Problem 1

A system obeys the following two equations of states.

$$U = PV \qquad \text{and} \qquad P = BT^2$$

where B is a constant. Find the fundamental relation for this system.

Problem 2

The fundamental equation for a system is given by

$$u = Ae^{b(v-v_0)^2} s^{\frac{4}{3}} e^{(s/3R)}.$$

Here A, b, v_0 are constants. (a) Show that this relation satisfies the third law of thermodynamics. (b) Show that c_v is proportional to T^3 at low temperature. (c) Show that c_v is proportional to $3k_B$ at high temperature, where k_B is the Boltzmann constant. (d) Calculate the volume expansion co-efficient at $P = 0$.

Problem 3

For a magnetic system with magnetic dipole moment I, the Euler equation is given by,

$$U = TS - PV + IB_e + \mu N$$

where B_e is the external magnetic field. Derive the following two Maxwell's relations for this system.

$$\left(\frac{\partial B_e}{\partial S}\right)_{I,V,N} = \left(\frac{\partial T}{\partial I}\right)_{S,V,N}$$

$$\left(\frac{\partial B_e}{\partial V}\right)_{S,I,N} = -\left(\frac{\partial P}{\partial I}\right)_{S,V,N}$$

Problem 4

One mole of a monatomic ideal gas is contained in a cylinder of volume 10^{-3} m^3 at a temperature of 400 K. The gas is to be brought to a final state of volume 2×10^{-3} m^3 and temperature 400 K. A thermal reservoir of temperature 300 K is available, as is a reversible work source. What is the maximum work that can be delivered to the reversible work source?

Problem 5

A system obeys the following two equations of states.

$$T = u/b \quad \text{and} \qquad P = avT,$$

where a and b are constants. Two such systems each of 1 mol, are initially at temperature T_1 and T_2 (with $T_2 > T_1$) and each has a volume v_0. The systems are to be brought to a common temperature T_f, with each at the same final volume v_f. The process is such that the delivered work is maximum to a reversible work source. (a) What is the final temperature T_f? (b) How much work can be delivered?

Problem 6

Given the thermodynamic potential $G = U - TS + PV$, establish the following relation.

$$dG = -SdT + VdP + \mu_1 dN_1 + \mu_2 dN_2 + ...$$

Problem 7

A system obeys the following fundamental relation.

$$(s - s_0)^4 = Avu^2$$

Calculate the Gibbs potential $G(T, P, N)$.

Problem 8

The Massieu function (which is the Legendre transformation on S) for $S[\frac{P}{T}]$ is given by

$$S\left[\frac{P}{T}\right] = -\frac{G}{T},$$

Show that $dS\left[\frac{P}{T}\right] = \frac{1}{T}U - Vd\left(\frac{P}{T}\right) - \left(\frac{\mu_1}{T}\right)dN_1 - \left(\frac{\mu_2}{T}\right)dN_2....$

Problem 9

The fundamental equation for a gas is given by $UN^{1/2}V^{3/2} = A(S - R)^3$, where $A = 2 \times 10^{-2}(K^3 m^{9/2} J^3)$. Two moles of this gas are used as the auxiliary system of a Carnot cycle, operating between two thermal reservoirs at 373K and 273K. In the first isothermal expansion 10^6J is extracted from the hotter reservoir. (a) Find the heat transfer and work transfer in each of the four steps of the Carnot cycle. (b) Calculate the efficiency of the cycle.