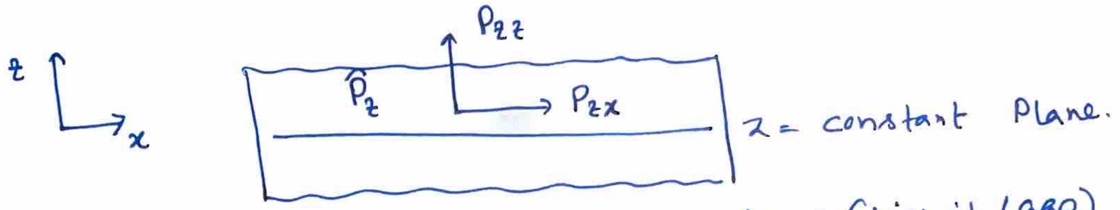
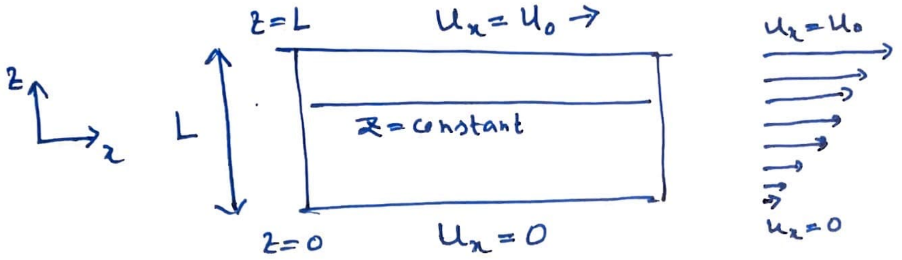
Viscosity



This box contains fluid (liquid / gas).

- Consider $z =$ constant plane inside the fluid.
- The fluid below this plane ($z =$ const.) exerts a mean force P_z per unit area ($\equiv$ stress) on the fluid above the plane.
- Similarly, according to Newton's third law, the fluid above the plane exets a stress $-P_z$ on the fluid below the plane.
- The z-component of P_z i.e, P_{zz} measure the ~~change to~~ mean force per unit area normal to the $z =$ constant plane, which is the mean pressure $\bar{p}$. $\therefore$ $\boxed{P_{zz} = \bar{p}}$
- Now if the fluid is at rest or have uniform motion, there is no ~~~~ net momentum along the x-direction i.e., $P_{zx} = 0$.
- However, under non-equilibrium situation, when the motion of the fluid is not uniform, $P_{zx} \neq 0$ and then there will be non-zero viscosity.
- Viscosity is the measure by which the fluid below the $z =$ const plane slow down the fluid above the plane.



The fluid is confined under two plates - one at $z=0$ and another at $z=L$.

$z=0$ plate has $u_x = 0$ and $z=L$ plate has $u_x = u_0$

Fluid inside the ~~...~~ above region between $z=0$ and $z=L$.

- Let us assume that the fluid has mean velocity u_x in the x-direction.

- u_x depends on z i.e., $u_x = u_x(z)$. This is because $z=0$ plane has zero mean velocity but $z=L$ has highest mean velocity $u_x = u_0$

- Therefore, molecules above the $z=$ constant plane have larger momentum than the molecules below it.

- As a result, as molecules cross back and forth this plane, they carry the momentum with them. Hence, the fluid below the $z=$ const. plane <u>gain momentum</u> in the x-direction from the molecules coming above it. Similarly, molecules above the plane <u>lose momentum</u> along x-direction.

- This change in momentum per unit time and per unit area $\equiv P_{zx}$ [along x-direction on the $z=$ const. plane].

- <u>Approximate calculation of P_{zx}:</u>

Let there be n molecules per unit volume with a mean velocity $\bar{v}$.

- Therefore, we can say, roughly $\frac{1}{3}$ of them have velocities along z-direction. Half of these or $\frac{1}{6}n$ molecules have mean velocities $\bar{v}$ along the $+z$ direction. The other half ($\frac{1}{6}n$) have mean velocity $\bar{v}$ along the $-z$ direction.

- Thus, on average there $\frac{1}{6}n\bar{v}$ molecules which in unit time cross a unit area of the $z=$ constant plane.

- If we take $l =$ mean free path of the molecules, the molecules which are crossing from above the $z=$ constant plane have experienced last collisions at $(z+l)$. Similarly, molecules which are crossing from below the $z=$ const. plane have experienced last collisions at $(z-l)$.

$\therefore$ The net increase in momentum per unit time per unit area

$$P_{zx} = \frac{1}{6}n\bar{v}\left[m u_x(z-l)\right] - \frac{1}{6}n\bar{v}\left[m u_x(z+l)\right]$$

$$= \frac{1}{6}n\bar{v}m\left[u_x(z-l) - u_x(z+l)\right]$$

$$= \frac{1}{6}n\bar{v}m\left[u_x(z) - \frac{\partial u_x}{\partial z}l - u_x(z) - \frac{\partial u_x}{\partial z}l\right]$$

$$= \frac{1}{6}n\bar{v}m\left(-2\frac{\partial u_x}{\partial z}l\right) \equiv -\eta\frac{\partial u_x}{\partial z}$$

$$\therefore \boxed{\eta = \frac{1}{3}n\bar{v}ml}$$ Co-efficient of viscosity.

We know that: $l \approx \frac{1}{\sqrt{2}n\sigma_0}$, $\bar{v} = \sqrt{\frac{8}{\pi}\frac{k_BT}{m}}$

$$\therefore \boxed{\eta = \frac{1}{3\sqrt{2}}\frac{m}{\sigma_0}\bar{v}}$$

$\sigma_0 =$ scattering cross-section $=$ constant

$\therefore$ η is independent of number density (n) and hence the pressure of the gas $p = nk_BT$.