OPTICS (PHY224)
Laboratory Manual

Semester 2024-2025-I

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Contents

0. Introduction

0.1 Reference Material

This course on Optics will simultaneously provide you the theoretical background in the lectures and this laboratory will introduce you to measuring optical phenomena. Please use this laboratory manual as a guide while using principally the following books and references.

1. E. Hecht, "Optics"
2. J. Peatross and M. Ware, "Physics of Light and Optics", freely available at http://optics.byu.edu
3. K.K. Sharma, "Optics: Principles and applications", Elsevier, 2006
4. M. Born and E. Wolf, "Principles of Optics", Cambridge Univ. Press.

0.2 General Instructions

1. Reading and understanding is almost half the experiment. So please come prepared to the laboratory. Remember that you will be questioned about your preparedness at the beginning of each experiment.

2. Discuss with your Instructors any conceptual issues as well as any technical issues.

3. **Kindly treat the equipment properly and carefully. Optical components are delicate and get destroyed if they are not handled carefully. You have to follow these cardinal rules - Never touch the optical surfaces with your fingers. You will leave behind your fingerprints. Always hold the optical elements at the edges, or on the non-optical surfaces. Never hold the optical components away from the table where they can drop and break. If in doubt, talk to your instructors and laboratory staff.**

4. Never put yourself in any position where your eyes approach the axis of a laser beam (even with eye protection on). Keep beam paths well below standing or sitting eye level. Do not direct the laser beam towards other people.

5. The laboratory will be held on Thursdays and Fridays 2.00-5.00 pm. Do not come late - anybody coming in later than 15 minutes will be marked absent for the lab and will get zero in the preparation quiz. Absence in the laboratory will not be allowed except with leave approved by the DUGC.

6. The first preparatory Experiment (Chapter 1) must be finished during the first week.

1. Preparatory Lab

1.1 Aims

1. To understand how to use the various components provided to you.
2. To align your laser along the required path.
3. To calibrate the neutral density (ND) filters provided to you.
4. To study shape and size of the laser beam, and determine the waist size of the beam when focused by a convex lens.

1.2 Understand how the equipment works

You will be given a brief demonstration of how to use each piece of equipment provided to you. It is necessary to familiarize yourself with all the mechanical parts and how and when to use them in your experimental setup. Your basic kit will consist of the following:

1. A He-Ne laser, suitably mounted
2. Convex lenses of different focal lengths
3. Neutral density (ND) filters of various optical densities (OD)
4. Several posts, post-holders and post-bases
5. M6 bolts of several lengths
6. A circuit containing a photodiode
7. A hand-held digital multimeter (DMM)
8. A pinhole
9. Mounts for the filters, pinhole and photodiode.

You will also be provided other equipment specific to the week's experiment.

1.3 Alignment of the Optical Bench and your laser

The first task is to align the laser beam path along the length of the optical bench. The way the laser is mounted may vary from table to table, but there will be means to vary the lateral position and height of the laser beam, and also the direction of the beam (horizontal and vertical tilt angles)

The principle is simple. Before switching on the laser, fix the laser such that the output will be roughly aligned such that the beam propagates along a row of holes on the optical bench (or breadboard). Place a beam block at the opposite end of the optical bench (or breadboard) such that when the laser is switched on, the beam will be completely blocked and will not propagate beyond your experimental setup. This will ensure your safety and the safety of others working in the lab.

Now switch on the laser and ensure that the beam indeed propagates roughly along a row of holes on the bench. Mount the pinhole close to the laser such that the mount is centred along the row of holes along which the laser propagates. Then,

1. Adjust the lateral (horizontal and vertical) of the pinhole within its mount, such that the laser beam passes through the aperture.

2. Mount the pinhole along the same row of holes farther away from the laser. Adjust the tilt (both horizontal and vertical) of the laser without changing the position of the pinhole, such that the laser beam passes through the hole. Now, move the pinhole back to the position of step 1. If you find that the position of the laser beam spot is away from the aperture, go to Step 1.

The steps 1 and 2 need to be repeated until precise alignment is achieved. This will be demonstrated to you.

1.4 Calibration of your set of filters and the photodiode

First, mount the photodiode circuit board in the mount provided in a manner similar to the pinhole. Connect the output of the DC power adaptor to the corresponding connector on the photodiode circuit board. Connect the output of the photodiode circuit to your DMM (configured to measure DC voltage). If the lights in the lab are switched on, and the photodiode is exposed to the ambient light, you should see a reading of a few tens to about a hundred millivolts. Adjust the lateral position of the photodiode such that the laser beam spot is centered around the photodiode or is completely within the area of the photodiode.

In your kit you will find a brass mount with threads on which filters can be stacked. Mount the brass filter holder in front of the laser in such a way that the laser beam passes roughly through its centre. Mount an ND filter (say, of OD 1.0) on it. You will notice that the filter absorbs a significant fraction of the optical power. Align the filter such that the light reflected form the surface of the filter falls very close to (but does not enter) the output aperture of the laser.

To calibrate your filters, stack several filters in the laser beam so that the photodiode gives an output of $\approx$ 20 mV (this depends on the detector configuration, you will receive instructions during the lab session). Ensure that you include all the filters with low OD (1.0 and below). Now take out one filter at a time and note down the enhancement factor (m) by which the photodiode output increases. The transmittance of this filter is simply $1/m$. Tabulate the value of transmittance of this filter and the OD marked on its mount.

Replace this filter in the path of the beam and take out another one, and measure its transmittance- this way you can calibrate most of the filters in your kit. It is important that you make these measurements carefully because for all your experiments, you are going to use these filters. Make sure that the value of the output in the above measurements does not ever exceed 250 mV. If the voltage reading is too high, your transmittance measurement may be unreliable (**why?**)

Now that you know the transmittance of the filters provided to you, use various combinations of them to produce output signals in the photodiode between 0 and 250 mV. Verify that the voltage signal varies roughly linearly with the power transmitted by the combination of filters (based on your calibration)

The photodiodes used for the measurements in this course give linear response if the laser power incident on the photodiode lies within certain limits. As a thumb rule, keep the photodiode output between 30 to 250 mV. Introduce as many filters as necessary to bring the photodiode output just below the upper limit of its linear response.

1.5 Fluctuations in the output power of the He-Ne laser

A laser consists of an optical cavity (essentially two mirrors facing each other, between which light propagates back and forth) with a *gain medium* inside it. As light propagates back and forth, its power gets amplified due to the gain medium. The energy required for this amplification is supplied externally, for example via an electrical discharge or a flash lamp. For sustained gain, light must travel a half-integer multiple of the wavelength over one round trip. Each admissible configuration, characterized by a specific path length over which light propagates, is termed a *longitudinal mode*.

The output power of the He-Ne laser provided to you fluctuates in time. The fluctuations are enhanced if a polarizer is placed in the path of the beam (**why?**). Changes in the output power of the laser can significantly affect the results of your measurements. It is therefore desirable to have some idea of the extent of these fluctuations.

There are fluctuations that happen over different time-scales:

- Very short time scales corresponding to hundreds of megahertz frequencies that are caused due to beating of the longitudinal modes that are active in the cavity. You will need a fast photo-diode and high bandwidth oscilloscope to measure these fluctuations and You will not be able to observe these fast fluctuations in your laboratory experiment.
- Fluctuations at the frequency or harmonics of the power line frequency ($\sim$ 50 Hz) caused by the power supply
- Fluctuations caused by mode cycling in the laser cavity due to thermal expansion of the laser tube. The longitudinal mode spacing depends on the laser tube length. As the laser tube length changes due to variation in temperature, the number of longitudinal modes falling under the gain bandwidth also changes, thereby affecting the number of longitudinal modes that are lasing. Typical values of the intensity fluctuation caused by this can be quite large. Study this phenomenon. Estimate the longitudinal mode spacing for your laser tube and see if you can understand this effect. The typical gain bandwidth for the He-Ne laser is about a Gigahertz.

Monitor the laser output for about an hour or so by recording the photodiode voltage every one or two minutes. Repeat the same experiment after inserting a polarizing sheet in the path of beam. Find the mean value and the standard deviation in each case.

1.6 Laser beam shape

It is not possible to focus light beam down to a geometric point. If you confine (focus) a beam in the transverse direction, it spreads (diverges) out along that direction due to diffraction (you will learn about it in detail later in the course). In this experiment, you will study the spatial energy distribution across a laser beam.

A complete knowledge of the laser beam properties is often needed for the interpretation of data in many experiments. Manufacturers usually specify only a few parameters such as the dominant mode, beam width and divergence. This information can often prove to be too limited, particularly for beams of semiconductor lasers, semiconductor laser arrays, flat-topped laser beams etc.

The full characterisation of a laser beam requires the specification of about 10 parameters. For a symmetric beam, this can be reduced to a much smaller set of quantities. The ISO standards prescribe the following quantities for industrial purposes:

1. Beam width, divergence angle and beam propagation constants
2. Geometrical laser beam parameters and their propagation
3. Power characteristics and energy density distribution
4. Spectral and temporal fluctuations.
5. Parameters for astigmatic and stigmatic beams,
6. Beam positional stability
7. Beam polarization
8. Beam spectrum
9. Shape of laser wavefront: phase distribution

In this experiment, we will study the transverse beam shape of the He-Ne laser.

Many low-power lasers operate in the TEM_{00} mode where the electric field distribution is given by

$$E(x,y,z) = E_0 \frac{w_0}{w(z)} \exp\left[i(kz-\phi)\right] \exp\left[ik\frac{x^2+y^2}{2R(z)}\right] \exp\left[-\frac{x^2+y^2}{w^2}\right] \tag{1.1}$$

where the beam waist is defined as

$$w^2(z) = w_0^2\left[1+\frac{z^2}{z_0^2}\right] \tag{1.2}$$

and the radius of curvature $R(z)$ is defined as

$$R(z) = z + \frac{z_0^2}{z} \tag{1.3}$$

The quantity z_0 is known as the Guoy phase, and is given by

$$\phi = \tan^{-1}\left(\frac{z}{z_0}\right) \tag{1.4}$$

While such exact expressions can be derived for Gaussian beams, it is not possible for more general beams. A measure of beam width for more general cases is the so-called $D_{4\sigma}$ beam width, defined as

$$D_{4\sigma} = 4\left\{\frac{\int_{-\infty}^{\infty}(x-\bar{x})^2 I(x,y)dxdy}{\int_{-\infty}^{\infty} I(x,y)dxdy}\right\} \tag{1.5}$$

where

$$\bar{x} = \frac{\int_{-\infty}^{\infty} xI(x,y)dxdy}{\int_{-\infty}^{\infty} I(x,y)dxdy} \tag{1.6}$$

is the x coordinate of the centroid of the beam.

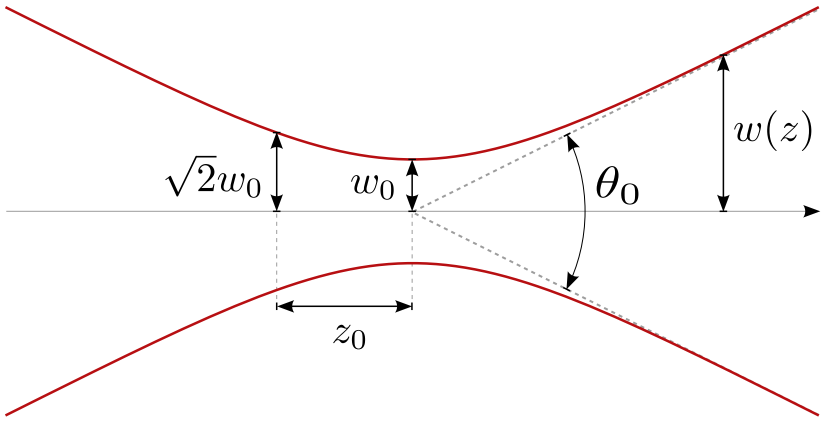

Figure 1.1: A schematic of the propagation of a Gaussian beam, showing the beam waist w_0, Rayleigh range z_0 and the divergence angle θ_0.

Far away from the beam focus, a Gaussian beam spreads in a hyperbolic shape, as shown in Figure 1.1. The asymptotes of the hyperboal define the far-field beam divergence such that the half angle at the $1/e^2$ radius in the intensity distribution is given by

$$\theta_0 = \lim_{z\to\infty} \frac{w(z)}{z} \tag{1.7}$$

Most real laser beams are not Gaussian although they might be close to Gaussian. The deviation away from a pure Gaussian beam affects the ability to focus them properly. The Beam Parameter Product (BPP) is defined as the product of beam radius (measured at the beam waist) and the beam divergence half-angle (measured in the far field). The BPP is often used to specify the beam quality of a laser beam: the higher the beam parameter product, the lower is the beam quality. The M^2 factor, also called beam quality factor or beam propagation factor, is the common measure of the beam quality of a laser beam. As per ISO Standards, the M^2 parameter is defined as the beam parameter product divided by λ/π, the latter being the beam parameter product for a diffraction-limited Gaussian beam with the same wavelength. In other words, the half-angle beam divergence is

$$\theta_0 = M^2 \frac{\lambda}{\pi w_0} \tag{1.8}$$

A diffraction-limited beam has an M^2 factor of 1, and is a Gaussian beam. Smaller values of M^2 are physically not possible due to diffraction (Gaussian has the minimum beam waist or focus spot size).

1.7 Measurement of the He-Ne laser beam waist

Since the He-Ne laser has a nearly Gaussian symmetric beam, it is almost entirely characterised by the beam waist and the Rayleigh range. One can measure the beam waist, its location and the Rayleigh range by a series of measurements of the beam width at various locations using a knife edge.

We will record the total transmitted power as a knife edge is translated through the beam as a function of the knife edge location. Mount a laser on a optical rail and direct the laser beam onto a large area photodetector. Take care to put the entire laser beam area into the photodetector -

you may need to sufficiently attenuate the beam to avoid saturation of the photodetector. Now using a translation stage, place a knife edge and translate it orthogonal to the beam direction, thereby slowly cutting off the beam. The photodetector records the integral of the Gaussian beam between ∞ and the position of the knife X. Note that if the Intensity of the Gaussian beam is

$$I(x,y) = I_0 \exp\left[-\frac{(x^2+y^2)}{w^2(z)}\right] \tag{1.9}$$

then the total power in the beam is

$$P_t = \int_{-\infty}^{\infty} I(x)dxdy = \frac{\pi}{4} I_0 w^2(z) \tag{1.10}$$

As a function of the location (X) of the knife-edge, using the properties of Gaussian integrals, the transmitted power is obtained to be

$$P(X) = \int_{X}^{\infty} I(x,y)dxdy \tag{1.11}$$

$$= P_t - \int_{\infty}^{X} I(x,y)dxdy \tag{1.12}$$

$$= \frac{P_t}{2}\left[1 - \text{erf}\left(\frac{2X}{w(z)}\right)\right] \tag{1.13}$$

where the error function $\text{erf}(x)$ is defined to be

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du \tag{1.14}$$

Plot the measured $P(X)$ versus X to determine the $1/e^2$ width of the beam. The width of the beam is also sometimes defined as the distance between the points of the measured curve that are 10% and 90% of the maximum signal. Relate this 10-90 width to the $D_{4\sigma}$ width.

As the location of the minimum waist-size (focus spot) is not known for the laser, let us assume it to be at a distance a from the face of the laser. Using the knife edge method to measure the width of the beam (10%-90%) at several (> 10) points along the laser beam. Use this data to determine the location as well the size of the beam waist of the laser. One can fit these measured data at the ten points to the equation

$$w^2(z) = w_0^2 + \left(\frac{M^2\lambda}{\pi w_0}\right)^2 (z - z_0)^2 \tag{1.15}$$

to determine the parameters w_0, M^2 and z_0. You may use MATLAB or Python or another software with similar capabilities to perform this analysis.

2. Fresnel coefficients for reflection at a air-glass interface

2.1 Introduction

When a plane electromagnetic wave is incident on a planar interface between two dielectric media, part of it is reflected and the remainder, transmitted into the second medium. Consider the geometry below.

The angle of reflection θ_r is equal to the angle of incidence θ_i. The angle of refraction θ_t is related to the angle of incidence via Snell's law.

$$n_2 \sin\theta_i = n_2 \sin\theta_t \tag{2.1}$$

Here, n_1 and n_2 are the refractive indices of the first and second medium, respectively.

The Fresnel equations relate the amplitudes of the reflected and transmitted waves with the incident wave amplitude. The expressions for the reflected (r) and transmitted (t) amplitudes differ for light polarized parallel (p) and perpendicular (s) to the plane of incidence.

$$r_\mathrm{s} = \frac{n_1\cos\theta_\mathrm{i} - n_2\cos\theta_\mathrm{t}}{n_1\cos\theta_\mathrm{i} + n_2\cos\theta_\mathrm{t}}, \tag{2.2a}$$

$$t_\mathrm{s} = \frac{2n_1\cos\theta_\mathrm{i}}{n_1\cos\theta_\mathrm{i} + n_2\cos\theta_\mathrm{t}}, \tag{2.2b}$$

$$r_\mathrm{p} = \frac{n_2\cos\theta_\mathrm{i} - n_1\cos\theta_\mathrm{t}}{n_2\cos\theta_\mathrm{i} + n_1\cos\theta_\mathrm{t}}, \tag{2.2c}$$

$$t_\mathrm{p} = \frac{2n_1\cos\theta_\mathrm{i}}{n_2\cos\theta_\mathrm{i} + n_1\cos\theta_\mathrm{t}}. \tag{2.2d}$$

From these amplitudes, the power reflectance (R) and transmittance (T) can be calculated as

$$R_i = |r_i|^2 \tag{2.3}$$

$$T_i = \frac{n_2\cos\theta_\mathrm{t}}{n_1\cos\theta_\mathrm{i}}|t_i|^2 \tag{2.4}$$

for $i = s, p$.

When $n_2 < n_1$, it is possible for all the light to be reflected, with nothing transmitted into the second medium. This phenomenon is termed *total internal reflection*. A second interesting case occurs for p-polarized light where, at a certain angle of incidence, termed the Brewster angle, all the light is transmitted into the second medium and none reflected. The angles of incidence at which these phenomena occur can be derived from the Fresnel formulae. (**Do this.**)

2.2 Aims

1. Verify the Fresnel coefficients of reflection for a air-glass interface
2. Determine the refractive index of the glass
3. Study total internal reflection of light

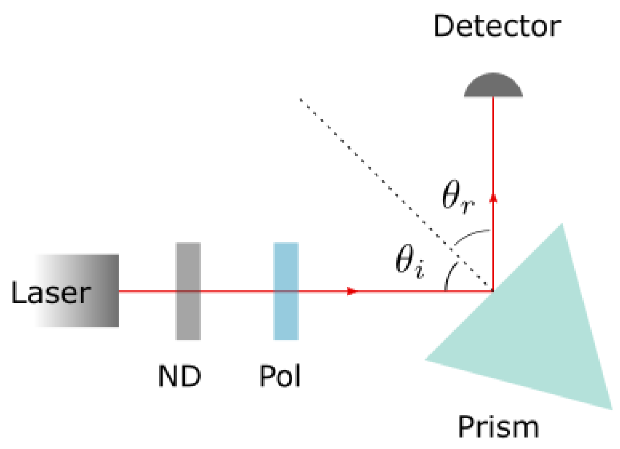

Figure 2.1: Schematic of the setup. ND: Neutral Density filter; Pol: Polarizer

2.3 Procedure

The experiment involves measuring the fraction of the incident laser power that is reflected off the surface of a glass prism.

Preliminary steps

Level the spectrometer and the prism table. Place the prism provided to you on the prism table. Align the laser at a convenient height such that the beam propagates horizontally. Add the neutral density filters and a polarizer whose axis orientation is precisely known. Mount the photodiode on the rotating arm of the spectrometer table. A schematic of the setup is shown in Figure 2.1.

2.3.1 Measuring reflectance and transmittance

With the polarizer axis horizontal (**does this orientation correspond to *s* or *p* polarization**), measure the reflected power as a function of angle of incidence. Use an angle step of 1° or smaller between measurements. Start at a small angle of incidence ($\theta_i \approx 10^\circ$) and continue to at least until ($\theta_i \approx 70^\circ$) . Remove the prism and measure the incident power for this polarization. Repeat the measurements with the polarization axis vertical. Plot the reflectance as a function of the incidence angle for both polarizations. Compare the results with the predictions of the Fresnel formulae.

2.3.2 Determination of the refractive index

For p-polarized light, you should see a minimum in the reflectance at a particular angle of incidence. This corresponds to the Brewster angle. Determine the refractive index of the prism from this measurement (**how?**). Analyse the error in your measurement.

2.3.3 Total internal reflection

Replace the equilateral prism with a right-angle prism. Adjust the geometry such that the light enters the prism through one of the shorter sides and exits through the hypotenuse. Now, reduce the angle of incidence on the first (air $\rightarrow$ glass) interface of the prism, thereby increasing the angle of incidence at the second (glass $\rightarrow$ air) interface. Beyond a certain angle, all the light is reflected.

Can you determine the angle of incidence at the glass $\rightarrow$ air corresponding to the onset of total internal reflection?

3. Polarization Optics

3.1 Introduction

Light propagating in a homogeneous isotropic dielectric is a transverse wave. A plane electromagnetic wave can be expressed as

$$\mathbf{E}(z,t) = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \tag{3.1}$$

The complex amplitude of this wave $\mathbf{E}_0$ is a vector quantity. If $\tilde{\mathbf{E}}_0$ points along a specific direction, the plane wave is said to be polarized along that direction.

Ordinarily, ambient light consists of a mix of several orientations of the electric field. For example, consider a light wave propagating along the z direction. We can express this as

$$\mathbf{E}(z,t) = (E_x\mathbf{x} + E_y\mathbf{y})e^{i(kz-\omega t)} \tag{3.2}$$

If the components E_x and E_y do not differ by a phase, or if the phase difference is π (i.e. E_x/E_y is real), the light is said to be *linearly polarized* or *plane polarized*. In such a situation, when measured in any plane perpendicular to the z-axis, the electric field is simply oscillates with a frequency ω along a fixed direction.

A *polarizer* is an optical element that allows only light whose electric field is oriented along a specific axis to pass through. Light with an electric field component perpendicular to this axis is rejected via absorption or reflection, depending on the type of polarizer. If plane polarized light of intensity I_0 is incident on the polarizer, and the axis of the polarizer makes an angle θ with the incident polarization, the transmitted intensity $I(\theta)$ is given by (Malus' law):

$$I(\theta) = I_0 \cos^2\theta, \tag{3.3}$$

In general, the complex amplitudes E_x and E_y can differ by a phase. This leads to more complex states of polarization, in which the instantaneous direction of the electric field vector changes its direction during every cycle of oscillation. Such light waves are said to be *elliptically polarized*. When $E_y = E_x e^{i\pm\pi/2}$, the light is said to be *circularly polarized*.

In practice one component of the electric field can be phase-shifted with respect to the other using a device known as a wave plate. A wave plate is made of a *birefringent* material — a material that exhibits different refractive indices for two orthogonal polarization states of light. Each wave-plate possesses a *fast axis* and a *slow axis*. The slow axis corresponds to a longer optical path. Consequently, light polarized along the slow axis acquires a larger phase than light polarized along the fast axis. A *half-wave plate* and *quarter-wave plate* impart a phase difference of π and $\pi/2$ respectively, between the slow and fast axes.

A half-wave plate whose fast axis is oriented at θ relative to the incident polarization, rotates the linear polarization by an angle 2θ. A quarter-wave plate converts the incident linear polarization to a variety of elliptical polarization states. When the quarter-wave plate fast axis makes an angle of $\pm\pi/4$ with the incident polarization, the output polarization is circular. The action of polarizers and wave-plates can be conveniently described by Jones matrices. You will learn about these in the theory lectures.

3.2 Aims

The current experiment aims at studying the properties of various optical components that alter the polarization of light. Combinations of these elements can be used to tailor the output polarization according to one's requirement.

1. To verify Malus' law using two polarizers
2. To study the action of a half-wave plate
3. To study the action of a quarter-wave plate
4. To build and study a simple optical isolator

3.3 Procedure

Follow the usual sequence of steps to align the laser such that the beam propagates parallel to a row of holes in the optical bench. Add a suitable combination of neutral density filters such that the signal detected on the photodiode is roughly 250 mV. Leave a space of at least 30 cm between the filters and the photodiode so that you can insert combinations of polarizers and wave plates in the beam path.

3.3.1 Preliminary Steps

Mount the reference polarizer provided to you such that the polarization axis is vertical (or horizontal). Place it in the path of the laser beam. Now, place one of the polarizers mounted on rotation mounts in the laser path. Let us call it P1. Rotate the polarizer to an orientation such that the signal is minimum (the axes of the two polarizers are perpendicular to each other). Now rotate the polarizer P1 by 90 degrees, such that its axis is now parallel to the reference polarizer. Remove the reference polarizer. Now the light beyond the polarizer is linearly polarized along the vertical (or horizontal) direction.

3.3.2 Verification of Malus' Law

Place a second polarizer in the laser beam path (let us call this P2) about 20 cm from the first polarizer. Rotate P2 from 0° to 360° in steps of 5° and measure the transmitted intensity using the photodiode at each step. Plot your measured signal as a function of the orientation of P2. Do your observations agree with Malus' law?

3.3.3 The half wave plate and quarter-wave plate

Rotate the polarizer P2 such that its axis is parallel to that of P1 (transmission is maximized). Now, place a half-wave plate (also mounted on a rotating mount) between P1 and P2. Let us call this HWP. Rotate the HWP from 0° to 360° in steps of 5° and measure the transmitted intensity, while keeping P1 and P2 fixed. Plot the measured signal as a function of HWP orientation. Justify your observations by deriving an expression for the transmitted intensity using Jones matrices.

Now, replace the HWP with a quarter-wave plate (QWP) and repeat the procedure. Again, justify your observations. How does the variation of signal with QWP orientation differ from the case of the HWP?

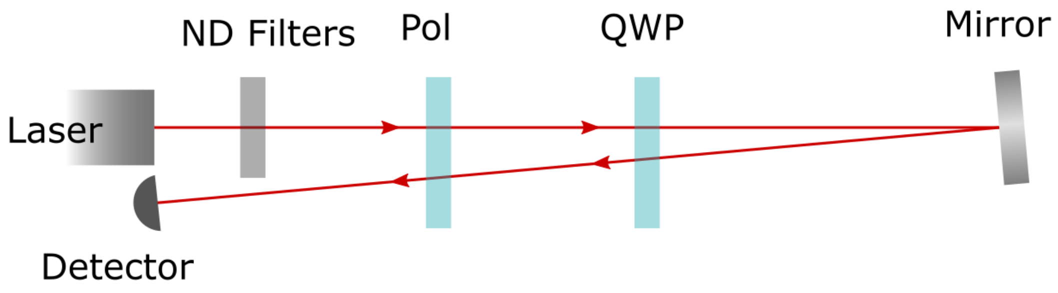

Figure 3.1: A simple optical isolator

3.3.4 An optical isolator

An optical isolator is a device that blocks back reflection. To build a simple isolator, assemble the setup shown in Figure 3.1. Light from the laser passes through the ND filters, the polarizer (Pol) and the QWP and is incident on the mirror at near-normal incidence. The reflected beam then passes through the same QWP and polarizer, and its power is measured by the photodiode. Adjust the orientation of the QWP axis and explore orientations for which no reflected light reaches the photodiode. Derive, using Jones matrices and vectors, the conditions where the reflected beam is not transmitted through the polarizer. Can you deduce how the mirror affects the light?

4. Linear and Circular Birefringence

4.1 Introduction

Linear birefringence

In many commonly occurring media such as glass, the polarization and displacement vector are proportional to the applied field. The latter can be described as:

$$\mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E} \tag{4.1}$$

where ε is the dielectric constant. Many crystalline materials can be more polarizable along certain directions than others, leading to a susceptibility (and dielectric constant and refractive index) that varies with the direction of the applied electric field. For such scenarios, the scalar dielectric constant ε must be replaced by the dielectric tensor $\boldsymbol{\varepsilon} = \varepsilon_{ij}$. This leads to a more complex relation between the displacement vector and electric field:

$$\mathbf{D} = \varepsilon_0 \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \mathbf{E}. \tag{4.2}$$

The dielectric tensor is symmetric ($\varepsilon_{ij} = \varepsilon_{ji}$) and diagonalizable by transforming the coordinate axes to match the *principal axes* of the crystal, which are related to the symmetries of the crystal.

Since the magniture of dielectric response of the medium now depends on the direction of the electric field, electromagnetic waves of different polarization may experience different refractive indices and travel with different propagation constants and velocities through the medium. The two orthogonal polarizations therefore acquire a different phase as they propagate through the medium, and this changes the state of polarization. Linearly polarized light, upon propagating through the film becomes elliptically polarized. Therefore, a system consisting of the birefringent film placed between two crossed polarizers transmits a finite fraction of the incident light. The transmitted intensity I_T can be expressed as

$$I_T = I_0 \sin^2(2\alpha) \sin^2(\Delta n k_0 \delta/2) \tag{4.3}$$

where I_0 is the incident intensity, Δn is the difference between refractive indices for two orthogonal polarizations, k_0 is the free space wave number, δ is the thickness of the sample, and α is the angle between the ordinary axis of the anisotropic medium and the polarization axis.

Circular Birefringence

A linear polarization can be expressed as a superposition of left circularly polarized (LCP) and right circularly polarized (RCP) light with equal amplitudes. By adding a phase to one of the circular polarizations, the linear polarization is rotated (**derive a relation between this phase shift and polarization rotation angle, using Jones matrices**). If a medium exhibits different refractive indices for LCP and RCP, the two circular polarizations acquire a different phase as they propagate through the medium. As a consequence, if linearly polarized light passes through the medium, it's plane of polarization is rotated. This phenomenon is termed circular birefringence or *optical activity*.

This occurs when the elementary constituents of the medium (molecules, unit cells in a crystal etc.) lack inversion symmetry (exhibit *handedness*) and respond differently to LCP and RCP light. Such molecules are termed *chiral*. In the current experiment, it is the sugar molecules that are chiral. Chiral molecules occur in two forms (which are mirror images of each other, or *enantiomers*). The two enantiomers rotate the plane of polarization in opposite directions. When looking towards the source of light, if the plane of polarization is rotated clockwise, the enantiomer is termed the $D-$enantiomer (for *dextro*, meaning right). The other enantiomer, which rotates the plane of polarization anti-clockwise when looking towards the source, is termed the $L-$enantiomer (for *laevo* meaning left).

This behaviour is exhibited by a large number physiologically relevant molecules, and measuring this rotation of polarization is a very important means of identifying the stereoisomers of molecules involved.

4.2 Aims

1. Determine the birefringence of a thin film of mica
2. Determine the difference in refractive indices for left and right circularly polarized light in a solution of dextrose.

4.3 Procedure

The basic setup is similar to Experiment 3. Align the laser beam, add the neutral density filters, add a polarizer whose axis is known, and add a second polarizer a sufficient distance (about 30 cm between polarizers should suffice) along the polarization direction so that components can be inserted between them.

4.3.1 Studying linear birefringence

Rotate the first polarizer so that its axis coincides with the x axis. Rotate the second polarizer such that its axis is orthogonal to the first polarizer and no light is transmitted. Introduce the optically anisotropic film sample (also mounted on a rotating mount) between the polarizers. Measure the transmission after the second polarizer for a full range of orientations of the anisotropic sample ($0°$ to $360°$ in steps of $5°$). Plot the transmission versus the angular orientation of the anisotropic sample. Analyze your data using Equation 4.3 and determine the value of Δn, the difference in refractive indices for the two orthogonal polarizations. Also obtain an estimate of the error in your value.

4.3.2 Studying circular birefringence

The setup here is similar to the previous subsection. Start with the first polarizer oriented along the x axis and the second one along y, so that no light is transmitted. Note the angular position of the second polarizer. Introduce the liquid cell containing a solution of dextrose between the polarizers. Now, rotate the second polarizer through a full $360°$ in steps of $5°$.

1. Confirm that the polarization of the light that exits the liquid cell is linear (**how?**).

2. Determine the angle through which the polarization has been rotated. From this measurement, determine the difference in refractive index for the two circular polarizations from your data.

3. Now, generate circular polarization by introducing a quarter-wave plate after the first polarizer. Check if the light remains circularly polarized after passing through the dextrose solution.

4. Repeat the Step 2 using at least two other different concentrations of dextrose solution.

5. Interference with two or more slits

5.1 Introduction

Interference

When two light waves are superposed, the resulting intensity at the point of observation can be expressed as follows.

$$I(\mathbf{r},t) = \frac{n\varepsilon_0 c}{2}\mathbf{E}(\mathbf{r},t)\cdot\mathbf{E}^*(\mathbf{r},t) \tag{5.1}$$

For two waves $\mathbf{E}_1(\mathbf{r},t) = \mathbf{E}_0 e^{i(\mathbf{k}_1\cdot\mathbf{r}-\omega t)}$ and $\mathbf{E}_2(\mathbf{r},t) = \mathbf{E}_0 e^{i(\mathbf{k}_2\cdot\mathbf{r}-\omega t)}$ of the same amplitude and frequency with a phase difference ϕ travelling along different directions, the resulting intensity is given by:

$$I(\mathbf{r},t) = n\varepsilon_0 c\mathbf{E}_0\cdot\mathbf{E}_0^*[1+\cos(\Delta\mathbf{k}\cdot\mathbf{r}+\phi)] \tag{5.2}$$

where $\Delta\mathbf{k} = \mathbf{k}_2 - \mathbf{k}_1$.

In regions where the waves are out of phase, the amplitudes cancel out each other, and in regions where they are in phase, the amplitudes add up. The two situations are termed *constructive* and *destructive* interference, respectively.

The double slit experiment

The double-slit experiment by Young was an important experiment in the development of optics, which showed that light is a wave. Consider the geometry shown in Figure 5.1. To achieve constructive interference at a point on the screen, the difference between path travelled by light from the two slits should be an integer multiple of the wavelength λ. For small angles θ and $d \gg a$, it is straightforward to obtain (**derive this.**):

$$I(\theta) = I(0)\cos^2\left(\frac{ka\sin\theta}{2}\right) \tag{5.3}$$

where $k = 2\pi/\lambda$. This represents a periodic sinusoidal fringe pattern.

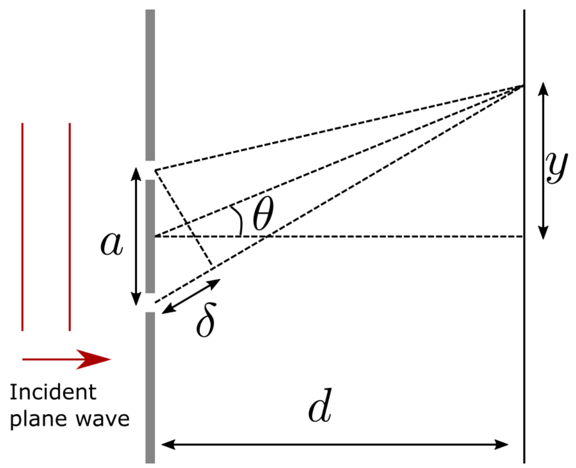

Figure 5.1: Geometry to study interference with two slits

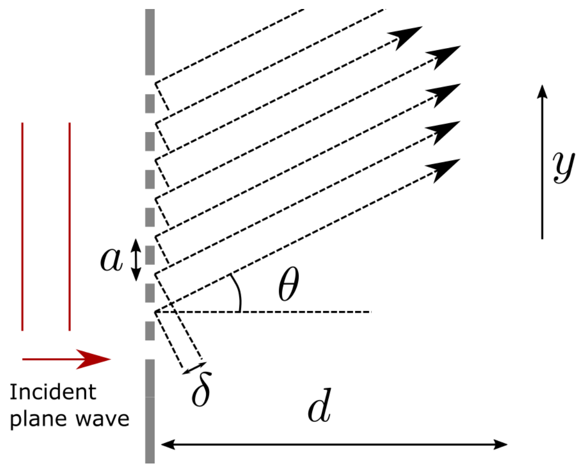

Figure 5.2: Geometry to study interference with n slits

Extension to n slits

When the number of slits is increased to n (equally spaced with a separation a), under similar assumptions of $d \gg a$ (the point of observation is far away), the path difference for light originating from any two adjacent slits is the same ($\delta = a \sin\theta$). This is shown in Figure 5.2.

For convenience we can define a quantity $\alpha = 2\pi\delta/\lambda$. In this approximation, the intensity distribution can be expressed as (**derive this expression and plot the function as an exercise**):

$$I(\alpha) \approx I(0) \left[\frac{\sin(n\alpha/2)}{n \sin(\alpha/2)} \right]^2 \tag{5.4}$$

The interference pattern consists of primary maxima with secondary maxima between them. The number of secondary maxima between consecutive primary maxima is given by $m = n - 2$, (**Derive this relation.**). By measuring m, is is therefore straightforward to determine the number of slits. The spacing between the slits can be determined by measuring the angles θ corresponding to the primary maxima.

Diffraction Grating

Consider an extension of the n slit case (Figure 5.2) to an infinite number of slits. As n becomes larger, the secondary maxima become less significant, and the interference pattern is dominated by the primary maxima. For normal incidence, angles corresponding to these maxima is given by the locations of the primary maxima in case of n slits:

$$a \sin\theta = m\lambda \tag{5.5}$$

The central maximum is located at $\theta = 0$, for any wavelength λ. However, the locations of other maxima depend on the wavelength. THerefore, for $m \neq 0$, the grating exhibits “dispersion” like a prism. Separating wavelengths is the primary application of gratings. Can you point out the difference between the dispersion characteristics of a prism and a grating?

5.2 Aims

1. Develop a qualitative understanding of the double-slit experiment

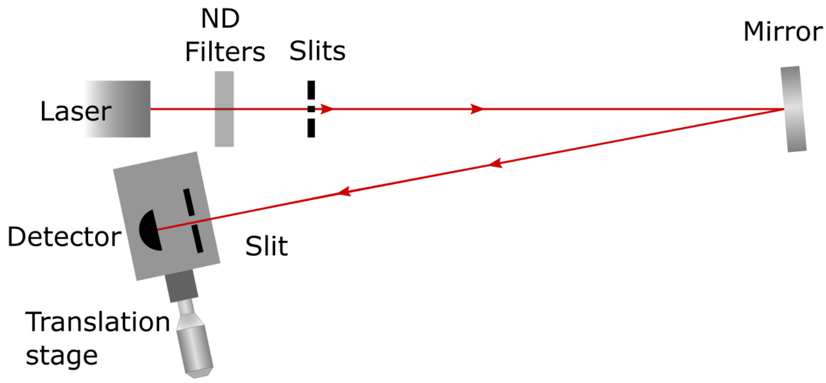

Figure 5.3: Schematic of the experimental setup to study double slit and multiple slit interference.

2. Record the interference pattern from a pair of slits, and determine the slit separation

3. Record the interference pattern from n closely spaced slits, and determine the number of slits and their separation

5.3 Procedure

Align the laser beam along a row of holes on the optical table (or breadboard) like you did in the previous experiments. Arrange a white screen at the far end of the table to view the interference pattern.

5.3.1 Double Slit

Introduce the double slit into the laser path such that the laser beam is centered between the slits, and equal power passes through both slits. Close the slits. Open a single slit and observe what happens. You should see a single slit diffraction pattern on the screen. Reduce the width of the slit gradually. You will see that the diffraction pattern broadens as the width of the slit is reduced (**why?**). When the slit is narrow enough, you will see that the central maximum spans several centimetres and the secondary maxima are quite faint. Now open the second slit. You will see that its diffraction pattern is superimposed over that of the first slit. Reduce the width of the second slit so that its diffraction pattern is as wide as that of the first slit. In the region where the two diffraction patterns are superimposed, you should see a periodic interference pattern.

Place a mirror at the far end of the screen, such that the light is reflected back with a small angle, and you can measure the intensity distribution by placing a detector beside the laser. Mount the photodiode on a translation stage and place it at the plane where you want to measure the interference pattern. if the photodiode is wider than the fringes, place a pinhole or slit in front of the detector so that only a narrow region of the interference pattern is sampled by the detector. A schematic of the setup is shown in Figure 5.3.

Record the intensity distribution of the interference pattern (at least 7-8 fringes), by moving the translation stage using the micrometer actuator, and measure the photodiode signal. Use the fringe spacing to determine the slit separation. Analyze the possible errors in this measurement.

5.3.2 *n* slits

Proceed as in the previous section. Illuminate the slits in such a way that light of the same intensity passes through each slit. Instead of the double slit, center the n-slit sample in the laser beam, and observe its interference pattern. You will notice that there is an intense central maximum with primary maxima on either side and secondary maxima between them. Measure the signal across the intensity distribution from the first primary maximum on one side of the central maximum to the first primary maximum on the other side, thereby recording the intensity variation across the secondary maxima. You will need at least five points per peak. Using this measurement, determine the number of slits and the separation between the slits.

5.3.3 Diffraction grating

Illuminate a transmission grating with the laser beam as in the case of *n* slits. You should now see a pattern of intensity maxima in the output. Now, Place a screen (preferably a graph paper) at the output at a convenient distance from the grating such that several intensity maxima are present on the screen. Measure the distance between the maxima and the distance between the screen and the grating. Using these measurements, and the density of lines on the grating, determine the wavelength of light. Does this value match what you expect? Perform an error-analysis for this measurement.

6. Michelson Interferometer

6.1 Introduction

The basic layout of a Michelson interferometer is shown in Figure 6.1. A plane wave is split into two replicas (of half the power) by a beamsplitter. One wave travels a distance d_1 to the mirror $M1$ and another wave travels a distance d_2 to the mirror $M2$. The waves are then completely reflected and return to the beam splitter. If $d_1 \neq d_2$, the difference in total path travelled results in a phase difference between the two waves. At the beam splitter, the waves returning from the mirrors are split in two again, one copy travels back towards the source, and another copy travels towards the screen.

Let us assume that the electric field amplitude of the input plane wave is $E_i = E_0 \exp(ikz - \omega t)$ with corresponding intensity I_0 and the beamsplitter is lossless and thin. We can also assume for convenience, without loss of generality, that $\exp(ik(|AB| + |BD|)) = 0$. the amplitude of the superposition at the detector plane is given by

$$E = \frac{E_0}{2}\left(\exp(ik \cdot 2d_1 - \omega t) + \exp(ik \cdot 2d_2 - \omega t)\right) \tag{6.1}$$

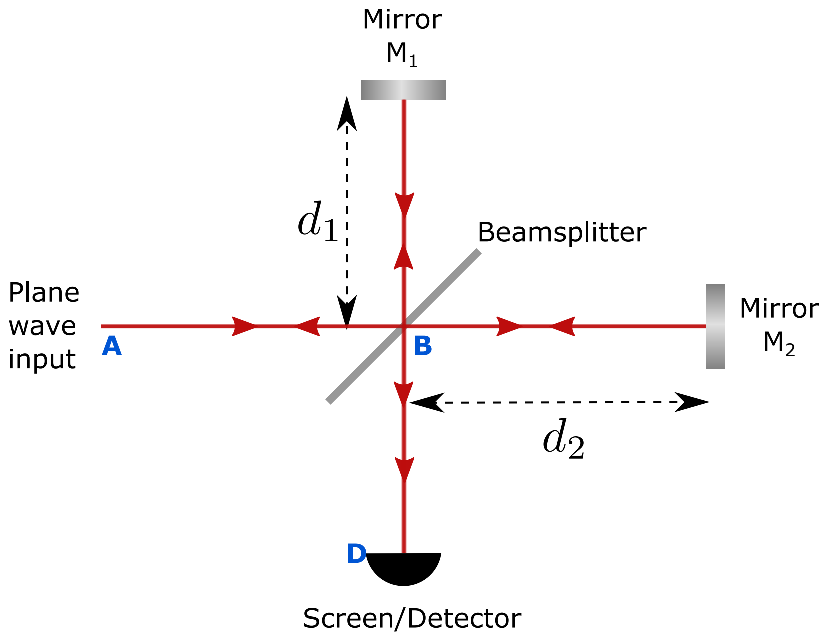

Figure 6.1: A schematic diagram of a Michelson interferometer with a plane wave input

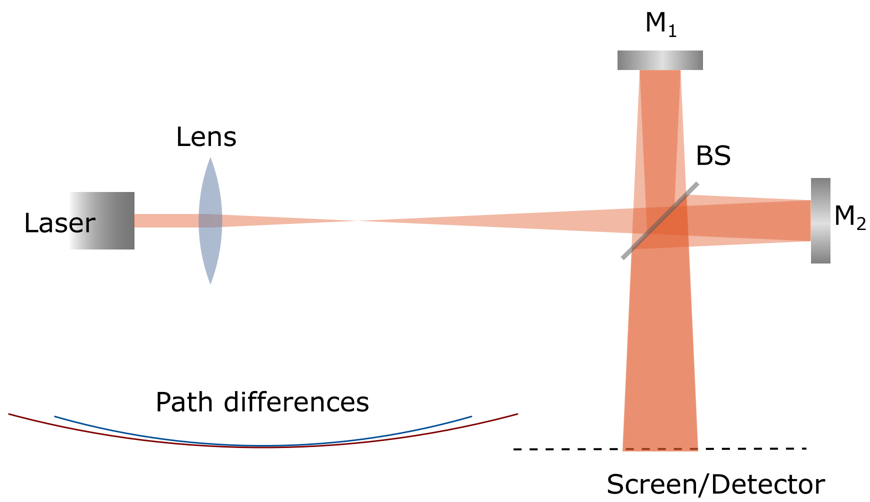

Figure 6.2: A schematic diagram of a Michelson interferometer with diverging beams. At the bottom left is a sketch of the wavefronts at the screen corresponding to the longer (red) and shorter (blue) arms.

The intensity is then equal to

$$\begin{aligned} I &= \frac{n\varepsilon_0 c E_0^2}{8} |\exp(ik \cdot 2d_1 - \omega t + \exp(ik \cdot 2d_2 - \omega t)|^2 \\ &= \frac{n\varepsilon_0 c E_0^2}{4}(1 + \cos(2k(d_2 - d_1))) \\ &= \frac{I_0}{2}(1 + \cos(2k(d_2 - d_1))) \\ &= \frac{I_0}{2}(1 + \cos k\Delta) \end{aligned} \tag{6.2}$$

where Δ is the path difference between the two superimposed waves. This treatment applies to the case where the interfering wavefronts are planar. When $k\Delta = 2n\pi$, all light propagates towards the detector and when $k\Delta = (2n+1)\pi$, no light reaches the detector (where n is an integer). For perfectly plane co-propagating waves, no fringes will be seen on the screen - just a uniform intensity.

In your lab experiment, you introduce a convex lens of short focal length in front of the laser, so that the wavefronts diverge from the focal plane of the lens, as shown in Figure 6.2. In this case, spherical wavefronts emerging from the focal plane of the lens travel along the arms of the interferometer. If there is a path difference between the arms of the interferometer ($d_1 \neq d_2$), the two wavefronts interfering at the screen will possess different radii of curvature. This leads to circular fringes. As you increase or decrease the path length on one of the arms of the interferometer, the fringes appear to either emerge from or collapse into the center (**can you determine when they emerge from the center and when they collapse into it?**)

6.2 Aims

1. To set up a Michelson interferometer align it to obtain a circular fringe pattern.
2. Adjust the lengths of the arms of the interferometer to obtain zero path delay

3. To calibrate the fine divisions of a micrometer actuator

6.3 Procedure

6.3.1 Preliminary alignment

For this experiment you will need two mirrors - one mounted on a post, and a second one mounted on a specialized stage. This stage contains the mirror on a platform that can be moved longitudinally by means of a micrometer and a lever. The distance corresponding to one fine division of the micrometer needs to be calibrated as part of the experiment.

1. Align the laser beam along one row of holes on the optical table, as in the case of all other experiments.

2. Place the beamsplitter in the path of the beam about 30 cm from the laser at a 45° angle. The laser beam will be partly reflected and partly transmitted, resulting in two beams propagating in perpendicular directions.

3. Mount the two mirrors at roughly the same distance from the beamsplitter such that they reflect the two beams back to the beamsplitter. You can estimate the distance by counting the number of holes on the breadboard between the mirror and the beamsplitter.

4. At the output arm, place a white screen. You should see laser beam spots on the screen resulting from reflections from both mirrors. You may see multiple spots, depending on the thickness of the beamsplitter.

5. Perform a fine adjustment of the alignment using the tilt adjustment screws on the beamsplitter and the mirrors. The positions of the two spots on the screen corresponding to reflected light from the mirrors should coincide.

The basic layout of the setup should resemble Figure 6.2.

6.3.2 Generation of a circular fringe pattern

1. Introduce a convex lens of focal length $f = 5$ cm in front of the lens and center it around the laser beam. Now the beam first converges until it reaches the focal plane of the lens then diverges towards the beamsplitter. You will see a much larger illuminated region in on the screen

2. Fine tune the alignment of the interferometer mirrors until you see a circular fringe pattern on the screen. Often there will be vibrations from people walking, ceiling fans, and faraway construction work, which cause the fringes to lose contrast or disappear (**why?**). Try to identify the sources of vibrations and minimize them.

6.3.3 Achieving a zero path difference

Make the path lengths in the arms of the interferometer as close to each other as possible by adjusting the position of the mirrors. As the path difference is reduced, the radius of the first fringe from the center increases in diamater (**why?**) When the path difference is zero, the wavefronts from both arms of the interferometer coincide exactly on the screen, and there is no change in path difference across the beam. Therefore ideally, there will be no fringes, and the central maximum (or minimum) covers the entire illuminated area.

6.3.4 Calibrating the micrometer divisions

1. Move one of the mirrors away from an equal path length configuration, so that a clear circular fringe pattern is visible on the screen.

2. Note the reading on the circular scale of the micrometer of the lever operated stage.

3. (**this step takes some practice**) Now rotate this micrometer by several divisions while simultaneously counting the number of fringes collapsing into the centre of the pattern (or emerging from it). Record the number of divisions moved and the number of fringes collapsed.

4. Repeat the previous two steps ≈ 10 times. Tabulate and plot the number of fringes collapsed versus the number of divisions moved.

5. With the knowledge that one fringe maximum collapsed corresponds to a path difference of one wavelength (λ), and therefore a mirror movement of $\lambda/2$, determine the calibration constant of the micrometer (distance moved per division) Perform the necessary analysis and estimate the error in your measurement.

7. Fabry-Perot Interferometer

7.1 Introduction

The Fabry-Perot Interferometer is made of two reflecting surfaces M_1 and M_2 facing each other separated by a distance d as shown in Figure 7.1.

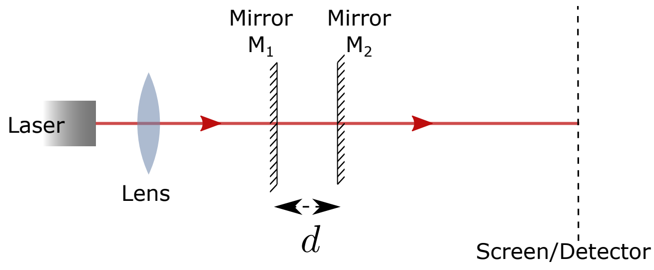

Figure 7.1: A schematic diagram of a Fabry Perot Interferometer setup

Unlike the Michelson interferometer, where each wavefront undergoes two reflections (one at the beamsplitter and one at a mirror), waves in a Fabry perot interferometer undergo an infinite number reflections back and forth between the mirrors, like in a thin film. As in the case of the Michelson interferometer, we can produce a circular fringes by using a diverging beam as the input by placing a lens in the path of the laser beam.

The interference maxima in transmission in case of a Fabry Perot interferometer look much sharper than those of the Michelson interferometer. You should be familiar with the reason for this from the last two theory classes. When illuminated by a plane wave, the transmittance of a Fabry Perot interferometer is proportional to

$$T \propto \frac{1}{1+F\sin^2(\delta/2)} \tag{7.1}$$

where δ is given by

$$\delta = \frac{4\pi n}{\lambda_0} d\cos\theta \tag{7.2}$$

Here, λ_0 is the vacuum wavelength, n is the refractive index of the medium between the two reflecting surfaces and θ is the angle at which the waves are propagating between the two mirrors, with respect to the axis of the interferometer. Here, F is the finesse of the interferometer.

As in case of the previous experiment, placing a positive (convex) lens between the laser and the first mirror M_1 leads to the formation of a diverging beam of light and therefore circular fringes. The radius of the observed fringes ξ_ν (where ν is the fringe index, with the exception of the central maximum/minimum) obeys the following dependence on the distance between the mirrors d.

$$d = n\frac{D^2\lambda}{\xi_n^2} \tag{7.3}$$

where $\xi_n^2 = \xi_{\nu+n}^2 - \xi_\nu^2$ and D is the distance between the second mirror M_2 and the screen.

7.2 Aims

1. To align and configure the Fabry Perot interferometer to obtain circular fringes.
2. To use the fringes to calibrate a fine micrometer.
3. To measure the spacing between the mirrors of the interferometer and determine the free spectral range.

7.3 Procedure

7.3.1 Alignment

You will be provided two mirrors mounted on adjustable (tilt) mounts. One of the mirrors can be moved using a lever-based fine micrometer stage. This is the micrometer actuator that you will calibrate, like you did in the previous experiment.

1. Align the laser along a row of holes in the breadboard like all the other experiments.
2. Place the two-mirror assembly in the path of the laser beam, about 30 cm from the laser. Place a screen beyond it.
3. Try to locate the beam spots corresponding to light reflected from both the mirrors, and try to redirect them towards the laser. As you improve the alignment of the mirrors, you will notice that there is a series of spots in the transmitted light. This is because of multiple reflections of the laser beam between the two mirrors. Gently adjust the tilt of the mirrors such that all these spots are superimposed (**this takes practice**). In such a configuration, light is reflected back and forth between the two mirrors at normal incidence.
4. Introduce a positive lens (say, of focal length 5 cm) into the laser beam path between the laser and the Fabry Perot mirror pair. Upon aligning and centering the lens, you should see circular fringes on the screen.

7.3.2 Calibrating the micrometer divisions

1. You will see that as you turn the fine micrometer (the one with the lever), the fringes collapse/emerge from the center of the pattern, as in the case of the Michelson Interferometer.
2. (**this step takes some practice**) Rotate this micrometer by several divisions while simultaneously counting the number of fringes collapsing into the centre of the pattern (or emerging from it). Record the number of divisions moved and the number of fringes collapsed, as you did in case of the Michelson interferometer.
3. Repeat the previous two steps ≈ 10 times. Tabulate and plot the number of fringes collapsed versus the number of divisions moved.
4. With the knowledge that switching from one maximum to the next corresponds to a mirror displacement of $\lambda/2$, determine the calibration constant of the micrometer (distance moved per division) and estimate the error in the value determined.

7.3.3 Determining the free spectral range

1. Measure the radii of a few circular fringes . You can do this using a micrometer stage and a pinhole or slit.

2. Measure the distance between the second mirror and the screen. Use Equation 7.3 to calculate the distance between the mirrors.

3. Using the above value of the distance between the mirrors, determine the free spectral range (FSR $= \frac{c}{2d}$)

8. Fraunhofer Diffraction

8.1 Introduction

You have learnt the basics of scalar diffraction theory in the theory classes. Consider the geometry shown in Figure 8.1, where light propagates along the positive z-axis. The field diffracted from a finite aperture located at $z = 0$, at a point (x, y, z_0) in the Fraunhofer regime, is given by

$$E(x,y,z) \propto \iint_{\text{Aperture}} E(x',y') \exp\left(-i\frac{2\pi}{\lambda z}(x'x + y'y)\right) dx'\,dy' \tag{8.1}$$

The diffractied field has the same functional form as a scaled Fourier transform of the field in the aperture plane. For a rectangular slit of width Δx and Δy in the x and y directions, this is quite straightforward to evaluate, and results in the following intensity distribution:

$$I(x,y,z_0) \propto \text{sinc}^2\left(\frac{\pi\Delta x}{\lambda z_0}x\right)\text{sinc}^2\left(\frac{\pi\Delta y}{\lambda z_0}y\right) \tag{8.2}$$

If the length of the slit along the y direction is long, you will not be able to discern the pattern along the y direction, and you will observe a one-dimensional diffraction pattern $\propto \text{sinc}^2\left(\pi x\Delta x/(\lambda z_0)\right)$. The diffraction pattern of an opaque wire of diameter Δx would also be similar, except in the forward direction.

8.2 Aims

1. To observe and characterize the Fraunhofer diffraction pattern from a single slit, and determine the width of the slit.

2. To observe and characterize the Fraunhofer diffraction pattern from a thin wire, and determine the diameter of the wire.

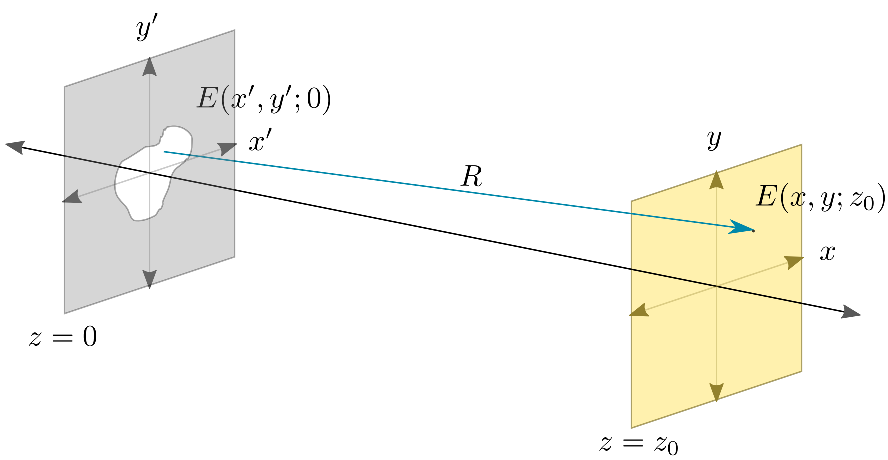

Figure 8.1: Geometry of the diffraction problem

8.3 Procedure

1. Align the laser along a row of holes in the breadboard like all the other experiments.
2. Use two convex lenses to build a telescope that increases the diameter of the laser beam to several mm, so that this better approximates a plane wave, and the slit is uniformly illuminated.
3. Introduce a variable-width slit provided to you in the path of the light. Observe the diffraction pattern far away from the laser. Ensure that the distance from the slit to the screen is long enough so that the fringe spacing can be conveneintly measured.
4. Using a photodiode and a pinhole or slit, measure the diffraction pattern of the single slit.
5. From the positions of the minima estimate the width of the slit. Estimate the error in your measurement.
6. Repeat the experiment using a thin wire instead of a slit. Record the diffraction pattern, and determine the thickness of the wire. Estimate the error in your measurement
7. Comment on the qualitative differences and similarities between the diffraction patterns obtained using the slit and the wire.