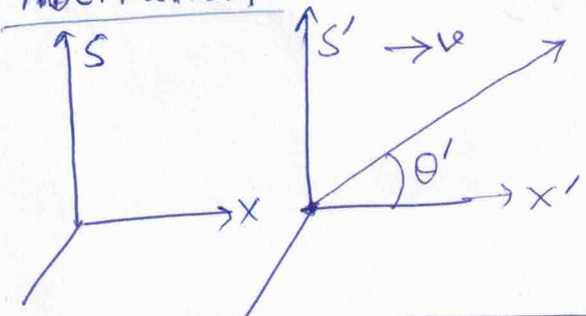
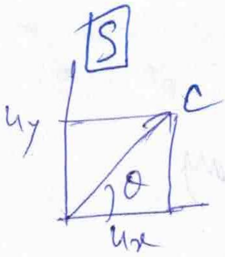
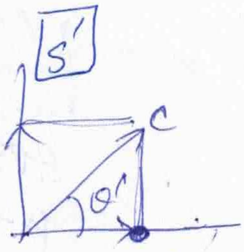
Q 3 **Aberration**







$c\cos\theta' = u_x'$

$c\sin\theta' = u_y'$

Velocity addition formula

$$u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}} = \frac{c\cos\theta' + v}{1 + \frac{\cancel{c}\cos\theta' v}{c^2}}$$

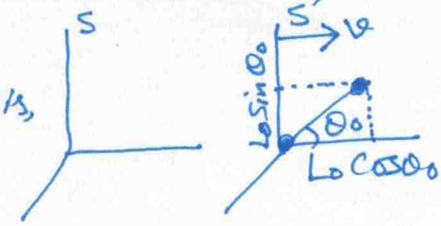
$$u_y = \frac{u_y'\sqrt{1 - v^2/c^2}}{1 + u_x'\left(v/c^2\right)} = \frac{c\sin\theta'\sqrt{1 - v^2/c^2}}{1 + \cancel{c}\cos\theta'\, v/c^2}$$

$$\tan\theta = \frac{u_y}{u_x} = \frac{c\sin\theta'\sqrt{1-\beta^2}}{\cancel{\left(1 + \cos\theta' \frac{v}{c}\right)}} \cdot \frac{\cancel{\left(1 + \cos\theta' v/c\right)}}{c\cos\theta' + v}$$

$$\boxed{\tan\theta = \frac{\sin\theta'\sqrt{1-\beta^2}}{\cos\theta' + \beta}}$$

Relativistic Aberration

Problem. A rod of length L_0 [illegible] along the horizontal direction. The rod makes an angle of θ_0 w.r.t the x' axis. (a) Show that the length of the rod as measured by a stationary observer is given by $L = L_0\sqrt{1-\beta^2\cos^2\theta_0}$. (b) Show that the angle that the rod makes with the x axis, is given by the expression $\tan\theta = \gamma\tan\theta_0$



Soln.

Projected lengths along X and Y axis,

$$\Delta x' = L_0\cos\theta_0$$
$$\Delta y' = L_0\sin\theta_0.$$

From length contraction $\Delta x' = \gamma\Delta x = \dfrac{\Delta x}{\sqrt{1-\beta^2}}$

$$\Delta y' = \Delta y$$

Length measured from S is L,

$$L = \sqrt{(\Delta x)^2 + \Delta y^2} = \sqrt{(1-\beta^2)\Delta x'^2 + \Delta y'^2}$$
$$= \sqrt{(1-\beta^2)L_0^2\cos^2\theta_0 + L_0^2\sin^2\theta_0}$$
$$= \sqrt{L_0^2(\cos^2\theta_0 + \sin^2\theta_0) - \beta^2 L_0^2\cos^2\theta_0}$$
$$= \sqrt{L_0^2 - \beta^2 L_0^2\cos^2\theta_0}$$
$$\Rightarrow \boxed{L = L_0\sqrt{1-\beta^2\cos^2\theta_0}}$$

angle is θ in S frame,

$$\tan\theta = \frac{\Delta y}{\Delta x} = \frac{\Delta y'}{\frac{\Delta x'}{\gamma}} = \gamma\frac{\Delta y'}{\Delta x} = \gamma\tan\theta_0.$$

So, $\boxed{\tan\theta = \gamma\tan\theta_0}$

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$\left\{ \begin{array}{l} \text{Distance measured in Lab frame} = 880 \text{ meter} \\ \cancel{\text{Muon}} \text{ Muon lifetime } \cancel{\text{sec}} = 2.2\times10^{-6} \text{ sec (in muon rest frame).} \end{array} \right.$

So, $v = \dfrac{880 \text{ meter}}{2.2\times10^{-6}} \cdot m/s \Rightarrow$ Frame mismatch.

Let's consider time dilation effect.

In Lab frame muon will last $\gamma * 2.2\times10^{-6}$ sec.

(i.e. $\gamma.\tau$).

$\tau = 2.2\times10^{-6}$ sec

$$\text{So, } v = \frac{d}{\gamma\tau} = \frac{d\sqrt{1-v^2/c^2}}{\tau}$$

$$\Rightarrow v^2 = \frac{d^2}{\tau^2}\left(1-\frac{v^2}{c^2}\right)$$

$$\Rightarrow v^2\tau^2 = d^2 - d^2\frac{v^2}{c^2}$$

$$\Rightarrow \frac{v^2}{c^2}\left(c^2\tau^2 + d^2\right) = d^2$$

$$\Rightarrow \left(\frac{v}{c}\right)^2 = \frac{d^2}{c^2\tau^2+d^2} = \frac{1}{\frac{c^2\tau^2}{d^2}+1}$$

Now

$$\frac{c\tau}{d} = \frac{3\times10^8 \text{ m/s} \; * \; 2.2\times10^{-6} \text{ second}}{880 \text{ meter}}$$

$$= \frac{660}{880} = \frac{3}{4}$$

$$\left(\frac{v}{c}\right)^2 = \frac{1}{\left(\frac{3}{4}\right)^2+1} = \frac{16}{25}$$

$$\boxed{\frac{v}{c} = \frac{4}{5}}$$

$$\text{So, } v = \frac{4}{5}\cdot c$$

$$\Rightarrow v = \frac{4}{5}\times 3\times10^8 \text{ m/s}$$

$$\Rightarrow v = 2.4\times10^8 \text{ m/s}$$