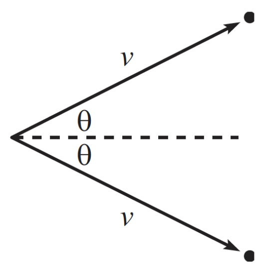
PHY226M, Problem Set 3
Special Theory of Relativity
April 2025

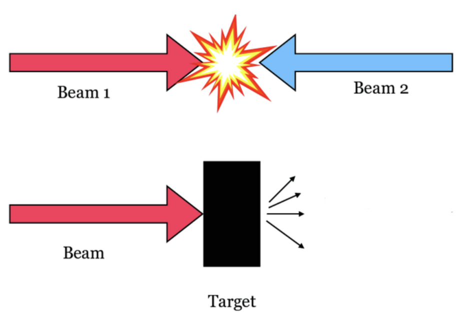
1. In the lab frame, two particles move with speed v along the paths shown in the figure. The angle between the trajectories is 2θ. What is the speed of one particle, as viewed by the other?



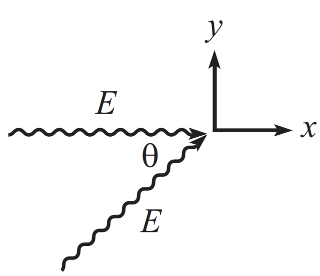
2. In Hamburg, Germany, there is an accelerator called HERA (**H**adron **E**lectron **R**ing **A**ccelerator).

   (a) In HERA, electrons (e) of energy 27.5 GeV are brought into head-on collision with protons (p) of energy 820 GeV. Calculate $\sqrt{s}$ (i.e. E_{total}) of e-p collisions at HERA. Recall that s is a Mandelstam variable. Assume $c = 1$.

   (b) Determine the beam energy that would be needed to produce e-p collisions with the same value of $\sqrt{s}$ (i.e. E_{total}) using electrons incident on a stationary proton target.



3. Two photons each have energy E. They collide at an angle and create a particle of mass M. What is M?



4. If $x_\mu = (x_0, x_1, x_2, x_3)$ and $x^\mu = (x^0, x^1, x^2, x^3)$, show that $x_0 = x^0$, $x_1 = -x^1$, $x_2 = -x^2$, $x_3 = -x^3$, where the Minkowski metric is $\mathsf{diag}(1, -1, -1, -1)$.

5. Assume $c = 1$ and show that $F = \gamma^3 m_0 a$, where F is force, a is acceleration, m_0 is rest mass.

6. A particle with mass M and energy E is moving along positive X axis. It decays into two identical particles. In the lab frame, one of them moves along the positive Y axis, and the other one makes an angle θ with positive X axis, measured clockwise. What are the energies of each of the created particles?

7. A particle at rest with four momentum p (where $p^2 = M^2$) decays to two other particles (say 1 and 2) with four momentum p_1 (where $p_1^2 = m_1^2$) and four momentum p_2 (where $p_2^2 = m_2^2$). Particle 1 and 2 have non-zero 3-momentum. We are assuming $c = 1$ in this problem.

   (a) Find the expressions of $(p_1.p_2)$, E_1, and E_2, where E_1 and E_2 are energies of particle 1 and 2 respectively.

   (b) Show that $|\vec{p_1}| = |\vec{p_2}|$.

8. Let's take an example where a particle called Higgs boson decays to two photons. It is written as: $H \rightarrow \gamma\gamma$. Assume $c = 1$.

   (a) If Higgs boson was at rest while decaying, can you compute the energy of each photon?

   (b) If Higgs boson was not at rest while decaying, then can you compute the angle between $\vec{p_H}$ and $\vec{p_\gamma}$?

9. In problem set 1, we introduced a quantity ϕ like this: $\tanh\phi = \beta$. The quantity ϕ is called rapidity, and clearly, it is defined as $\phi = \tanh^{-1}\beta$.

   (a) Show that ϕ can also be expressed as: $\phi = \frac{1}{2}\ln\left(\frac{1+\beta}{1-\beta}\right)$

   (b) Show that ϕ can also be expressed as: $\phi = \frac{1}{2}\ln\left(\frac{E+pc}{E-pc}\right)$

   (c) Show that rapidity is additive, i.e., $\phi = \phi_1 + \phi_2$, where $\beta = \dfrac{u_x}{c}$, $\beta_2 = \dfrac{u'_x}{c}$ and $\beta_1 = \dfrac{v}{c}$. As usual, v, u_x and u'_x are related via the Einstein's velocity addition rule.