Course: PHY312, Tutorial 3

1. Derive the expression for the density of state, $n_{1D}(E)$, for a 1-dimensional system. For a free electron, calculate the density of states at E = 5 meV and E = 10 meV from the effective conduction band edge.

2. At T = 300 K, what is the occupation probability of a fermion for a state 0.20 eV above E_F?

3. Using the density of state for a 2-dimensional system, derive an expression for the occupation number of electrons (per unit area). Consider, effective mass $m_e^* = 0.067\ m_0$, E_F = 10 meV and chemical potential, E_F, doesn't change with temperature. Calculate the occupation number of electrons in the band at T = 0 and T = 300 K.

4. There is a 2D material system with an effective bandgap of 1.1 eV. The chemical potential is located at the middle of the bandgap. [Effective mass of electron is $m_e = 0.06\ m_0$ and holes $m_h = 0.6\ m_0$]

   i. Calculate electron and hole occupancy in the conduction and valence band, respectively, at liquid helium temperature and at room temperature.
   ii. Compare the electron and hole occupation numbers at room temperature. Discuss whether this is feasible for a real physical system.

5. Consider there is a 1-dimensional system (quantum wire) of size l_x = 50 nm, l_y = 50 nm, and $l_z = \infty$. The effective mass of the electron is $0.06\ m_0$.

   i. Calculate the ground state, first excited state, and second excited state energy with respect to the conduction band edge.
   ii. If chemical potential, E_F, is located 0.1 eV below its bulk conduction band edge, calculate the total number of electron occupancy in the band at 300 K. [use $\int_0^\infty u^{-1/2} e^{au} du = \sqrt{\pi/a}$]

6. In the problem 5, if l_y = 100 nm instead, calculate the energy of the first three energy levels of this system. In which case (previous problem or this one), you expect the total electron occupancy to be higher, discuss qualitatively?

7. A Si sample has a free electron density in the conduction band of 10^{17} /cm^3. Where is E_F relative to the conduction band edge?

8. Given the Bose-Einstein distribution function $f_{BE}(E, \mu, T) = 1/[\exp\{(E-\mu)/k_BT\}-1]$, and the chemical potential of phonon μ is zero,

   i. Calculate the average number of phonons in a state with energy E = 25 meV at a temperature of T = 300 K.
   ii. Calculate the average number of phonons in the same state if the temperature is lowered to 10 K.
   iii. Based on your results, discuss how the average phonon occupancy changes with temperature.