PHY226M, Problem Set 5
Special Theory of Relativity
April 2025

1. Lorentz force can be written as the following:

$$\frac{dP^\mu}{d\tau} = qF^{\mu\nu}U_\nu$$

where U_ν is 4-velocity, P^μ is 4-momentum and $F^{\mu\nu}$ is electromagnetic tensor. From this, find Lorentz force in its usual form, i.e. $\vec{F} = q(\vec{E} + \vec{u} \times \vec{B})$

2. Assume that S' frame is moving w.r.t S frame with constant velocity v along the common $X(X')$ axis. Use electromagnetic tensor ($F^{\mu\nu}$) to prove the following relations:

$$B'_x = B_x \tag{1}$$

$$B'_y = \gamma(B_y + \frac{v}{c^2}E_z) \tag{2}$$

$$B'_z = \gamma(B_z - \frac{v}{c^2}E_y) \tag{3}$$

As usual, the primed quantities are in the primed frame, and the unprimed quantities are in the unprimed frame.

3. In A's frame, B moves to the right with speed u, and C moves to the left with speed v. Show that, the speed (w) of B as seen from C's frame is the following:

$$w = \frac{u+v}{1+uv}$$

Use four-vector notation to solve this problem and assume $c = 1$.

4. Start from $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and find the (3,0)th component and (2,3)th component of $F_{\mu\nu}$, i.e F_{30} and F_{23}, in terms of the components of electric field and magnetic field.

5. Starting from $F^{\mu\nu} = \eta^{\mu\rho}\eta^{\nu\sigma}F_{\rho\sigma}$, find the relation between F^{32} and F_{32} , where Minkowski metric ($\eta^{\mu\nu}$) is $\mathsf{diag}(1, -1, -1, -1)$