PHY226M, Problem Set 6
Special Theory of Relativity
April 2025

1. If Λ is a 4×4 matrix representing a Lorentz transformation, then transformations of the type
$$x' = \Lambda x + a$$
are known as Poincaré transformations. We will denote it as (Λ, a). Show that the result of two Poincaré transformations (Λ_1, a_1) and (Λ_2, a_2) applied successfully, is also a Poincaré transformation, and it looks like this:
$$(\Lambda_2\Lambda_1, \Lambda_2 a_1 + a_2)$$

2. A particle of charge q is moving with uniform velocity u in S frame along X-axis. Choose a S' frame in a way that the charge q is at rest at S'. Show that:
$$E_y = \frac{\gamma q y}{4\pi\epsilon_0[\gamma^2(x-ut)^2 + y^2 + z^2]^{3/2}} \tag{1}$$
$$B_y = -\frac{u}{c^2}E_z \tag{2}$$
$$B_z = \frac{u}{c^2}E_y \tag{3}$$

3. Show that, in the laboratory frame with particle X at rest, the reaction $\nu + X \to l + Y$ can only happen if the incoming neutrino (ν) has an energy above a threshold as given below:
$$E_\nu \geq \frac{(m_l + m_Y)^2 - m_X^2}{2m_X}$$
Assume that neutrino's rest mass is zero.

4. Write down the Lorentz boost matrix when the relative velocity v_1 between frames is in the X-direction. We will call it L_1. Also write down the Lorentz boost matrix when the relative velocity v_2 between frames is in the Y-direction. We will call it L_2. Show that L_1 and L_2 do not commute, i.e, $L_1 L_2 \neq L_2 L_1$.

5. Show that $\dfrac{d^3p}{E}$ is Lorentz invariant, i.e,
$$\frac{dp_x dp_y dp_z}{E} = \frac{dp'_x dp'_y dp'_z}{E'}$$

6. The rest mass and charge of a particle are m and q respectively. Explain why the combination (m, q, m, q) is not a four-vector.