PHY226M, Quiz 1, Special Theory of Relativity
Date: 21st March 2025, Total time: 40 minutes (11:05 – 11:45), Total marks: 25

Clearly write your full name and roll number in your answer paper.

In your answers, jumping steps will lead to deduction of marks. Please ensure that all steps are explicitly stated and clearly explained.

All the conventions and notations used here have the same meaning as discussed during the lectures.

1. In a laboratory experiment, a muon is observed to travel 880 meter before disintegrating. A graduate student looks up the lifetime of a muon on internet, and found it to be 2.2×10^{-6} second. The student concluded that the speed of the muon was:

$$v = \frac{880\,\text{meter}}{2.2 \times 10^{-6}\,\text{second}} = 4 \times 10^{8}\,\text{meter/second}.$$

But this is faster than light! Identify the student's mistake, and find the actual speed of this muon. `[Marks 8]`

2. A rod of length L_0 moves with a speed v along the horizontal direction, which is the common XX' axis. The rod makes an angle of θ_0 w.r.t. the X' axis.

   (a) Show that the length of the rod as measured by a stationary observer is given by $L = L_0\sqrt{1-\beta^2\cos^2\theta_0}$. `[Marks 5]`

   (b) Show that the angle that the rod makes with the X axis is given by the expression $\tan\theta = \gamma\tan\theta_0$. `[Marks 4]`

3. Use velocity addition theorem in relativity (which means the transformation of velocity across inertial frames) to prove the relativistic aberration formula:

$$\tan\theta = \frac{\sin\theta'\sqrt{1-\beta^2}}{\cos\theta'+\beta}$$

You can assume that the primed frame is moving with velocity v w.r.t the unprimed frame along the common XX' axis. `[Marks 8]`

Expressions that you might need

$$u_x = \frac{u'_x + v}{1 + \dfrac{u'_x v}{c^2}} \quad \text{and} \quad u_y = \frac{u'_y\sqrt{1-\beta^2}}{1 + \dfrac{u'_x v}{c^2}}$$