Course: PHY312, Tutorial 1

1. Calculate the de Broglie wavelength of a free electron at room temperature.
   i. If a quantum well (QW) has a width equal to this de-Broglie wavelength, calculate the energy separation between its ground and first excited state (in eV).
   ii. Comment whether these two states can be probed separately (isolated/merged state).
   iii. Calculate the variance of an electron occupied at the first excited state of this QW.
   iv. Using the above result, calculate the uncertainty in momentum of this electron.
   v. What is the percentage uncertainty in momentum?
2. Show that $\sigma_A^2\sigma_B^2 \geq \left(\frac{1}{2i}\langle[A,B]\rangle\right)$, where σ_A and σ_B are the variances of two operators A and B.
3. Show that the eigenvalues of a Hermitian operator are real and discuss why this is necessary for the operator to represent a physically measurable quantity.
4. Show that if a Hamiltonian commutes with an operator A, then the expectation value of A is time independent.
5. Why do the Pauli matrices have both diagonal and off-diagonal elements? How do these relate to spin measurements along different axes?
6. In terms of spinor algebra, what does a global phase factor in a spinor mean physically?
7. For a spinor in state $|\chi_+^{(z)}\rangle = \begin{bmatrix}1\\0\end{bmatrix}$, calculate variances ΔS_y and ΔS_z and verify the spin uncertainty relation: $\Delta S_y\,\Delta S_z \geq \hbar/2 * |\langle S_x\rangle|$. Comment on what is the physical significance of this expression. Here, $|\chi_+^{(z)}\rangle$ represents spin up with respect to z-basis.
8. Construct the projection operator $P_+^{(x)}$ for spin-up along x. Verify that $P_+^{(x)}\,|\chi_+^{(x)}\rangle = |\chi_+^{(x)}\rangle$ and $P_+^{(x)}|\chi_+^{(x)}\rangle = 0$.
9. A spin-1/2 particle in a uniform magnetic field B = B_0 starts in $|\chi_+^{(z)}\rangle$. Derive the time evolution of the spinor and show that it precesses about the z-axis.
10. Using $R_z(\theta) = e^{-i\theta\sigma_z/2}$, show that $R_z(\pi)\,|\chi_+^{(x)}\rangle = -i|\chi_-^{(x)}\rangle$.
11. Given $|\chi_+^{(z)}\rangle = \begin{bmatrix}1\\0\end{bmatrix}$, and $|\chi_-^{(z)}\rangle = \begin{bmatrix}0\\1\end{bmatrix}$, express $|\chi+\rangle$ and $|\chi-\rangle$ in the x-basis.
12. If a spin-1/2 particle is in state $|\psi\rangle = \frac{1}{\sqrt{3}}\,|\chi+^{(z)}\rangle + \sqrt{\frac{2}{3}}|\chi-^{(z)}\rangle$, calculate $\langle S_x\rangle$, $\langle S_y\rangle$ and $\langle S_z\rangle$.