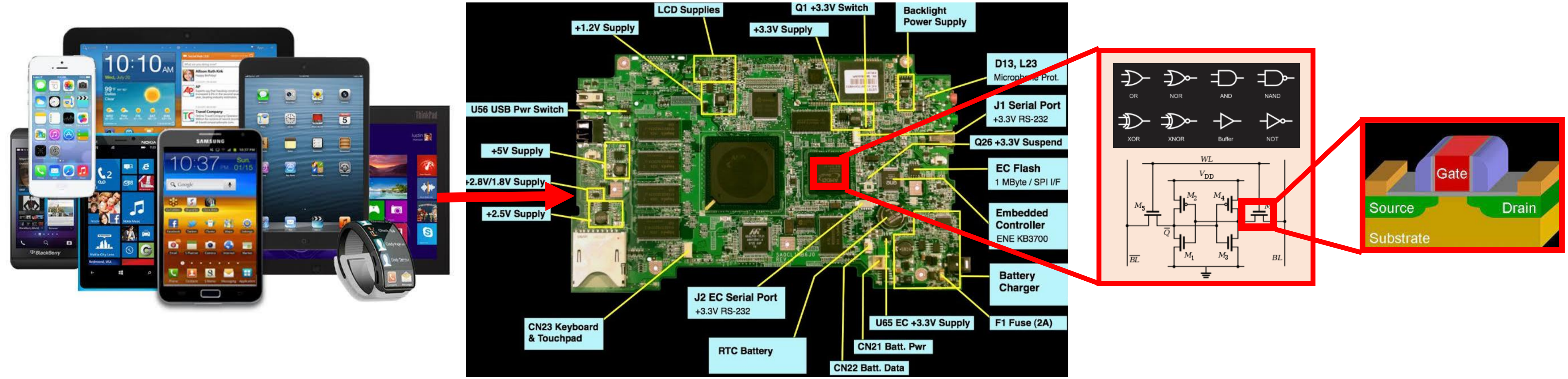


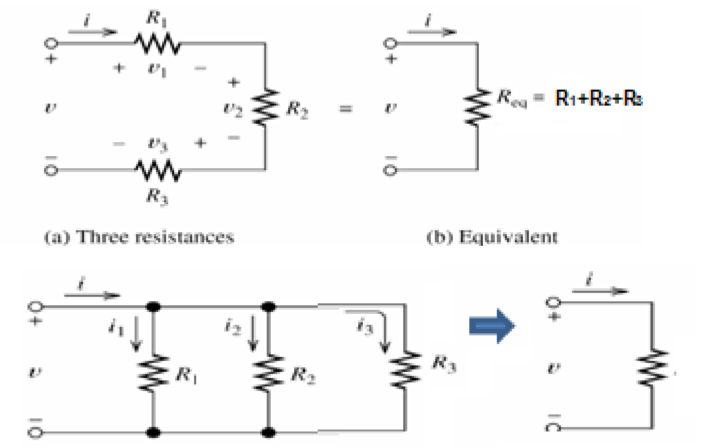

# ESC201: Introduction to Electronics

## Module 2: Elements with Memory

Dr. Shubham Sahay,
Assistant Professor,
Department of Electrical Engineering,
IIT Kanpur

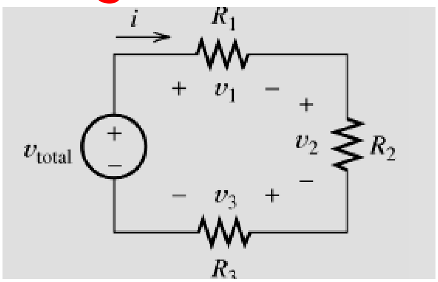
# Summary

## Series/Parallel resistances




$$R_{eq} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3}$$

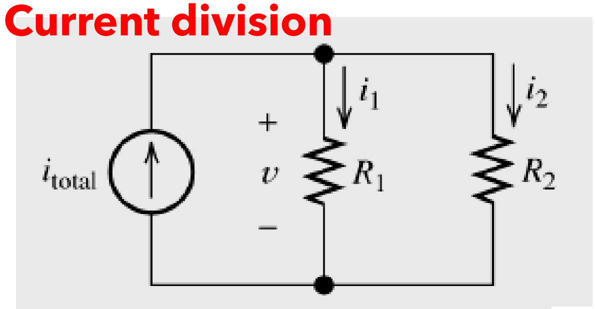
## Voltage division



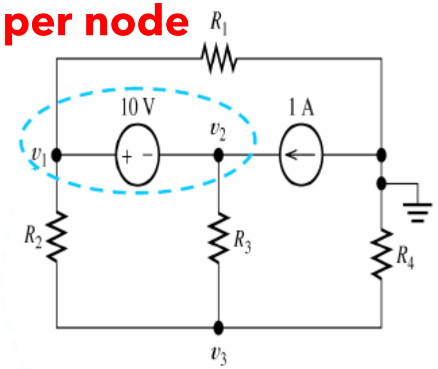
$$v_2 = R_2 i = \frac{R_2}{R_1 + R_2 + R_3} v_{\text{total}}$$

## Current division

$$i_2 = \frac{v}{R_2} = \frac{R_1}{R_1 + R_2} i_{\text{total}}$$



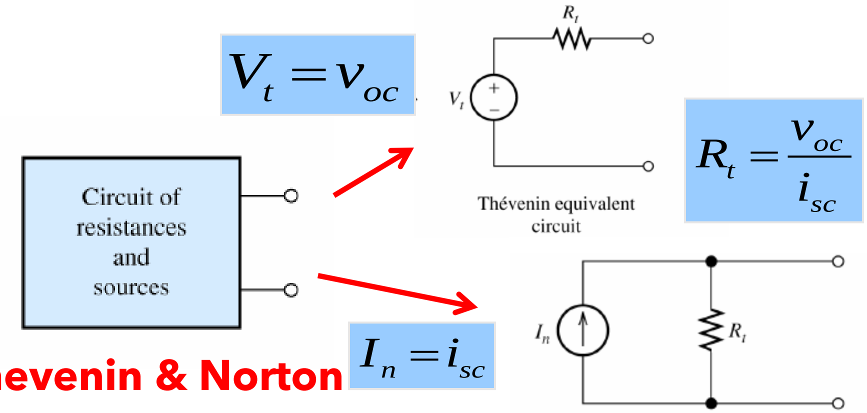
## Super node




## Nodal Analysis:

1. Identify and number the nodes
2. Choose a reference node
3. Write KCL for each node such that Sum of currents leaving a node is zero

## Thevenin & Norton




$$V_t = v_{oc}$$

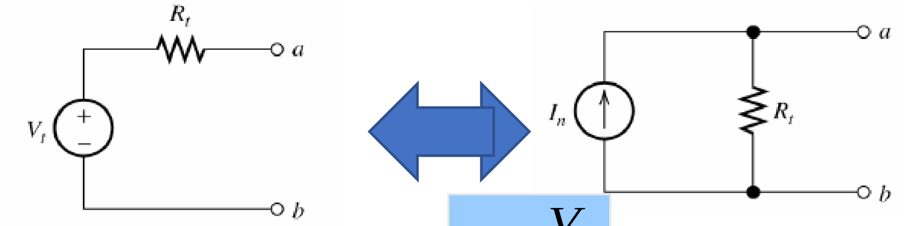
$$R_t = \frac{v_{oc}}{i_{sc}}$$

$$I_n = i_{sc}$$

## Mesh Analysis

1. Assign mesh currents $i_1$, $i_2$, …, in to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

## Source Transformation



$$V_t = I_n \times R_t$$

$$I_n = \frac{V_t}{R_t}$$

The **superposition principle** states that the total response is the sum of the responses to each of the independent sources acting individually.

# Storage devices

- Story so far:
  - Resistors, Independent and dependent voltage and current sources
  - Output depends on current input

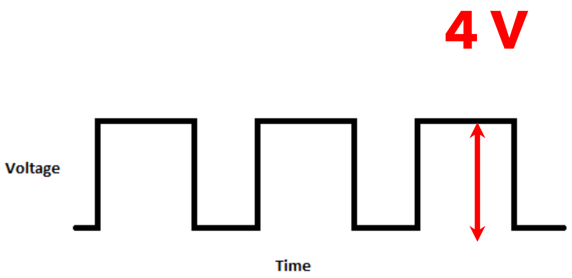
$$V_O(t) = f(V_I(t))$$

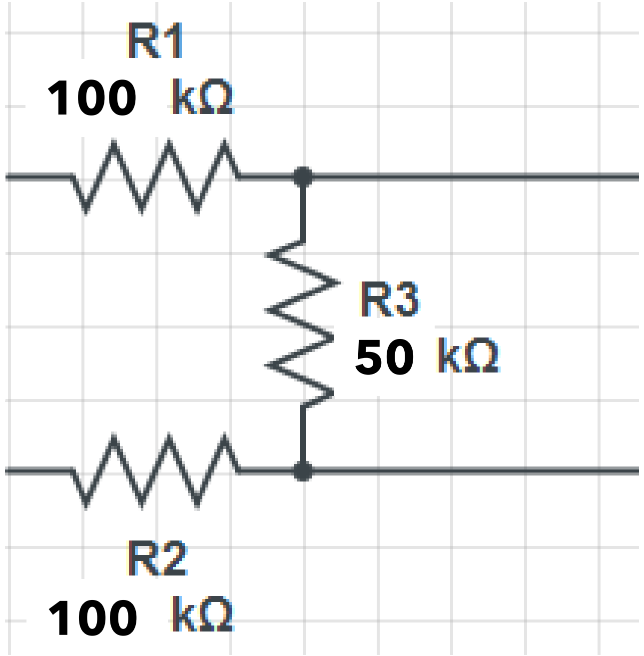
  - Memory-less!

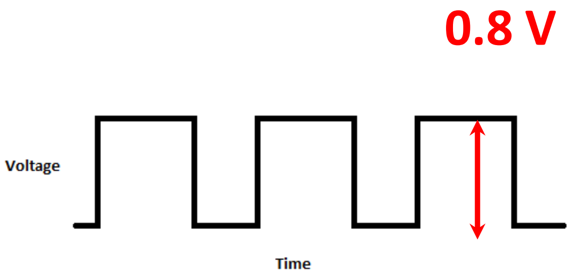
- Capacitors & Inductors
  - $V_O(t)$ depends on $V_I(\tau)$ for all $\tau \leq t$

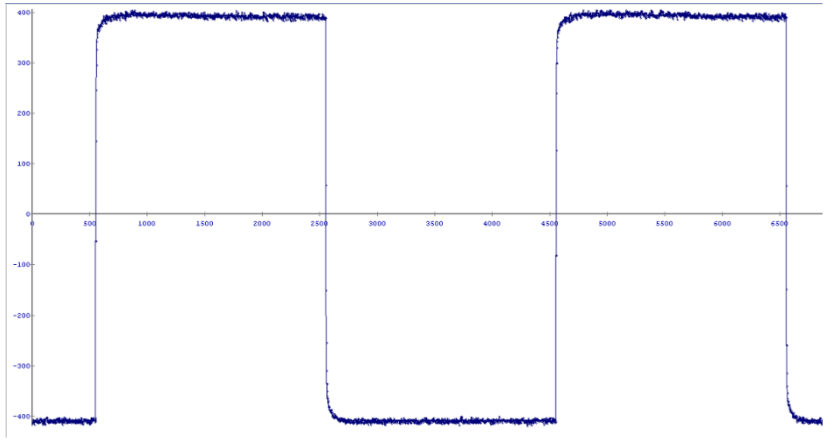
- Elements with memory
  - Depends on the rate of change of input signals

# Example: potential divider

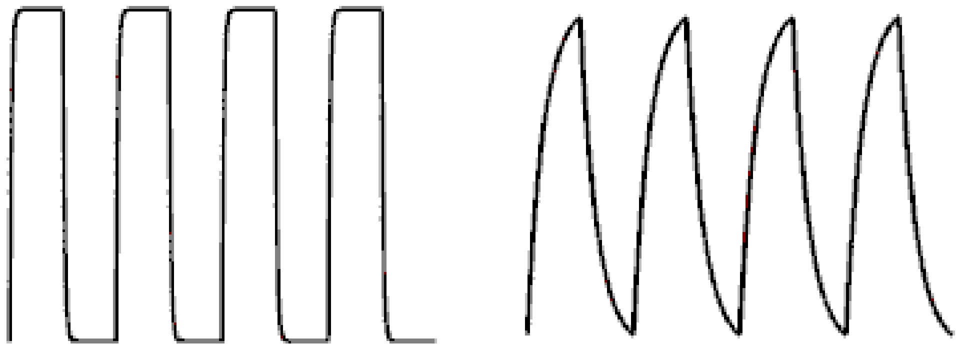


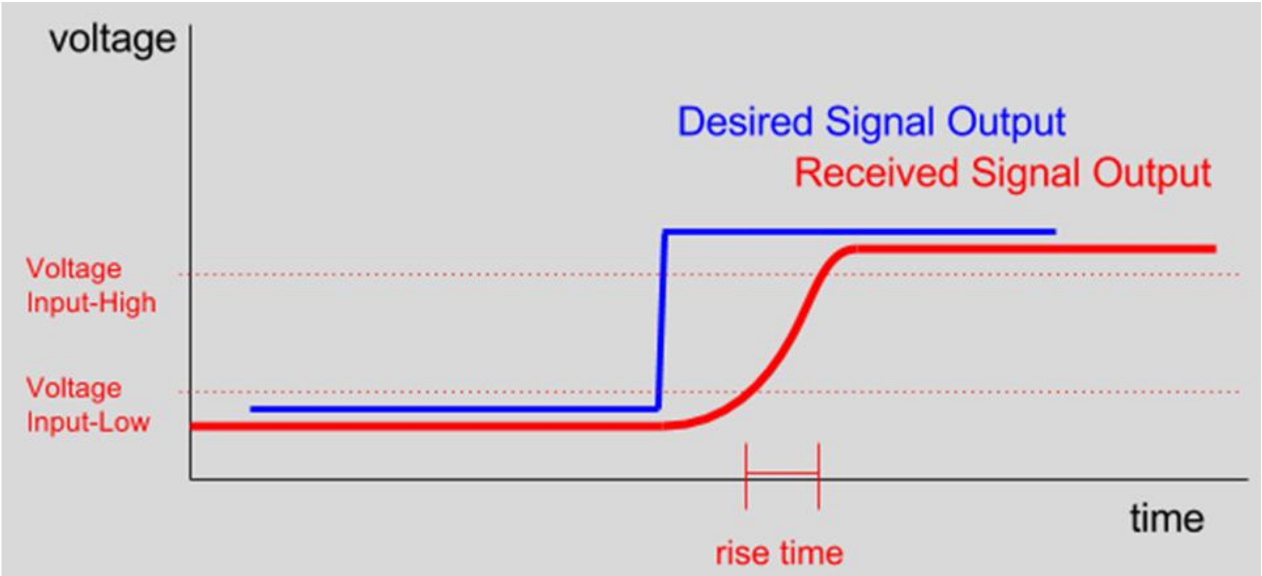



# Non-zero rise time



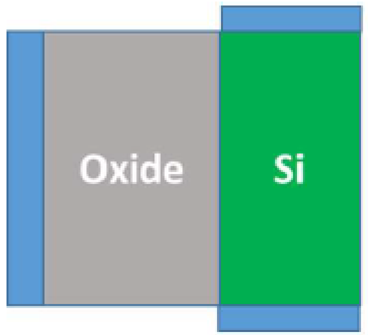


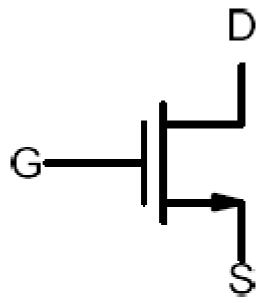

| Intel i7 Mobile Processors | Freq. (GHz) | Year |
|---|---|---|
| Clarksfield (45 nm) | 2.13 | 2009 |
| Arrandale (32 nm) | 2.67 | 2010 |
| Sandy bridge (32 nm) | 2.7 | 2011 |
| Ivy bridge (22 nm) | 2.9 | 2012 |
| Haswell (22nm) | 2.9 | 2013 |
| Broadwell (14 nm) | 3.1 | 2014 |
| Skylake (14 nm) | 3.3 | 2015 |
| Kaby-Lake (14 nm) | 3.5 | 2017 |

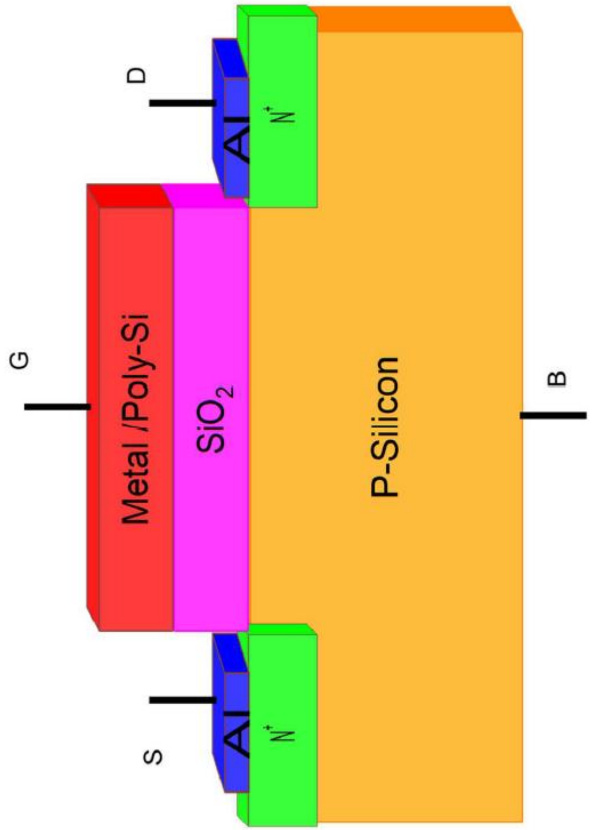
Elements may oppose any change in voltage or current!

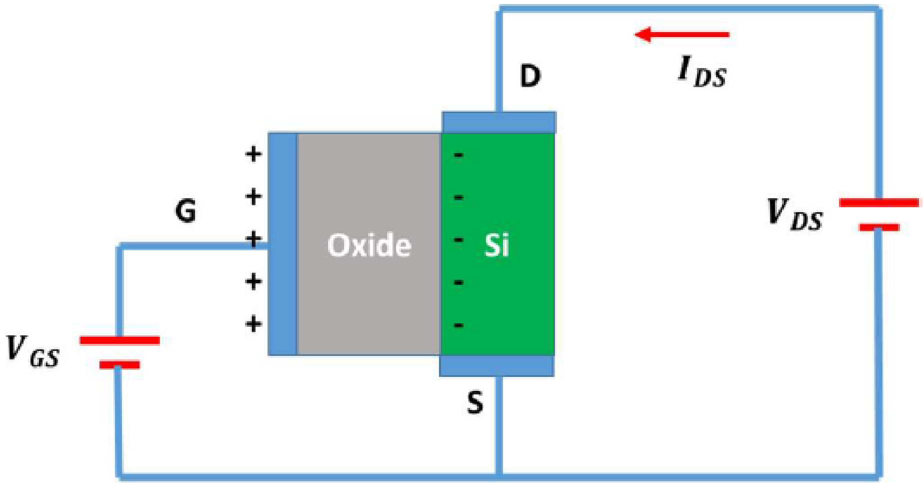
# MOSFETs: Workhorse of Semiconductor Industry

MOSFET : Metal Oxide Semiconductor Field effect Transistor

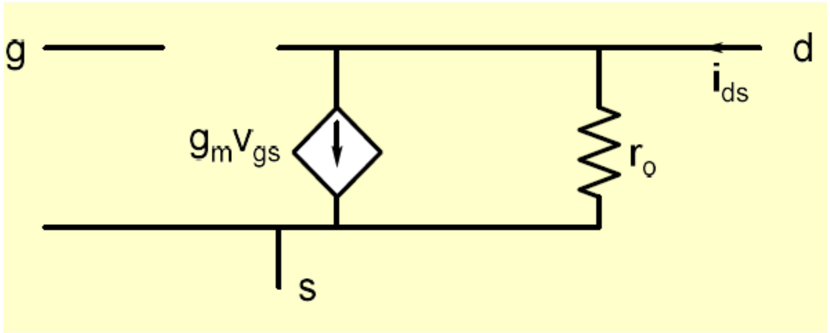


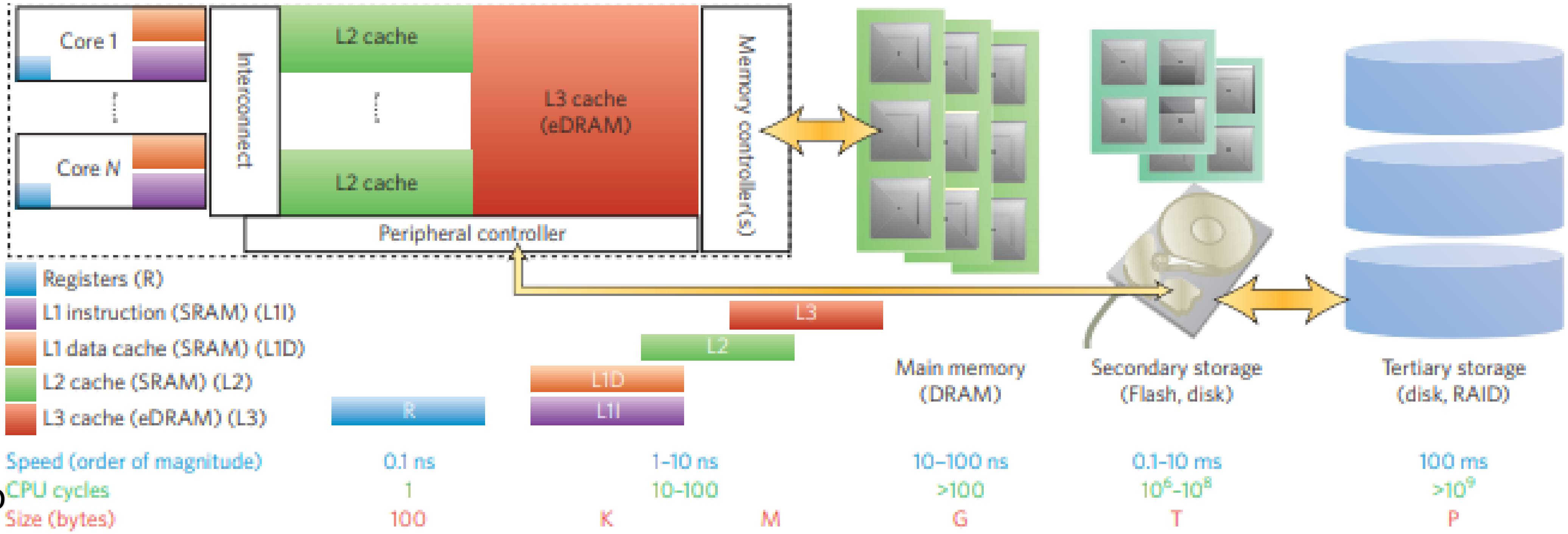




Drain current is controlled by gate voltage




Voltage controlled current source

# Memory Organization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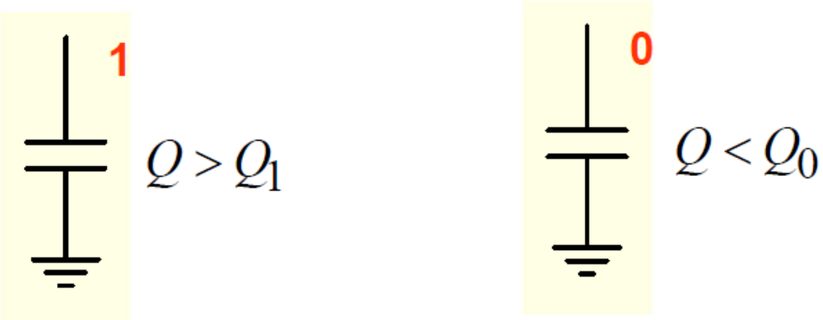
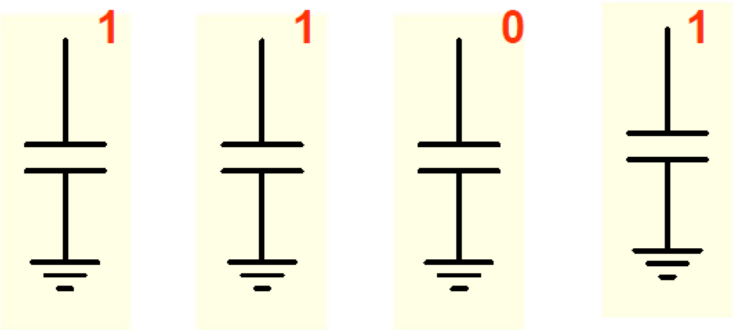
source: H.S.P. Wong *et. al.*, Nature Nanotechnology, Vol. 10, March 2015

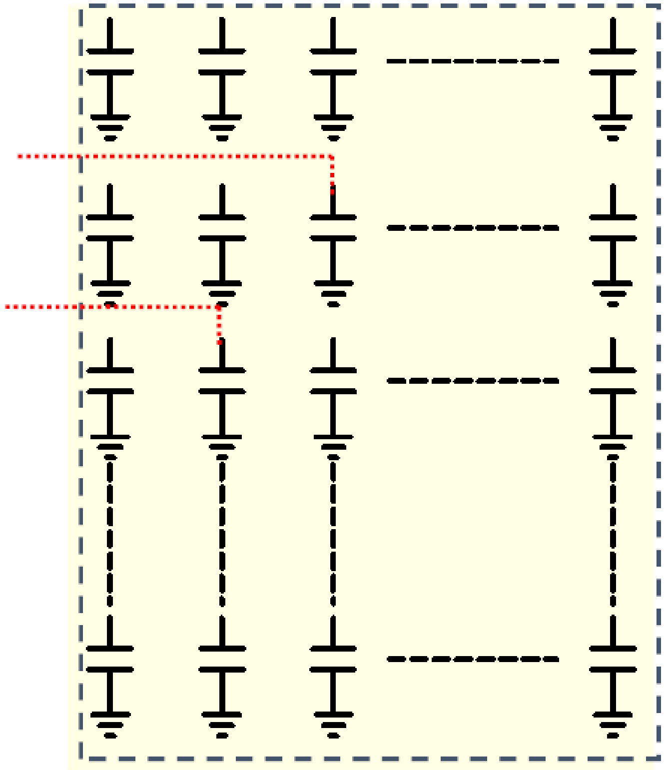
# Dynamic RAM

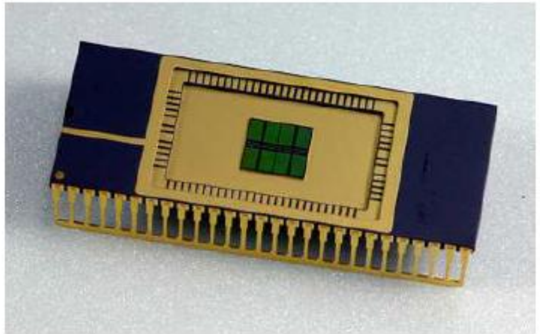



1101 in binary number format represents number 13

Each capacitor can store 1 bit of information

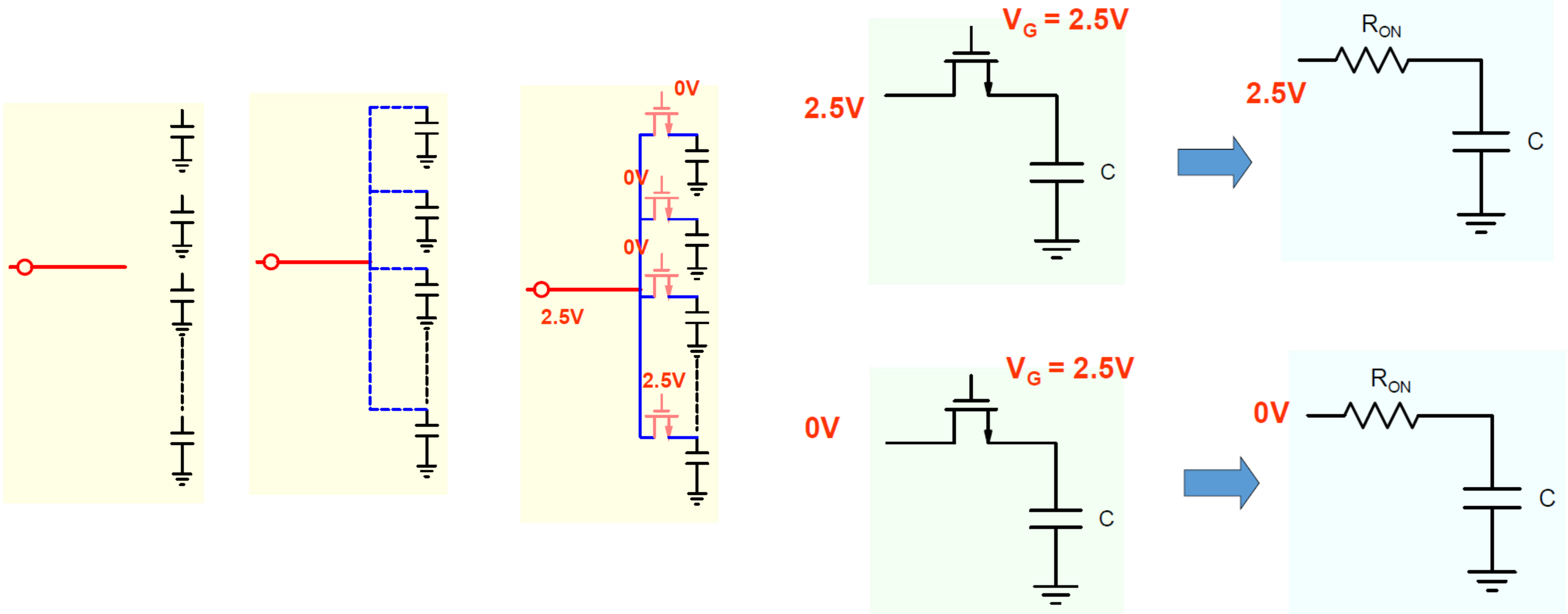




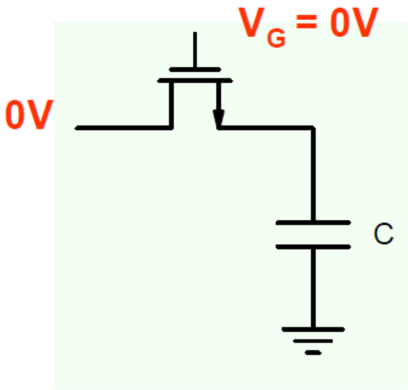
We do not have enough pins to individually access each capacitor !

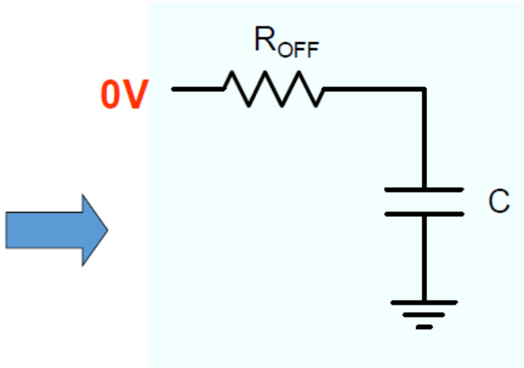
One pin has to be connected to several capacitors !

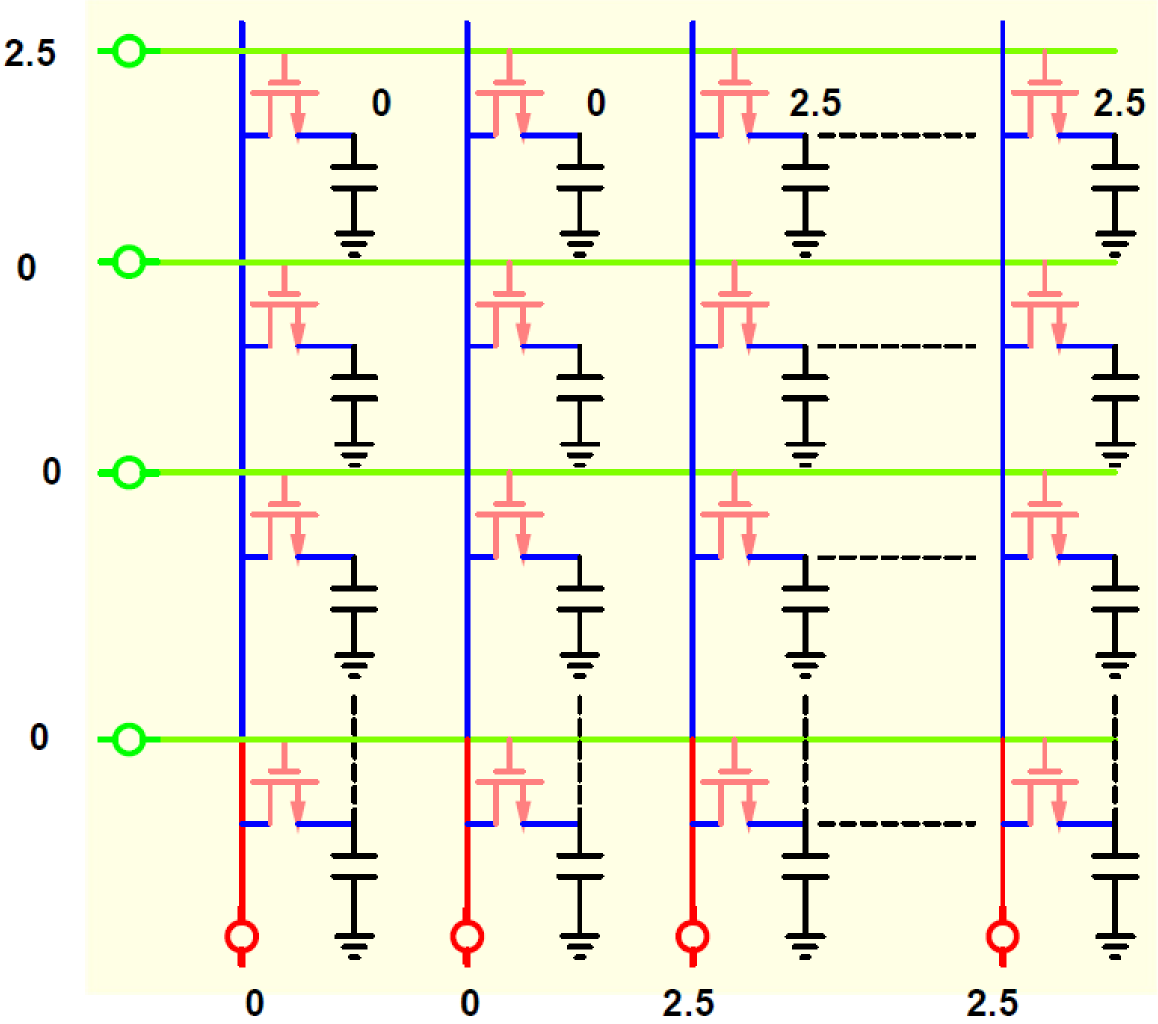
# Reducing pins in DRAMs



# Crossbar Organization



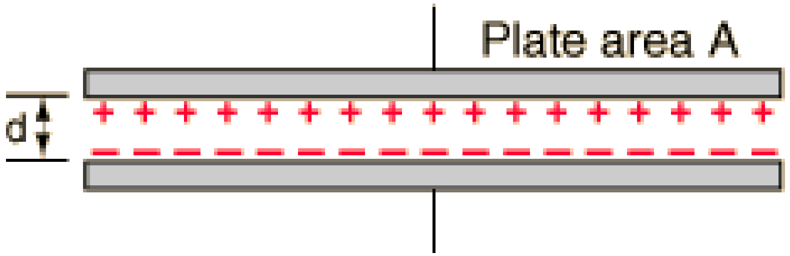
# Capacitor

- Two sheets of conductors separated by a layer of insulating material
  - Insulating material is called dielectric - could be air, polyester, …

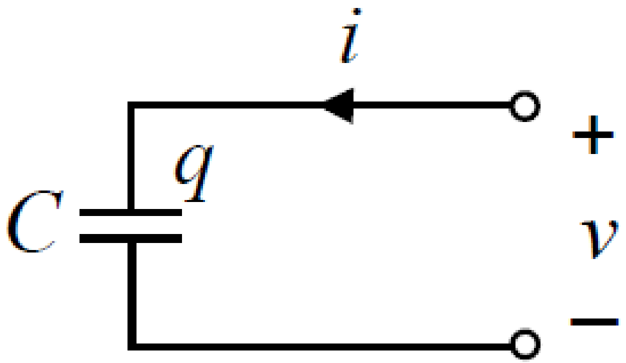
# Capacitors

SPRAGUE
POWERLYTIC®
36DE
81000-40DC
8726L 2201
MADE IN USA
+95°C
ELNA
CE-W
75V 8000μF
BLACK-NEG

# Ideal Capacitor




$$C = \frac{\varepsilon A}{d} = \frac{k\varepsilon_0 A}{d}$$

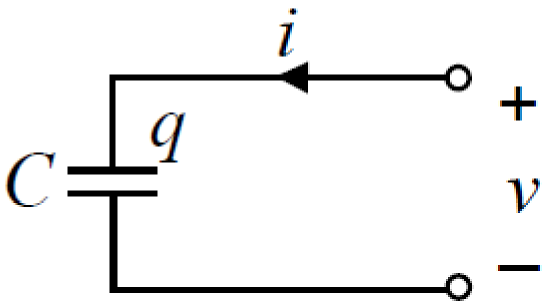



$$q = C\ v$$

coulombs farads volts

Lumped circuit assumption

# Ideal Capacitor




$$q = C\,v$$

$$i = \frac{dq}{dt}$$

$$= \frac{d(Cv)}{dt}$$

$$= C\frac{dv}{dt}$$

$$\boxed{i = C\frac{dv}{dt}}$$

$$E = \frac{1}{2}Cv(t)^2$$

# Capacitor in steady state

$$i = C\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int_{t_o}^{t} i\,dt + v(t_o)$$

For dc or steady state when the voltage does not vary with time. $i = 0$

A capacitor under DC or steady state acts like an open circuit.

# Are capacitors linear?

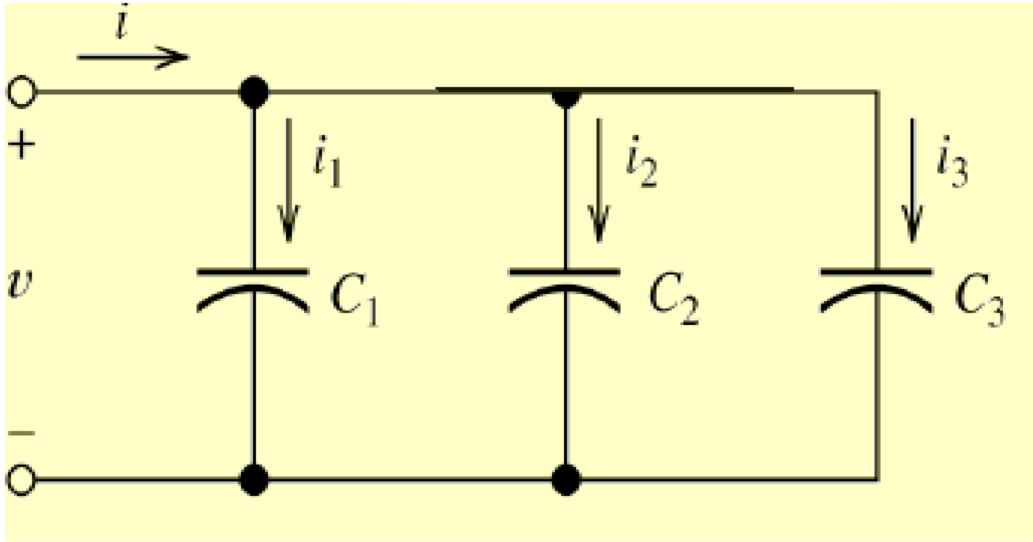
- Yes! Differentiation and integration are linear operators
- Quick check:

$$i_1(t) = C\frac{dv_1(t)}{dt} \qquad\qquad i_2(t) = C\frac{dv_2(t)}{dt}$$

$$\alpha i_1(t) + \beta i_2(t) = C\frac{d}{dt}(\alpha v_1(t) + \beta v_2(t)) = \alpha C\frac{dv_1(t)}{dt} + \beta C\frac{dv_2(t)}{dt}$$

- Consequence: can use superposition, combination rules, Thevenin/Norton

# Capacitor: series/parallel



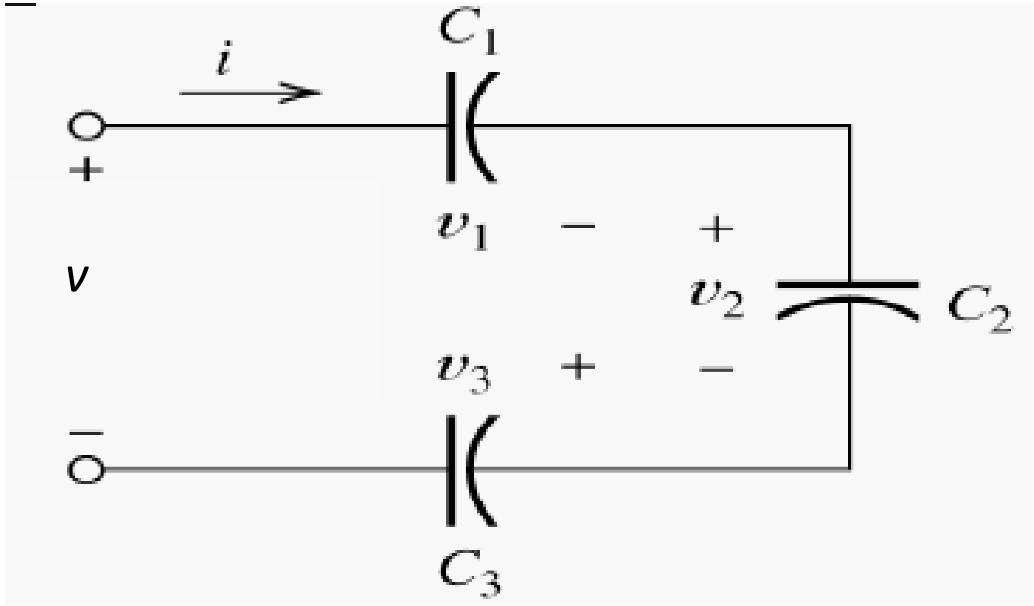

$$i_1 = C_1 \frac{dv}{dt} \qquad i_2 = C_2 \frac{dv}{dt} \qquad i_3 = C_3 \frac{dv}{dt}$$

$$i = i_1 + i_2 + i_3 \quad = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt}$$

$$i = (C_1 + C_2 + C_3) \frac{dv}{dt}$$

$$\boxed{i = C_{eq} \frac{dv}{dt}}$$

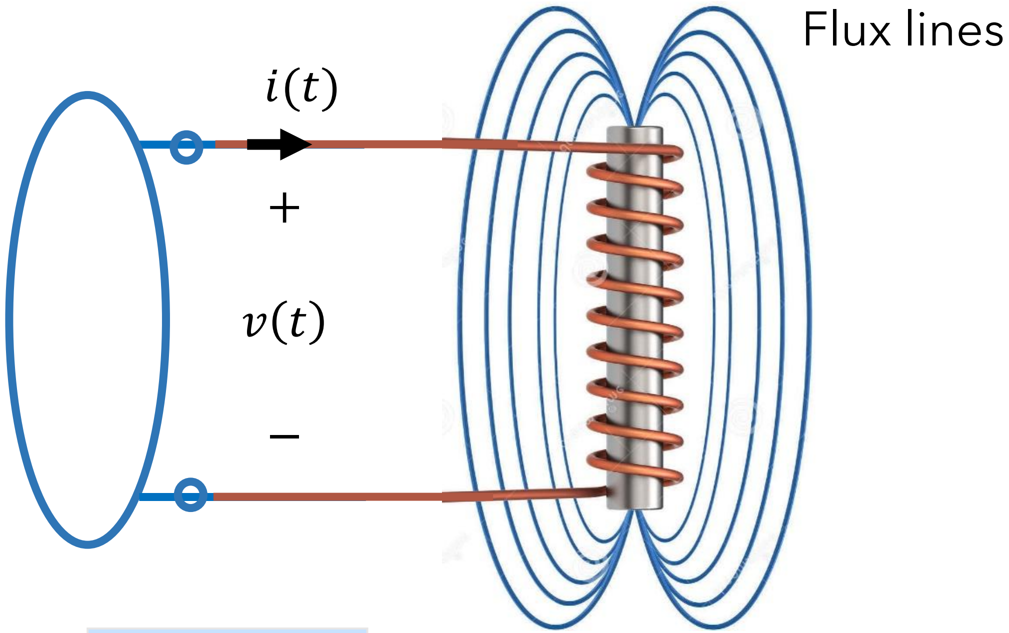
$$\boxed{C_{eq} = C_1 + C_2 + C_3}$$

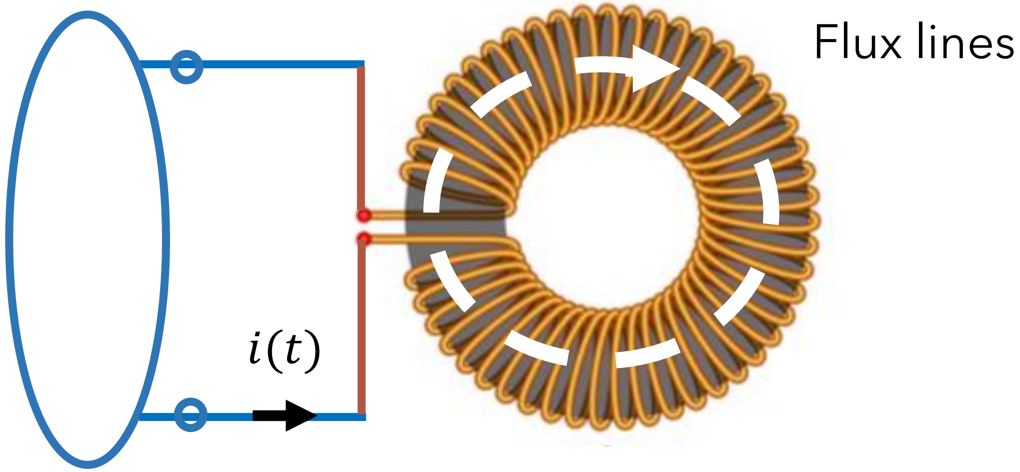



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

# Inductors

# Inductors



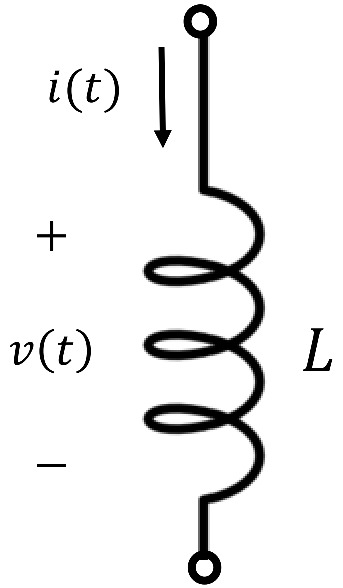




$$\varphi = L \times i$$

$$v = \frac{d\varphi}{dt} = L \times \frac{di}{dt}$$

$$E = \frac{1}{2} L\, i(t)^2$$

# Inductance




$$v = L\,\frac{di}{dt}$$

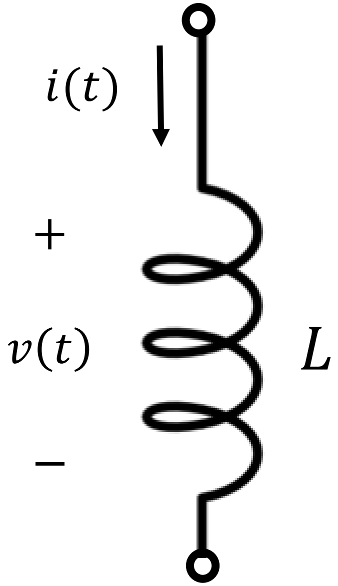
**Henry**

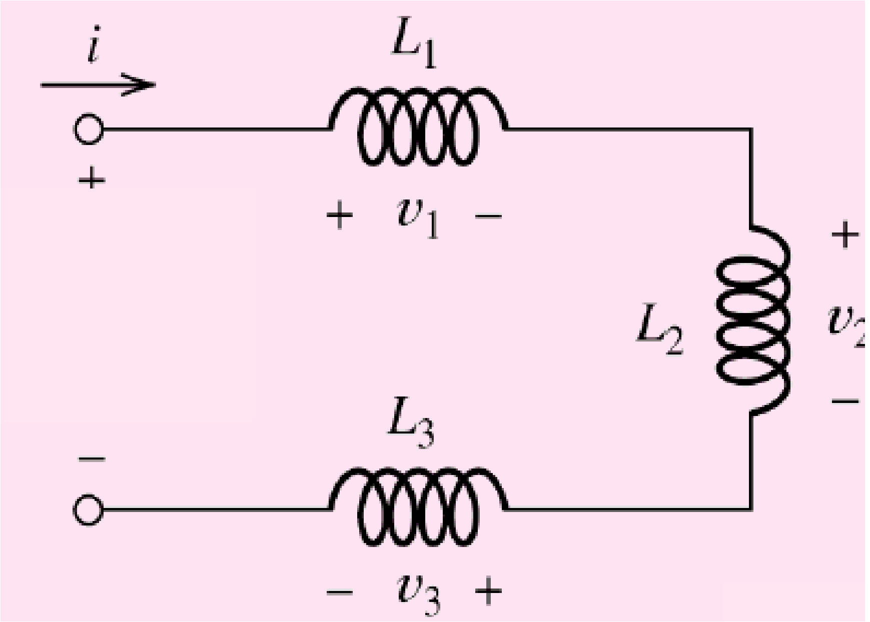
For dc or steady state when the current does not vary with time.

$$v = 0$$

An inductor under DC or steady state acts like a short circuit

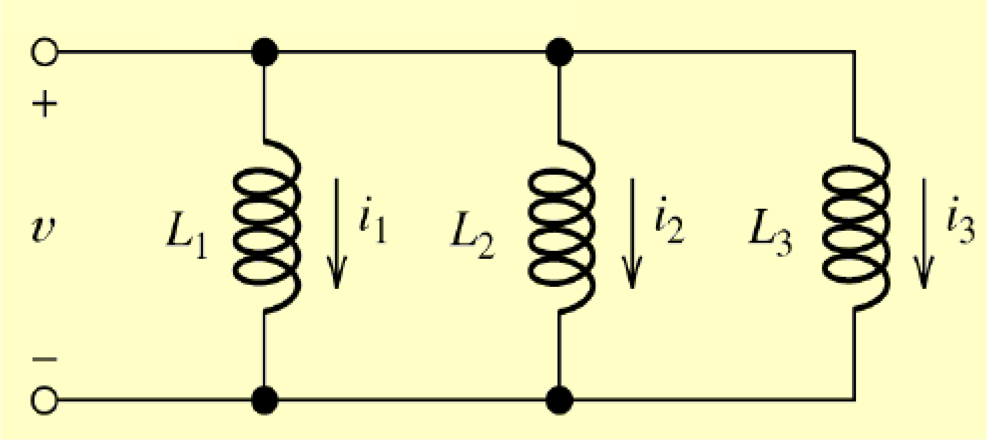
# Inductors: series/parallel



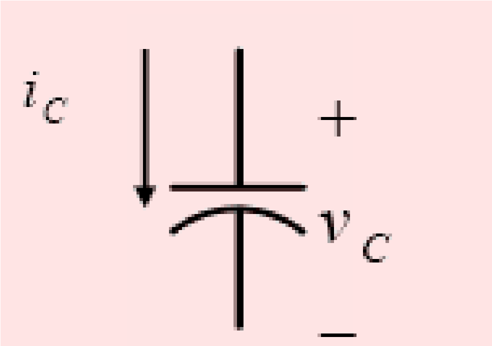

$$L_{eq} = L_1 + L_2 + L_3$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$



# Two things to remember

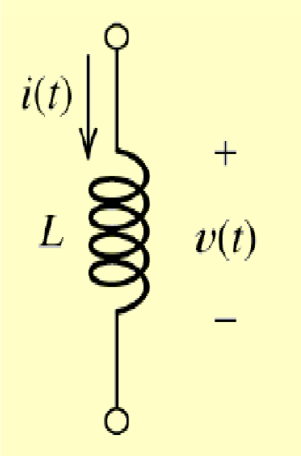
## Voltage across a capacitor cannot change instantaneously




$$i_c = C\frac{dv_c}{dt}$$

Instantaneous change in voltage implies infinite current!

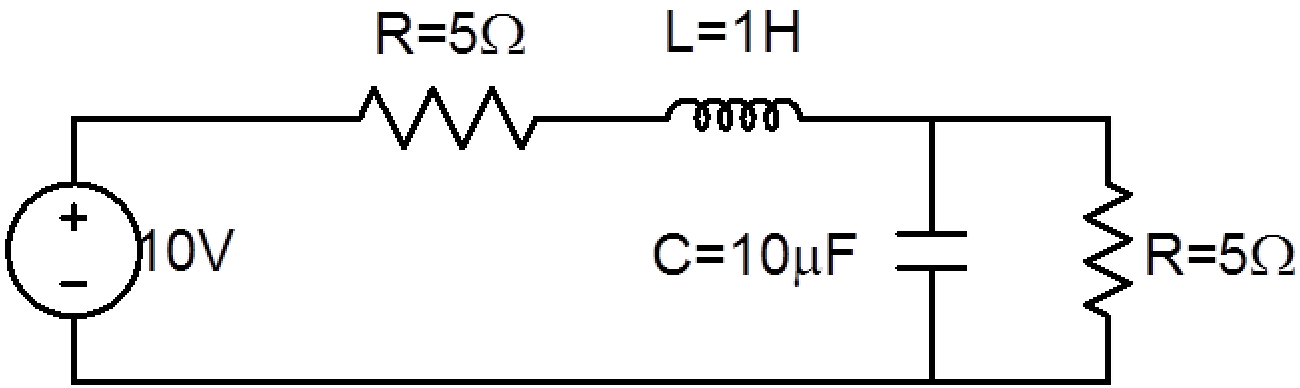
## Current through an inductor cannot change instantaneously




$$v = L\frac{di}{dt}$$

Instantaneous change in current implies infinite voltage!

# Circuit analysis: Example 1

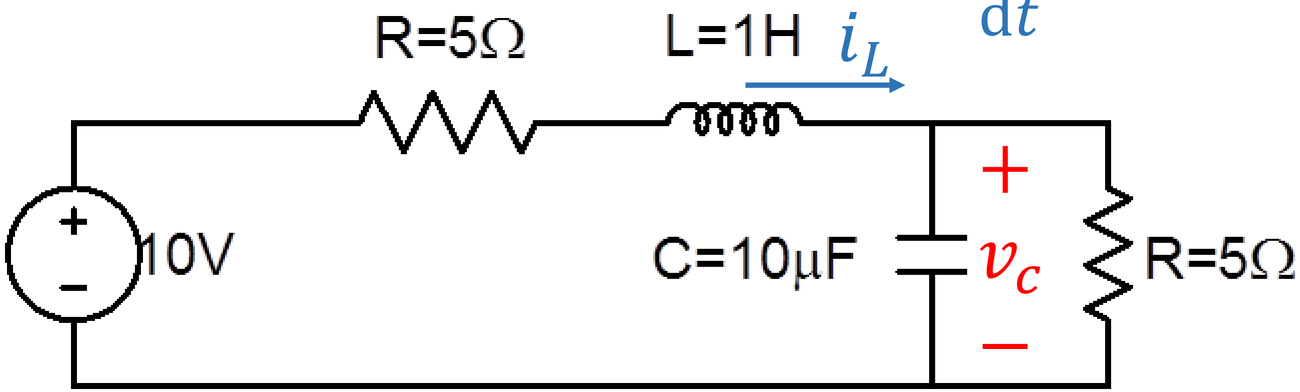



What is current through the inductor or voltage across the capacitor ?

We cannot have an answer unless we have some knowledge of the past state of the circuit.

Suppose we are told that circuit has been in this state for a very long time.

# Circuit analysis: Example 1

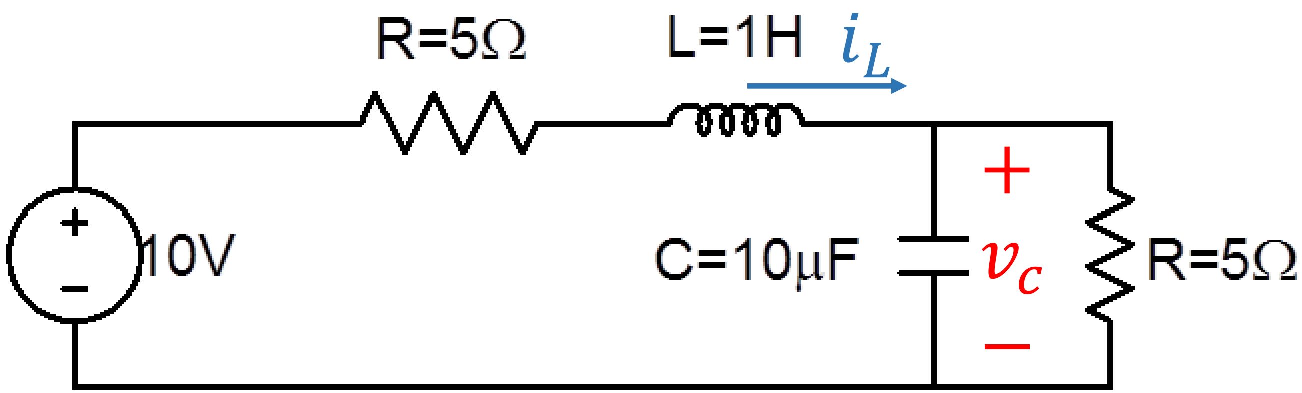


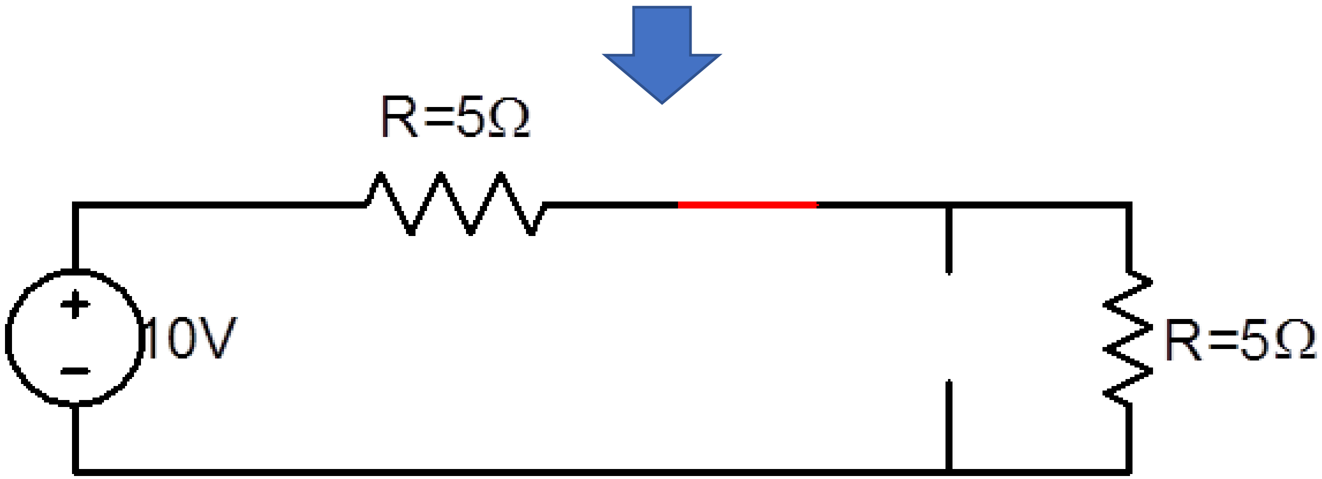

$$\frac{\mathrm{d}i_L}{\mathrm{d}t} = 0 \qquad v_L = L\frac{\mathrm{d}i_L}{\mathrm{d}t} = 0$$

$$\frac{\mathrm{d}v_c}{\mathrm{d}t} = 0 \qquad i_C = C\frac{\mathrm{d}v_c}{\mathrm{d}t} = 0$$

Suppose we are told that circuit has been in this state for a very long time.

# Example 1: Steady state behavior




$$\frac{di_L}{dt} = 0 \qquad v_L = L\frac{di_L}{dt} = 0$$

$$\frac{dv_c}{dt} = 0 \qquad i_C = C\frac{dv_c}{dt} = 0$$

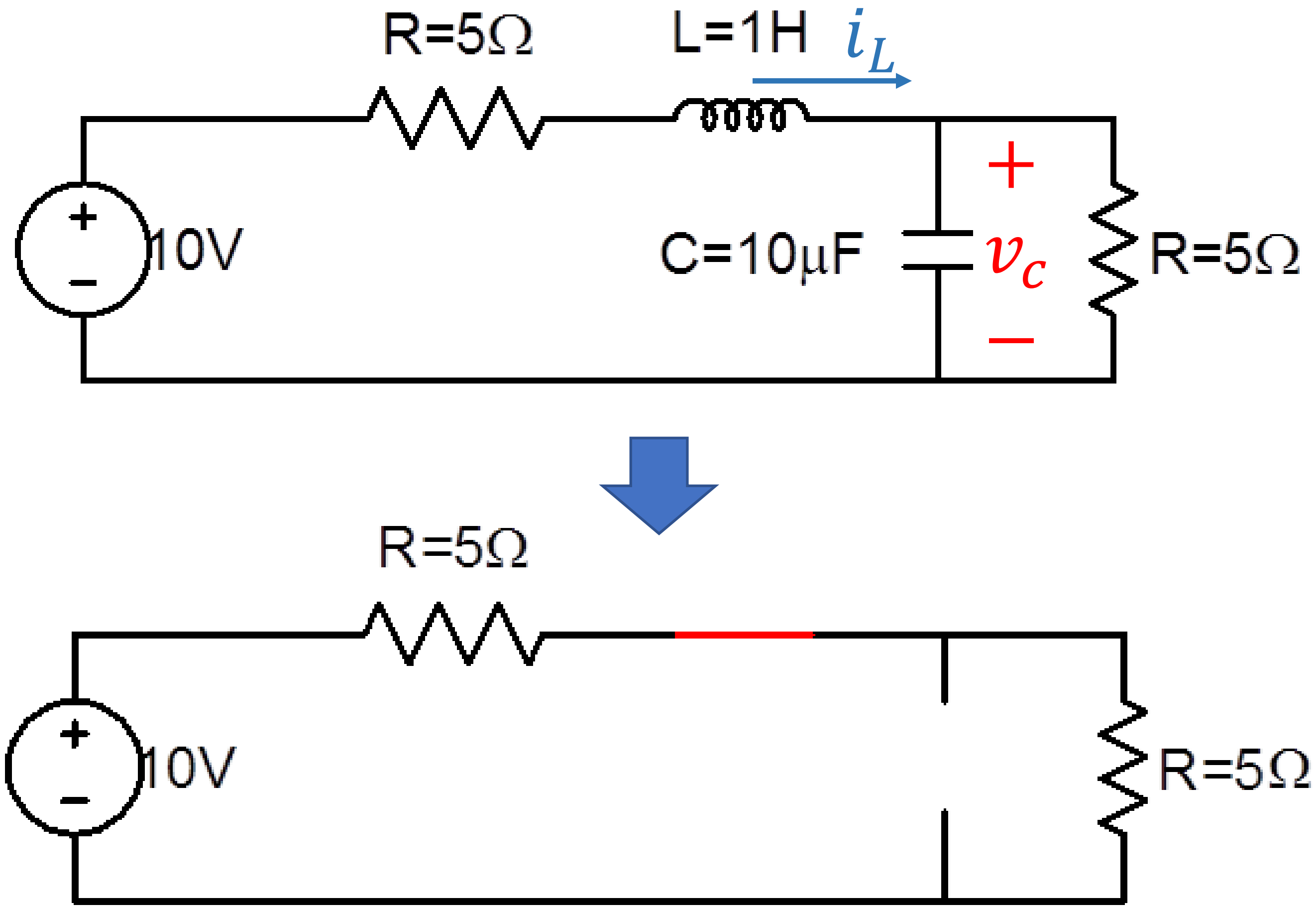
A capacitor under dc or steady state acts like an open circuit.

An inductor under dc or steady state acts like a short circuit

Suppose we are told that circuit has been in this state for a very long time.

Steady state behavior

# Example 1: Steady state behavior




$$\frac{di_L}{dt} = 0 \qquad v_L = L\frac{di_L}{dt} = 0$$

$$\frac{dv_c}{dt} = 0 \qquad i_C = C\frac{dv_c}{dt} = 0$$

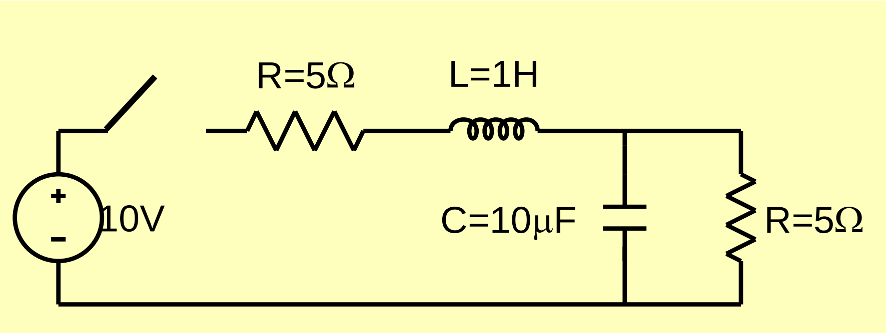
A capacitor under dc or steady state acts like an open circuit.

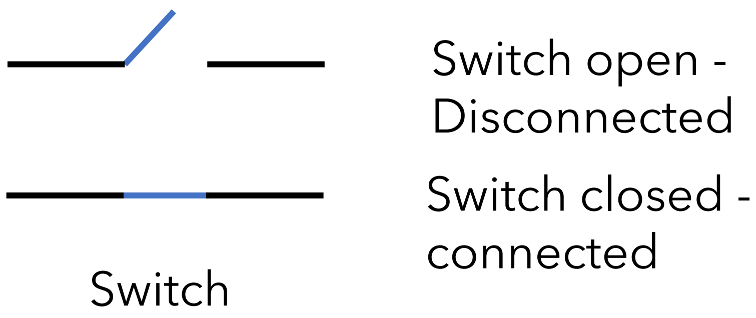
An inductor under dc or steady state acts like a short circuit

Suppose we are told that circuit has been in this state for a very long time.

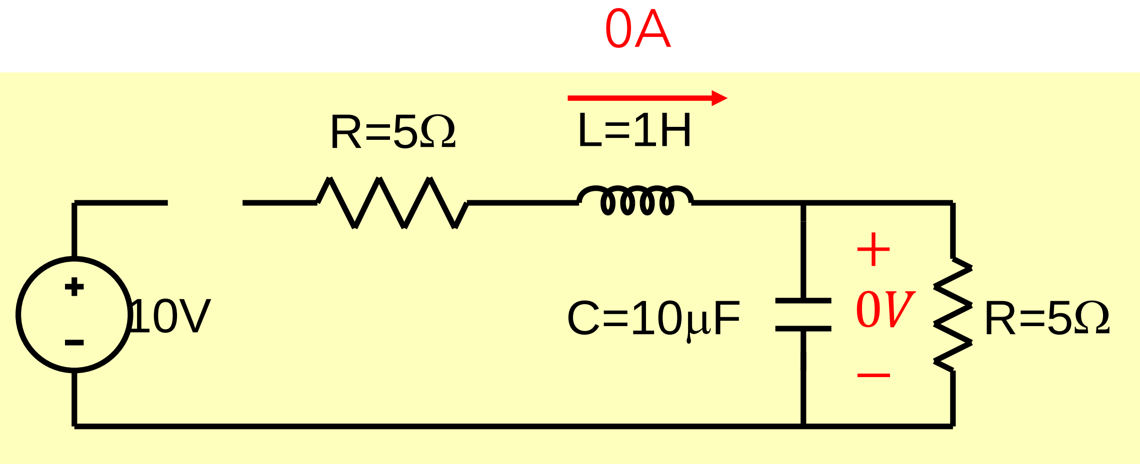
Steady state behavior

# Example 2: Immediate behavior after a change



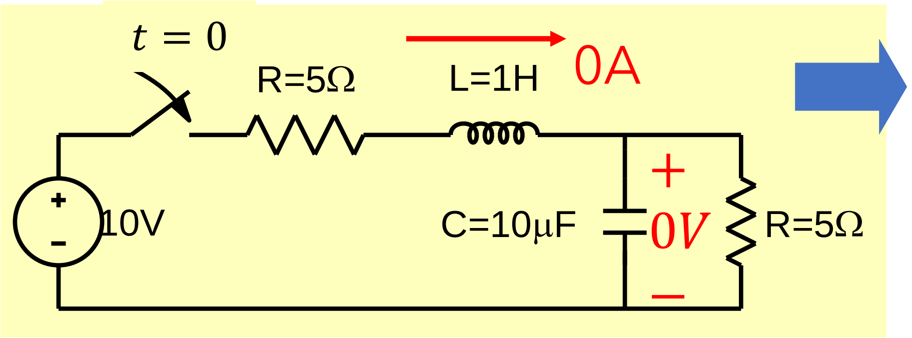



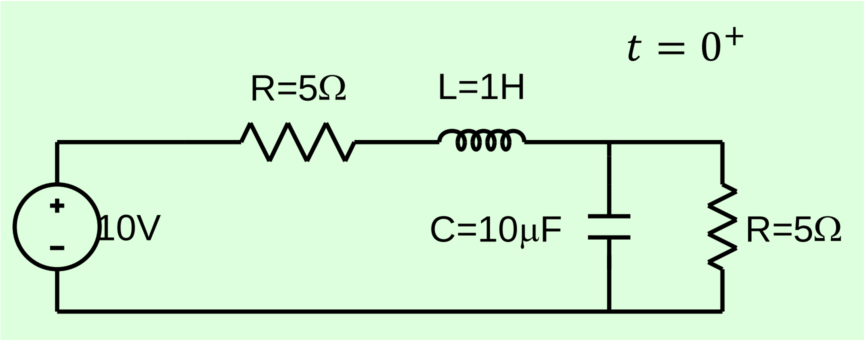

Circuit before switching



# Example 2: Immediate behavior after a change

Suppose the circuit was switched on at $t = 0$ second.

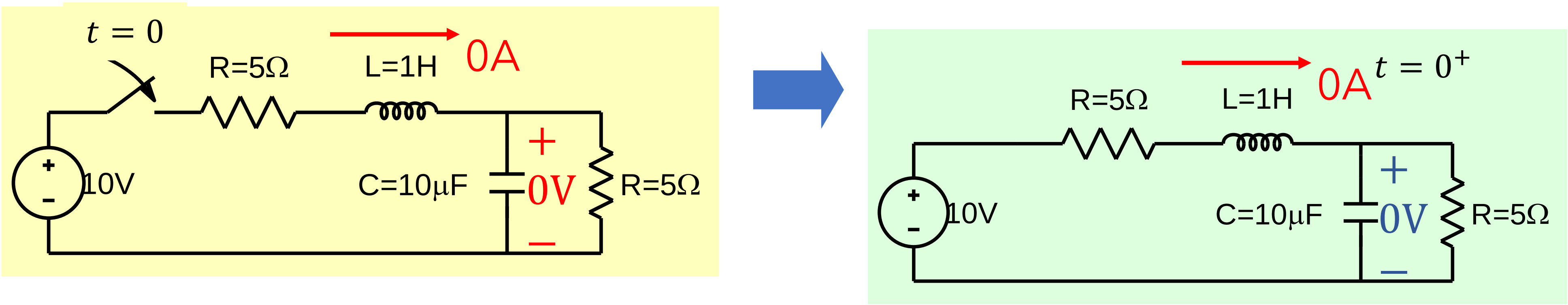



Find voltage and current immediately after closing the switch.

# Example 2: Immediate behavior after a change

Suppose the circuit was switched on at $t = 0$ second.




Find voltage and current immediately after closing the switch.

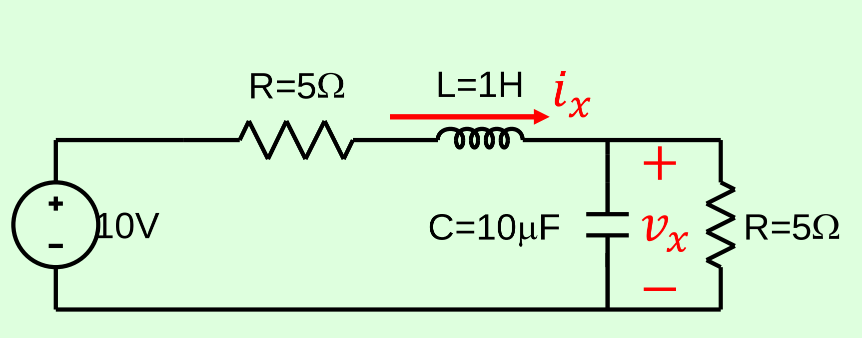
Current through an inductor cannot change instantaneously

Voltage across a capacitor cannot change instantaneously

# Example 2: Steady state behavior after a change

Find voltage and current a long time after closing the switch

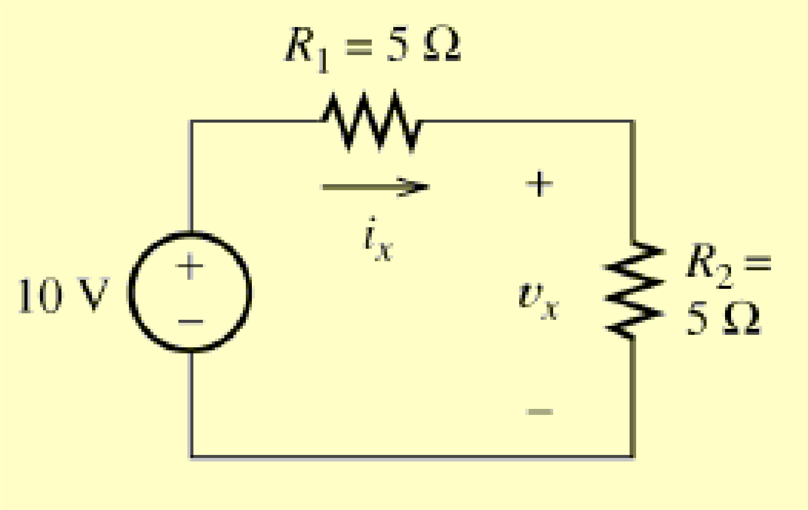
$t \rightarrow \infty$




Recall

A capacitor under dc or steady state acts like an open circuit.

An inductor under dc or steady state acts like a short circuit




$$i_x = \frac{10}{R_1 + R_2} = 1\text{ A}$$

$$v_x = R_2 i_x = 5\text{ V}$$

$$i_L(t \rightarrow \infty) = 1 \quad \text{A}$$
$$v_c(t \rightarrow \infty) = 5 \quad \text{V}$$