13|11|2024

1) $$E(y) = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 2 \end{bmatrix}}_{X} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

BLUE of $\ell^T\beta$ $\quad \underline{\ell^T(X^TX)^{-1}X^Ty}$

$$v(C^Ty) = Var(c_1y_1 + c_2y_2 + \dots + c_ny_n)$$
$$= Var(c_1y_1) + Var(c_2y_2) + \dots + Var(c_ny_n)$$
$$= c_1^2 Var(y_1) + \dots + c_n^2 Var(y_n)$$
$$= \sigma^2 \sum_{i=1}^{n} c_i^2$$

$$\boxed{\begin{aligned} &Var(ax+by) \\ &= a^2 Var(x) \\ &+ b^2 Var(y) \\ &+ 2ab\,Cov(x,y) \end{aligned}}$$

2) $$Var(\text{blue of } \beta) = \boxed{\sigma^2 (X^TX)^{-1}}$$

$$X = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ 1 & \alpha \end{bmatrix} \qquad X = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$(X^TX) = \begin{bmatrix} 6 & 1+\alpha \\ 1+\alpha & 2+\alpha^2 \end{bmatrix}$$

$$(X^TX)^{-1} = \frac{1}{5\alpha^2 - 2\alpha + 11} \begin{bmatrix} 2+\alpha^2 & -1-\alpha \\ -1-\alpha & 6 \end{bmatrix}$$

$$\beta^T = (\beta_1, \dots \beta_p)$$

$$\mathrm{Var}(\text{blue of } \beta) = \sigma^2 \left[(X^T X)^{-} \right]_{p \times p}$$

$$= \sigma^2 \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ \vdots & & & \\ a_{p1} & a_{p2} & \cdots & a_{pp} \end{bmatrix}$$

$$\mathrm{Var}(\text{blue of } \beta_1) = a_{11} \sigma^2$$
$$\mathrm{Var}(\text{blue of } \beta_2) = a_{22} \sigma^2$$
$$\mathrm{Cov}(\text{blue of } \beta_1, \text{blue of } \beta_2) = \sigma^2 a_{12} \; (= \sigma^2 a_{21})$$

Consider blue of β_1, $c_1 y_1 + c_2 y_2 + c_3 y_3$

$$\mathrm{Var}(c_1 y_1 + c_2 y_2 + c_3 y_3)$$
$$= \sigma^2 \sum_{i=1}^{3} c_i^2$$

$c_2 + c_3 = d$

$c_2 > c_3$

$\sigma^2 (c_1^2 + c_2^2 + c_3^2)$
where $c_2 + c_3 = d$

$$= \sigma^2 \left(c_1^2 + \frac{d^2}{4} + \frac{d^2}{4} \right)$$

$$= \sigma^2 \left(c_1^2 + c_2^2 + (d - c_2)^2 \right)$$

$$X = \begin{bmatrix} \; \end{bmatrix}$$

$l^T \in R(X)$

$(X)_{n \cdot}$ n dependent

$$\begin{bmatrix} y_1 \\ \vdots \\ y_{n-1} \end{bmatrix} = \begin{bmatrix} x_{11} & \cdots & x_{1p} \\ \vdots & & \\ x_{n-1,1} & \cdots & x_{n-1,p} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_p \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_{n-1} \end{bmatrix}$$

$$x = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix} \to l^T\beta \xrightarrow{BLUE} c^T y \quad (d_1\ d_2\ 0)\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$\bar{X} = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} \to l^T\beta \xrightarrow{BLUE} (d_1\ d_2)\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\text{Var}\left(\quad \right) < \text{Var}(c^T y)$$

4)

$$X = \begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ \vdots & & & & \\ 1 & 1 & 0 & \cdots & 0 \\ \hline 1 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \\ 1 & 0 & 1 & \cdots & 0 \\ \hline \vdots & \vdots & \vdots & & \vdots \\ \hline 1 & 0 & 0 & \cdots & 1 \\ \vdots & & & & \\ 1 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Set of estimable $l^T = \left\{ (l_1, l_2, \ldots, l_{k+1}) \mid l_1 = \sum_{i=2}^{k+1} l_i \right\}$

$$\mu + \alpha_i \overset{BLUE}{=} \bar{y}_i \qquad \bar{y}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} y_{ij}$$

$$\beta^{LS} = (X^T X)^- X^T y$$

• Here μ is not estimable and in β^{LS}, if we put different choices of $(X^T X)^-$, we will obtain different estimators of μ.

7) Suppose $E(p^T y) = l^T \beta$ where $\underline{p \neq d}$

$$E(p^T y - d^T y)$$
$$= E(p^T y) - E(d^T y)$$
$$= l^T\beta - l^T\beta = 0$$

$$(p^T - d^T) y$$

As $d^T y$ is the unique unbiased estimator of $l^T\beta$, $d^T y$ is the BLUE.

8) $E(\text{Blue of } p^T\beta - u^T y) = 0$

Blue of $p^T\beta - u^T y = \alpha d^T y$