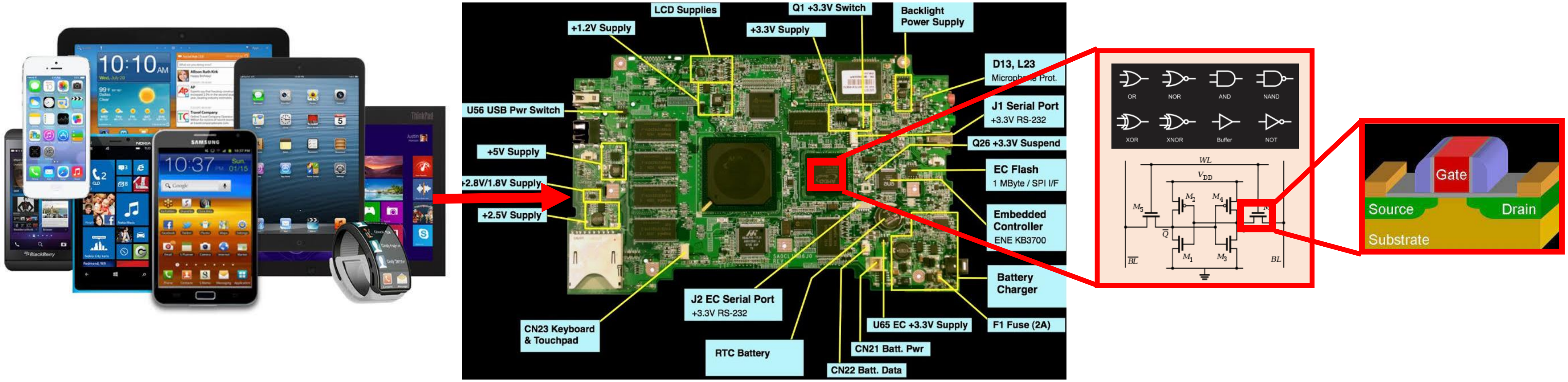


ESC201: Introduction to Electronics

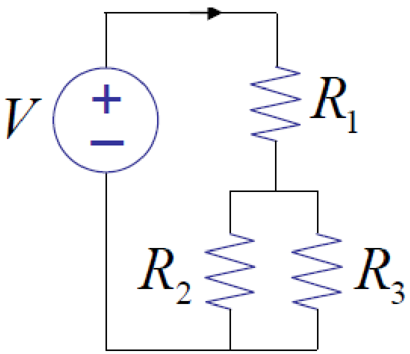
Module 1: Circuit Analysis

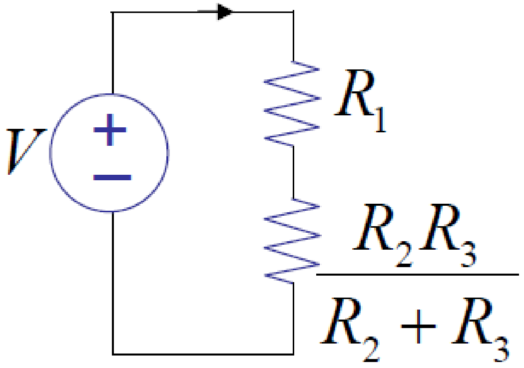
Dr. Shubham Sahay,
Assistant Professor,
Department of Electrical Engineering,
IIT Kanpur

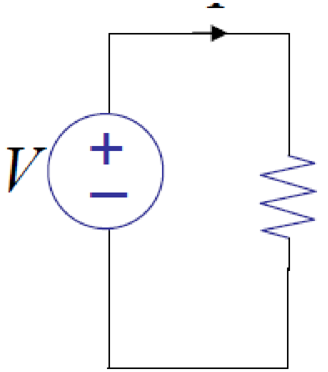
Recap

Solve circuits (i.e., find currents and voltages of interest)

Combine resistances in series and parallel

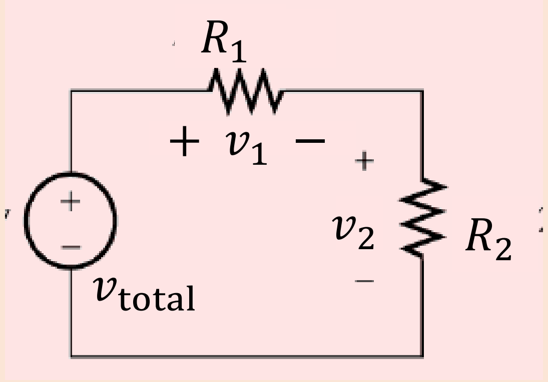






$$R = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

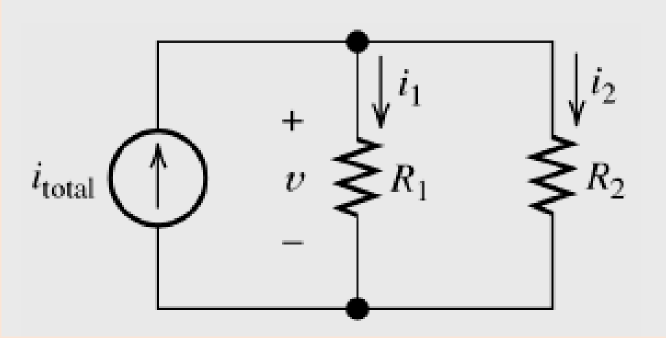
Apply the voltage-division and current-division principles



$$v_1 : v_2 = R_1 : R_2$$

$$v_1 = \frac{R_1}{R_1 + R_2} v_{\text{total}}$$

$$v_2 = \frac{R_2}{R_1 + R_2} v_{\text{total}}$$



$$i_1 : i_2 = R_2 : R_1$$

$$i_1 = \frac{R_2}{R_1 + R_2} i_{\text{total}}$$

$$i_2 = \frac{R_1}{R_1 + R_2} i_{\text{total}}$$

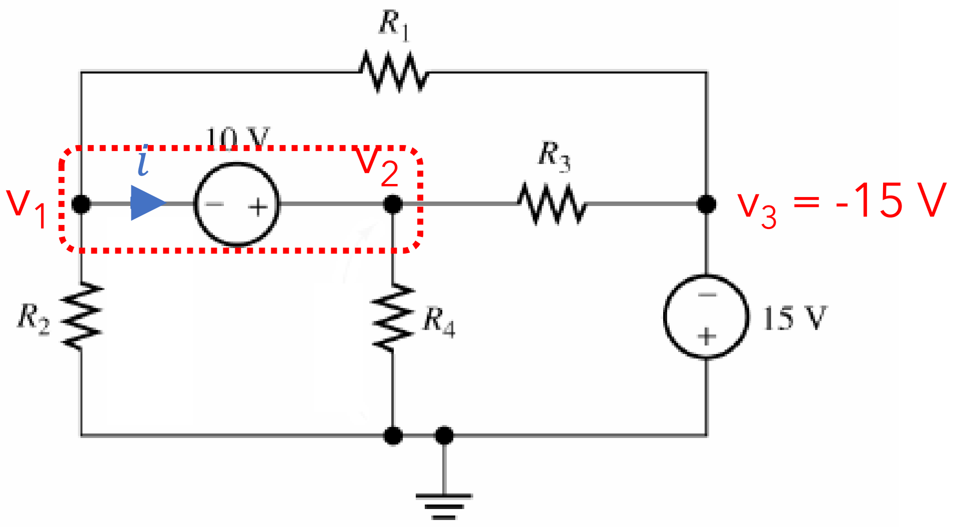
Nodal Analysis

- In nodal analysis, the variables used to describe the circuit will be "Node Voltages" (Recall Nodes!)
  - Nodal voltage are the voltages of each node with respect to a pre-selected reference node
- Steps:
  - Designate a node as reference or ground
  - Label voltages of remaining nodes (unknown variables)
  - Use KCL for all nodes except ground,
    - Write currents in terms of node voltages (using I-V equations)
  - Solve for node voltages
  - Back solve for branch voltages, if required

Example 4: hanging nodes

Circuits with voltage sources that are not connected to ground

Super node



Node 1

i

$$\frac{v_1}{R_2}+\frac{v_1-v_3}{R_1}+\boxed{\frac{v_1-v_2}{?}}=0$$

Node 2

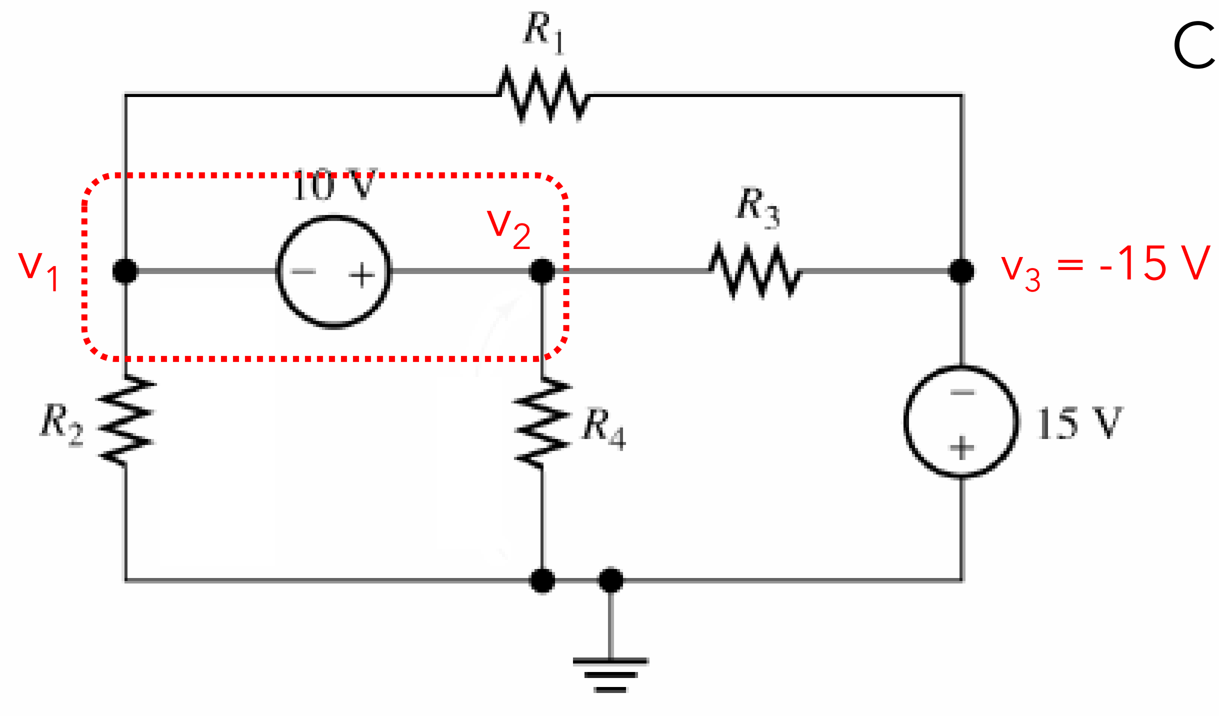
$-i$

$$\frac{v_2}{R_4}+\frac{v_2-v_3}{R_3}+\boxed{\frac{v_2-v_1}{?}}=0$$

$$v_2-v_1=10$$

$$\frac{v_1}{R_2}+\frac{v_1-v_3}{R_1}+\frac{v_2}{R_4}+\frac{v_2-v_3}{R_3}=0$$

Super node



Current entering the super node

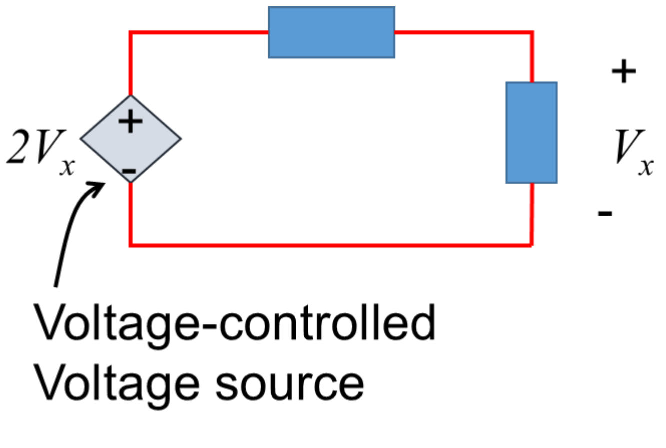
= current leaving the super node

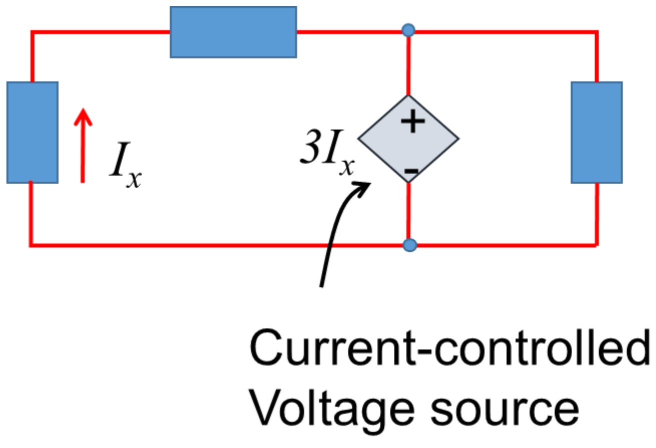
Node 1 and node 2 are merged together into a super node.

KCL is applied to the super node

$$\frac{v_1}{R_2} + \frac{v_1 - v_3}{R_1} + \frac{v_2}{R_4} + \frac{v_2 - v_3}{R_3} = 0$$

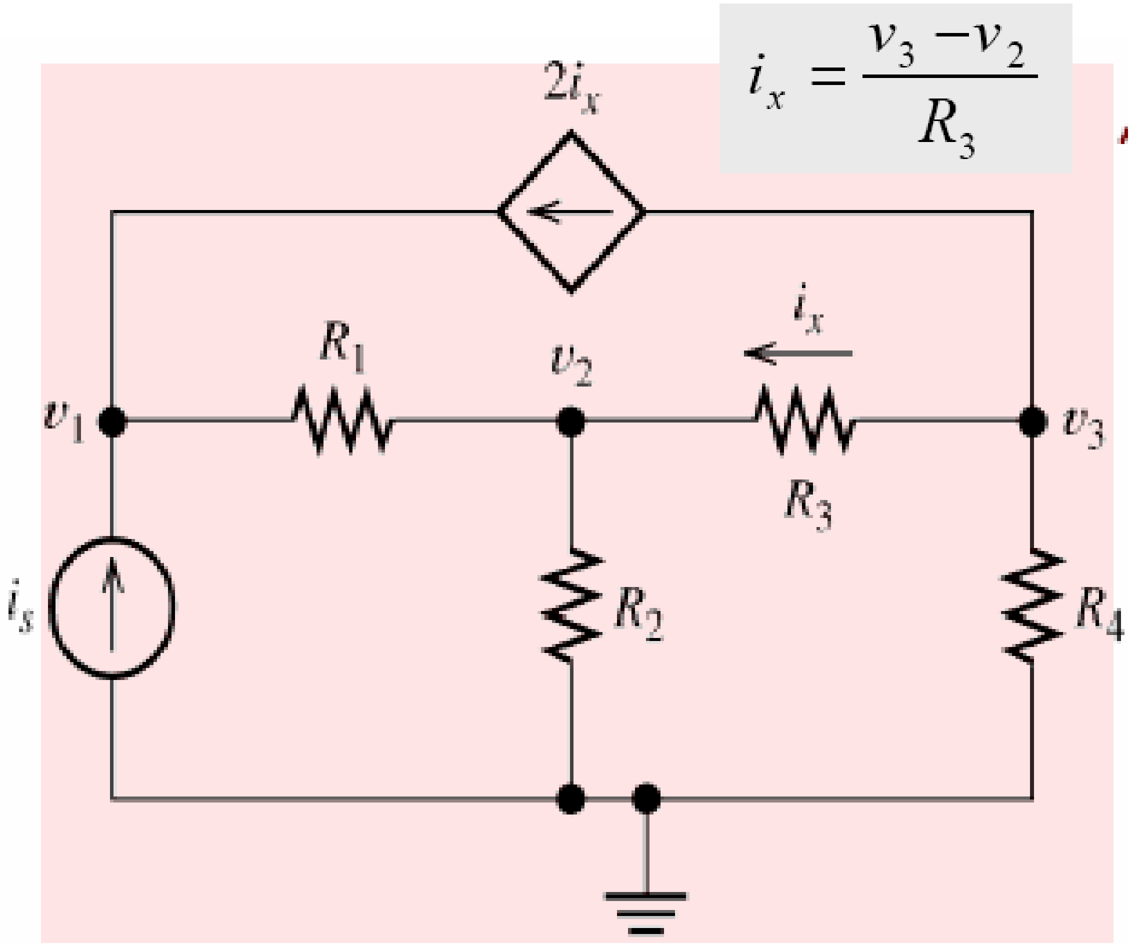
Example 5: dependent sources





Super nodes also useful for analyzing such circuits

Node-Voltage Analysis with a Dependent Source



At Node 1

$$\frac{v_1 - v_2}{R_1} - i_s - 2i_x = 0$$

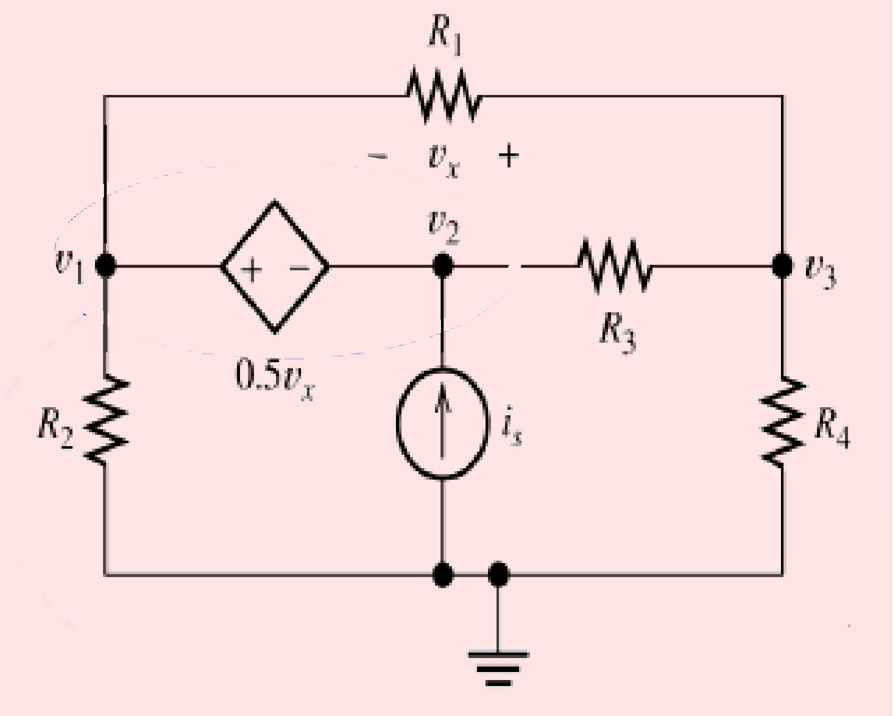
At Node 2

$$\frac{v_2 - v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_2 - v_3}{R_3} = 0$$

At Node 3

$$\frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} + 2i_x = 0$$

Dependent node: example



Contolling variable

$$v_x = v_3 - v_1$$

Dependent voltage source :

$$v_1 - v_2 = 0.5v_x$$

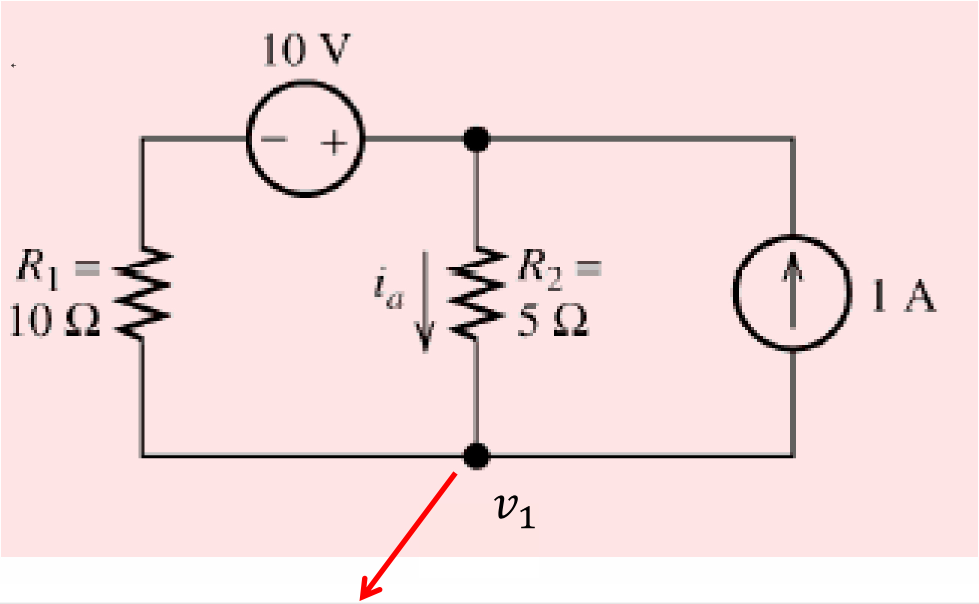
Node 3

$$\frac{v_3}{R_4} + \frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_1} = 0$$

supernode (nodes 1 and 2)

$$\frac{v_1}{R_2} + \frac{v_1 - v_3}{R_1} + \frac{v_2 - v_3}{R_3} = i_s$$

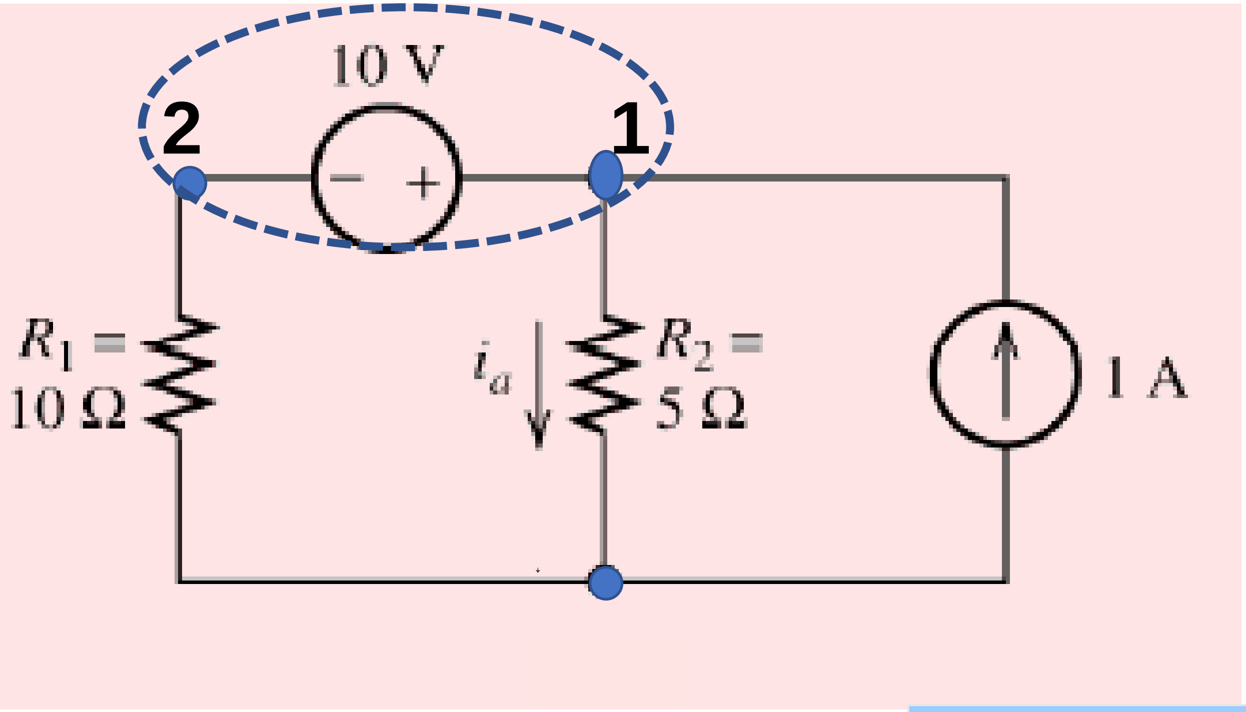
Choosing a reference node



$$\frac{v_1}{10}+\frac{v_1-10}{5}+1=0 \Rightarrow v_1=\frac{10}{3}$$

$$i_a=\frac{10-v_1}{5}=1.33A$$

Choosing a reference node



Super node: $$v_1 - v_2 = 10V$$

KCL at super node: $$\frac{v_1}{5} - 1 + \frac{v_2}{10} = 0$$

Mesh Analysis

1. Mesh analysis provides another general procedure for analyzing circuits using mesh currents as the circuit variables.

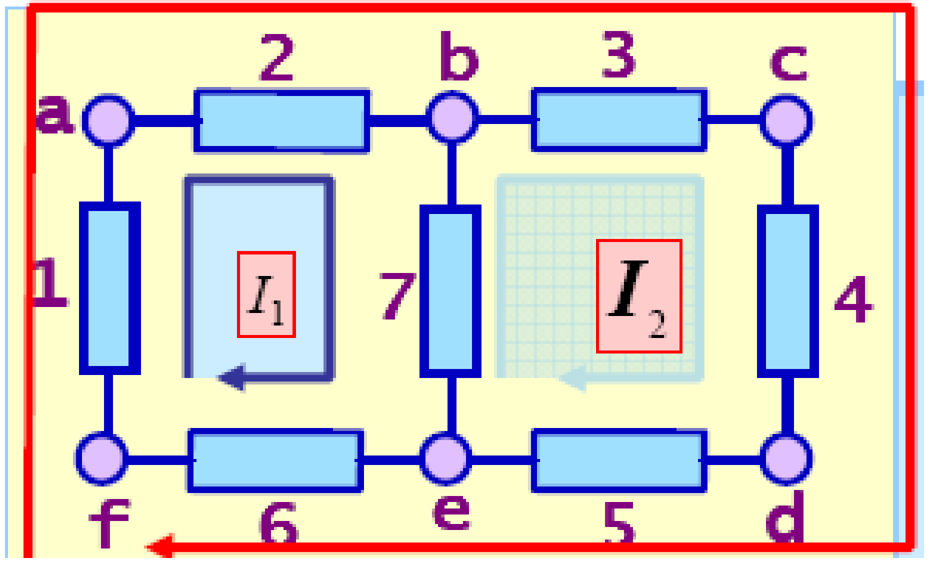
2. Mesh analysis applies KVL to find unknown currents.

3. A mesh is a loop which does not contain any other loop within it.

Mesh Analysis

A loop is a closed path through circuit elements that does not pass through any node more than once.

A mesh is a loop that does not include any other loop.



3 loops in above circuit.

fabef   ebcde   ~~fabcdef~~

2 meshes in above circuit.

Mesh Currents

How many meshes?

I_1, I_2 and I_3 are branch current

real, measurable directly

i_1 and i_2 are mesh current

imaginary,

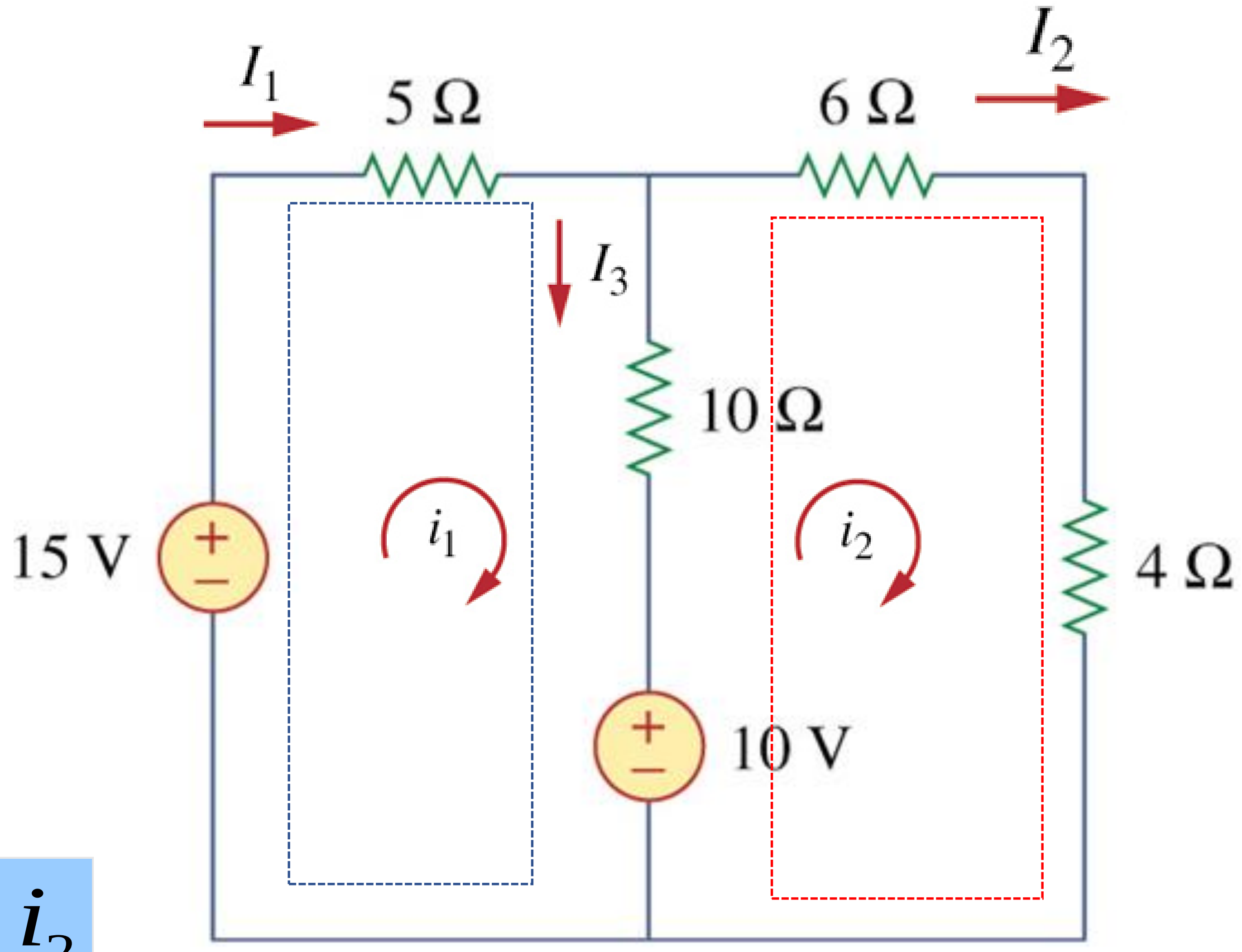
may not be measurable directly

$$I_1 = i_1 \qquad I_2 = i_2$$

From KCL

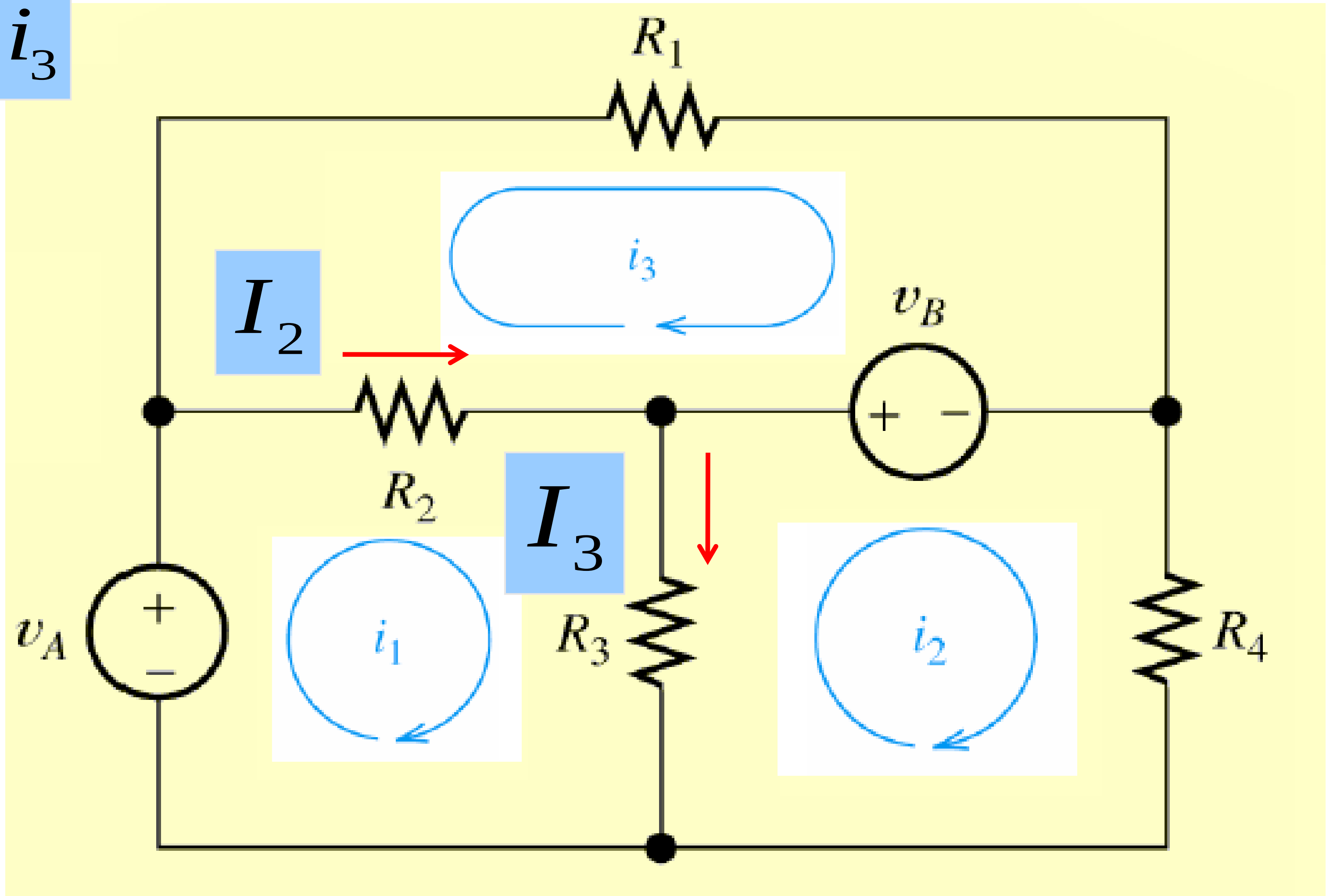
$$I_1 - I_2 - I_3 = 0$$

$$I_3 = i_1 - i_2$$



Mesh Currents

$$I_2 = i_1 - i_3$$



$$I_3 = i_1 - i_2$$

Mesh Analysis

Steps to determine the mesh currents:

1. Assign mesh currents i_1, i_2, …, in to the n meshes.

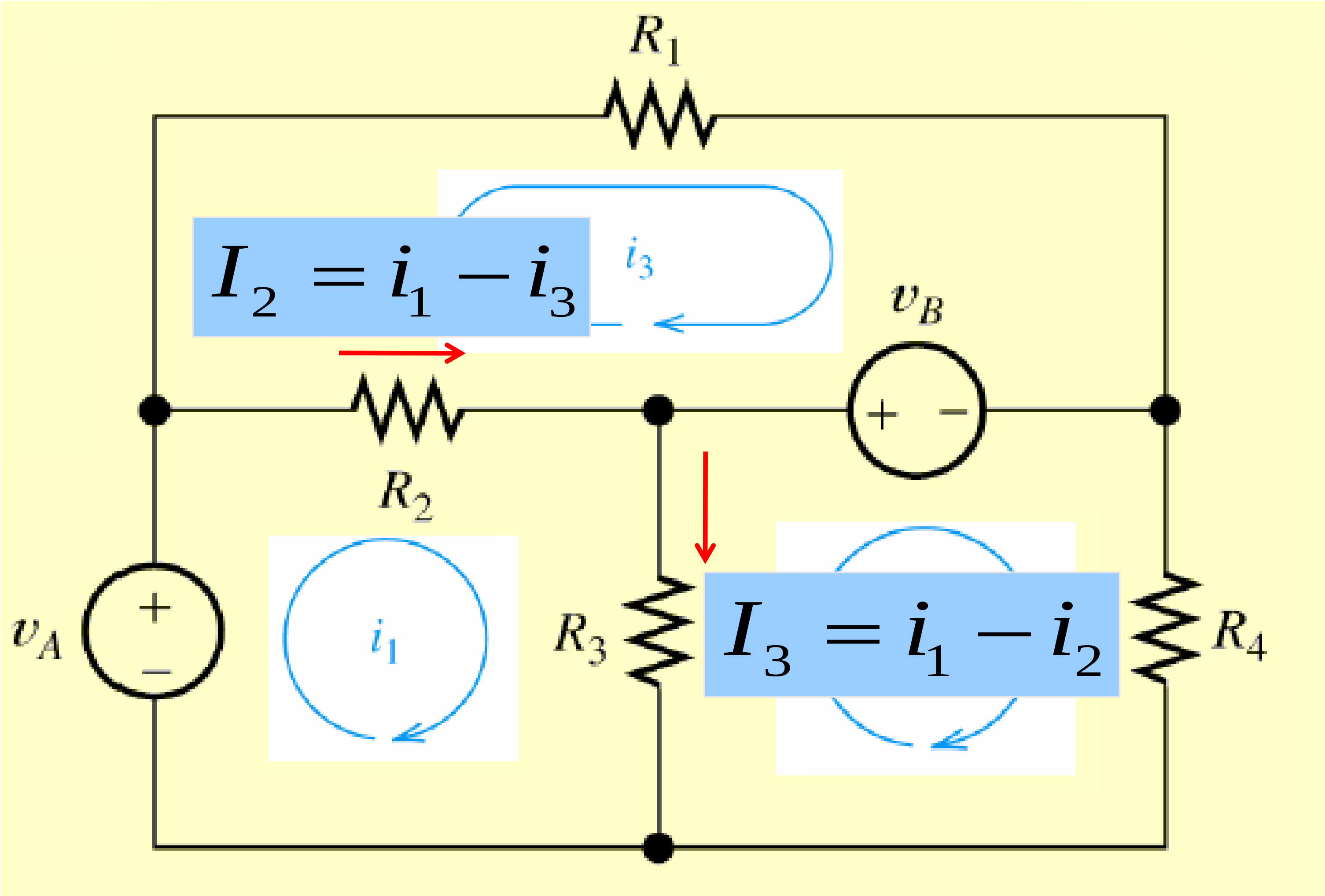
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.

3. Solve the resulting n simultaneous equations to get the mesh currents.

Apply KVL to each mesh

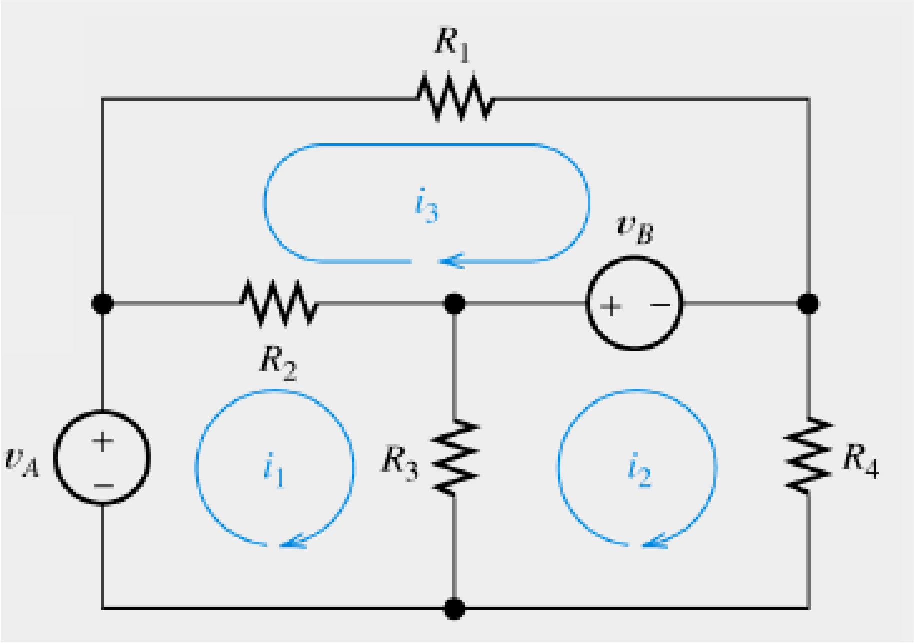
$$-v_A + I_2R_2 + I_3R_3 = 0$$

$$R_2(i_1 - i_3) + R_3(i_1 - i_2) - v_A = 0$$



Apply KVL to Each Mesh

$$R_2(i_3 - i_1) + R_1 i_3 - v_B = 0$$

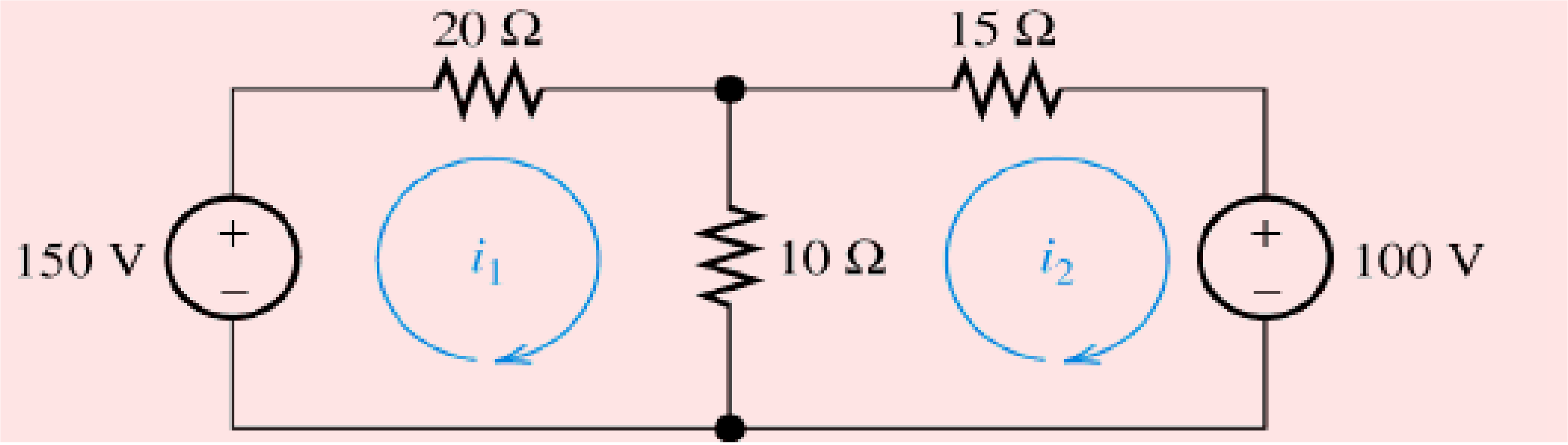


$$R_2(i_1 - i_3) + R_3(i_1 - i_2) - v_A = 0$$

$$R_3(i_2 - i_1) + v_B + R_4 i_2 = 0$$

Mesh Analysis: Example 2

Compute currents in each element of the following circuit.



$$\text{mesh 1}: 20i_1 + 10(i_1 - i_2) - 150 = 0$$

$$\text{mesh 2}: 10(i_2 - i_1) + 15i_2 + 100 = 0$$

$$30\,i_1 - 10\,i_2 = 150$$

$$-10\,i_1 + 25\,i_2 = -100$$

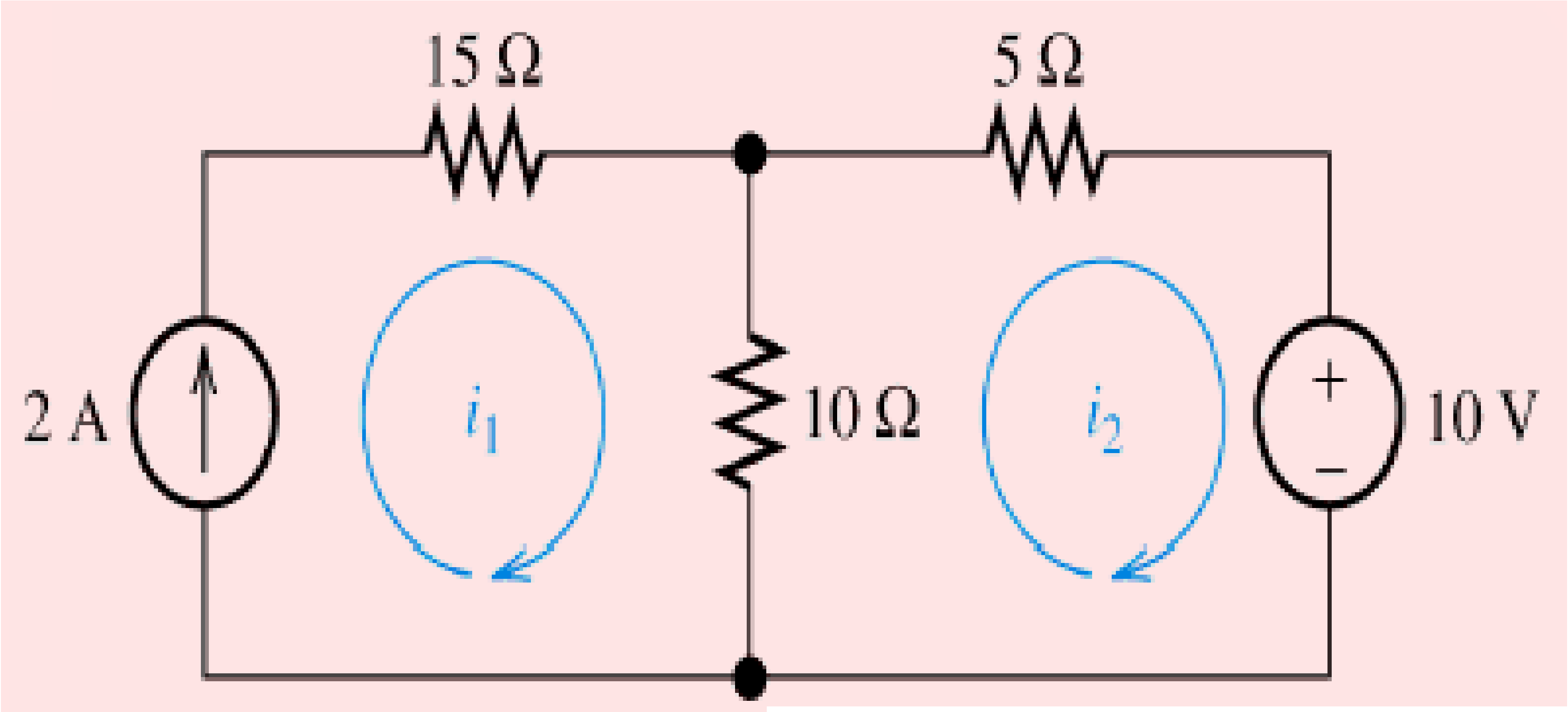
$$i_1 = 4.231\,A$$

$$i_2 = -2.308\,A$$

The current in the 10 - Ω is i_1-i_2 = 6.539 A

Mesh Analysis: Example 3

Mesh Currents in Circuits Containing Current Sources



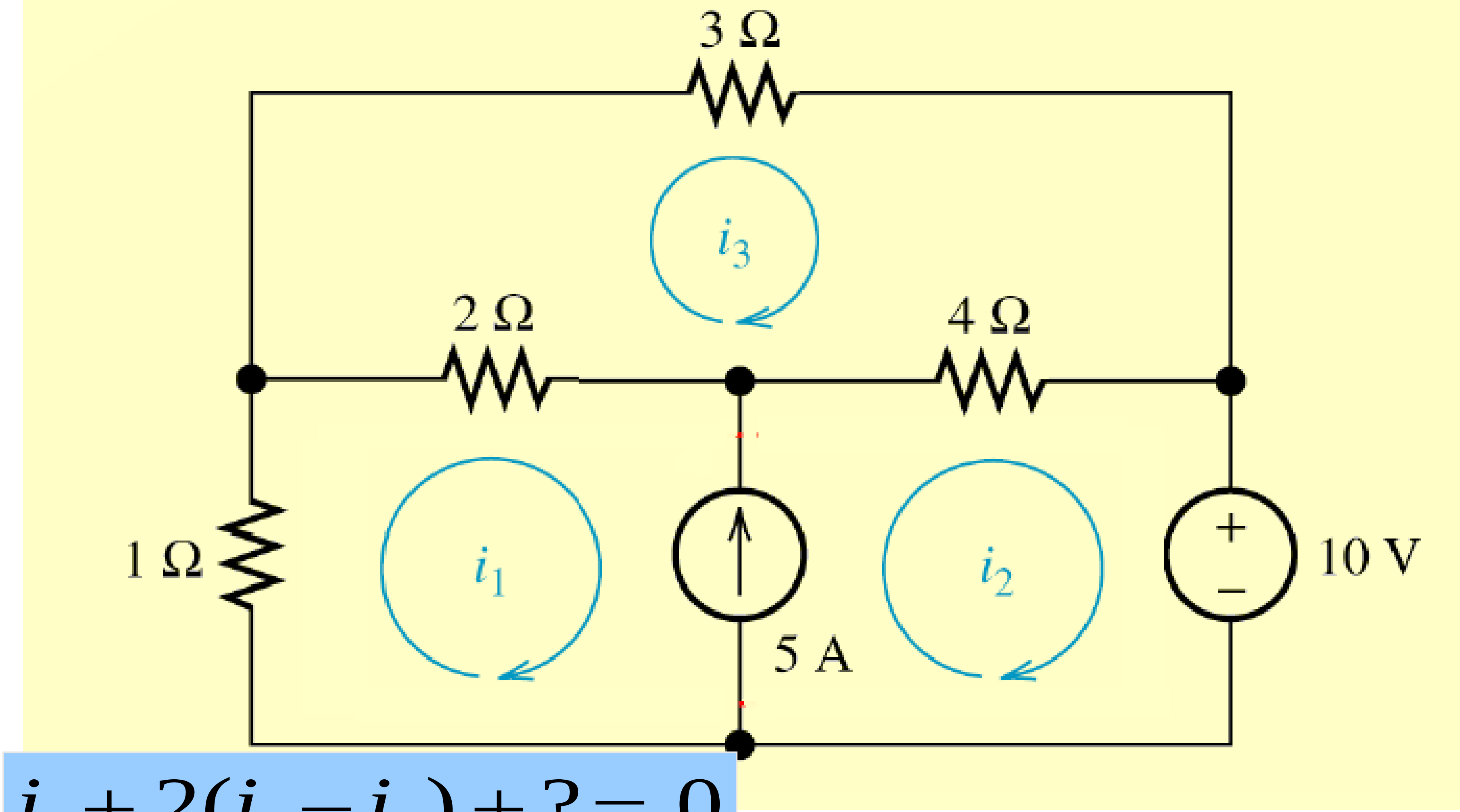
$$15i_1 + 10(i_1 - i_2) + ? = 0$$

$$10(i_2 - i_1) + 5i_2 + 10 = 0$$

$$i_1 = 2\text{A}$$

Mesh Analysis: Example 4

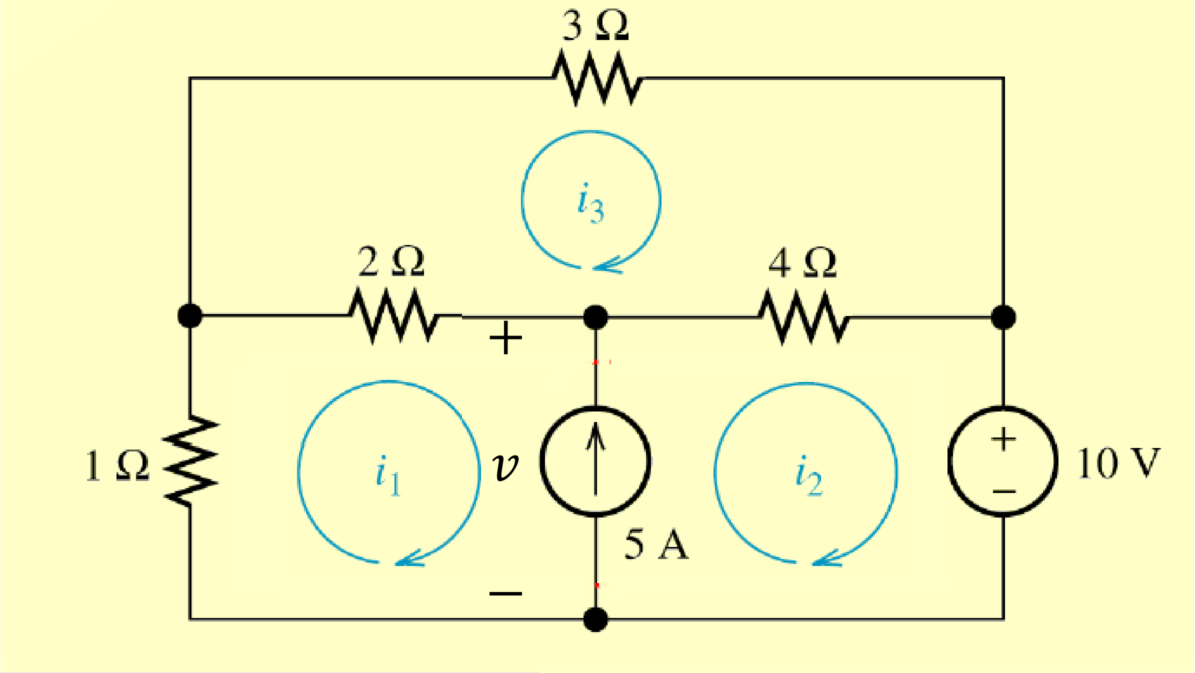
Current source common to 2 mesh



$$i_1 + 2(i_1 - i_3) + ? = 0$$

Mesh Analysis: Example 4

Current source common to 2 mesh



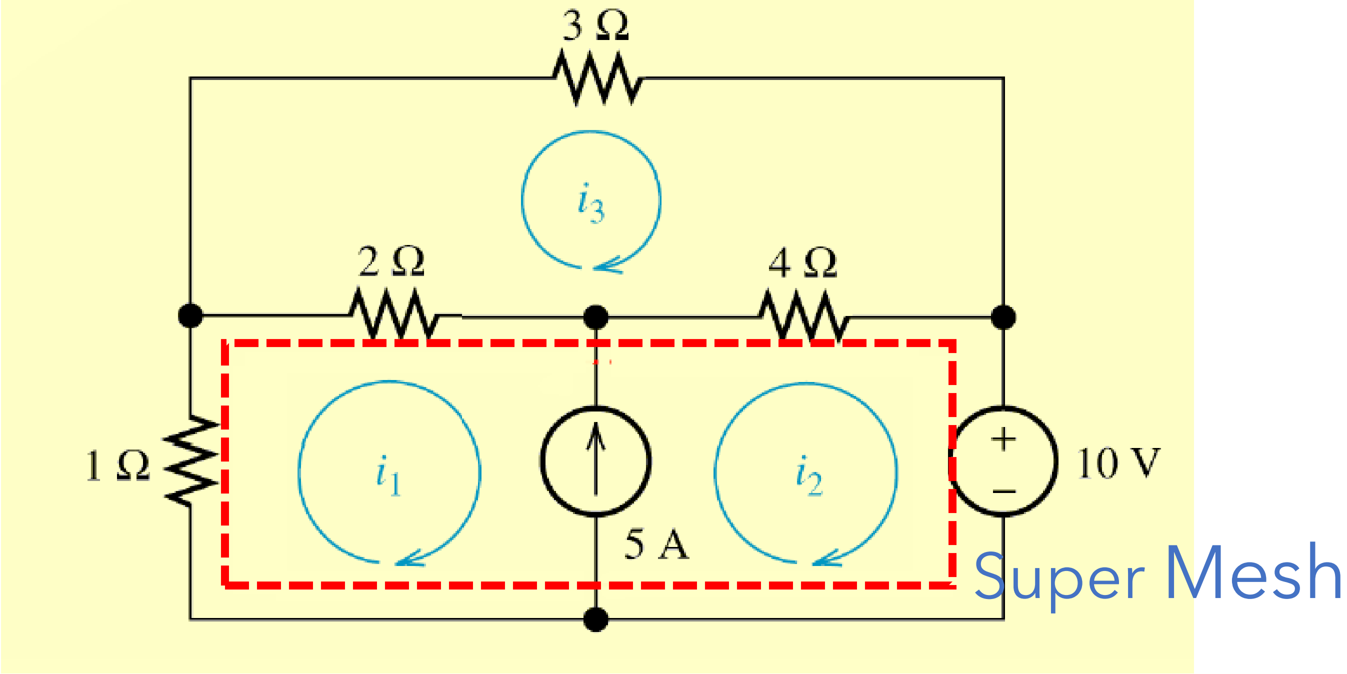
Super Mesh

$$i_1 + 2(i_1 - i_3) + ? = 0$$

$$i_1 + 2(i_1 - i_3) + v = 0 \qquad -v + 4(i_2 - i_3) + 10 = 0$$

$$i_1 + 2(i_1 - i_3) + 4(i_2 - i_3) + 10 = 0$$

Mesh Analysis: Example 4



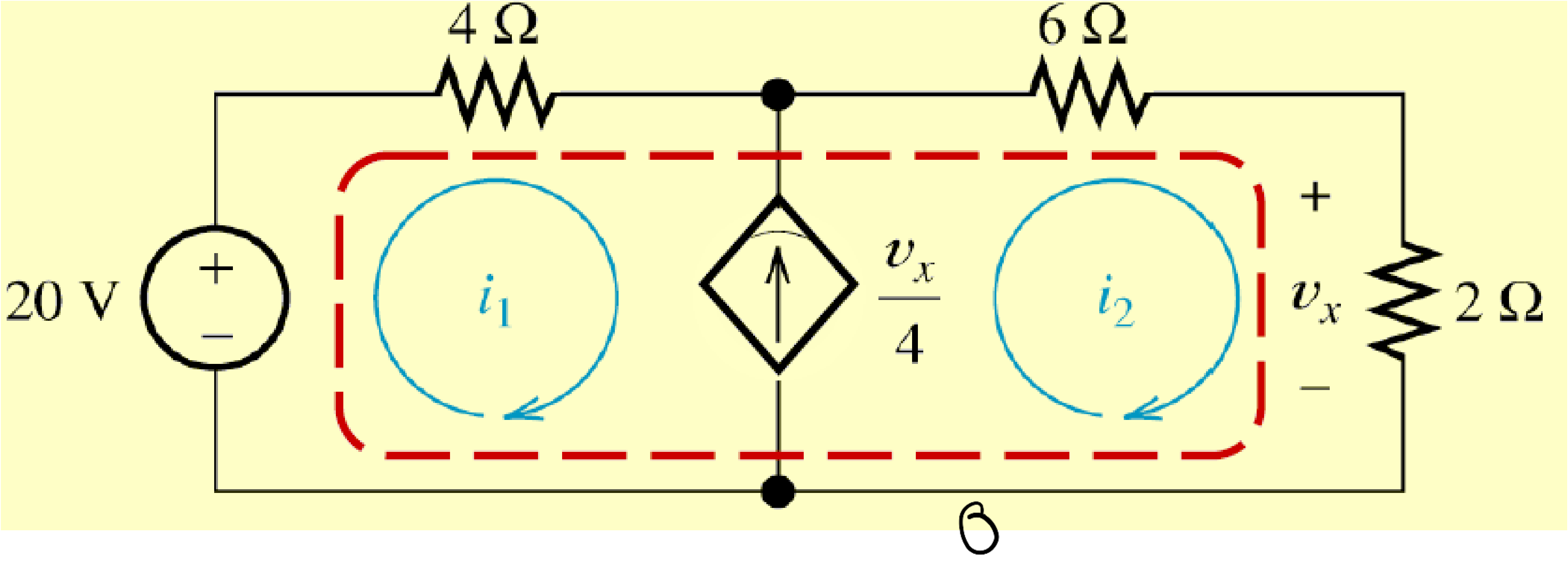
$$i_1 + 2(i_1 - i_3) + 4(i_2 - i_3) + 10 = 0$$

$$i_2 - i_1 = 5$$

Mesh-3

$$3i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) = 0$$

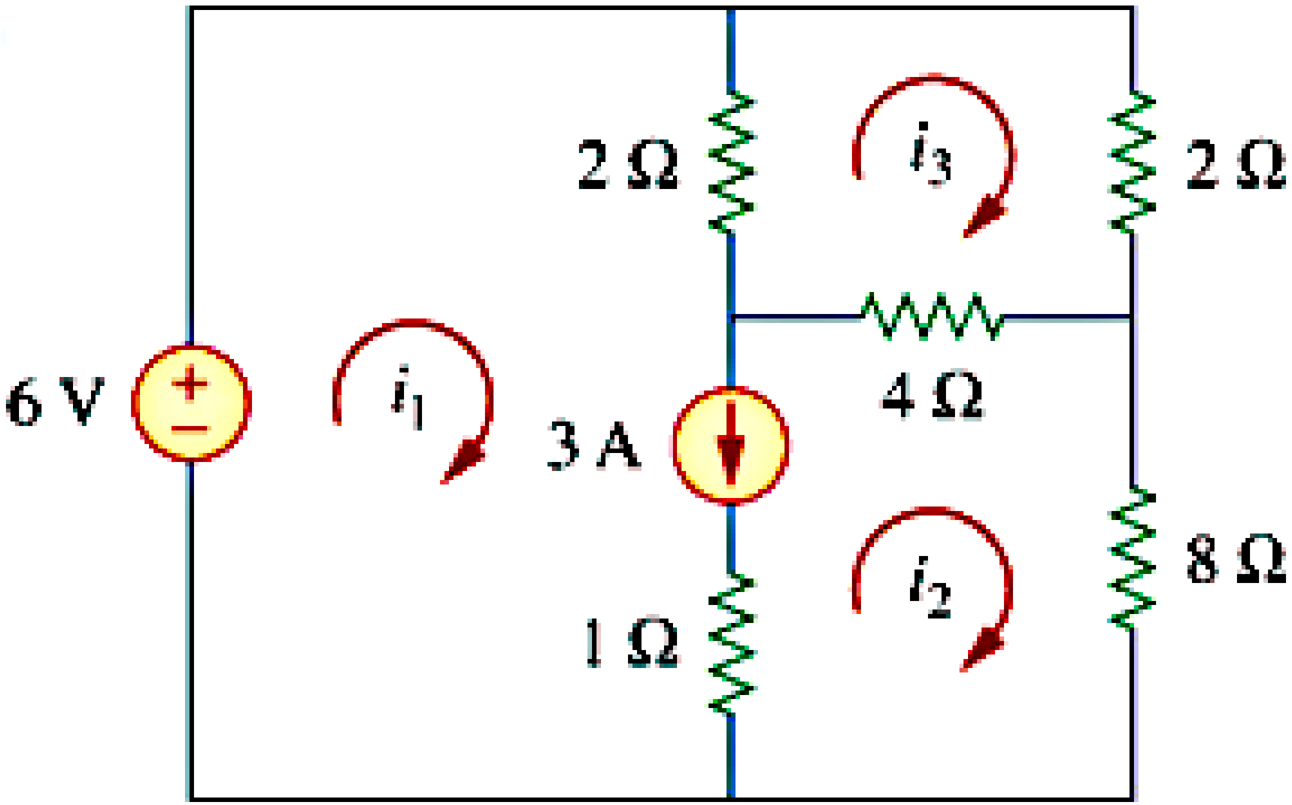
Example 5: Dependent Sources



$$-20+4i_1+6i_2+2i_2=0$$

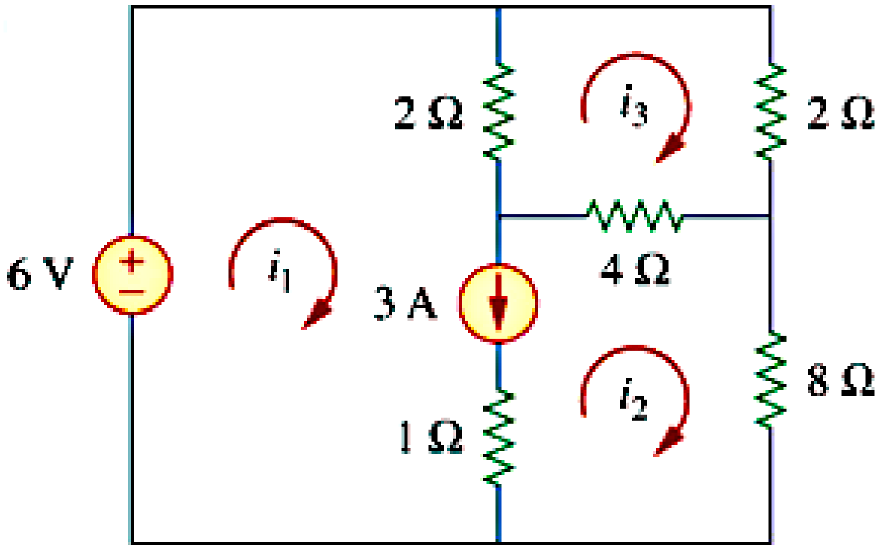
$$\frac{v_x}{4}=i_2-i_1 \qquad v_x=2i_2$$

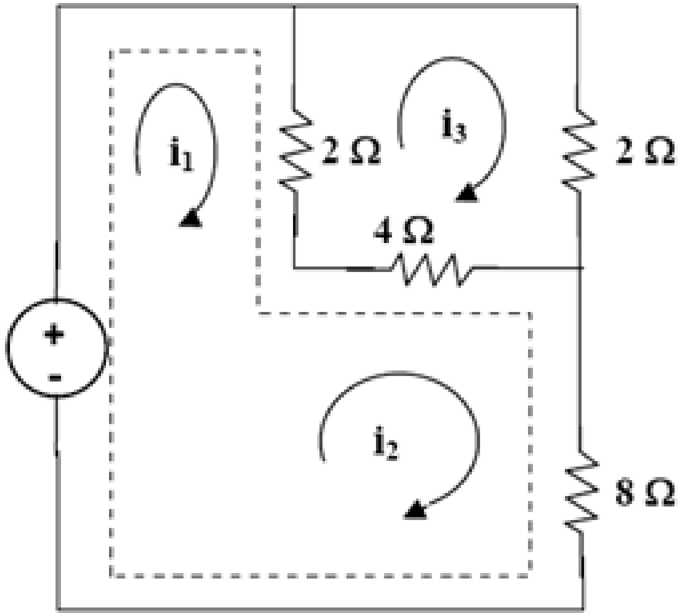
Example 6:



Identify the super mesh

Example 6





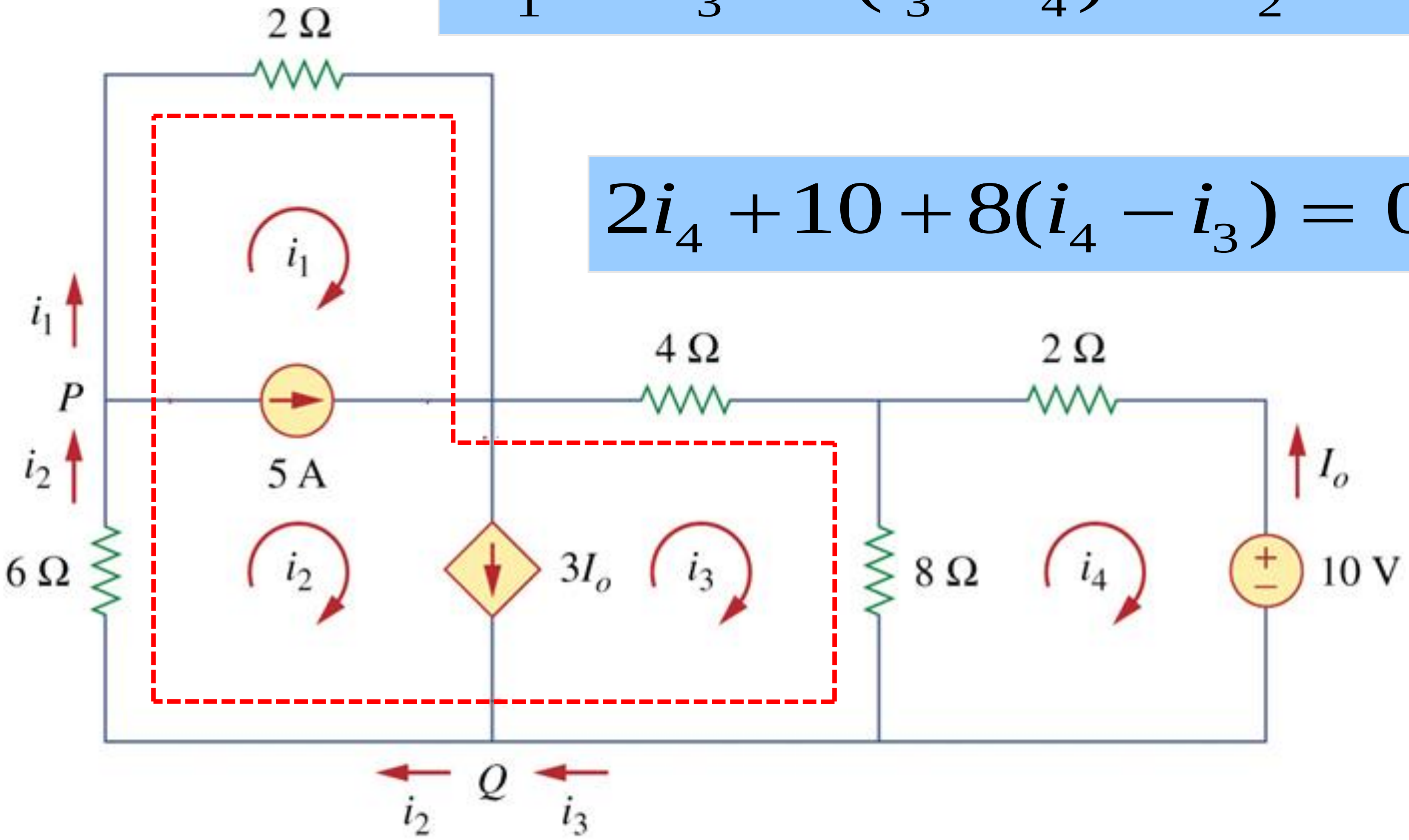
$$-6 + 2(i_1 - i_3) + 4(i_2 - i_3) + 8i_2 = 0$$

$$2i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) = 0$$

$$i_1 - i_2 = 3$$

Example 7

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$



$$2i_4 + 10 + 8(i_4 - i_3) = 0$$

$$i_2 - i_1 = 5 \qquad i_2 - i_3 = 3I_O \qquad I_O = -i_4$$

Nodal vs. Mesh Analysis

To select the method that results in simpler or the smaller number of equations.

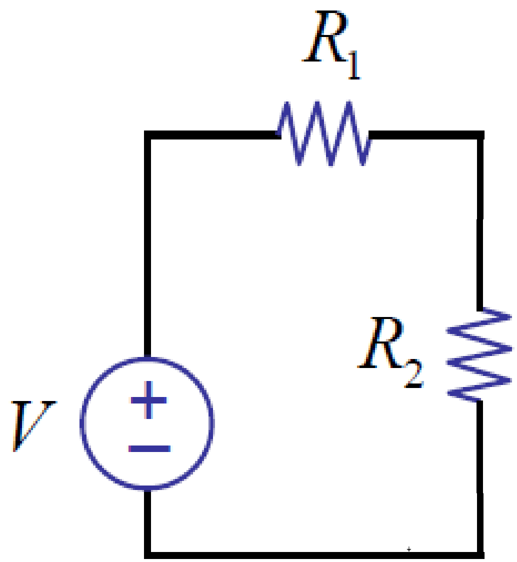
1. Choose nodal analysis for circuit with fewer nodes than meshes.
   - Choose mesh analysis for circuit with fewer meshes than nodes.

2. Circuit components
   - Networks that contain many series connected elements, voltage sources, or supermeshes are more suitable for mesh analysis.

   - Networks with parallel-connected elements, current sources, or supernodes are more suitable for nodal analysis.

3. If node voltages are required, it may be expedient to apply nodal analysis. If branch or mesh currents are required, it may be better to use mesh analysis.

Linearity

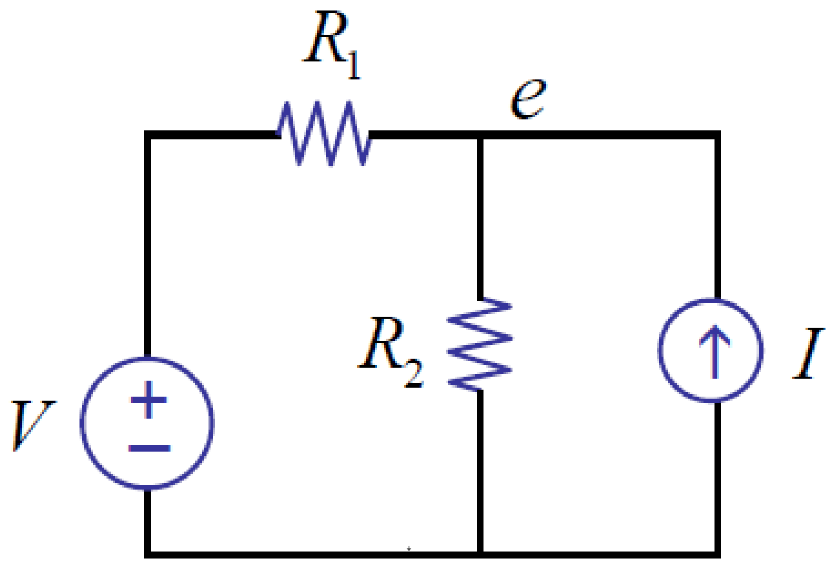


$$i = \frac{1}{R_1 + R_2} V$$

$$v_2 = \frac{R_2}{R_1 + R_2} V$$

- Linear in I
- No terms of the form I^2 or higher powers

Linearity



$$\frac{e-V}{R_1}+\frac{e}{R_2}-I=0$$

- Linear in I and V
- No terms of the form, I^2 I/V or I*V

Linearity

$$\frac{e-V}{R_1}+\frac{e}{R_2}-I=0$$

- Rearranging:

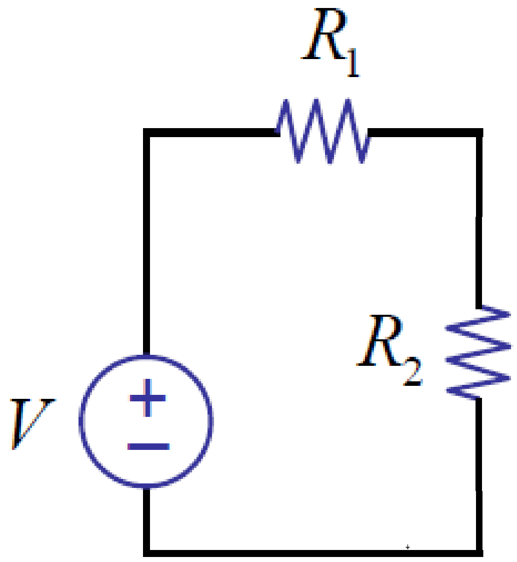
$$e=\frac{R_2}{R_1+R_2}V+\frac{R_1R_2}{R_1+R_2}I$$

- In general, for such circuits we have that

$$\boxed{e=a_1V_1+a_2V_2+\ldots+b_1I_1+b_2I_2+\ldots}$$

Linear!

Additivity



Suppose

$$v_2 = a \text{ Volt for } V = 3 \text{ Volt}$$

$$v_2 = b \text{ Volt for } V = 1 \text{ Volt}$$

$$v_2 = \frac{R_2}{R_1 + R_2} 3 = a$$

$$v_2 = \frac{R_2}{R_1 + R_2} 1 = b$$

Then for

$$V = (3 + 1) \text{ Volt}$$

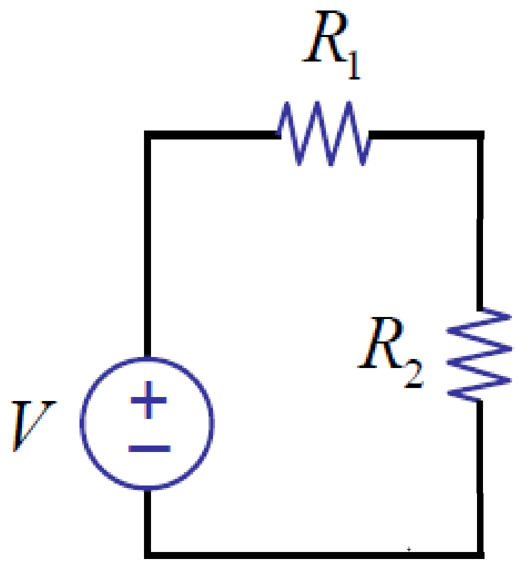
$$v_2 = \frac{R_2}{R_1 + R_2} (3 + 1)$$

$$= (\mathrm{a} + \mathrm{b}) \text{ Volt}$$

$$i = \frac{1}{R_1 + R_2} V$$

$$v_2 = \frac{R_2}{R_1 + R_2} V$$

Homogeneity

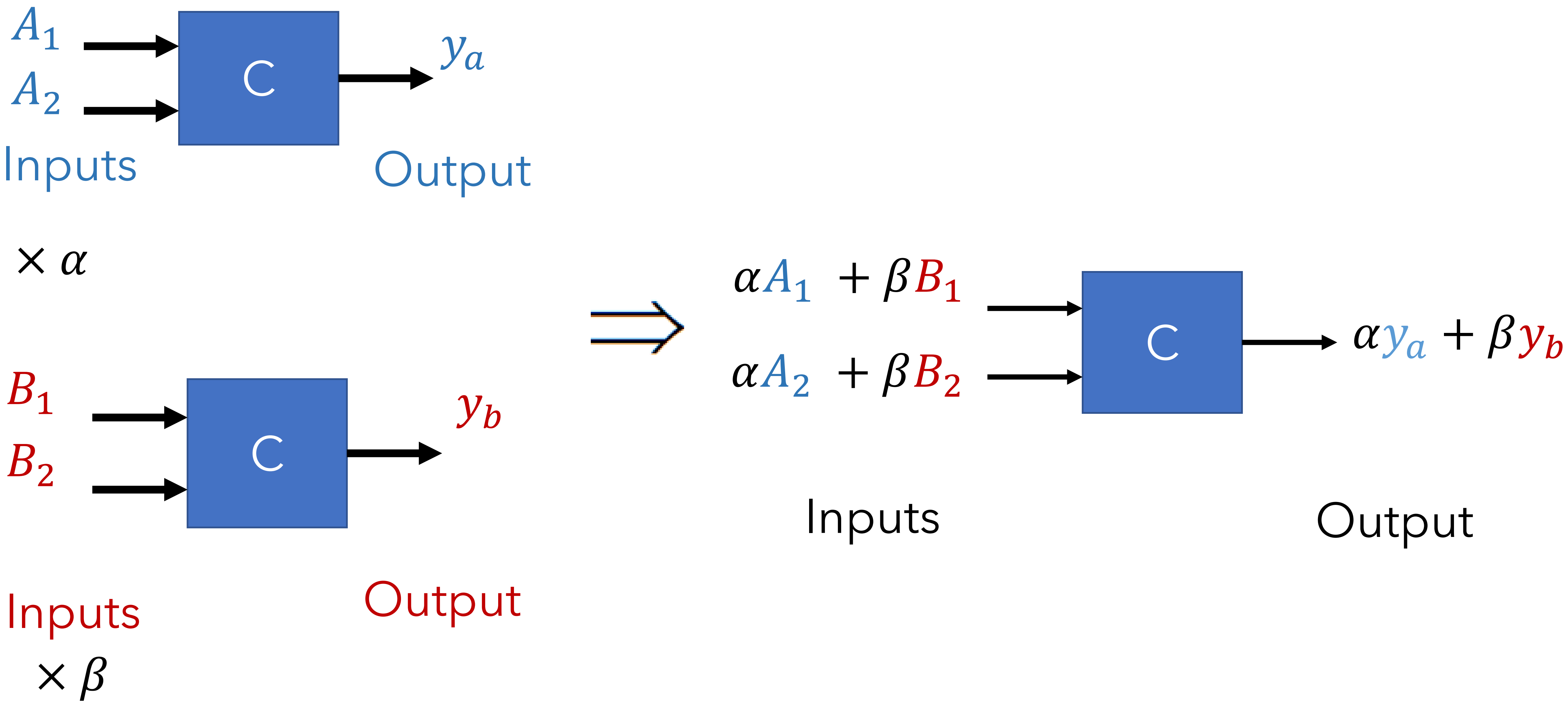


$$i = \frac{1}{R_1 + R_2} V$$

$$v_2 = \frac{R_2}{R_1 + R_2} V$$

- Double the voltage

Linear Systems: Superposition Principle



Example: Our favorite element 'Resistor'

$$V = IR$$

Linear Element: Current as input, Voltage as output.

Increasing the current by a constant *k*, the new voltage across is

$$kIR = kV$$

Homogeneity

Response to two excitations:

$$V_1 = I_1 R \qquad V_2 = I_2 R$$

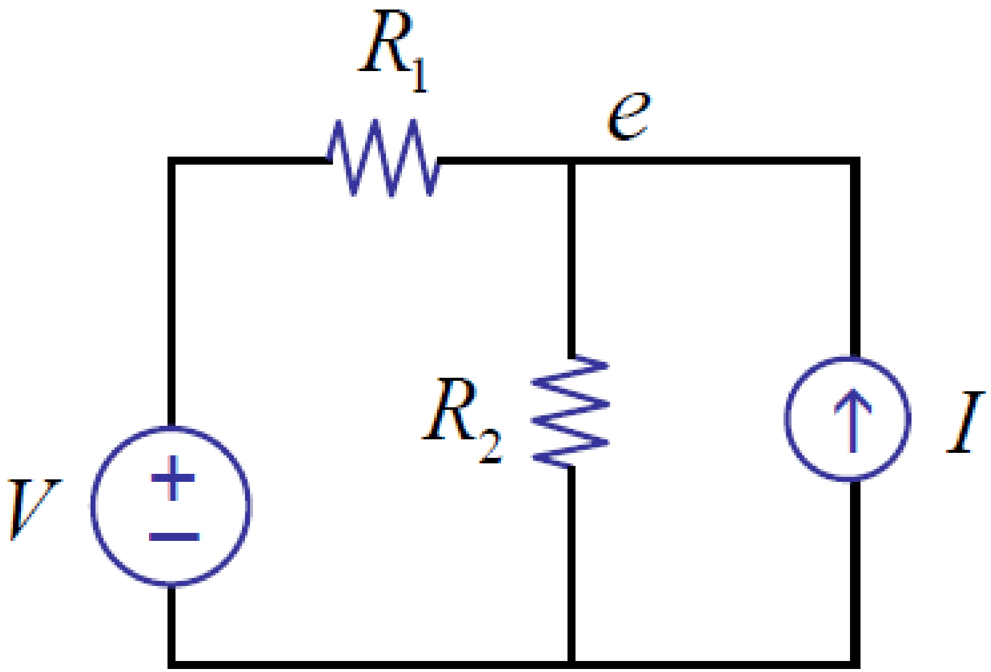
$$V = (I_1 + I_2)R = I_1 R + I_2 R = V_1 + V_2$$

Additivity

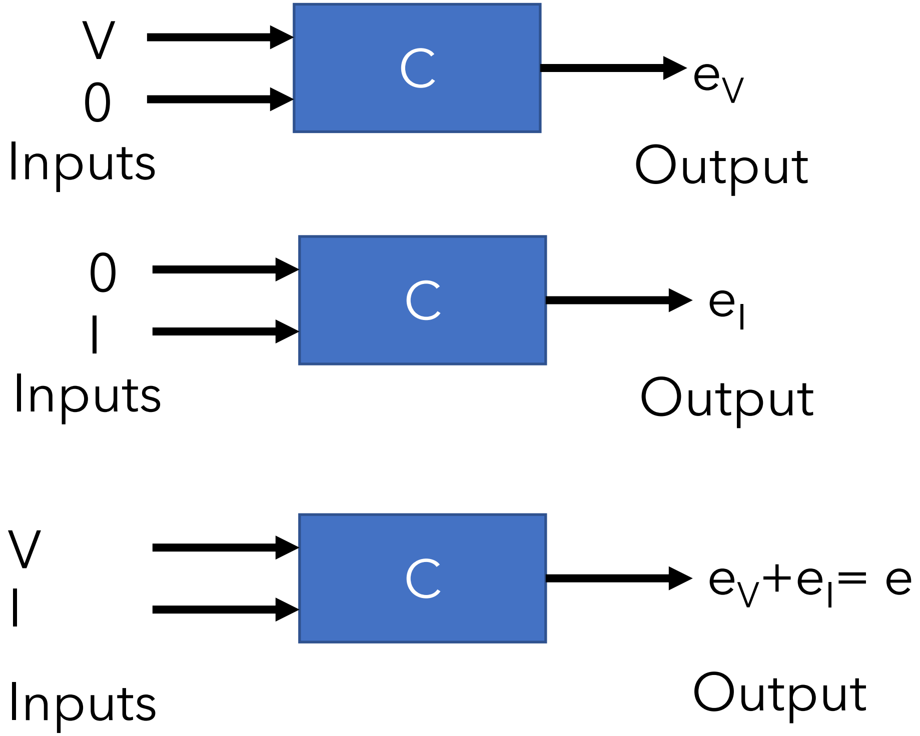
Superposition

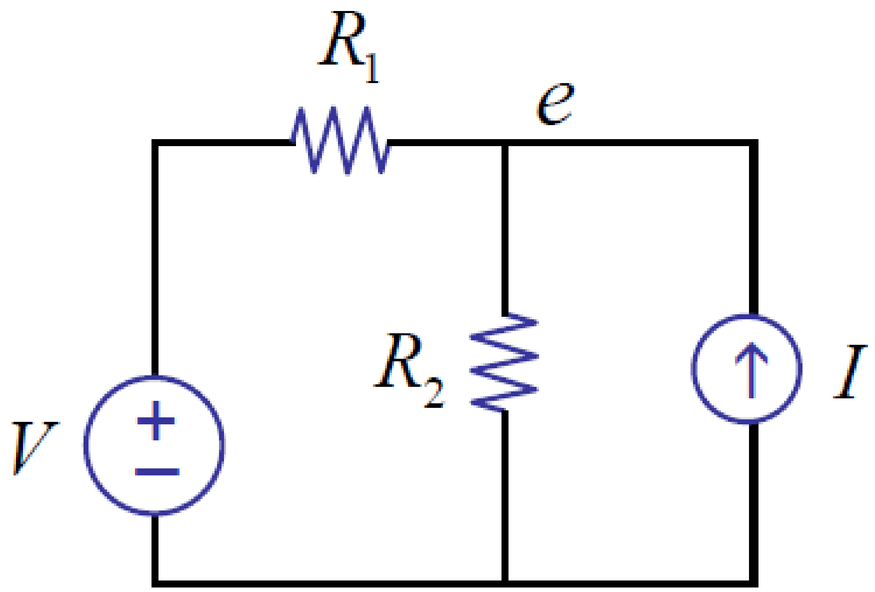
Superposition Principle

The superposition principle states that
The total response is the sum of the responses to each of the independent sources acting individually.



Superposition Method

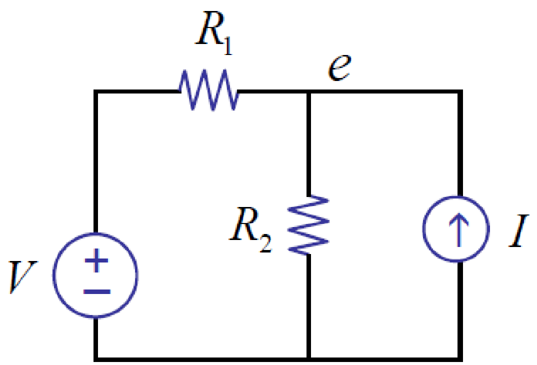




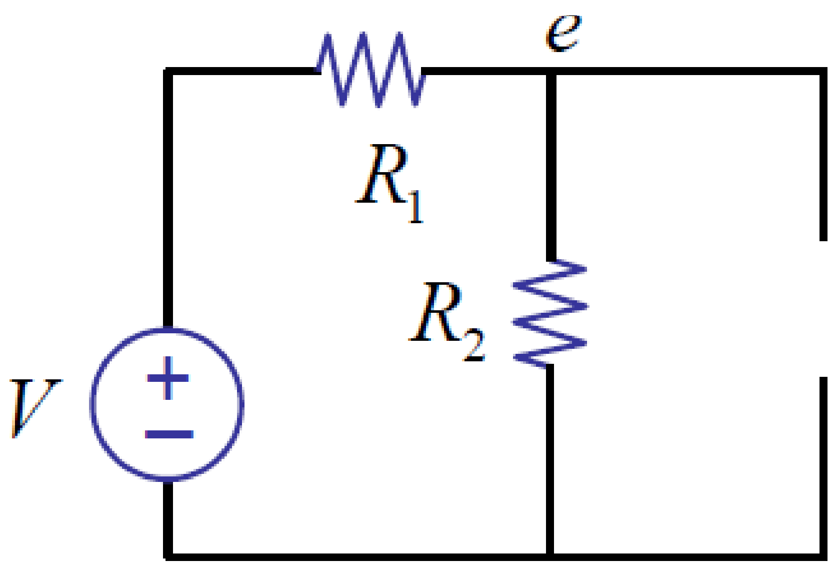
1. Find circuit response to each source acting alone

2. Sum up the individual/partial responses to get the total response

Example

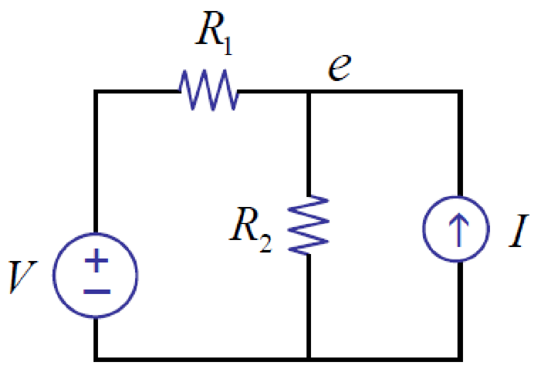


Response to voltage source only

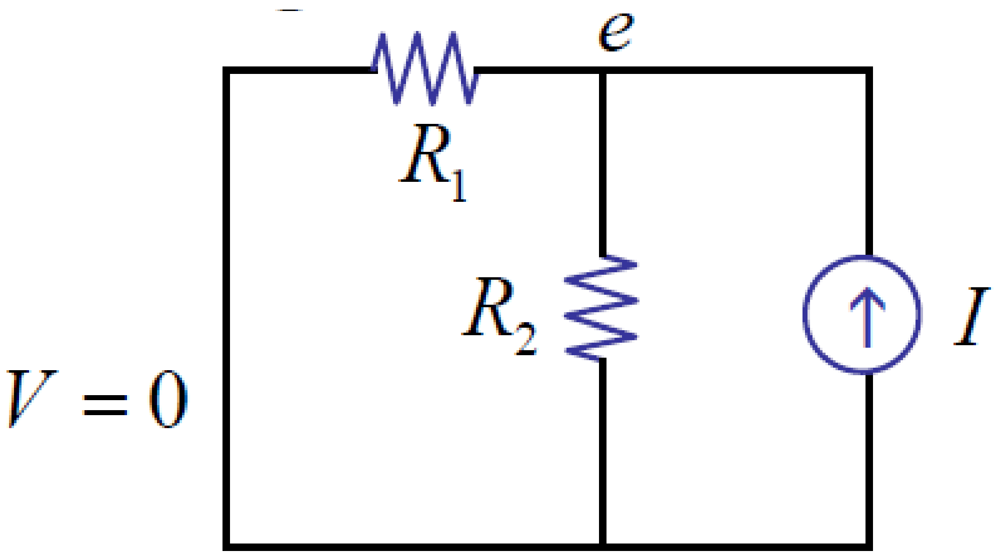


$I = 0 \quad e_V = \dfrac{R_2}{R_1 + R_2} V$

Example

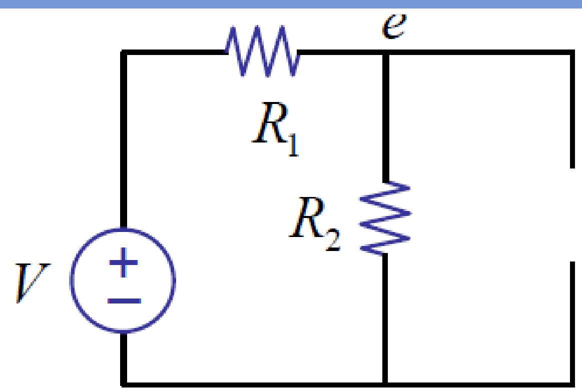


Response to current source only

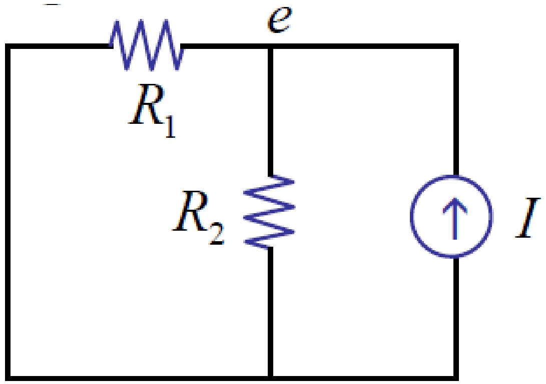


$$e_I = \frac{R_1 R_2}{R_1 + R_2} I$$

Example



$I = 0 \quad e_V = \frac{R_2}{R_1 + R_2} V$

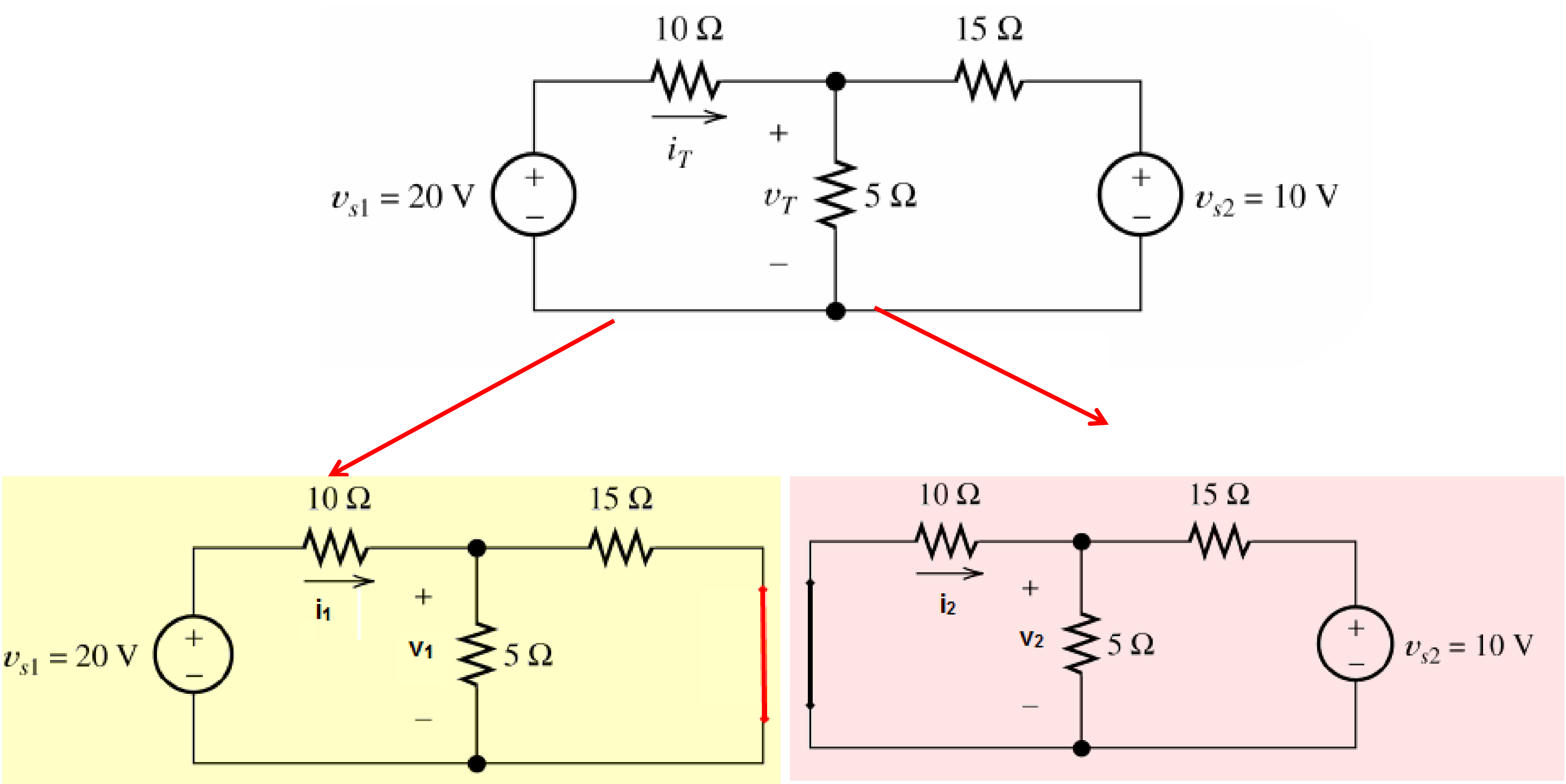


$V = 0 \qquad e_I = \frac{R_1 R_2}{R_1 + R_2} I$

Superposition

$$e = e_V + e_I = \frac{R_2}{R_1 + R_2} V + \frac{R_1 R_2}{R_1 + R_2} I$$

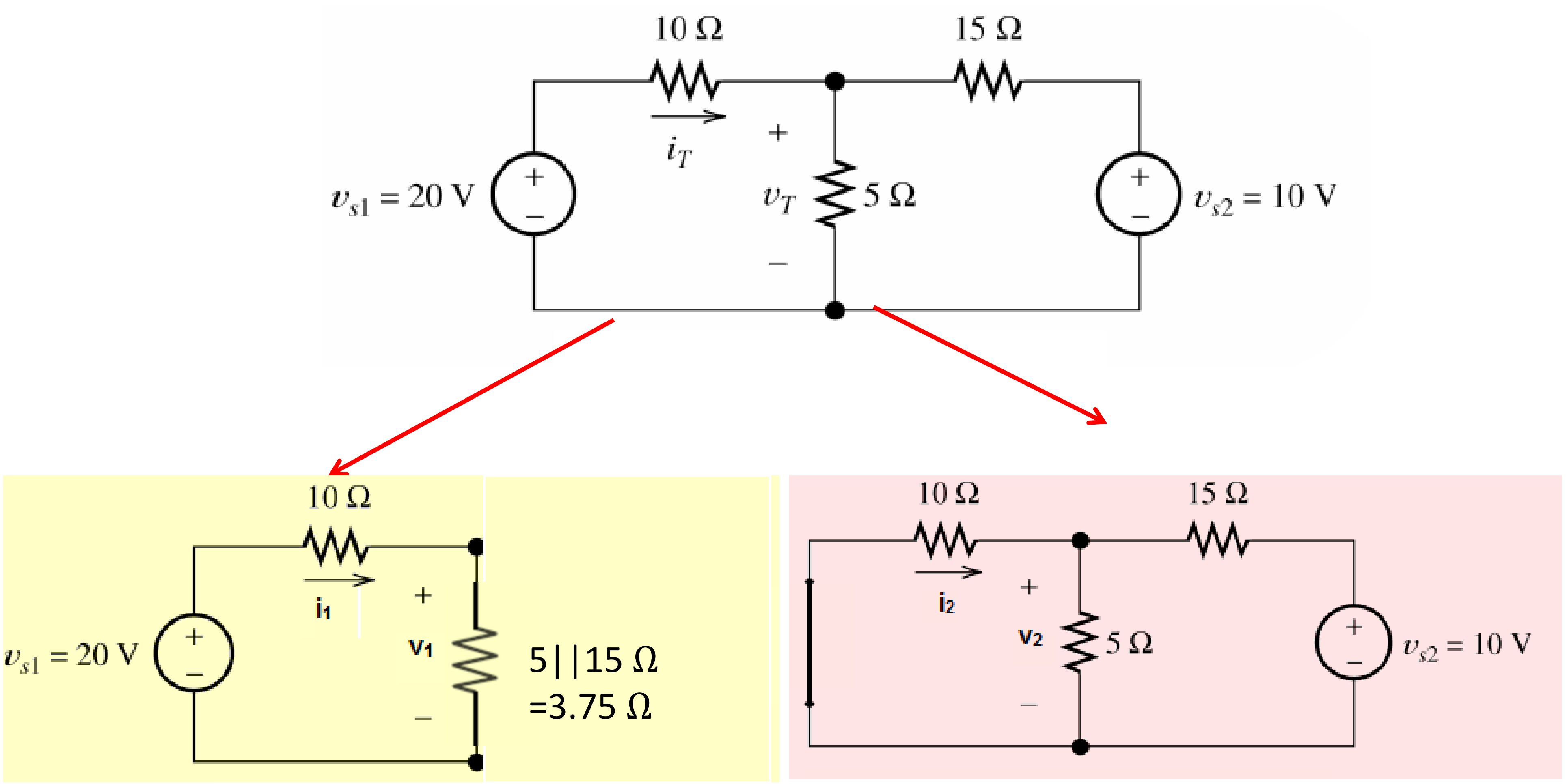
Example 2



$$i_T = i_1 + i_2$$

$$v_T = v_1 + v_2$$

Example 2

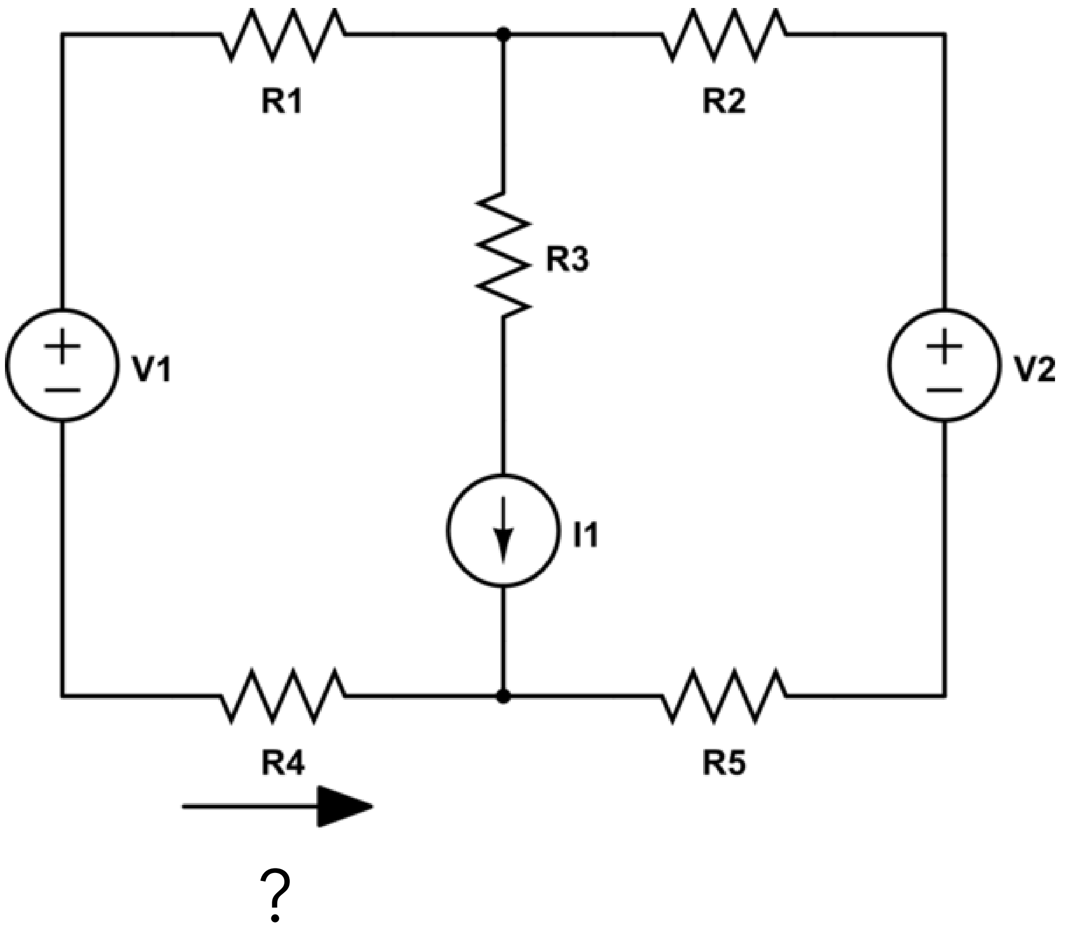


$$i_T = i_1 + i_2$$

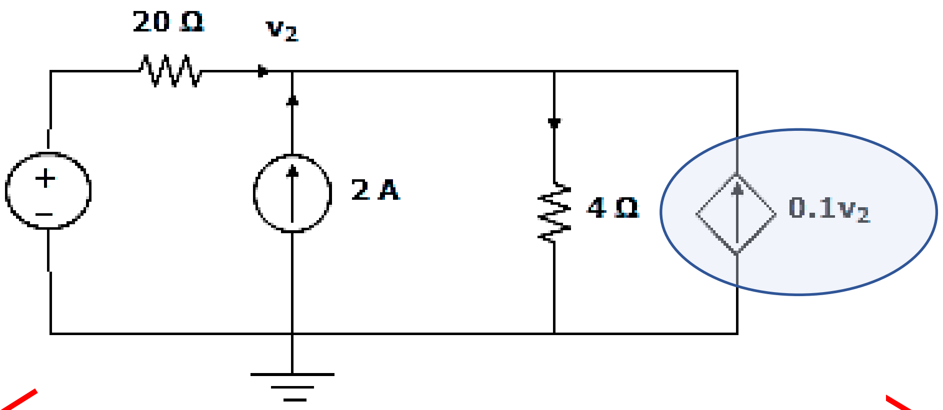
$$v_T = v_1 + v_2$$

Example 3

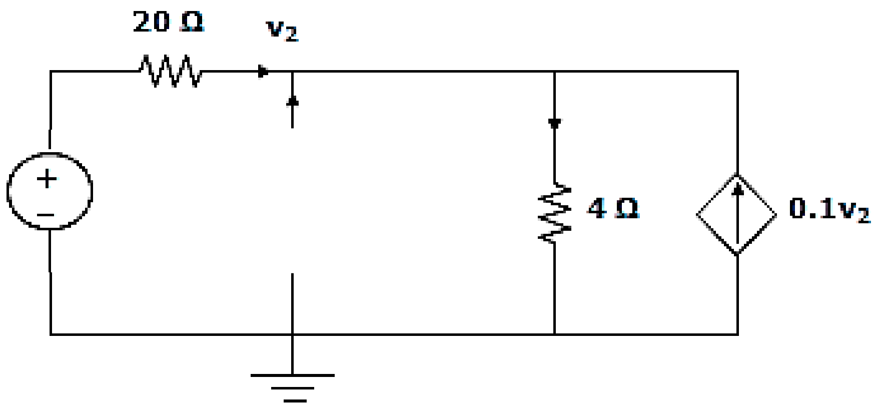
Source	Response
V_1	$-\dfrac{V_1}{R_1+R_2+R_5+R_4}$
I_1	$\dfrac{V_2}{R_2+R_1+R_4+R_5}$
V_2	$-I_1\dfrac{R_2+R_5}{R_1+R_4+R_2+R_5}$

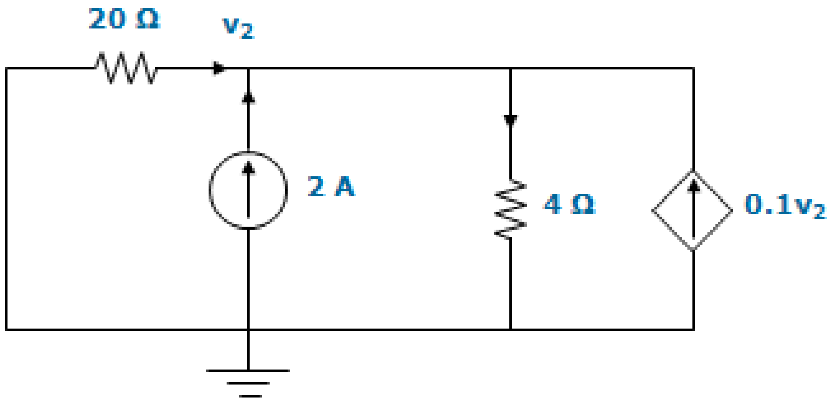


Example 4: Dependent sources



Dependent sources remain unchanged

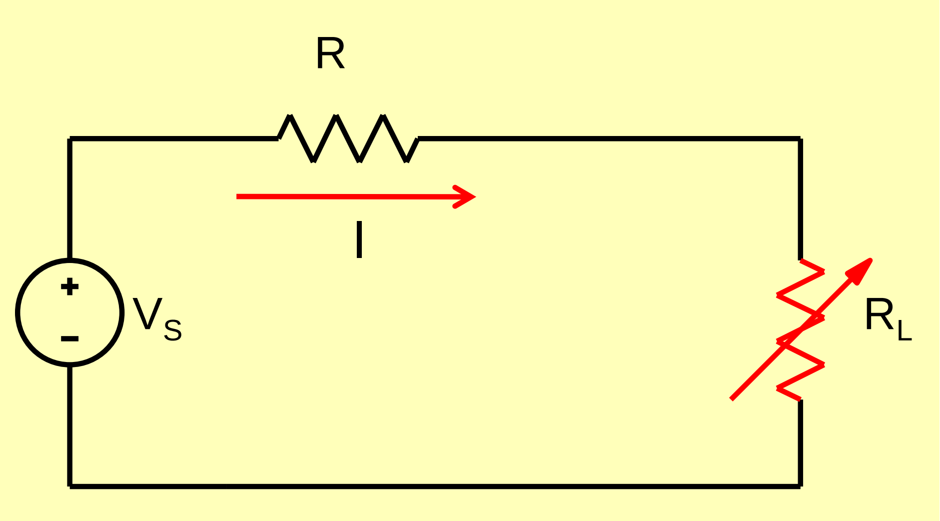




$$\frac{v_1 - 10}{20} + \frac{v_1}{4} - 0.1v_1 = 0$$

$$\frac{v_2}{20} - 2 + \frac{v_2}{4} - 0.1v_2 = 0$$

Max. Power Transfer for DC Circuits



What value of R_L will give rise to maximum load power ?

$$I = \frac{V_S}{R + R_L}$$

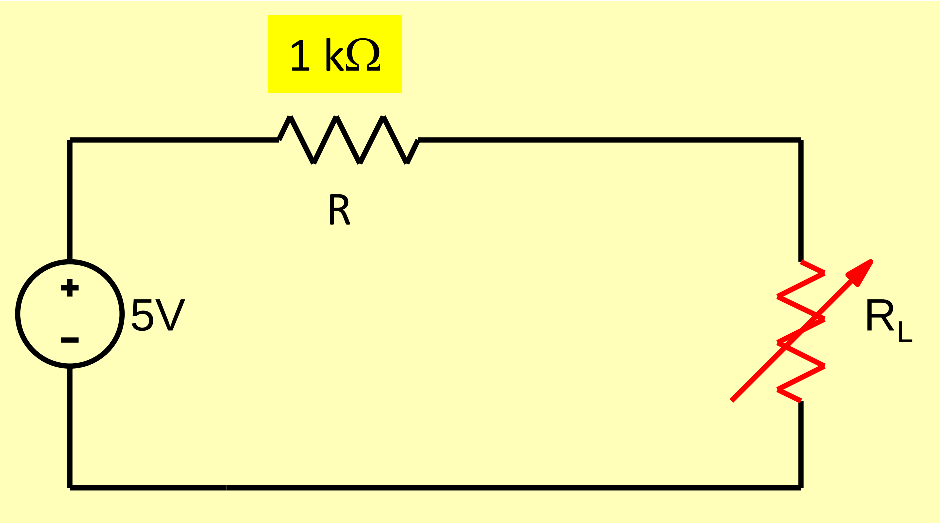
$$P_L = I^2 R_L = V_S^2 \times \frac{R_L}{(R + R_L)^2}$$

$$\frac{\partial P_L}{\partial R_L} = 0$$

$$R_L = R$$

$$P_{L\max} = \frac{V_S^2}{4R_L}$$

Example



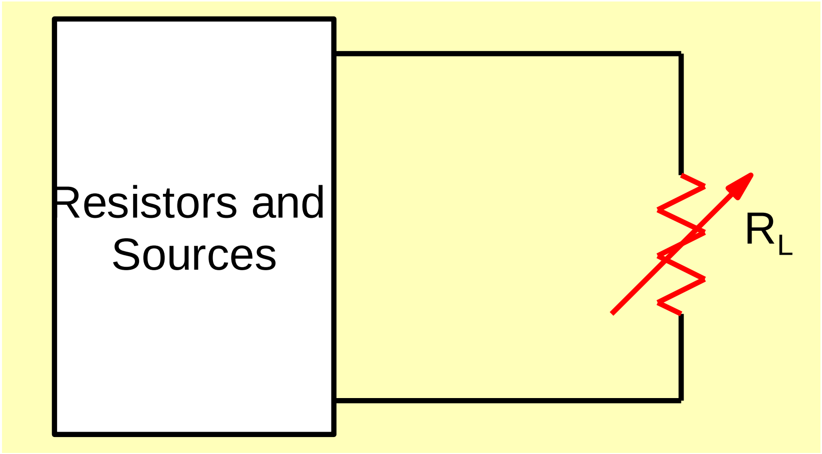
R_L = 1 kΩ → P_L = 6.25 mW

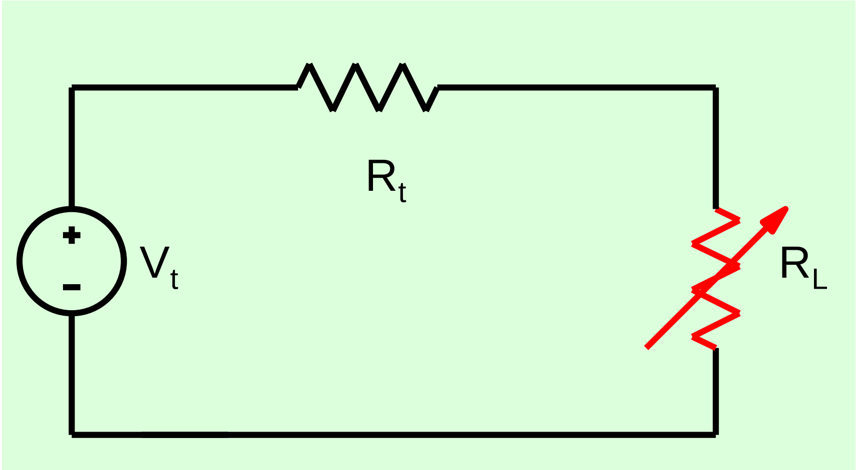
R_L = 10 kΩ → P_L = 2 mW

R_L = 0.2 kΩ → P_L = 3.47 mW

Maximum power is delivered to the load when **$R_L = R$**

General Case





Maximum power is delivered to the load when $\mathbf{R_L = R_t}$