OPTICS (PHY224)
Temporal coherence basics

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Two-wave interference

Consider a typical two wave interference experiment. Let us denote the two fields superimposed on the detector by $\mathbf{E}_1$ and $\mathbf{E}_2$. The irradiance is therefore,

$$\begin{aligned} I &\propto \left\langle |\mathbf{E}_1 + \mathbf{E}_2|^2 \right\rangle \\ &= \langle (\mathbf{E}_1 + \mathbf{E}_2) \cdot (\mathbf{E}_1^* + \mathbf{E}_2^*) \rangle \\ &= \left\langle |\mathbf{E}_1|^2 + |\mathbf{E}_2|^2 + 2\mathrm{Re}(\mathbf{E}_1^* \cdot \mathbf{E}_2) \right\rangle \end{aligned} \tag{1}$$

Here, $\langle \cdots \rangle$ denote the time average defined as follows.

$$\langle f(t) \rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} f(t) dt \tag{2}$$

We assume *stationarity*, which means the average is independent of the choice of the origin on the time axis. We additionally assume that the fields have the same polarization, so we can ignore their vector nature. In such a situation, the expression for irradiance becomes

$$I = I_1 + I_2 + 2\mathrm{Re}\langle E_1 E_2^* \rangle \tag{3}$$

where $I_1 = |E_1|^2$ and $I_2 = |E_2|^2$.

Let us define the cross-correlation of the two fields E_1 adn :

$$\Gamma_{12}(\tau) = \langle E_1^*(t) E_2(t+\tau) \rangle \tag{4}$$

It is straightforward to see that $\Gamma_{11}(0) = I_1$ and $\Gamma_{22}(0) = I_2$

The Michelson Interferometer

In case of the basic Michelson interferometer shown in Figure 1, the amplitude of plane waves from the source is split equally between the two arms of the interferometer. In such a situation, both the fields are copies of each other, one shifted in time τ relative to the other.

$$E_2(t) = E_1(t+\tau) \tag{5}$$

In this case, the interference term in Equation 3 becomes an *autocorrelation* of E_1, defined as

$$\Gamma_{11}(\tau) = \langle E_1^*(t) E_1(t+\tau) \rangle \tag{6}$$

We can now define the irradiance in terms of this autocorrelation.

$$I = 2(I_1 + \mathrm{Re}\Gamma_{11}(\tau)) \tag{7}$$

It is often more convenient to define a normalized form of Γ_{11} as follows.

$$\gamma_{11}(\tau) = \frac{\Gamma_{11}(\tau)}{I_1} \tag{8}$$

Using γ_{11}, the interference pattern is simply:

$$I = 2I_1(1 + \mathrm{Re}\gamma_{11}(\tau)) \tag{9}$$

γ_{11} is typically a complex oscillating function of τ. An interference pattern results for any nonzero value of $|\gamma_{11}|$. More specifically, we can describe three distinct cases that we commonly encounter in experiments:

$$|\gamma_{11}| = 1 \rightarrow \text{Fully coherent}$$

$$0 < |\gamma_{11}| < 1 \rightarrow \text{Partially coherent}$$

$$|\gamma_{11}| = 0 \rightarrow \text{Fully incoherent}$$

The intensity of the interference pattern varies between $I_{\mathrm{max}} = 2I_1(1 + |\gamma_{11}|)$ and $I_{\mathrm{min}} = 2I_1(1 - |\gamma_{11}|)$. These fringes exhibit a contrast given by

$$\eta = \frac{I_{\mathrm{max}} - I_{\mathrm{min}}}{I_{\mathrm{min}} + I_{\mathrm{min}}} \tag{10}$$

It is quite straightforward to see that

$$\eta = |\gamma_{11}| \tag{11}$$

The fringe contrast in a Michelson interferometer is therefore a direct measure of the temporal coherence of the input light waves.

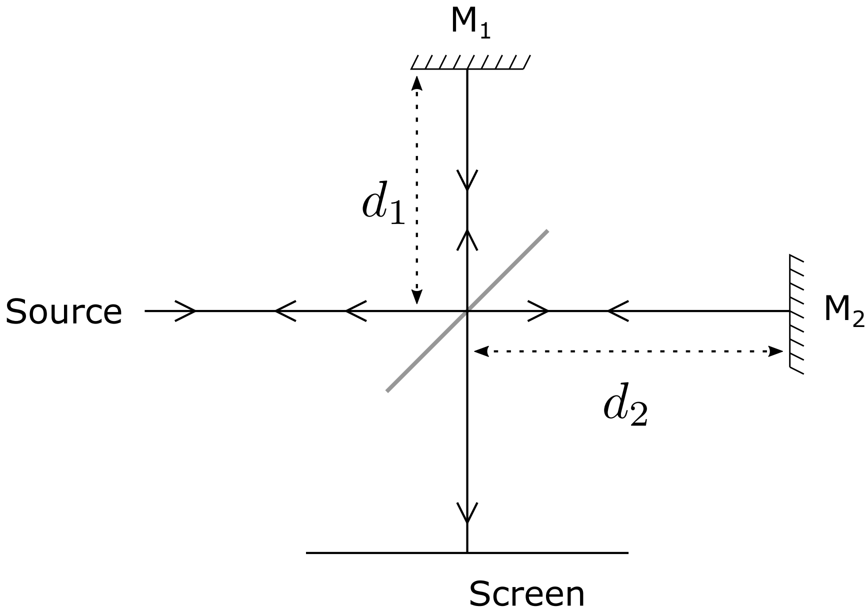

Figure 1: A Michelson Interferometer