OPTICS (PHY224)
Problem Set 2

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1. Two y polarized plane waves described by fields $E_1(\mathbf{r},t) = E_0 \exp[i(\mathbf{k}_1 \cdot \mathbf{r} - \omega t)]$ and $E_2(\mathbf{r},t) = E_0 \exp[i(\mathbf{k}_2 \cdot \mathbf{r} - \omega t)]$ propagate towards the plane of observation, $z = 0$. $\mathbf{k}$ lies in the x-z plane.

   (a) Calculate the phase difference $\delta(x)$ between the two waves at the plane $z = 0$.

   (b) The observed interference pattern can be quantified in terms of the irradiance, defined as $I(\mathbf{r},t) = \mathbf{E}(\mathbf{r},t)\mathbf{E}^*(\mathbf{r},t)$. Show that the interference pattern, in terms of the irradiance $I(x)$ is given by $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta(x)$ where $\delta(x)$ is the phase difference between the two waves and I_1 and I_2 are the irradiances of the first and second wave, respectively.

   (c) Determine the fringe separation of the interference pattern.

   (d) What is the smallest fringe separation that can be obtained in such an interference pattern?

   (e) What would have happened if one of the waves was of a different frequency, say $E_2(\mathbf{r},t) = E_0 \exp[i(\mathbf{k}_2 \cdot \mathbf{r} - \omega' t)]$

2. Consider the Michelson interferometer with a 50:50 beam splitter shown in Figure 1. The light from the source is horizontally polarized and consists of two co-propagating frequencies and can be described by

$$E(\mathbf{r},t) = E_0 \left[\exp[i(\mathbf{k}_1 \cdot \mathbf{r} - \omega_1 t)] + \exp[i(\mathbf{k}_2 \cdot \mathbf{r} - \omega_2 t)]\right]$$

   (a) Derive an expression for the intensity/irradiance at the the screen in terms of the source irradiance I and $d = d1 - d2$

   (b) At what value(s) of $d = d_1 - d_2$ does the intensity reach its maximum and minimum value? What are these values?

3. Consider the Michelson interferometer illustrated below in Figure 2, illuminated by a monochromatic plane wave of wavelength λ_0 at the input port. One of the mirrors, M_1, is tilted by a small angle θ.

   (a) Describe the shape of the fringes on the screen, and their spacing. **[3 Marks]**

   (b) What happens when the second mirror M_2 is moved a distance Δ away from the beamsplitter? Be quantitative in your answer.

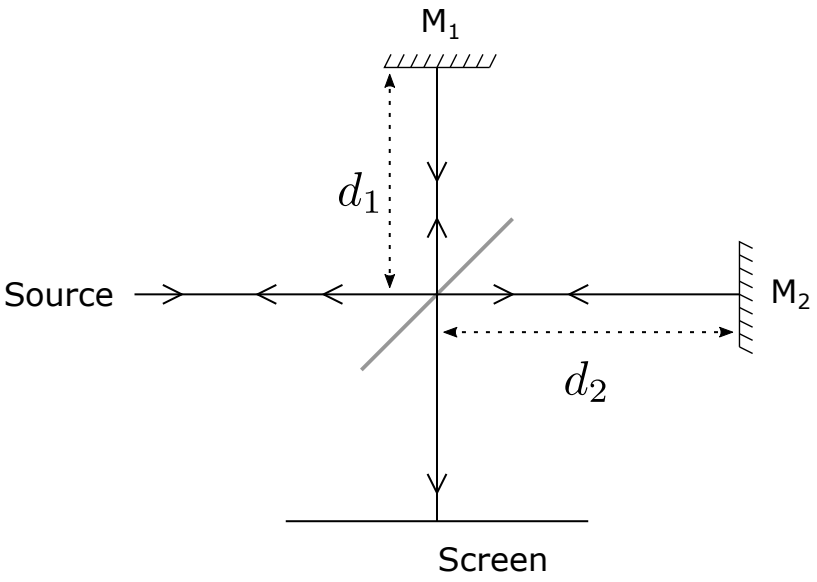

Figure 1

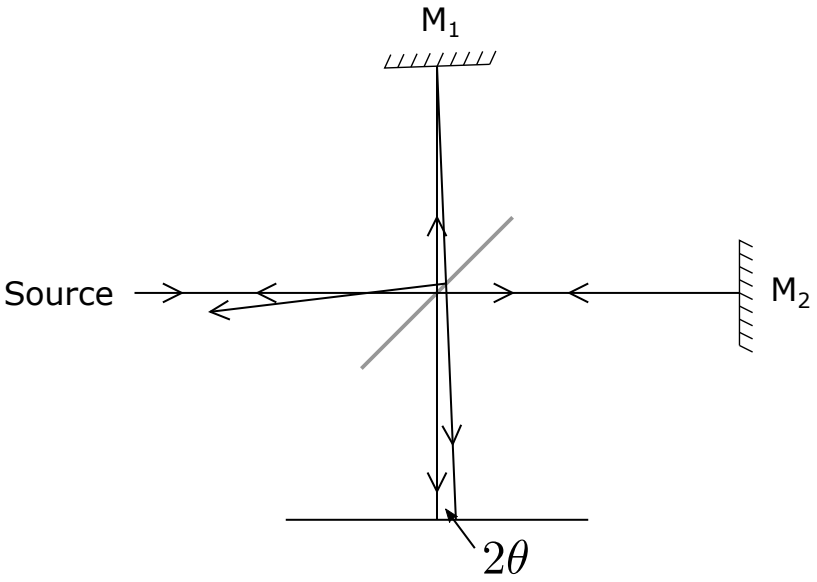

Figure 2