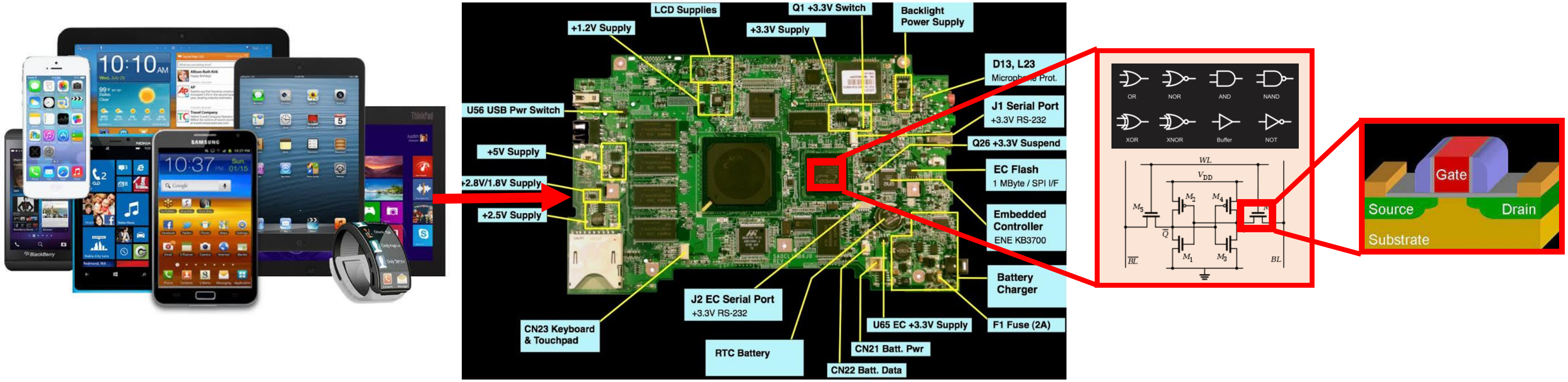

ESC201: Introduction to Electronics

Module 3: Frequency Domain Analysis

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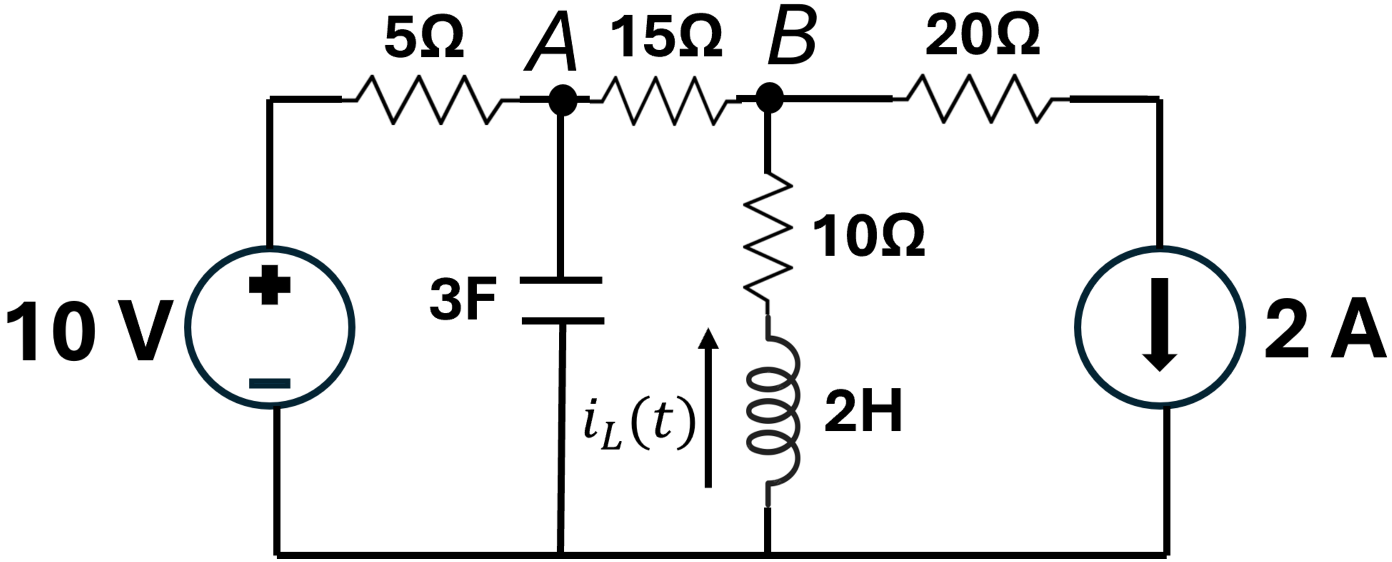
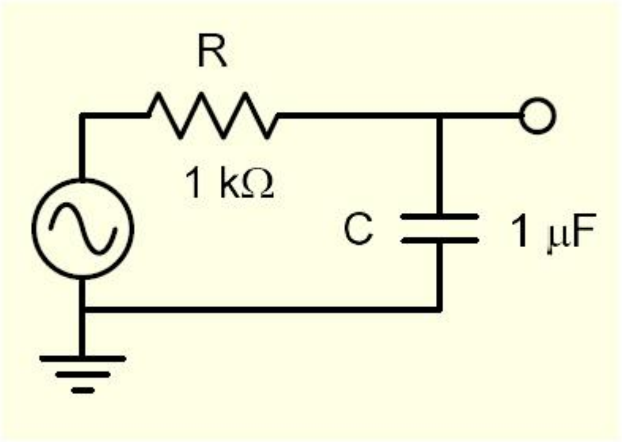
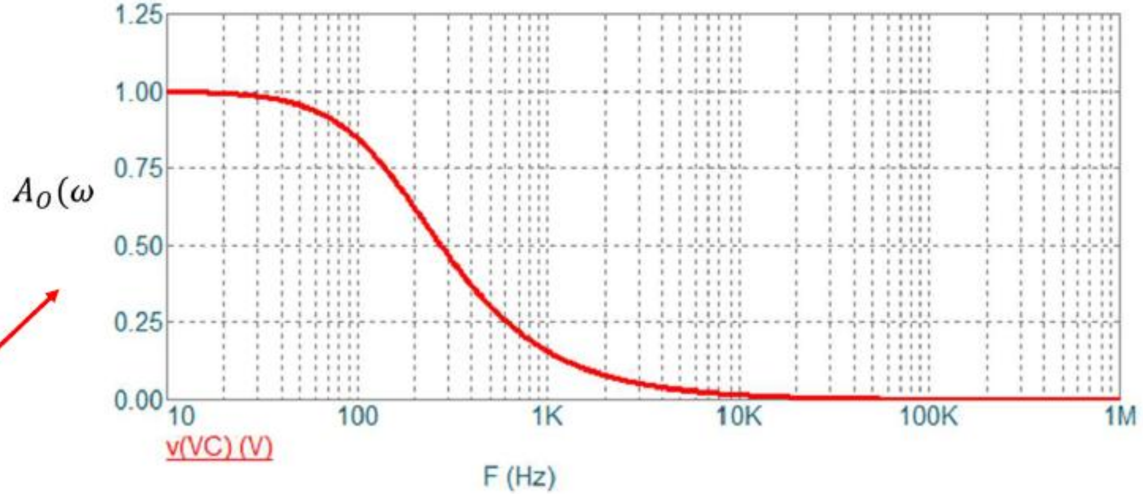
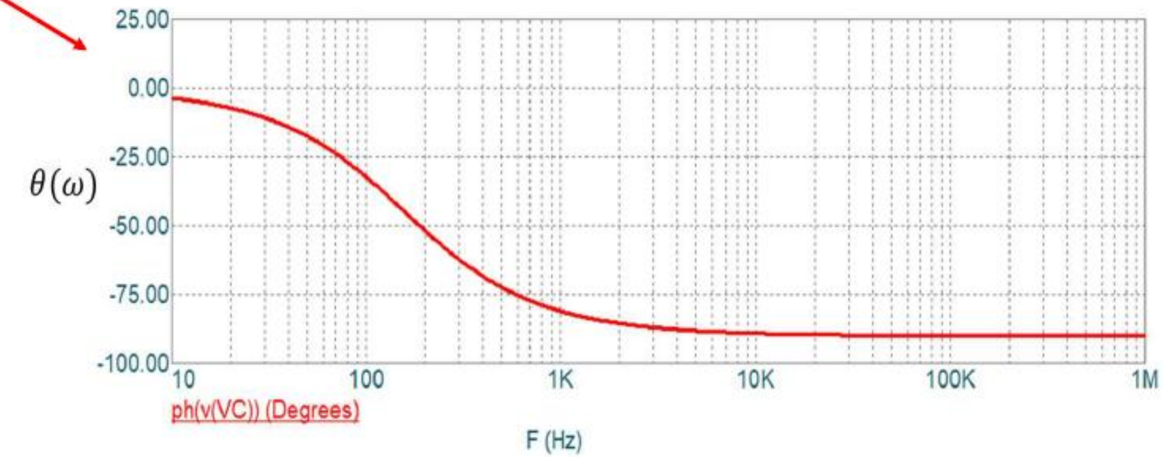
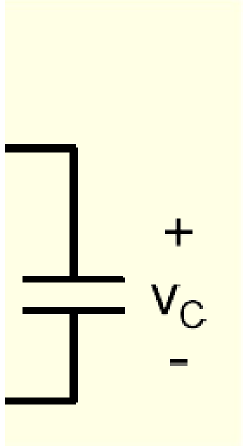


Why Sinusoidal Steady-state Response



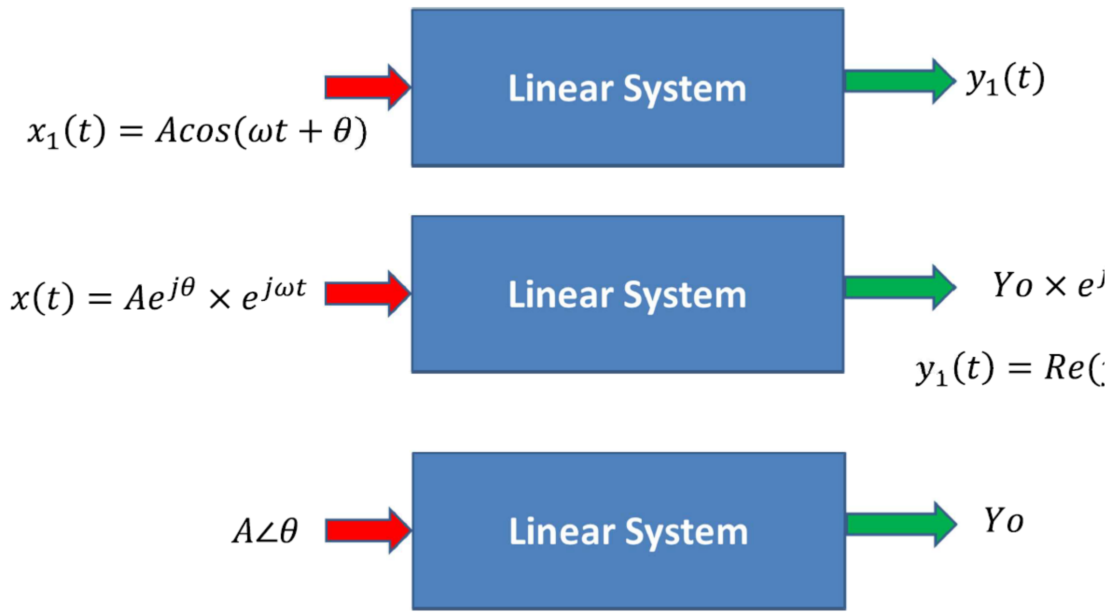


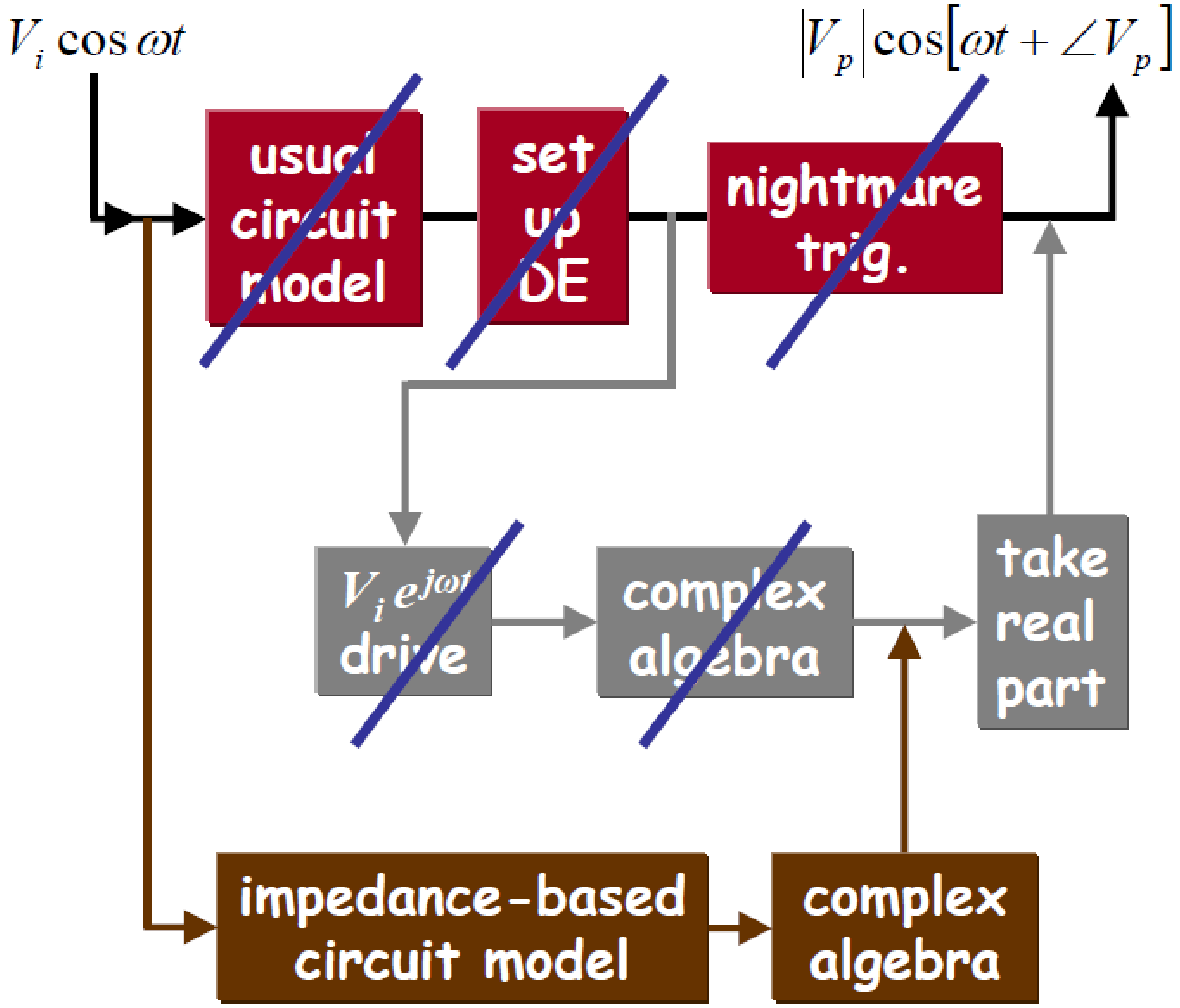




system

Domain Transformation and Solution



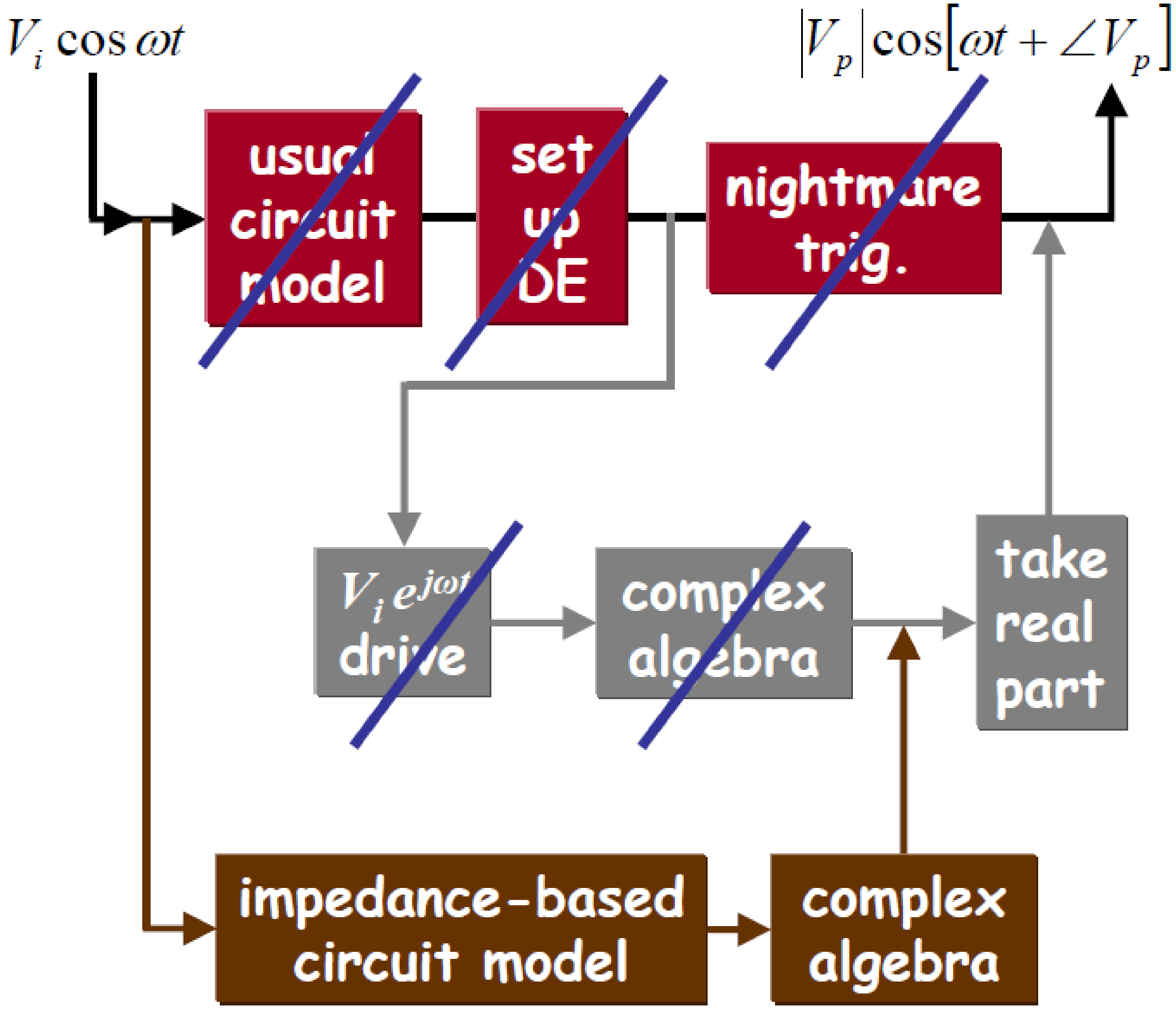


Current through the inductor lags the voltage b

$$V_L = j\omega L \times I_L$$

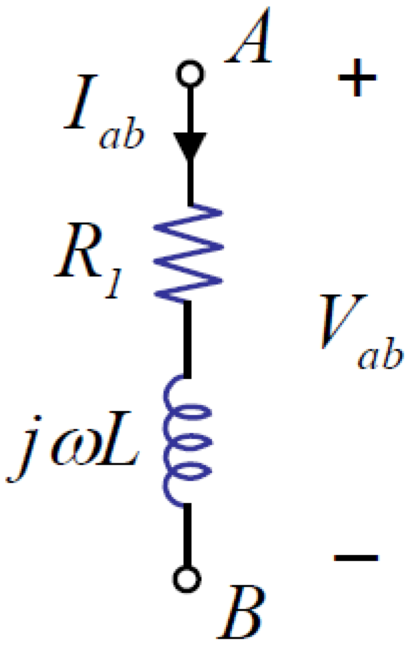
In a capacitor, current leads voltage by 9(

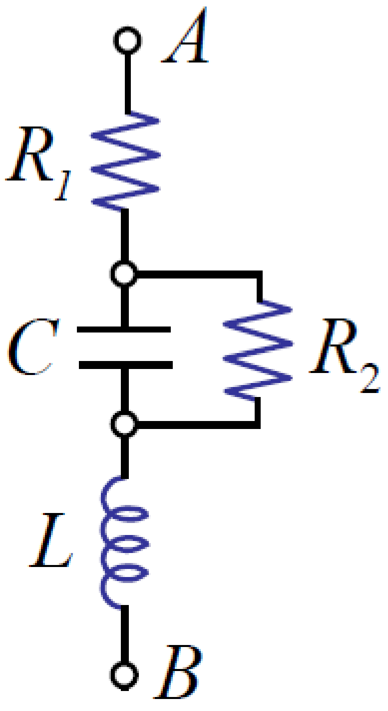
Big Picture



$V_i \cos \omega t$
$|V_p| \cos[\omega t + \angle V_p]$
usual circuit model
set up DE
nightmare trig.
$V_i e^{j\omega t}$ drive
complex algebra
take real part
impedance-based circuit model
complex algebra

Series-parallel operations



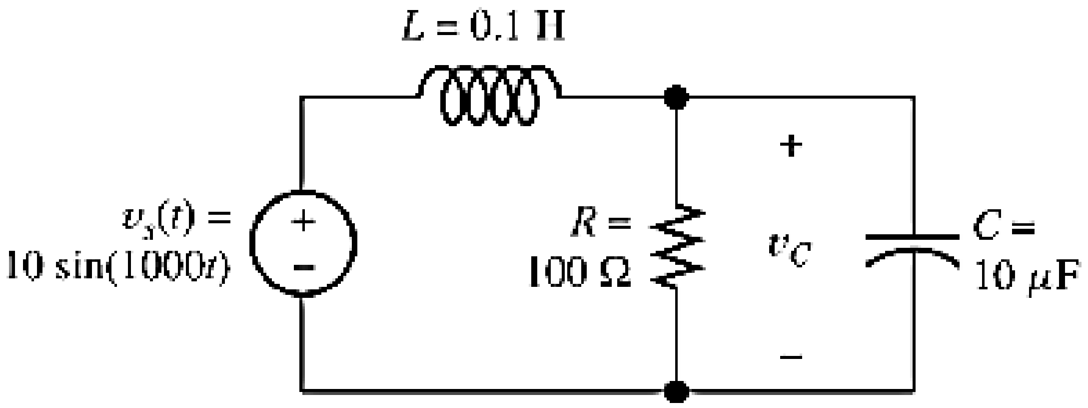


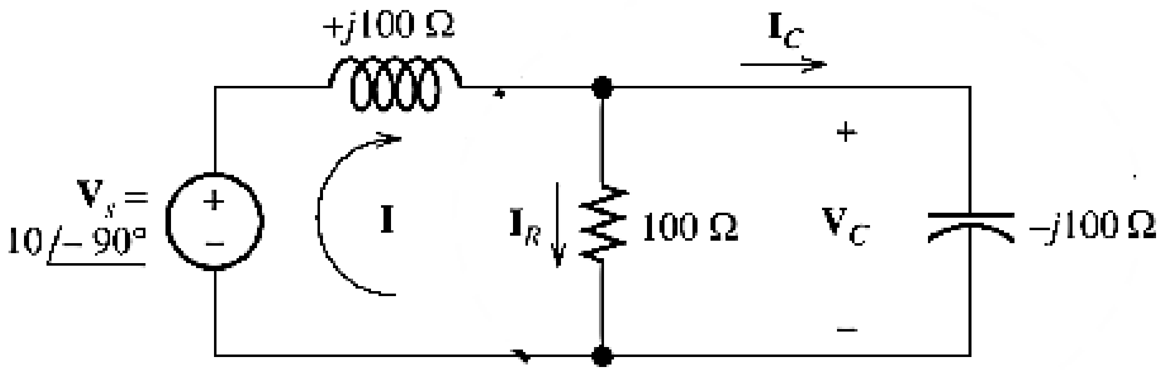
$$Z_{AB} = \frac{V_{ab}}{I_{ab}} = R_1 + j\omega L$$

$$\begin{aligned} Z_{AB} &= R_1 + Z_C \,||\, R_2 + Z_L \\ &= R_1 + \frac{Z_C R_2}{Z_C + R_2} + Z_L \\ &= R_1 + \frac{R_2}{1 + j\omega C R_2} + j\omega L \end{aligned}$$

Example

Find the voltage across capacitor in steady state





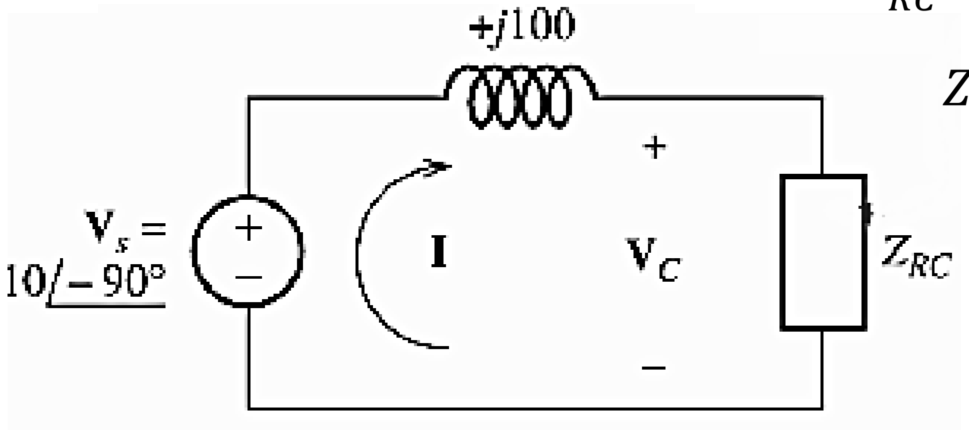
$$V_c = V_s \frac{Z_{RC}}{100j + Z_{RC}}$$

$$V_c = 10\angle -90 \ \frac{50-50j}{100j+50-50j} = -10\text{V}$$

$$v_c(t) = -10\cos(1000t)$$

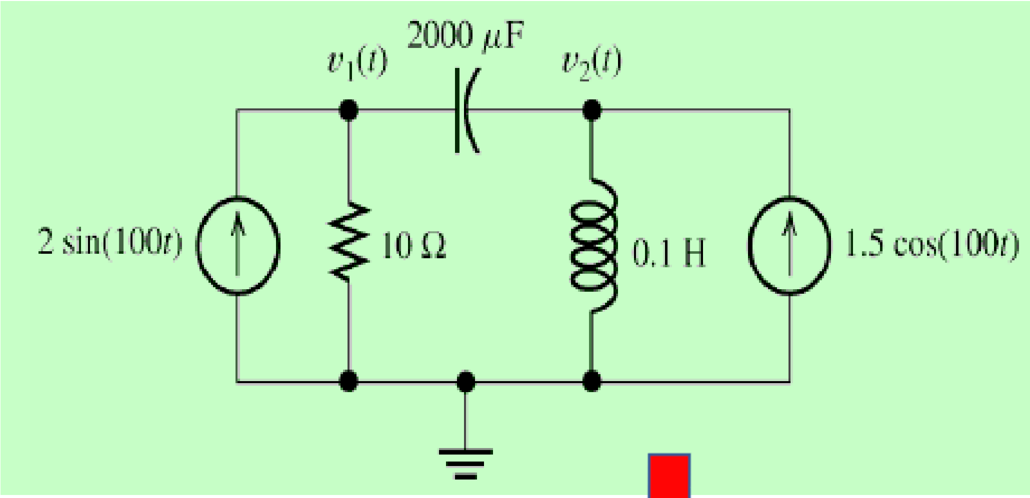
$$\frac{1}{Z_{RC}} = \frac{1}{100} + \frac{1}{-j100}$$

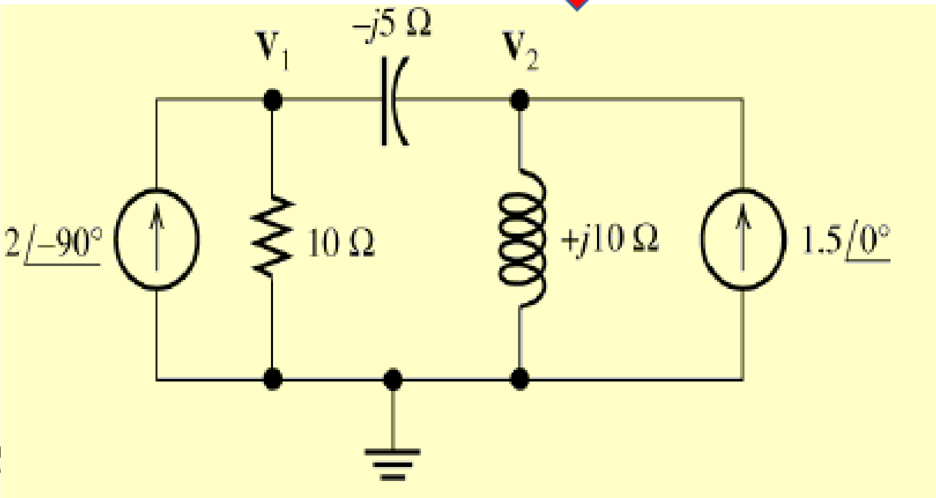
$$Z_{RC} = 50 - 50j$$



Example 2

Use nodal analysis to find $v_1(t)$ in steady state





$$\frac{\mathbf{V_1}}{10}+\frac{\mathbf{V_1}-\mathbf{V_2}}{-j5}=2\angle -90°$$

$$(0.1+j0.2)\mathbf{V_1}-j0.2\mathbf{V_2}=-j2$$

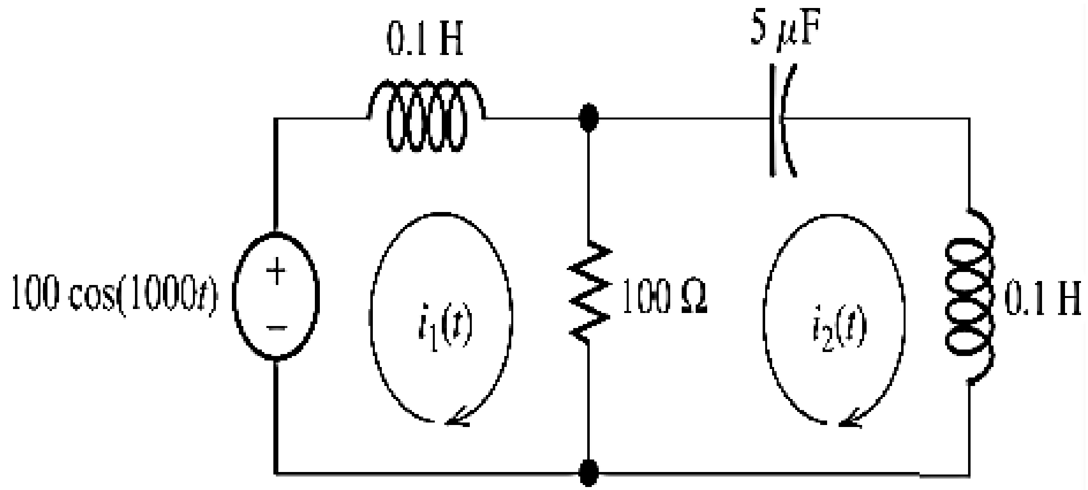
$$\frac{\mathbf{V_2}}{j10}+\frac{\mathbf{V_2}-\mathbf{V_1}}{-j5}=1.5\angle 0°$$

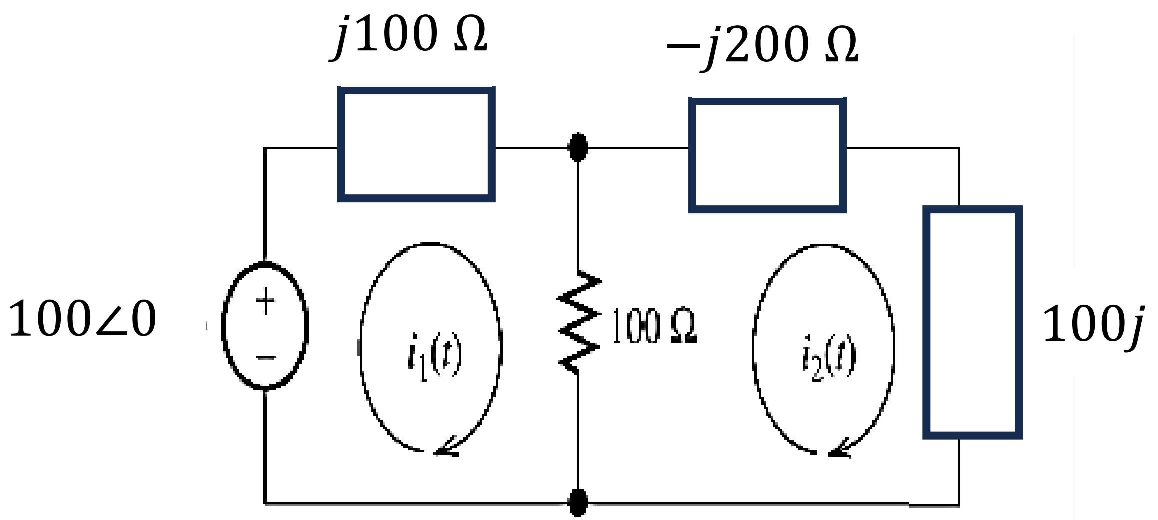
$$-j0.2\mathbf{V_1}+j0.1\mathbf{V_2}=1.5$$

$$V_1=16.1\angle 29.7°$$

$$v_1(t)=16.1\cos(100t+29.7°)$$

Mesh Analysis



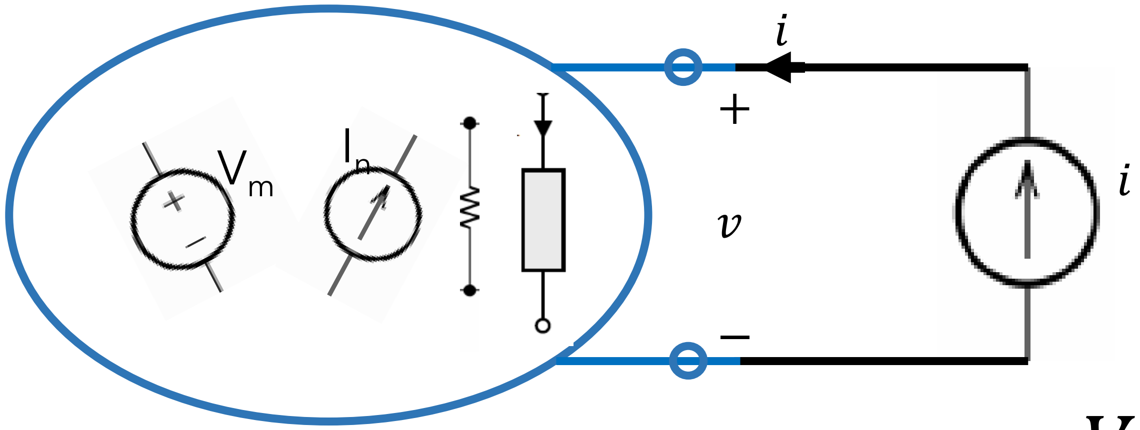


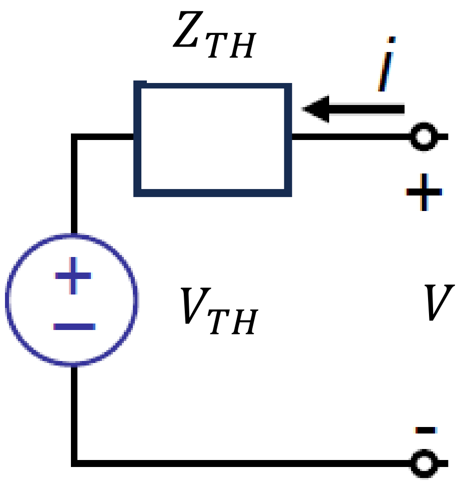
$$j100I_1 + 100(I_1 - I_2) = 100 \qquad - j200I_2 + j100I_2 + 100(I_2 - I_1) = 0$$

$$(100 + j100)I_1 - 100I_2 = 100 \qquad -100I_1 + (100 - j100)I_2 = 0$$

$$I_1 = 1.414\angle -45^\circ \text{ A and } I_2 = 1\angle 0^\circ \text{ A}$$

Thevenin/Nortan Equivalent with Impedances

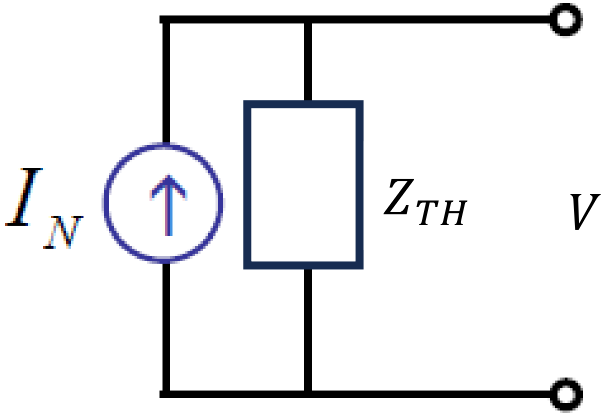




$$V = V_{TH} + Z_{TH} i$$

Can be modeled as a phasor voltage source in series with an impedance!

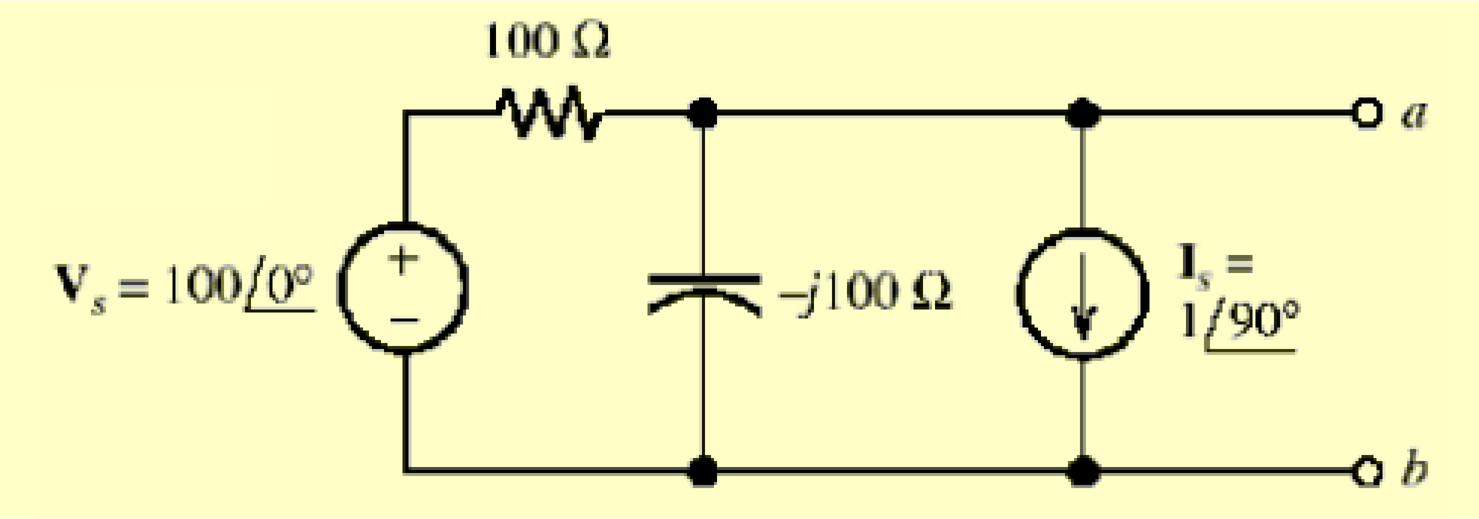
$$I = \frac{V}{Z_{TH}} - I_N$$



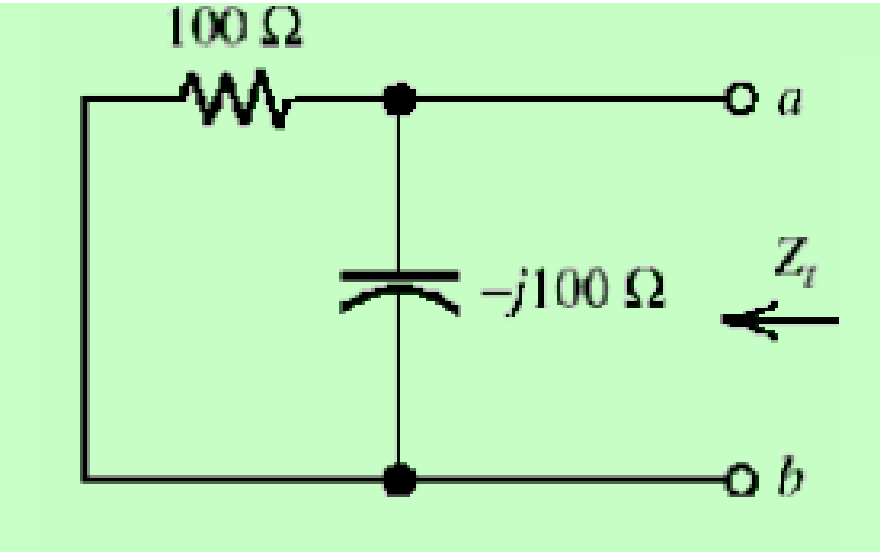
Can be modeled as a phasor current source in parrallel with an impedance!

Example

Find the Thevenin/Nortan Equivalent



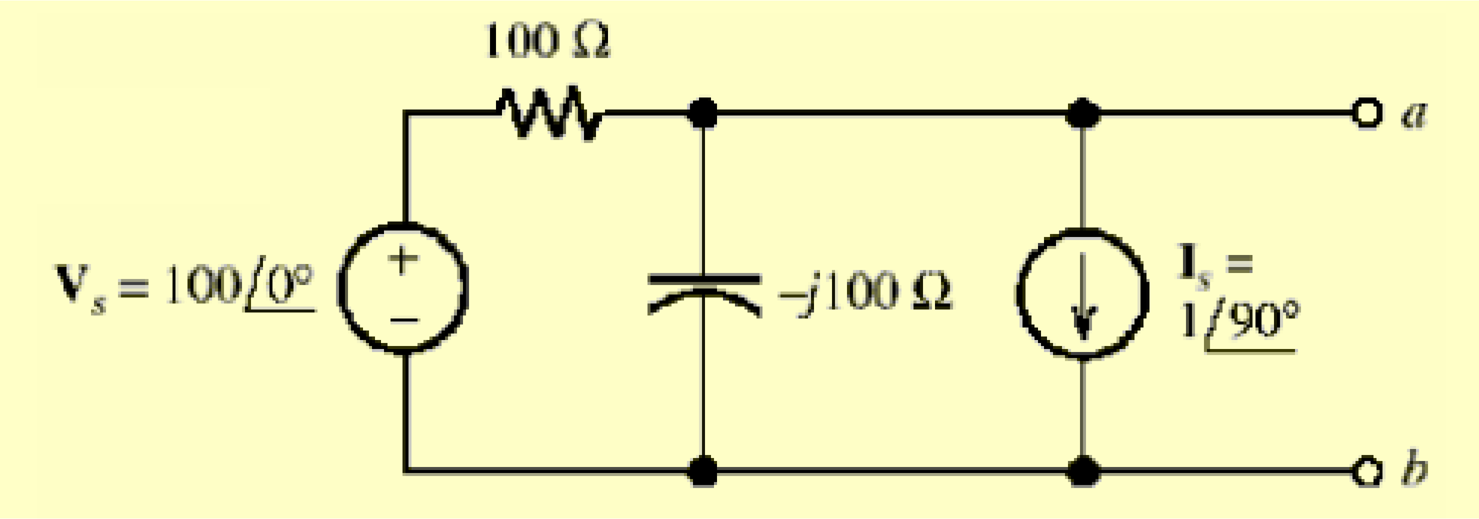
Thevenin Impedance: All sources to be zeroes



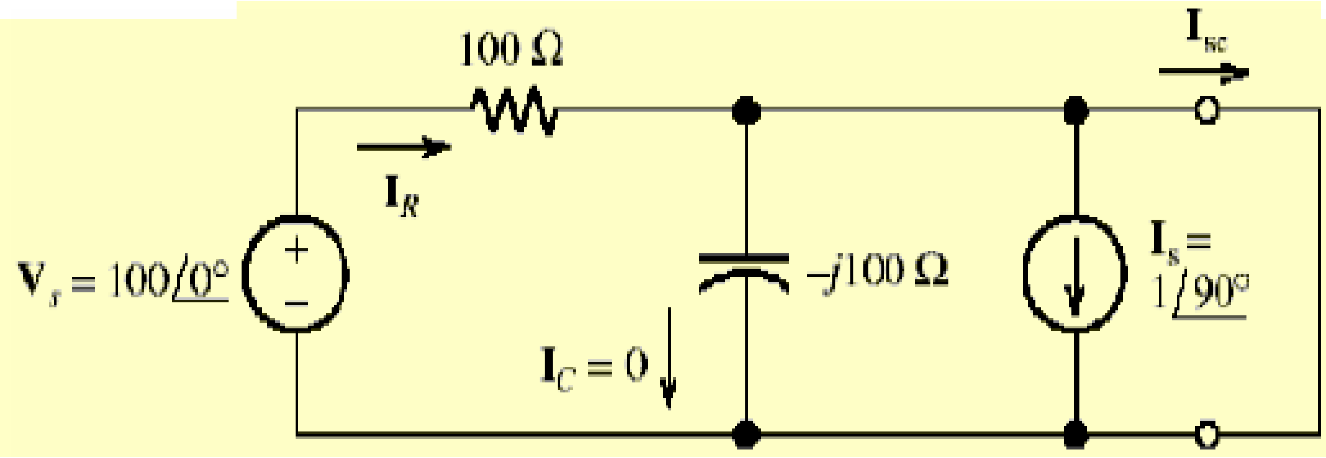
$$\frac{1}{Z_{TH}} = \frac{1}{100} + \frac{1}{-j100}$$

$$Z_{TH} = (50 - 50j)\Omega$$

Example



Nortan Current: Short circuit the terminals



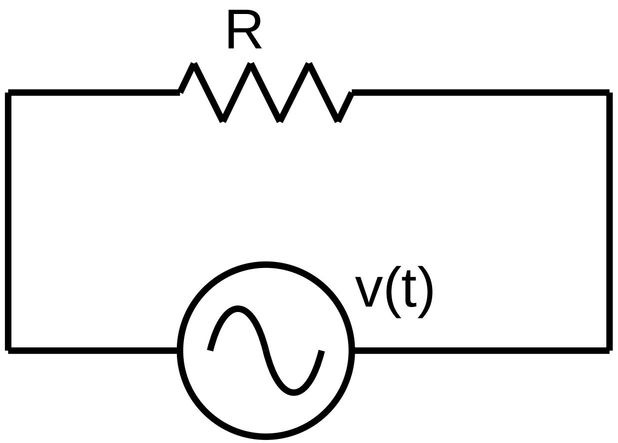
$$I_{sc} = I_R - I_s = \frac{100\angle 0°}{100} - 1\angle 90°$$

$$= 1 - j = 1.414\angle -45°$$

Thevenin Voltage

$$V_{TH} = I_{SC} Z_{TH} = 100\angle -90^o \text{ V}$$

Power dissipation with sinusoidal Voltage

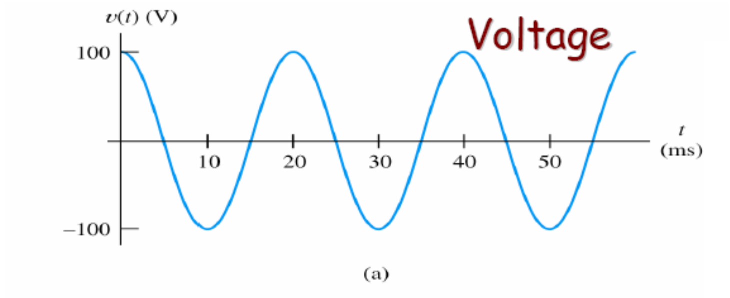


$$p_{avg} = \frac{\frac{1}{T}\int_0^T v(t)^2 dt}{R}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2 dt}$$

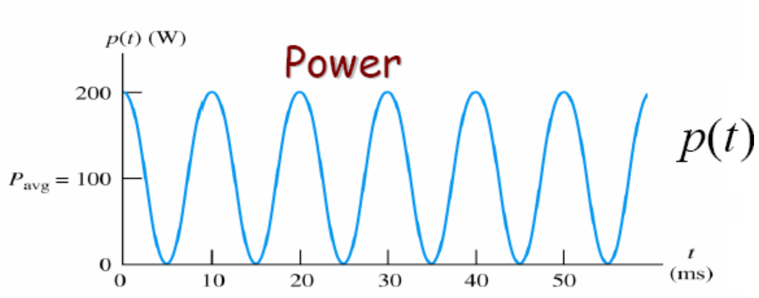
$$p_{avg} = \frac{V_{rms}^2}{R}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$



$$p_{avg} = I_{rms}^2 R$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

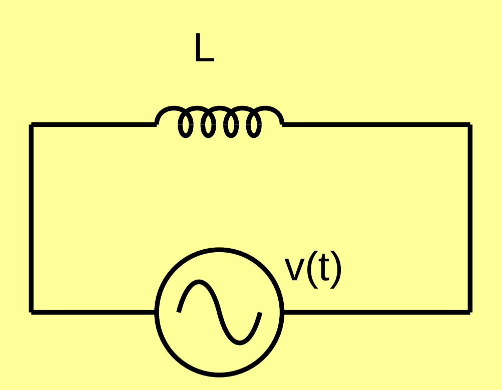


$p(t)$

$$p(t) = v(t)i(t) = \frac{v(t)^2}{R}$$

$$p(t) = 200\cos^2 100\pi t \ \text{W}$$

Average power in Inductor

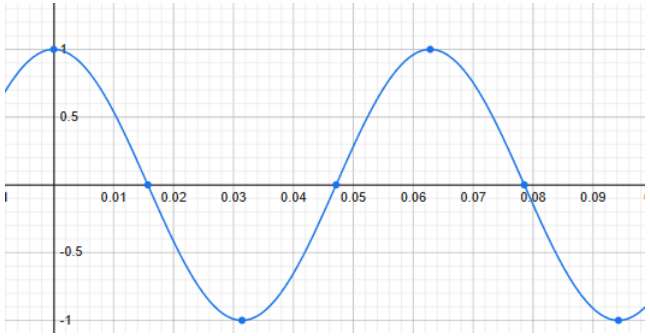


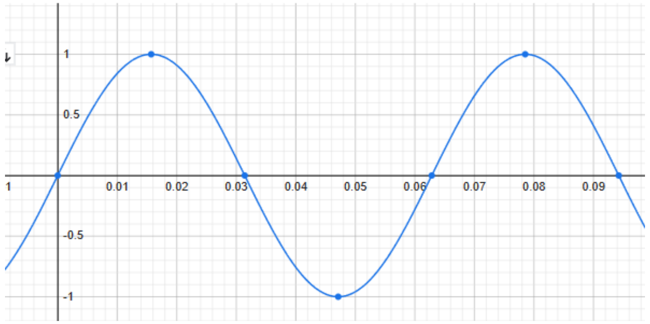
$$v(t) = V_m \cos(\omega t)$$

$$i(t) = I_m \cos(\omega t - 90°) = I_m \sin(\omega t)$$

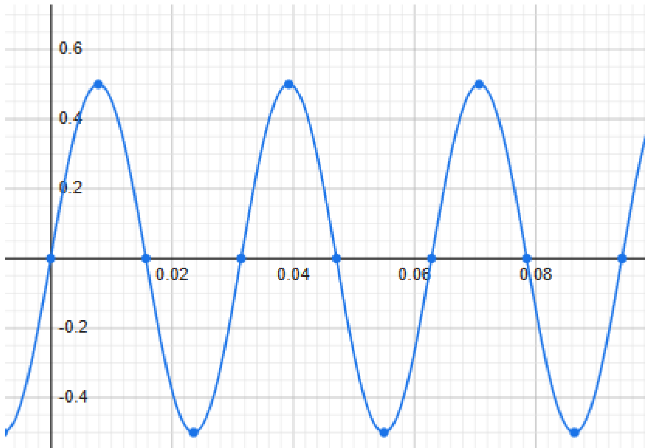
$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t)\sin(\omega t)$$

$$= \frac{V_m I_m}{2}\sin(2\omega t)$$



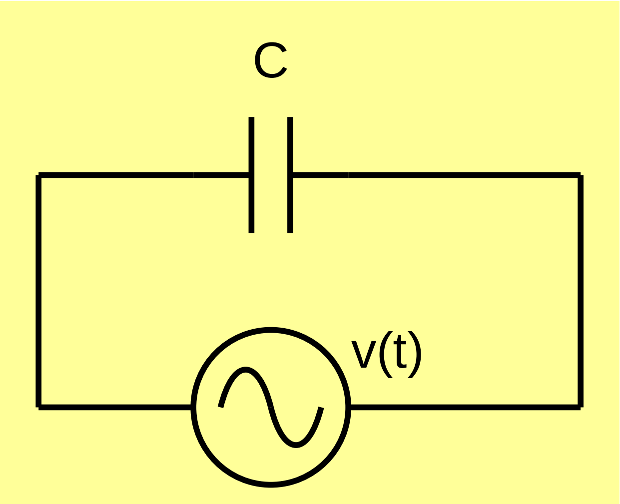


$$\boxed{p_{avg} = 0}$$



Average power absorbed by inductor is zero

Average power in Capacitor



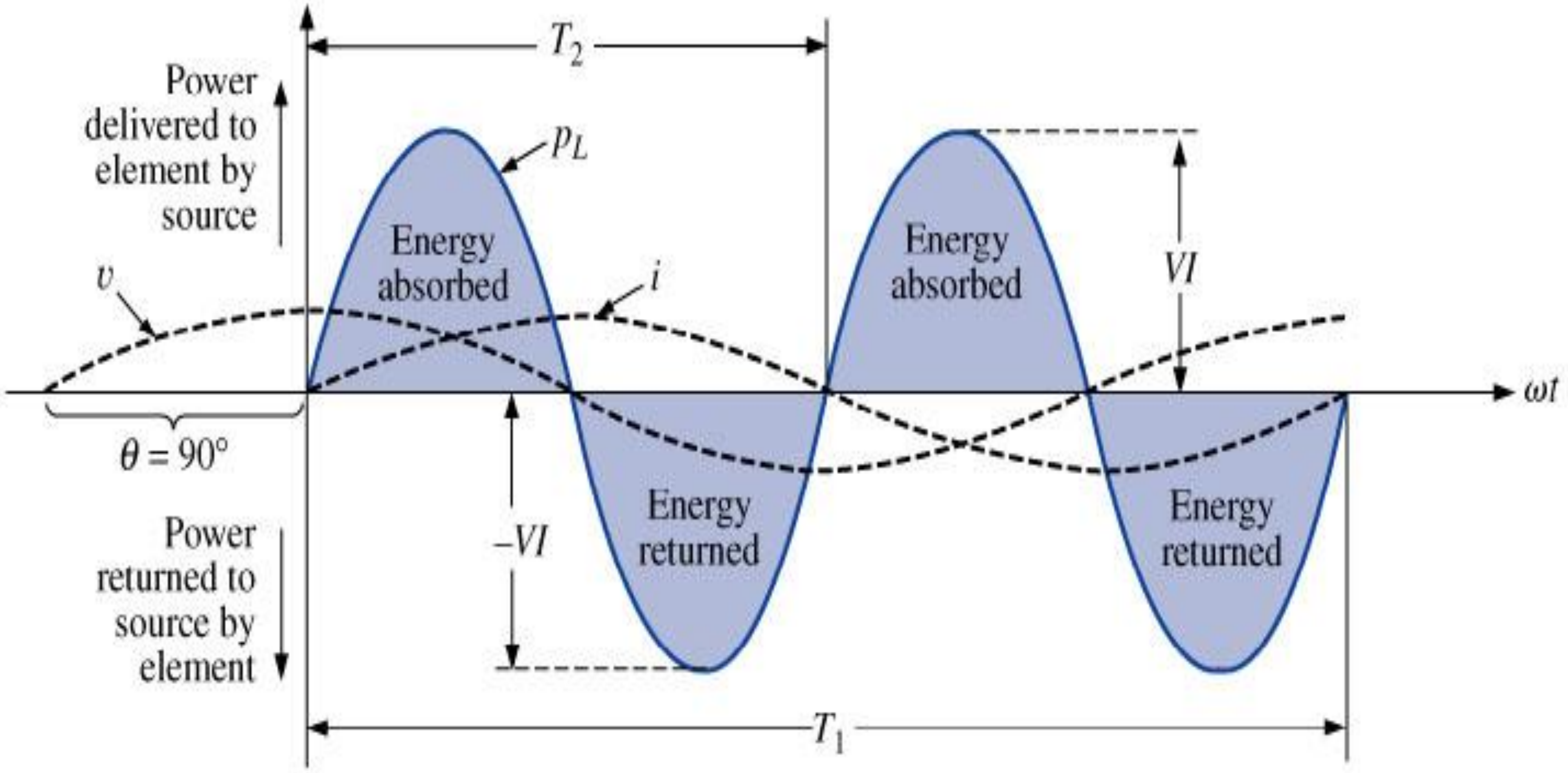
$$p(t) = v(t)i(t) = -V_m I_m \cos(\omega t)\sin(\omega t)$$

$$= -\frac{V_m I_m}{2}\sin(2\omega t)$$

$$p_{avg} = 0$$

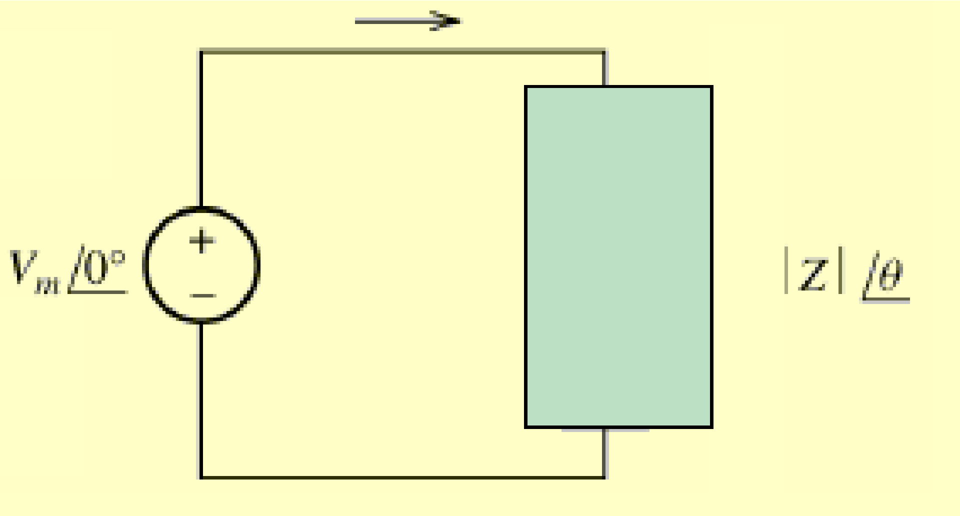
Average power absorbed by capacitor is zero

Inductive and Capacitive loads



Average Power in Circuits with Impedances

$$I = \frac{V}{Z} = \frac{V_m \angle 0_\circ}{|Z| \angle \theta} = I_m \angle -\theta$$



$$v(t) = V_m \cos(\omega t)$$

$$i(t) = I_m \cos(\omega t - \theta)$$

$$\frac{1}{T}\int_0^T \cos\omega t \cos\omega t + \theta \;\; dt = \frac{1}{2}\cos\theta$$

$$p_{avg} = \frac{1}{T}\int_0^T v(t) i(t)\, \mathrm{dt}$$

$$p_{avg} = \frac{1}{T}\int_0^T V_m \cos(\omega t) I_m \cos(\omega t - \theta) \mathrm{dt}$$

$$p_{avg} = V_m I_m \frac{1}{2}\cos\theta \qquad = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\theta$$

$$p_{avg} = V_{rms} I_{rms} \cos\theta$$

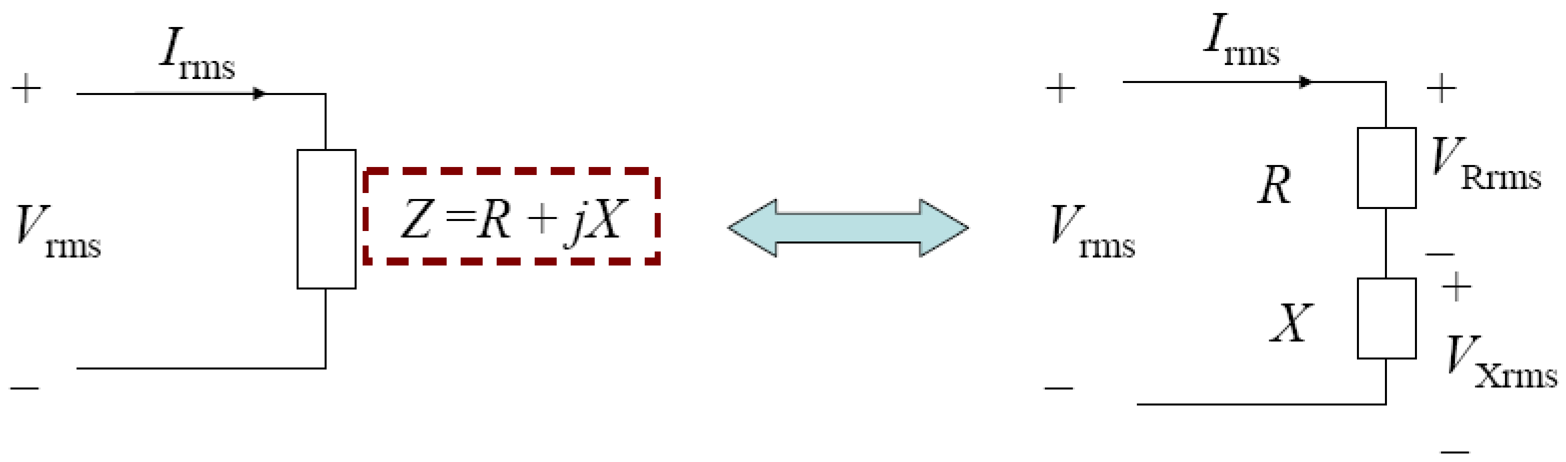
Power factor

$$I_m = \frac{V_m}{|Z|}$$

$$I_{rms} = \frac{V_{rms}}{|Z|}$$

PF = 1: R
PF = 0: L or C

Example



$$\cos\theta = \frac{R}{\sqrt{R^2 + X^2}}$$

Average power consumed by Z

$$p_{avg} = V_{rms} I_{rms} \cos\theta \;= I_{rms} |Z| I_{rms} \frac{R}{\sqrt{R^2 + X^2}}$$

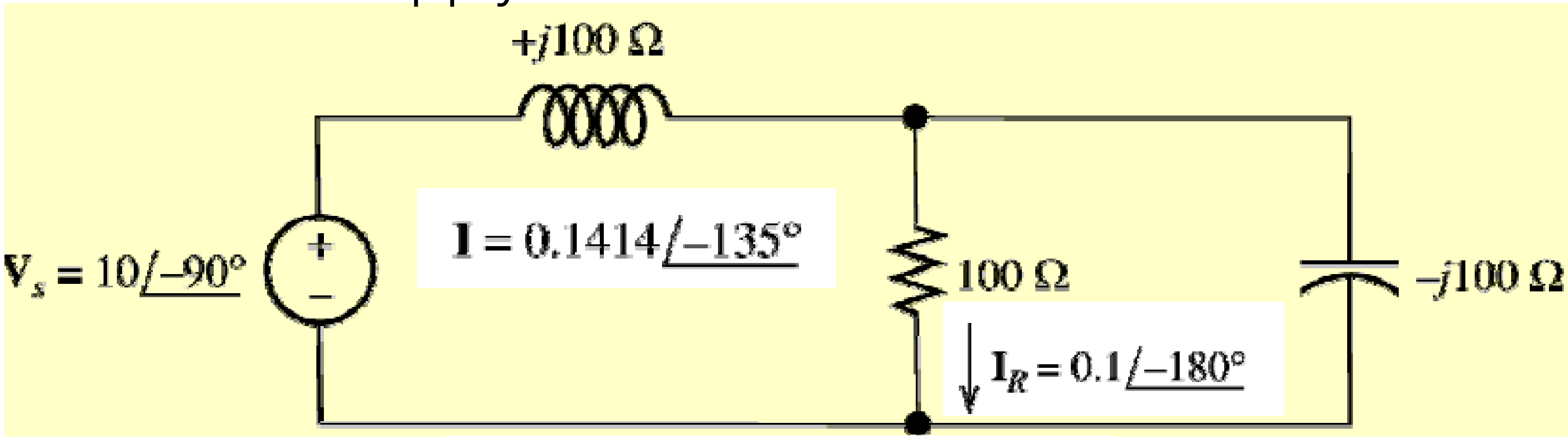
$$= I_{rms}^2 R \;= \frac{V_{R,rms}^2}{R}$$

RMS Voltage across R

=Average power consumed by R

Example

Find the average power drawn from the supply.



$$P = V_{srms} I_{rms} \cos(\theta)$$

$$V_{rms} = \frac{|\mathbf{V_s}|}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.071V$$

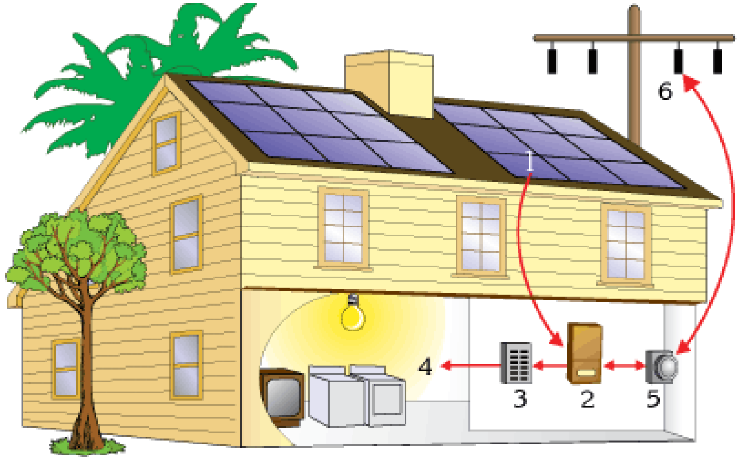
$$P = V_{srms} I_{rms} \cos(\theta)$$
$$= 7.071 \times 0.1 \times \cos(45^\circ)$$
$$= 0.5\ \mathbf{W}$$

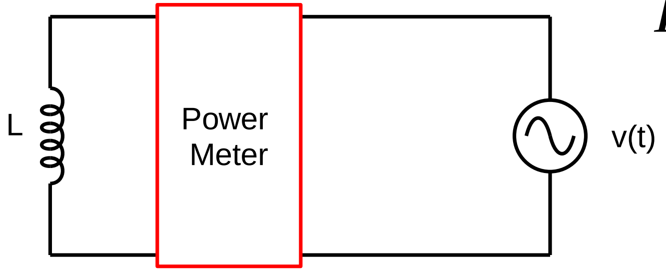
$$I_{rms} = \frac{|\mathbf{I}|}{\sqrt{2}} = \frac{0.1414}{\sqrt{2}} = 0.1A$$

Where is this power dissipated?

$$I_{Rrms} = \frac{0.1}{\sqrt{2}} = 0.071 \qquad P = I_{Rrms}^2 \times R = 0.5W$$

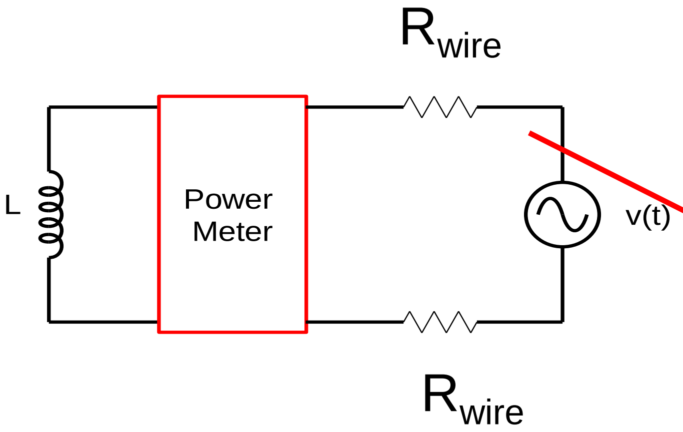
Charge for reactive power?





$$p_{avg} = 0$$

Should a power company charge a person even though power consumed is zero?



Power is dissipated
There is a cost for generation of this power.

Average & reactive power

Average power: $P = V_{\text{rms}} I_{\text{rms}} \cos\theta$

Reactive power: $Q = V_{\text{rms}} I_{\text{rms}} \sin\theta$

(Volt Amperes Reactive (VAR))

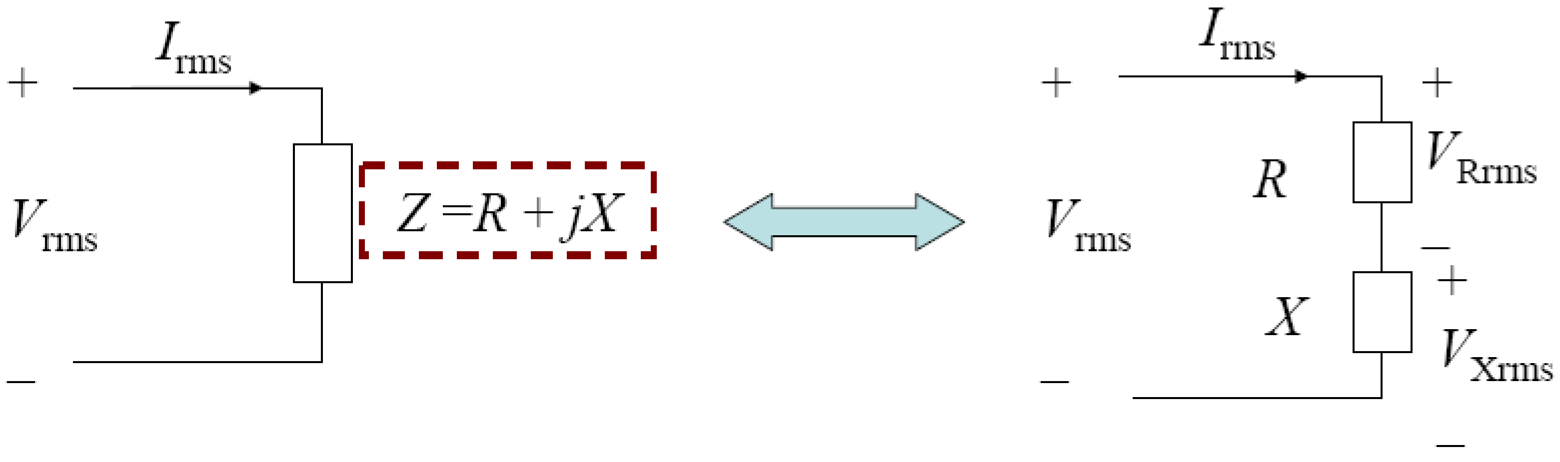
Apparant power: $V_{\text{rms}} I_{\text{rms}}$, Units: volt-amperes (VA)

$$P^2 + Q^2 = V_{rms}^2 I_{rms}^2 \cos^2(\theta) + V_{rms}^2 I_{rms}^2 \sin^2(\theta)$$
$$= (V_{rms} I_{rms})^2$$
$$= (\text{ Apparant power })^2$$

$$\boxed{\text{Apparant power} = \sqrt{P^2 + Q^2}}$$

- No average power is consumed in a pure inductive/capacitive load
- But reactive power has current associated with it and causes loss of power in transmission lines
- Power companies charge their industrial customers for reactive power.

Apparent Power



$$\cos\theta = \frac{R}{\sqrt{R^2 + X^2}}$$

$$\sin\theta = \frac{X}{\sqrt{R^2 + X^2}}$$

Reactive power

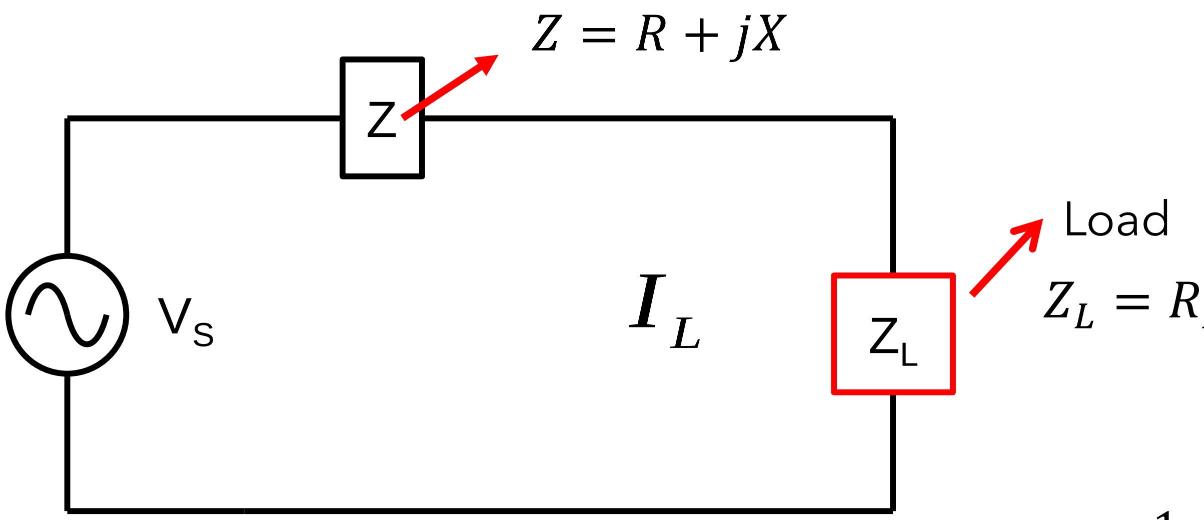
$$q_{avg} = V_{rms} I_{rms} \sin\theta \qquad = I_{rms}|Z| I_{rms} \frac{X}{\sqrt{R^2 + X^2}}$$

$$= I_{rms}^2 X \qquad = \frac{V_{X,rms}^2}{X}$$

RMS Voltage across X

= Reactive power at X

Maximum power transfer for sinusoidal input



$Z_L = R_L + jX_L$

$$I_L = \frac{V_S}{R + R_L + j(X + X_L)}$$

$$I_{L,rms} = \frac{1}{\sqrt{2}} \frac{V_S}{|R + R_L + j(X + X_L)|}$$

$$P_L = \frac{1}{2} \frac{V_S^2}{(R + R_L)^2 + (X + X_L)^2} R_L$$

To maximize P_L

$$X_L = -X$$

$$P_L = \frac{1}{2} \frac{V_S^2}{(R + R_L)^2} R_L$$

$$\boxed{Z_L = \bar{Z}}$$

$$R_L = R$$

$$P_L = \frac{1}{8} \frac{V_S^2}{R}$$