PHY226M, Problem Set 2
Special Theory of Relativity
March 2025

1. **The hypothetical photon rocket.** Enthusiasts for space travel proposed the use of radiation as a propellant for a spaceship. Assume that m_0 is the rest mass of a rocket which has a payload of rest mass fm_0, where f is a fraction. Show that, if $\gamma = 10$, then f is ~ 0.05.

2. **Motion under a constant force.** A particle of mass m is subject to a constant force F. If it starts from rest at the origin, at time $t = 0$, find its position (x), as a function of time.

3. **Decay of charged pion.** A charged pion at rest decays into a muon and a neutrino. Find the energy of the outgoing muon, in terms of the two rest masses, m_π and m_μ. Assume $m_\nu = 0$.

4. **Absorption and emission of photon**

   (a) Suppose a stationary atom (or nucleus) of rest mass M_0 is struck by a photon of energy Q, which is completely absorbed. The combined system will have (relativistic) mass M' and will recoil with a velocity v. Show that:

   $$\beta = \frac{v}{c} = \frac{Q}{M_0c^2 + Q} \tag{1}$$

   (b) Suppose a stationary atom of rest mass M_0 emits a photon of energy Q. The emitting atom, which undergoes recoil, has (relativistic) mass M', rest mass M_0' and velocity v. Prove that:

   $$Q = Q_0\left(1 - \frac{Q_0}{2M_0c^2}\right) \tag{2}$$

   where Q_0 is the difference between the initial rest energy of the atom before recoil and the final rest energy of the atom after recoil.

5. In the Large Hadron Collider (LHC), each proton reaches an energy of 7 TeV. What is the value of Lorentz factor (γ) when proton reaches that energy?

6. For what value of β will the relativistic mass of a particle exceed its rest mass by a fraction f?

7. **Compton scattering.** A photon of energy E_0 bounces off an electron, initially at rest. Find the energy E of the outgoing photon, as a function of the scattering angle θ. Write the expression in terms of λ also.