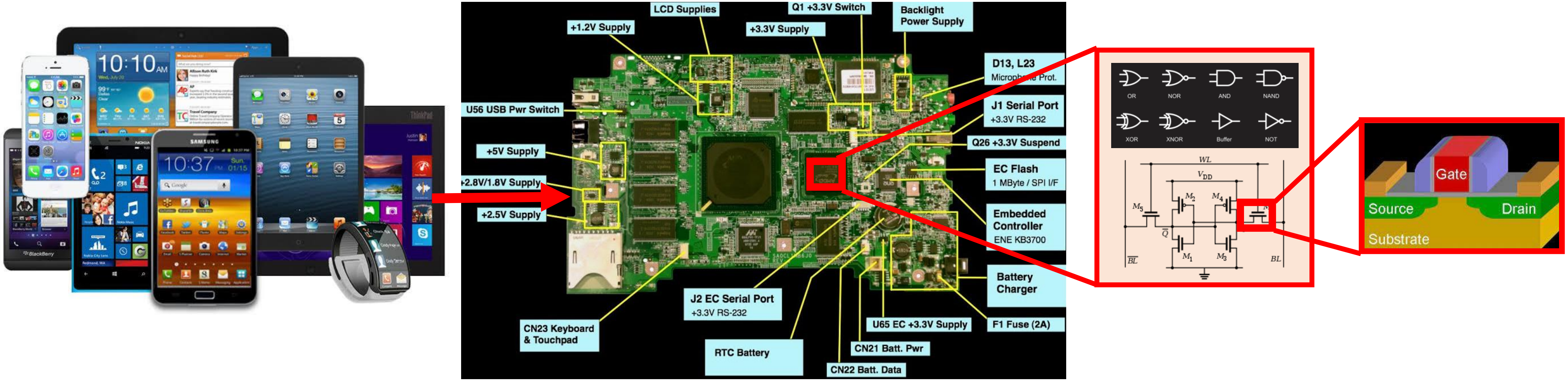


ESC201: Introduction to Electronics

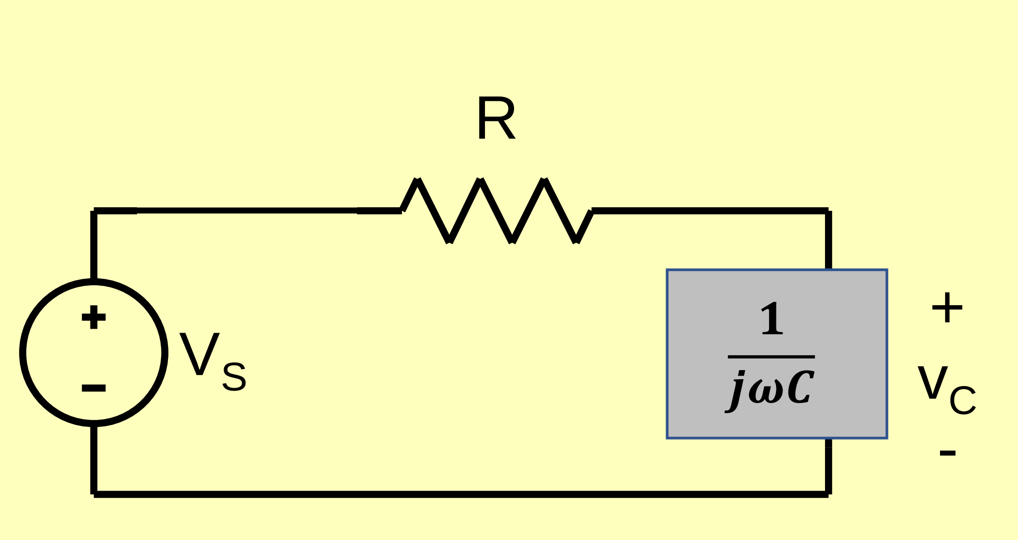
Module 3: Frequency Domain Analysis

Dr. Shubham Sahay,
Assistant Professor,
Department of Electrical Engineering,
IIT Kanpur

Frequency Response

$$V_c = V_s \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = V_s \boxed{\frac{1}{j\omega RC + 1}}$$

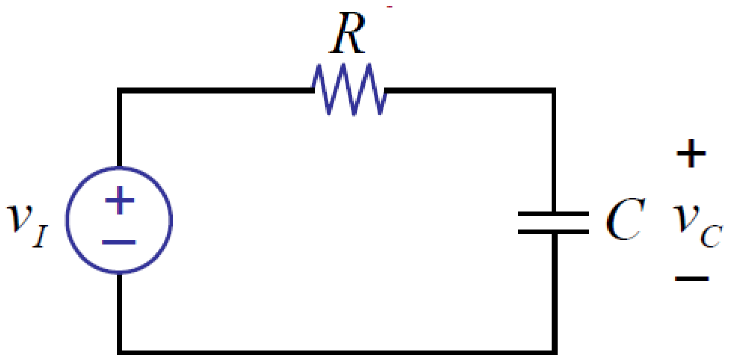
$H(\omega)$



$$V_c(\omega) = V_s(\omega)H(\omega)$$

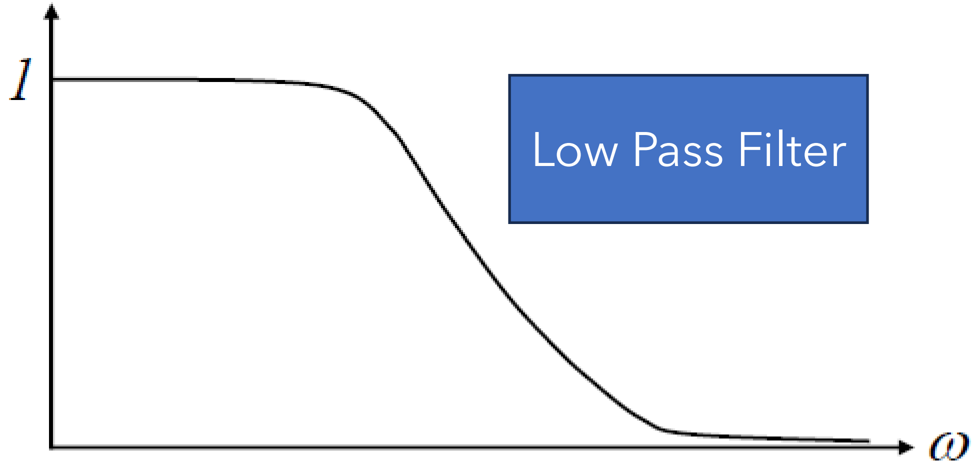
$$H(\omega) = \frac{V_c(\omega)}{V_s(\omega)}$$

Transfer function of RC



$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$



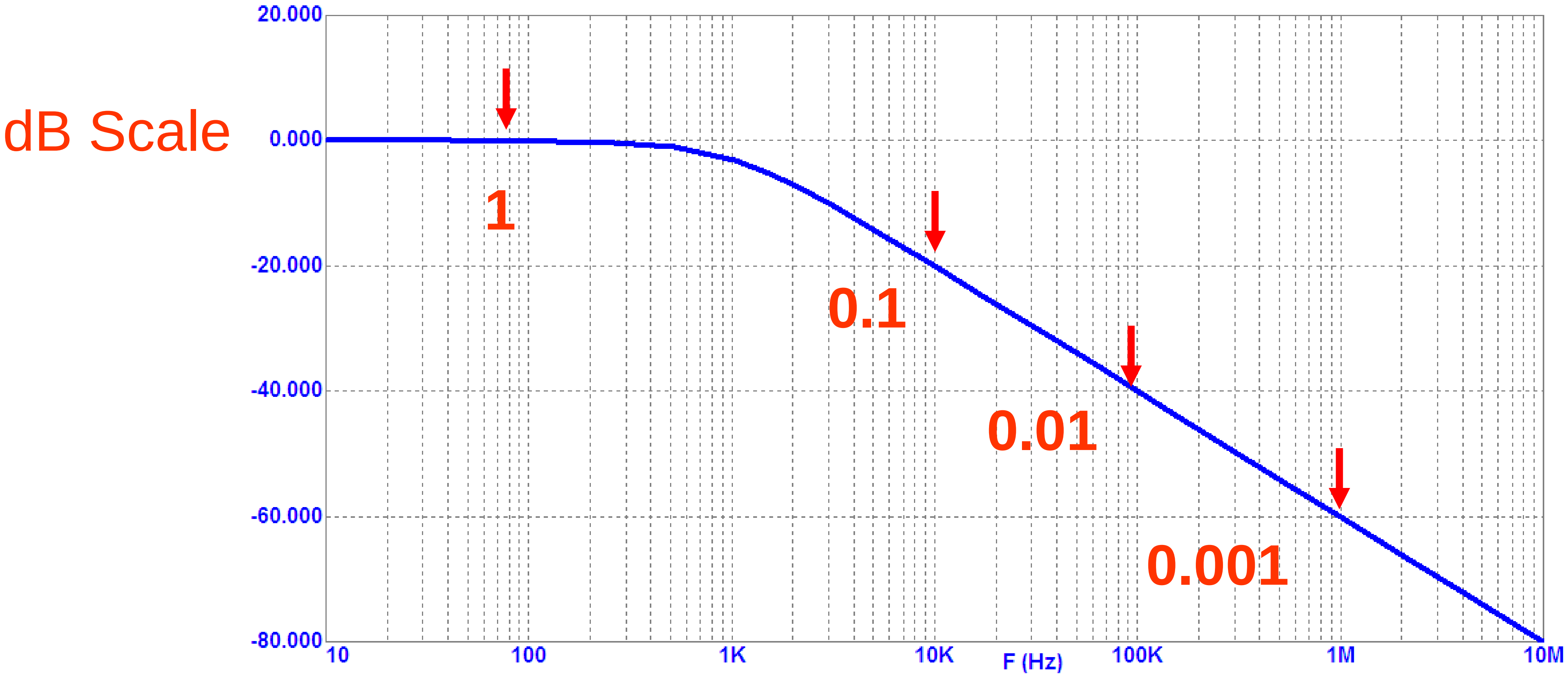
$$G_{dB} = 20\log_{10}\left(\frac{V_2}{V_1}\right)$$

$$\left|H(j\omega)\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$\phi(\omega) = -\tan^{-1}(\omega CR)$$

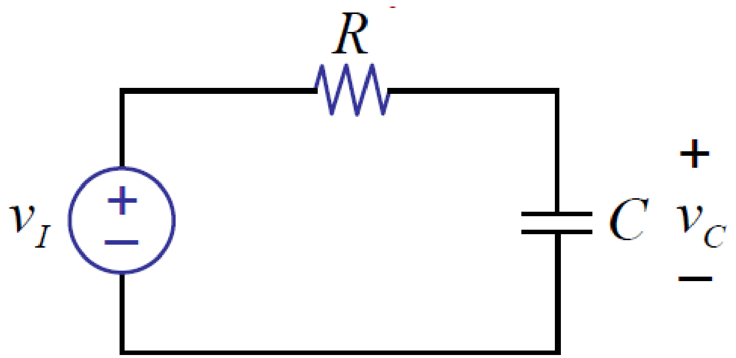
At low ω	~ 1	$\sim 0^o$
At high ω	$\sim \frac{1}{\omega}$	$\sim -90^o$

Bode Plot



A plot of the decibel magnitude of transfer function versus frequency using a logarithmic scale for frequency is called a **Bode plot**

Plotting RC transfer function



$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$\phi(\omega) = -\tan^{-1}(\omega CR)$$

$$|H|_{dB} = -20\log_{10}\sqrt{1+(\omega RC)^2}$$

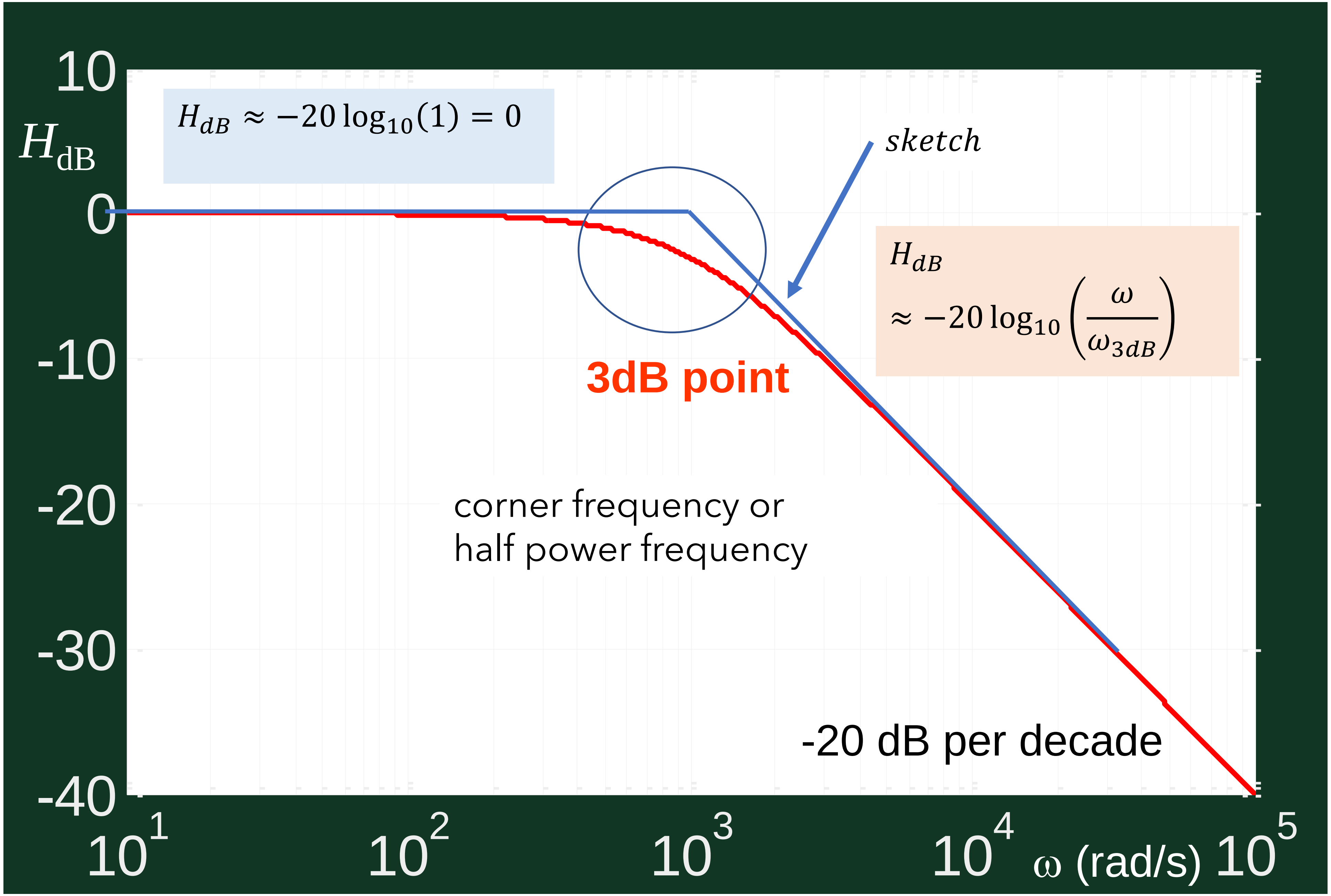
$$\omega = \frac{1}{RC} \Rightarrow |H(j\omega)| = \frac{1}{\sqrt{1+(1)^2}} = \frac{1}{\sqrt{2}} = -3dB$$

ω_{dB}

$$\boxed{|H|_{dB} = -20\log_{10}\sqrt{1+\left(\frac{\omega}{\omega_{3dB}}\right)^2}}$$

$\omega << \omega_{3dB}$ | $|H|_{dB} \approx -20\log_{10}(1) = 0$

$\omega >> \omega_{3dB}$ | $|H|_{dB} \approx -20\log_{10}\left(\frac{\omega}{\omega_{3dB}}\right)$



$$H_{dB} = -20\log_{10}\sqrt{1+\left(\frac{\omega}{\omega_{3dB}}\right)^2}$$

$$\omega_{3dB} = \frac{1}{RC} = 10^3$$

ω	H_{dB}
ω_{3dB}	0
10 ω_{3dB}	-20
100 ω_{3dB}	-40

20dB/ decade decrease

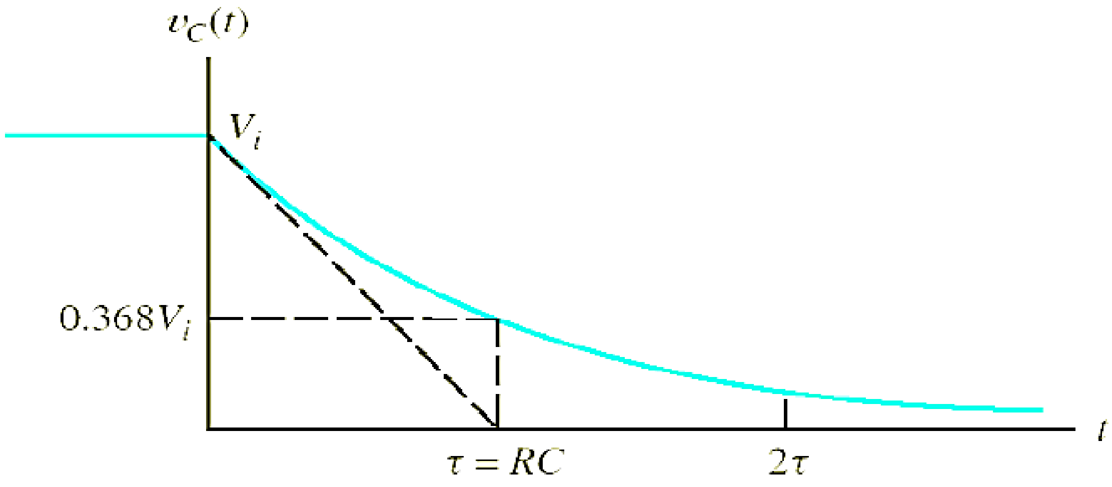
$\sim H \propto \frac{c}{\omega}$

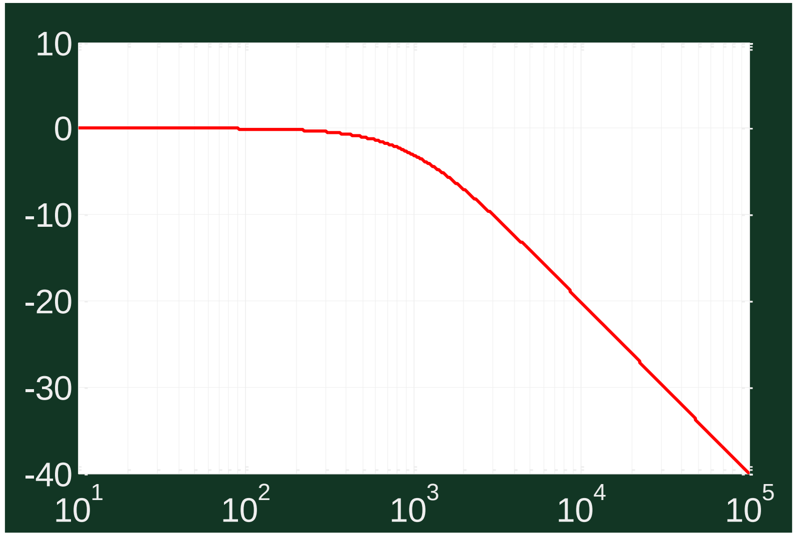
$H(dB) \propto -20\log \omega/c$

Relationship between time constant and 3dB frequency

$$H_{dB} = -20\log_{10}\sqrt{1+\left(\frac{\omega}{\omega_{3dB}}\right)^2}$$

$$\omega_{3dB} = \frac{1}{RC}$$



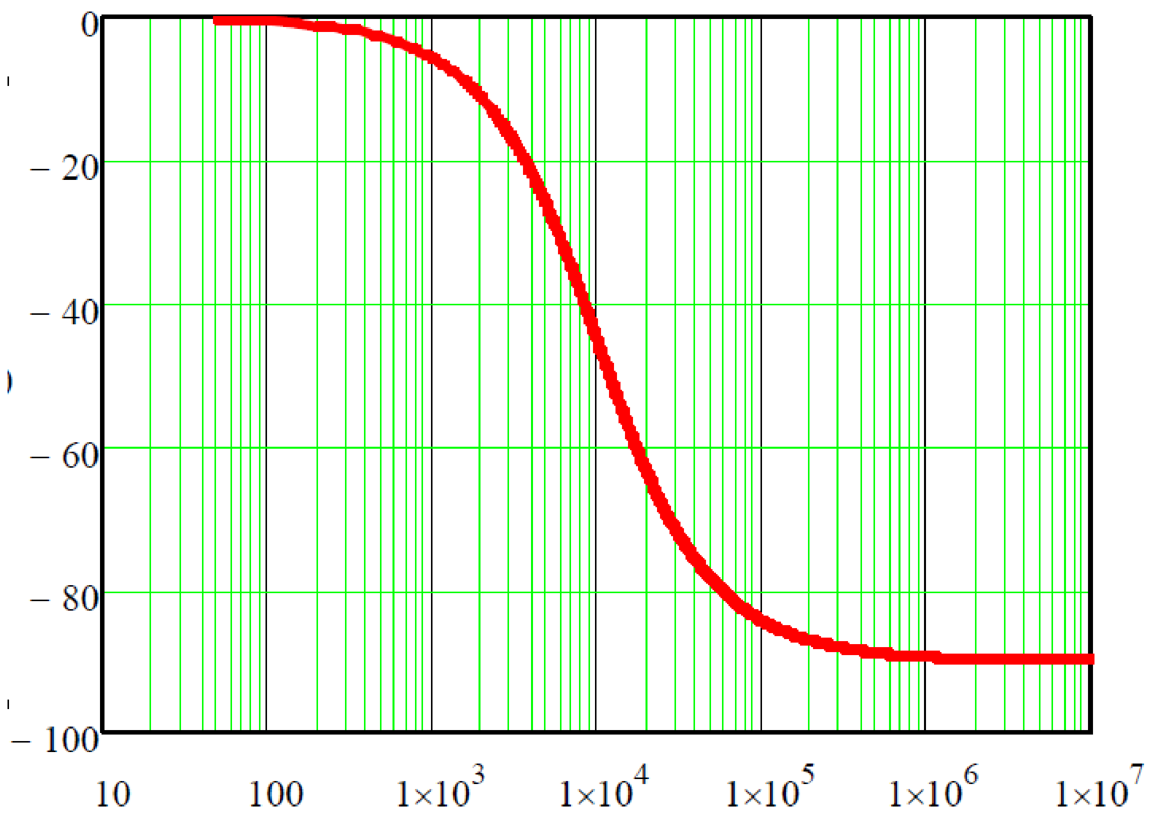


Phase plot

$$\varphi(\omega) = -\tan^{-1}(\omega/\omega_O)$$

$\omega \to 0, \phi \to 0 \quad \omega \to \infty, \phi \to -90°$

$\omega = \omega_O, \phi \to -45°$



Sketching Bode Plots

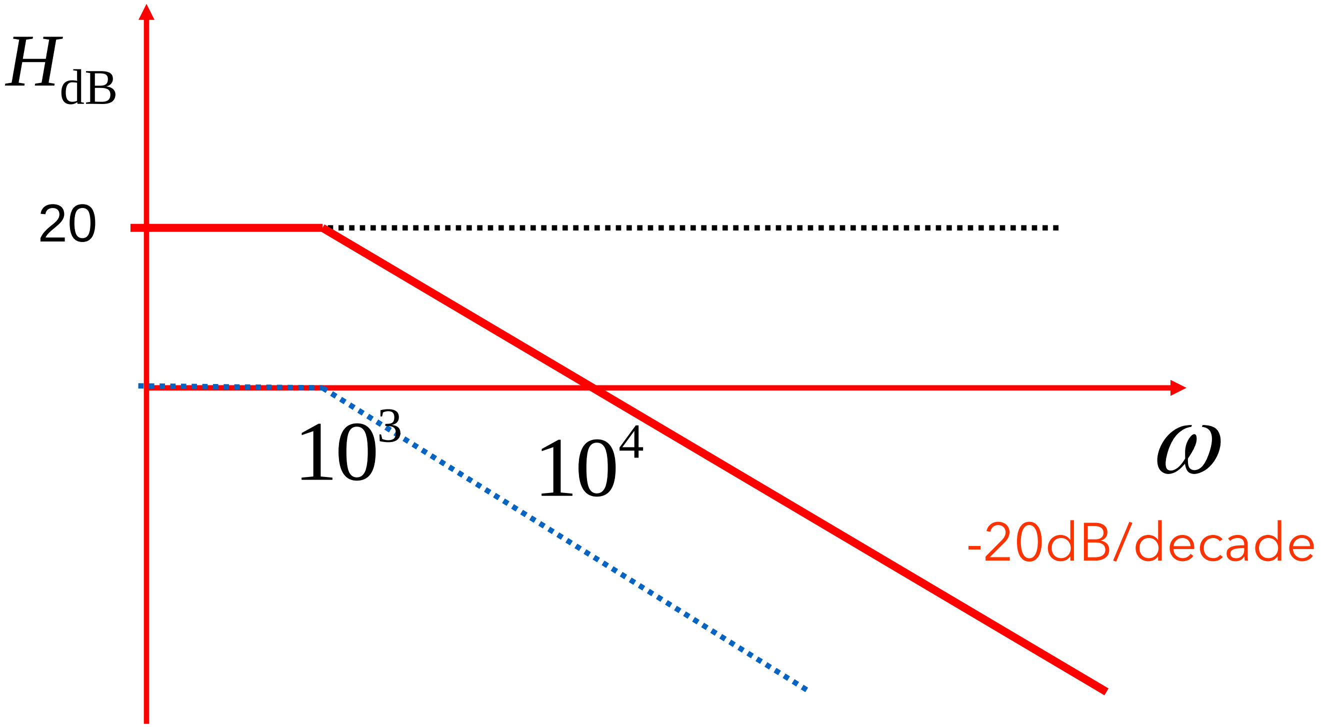
$$H(j\omega) = \frac{10}{1 + j\omega 10^{-3}}$$

$$\omega_{3dB} = 10^3$$

$$H_{dB} = 20 - 20\log_{10}\sqrt{1 + \left(\frac{\omega}{10^3}\right)^2}$$

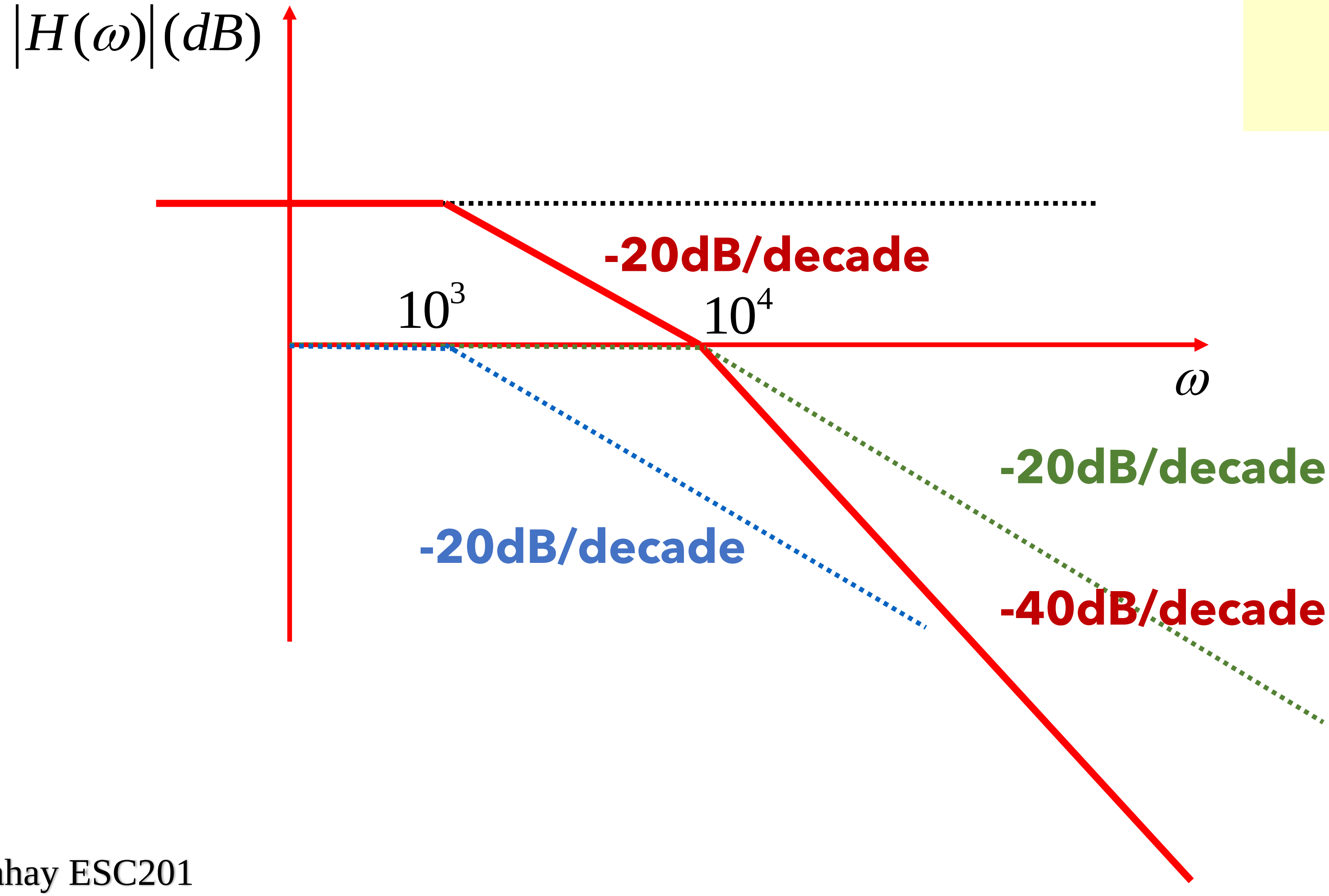
$$\omega << 10^3 : 0dB$$

$$\omega >> 10^3 : -20\log_{10}\frac{\omega}{10^3}$$



Sketching Bode plots

$$H(\omega) = \frac{10}{1 + j\dfrac{\omega}{10^3}} \times \frac{1}{1 + j\dfrac{\omega}{10^4}}$$



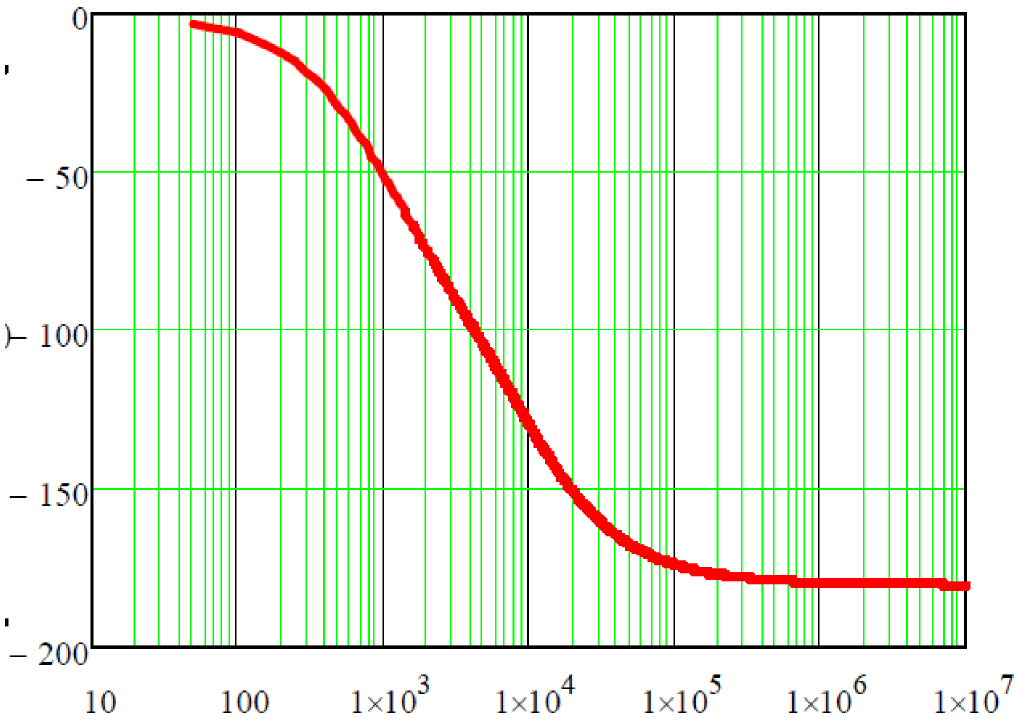
Phase plot

$$H(\omega) = \frac{10}{1 + j\frac{\omega}{10^3}} \times \frac{1}{1 + j\frac{\omega}{10^4}}$$

$\omega \to 0, \phi \to 0 \quad \omega \to \infty, \phi \to -90°$

$\omega = \omega_O, \phi \to -45°$

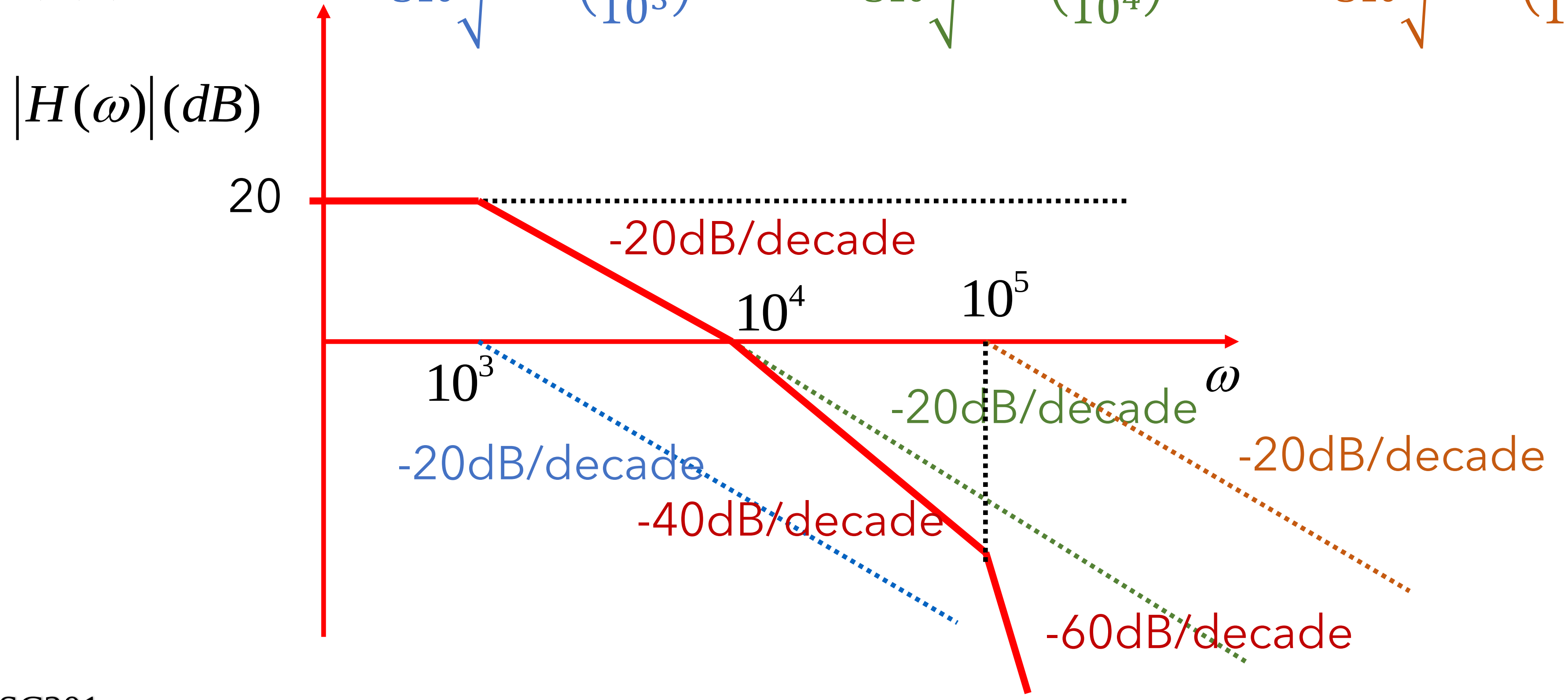
$$\varphi(\omega) = -\tan^{-1}(\omega/10^3) - \tan^{-1}(\omega/10^4)$$



Sketching Bode plots

$$H(\omega) = \frac{10}{1 + j\dfrac{\omega}{10^3}} \times \frac{1}{1 + j\dfrac{\omega}{10^4}} \times \frac{1}{1 + j\dfrac{\omega}{10^5}}$$

$$20\mathrm{Log}_{10}(|H(\omega)|) = 20 - 20\log_{10}\sqrt{1 + \left(\frac{\omega}{10^3}\right)^2} - 20\log_{10}\sqrt{1 + \left(\frac{\omega}{10^4}\right)^2} - 20\log_{10}\sqrt{1 + \left(\frac{\omega}{10^5}\right)^2}$$

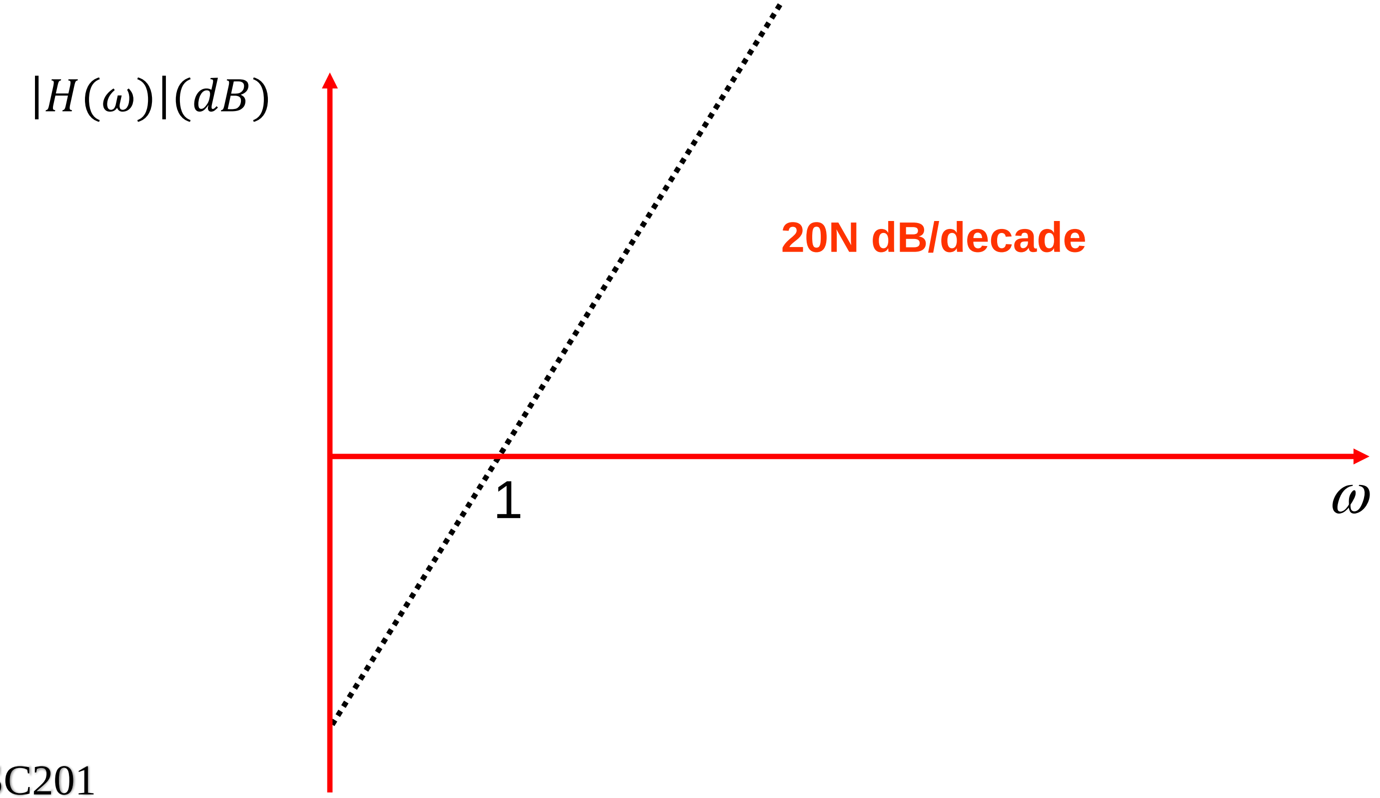


Sketching Bode Plot

$$H(\omega) = (j\omega)^N$$

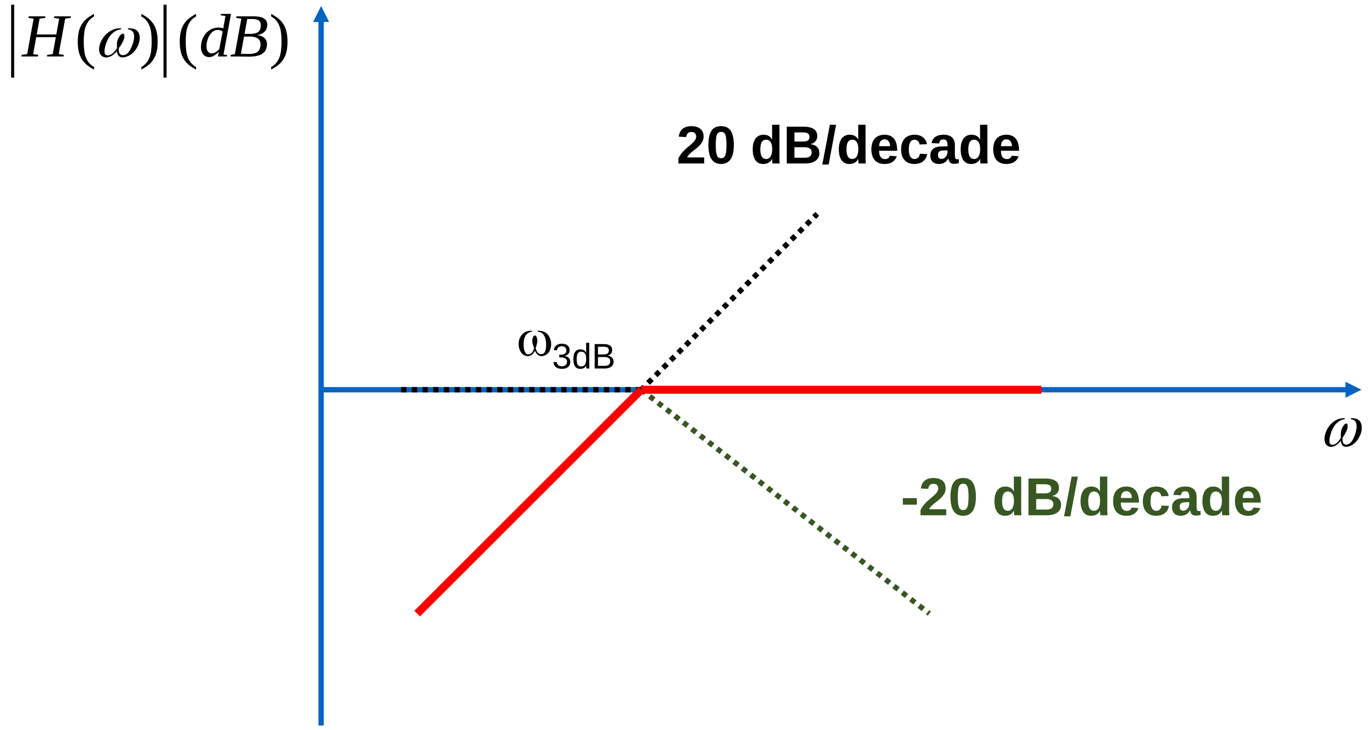
In dB scale

$$20\log_{10}(|H(\omega)|) = 20N\log_{10}(\omega)$$



Sketching Bode Plot

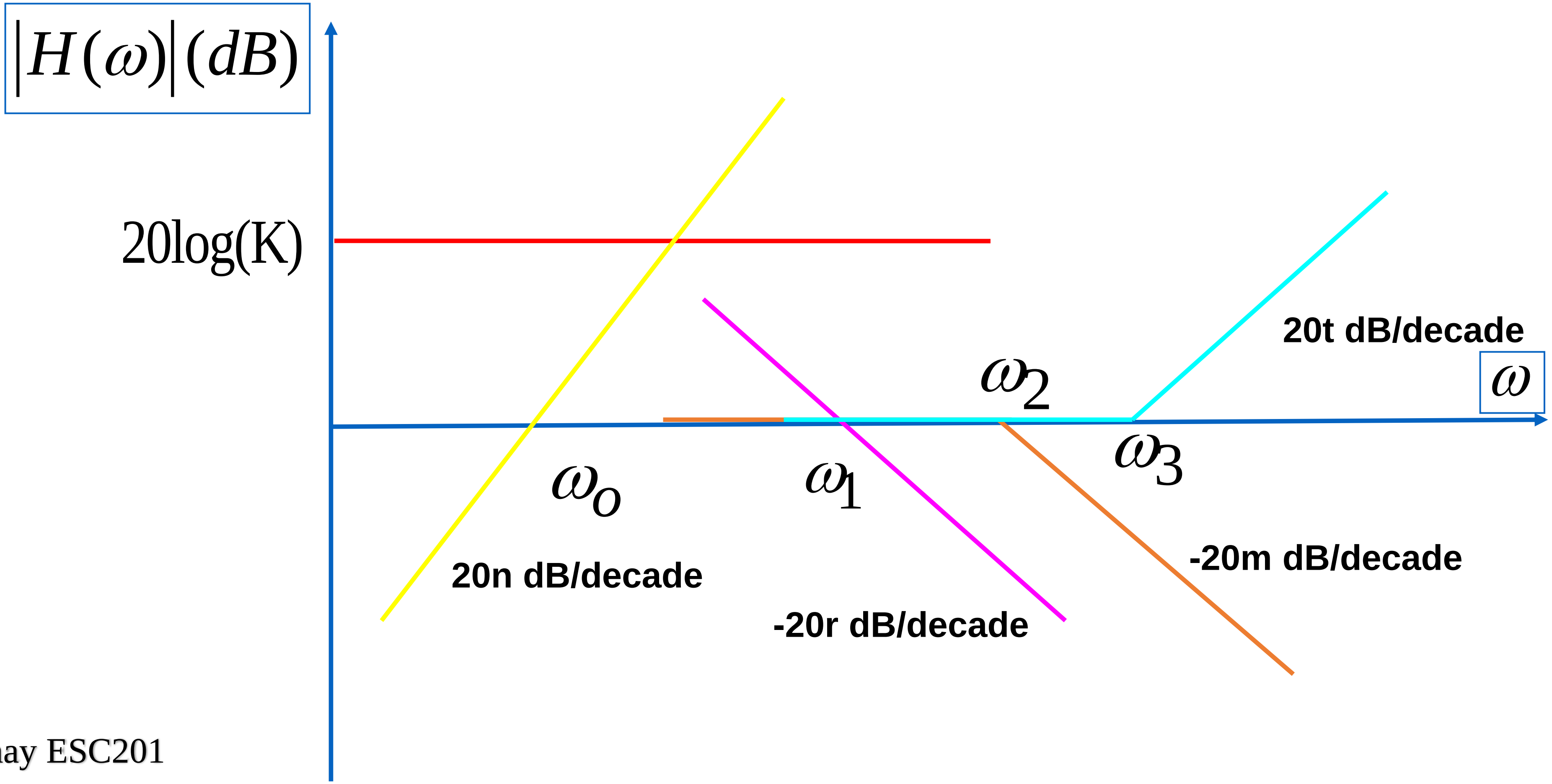
$$20\mathrm{Log}_{10}(|H(\omega)|) = 20\log_{10}\left(\frac{\omega}{\omega_{3dB}}\right) - 20\log_{10}\sqrt{1+\left(\frac{\omega}{\omega_{3dB}}\right)^2}$$



High Pass Filter

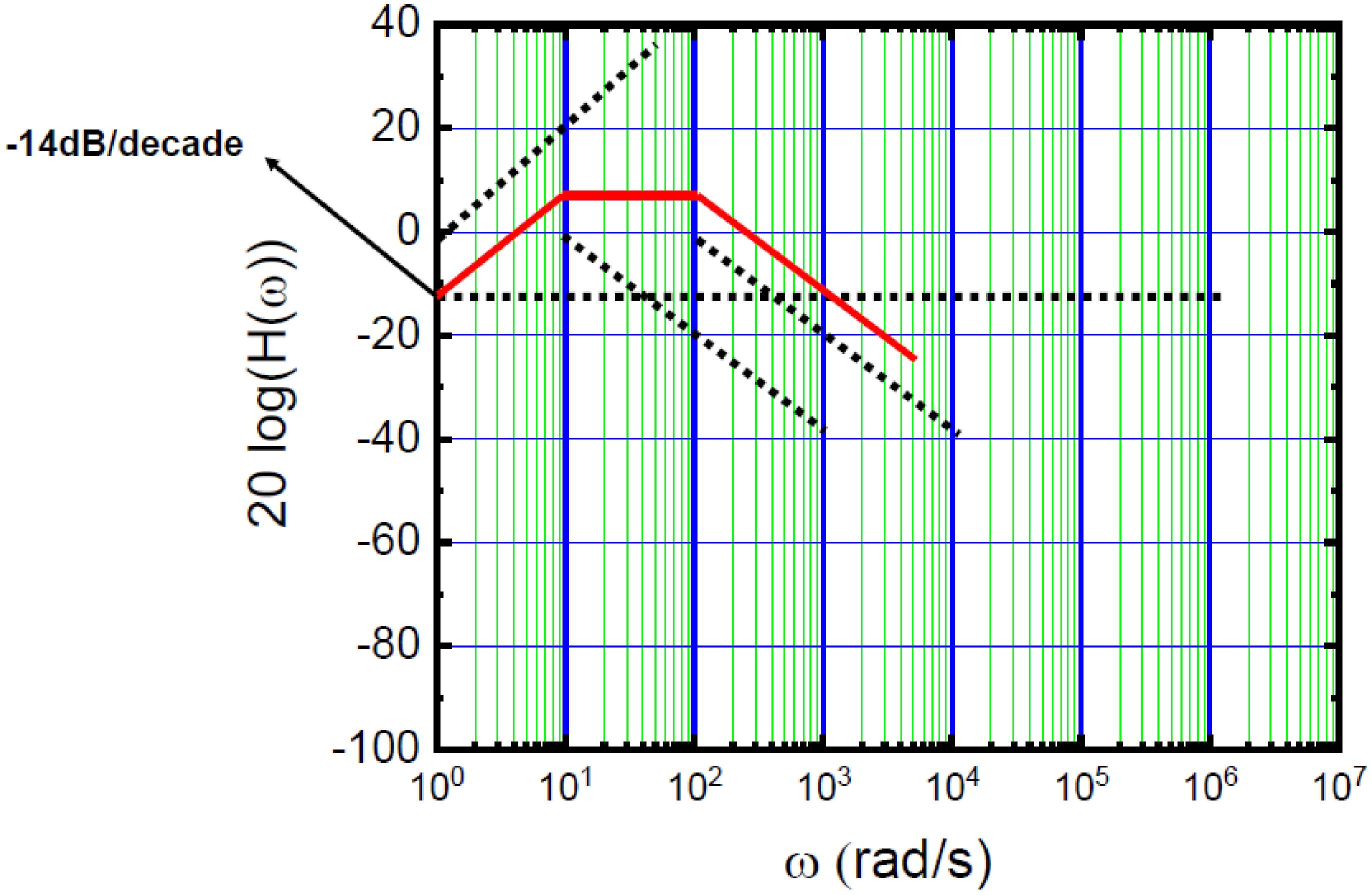
Bode Plot

$$H(\omega) = K \times j(\omega/\omega_o)^n \times \frac{1}{j(\omega/\omega_1)^r} \frac{1}{\{1 + j(\omega/\omega_2)\}^m} \times \{1 + j(\omega/\omega_3)\}^t$$

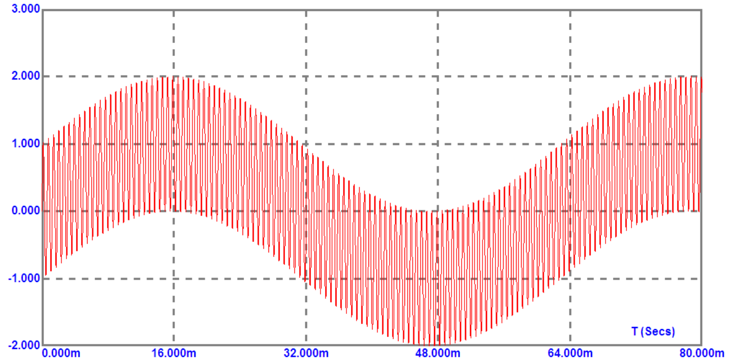


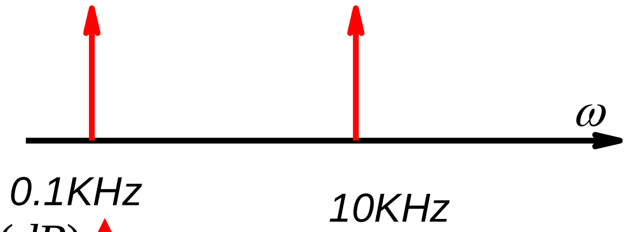
Bode Plot

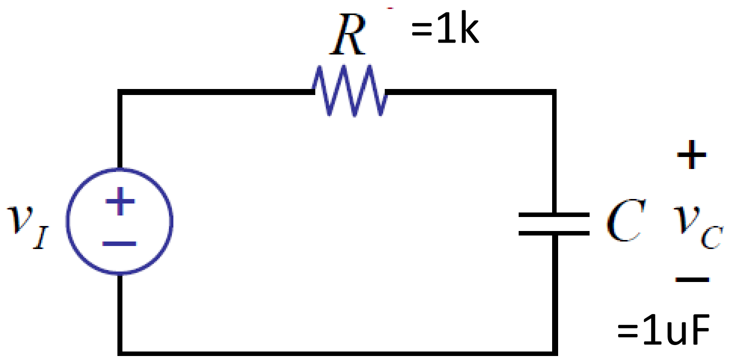
$$H(\omega) = 200 \times j\omega \times \frac{1}{10 + j\omega} \times \frac{1}{100 + j\omega} \qquad = 0.2 \times j\omega \times \frac{1}{1 + j\frac{\omega}{10}} \times \frac{1}{1 + j\frac{\omega}{100}}$$



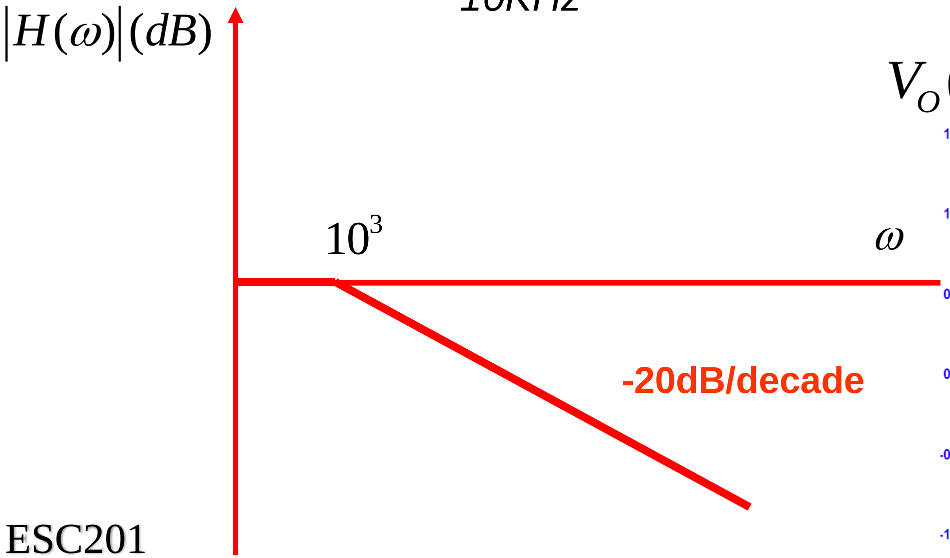
Recall Example



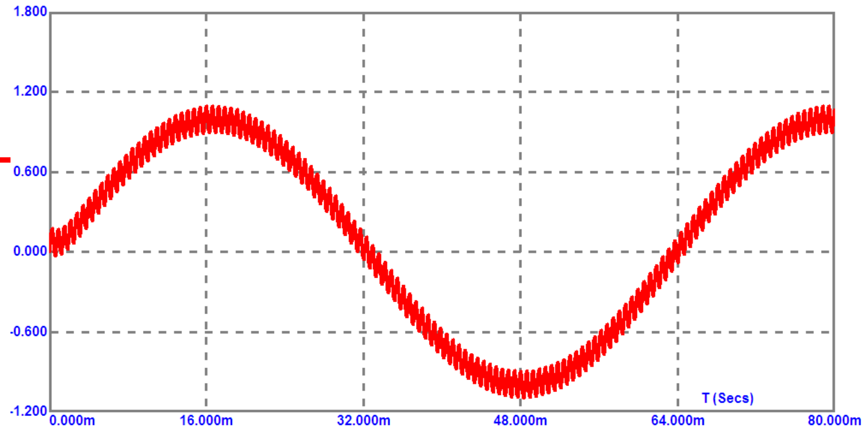




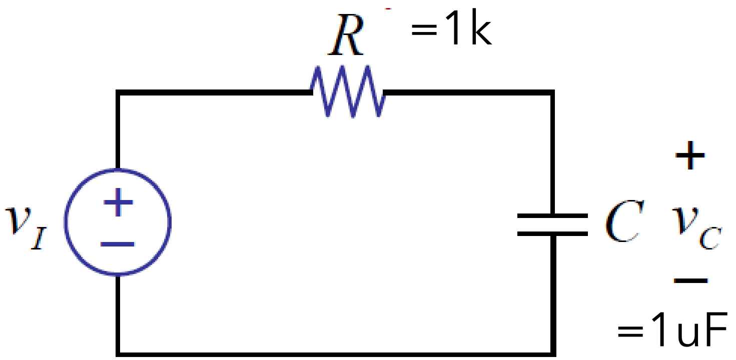
$$H(\omega) = \frac{1}{1 + j\omega 10^{-3}} = \frac{1}{1 + j\dfrac{\omega}{10^3}}$$

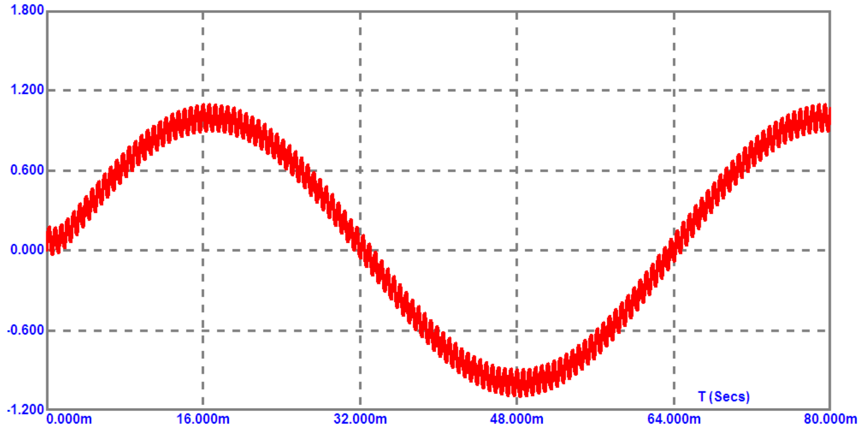


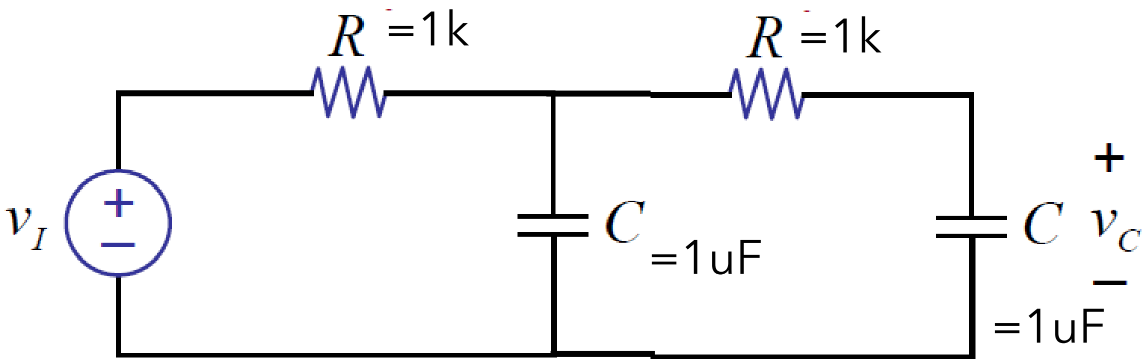
$$V_O(t) = 1Sin(100t) + 0.1Sin(10^4 t)$$

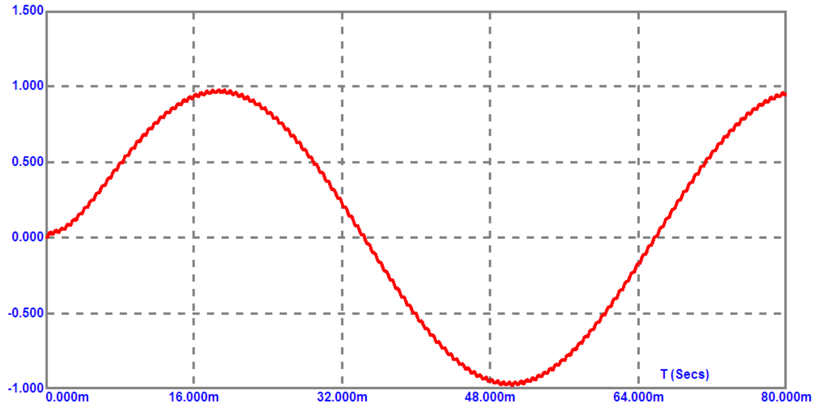


Example: low pass filter…

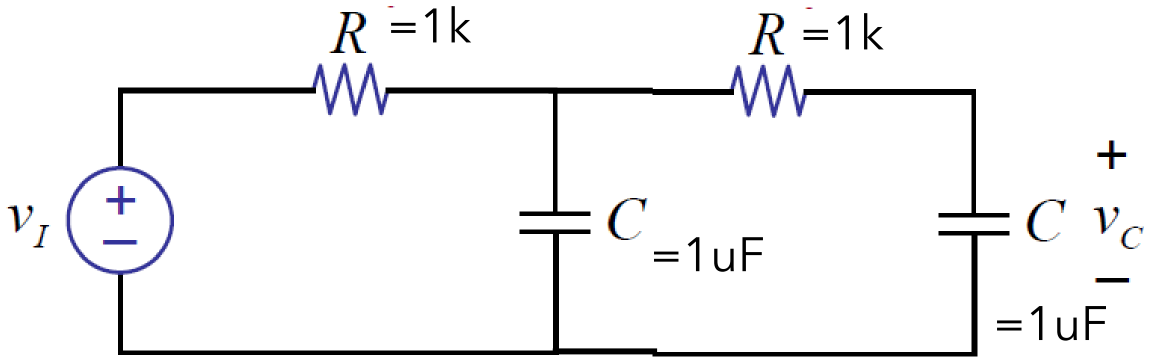








Analysis of two-stage RC

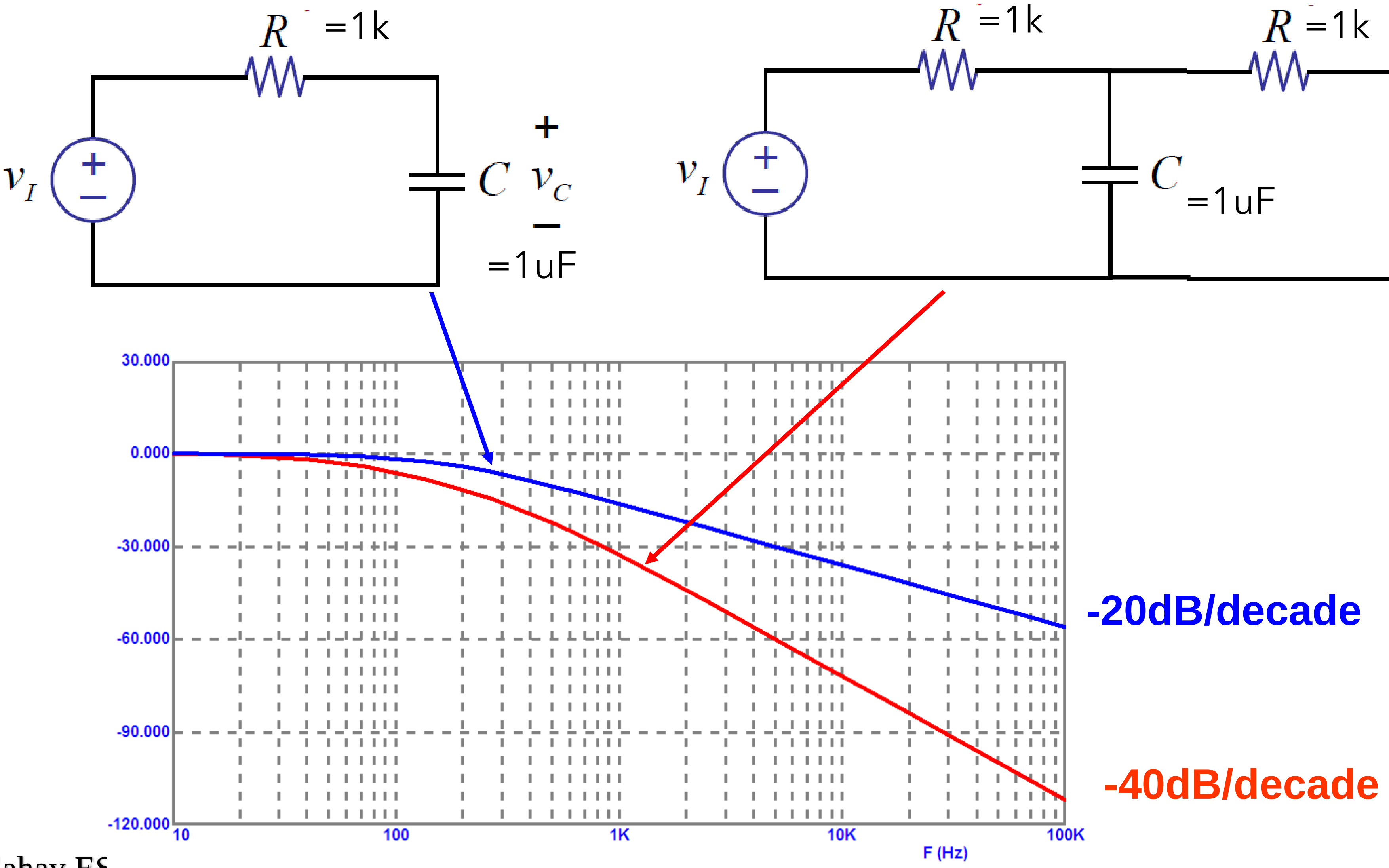


$$H(\omega) = \frac{\left(R + \frac{1}{j\omega C}\right) || \frac{1}{j\omega C}}{R + \left(R + \frac{1}{j\omega C}\right) || \frac{1}{j\omega C}} \times \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$= \frac{1}{1 - \omega^2 R^2 C^2 + 3j\omega RC}$$

$$\approx \frac{1}{1 + 3j\omega RC} \times \frac{1}{1 + \frac{j\omega RC}{3}}$$

Example: low pass filter…



Adding more RC stages, makes the characteristics sharper