Name	Roll Number	Signature

1. Using the rotation operator, $R_z(\theta) = e^{-i\theta\sigma_z/2}$, show that $R_z(\pi)\ |\chi_+^{(x)}\rangle = -i|\chi_-^{(x)}\rangle$. Where, $|\chi_+^{(x)}\rangle$ and $|\chi_-^{(x)}\rangle$ are the spinors corresponding to the x-basis, and the Pauli spin matrix along the z-direction is $\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. [5 marks]

2. Given the energy dispersion for a 1D crystal $E(k) = E_0 - 2t\times\cos(ka)$, where t = 1 eV and lattice constant a = 6 Å, calculate the effective mass at $k = \pi/a$. Plot the velocity of an electron as a function of wavevector within the first Brillouin zone. Why does the velocity decrease with an increase in wave vector close to $k = \pi/a$ in the first Brillouin zone? [Marks: 2 + 2 + 2]

3. There is a quantum well of length 10 nm (2D system). Calculate the density of states at E = 5 meV and 20 meV from the bottom of the Quantum well. If the chemical potential (Fermi level) of the system is 10 meV below the bottom of the well, what is the probability of finding an electron at the ground state? [Consider, $m^* = m_0$, T = 300 K, $k_B = 1.38\times10^{-23}$ m^2 kg s^{-2} K^{-1}] [Marks: 4 + 2]

4. Why honeycomb lattice is not a Bravais lattice? What is the Bravais lattice by all the points with the Cartesian coordinate (n_1, n_2, n_3) if n_i are either all even or all odd? Find rotational and mirror symmetry axes or planes for the following unit cells. Draw (111) plane and denote the direction of the plane in the first object. [2 + 3 + 3 + 1]