1. Show (100), (111), (110) and (121) planes within a simple cubic structure. Show the direction of the planes with proper notations.

2. A cubic crystal is strained so that the lattice vectors change to a_1 = (a 0 0), a_2 = (0 a 0), a_3 = (αa 0 βa) with small α, and β. (a) For the original (unstrained) plane (2,1,0) determine the new intercepts and compute the new Miller-like indices (to first order in α,β). (b) Show how a family of equivalent directions ⟨100⟩ splits under this distortion.

3. What is the Bravais lattice formed by all points with Cartesian coordinates (n_1, n_2, n_3) if:

   (i) The n_i are either all even or all odd?

   (ii) The sum of the n_i is required to be even?

4. Show that the *c/a* ratio for an ideal hexagonal close-packed structure is *c/a* = $\sqrt{8/3}$.

5. Prove that the Wigner-Seitz cell for any two-dimensional lattice is either a hexagon or a rectangle. Show that the Wigner-Seitz cell volume for bcc is $a^3/2$.

6. Sketch a bcc unit cell with a monoatomic basis. If you draw (110) plane within this unit cell, what is the atomic density per unit area on this plane? If the atomic density of this semiconductor is 1.6×10^{22} cm^{-3}, calculate the lattice constant.

7. The electron affinity (χ_A) of GaAs and $Al_{0.3}Ga_{0.7}As$ is 4.07 eV and 3.74 eV respectively. If these two semiconductors were used to form a heterostructure, what would be the valence band offset at the junction? Which type of band alignment is it?
   [Consider, $E_g^{GaAs} = 1.427$ eV and $E_g^{Al_{0.3}Ga_{0.7}As} = 1.82$ eV]

8. The bandgap and lattice constant of GaAs is 1.427 eV and 5.653 Å, and that for InAs is 0.35 eV and 6.058 Å. With this knowledge, how can you achieve a semiconductor with bandgap of 0.9 eV? What would be the lattice constant of this semiconductor? What is the lattice mismatch of this semiconductor with respect to the GaAs?
   [Consider the bowing parameter, b = 0.477 eV]

9. The intrinsic level (E_i) is the Fermi level of an undoped semiconductor. It is defined as the energy level where the number of electrons in the conduction band equals the number of holes in the valence band. Derive an expression relating E_i to the center of the band gap $E_g/2$. Calculate the displacement of E_i from $E_g/2$ for Si at 300 K, assuming the effective mass values for electrons and holes are 1.1 m_0 and 0.56 m_0, respectively.

10. Two semiconductors with different doping concentrations N_d^1 and N_d^2 are brought into contact at T = 300 K. Initially, their Fermi levels differ by 0.2 eV. Explain why this difference implies a current would flow. Why must the Fermi levels align at equilibrium?

11. What is quasi bound state in low-dimensional systems? We have two semiconductors, α and β, with bandgaps E_g^α = 3.4 eV and E_g^β = 0.5 and electron affinity $\chi_A{}^\alpha$ = 0.4 eV and $\chi_A{}^\beta$ = 0.8 eV. Can you make quasi bound state for electron and holes using these two semiconductors, if yes how?

12. For Si at 300 K, intrinsic carrier density $n_i = 1.5\times10^{10}$ cm^{-3}. If the Si is doped with donor concentration $N_d = 10^{16}$ cm^{-3}. Calculate the equilibrium electron and hole concentrations. Verify mass action law for this 3D system.

13. For Si at 300 K, calculate the critical doping density at which the Fermi level enters the conduction band. Assume $N_c = 2.8\times10^{19}$ cm^{-3}. Can you use Boltzmann statics instead of Fermi-Dirac distribution function for this semiconductor, discuss.

14. Phosphorus-doped Si has donor concentration $N_d = 10^{15}$ cm^{-3}. The donor ionization energy is 45 meV.

    i. At 77 K, estimate the fraction of ionized donors using the Boltzmann approximation.

    ii. At 300 K, assume complete ionization and calculate the electron concentration and fermi level with respect to the conduction band edge.

15. A new semiconductor has $N_c = 10^{19}$ cm^{-3}, $N_v = 5 \times 10^{18}$ cm^{-3}, and $E_g = 2$ eV. If it is doped with 10^{17} donors (fully ionized), calculate the electron, hole, and intrinsic carrier concentrations at 627°C. Draw the engineering band diagram, showing the position of E_f.

16. A p-n junction is formed by joining p-type Si with $N_a = 10^{17}$ cm^{-3} and n-type Si with $N_d = 10^{16}$ cm^{-3}. The intrinsic carrier concentration is $n_i = 1.5\times10^{10}$ cm^{-3}. Calculate the initial Fermi level difference before contact. Show that, after equilibrium, this difference equals the built-in potential:

$$V_{bi} = \frac{kT}{q} ln\left(\frac{N_a N_d}{n_i^2}\right)$$