PHY226M, Problem Set 4
Special Theory of Relativity
April 2025

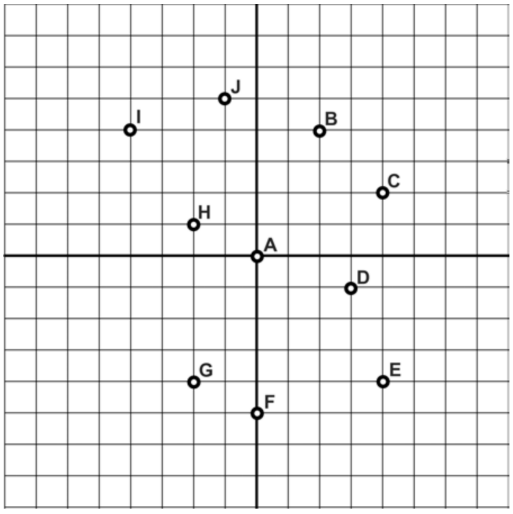
1. P is an object at rest in laboratory frame. Draw its world-line in laboratory frame.

2. In laboratory frame, A and B both are moving along positive X direction with velocity u_A and u_B respectively, where $u_A < c$, $u_B < c$ and $u_A < u_B$. Draw the world-lines of A and B in laboratory frame. Also draw the world-line of a light ray, moving along positive X axis, in the same laboratory frame.

3. (For this problem, use space-time diagram where the axes in frame S are orthogonal.) S' moves at speed v w.r.t S along positive X direction. A meter stick lies along X axis and is at rest in S. If S' measures its length, what is the result?

4. In a given reference frame, event 1 happens at $x = 0$, $ct = 0$, and event 2 happens at $x = 2$, $ct = 1$ in some length unit. Find a frame in which the two events are simultaneous. Draw the space-time diagram and find the relative velocity (v) between the two frames.

5. This is a space-time diagram where horizontal axis is x-axis and vertical axis is ct axis.



Answer the following questions with proper arguments.

(a) Which events are simultaneous in this reference frame?

(b) Which events are taking place at the same position in this reference frame?

(c) Which events are light-like separated?

(d) List all events that could be caused by A.

(e) List all events that could have caused A.

6. In an inertial frame of reference, four events A, B, C,D have the following coordinates:
   event A: $x = 0$, $ct = 0$
   event B: $x = L$, $ct = 2L$
   event C: $x = L$, $ct = -L/2$
   event D: $x = 0$, $ct = 2L$
   Answer the following questions with proper arguments.

   (a) Draw a space-time diagram showing the events A, B, C, D.

   (b) Is it possible that event A has caused event B?

   (c) Is it possible that event A has caused event C?

   (d) Is it possible that event C has caused event A?

   (e) Is it possible that event C has caused event B?

7. A hare and a tortoise decide to have a race. They start from the same point and race in the same direction, but the rule of the race is that the tortoise have to run only half the distance as shown in the figure. The race takes place and the referee, who is at rest with respect to the ground, sees both animals cross their respective finish lines at the same time. The trajectory of the hare and the tortoise are shown on the space-time diagram. Answer the following questions:

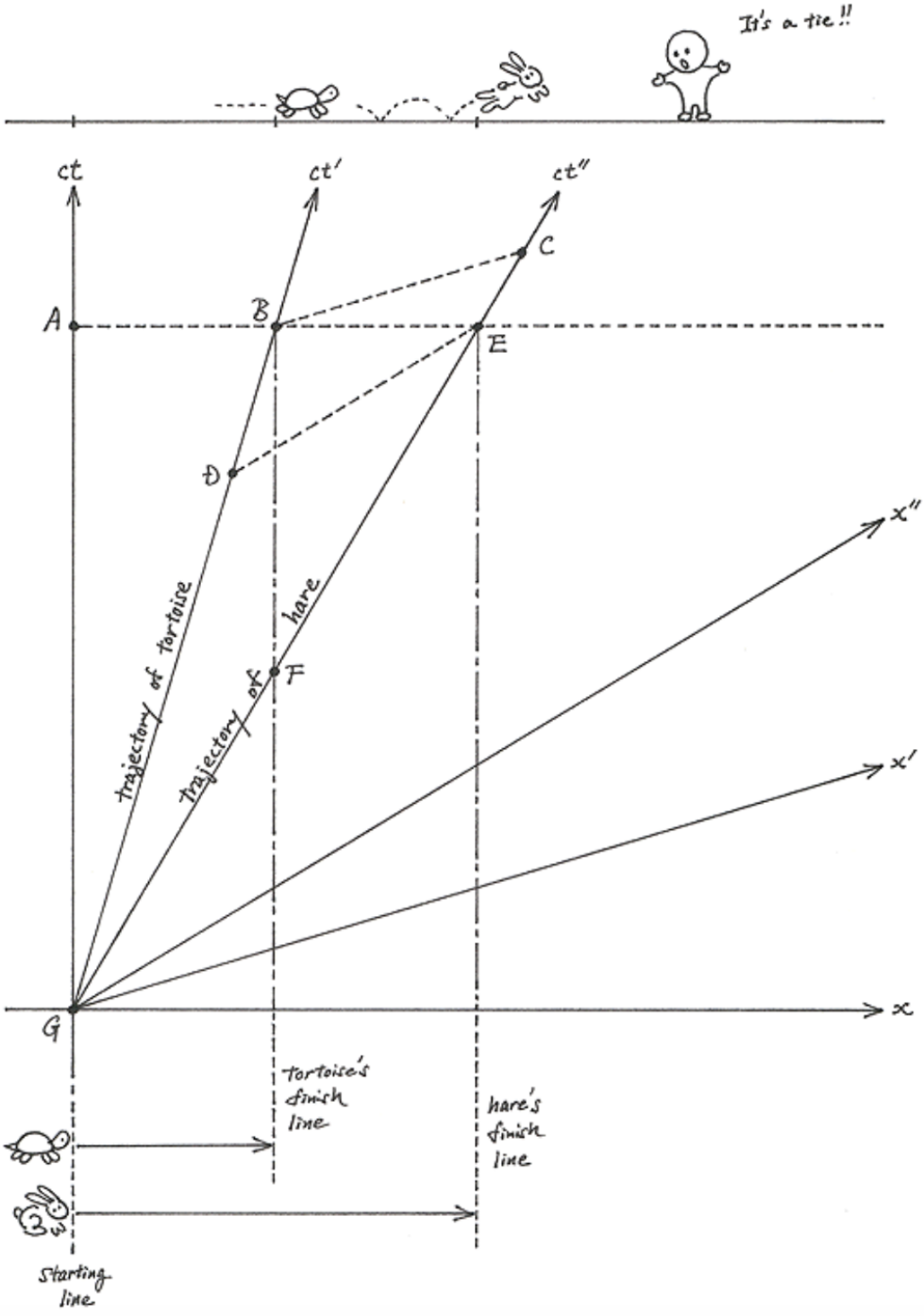
   (a) At which of the points labeled A through G in the space-time diagram does the hare cross its finish line?

   (b) At which point does the tortoise cross its finish line?

   (c) At which point is the tortoise when the hare crosses its finish line in the hare frame?

   (d) At which point is the hare when the tortoise crosses its finish line in the tortoise frame?

   (e) What is the result of the race in the respective frames of the hare and the tortoise? Do they agree or disagree?


It's a tie!!
ct
ct′
ct″
A
B
C
D
E
F
G
trajectory of tortoise
trajectory of hare
x″
x′
x
tortoise's finish line
hare's finish line
Starting line