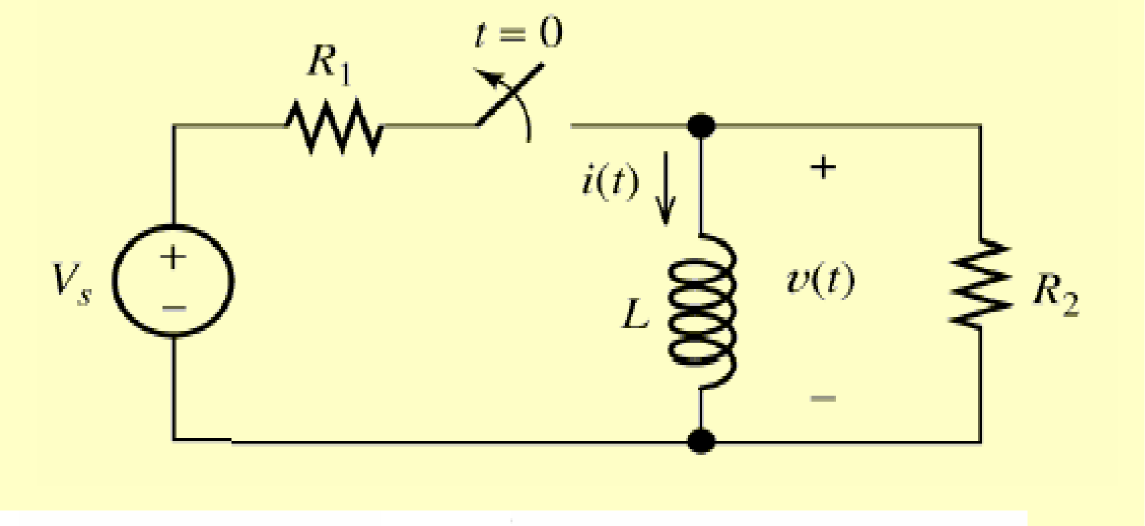
$$i(t \to \infty) = 0$$

$$i(t) = i(\infty) + \{i(0^+) - i(\infty)\} \times e^{-\frac{t}{\tau}}$$

$$\tau = \frac{L}{R_2}$$

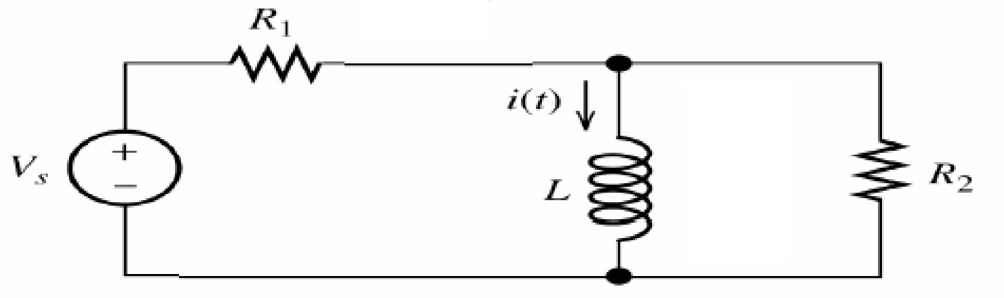
$$i(t) = i(0^+) \times e^{-\frac{t}{\tau}}$$

Example 2

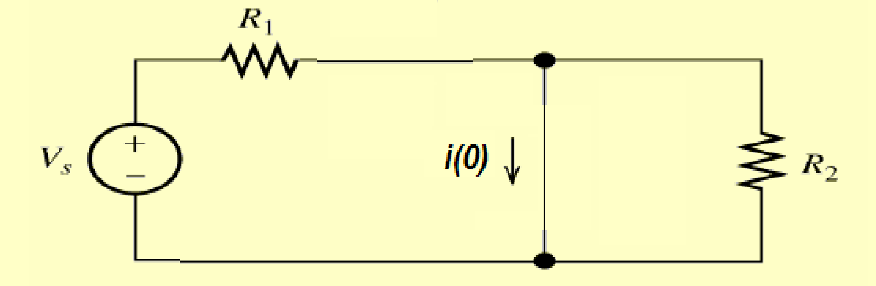


Initial condition

Circuit for $t \leq 0$



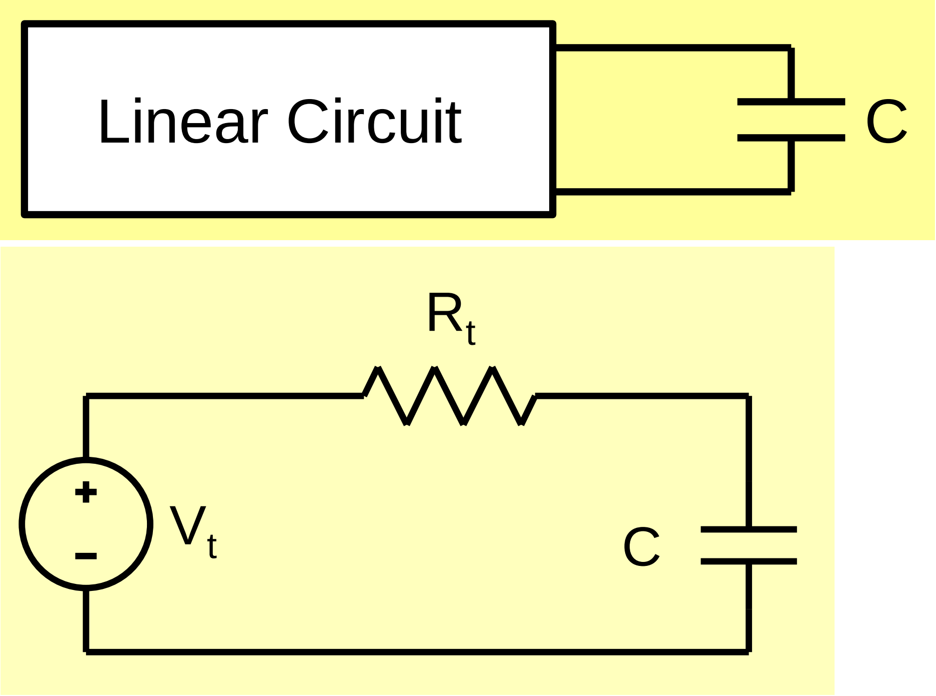
$$i(0^+) = i(0^-) = \frac{V_S}{R_1}$$



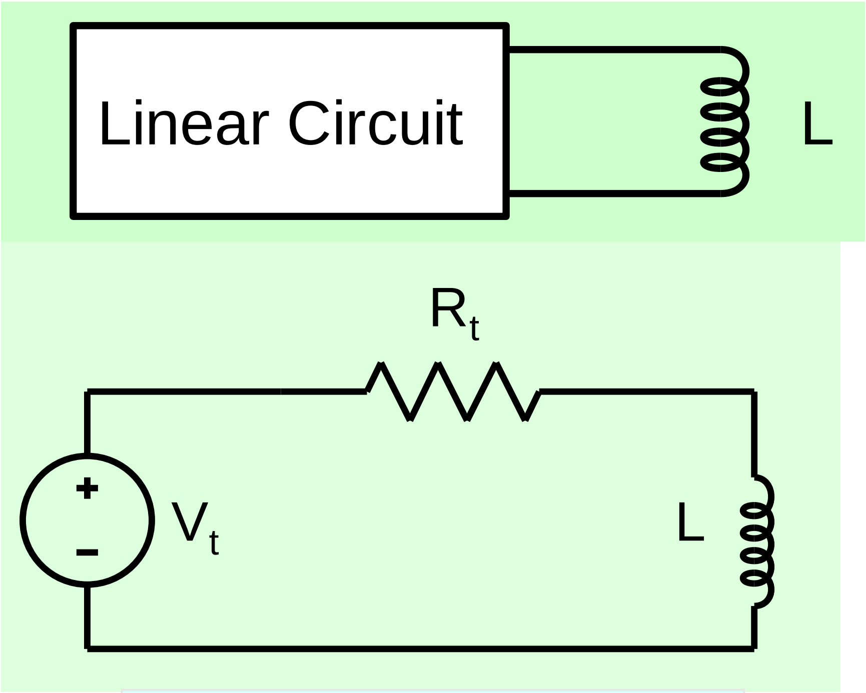
$$i(t) = \frac{V_S}{R_1} e^{-\frac{R_2}{L}t}$$

Circuit analysis with Inductor/Capacitor

Easy if the circuit contains a single L or C: Thevenin equivalent

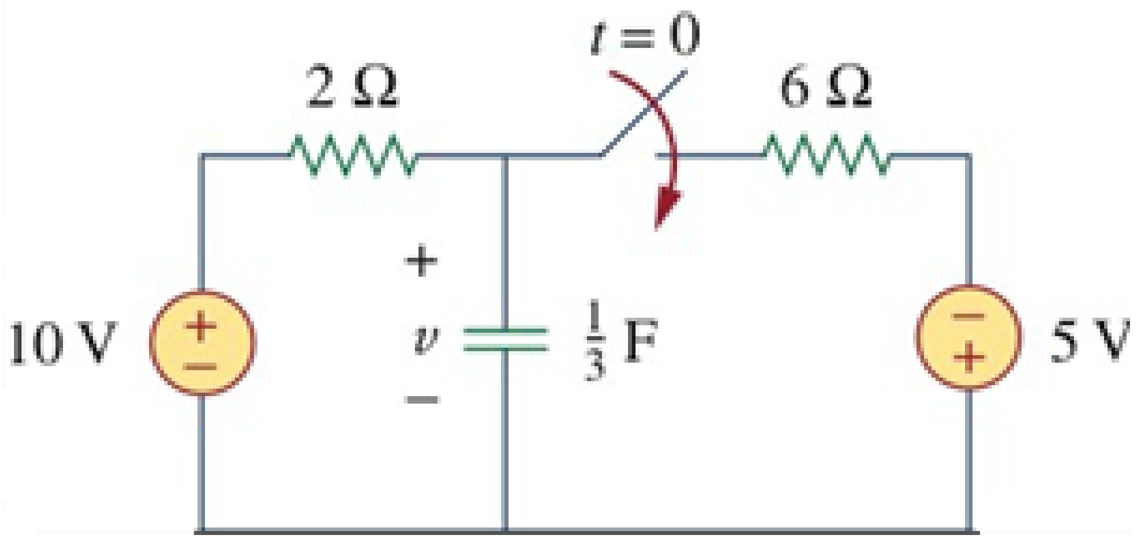


$$x(t) = x(\infty) + \{x(0^+) - x(\infty)\}e^{-\frac{t}{\tau}}$$

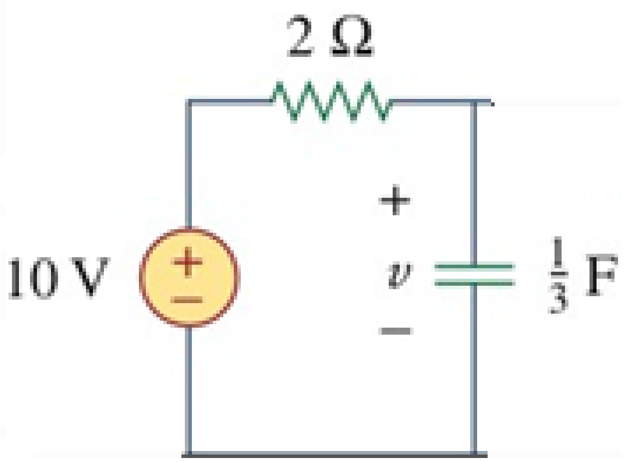


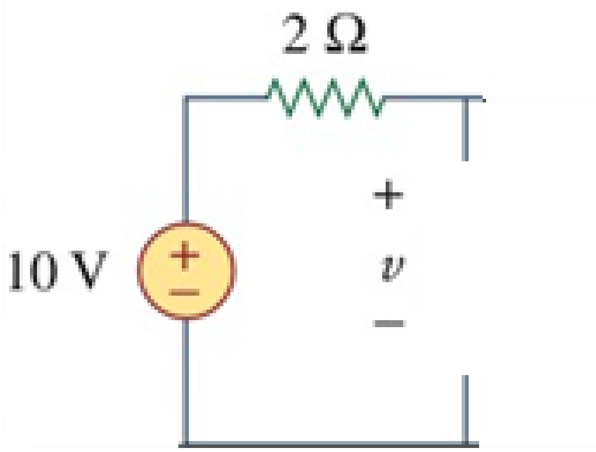
$$\tau = \frac{L}{R_{eq}} \ or \ R_{eq}C$$

Example



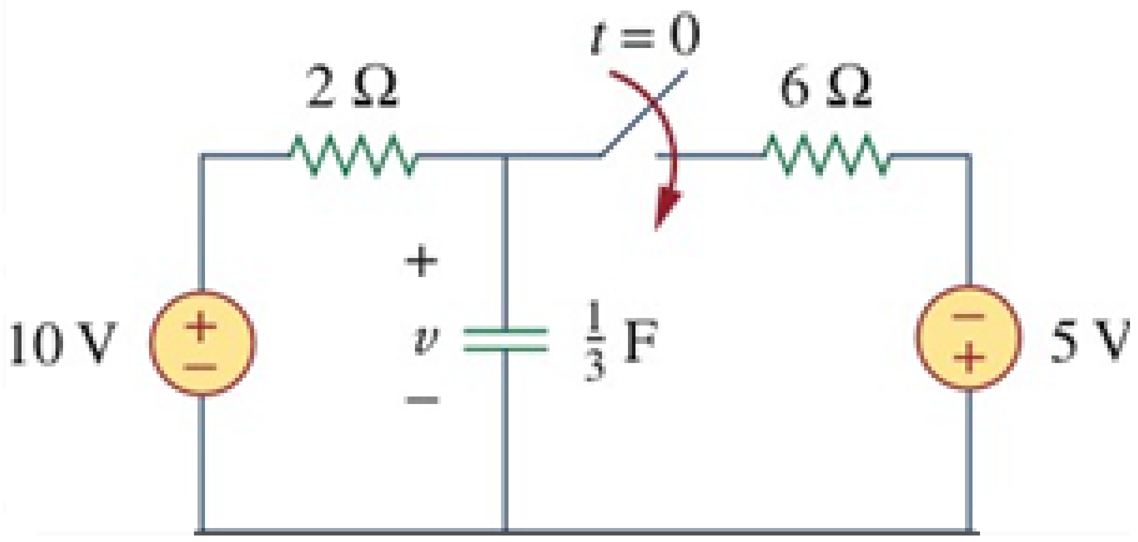
Circuit for t < 0



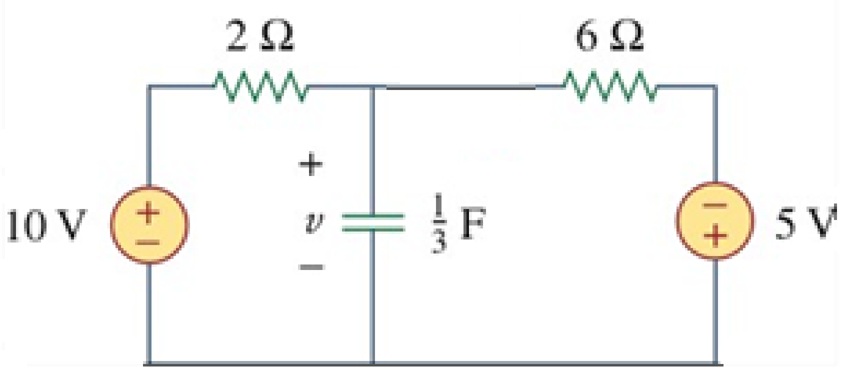


$v(0^+) = 10\text{V}$

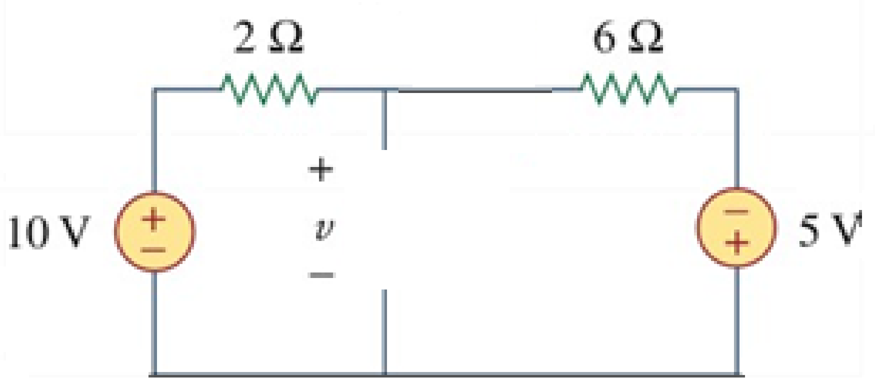
Example



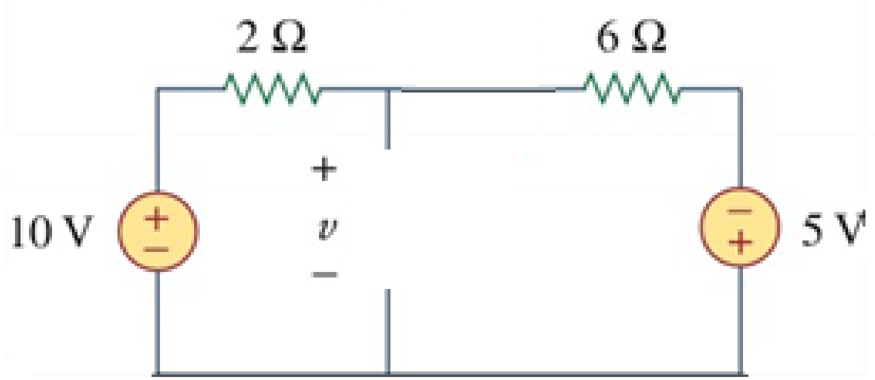
Circuit for t > 0



Determine the Thevenin equivalent, as seen by the capacitor:



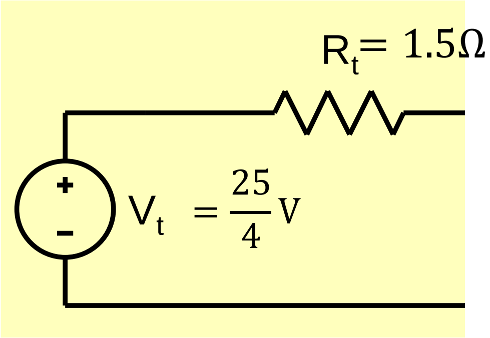
Example



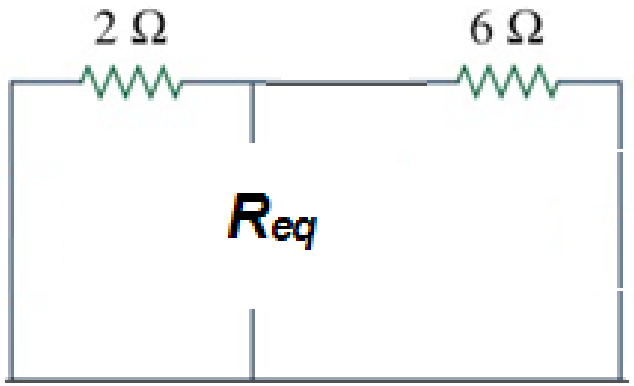
Determine the Thevenin equivalent, as seen by the capacitor:

Open circuit voltage

$$v_{OC} = \frac{25}{4}\text{V}$$



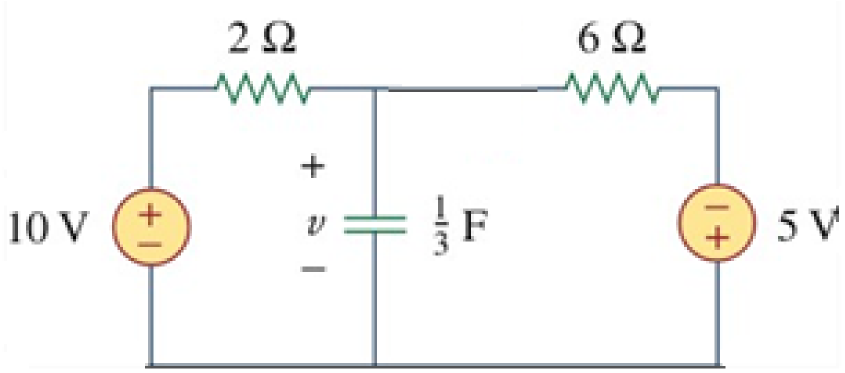
Thevenin resistance

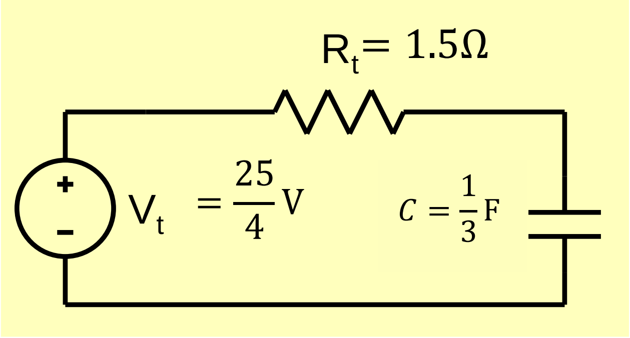


$$R_{eq} = 2||6 = 1.5\Omega$$

Example

Circuit for t > 0





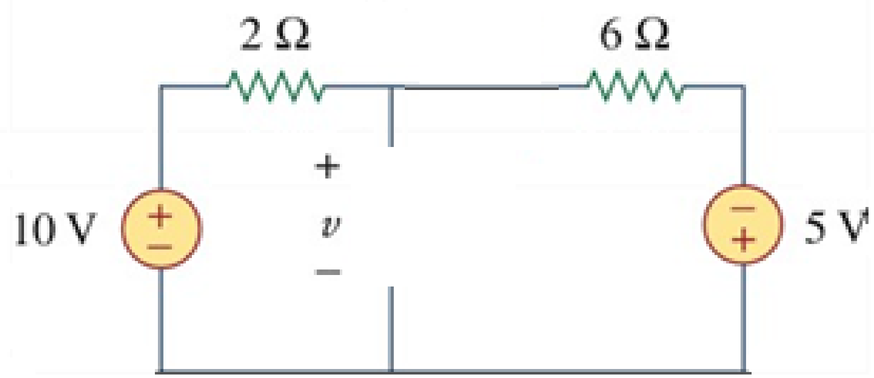
$$v(t) = v(\infty) + \{v(0^+) - v(\infty)\}e^{-\frac{t}{\tau}}$$

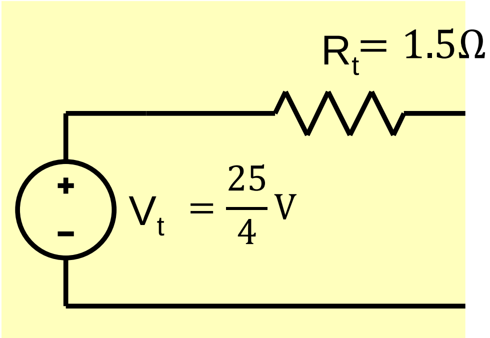
$$v(0^+) = 10\text{V}$$

$$v(\infty) = \frac{25}{4}\text{V}$$

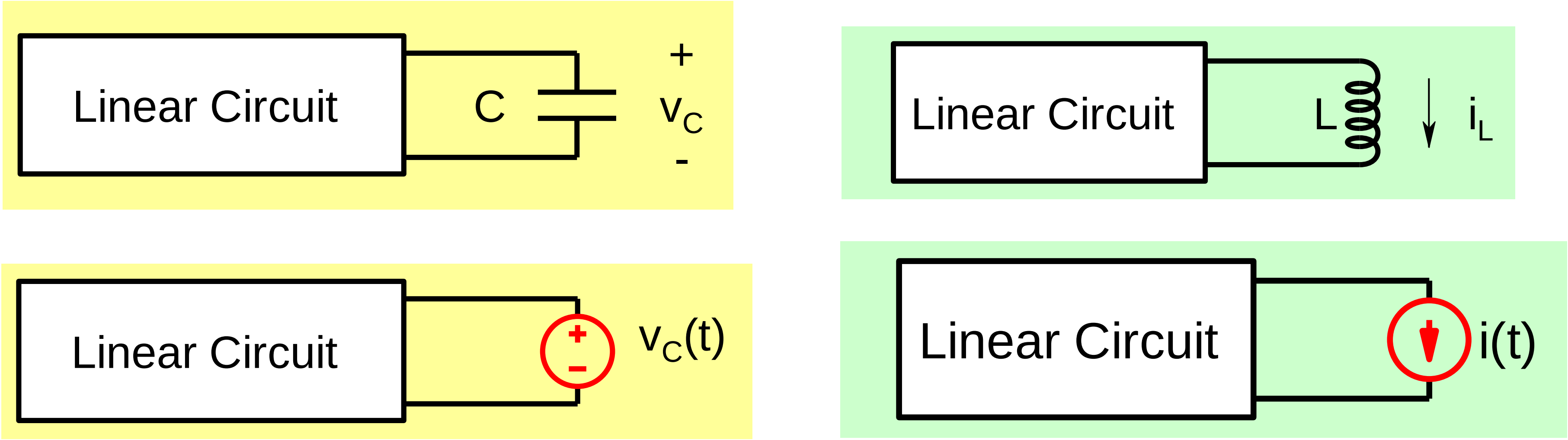
$$\tau = C \times R_{eq} = \frac{1}{3} \times 1.5 = 0.5s$$

$$v(t) = \frac{25}{4} + \frac{15}{4}e^{-2t}$$

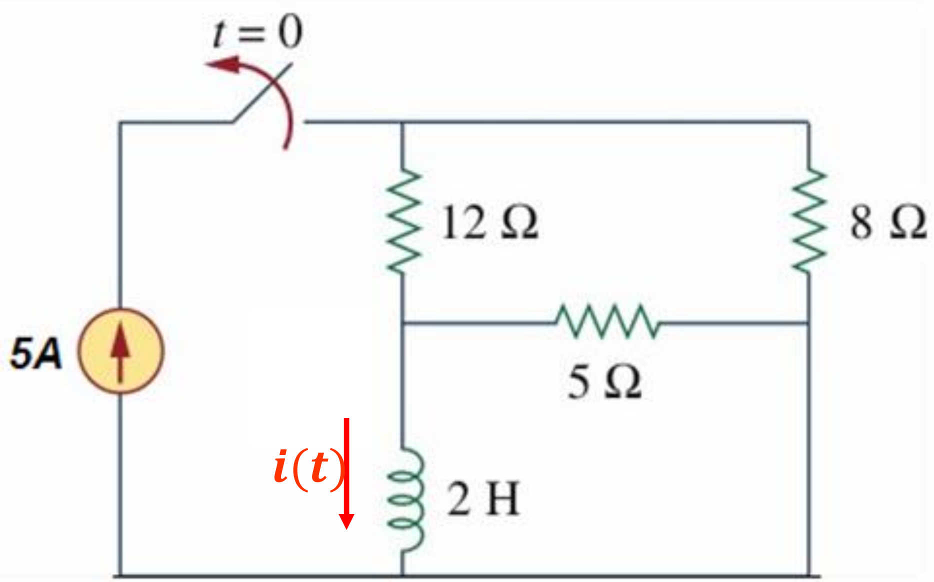


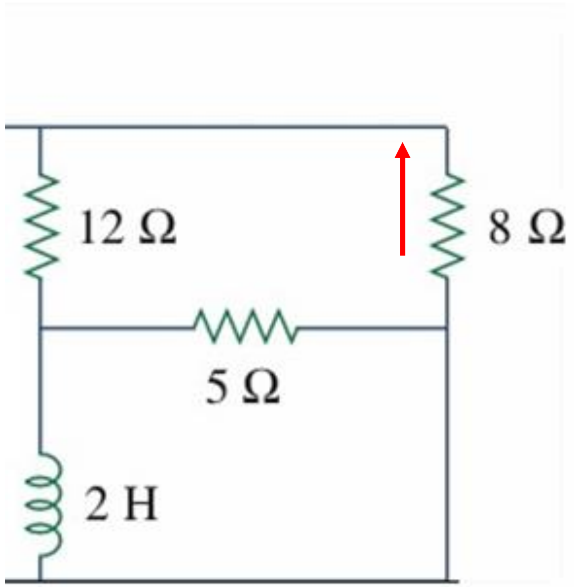


Voltages & currents inside the circuit?

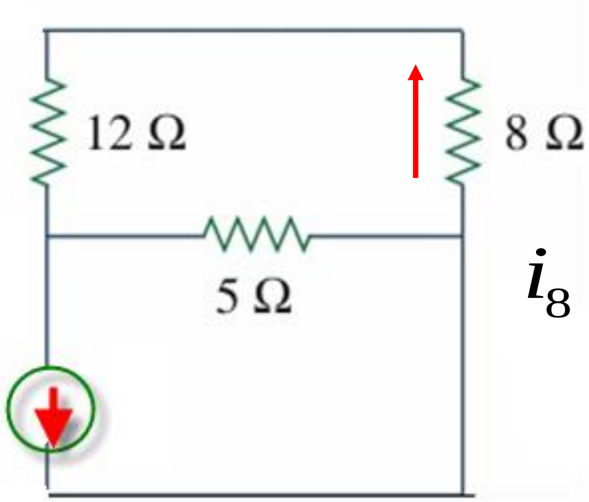


Example 3…





$$i(t) = 2 \times e^{-2t} \text{ A}$$



$$i_8 = i(t) \times \frac{5}{5+20} = 0.4 \times e^{-2t}$$