PHY226M, Problem Set 1
Special Theory of Relativity
March 2025

1. (a) Write Lorentz transformation in matrix form.

   (b) Use the property $(\gamma^2 - \gamma^2\beta^2) = 1$ to rewrite the Lorentz transformation matrix in terms of $\cosh\phi$ and $\sinh\phi$, where $\tanh\phi = \beta$. We will call the resulting matrix $\Lambda(\phi(\beta))$.

   (c) Show that $\Lambda(\phi(\beta_1))\Lambda(\phi(\beta_2)) = \Lambda(\phi(\beta_1) + \phi(\beta_2))$

2. Muons are often created when cosmic rays interact with the upper atmosphere. Let's say muons are moving with velocity close to the speed of light; $\beta = 0.9999$. Will these muons reach the ground, given an atmospheric thickness of about 10 km? Discuss *classical* result and the relativistic result.
   Side note: thanks to time dilation, it may be possible some day to build a muon-collider. `https://home.cern/science/accelerators/muon-collider` .

3. Two lumps of clay, each of rest mass m_0, collide head on at speed $\frac{3}{5}c$. They stick together and come to rest. The mass of the composite lump is M_0. Show that $M_0 > 2m_0$.

4. Start from the relation $E^2 = c^2p^2 + m_0^2c^4$ and show that $\frac{dE}{dp} = u$, where u is the velocity of the object and m_0 is the rest mass of the object.

5. Using the velocity addition rule, show that

   (a) $\beta = \frac{\beta_1+\beta_2}{1+\beta_1\beta_2}$, where $\beta_1 = v/c$, $\beta_2 = u'_x/c$ and $\beta = u_x/c$

   (b) Write the expression of $1-\beta$ and prove that value of β will be within 0 to 1, if values of β_1 and β_2 are within 0 to 1.

6. The passage of light through a medium (eg. water) is characterised by a refractive index n. Velocity of light relative to the medium is c/n. Suppose that such a medium is moving with speed v parallel to the direction of the light. What will be the observation of a stationary observer about the velocity (V) of light? You can assume $v << c$.

7. Frame S' has a speed $v = 0.6c$ relative to S. Clocks are adjusted so that $t = t' = 0$ at $x = x' = 0$. Two events occur. Event 1 occurs at $x_1 = 10$ m, $t_1 = 2 \times 10^{-7}$ sec ($y_1 = 0$, $z_1 = 0$). Event 2 occurs at $x_2 = 50$ m, $t_2 = 3 \times 10^{-7}$ sec ($y_2 = 0$, $z_2 = 0$).

   (a) What is the distance between the events as measured in S'?

   (b) What is the time between the events as measured in S'?

8. Write down the expression of relativistic kinetic energy (K) of an object in terms of rest mass (m_0) of the object, velocity (u) of the object and the speed of light (c). Show that if we assume $u << c$, then the expression of K becomes similar to what we expect from Newtonian mechanics, i.e., $K = \frac{1}{2}m_0u^2$.

9. Write down the relation between kinetic energy (K) and momentum ($|\vec{p}| = p$) of an object in relativity. From there, show that if the velocity of the object is negligible compared to c, then we get back the Newtonian relation between K and p, i.e., $p = \sqrt{2m_0K}$.