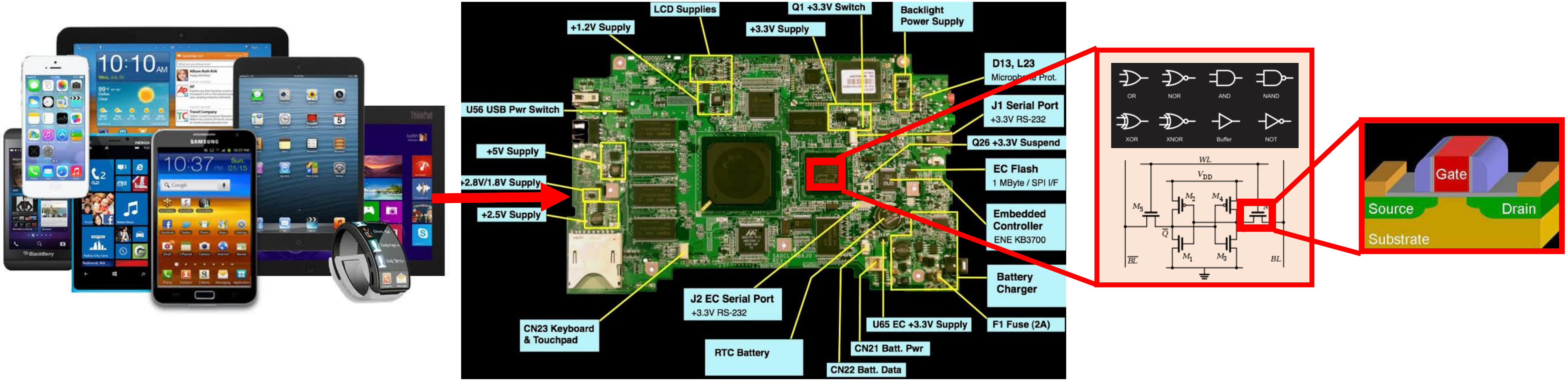
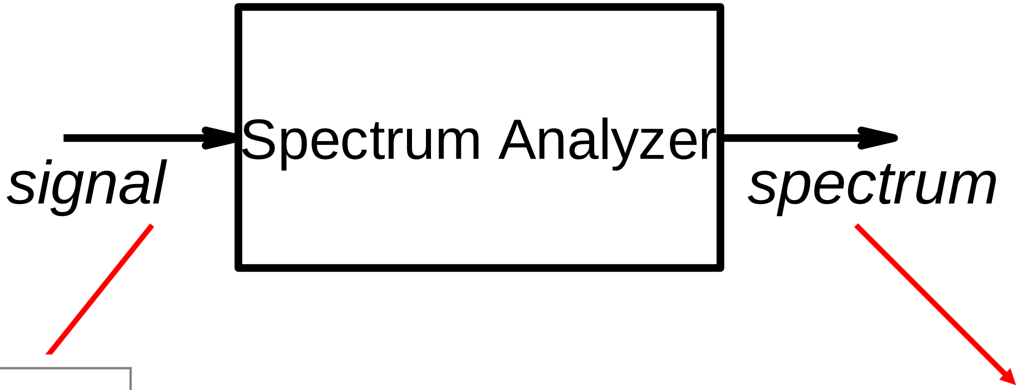


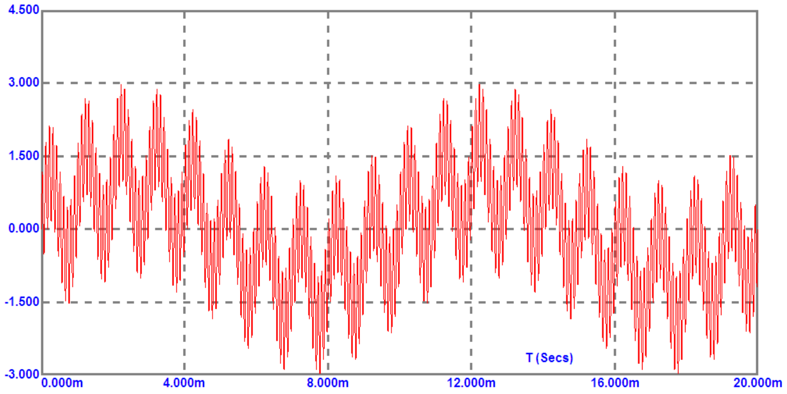
ESC201: Introduction to Electronics

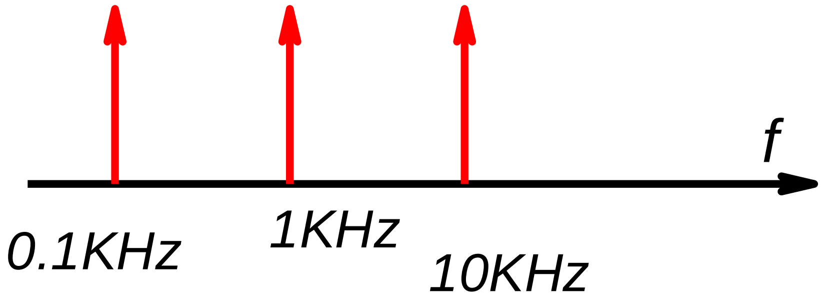
Module 3: Frequency Domain Analysis

Dr. Shubham Sahay,
Assistant Professor,
Department of Electrical Engineering,
IIT Kanpur

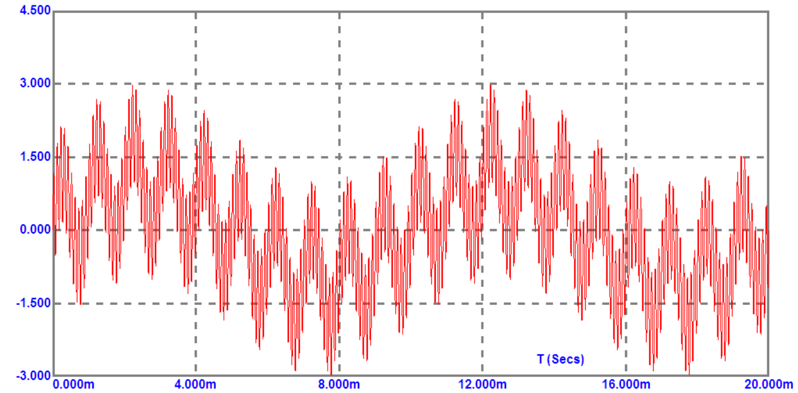
Spectrum


signal
Spectrum Analyzer
spectrum


4.500
3.000
1.500
0.000
-1.500
-3.000
0.000m
4.000m
8.000m
12.000m
16.000m
20.000m
T (Secs)


f
0.1KHz
1KHz
10KHz

Frequency Domain



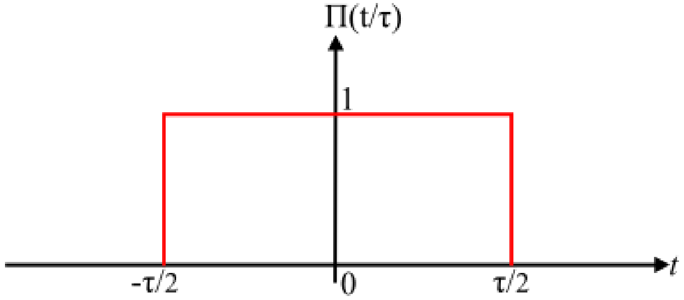
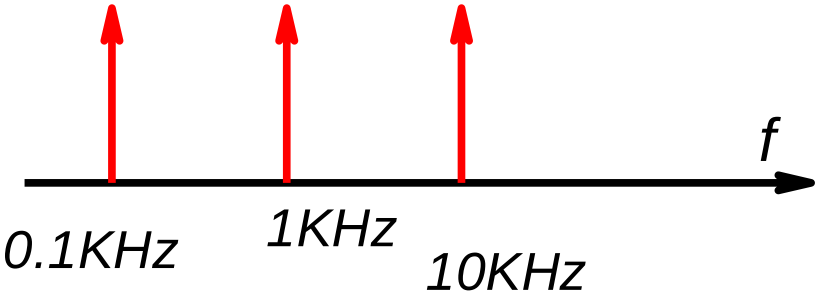
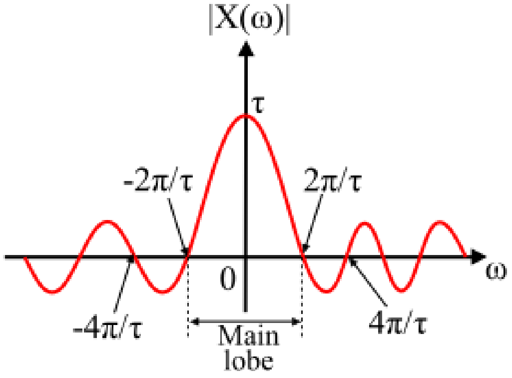


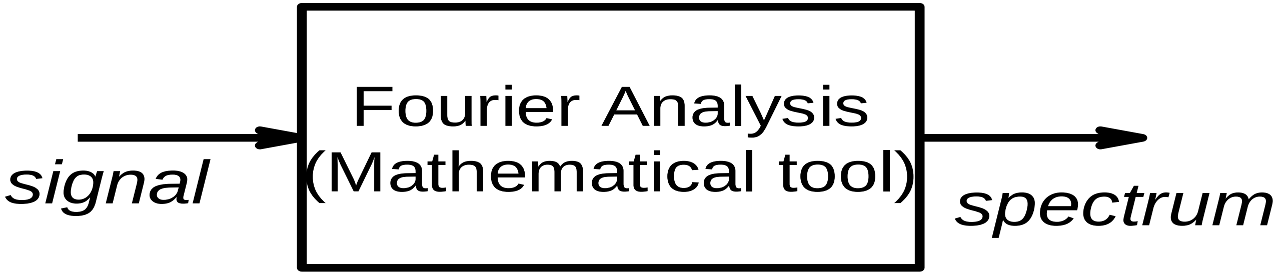
Figure-1



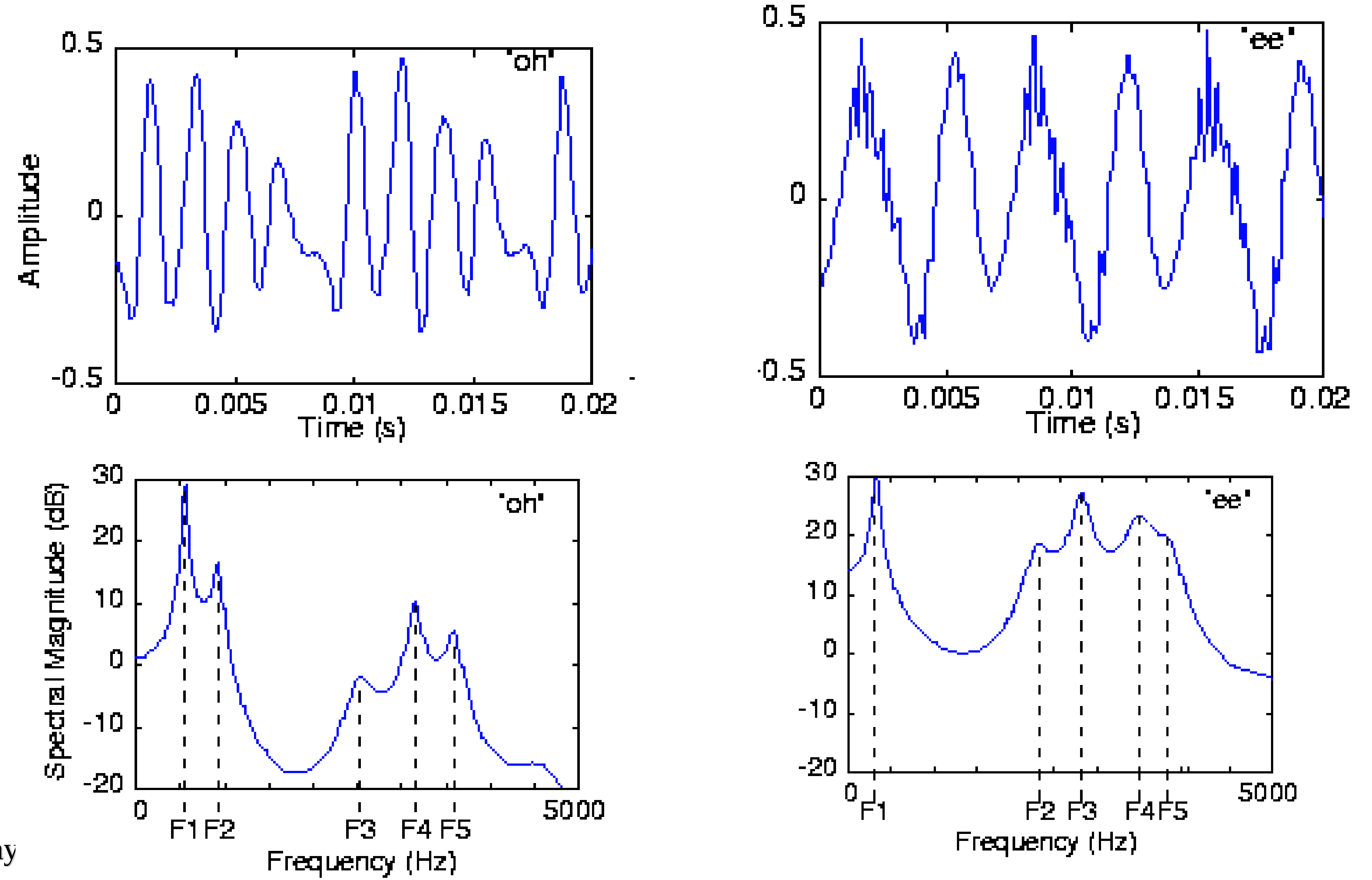


Magnitude Spectrum

Any signal can be written as the summation of many sinusoids

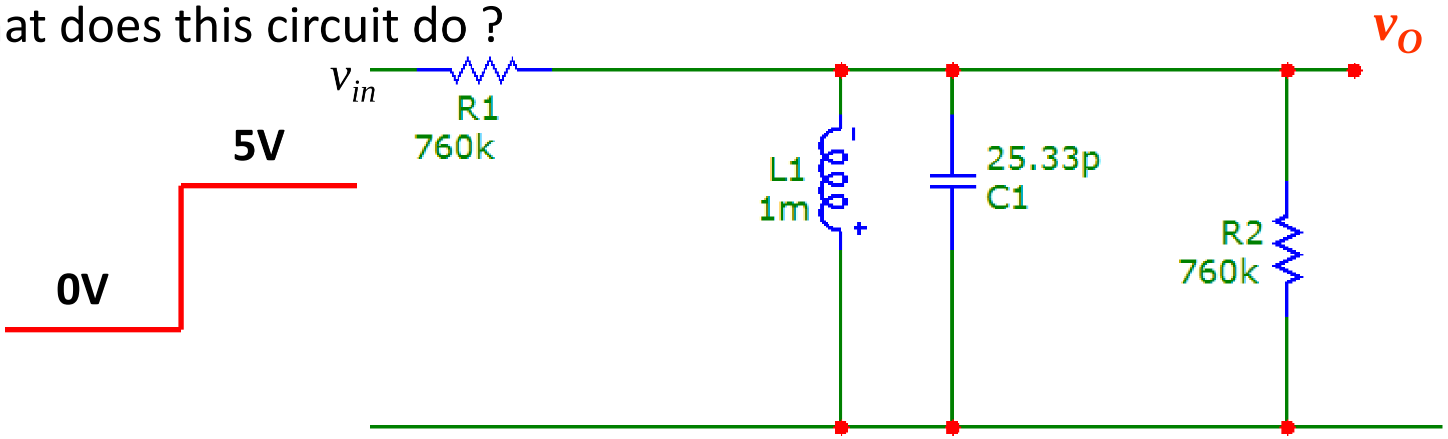


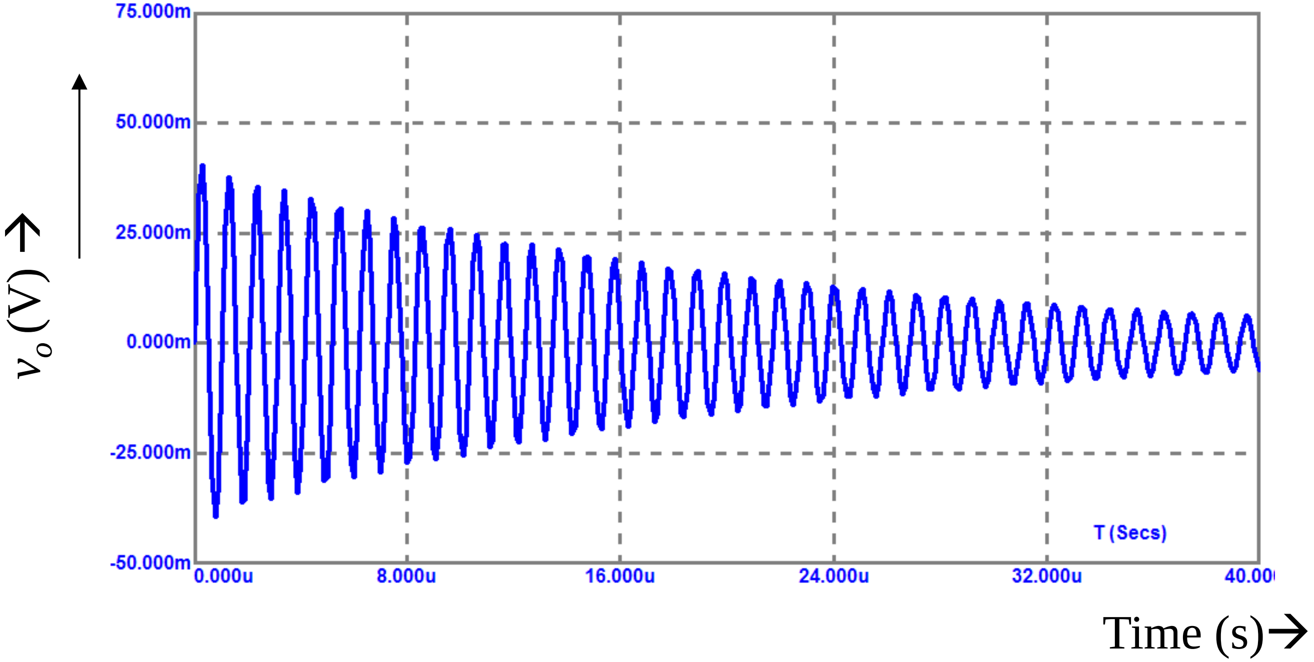
Example: Speech Signals



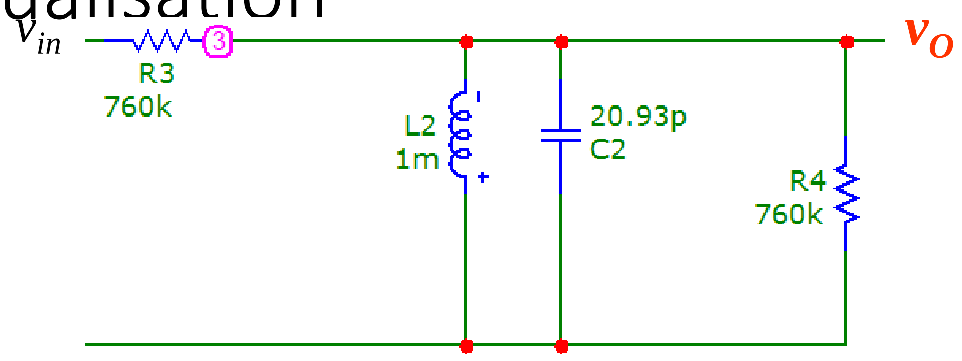
System Response in Time Domain

What does this circuit do ?



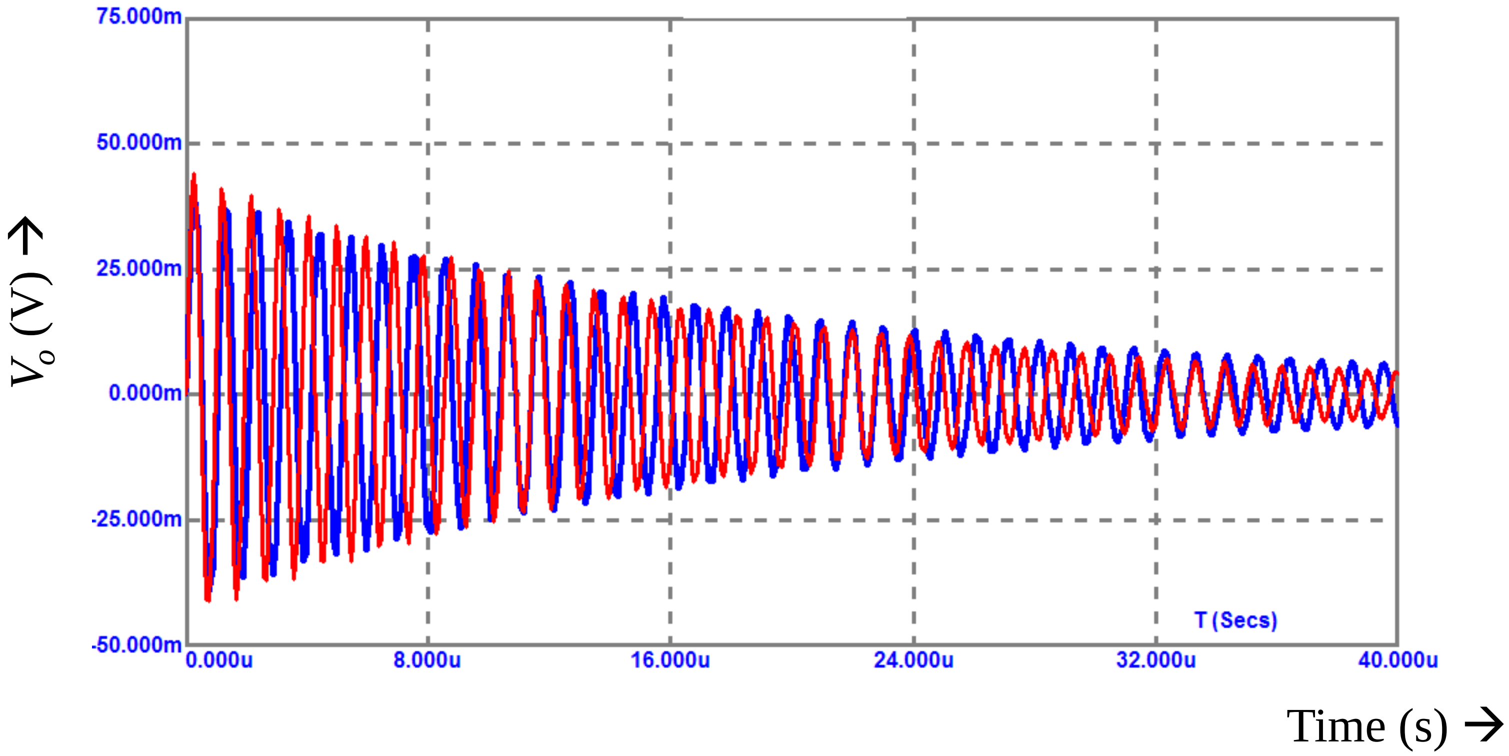


Limitations of Time Domain Visualisation



Capacitance was ~25 pF earlier

Suppose capacitance is reduced to ~21 pF

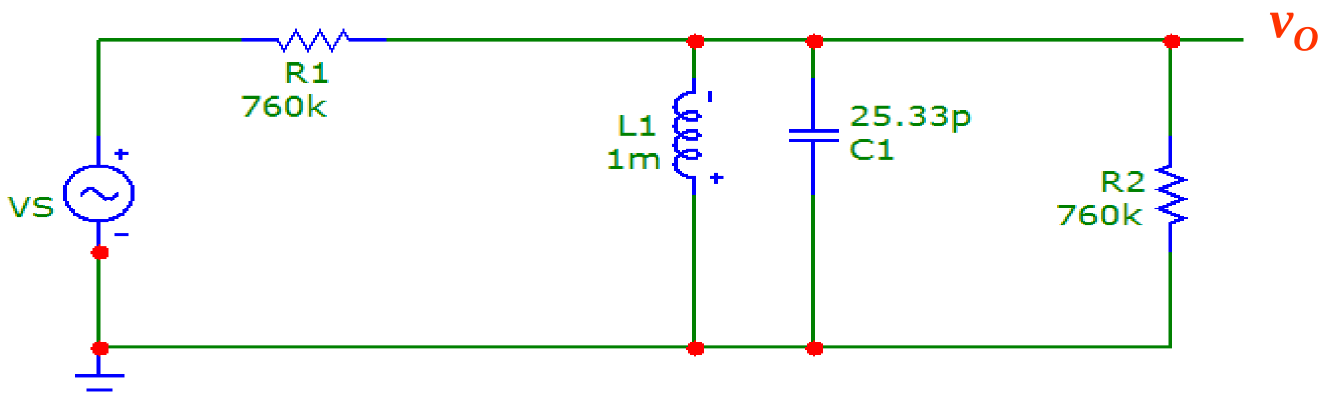


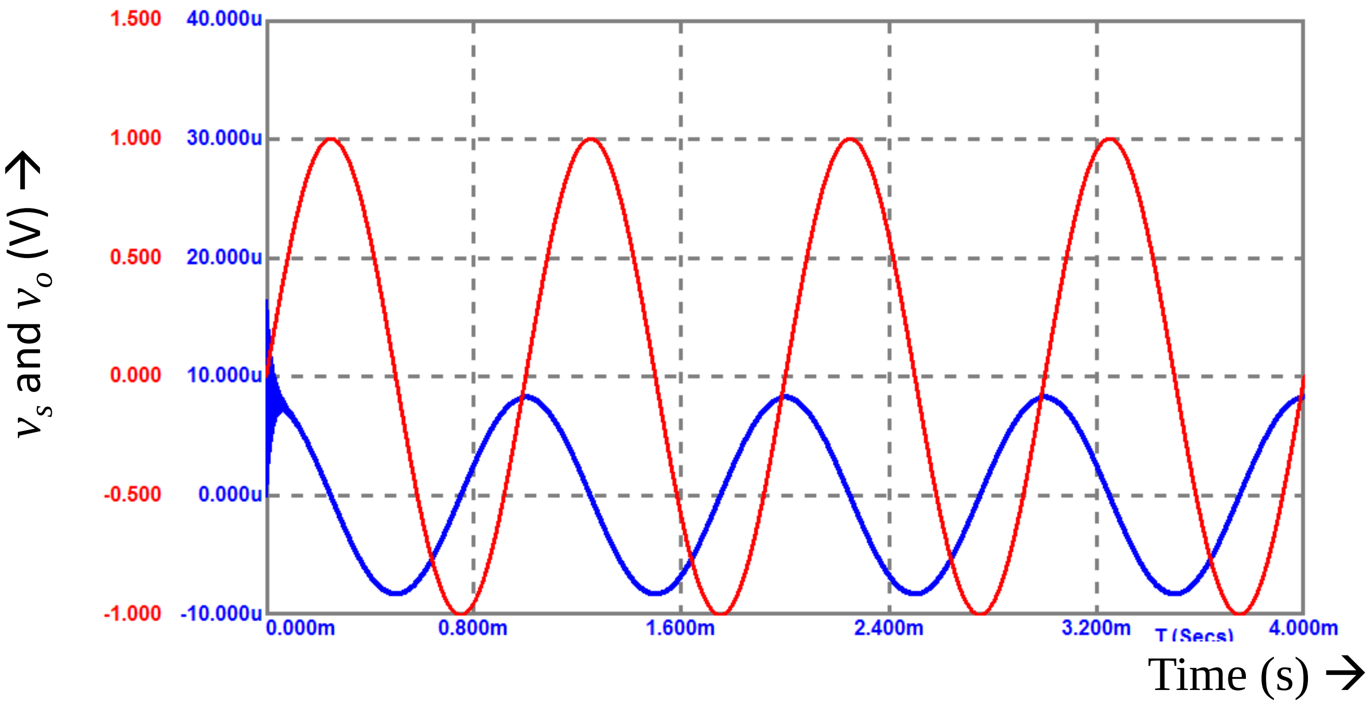
It is hard to find out what impact the change in capacitor has on circuit behaviour

Response for Sinusoidal Signal

Apply a pure sinusoidal signal

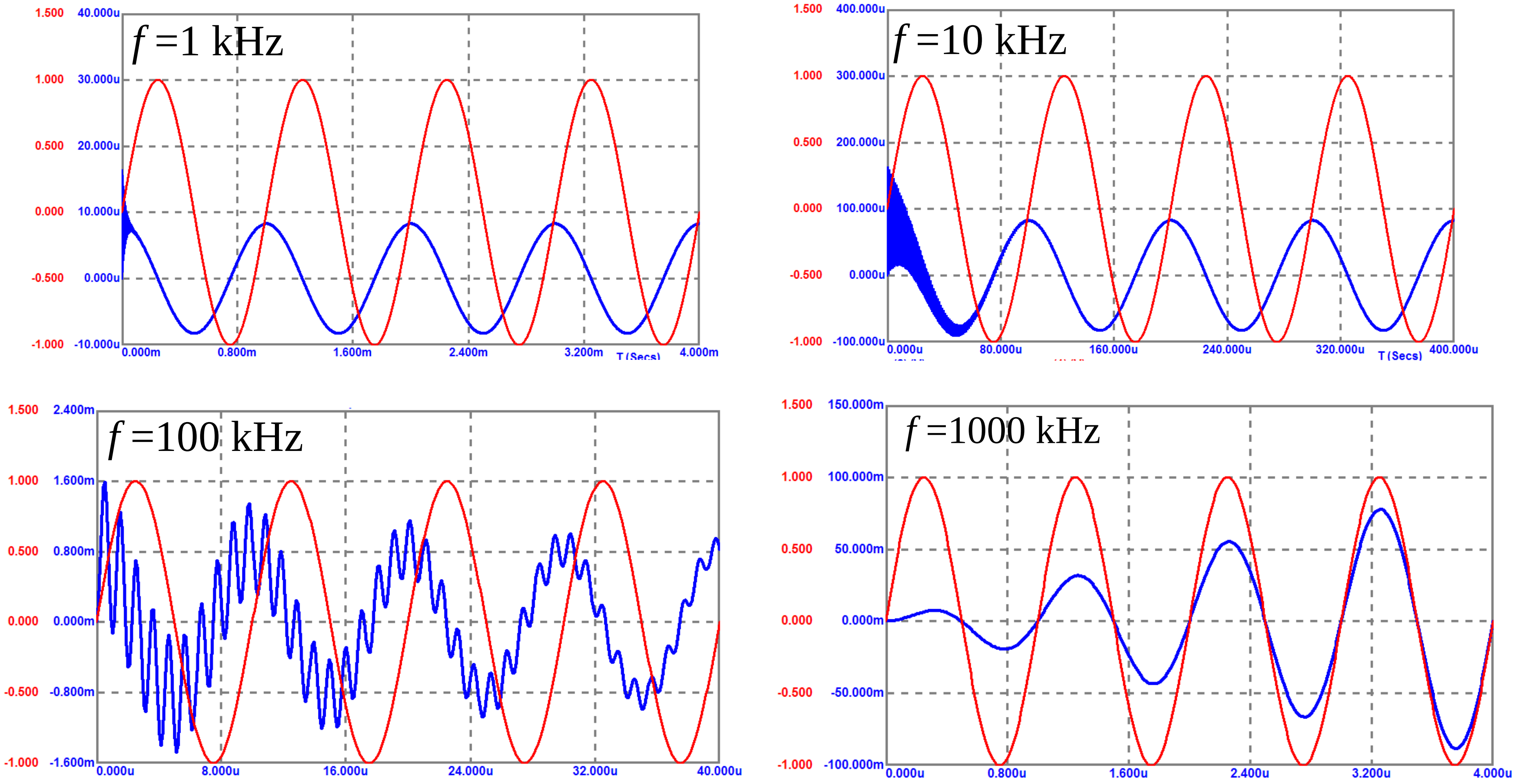
$f = 1$ kHz





Different Frequency Sinusoidal Inputs

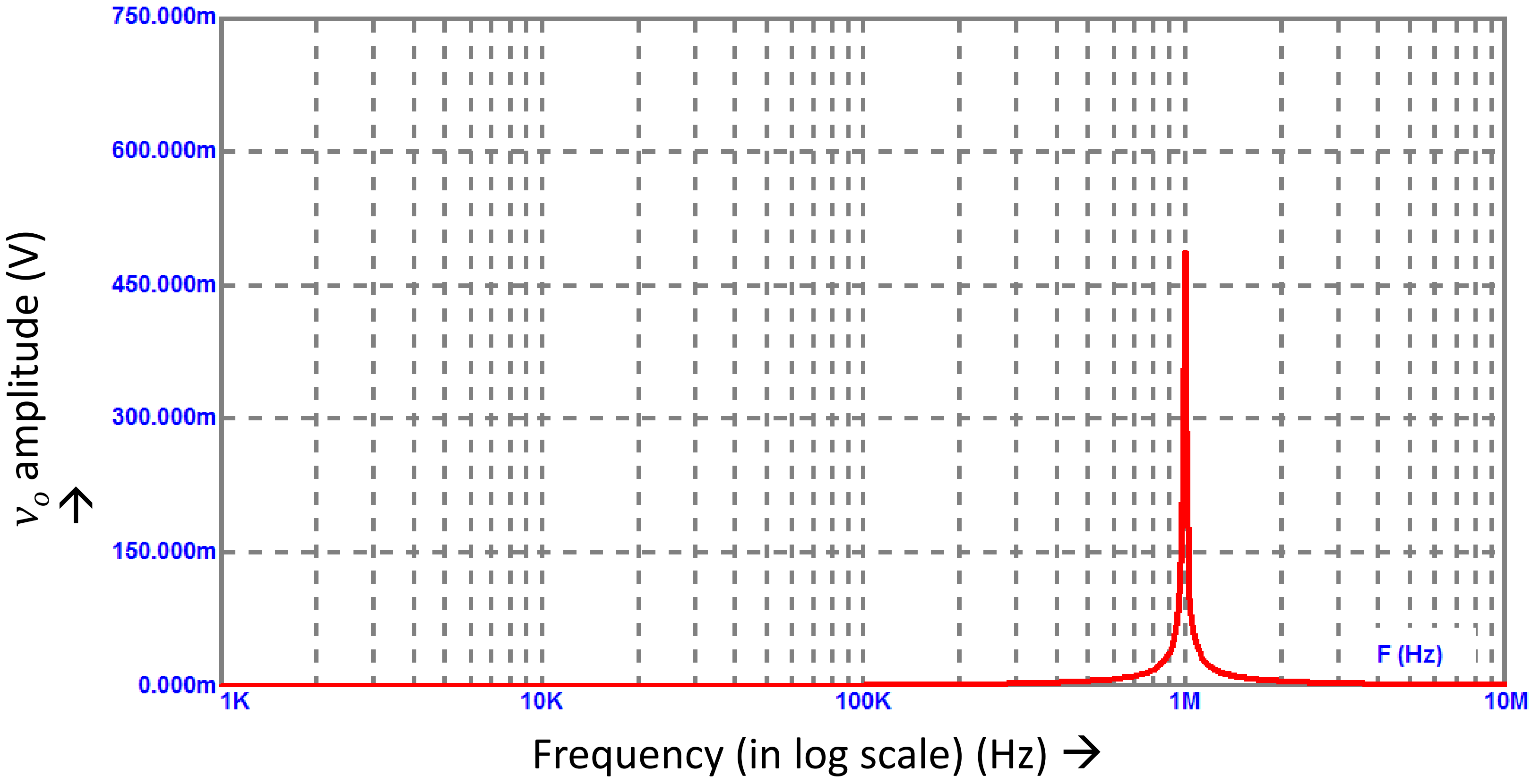
Measure response at many different frequencies for a constant input amplitude



Why not Plot the amplitude and phase as a function of frequency?

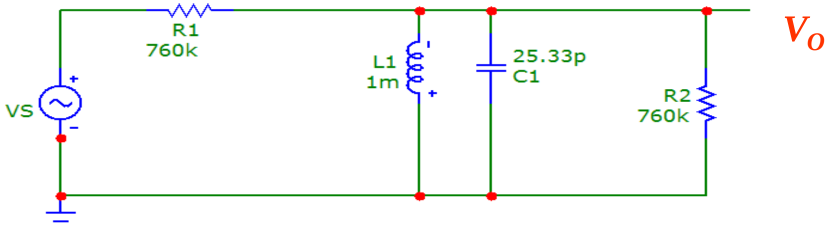
Output Amplitude with Frequency

Amplitude as a function of frequency

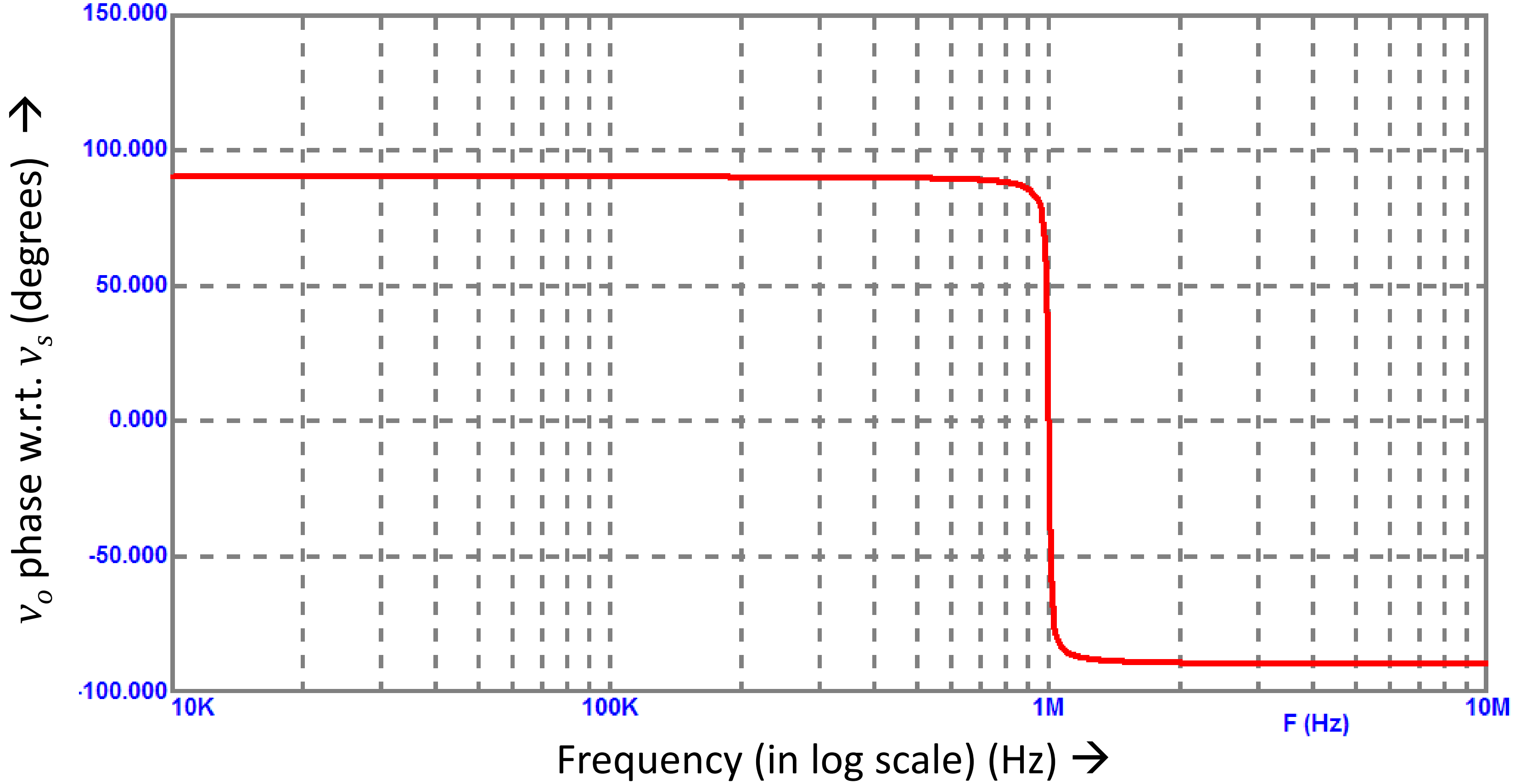


- One can clearly see the frequency selective nature of the circuit
- The frequency selective nature if often called a filter

Output Phase with Frequency



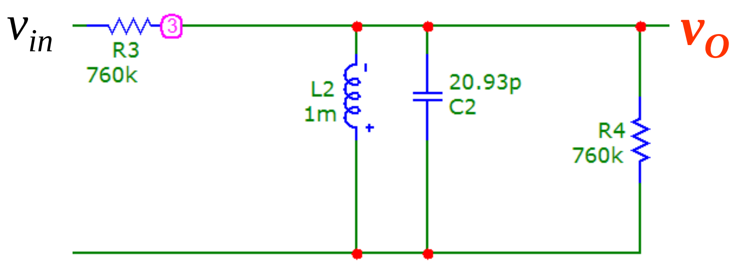
Phase as a function of frequency



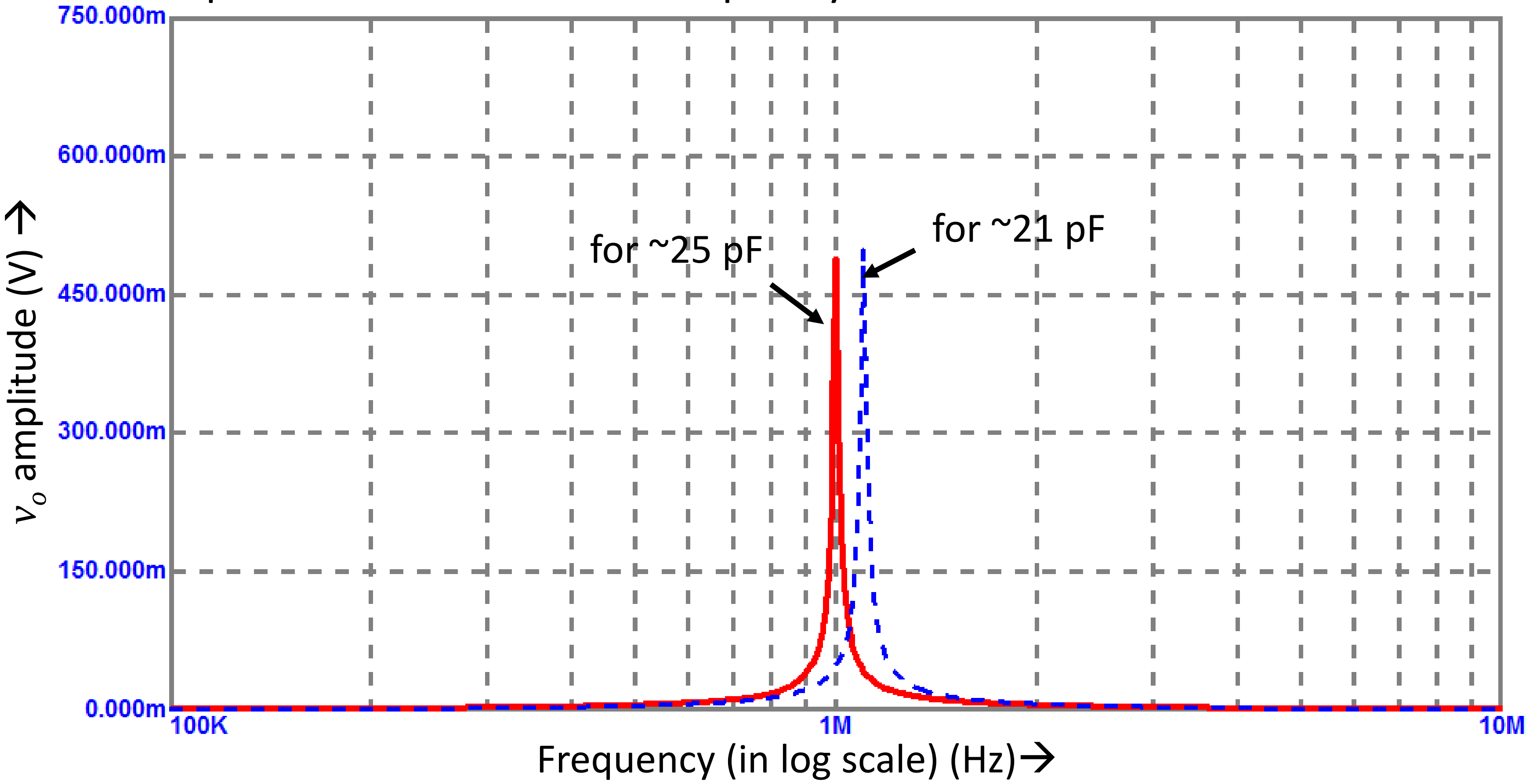
Effect of Tuning the Circuit

Capacitor was ~25 pF.

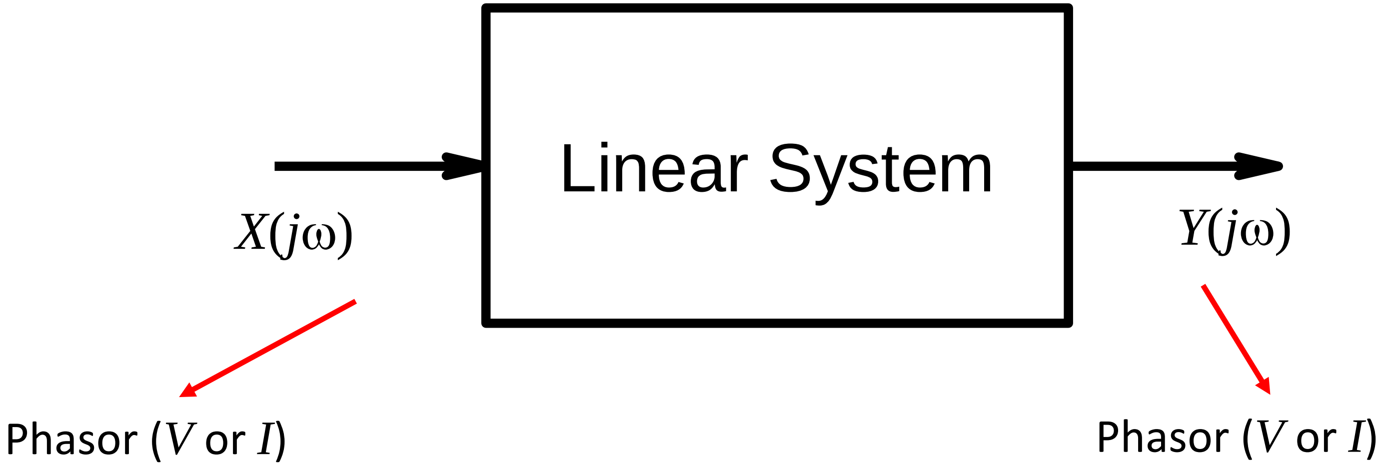
Suppose the capacitor is reduced to ~21 pF.



Amplitude as a function of frequency



Frequency Domain Analysis



Transfer function: $H(j\omega) = \dfrac{Y(j\omega)}{X(j\omega)}$

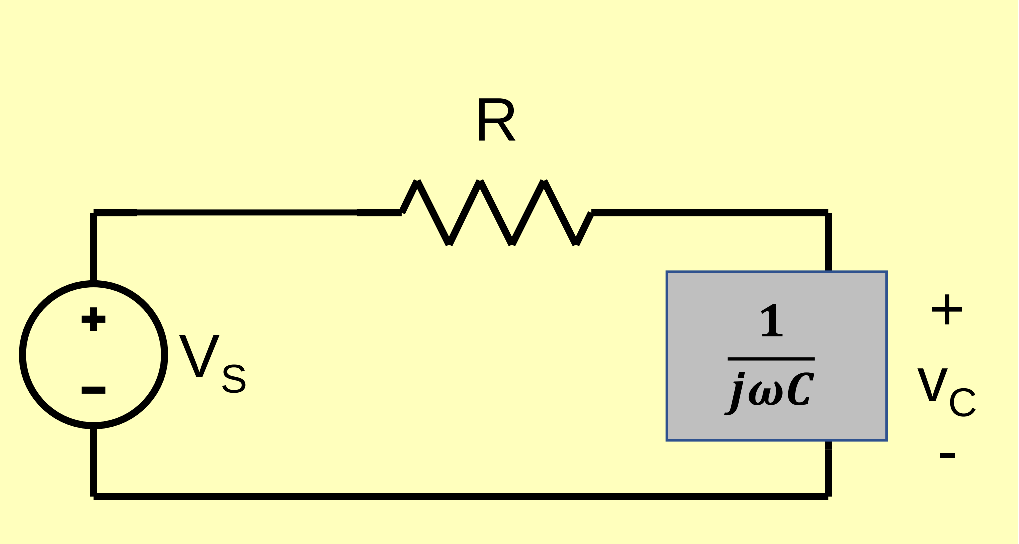
- Transfer function has a magnitude and a phase
- Transfer function is a useful tool for finding the frequency response of a system

Frequency Response

$$V_c = V_s \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = V_s \boxed{\frac{1}{j\omega RC + 1}}$$

$H(\omega\ )$

$$V_c(\omega) = V_s(\omega)H(\omega)$$



$$H(\omega) = \frac{V_c(\omega)}{V_s(\omega)}$$

Frequency Response

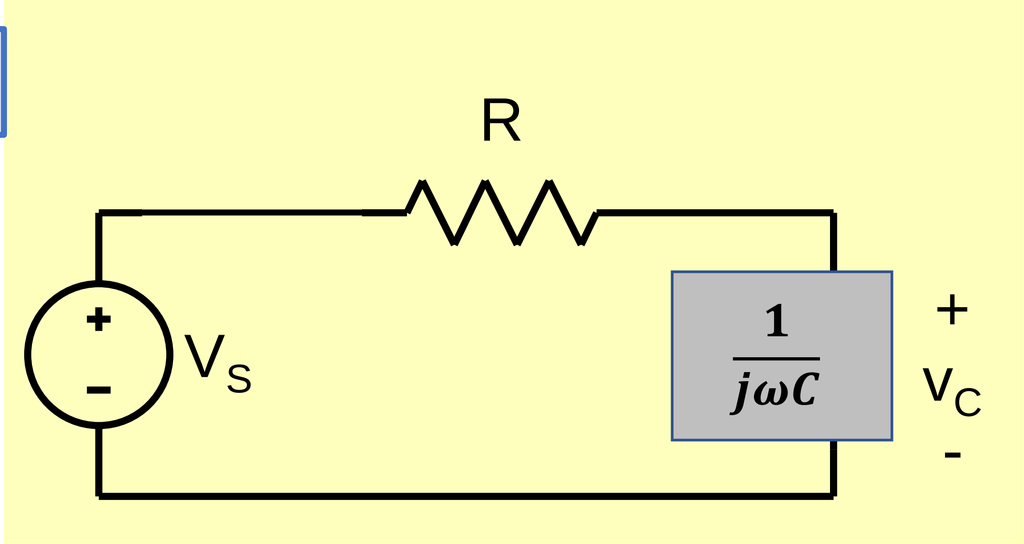
Transfer Input to Output:
Transfer function

Behavior of Circuit in Frequency Domain

$$v_s(t) = v_{s1}\cos(\omega_1 t) + v_{s2}\cos(\omega_2 t)$$

$$V_c = V_s \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = V_s \boxed{\frac{1}{j\omega RC + 1}}$$

$H(\omega)$



$$v_c(t) = v_{s1}|H(\omega_1)|\cos\left(\omega_1 t + \angle H(\omega_1)\right) + v_{s2}|H(\omega_2)|\cos\left(\omega_2 t + \angle H(\omega_2)\right)$$

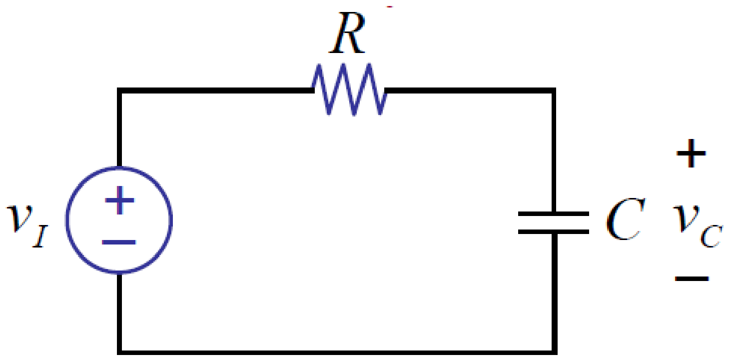
Frequency Domain Analysis

What if we give signal consisting of two sinusoids of frequencies ω_1 and ω_2

Linearity

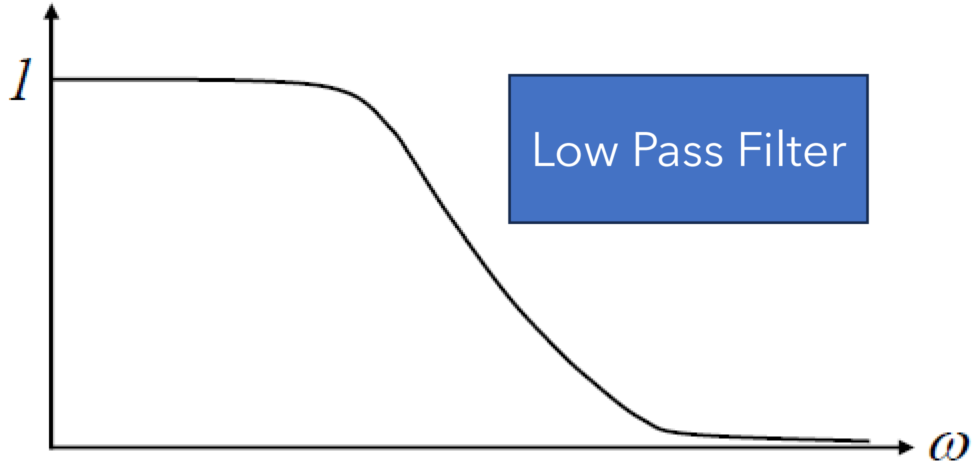
Response is sum of individual responses

Transfer function of RC



$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

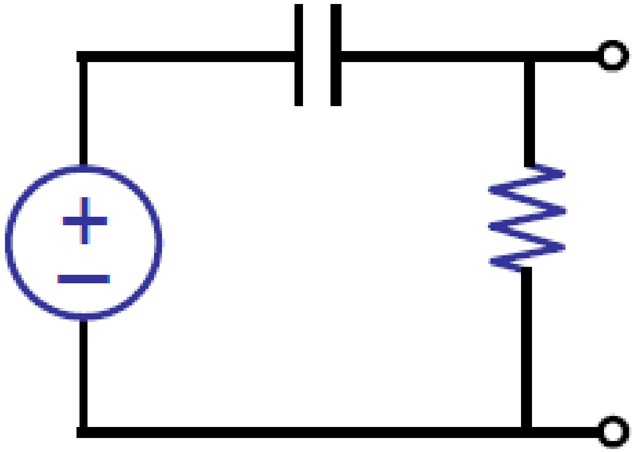


$$\left|H(j\omega)\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

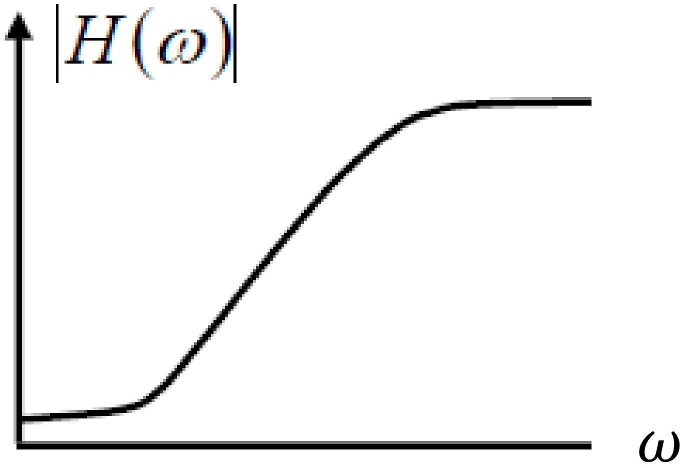
$$\phi(\omega) = -\tan^{-1}(\omega CR)$$

At low ω	~ 1	$\sim 0^o$
At high ω	$\sim \frac{1}{\omega}$	$\sim -90^o$

High-pass filter using RC Circuit



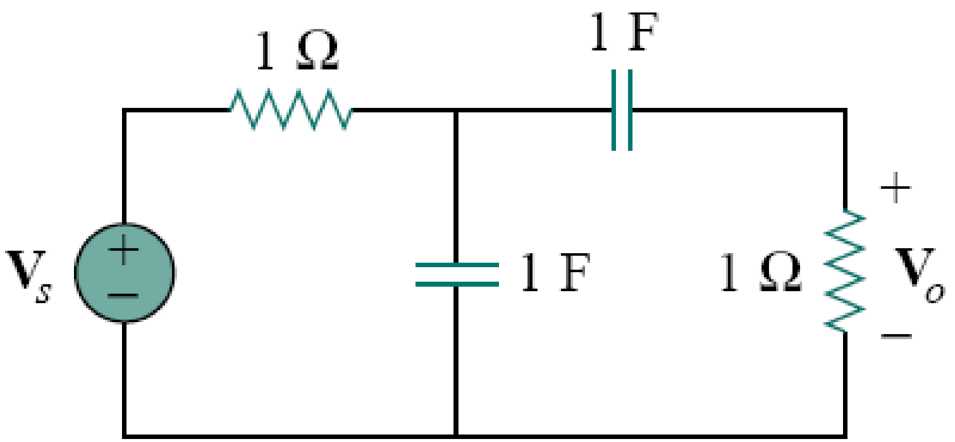
$$V_c = V_s \frac{R}{R + \frac{1}{j\omega C}} = V_s \frac{j\omega RC}{j\omega RC + 1}$$



High Pass Filter

$$\varphi(\omega) = 90 - \tan^{-1}(\omega CR)$$

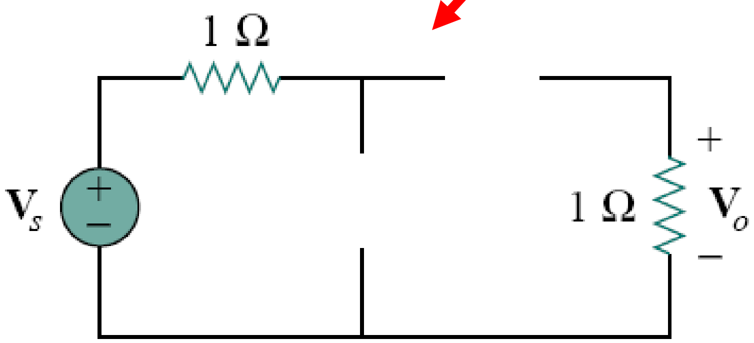
Example

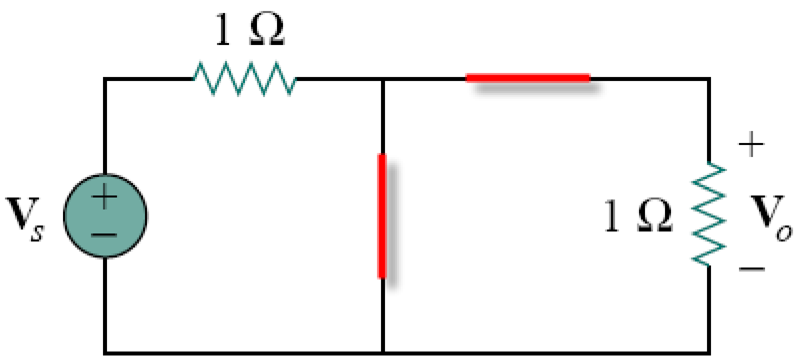


A capacitor offers
very high impedance at low frequencies and
very low impedance at high frequencies.

Low *f*

High *f*





Try writing its
Exact response

bandpass filter

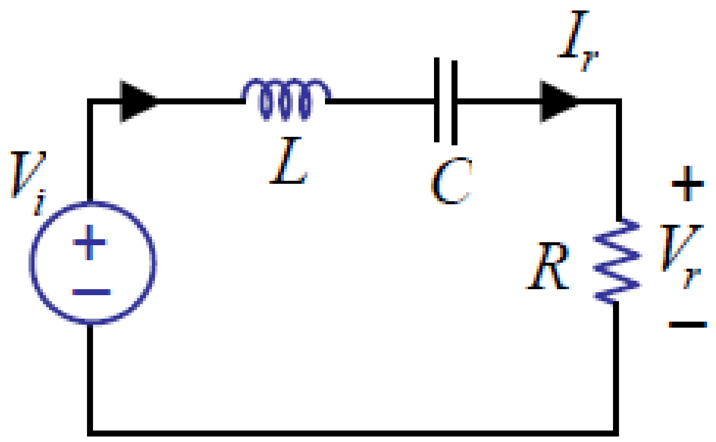
Example: RLC Circuit

$$\frac{V_r}{V_i} = \frac{R}{j\omega L + \dfrac{1}{j\omega C} + R}$$

$$= \frac{j\omega RC}{1 - \omega^2 LC + j\omega RC}$$

$$= \frac{j\omega RC}{\left(1-\omega^2 LC\right) + j\omega RC} \cdot \frac{\left(1-\omega^2 LC\right) - j\omega RC}{\left(1-\omega^2 LC\right) - j\omega RC}$$

$$\left|\frac{V_r}{V_i}\right| = \frac{\omega RC}{\sqrt{\left(1-\omega^2 LC\right)^2 + (\omega RC)^2}}$$



Observe:

$$\text{Low } \omega: \quad \approx \omega RC$$

$$\text{High } \omega: \quad \approx \frac{R}{\omega L}$$

$$\omega\sqrt{LC} = 1: \quad \approx 1$$

Transfer function of RLC

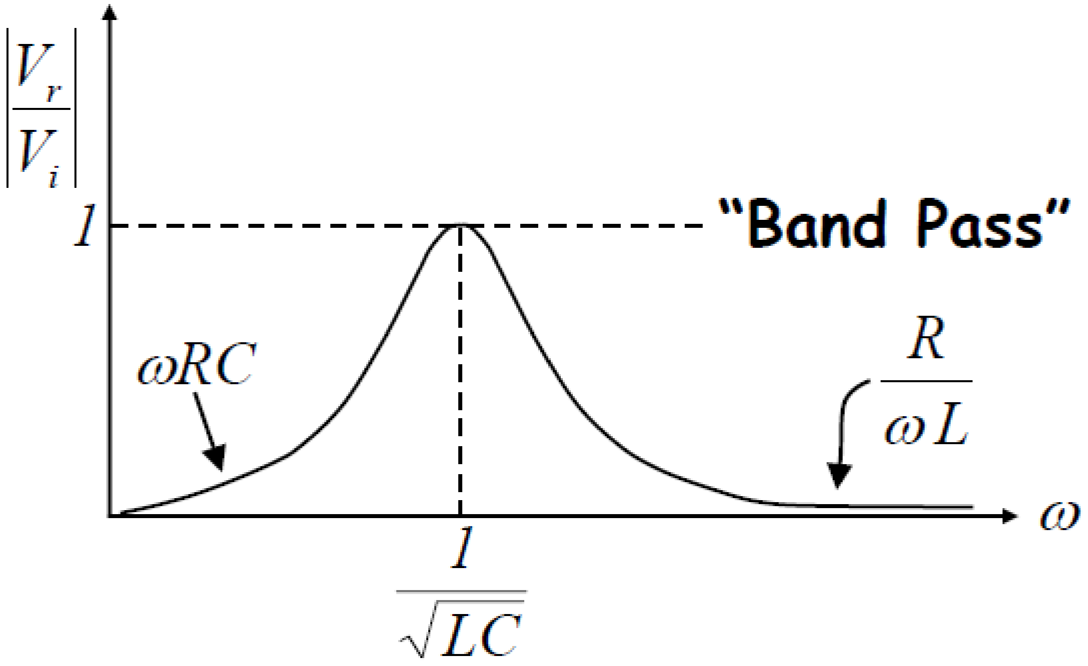
Observe:

$$\text{Low } \omega: \quad \approx \omega RC$$

$$\text{High } \omega: \quad \approx \frac{R}{\omega L}$$

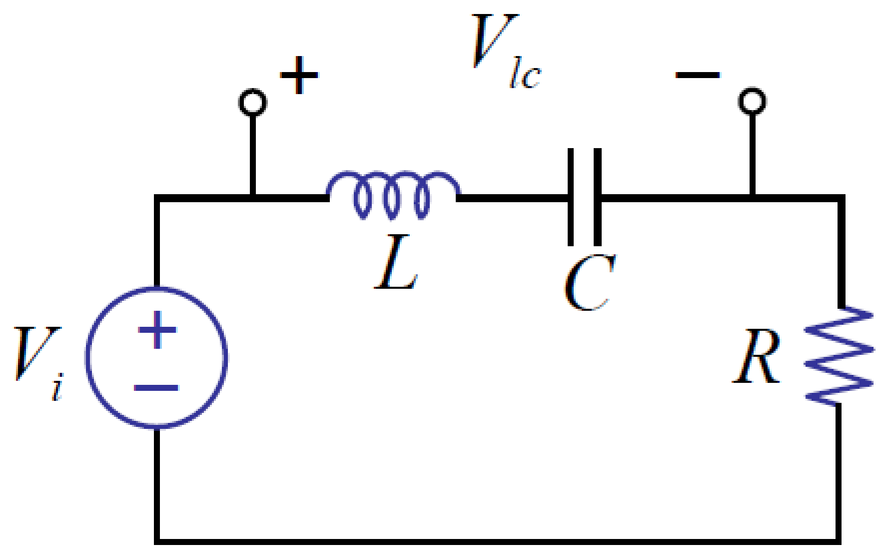
$$\omega\sqrt{LC} = 1: \quad \approx 1$$

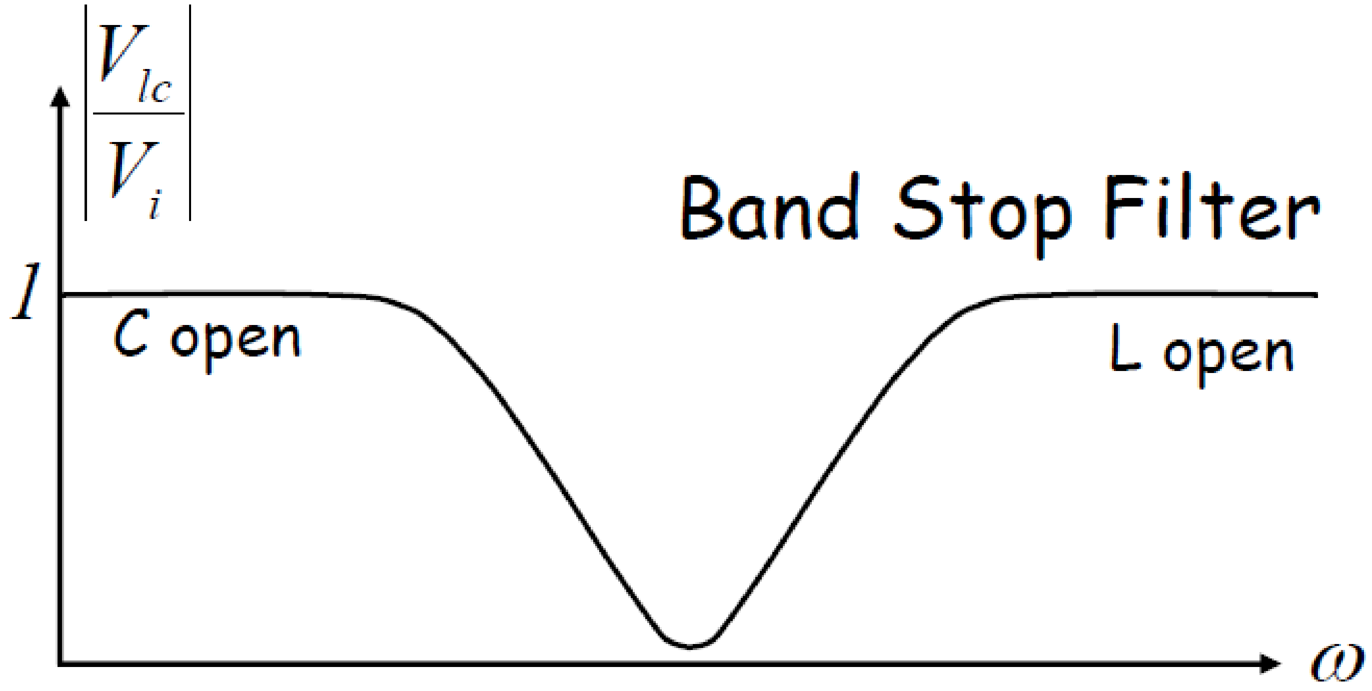
$$\left|\frac{V_r}{V_i}\right| = \frac{\omega RC}{\sqrt{\left(1-\omega^2 LC\right)^2 + \left(\omega RC\right)^2}}$$



Remember this trick to sketch the form of transfer functions quickly.

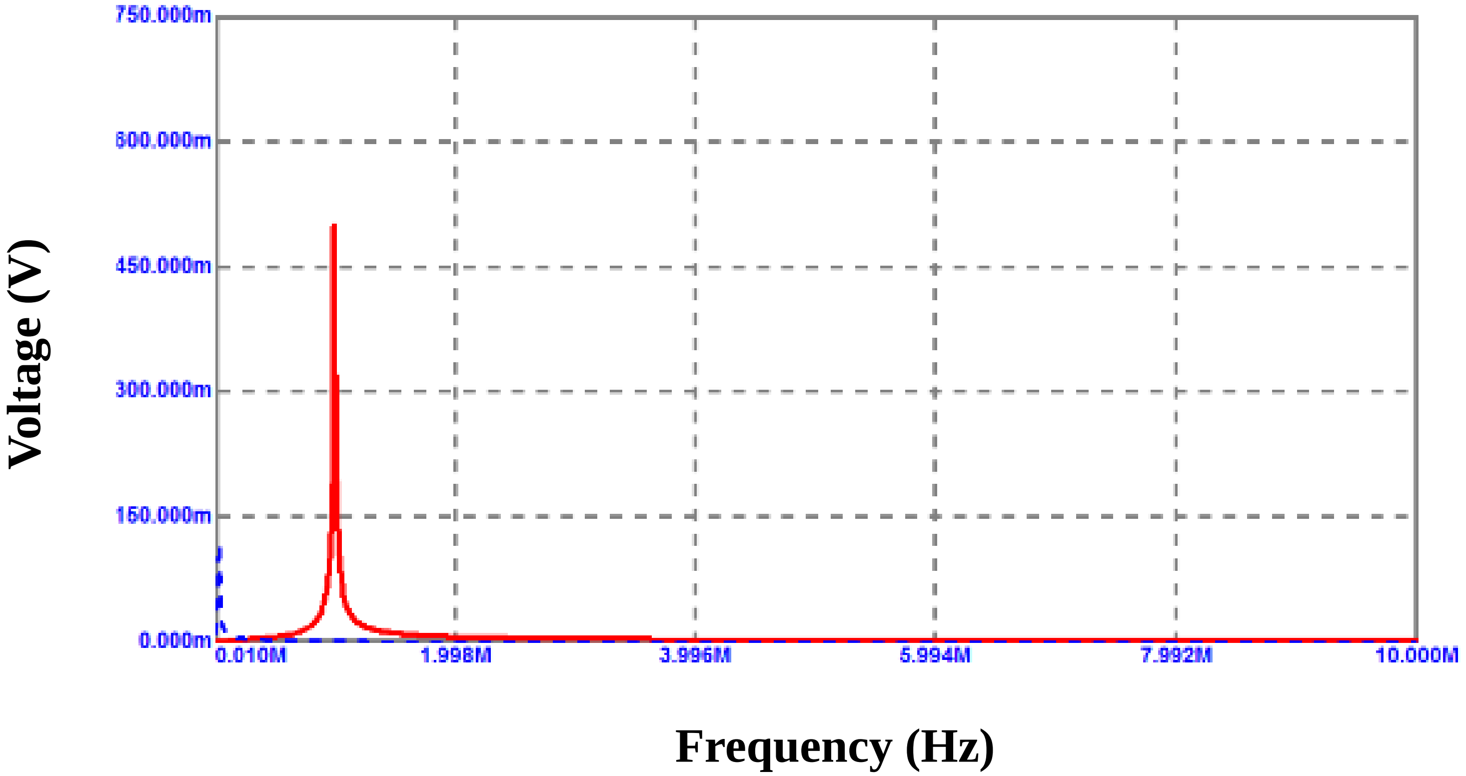
Example: RLC Circuit





Response at Output

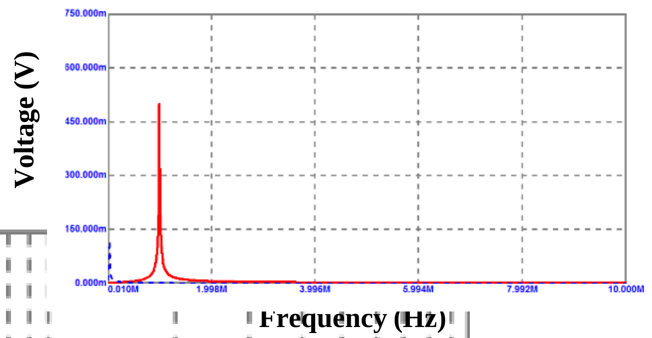
What if we plot output value on a linear-linear scale?

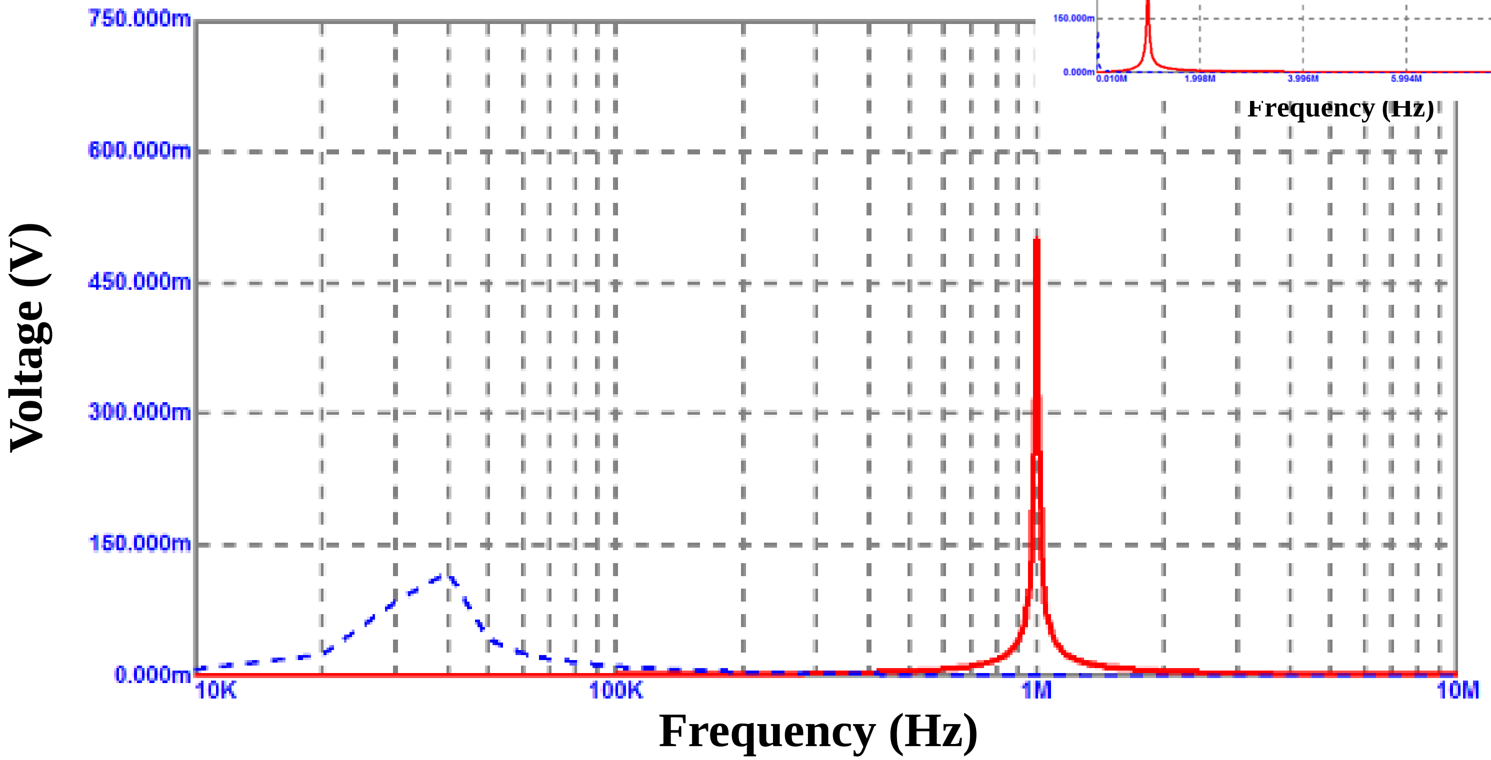


- Only higher frequency data is visible in linear scale
- Information of what happens at low frequency remains hidden due to the scale

Frequency on Abscissa in Log Scale

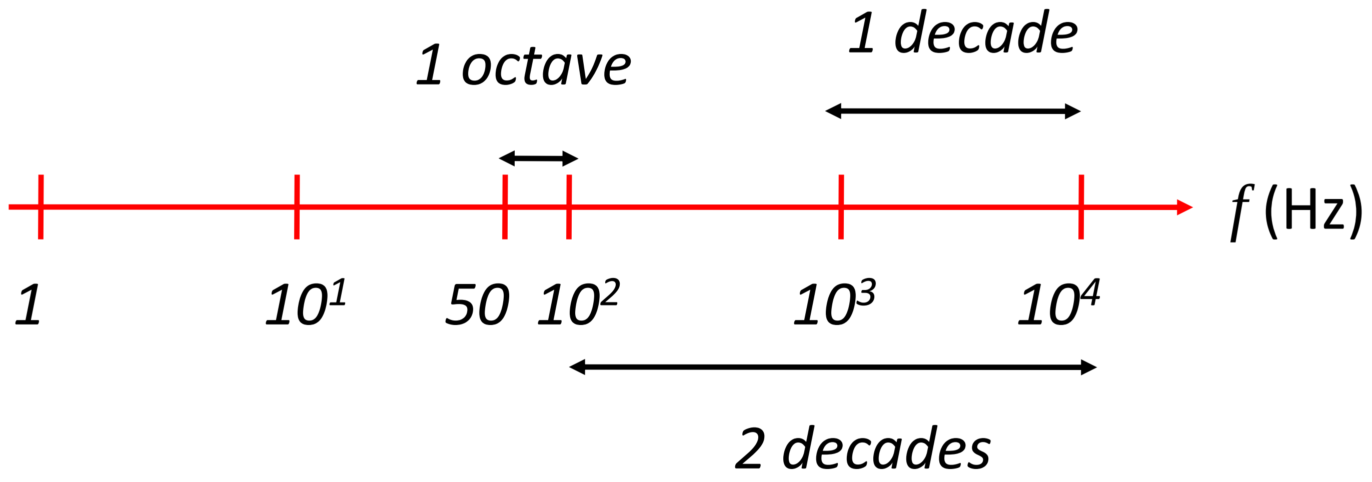
Plot output value on linear-log scale





- Now we are able to see what is happening at the lower frequencies
- But how does the response change in between?

Logarithmic Frequency Scale



$$\text{Number of decades} = \log_{10}\left(\frac{f_2}{f_1}\right)$$

$$\text{Number of octaves} = \log_2\left(\frac{f_2}{f_1}\right) = \frac{\log_{10}(\frac{f_2}{f_1})}{\log_{10}(2)}$$

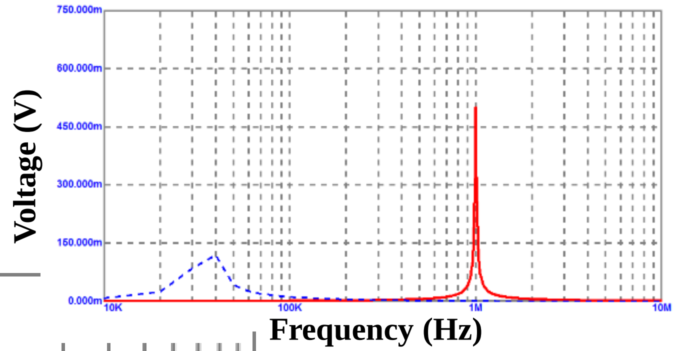
"Octave" comes from the eight notes in western classical music. Even in Indian classical music, while practicing the 'Saptaka', eight notes (including the start of the next octave) is sung.

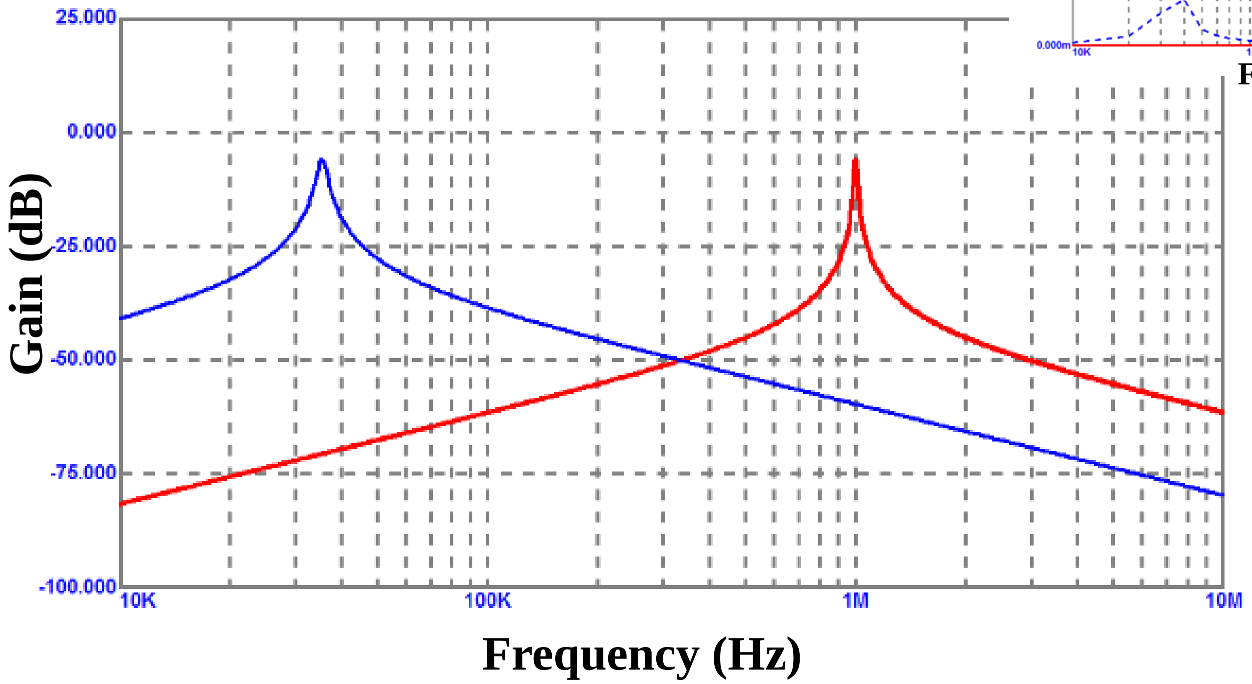
- In engineering, we often use $\omega = 2\pi f$ as the variable
- This just scales the axis by 2π; ratios remain the same!

What About the Ordinate?

Why not go for log-log representation?

If dimensionless ratio is possible,
we go for decibel representation

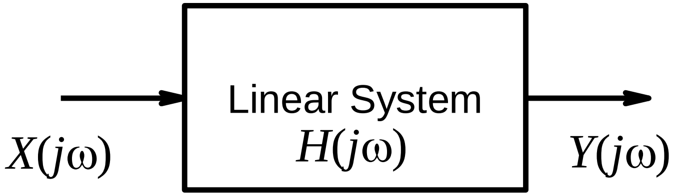




- Behaviour of the functions at intermediate frequencies are clearer now!
- Dimensional quantities, the log values indicate the ratios and are meaningful

Forms of Transfer Function

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$



$$H(j\omega) = \text{Voltage gain} = \frac{V_o(j\omega)}{V_i(j\omega)}$$

$$H(j\omega) = \text{Current gain} = \frac{I_o(j\omega)}{I_i(j\omega)}$$

Dimensionless quantities

$$H(j\omega) = \text{Transfer impendence} = \frac{V_o(j\omega)}{I_i(j\omega)}$$

$$H(j\omega) = \text{Transfer admittance} = \frac{I_o(j\omega)}{V_i(j\omega)}$$

Decibel

The magnitude of gain transfer function is often specified in **decibels**

$$G_{dB} = 10\log_{10}\left(\frac{P_2}{P_1}\right)$$

A popular unit to indicate increase in power as a ratio of base power

In Electrical analysis, power is proportional to V^2 or I^2

Voltage gain and Current gain in decibels is specified as

$$G_{dB} = 20\log_{10}\left(\frac{V_2}{V_1}\right) \qquad G_{dB} = 20\log_{10}\left(\frac{I_2}{I_1}\right)$$

Electrical systems we often represent gain in current or voltage

This is reason decibels when describing electrical systems have a factor of 20

dBm, dBm_w and dBW Units

Electronic engineers use some practical units which are not formal:

- dBm or dB_{mW}: Power with reference power as 1 mW

- dBW: Power with reference power as 1 W

- In telephone, reference resistance considered is 600 Ω
- In radio frequency world, it is 50 Ω

- Use of dB or dBm as absolute unite is discouraged in SI standards
- They supposed to be dimensionless unit for comparison

Representing Sound

Decibel scale is more convenient for our perception of hearing

Change in sound pressure level	Apparent change in loudness
3 dB	Just noticeable
5 dB	Clearly noticeable
10 dB	Twice or half as loud
20 dB	4 times or ¼ as loud

Sometimes the unit dB(A) with ambiance level noise as reference is used

The Central Pollution Control Board (CPCB) in India has laid down the permissible noise levels in India for different areas. Noise pollution rules have defined the acceptable level of noise in different zones for both daytime and night time.

In industrial areas, the permissible limit is 75 dB for daytime and 70 dB at night.

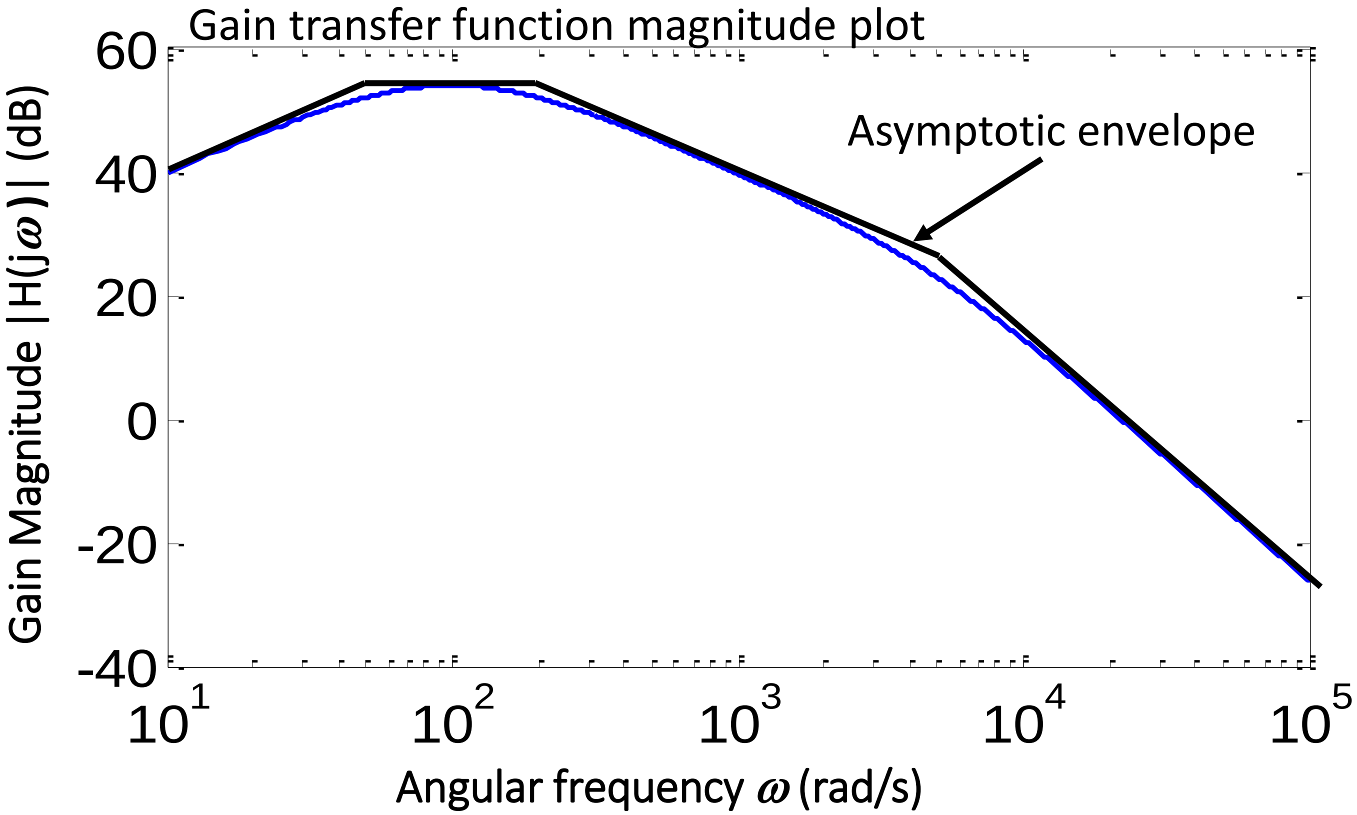
In commercial areas, it is 65 dB and 55 dB, while in residential areas it is 55 dB and 45 dB during daytime and night respectively.

https://www.downtoearth.org.in/blog/pollution/noise-pollution-violations-new-fines-proposed-by-cpcb-step-in-right-direction-72415

Decibel Scale

|H|	20Log10(|H|)
1000	60
100	40
10	20
2	6
$\sqrt{2}$	3
1	0
$1/\sqrt{2}$	-3
0.5	-6
0.1	-20
0.01	-40

Bode Plot



Hendrik Wade Bode

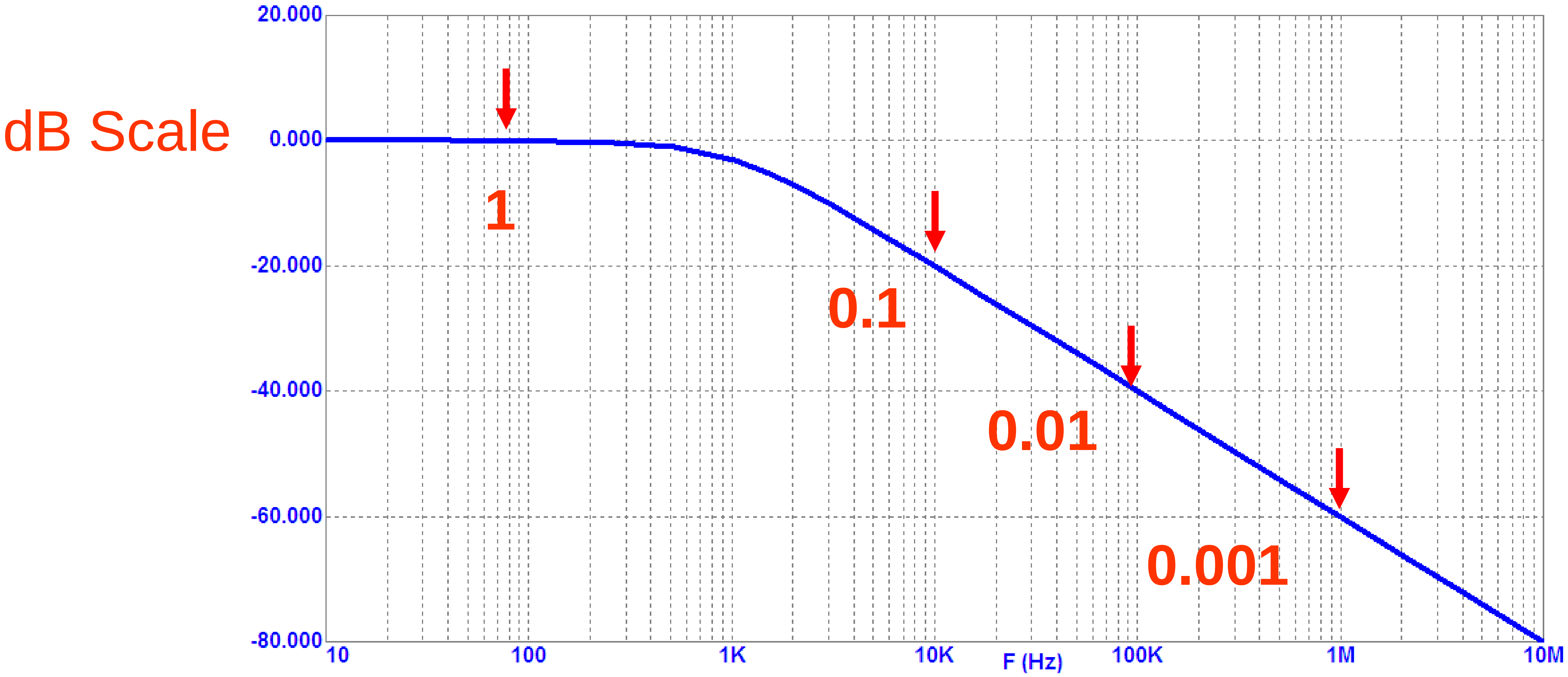
Wikipedia

Plots for rad/s and Hz will shift by a factor $\log_{10}(2\,\pi)$ laterally

Hendrik Bode developed a method to analyse these plots elegantly with asymptotes

The asymptotic plots or the plots themselves are often called Bode plot

Bode Plot



A plot of the decibel magnitude of transfer function versus frequency using a logarithmic scale for frequency is called a **Bode plot**