MODERN PHYSICS LABORATORY (PHY315A)

LABORATORY MANUAL

Session: 2025-2026 1st Semester

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About the Modern Physics Laboratory Course

The Modern Physics Laboratory runs on two days in a week (Tuesday and Wednesday) from 2:00PM to 5:00PM. The lab reports should be submitted before the allotment of second next experiment by 5:00 PM to the lab office. Please do not delay the submission of lab reports. Come prepared so that you can finish the experiments and submit the report on time. Laboratory work is an important part of the course and satisfactory completion of it is required. Your performance in the laboratory is taken into account in evaluating the performance in the course. The laboratory grade is based on your performance in the weekly sessions, the reports you write, and your performance in examinations on laboratory work.

You will be provided with write-ups containing instructions on various experiments. You are also expected to read the matter from the references (if any) given in the write-ups or suggested by your instructor.

On reaching the laboratory you should check the apparatus provided and ascertain if there are any shortages or malfunctions. Set up the equipment in accordance with the instructions. Proceed carefully and methodically. Remember that scientific equipment is expensive and quit susceptible to damage. So handle it carefully. If the apparatus is complicated ask the instructor to inspect before you proceed with the actual performance of the experiment. Make the measurements required and record them neatly in tabular form. Double-check to make sure that you have recorded all necessary date.

It is more important to see what result you get with given apparatus rather than what is the correct result. The apparatus given to you is capable of certain accuracy and your result may be completely acceptable even if it differs from correct result. You must learn to do things on your own even if you might make mistakes some time. In case result are to be found graphically; each graph should occupy one complete sheet; the information as to quantities plotted, scale chosen and units should be mentioned clearly on graph in ink.

Following is the format of the Report:

Lab reports are to be submitted before beginning a second new experiment (only one backlog allowed).The students are expected (but not required) to spend about two or in some cases three turns on one experiment. This is an experimental course and we test the originality and systematicness in carrying out the experiment and reporting the data, and thoroughness in analyzing them to reach the appropriate result. It's a good practice to keep a separate lab-book with raw data and a photocopy of the relevant data pages should be attached to the report. The report may be divided into the following sections:

1. Your name, roll number, instructor's name, date title or the experiment.
2. Aim/goal (no abstract) (data of the experiment).
3. Theory (Brief)/principle
4. Procedure/apparatus/method/schematics
5. Data /observations (However ugly, show the raw data)
6. Graphs/ Analysis, and calculations (includes error analysis)
7. Result and errors/conclusions
8. Suggestions/Precautions/Difficulties faced/discussion/comments

Remember highest weightage in a report is given to 5, 6 and 7 parts as listed above. Also a hand written report is recommended over the computer printout (other than the graphs).

Project:

In the last one month of the semester after each group has finished all the experiments a small project has to be chosen by student after brief literature search (eg., American Journal of Physics). These may be carried out in research labs, using central facilities or some simple experiments may be set-up in the PHY315 lab itself.

Before coming to the lab read about the day's experiments and try to understand them thoroughly. Please remember that if you have any doubts or questions about the experiments you are going to do, or have already done, you can raise them without any hesitation during the lecture or tutorial hours. Note that you get only two or three turn to do an experiment. Try to finish the experiment within the allotted time. The lab report Sheet should be ready with all relevant diagrams, graph sheets, empty tables etc before you enter the lab. As you do the experiment record all your observations in the lab report, make the necessary graphs and fill up the tables. The lab report grades will be based on the reported observations and quantitative details or analysis. Don't waste your time writing more detailed introduction and theory in the lab report.

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<u>Error Analysis</u>

"To error is human; to evaluate and analyze the error is scientific".

Introduction: Every measured physical quantity has an uncertainty or error associated with it. An experiment, in general, involves (i) direct measurement of various quantities (primary measurements) and (ii) calculation of the physical quantity of interest which is a function of the measured quantities. An uncertainty or error in the final result arises because of the errors in the primary measurements (assuming that there is no approximation involved in the calculation).For example , the result of a recent experiment to determine the velocity of light (Phys. Rev. Lett. 29 , 1346(1972)was given as

$$C = (299.792.456.2 \pm 1.1)\ \text{m/sec}$$

The error in the value of C arises from the errors in primary measurements viz., frequency and wavelength.

Error analysis therefore consists of (i) estimating the errors in all primary measurements, and (ii) propagating the error at each step of the calculation. This analysis serves two purposes. First, the error in the final result (± 1.1 m/sec in the above example) is an indication of the precision of the measurement and, therefore an important part of the result. Second, the analysis also tells us which primary measurement is causing more error than others and thus indicates the direction for further improvement of the experiment.

For example ,in measuring 'g' with a simple pendulum, if the error analysis reveals that the errors in 'g' caused by measurements of 1 (length of the pendulum) and T (time period) are 0.5 cm/sec^2 and 3.5 cm/ sec^2 respectively, then we know that there is no point in trying to devise a more accurate measurement of 1. Rather, we should try to reduce the uncertainty in T by counting a larger number of periods or using a better device to measure time. Thus error analysis prior to the experiment is an important aspect of planning the experiment.

Nomenclature:

(i) 'Discrepancy' denotes the difference between two measured values of the same quantity.

(ii) 'Systematic errors' are errors which occur in every measurement in same way- often in the same direction and of the same magnitude –for example, length measurement with a faulty scale. These errors can in principle, be eliminated or corrected for.

(iii) 'Random errors' are errors which can cause the result of a measurement to deviate in either direction from its true value. We shall confine our attention to these errors, and discuss them under two heads: estimated and statistical errors.

II Estimated Errors

Estimating a primary error: An estimated error is an estimate of the maximum extent to which a measured quantity might deviate from its true value. For a primary measurement, the estimated error is often taken to be the least count of the measuring instrument. For example, if the length of a string is to be measured with a meter rod, the limiting factor is the accuracy in the least count, i.e. 0.1 cm. Two notes of caution are needed here.

(I) What matters really is the effective least count and not the nominal least count. For example, in measuring electric current with an ammeter, if the smallest division corresponds to 0.1 amp., but the marks are far enough apart so that you can easily make out a quarter of a division, then the effective least count will be 0.025 amp. On the other hand if you are reading a vernier scale where three successive marks on the vernier scale (say, 27^{th}, 28^{th}, 29th) look equally well in coincidence with the main scale, the effective least count is 3 times the nominal one. Therefore, make a judicious estimate of the least count.

(II) The estimated error is, in general. to be related to the limiting factor in the accuracy. This limiting factor need not always be the least count. For example, in a null-point electrical measurement, suppose the deflection in the galvanometer remains zero for all values of resistance R from 351Ω to 360Ω. In that case, the uncertainty in R is 10 Ω. Even though the least count of the resistance box may be less.

Propagation of estimated errors:

How to calculate the error associated with f, which is a function of measured quantities a, b and c? Let

$$f = f(a.\ b.\ c) \qquad (1)$$

From differential calculus (Taylor's series in the 1^{st} order)

$$df = \frac{\partial f}{\partial a}\, da + \frac{\partial f}{\partial b}\, db + \frac{\partial f}{\partial c} dc. \qquad (2)$$

Eq. (2) relates the differential increment in f resulting from differential increments in a, b, c. Thus if our errors in a, b, c(denoted as $\delta a\,, \delta b\,, \delta c$) are small compared to a, b, c, respectively, then we may say

$$\delta f = |\frac{\partial f}{\partial a}|\delta a + |\frac{\partial f}{\partial b}|\delta b + |\frac{\partial f}{\partial c}|\delta c \quad (3)$$

Where the modulus signs have been put because errors in a, b, and c are independent of each other and may be in the positive or negative direction. Therefore the maximum possible error will be obtained only by adding absolute values of all the independent contributions. (All the $\delta' s$ are considered positive by definition). Special care has to be taken when all errors are not independent of each other. This will become clear in special case (V) below.

Some simple cases:

(i) For addition or subtraction, the absolute errors are added, e.g.

If f = a+ b – c, then

$$\delta f = \delta a + \delta b + \delta c\,. \quad (4)$$

(ii) For multiplication and division, the fractional (or percent) errors are added, e.g.,

If f = ab/ c, then

$$|\frac{1}{f}|\delta f = |\frac{1}{a}|\delta a + |\frac{1}{b}|\delta b + |\frac{1}{c}|\delta c. \quad (5)$$

(iii) For raising to constant powers, including fractional powers, the fractional error is multiplied by the power, e.g.,

If f = $a^{3.6}$, then

$$\left|\frac{1}{f}\right| \delta f = \left|3.6 \times \frac{1}{a}\right| \delta a. \quad (6)$$

(iv) In mixed calculations, break up the calculation into simple parts, e.g.,

If f =a/b – $c^{3/2}$,then

$$\delta f = \delta\left(\frac{a}{b}\right) + \delta(c^{3/2})$$

Since , $|b/a|\delta(a/b) = \left|\frac{1}{a}\right| \delta a + \left|\frac{1}{b}\right| \delta b, \left|1/c^{\frac{3}{2}}\right| \delta\left(c^{\frac{3}{2}}\right) = \left|\frac{3}{2c}\right| \delta c$ (7)

So, $\delta f = \left|\frac{1}{b}\right| \delta a + |a/b^2|\delta b + \left|\frac{3}{2}\ c^{1/2}\right| \delta c.$

Note that the same result could have been derived directly by differentiation.

(v) Consider f =ab/c – a^2.

The relation for error, before putting the modulus signs, is

$$\delta f = \left(\frac{b}{c}\right)\delta a + \left(\frac{a}{c}\right)\delta b - \left(\frac{ab}{c^2}\right)\delta c - 2a\delta a.$$

Note that the δa factors in the first and fourth terms are not independent errors. Therefore, we must not add the absolute values of these two terms indiscriminately. The correct way to handle it is to collect the coefficients of each independent errors before putting modulus signs, i.e.,

$$\delta f = \left|\frac{b}{c} - 2a\right| \delta a + \left|\frac{a}{c}\right| \delta b + \left|\frac{ab}{c^2}\right| \delta c. \qquad (8)$$

III Statistical Errors

Statistical distribution and standard deviation: Statistical errors arise when making measurements on random processes.e.g. counting particles emitted by a radioactive source. Suppose we have a source giving off 10 particles/sec. on the average. In order to evaluate this count rate experimentally, we count the number of particles for 2 seconds. Shall we get 20 counts? Not necessarily. In fact, we may get any number between zero and infinity. Therefore, in a measurement on a random process, one cannot specify a maximum possible error. A good measure of uncertainty in such a case is the standard deviation (s.d.) which specifies the range within which the result of any measurement is “most likely” to be.

The exact definition of “most likely” depends on the distribution governing the random events. For all random processes whose probability of occurrence is small and constant. Poisson distribution is applicable, i.e.,

$$P_n = \frac{m^n}{n!} e^m \qquad (9)$$

Where, P_n is the probability that you will observe a particular count n. when the expectation value is m.

It can be shown that if an infinite number of measurements are made, (i) their average would be m and (ii) their standard deviation (s.d.) would be $\sqrt{m}$, for this distribution. Also if m is not too small then 68% or nearly two- thirds of the measurements would yield numbers within one s.d. in the range $m \pm \sqrt{m}$.

In radioactive decay and other nuclear processes, the Poisson distribution is generally valid. This means that we have a way of making certain conclusions without making an infinite number of measurements. Thus, if we measure the number of counts only once, for 100 sec, and the number is say 1608, then (i) our result for average count rate is 16.08/sec, and (ii) the standard deviation is $\sqrt{1608} = 40.1$ counts which correspond to 0.401/sec. So our result for the count rate is (16.08$\pm$ 0.40 /sec^{-1} . The meaning of this statement must be remembered. The actual count rate need not necessarily lie within this range, but there is 68% probability that it lies in that range.

The experimental definition of s .d. for k measurements of a quantity x is

$$\sigma_X = \sqrt{\sum_{n=1}^{k}(\delta x_n^2)/k-1.} \tag{10}$$

Where δ_n^x is the deviation of measurement x_n from the mean. However since we know distribution, we can ascribe the s.d. even to a single measurement.

Propagation of statistical errors: For a function f of independent measurements a,b,c, the statistical error σ_f is

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial a}\sigma_a\right)^2 + \left(\frac{\partial f}{\partial b}\sigma_b\right)^2 + \left(\frac{\partial f}{\partial c}\sigma_c\right)^2} \tag{11}$$

A few simple cases are discussed below.

(i)For addition or subtraction , the squares of errors are added e.g.

If f = a+ b – c

Then , $\sigma_f^2 = \sigma_a^2 + \sigma_b^2 + \sigma_c^2$. (12)

(ii) For multiplication or division, the squares of fractional errors are added, e.g.

If f = ab/c, then

$$\left(\frac{\sigma_f}{f}\right)^2 = \left(\frac{\sigma_a}{a}\right)^2 + \left(\frac{\sigma_b}{b}\right)^2 + \left(\frac{\sigma_c}{c}\right)^2 \tag{13}$$

(iii) If a measurement is repeated n times, the error in the mean is a factor $\sqrt{n}$ less than the error in a single measurement. i.e.,

$$\sigma_{\bar{f}} = \frac{\sigma_f}{\sqrt{n}} \tag{14}$$

Note that Eqs. (11-14) apply to any statistical quantities a,b etc. i.e., primary measurements as well as computed quantities whereas

$$\sigma_m = \sqrt{m} \tag{15}$$

Applies only to a directly measured number. Say, number of α- particle counts but not to computed quantities like count rate.

(IV) Miscellaneous

Repeated measurements: Suppose a quantity f, whether statistical in nature or otherwise is measured n times . The best estimate for the actual value of f is the average f of all measurements. It can be shown that this is the value with respect to which the sum of squares of all deviations is a minimum. Further, if errors are assumed to be randomly distributed, the error in the mean value is given by

$$\delta_{\bar{f}} = \frac{\delta_f}{\sqrt{n}}. \qquad (16)$$

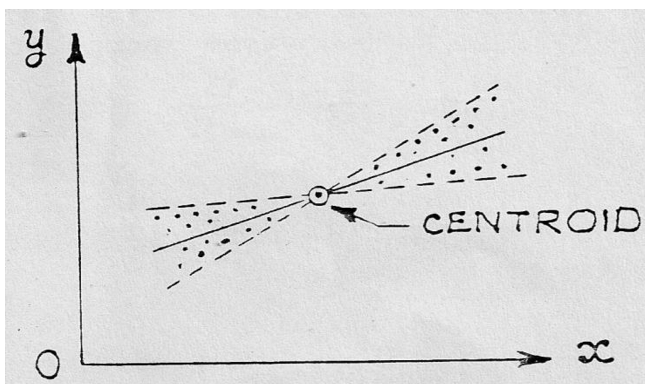
Where δ_f is the error in one measurement. Hence one way of minimizing random errors is to repeat the measurement many times.

Combination of statistical and estimated errors: In cases where some of the primary measurements have statistical errors and others have estimated errors, the error in the final result is indicated as a s.d. and is calculated by treating all errors as statistical.

Errors in graphical analysis: The usual way of indicating errors in quantities plotted on graph paper is to draw error bars. The curve should then be drawn so as to pass through all or most of the bars.

Here is a simple method of obtaining the best fit for a straight line on a graph. Having plotted all the points (x_1, y_1),……………….. (x_n, y_n) , plot also the centroid (x, y).

Then consider all straight lines through the centroid (use a transparent ruler) and visually judge which one will represent the best mean.



Having drawn the best line, estimate the error in slope as follows. Rotate the ruler about the centroid until its edge passes through the cluster of points at the 'top right' and the 'bottom left'. This new line gives one extreme possibility; let the difference between the slopes of this and the best line be Δm_1 .Similarly determine Δm_2 corresponding to the other extreme. The error in the slope may be taken as

$$\Delta m = \frac{\Delta m_1 + \Delta m_2}{2} \frac{1}{\sqrt{n}}$$

Where n is the number of points. The factor $\sqrt{n}$ comes because evaluating the slope from the graph is essentially an averaging process.

It should be noted that if the scale of the graph is not large enough, the least count of the graph may itself become a limiting factor in the accuracy of the result. Therefore, it is desirable to select the scale so that the least count of the graph paper is much smaller than the experimental error.

Significant figures: A result statement such as f = 123.4678±1.2331 cm contains many superfluous digits. Firstly, the digits 678 in quantity f do not mean anything because they represent something much smaller than the uncertainty δf .Secondly δf is an approximate estimate for error and should not need more than two significant figures. The correct expression would be f = 123.5±1.2 cm.

<u>(V) Instructions</u>

1. Calculate the estimated/ statistical error for final result. In any graph you plot, show error bars. (If the errors are too small to show up on the graph, then write them somewhere on the graph).
2. If the same quantity has been measured/ calculated many times, you need not determine the error each time. Similarly one typical error bar on the graph will be enough.

3. In propagating errors, the contributions to the final error from various independent measurements must be show. For example if

$$f \;= ab\;;\;\frac{\delta f}{|f|} = \frac{\delta a}{|a|} + \frac{\delta b}{|b|}\;,\;\; a = 10.0 \pm 0.1\;, b = 5.1 \pm 0.2\;.$$

   Then,

$$\delta f = 51.0\left[\frac{0.1}{10.0} + \frac{0.2}{5.1}\right]$$

$$0.51 + 2.0 = 2.5$$

   Therefore, f = 51.0±2.5.

   Here the penultimate step must not be skipped because it shows that the contribution to the error from δb is large.
4. Where the final result is a known quantity (for example, e/m), show the discrepancy of your result from the standard value. If this is greater than the estimated error, this is abnormal and requires explanation.
5. Where a quantity is determined many times, the standard deviation should be calculated from Eq.(10). Normally, the s.d. should not be more than the estimated error. Also the individual measurements should be distributed only on both sides of the standard value.

(VI) Mean and Standard Deviation

If we make a measurement x_1 of a quantity x, we expect our observation to be close to the quantity but not exact. If we make another measurement we expect a difference in the observed value due to random errors. As we make more and more measurements we expect them to be distributed around the correct value, assuming that we can neglect or correct for systematic errors. If we make a very large number of measurements we can determine how the data points are distributed in the so-called parent distribution. In any practical case, one makes a finite number of measurements and one tries to describe the parent distribution as best as possible.

Consider N measurements of quantity x, yielding values x_1, x_2, ……………..x_N. One defines

$$\text{Mean } \bar{x} = \lim_{N\to\infty}\left[\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)\right] \qquad (1)$$

Which is equivalent to the centroid or average value of the quantity x.

Deviations: The deviation d_i of any measurement x_i from the mean $\bar{x}$ of the parent distribution is defined as

$$d_i = x_i - \bar{x} \qquad (2)$$

Note that if the $\bar{x}$ is the true value of the quantity being measured, d_i is also the true error in x_i.

The arithmetic average of the deviations for an infinite number of observations must vanish, by definition of $\bar{x}$ (Eq (1)).

$$\mathrm{Lim}_{N\to\infty}\left[\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})\right] = Lim_{N\to\infty}\left[\frac{1}{N}\sum_{i=1}^{N} x_i\right] - \bar{x} = 0 \qquad (3)$$

There are several indices one can use to indicate the spread (dispersion) of the measurements about the central value, i.e., the mean value. The dispersion is a measure of precision. One can define average deviation d as the average of the magnitudes of the deviations (absolute values of the deviations).

$$d = Lim_{N\to\infty}\left[\frac{1}{N}\sum_{i=1}^{N}(|x_i-\bar{x}|)\right]$$

This can be used as a measure of the dispersion of the expected observation about the mean. However, a more appropriate measure of the dispersion is found in the parameter called standard deviation σ, defined as

$$\sigma^2 \quad = \quad \mathrm{Lim}_{N\to\infty}\left[\frac{1}{N}\ \sum_{i=1}^{N}(x_i-\bar{x})^2\right] = Lim_{N\to\infty}\left(\frac{1}{N}\sum(x_i)^2-(\bar{x})^2\right) \qquad (4)$$

σ^2 is known as VARIANCE and STANDARD DEVIATION σ is the square root of the variance. In other words it is the root mean square (rms) of deviations. That is

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} d_i^2}{N}} \qquad (5)$$

The expression derived from a statistical analysis is

$$\sigma = \sqrt{\sum_{i=1}^{N} d_i^2 \Big/ (N-1)} \qquad (6)$$

Where, the denominator is N-1instead of N. In practice the distinction between these formulae is unimportant. According to the general theory of statistics the reliability of a result depends upon the Number of measurements and in general, improves with the square root of the number.

Significance: The mean$\bar{x}$, is a parameter which characterizes the information we are seeking when we perform an experiment. The mean is, of course, not the most probable value if the parent distribution is not symmetrical but nevertheless it is a reasonable parameter to characterize the distribution. In place of mean, one can characterize the distribution in terms of median or most probable value. It can be proved that if we use the average (mean) of the measured values for calculating the deviations, the sum of the square of the deviations is a minimum. The standard deviation is simply related to this minimum value of the square of the deviations and is used fir specifying error quantitatively.

The standard deviation characterizes the uncertainties associated with our experimental attempts to determine the "true" value- mean value (defined by Eq.(1) for all practical purposes. σ , for a given finite number of observations is the uncertainty in determining the mean of the parent distribution. Thus it is an appropriate measure of the uncertainty in the observations.

(VII) Method of Least Squares

Our data consist of pairs of measurements (x_i, y_i) of an independent variable x and a dependent variable y. We wish to fit the data to an equation of the form

$$y = a + bx \qquad (1)$$

By determining the values of the coefficients a and b such that the discrepancy is minimized between the values of our measurements y_i and the corresponding values $y = f(x_i)$ given by Eq.

(1). We cannot determine the coefficients exactly with only a finite number of observations, but we do want to extract from these data the most probable estimates for the coefficients.

The problem is to establish criteria for minimizing the discrepancy and optimizing the estimates of the coefficients. For any arbitrary values of a and b, we can calculate the deviations δy_i between each of the observed values y_i and the corresponding calculated values

$$\delta y_i = y_i - a - bx_i \tag{2}$$

If the coefficients are well chosen, these deviations should be relatively small. The sum of these deviations is not a good measure of how well we have approximated the data with our calculated straight line because large positive deviations can be balanced by large negative ones to yield a small sum even when the fit is bad. We might however consider summing up the absolute values of the deviations, but this leads to difficulties in obtaining an analytical solution. We consider instead the sum of the squares of deviations. There is no unique correct method for optimizing the coefficients which is valid for all cases. There exists, however, a method which can be fairly well justified, which is simple and straightforward, which is well established experimentally as being appropriate, and which is accepted by convention. This is the method of least squares which we will explain using the method of maximum likelihood.

Method of maximum likelihood: Our data consist of a sample of observations extracted from a parent distribution which determines the probability of making any particular observation. Let us define parent coefficients a_0 and b_0 such that the actual relationship between y and x given by

$$y(x) = a_0 + b_0 x \tag{3}$$

For any given value of $x = x_i$, we can calculate the probability P_i for making the observed measurement y_i assuming a Gaussian distribution with a standard deviation σ_i for the observations about the actual value,$y(x_i)$

$$P_i = \frac{1}{\sigma_{i\sqrt{2\pi}}} \exp\left[-\frac{1}{2}\left\{\frac{y_i - y(x_i)}{\sigma_i}\right\}^2\right]$$

The probability for making the observed set of measurements of the N values of y_i is the product of these probabilities

$$P(a_0, b_0) = \Pi P_i = \Pi\left[\frac{1}{\sigma_{i\sqrt{2\pi}}} \exp\left[-\frac{1}{2}\left\{\frac{y_i - y(x_i)}{\sigma_i}\right\}^2\right]\right. \tag{4}$$

Where the product Π is taken for i ranging from 1 to N.

Similarly, for any estimated values of the coefficients a and b, we can calculate the probability that we should make the observed set of measurements

$$P(a,b) = \Pi\left(\frac{1}{\sigma_{i\sqrt{2\pi}}}\right)\, exp\left[-\frac{1}{2}\left(\frac{\delta y_i}{\sigma_i}\right)^2\right] \tag{5}$$

The method of maximum likelihood consists of making the assumption that the observed set of measurements is more likely to have come from the actual parent distribution of Eq. (3) than from any other similar distribution with different coefficients and, therefore, the probability of Eq. (4) is the maximum probability attainable with Eq. (5) The best estimates for a and b are therefore those values which maximize the probability of Eq.(5).

The first term of Eq. (5) is a constant, independent of the values of a or b. thus, maximizing the probability P(a, b) is equivalent to minimizing the sum in the exponential. We define the quantity x^2 to be this sum

$$x^2 = \sum\left(\frac{\delta y_i}{\sigma_i}\right)^2 = \sum\left[\frac{1}{\sigma_i^2}(y_i - a - bx_i)^2\right], \tag{6}$$

Where $\sum$ always implies $\sum_{i=1}^{N} and$ consider this to be the appropriate measure of the goodness of fit.

Our method for finding the optimum fit to the data will be to minimize this weighted sum of squares of deviations and, hence, to find the fit which produces the smallest sum of squares or the least-squares fit.

Minimizing x^2: In order to find the values of the coefficients a and b which yield the minimum value for x^2, we use the method of differential calculus for minimizing the function with respect to more than one coefficient. The minimum value of the function x^2 of Eq.(6) is one which yields a value of zero for both of the partial derivatives with respect to each of the coefficients.

$$\begin{aligned} \frac{\partial}{\partial a}x^2 &= \frac{\partial}{\partial a}\left[\frac{1}{\sigma_i^2}(y_i - a - bx_i)^2\right] \\ &= -\frac{2}{\sigma^2}\sum(y_i - a - bx_i) = 0 \\ \frac{\partial}{\partial b}x^2 &= -\frac{2}{\sigma^2}\sum[x_i(y_i - a - bx_i)] = 0 \end{aligned} \tag{7}$$

Where we have for the present considered all of standard deviation equal, $\sigma_i = \sigma$. in other words, errors in y's are assumed to be same for all values of x.

These equations can be rearranged to yield a pair of simultaneous equations

$$\begin{aligned} \sum y_i &= aN + b\sum x_i \\ \sum x_i\, y_i &= a\sum x_i + b\sum x_i^2 \quad , \end{aligned} \tag{8}$$

Where we have substituted Na for $\sum_{i=1}^{N} a$ since the sum runs from i = 1 to N.

We wish to solve Eqs.(8) for the coefficients a and b. This will give us the values of the coefficients for which x^2 , the sum of squares of the deviations of the data points from the calculated fit, is a minimum. The solutions are:

$$a = \frac{1}{\Delta}(\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i)$$

$$b = \frac{1}{\Delta}(N \sum x_i y_i - \sum x_i\, y_i) \qquad (9)$$

$$\Delta = N \sum x_i^2 - (\sum x_i)^2$$

Errors in the coefficients a and b: Now we enquire what errors should be assigned to a and b. In general the errors in y's corresponding to different values of x will be different. To find standard deviation in 'a', say S_a, we approach in the following way. The deviations in 'a' will get contributions from variations in individual y_i's. The contributions of the deviation of a typical measured value δy_n to standard deviation S_a is found using Eq. 9 reproduced below

$$a = \frac{\sum x_n^2 \sum y_n - \sum(x_n y_n)}{N \sum x_n^2 - (\sum x_n)^2} \quad .$$

By differentiating it partially with respect to y_i we get

$$\frac{\partial a}{\partial y_i}\delta y_i = \frac{\sum x_n^2 - (\sum x_n)x_i}{N \sum x_n^2 - (\sum x_n)^2}\delta y_i \quad .$$

Since δy_i is assumed statistically independent of X_n we may replace δy_i by its average value

$$\overline{\delta y_i} = \sigma_y = \sqrt{\sum \frac{(\delta y_i)^2}{N}} \quad .$$

Thus this contribution becomes

$$\frac{\partial a}{\partial y_i}\delta y_i = \sigma_y \left[\frac{\sum x_n^2 - (\sum x_n)x_i}{N \sum x_n^2 - (\sum x_n)^2}\right] \quad .$$

The standard deviation S_a is found by squaring this expression, summing over all measured values of y (that is, summing the index j from 1 to N) and taking the square root of this sum. Also it should be realized that $\sum x_i = \sum x_n$, and $\sum x_j^2 = \sum x_n^2$. The result of this calculation is

$$S_a = \sigma_y \sqrt{\frac{\sum x_n^2}{[N \sum x_n^2 - (\sum x_n)^2]}}$$

In a similar manner, the standard deviation of the intercept S_b can be found and

$$S_b = \sigma_y \sqrt{\frac{N}{[N \sum x_n^2 - (\sum x_n)^2]}} \quad .$$

1. Photoelectric Effect Experiment

Objective:

1. To verify the Photoelectric effect.
2. To determine the plank constant and work function of the materials by the photoelectric effect.
3. To show that kinetic energy of electrons is independent of the intensity of the light.

Theory:

If a photon of frequency *f* strikes the cathode, then an electron can be ejected from the metal (external photoelectric effect) if there is sufficient energy. Some of the electrons thus ejected reach the (unilluminated) anode so that a voltage is set up between anode and cathode, which reaches the limiting value V after a short (charging) time. The electrons can only run counter to the electric field set up by the voltage V if they have the maximum kinetic energy, determined by the light frequency,

$$hf - A = \frac{mv^2}{2} \qquad \text{(Einstein equation)}$$

where *A* = work function from the cathode surface, v = electron velocity, *m* = rest mass of the electron.

Electrons will thus only reach the anode as long as their energy in the electric field is equal to the kinetic energy:

$$eV = \frac{mv^2}{2}$$

Where, *e* = electron charge: $1.602 \cdot 10^{-19}$ As.

An additional contact potential f occurs because the surfaces of the anode and cathode are different:

$$eV + \emptyset = \frac{mv^2}{2}$$

If we assume that *A* and ø are independent of the frequency, then a linear relationship exists between the voltage V (to be measured at high impedance) and the light frequency *f*:

$$V = -\frac{A + \emptyset}{e} + \frac{hf}{e}$$

If we assume V = a + *bf* to the values measured from the graph (V vs f) we can obtain *h*.

Literature value: *h* = 6.62 · 10-34 Js.

Specifications:

Light Source: 12V/35W tungsten –halogen lamp

Sensor: Sensitive Cesium – type vacuum phototube

Dark- current: less than .003 mA

Precision of the accelerated voltage: less than $\pm 2\%$

Measuring error: less than $\pm$ 8% compared with the recognized value ($h = 6.62619 \times 10^{34}$

Power supply: 220V/50 Hz $\pm$ 1 Hz

Caution: In addition to color filters, diodes (LED) are also used for more precise wavelength.

Wavelength of the glass filters

RED……………………………….. from 625 nm -635 nm

ORANGE………………………… from 575 nm – 585 nm

YELLOW (Dark)…………………. from 545 nm – 555 nm

YELLOW (Light)………………… from 505 nm – 515nm

GREEN…………………………… from 515nm-525nm

BLUE……………………………. from 465nm – 475 nm

Wavelength of LED

RED…………………………….. from 618 nm – 622 nm

ORANGE…………………. ….. from 584 nm- 588 nm

GREEN……………………….. from 528 nm -532 nm

BLUE…………………………. from 483 nm- 487 nm

Structure:

The structure of the instrument is showed in Fig1.

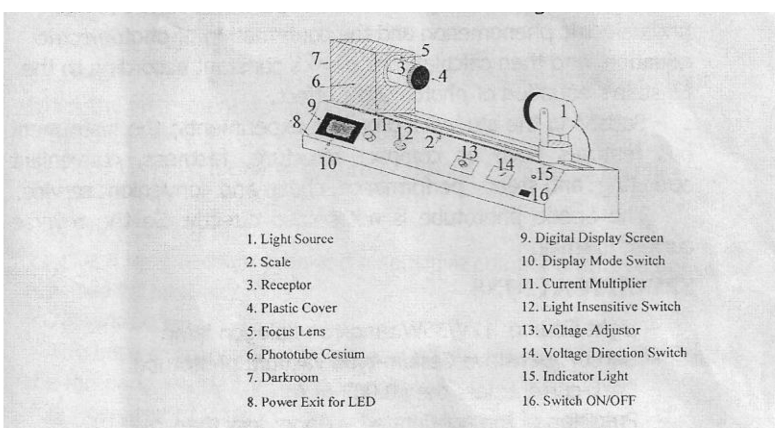


Figure 1. Sketch map of the plank's measuring instrument

Showed as Fig.1, the instrument is mainly composed of a light source (1), a light receiving darkroom (7) (photodiode insider), a DC amplifier, six pieces of different color filters and four types of LED diodes. The light source (1) and the receiving darkroom (7) are installed on a coattail guide. The distance between them may be adjusted from 10-40 cm along the guide wherever it appears the exact distance by changing the position of the light source (1).

1) **Light source:**

Tungsten –halogen 12V/35 W. By moving the light source (1) along the coattail guide, the distance between the light source and phototube can be adjusted. By vertically rotating the light body on its base, there is also possible to change the angle of the light beam irradiating on the photo tube.

2) Scale:

It is a graduated scale. Its overall length is 400 mm. The centre of phototube vacuum is used as the zero point.

3) Receptor: The receptor is used for the installation of filters or diodes (LED) known wavelength. A lens is fixed in its back end to focus the light beams.

4) Plastic cover: Used to cover the receptor and protect the phototube from light when the instrument is not used.

5) Focus Lens: Used to make a clearer picture of the light source or LED diode on the cathode of phototube.

6) Phototube cesium: It is a light vacuum tube and a sensitive component. It can be replaced without any tools.

7) Darkroom: The base of the vacuum phototube is built into the darkroom. In the forepart, a receptor (pipe) is installed.

8) Power exit for LED: 5V output for the power supply of LED diodes.

9) Digital display screen: It displays the intensity of the current (μA) or voltage (V) displaying in the indications screen.

10) Display mode switch: It is a switch to select current ((μA) or voltage (V) displaying in the indications screen.

11) Current multiplier: The knob is used to adjust the current amplifier. There are four flying locations:

10^{-6} A photocurrent can be measured when we choose the position "x1".

10^{-7} A photo current can be measured when we choose the position "× 0.1",

10^{-8} A photocurrent can be measured when we choose the position "× 0.01",and

10^{-9} A photo current can be measured when we choose the position "× 0.001",.

12) Light insensitive switch: It is potentiometer to regulate the intensity of the light source. Up is of STRONG, middle is of OFF, down is of WEAK.

13) Voltage adjuster: It is a potentiometer for regulating the accelerated voltage.

14) Voltage direction switch: It is a Switch for choosing voltage direction. $\pm$15V accelerated voltage is provided.

15) Indicator light for power.

16) Power switches ON/OFF.

Installation:

Put the instrument on the experimental table and turn on the power switch (16). Put the light source (1) at the position of 250 mm distance and set the light insensitive switch (12) at lower position to select weaker intensity light.

Loose the screws on the darkroom (7), remove the darkroom (7) away. Change the distance between the light source (1) and the vacuum phototube (6), adjust the position of the phototube base to make a clear picture of the image of the light source (1) on the cathode board, then put the darkroom (7) and tighten the screws.

Electrical adjustments:

Select the display mode of the digital screen with the display mode switch (10). When indicating presentation voltage, adjust the accelerate voltage adjustor (13) to get a stable voltage about$\pm 15V$.

Put the plastic cover (4)into the drawtube of the receptor (3), and let no light into the darkroom (7).Adjust the current multiplier (11)to choose “x1”, “x.1”, “x.001” and keep the dark current less than .003mA.

Change the light intensity switch (12) to get a different light intensity. That is strong off and weak. Setting the best work situation: the light must focus on the middle area of the phototube’s cathode plate instead of on the anode. The user can make arrangements to get a maximum current display with no changing of the other condition, and this is the best work situation. Up to now, all parts of the instrument have been tested and adjusted.

Operation of the equipment:

For the functioning of the instrument, it requires no additional device such as multimeters, power supplies and cables.

For the best results, the first measurements must be taken at least ten minutes after opening the instrument.

(1) Slide the light source (1) 250 mm in position. Open the switch ON/OFF (16) and turn on the power. After 5 minutes pre – heating, set the current multiplier (11) at the position“x1”.

(2) Insert the red color filter (625 nm – 635 nm) into the drawtube of the receptor (3). Select the light intensity switch (12) at weak light ; Select the voltage direction switch (12) at weak light ; Select the voltage direction switch (14) at “+” position ; Select the current multiplier (11) at “x1” or “x0.1”; Adjust the accelerated voltage adjuster (13) to intensify gradually the photo current until it reaches saturating , measure the voltage of saturate – current . Use the display mode switch (10) for choosing current or voltage display while checking current or voltage.

(3) Moving slowly the light source (1) to change the distance between the light source (1) and receiving filter, we can observe fall of the photo current. Quit the passing light from the entry hole by hand, photo current will disappear immediately, and then removed the hand, photo current appears immediately. The emergence of photocurrent forms very quickly and the process will not exceed the ever 10^{-9} sec. The same phenomena will appears when the light source (1) removes from the phototube.

(4) Change the distance (R) between the light source (1) and the vacuum phototube (6), take down the value of R and the current (I), draw the I-I/R^2figure, it will be a straight line. That shows the correlation between me and me/R2 or photocurrent and light intensity is direct ratio.

(5) Set the voltage adjuster (13) to increase gradually until it reaches zero in the current, and then measure the electrical voltage (voltage cut-off). Use the display mode switch (10) of the screen (9) for selecting indication of current or voltage.

(6) Insert the red color filter (625nm-635nm) into the drawtube of the receptor (3). Set the light intensity switch (12) at strong light. Set the voltage direction switch (14) at “-”.

Set the display mode switch (10) at current display. Adjust the accelerate voltage to about 0V.And set the current multiplier (11) at “x.001”. Adjust the accelerate voltage to decrease the photocurrent to zero, and take down the accelerate voltage value which is the close voltage Vj of 635 mm wavelength. Get the Vj of other four wavelengths by the same way. Input this five data of Vj and wavelength (λ) to a calculator, it's easy to get the plank's constant (h) by linear regress equation.

(7) For the measurements with LED light source , put it into the receptor (3) , and put its supply plug into the power exit for LED (8).Repeat the same process as the step 6) to get the plank's constant (h) by linear regress equation.

Indicative of the measurements results: Using the six glass color filters and four LED, take the measurements results – voltage (V_k) down as shown in the table below:

Filters or LED	Wavelength λ(nm)	Range V_k (V)	Average Va (V)
RED			
ORANGER			
YELLOW (dark)			
YELLOW (light)			
GREEN			
BLUE			
SOURCES LED			
RED			
ORANGER			
GREEN			
BLUE			

By processing the above test results show the following results:

Calculation of export project on the basis of pilot result:				
	Wavelength of light λ(10^{-7}m)	Trend –off Va (V)	Frequency of light	Photoelectron energy

			f (10^{15}Hz)	q.V_k(10^{-19} joule)
FILTER				
Red				
Orange				
Dark yellow				
Yellow				
Green				
Blue				
LED				
Red				
Orange				
Green				
blue				

Based on the results of measurements, we have three graphic representations of the photoelectric equation. The first (A) is the photoelectric equation with using the filters. The second (B) is the photoelectric equation with using the sources LED. The third (C) is the photoelectric equation with ten experimental results (6 filters and 4 LED).

Plot all three graphic representations and shows that the photo equation is of the form $y = a - bx$; also determine the Plank constants from the plotted graphs and the error in the measurements.

Safety Measures for Maintenance:

This instrument must be used in a dry environment indoors and far away from corrosive substances. The environment temperature must be between 0-40 ^{0}C.

When it is put on the laboratory table, make sure the photo tube not face the stray light (such that (such as sunlight) directly.

As soon as finish the experiment, please cover the entry hole with the black plastic cap to protect the phototube from ageing. If the sensitivity of photo tube reduces obviously, replace it with a new one. Save the instrument in a position away from dust moisture. Remove any dust on phototube, the focus lens and filters in time with absorbent cotton. When there is no solution to clean, please use alcohol and aether.

During the experiment, it must avoid overloading. When the gauge of the current or voltage is unknown, adjusting the selector at a peak measurement.

After use, do not forget to turn off the general switch, and to place the black cover into the entrance of the darkroom.

The instrument does not require regular maintenance.

In the event of significant deviations from the indicative measurements, make sure cleaning the filters, the focus lens and phototube.

Operating conditions:

Temperature environment: 0-40^{0}C.

The highest relative humidity: 80 %.

Power supply: 220 V +/- 10 V

Questions:

1. How does the change in light intensity affect the current and not affect the stopping volatge?
2. What are the experimental features of the photoelectric effect that Newtonian mechanics and classical electromagnetic theory cannot explain?
3. Compare the energies and stopping voltages of the blue light and the red light photons?
4. What will be the relationship between the frequency of light causing the current and the voltage necessary to stop that current?
5. What is the relationship between *wavelength* and stopping voltage?
6. Why is the stopping voltage so similar for white and far blue light, even though the white light has about 100 times as much light as the light through any of the filters?
7. How does the photoelectric effect in general, and your data in particular, demonstrate the quantized nature of light?
8. What property (or properties) of *waves* did we use in this experiment demonstrating the quantized nature of light (i.e., the existence of *photons*)?

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6. Y. Kraftmakher. Am. J. Phys., 2, 2006.

2. Electron Diffraction Experiment

Objectives:

1. To measure the diameter of the two smallest diffraction rings at different anode voltages.
2. To calculate the wavelength of the electrons from the anode voltages.
3. To determine the interplanar spacing of graphite from the relationship between the radius of the diffraction and the wavelength

Equipment: Electron diffraction tube with mounting, High voltage power supply(0-1KV), High value Register(10 MΩ), Power supply(0-600V_{DC}), Vernier caliper, Connecting cord for 50 KV, Norma connecting cords.

Principle and Task:

Fast electrons are diffracted from a polycrystalline layer of graphite: interference rings appear on a fluorescent screen. The interplanar spacing in graphite is determined from the diameter of the rings and the accelerating voltage.

Set-up and procedure:

Set up the experiment as shown in Fig. 1. Connect the sockets of the electron diffraction tube to the power supply as shown in Fig. 2. Connect the high voltage to the anode G3 through a 10 MV protective resistor.

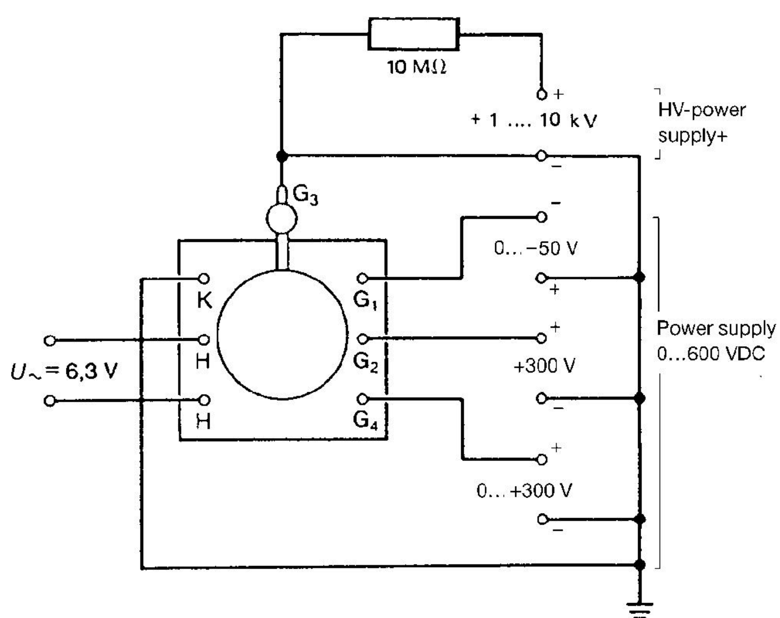


Fig. 1: Experimental set-up: electron diffraction.

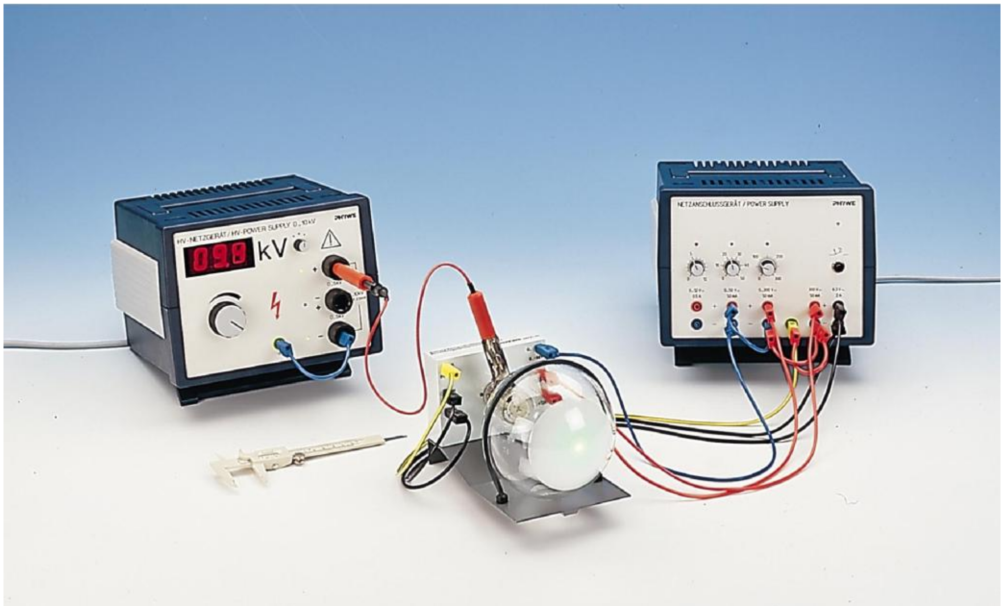
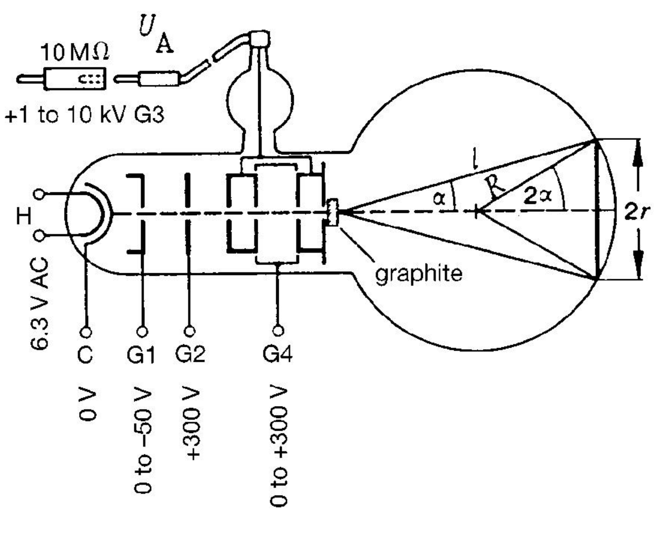


Fig. 2: Set-up and power supply to the electron diffraction tube.



Set the Wehnelt voltage G1 and the voltages at grid 4 (G4) and G3 so that sharp, well-defined diffraction rings appear. Read the anode voltage at the display of the HV power supply.

To determine the diameter of the diffraction rings, measure the inner and outer edge of the rings with the vernier caliper (in a darkened room) and take an average. Note that there is an-other faint ring immediately behind the second ring.

Theory:

To explain in the interference phenomenon, a wavelength λ, which depends on momentum, is assigned to the electrons in accordance with the de Broglie equation:

$$\lambda = \frac{h}{p} \qquad (1)$$

Where $h = 6.625 \cdot 10^{-34}$ Js, Planck's constant.

The momentum can be calculated from the velocity v that the electrons acquire under acceleration voltage U_A:

$$\frac{1}{2}mv^2 = \frac{p^2}{2m} = e.U_A \qquad (2)$$

The wavelength is thus,

$$\lambda = \frac{h}{\sqrt{2meU_A}} \qquad (3)$$

Where $e = 1.602 \cdot 10^{-19}$ as (the electron charge) and $m = 9.109 \cdot 10^{-31}$ kg

(Rest mass of the electron).

At the voltages U_A used, the relativistic mass can be replaced by the rest mass with an error of only 0.5%.The electron beam strikes a polycrystalline graphite film deposite on a copper grating and is reflected in accordance with the Bragg condition:

$$2d \sin\theta = n \cdot \lambda, \; n = 1, 2, \ldots \qquad (4)$$

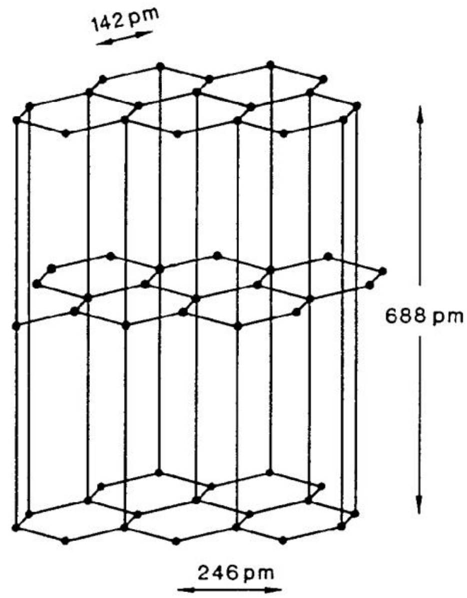


Fig. 3: Crystal lattice of graphite.

Where d is the spacing between the planes of the carbon atoms and θ is the Bragg angle (angle between electron beam and lattice planes).

In polycrystalline graphite the bond between the individual layers (Fig. 3) is broken so that their orientation is random. The electron beam is therefore spread out in the form of a cone and produces interference rings on the fluorescent screen.

The Bragg angle θ can be calculated from the radius of the interference ring but it should be remembered that the angle of deviation α (Fig. 2)

is twice as great:

$$\alpha = 2\theta.$$

From Fig. 2 we read off

$$\sin 2\alpha = r/R \qquad (5)$$

Where R = 65 mm, radius of the glass bulb.

Now, $\sin 2\alpha = 2 \sin\alpha \cos\alpha$.

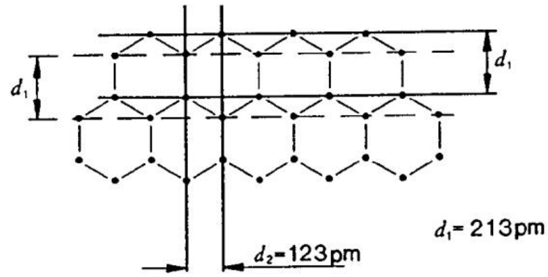


Fig. 4: Graphite planes for the first two interference rings.

For small angles α (cos 10^0 = 0.985) can put

$$\sin 2\alpha \approx 2 \sin \alpha \qquad (6)$$

so that for small angles θ we obtain

$$\sin \alpha = \sin 2\theta \approx 2 \sin \theta \qquad (6a)$$

With this approximation we obtain

$$r = \frac{2R}{d} n\,\lambda \qquad (7)$$

The two inner interference rings occur through reflection from the lattice planes of spacing d_1 and d_2 (Fig. 4), for $n = 1$ in (7).

Calculated the wavelength from the anode voltage in accordance with (3) and plot the radii of the first two interference rings as a function of the wavelength of the electrons.

Notes

The intensity of higher order interference rings is much lower than that of first order rings. Thus, for example, the second order ring of d_1 is difficult to identify and the expected fourth order ring of d_1 simply cannot be seen. The third order ring of d_1 is easy to see because graphite always has two lattice planes together, spaced apart by a distance of d1/3. (Fig. 5)

In the sixth ring, the first order of ring of d_4 clearly coincides with the second order one of d_2.

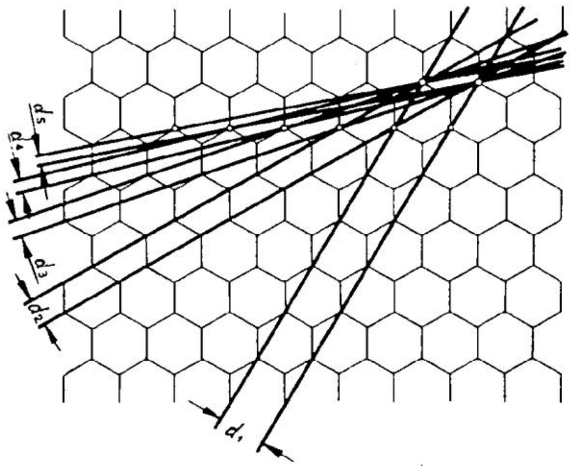


Fig. 5: Interplanar spacing in graphite

– The visibility of high order rings depends on the light intensity in the laboratory and the contrast of the ring system which can be influenced by the voltages applied to G1 and G4.

– The bright spot just in the center of the screen can damage the fluorescent layer of the tube. To avoid this reduce the light intensity after each reading as soon as possible.

References:

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3. Franck – Hertz Experiment

Objective:

Electrons are accelerated in a glass tube filled with mercury vapour. Depending on the acceleration voltage, a variable electron flux passes through the mercury vapor to the collector electrode.

Equipments: COBRA3 Basic Unit(1), Power Supply, 12 V_(1),RS232 Data cable(1),Cobra3 Universal Plotter Software(1),Franck-Hertz Oven(1),Power supply unit for F.-H. tube(1),Power supply, 0…600V-(1),DC measuring amplifier(1),Digital Thermometer(1),Ni Cr-Ni thermocouple(1), BNC cable(2), Short –circuit plug(1),Plate, ceramic fiber(1),Connecting cords(12).

Set- up:

- In accordance with Figs. 1 and 2.
- The power supply for the Franck – Hertz tube is required to generate a 0 to -50 V rising voltage from a constant supply voltage (50V-). As long as the switch S (short –circuit plug) is closed, the anode voltage is approximately 0.5 V. After the switch has been opened, the voltage increased logarithmically.
- Over a voltage divider (2x100 kΩ) that is integrated in the control unit , the voltage across the cathode and the grid U_A is divided in a ratio of 1:2(i.e.$U_A/2$) so that it can be measured by the Cobra3 interface(Measuring range $\pm 30V,$ Analog IN2). Subsequent to the measurement, the divided voltage is multiplied by two to obtain the true voltage across the cathode and grid.
- Over the voltage divider that is also integrated in the control unit, the 0……12V voltage is split down in order to be able to apply it as counter voltage to the collector electrode on the Franck – Hertz tube. When U=12 V, the split down voltage Us=3 V. The counter voltage can thus be regulated in the region from 0 to 3 V. Us=0.5V should be applied at the beginning of the experiment.
- The electron current that is conducted through the tube is on the order of 10^{-9}A. It is amplified by the DC measuring amplifier (10 nA…. 10μA measuring range) and feed to the Analog IN 1 Channel on the Cobra3 unit.
- Heat the oven up to approximately 160^0C. The tip of the temperature sensor is located at an intermediate tube height, exposed to the air. After approximately 10 to 15 min, the apparatus is ready for measurements.

Procedure:

- Start the "Universal Plotter" and set the parameters according to Fig.3.
- When the < Continue> icon has been clicked up on, two digital displays, which show the current voltage without permanently recording the values, become visible. Using the set knob on the DC measuring amplifier, set the voltage measured on the Analog channel with the switch S (short circuit switch) closed to approximately 0 V.
- Then click on the "Start measurement" icon and open switch S (Pull the short circuit plug).It now takes approximately 1.5 minutes to complete the measurement. Finally, the digital panel for Analog channel IN2 indicates approximately 25 V. Now, approximately 50 V lie across the cathode and the grid. Click on the < Stop Measurement> icon.

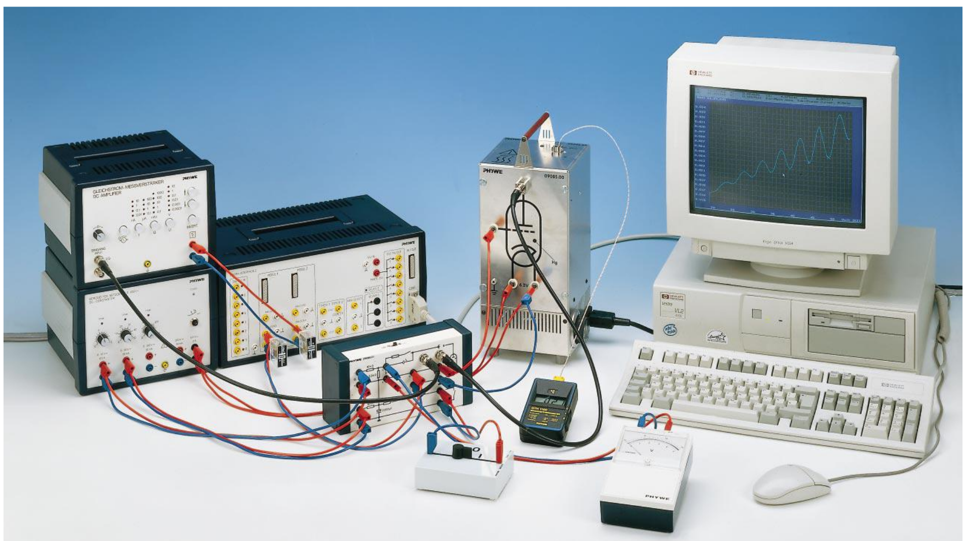


Fig.1: Experimental setup.

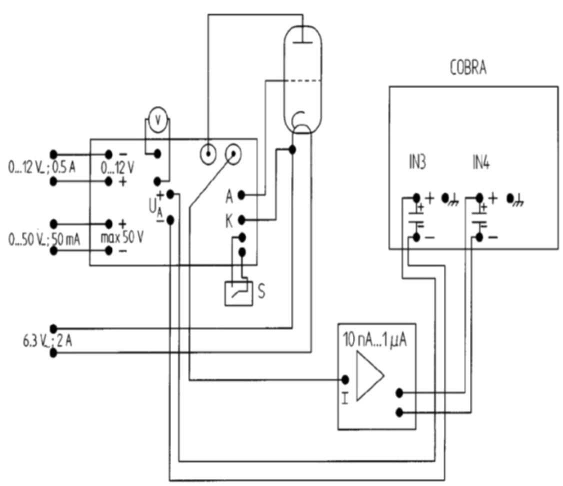


Fig.2: Circuit diagram for the Franck-Hertz experiment.

Caution:

- During the measurement processes watch the tube! If a bright bluish light suddenly appears between the cathode and the grid, the tube has "ignited". Shut switch S immediately (Plug in the short circuit switch!)to eliminate the intense light , which can damage the cathode . If this occurs, increase the oven temperature and restart the measurement.

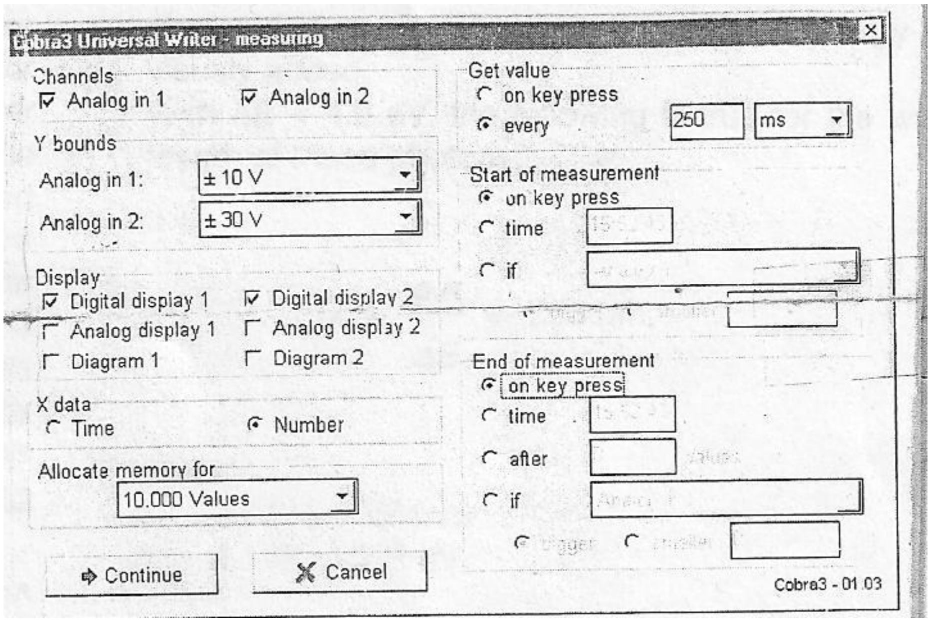


Fig .3: Measurement Parameters.

A weak greenish light that is arranged in horizontal layers is harmless: it shows the shock fronts of the electrons with the mercury atoms.

- Convert the Analog channel IN 2 in the Analysis / Channel modification window, using the formula x: = x*2, into the true voltage that lay across the cathode and the grid during the measurement. (The voltage splitter ratio of 1:2 is mathematically reversed!).
- Change the unit of the Analog channel IN 1 into current in Measurement / Information …/ Channels window: Symbol, I; Unit, nA.
- In the Measurement / Channel Manager window, select the following: for the x axis, Analog channel IN 2: and for the y axis, the current channel (Fig.5).

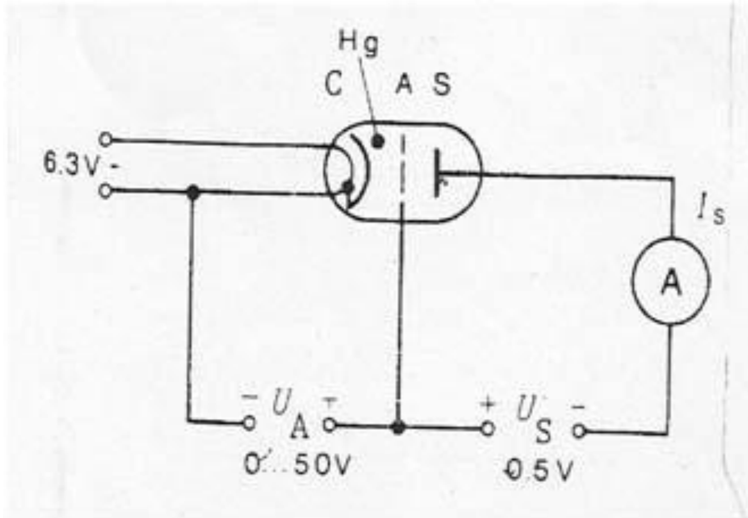


Fig.4: Measuring principle.

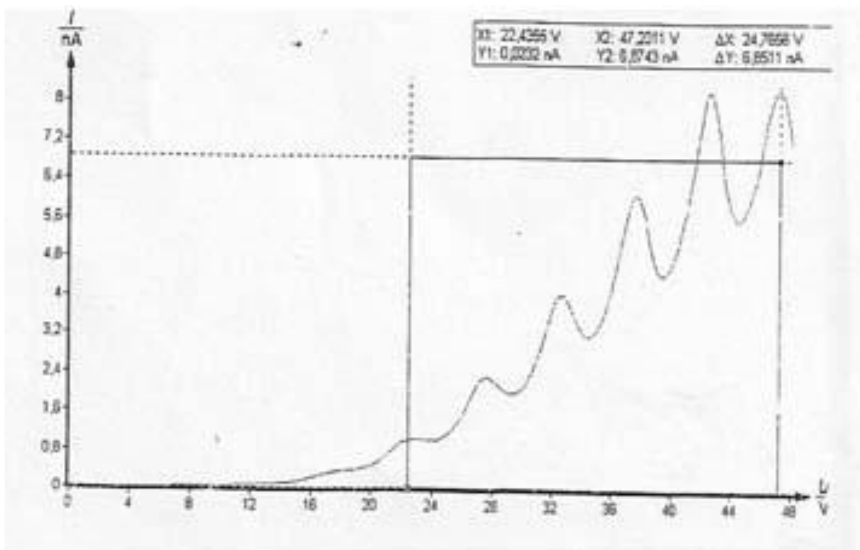


Fig. 5: Typical example of a Franck –Hertz curve. The energy difference determined for the adjacent maxima is ΔE = 4.95V.

Results:

- (Fig.4) Electrons emerge from a heated wire (cathode, C) and are accelerated by a voltage to the grid (anode, A). The electrons can pass through holes in the grid to be detected by the collector electrode S as current. During their passage through the tube the electrons collide with gaseous mercury atoms.
- Above an acceleration voltage of 4.9 V, the kinetic energy of the electrons is sufficiently large to excite electrons in the outer shell of the mercury atom by inelastic collisions (energy level 6 $^3P_1$).

- In this instant the colliding electron looses energy and can no longer move against the counter voltage U_S between the anode A and the collector electrode S. The current I_S becomes clearly weaker. Now, with a further increase in acceleration voltage U_A the kinetic energy again rises so that more electrons can reach the collector electrode S; current I_S increases. With an even greater increase in the acceleration voltage, the voltage 2 X 4.9 V = 9.8 V is reached. Now, the electrons can excite the electrons in the shell of the mercury atom twice on their way from the cathode to the anode. Thereafter they can no longer reach the collector electrode S, and the current I_S again drops. This procedure repeats itself when the acceleration voltage is repeatedly raised by 4.9 V. The I_S (U_A) graph (Fig. 5) shows maxima and minima that are equidistant from one another; the distance between them is measured.
- The following equation is valid for the voltage U_A between the anode and cathode:

$$U_A = U + (\phi_A - \phi_C)$$

Where U= Applied voltage, Φ_A =Work of emission at the anode, ϕ_C = Work of emission at the cathode

Since voltage differences are analyzed in this case, the works of emission must on be further considered. According to classical theory the energy level of the electrons in an atom should be continuously distributed. All the energy coming from the outside in the form of impulses should thus be absorbable by the atom. In other words, it should not be possible for the electrons to reach the collector electrode since they are continuously broken in the layer of mercury vapour. However, this experiment demonstrates that the energy levels in the electron envelope of the mercury atom are discreet. This is the statement of quantum mechanics. Only when a specific kinetic energy has been reached can the free electrons give up an exactly defined fraction of their energy to the shell electrons of the mercury atom. The mercury atom ultimately releases the energy supplied by emitting a photon that carries exactly the quantity of energy previously added.

With ΔE = 4.9 eV, the following is true for the wave length of these photons:

$$\lambda = \frac{ch}{\Delta E} = 253\ nm,$$

Where,

C= 2.9979 x 10^8 m/s , h = 4.136 x 10^{-15} eV,

It lies in the UV region.

Remarks:

- In general the following are true:
- The low energy maxima and minima can be better observed at low tube temperatures. However, the tube ignites extremely easily under these conditions. The Higher –energy maxima and minima can be better observed at high temperatures. At high temperatures the energy difference between a minimum and its adjacent maximum becomes continuously smaller.
- If the oven continues to heat up during the recording of the measure values, the high – energy peaks on the curve can easily migrate downward. However, for the evaluation this effect is inconsequential as only the voltage differences of the peaks are measured and not their amplitudes.
- If an older power supply that has no output jack for $U_A/2$ is used for the Franck – Hertz tube ; the following equipment is required:
- Switch box(1),Resistor in plug – in box , 100 k Ω(2),Connection cord(2)

References:

- G. Rapior, K. Sengstock and V. Baeva, "New Features of Franck-Hertz Experiment". Am. J. Phys. 74(5), 423-28 (2006).
- J.S. Huebner, "Comments on the Franck-Hertz Experiment. Am.J. Phys. 44,302-03(1976).
- H. Haken and H.C. Wolf, The Physics of Atoms and Quanta,6th Ed. Springer, Heidelberg,P.305(2000).

NiCr- Ni

Table of operative temperatures, tolerances and EMF values in mV at different temperatures for thermocouples NiCr- Ni DIN 43710

^{0}C	0	10	20	30	40	50	60	70	80	90	100	mv/^{0}C	Toll.in ^{0}C
0	0	.40	0.80	1.20	1.61	2.02	2.43	2.85	3.2	3.68	4.10	0.041	
100	4.10	4.51	4.92	5.33	5.73	6.13	6.53	6.93	7.33	7.73	8.13	0.040	
200	8.13	8.54	8.94	9.34	9.75	10.16	10.57	10.98	11.39	11.80	12.21	0.041	±3
300	12.21	12.63	13.04	13.46	13.88	14.29	14.71	15.13	15.55	15.98	16.40	0.042	
400	16.40	16.82	17.24	17.67	18.09	18.51	18.94	19.36	19.79	20.22	20.65	0.042	
500	20.65	21.07	21.50	21.92	22.35	22.78	23.20	23.63	24.06	24.49	24.91	0.043	±4.5
600	24.91	25.34	25.76	26.19	26.61	27.03	27.45	27.87	28.29	28.72	29.14	0.042	
700	29.14	29.56	29.97	30.39	30.81	31.23	31.65	32.06	32.48	32.89	33.30	0.042	±6

800	33.30	33.71	34.12	34.53	34.93	35.34	35.75	36.15	36.55	36.96	37.36	0.041	
900	37.36	37.76	38.16	38.56	38.95	39.35	39.75	40.14	40.53	40.92	41.31	0.040	
1000	41.31	41.70	42.09	42.48	42.87	43.25	43.63	44.02	44.40	44.78	45.16	0.039	
1100	45.16	45.54	45.92	46.29	46.67	47.04	47.41	47.78	48.15	48.52	48.89	0.037	± 8
1200	48.89	49.25	49.62	49.98	50.34	50.69	51.05	51.41	51.76	52.11	52.46	0.036	

4. Solar Cell Experiment

Objectives:

1. Find I_o and n from Dark I-V.
2. Find V_{oc} .I_{sc} and FF from illuminated I-V.
3. Find the efficiency η of the solar cell.

Introduction:

Crystalline Si p-n Solar Cell:

A solar cell or a photovoltaic cell is a semiconductor device to convert light into electricity. The cell consists of a p-n junction of a semiconductors material having a band gap E_g. When the cell is exposed to light, a photon with energy, $h\nu$ less than E_g makes no contribution to the cell output. However a photon with energy, $h\nu$ greater than E_g contributes an energy E_g to the cell output and the excess over E_g is dissipated as heat.

Theory:

Dark current of a p-n junction is given by,

$$I_D = I_S(exp\frac{qv}{nkt} - 1)$$

Where n is the ideality factor and varies from n=1 to n=2 . I_s is the dark saturation current and V is the applied voltage. Hence k is Boltzmann constant and T is the operating temperature of junction.

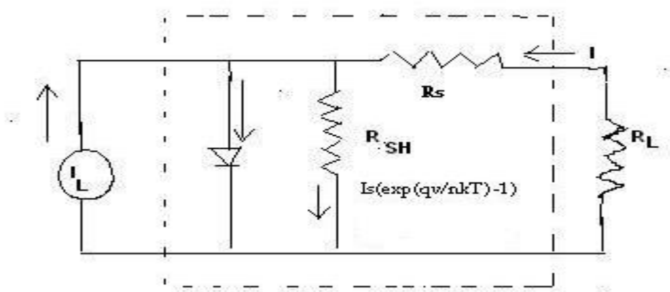
```
                    ______________________________Ec
                   /
          n       /
Ec_____________  /
Ef- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -Ef

                    ______________________________Ev
                   /
                  /                        p
Ev_____________  /
```

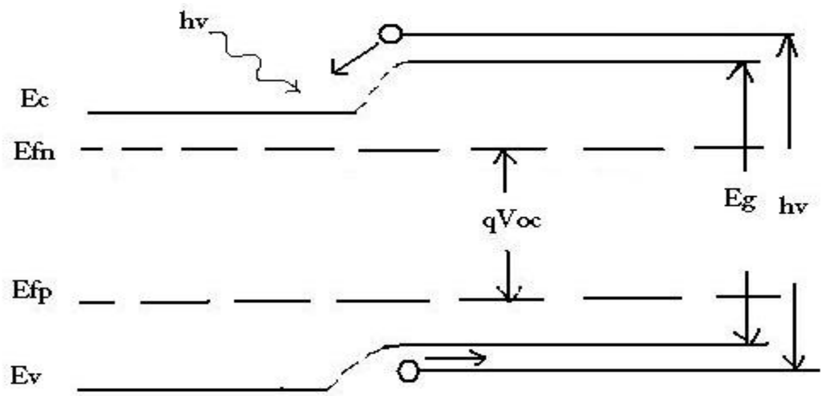
Fig-1: Band diagram of a p-n junction

When light of energy greater than or equal to E_g is shown on the cell, electron –hole pairs are generated because of absorption of light. Due to the inbuilt field in the p-n junction the photo generated electrons and holes get separated and contribute to the current output if the cell is short circuited. Under open circuit condition a voltage is generated across the solar cell called the open circuit voltage. The equivalent circuit of the solar cell and the band diagram of a p-n junction under illumination are shown in fig.2.

\



(a)



(b)

Fig.2: (a) Equivalent circuit of a solar cell and (b) The diagram of a p-n junction under illumination.

The I-V characteristics of an ideal solar cell is given by

$$I = I_S\left(e^{\frac{qv}{nkT}} - 1\right) - \quad I_L$$

The I-V characteristics of a real solar cell is given by

$$I - \left[\frac{V - I\,R_S}{R_{sh}}\right] = I_s\left[exp\frac{q(V - IR_s)}{nkT} - 1\right] - I_L$$

Here the current source I_L results from the photo generated carriers. I_s is the diode saturation current and R_L the load resistance. R_{sh} and R_s are the shunt and series resistances of the solar cell. The I-V characteristic under illumination is as shown in Fig-3.

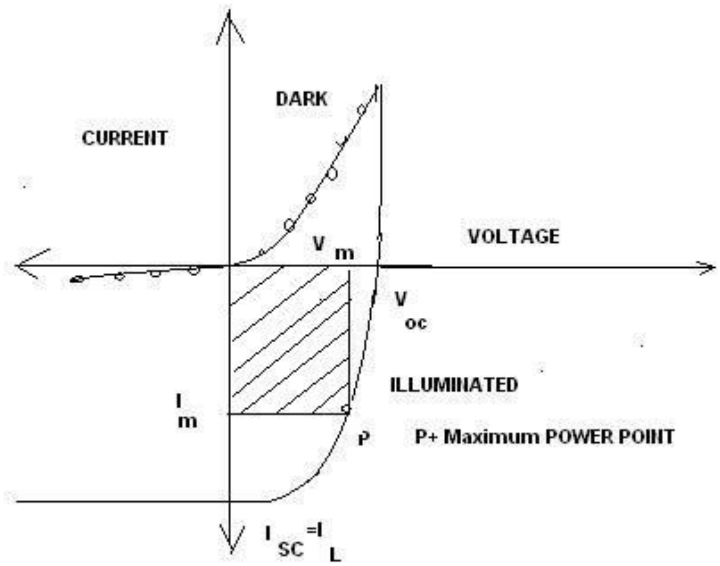


Fig. 3: The I-V characteristics under dark (0-0-0) and under illumination (____).

For an ideal solar cell R_s =0 and R_{sh}= ∞. It can be seen that a shunt resistance even as low as 100 ohm does not appreciably change the power output of the device. Whereas a series resistances of only 5ohm reduces the available power to less than 30% of the optimum power with Rs=0. We can thus neglect the effect of R_{sh}. The output current and output power are given by,

$$I = I_S\left\{\exp\frac{q(V-IR_S}{nkT} - 1\right\} - I_L$$

$$P = |\,IV| = I\,[\frac{nkT}{q}\,L_n(\frac{I+I_L}{I_S} + 1) + IR_S]$$

The Fill factor, FF is defined as

$$FF = \frac{I_m V_m}{I_{sc}.V_{oc}}$$

Where, Isc and Voc are shown in Fig. 3.

The Conversion efficiency, η**,** is defined as the ratio of max. Power output to the incident power (P_{in}).

$$\eta = \frac{I_m V_m}{P_{in}} = \frac{FF.I_{sc}V_{oc}}{P_{in}}$$

The fill factor can be determined graphically.

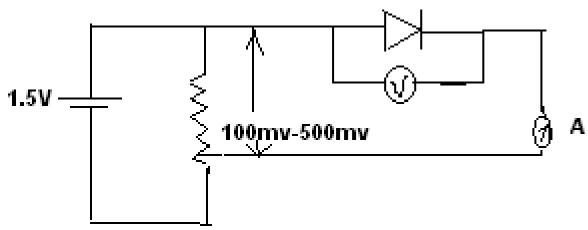


Fig. 4: Connections for Dark characteristics of the solar cell.

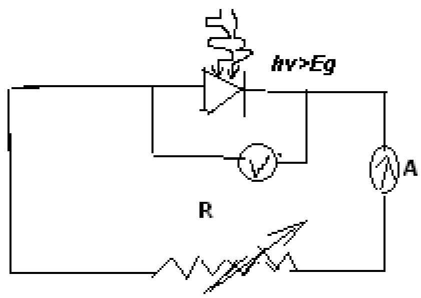


Fig. 5: Electrical circuits for measuring I-V characteristics under illumination.

Connect the circuit as shown in fig.4. Measure I-V characteristics under dark. Now connect the circuit as shown in fig-4. Measure the intensity of the light, measure V_{oc} and $I_{sc.}$ Take the I-V characteristics by varying the resistance under illumination.

Reference:

Physics of Semiconductor devices. SM Sze. IInd edition.

Thin Film Solar Cells: K.L. Chopra and S. R. Das

5. van der Pauw Experiment

Objectives:

1. To measure the resistivity of the given semiconductor sample.

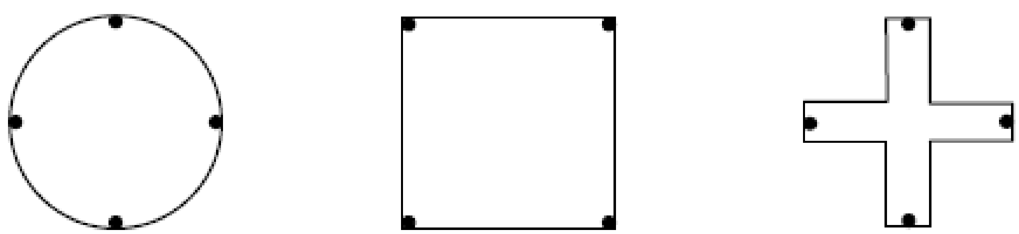
2. To measure the Hall coefficient of the given semiconductor sample.

Equipment: van der Pauw set-up, sample holder, constant current power supply, electromagnet.

Theory and principle:

The van der Pauw technique is a simple technique to determine electrical parameters like resistivity, mobility, carrier density and Hall Effect of thin flat homogeneous plate samples of arbitrary shape. One uses singly connected (not having any isolated holes or non conducting islands or inclusions) thin plate sample of uniform thickness containing four very small electrical contacts placed on the corners or the periphery off the sample plate. If the above conditions are fulfilled, there is no need to know the current pattern in the sample. The accuracy of the results depends on the smallness of the electrical contacts relative to the size of the sample plate.

Some common van der Pauw geometries of the samples under this configuration are shown below:



In order to reduce errors in the calculations it is preferable that the sample is symmetrical. A simple square/ rectangular shape under this configuration is shown in Fig. 1.

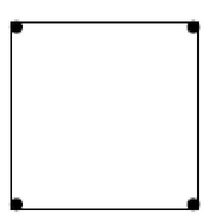


Fig. 1

Procedure:

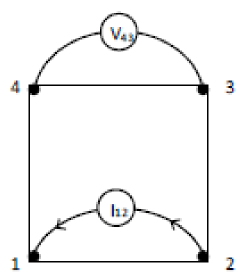
RESISTIVITY MEASUREMENT

Consider a sample plate (Fig.1) of sheet resistance R_s and thickness d. If ρ is the bulk resistivity of the material of the plate, then $\rho = R_s d$. The Van der Pauw's relation connects the sheet resistance R_s to the two transverse characteristic resistances $R_A^{'}$ and $R_B^{'}$ through the relation

$$e^{-(\pi R_A^{'}/R_s)} + e^{-(\pi R_B^{'}/R_s)} = 1 \quad \text{......................(1)}$$

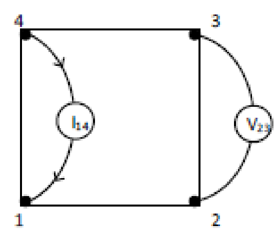
The characteristic resistances $R_A^{'}$ and $R_B^{'}$ are evaluated by measuring currents and voltages as shown in Figs 2.1 and 2.2.

In an actual experimental set-up, even if the set-up 1234 is a square, there will naturally be variations in the contacts at 1, 2, 3 & 4 and the lengths 12, 23, 34 & 41. In order to average and thereby eliminate the effect of these variations, six more combinations [Fig. 2.1(a, b, c) and 2.2 (a, b, c)] in addition to the above two are used.



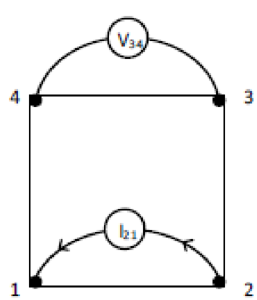
$$R_A^{'} = \frac{V_{43}}{I_{12}}$$

Fig. 2.1



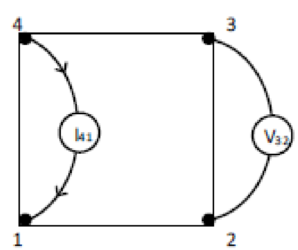
$$R_B^{'} = \frac{V_{23}}{I_{14}}$$

Fig. 2.2



$$R_A^{''} = \frac{V_{34}}{I_{21}}$$

Fig. 2.2 (a)



$$R_B^{''} = \frac{V_{32}}{I_{41}}$$

Fig. 2.2 (a)

The resistances 'A, correspond to currents 12, 21, 34 & 43 and voltages 43, 34, 21 & 12 and resistances 'B' correspond to currents 14, 41, 23 & 32 and voltages 23, 32, 14 & 41.

The resistances, $R_A^{'}, R_A^{''}, R_A^{'''}, R_A^{iv}$ and $R_B^{'}, R_B^{''}, R_B^{'''}, R_B^{iv}$ are separately averaged,

$$R_A = \frac{1}{4}(R_A^{'} + R_A^{''} + R_A^{'''} + R_A^{iv}) \qquad (2)$$

$$R_B = \frac{1}{4}(R_B^{'} + R_B^{''} + R_B^{'''} + R_B^{iv}) \qquad (3)$$

The characteristic resistance R_A and R_B are the ones which have been averaged over the configuration and oven current reversal. The Van der Pauw Eq. (1) connecting characteristic resistances to the sheet resistance R_S of the sample plate now reduces to

$$e^{-(\pi R_A/R_S)} + e^{-(\pi R_B/R_S)} = 1 \qquad (4)$$

This equation can be numerically solved iteratively for R_S (see Appendix)

If c is the size of the electrical contacts and l is the length of a side of the square/rectangular sample plate, the accuracy of the result on

$$\frac{\Delta R_S}{R_S} \sim \left(\frac{c}{l}\right)^2$$

Alternatively, following simple procedure leading to results within the error limits could be adopted. Take average of R_A and R_B

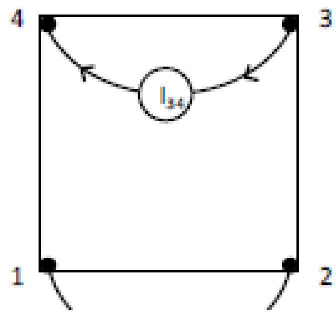
$$R = \frac{1}{2}(R_A + R_B) \qquad (5)$$

and use it in place of R_A and R_B in Eq. (4) to evaluate the sheet resistance R_S.

$$e^{-(\pi R/R_S)} + e^{-(\pi R/R_S)} = 1$$

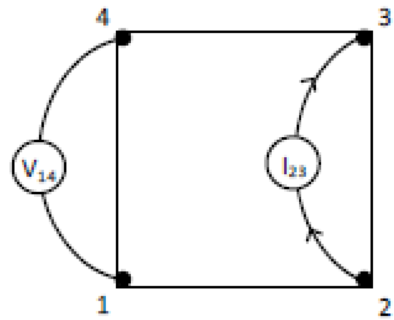
$$\frac{\pi R}{R_S} = \ln 2$$

$$R_S = \frac{\pi R}{l_n 2} \qquad (6)$$



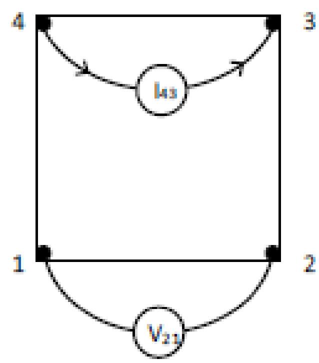
$$R_A^{'''} = \frac{V_{12}}{I_{34}}$$

Fig. 2.1(b)



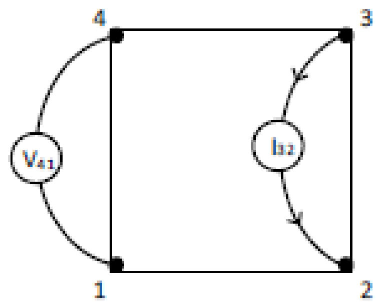
$$R_B^{'''} = \frac{V_{14}}{I_{23}}$$

Fig. 2.2(b)



$$R_A^{iv} = \frac{V_{21}}{I_{43}}$$

Fig. 2.1(c)



$$R_B^{iv} = \frac{V_{41}}{I_{32}}$$

Fig. 2.2(c)

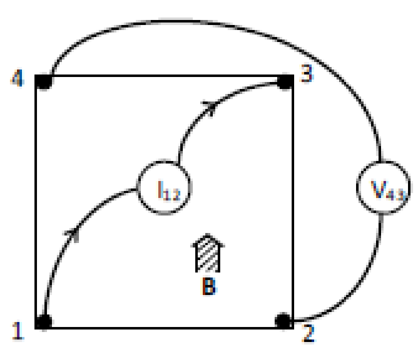


Fig. 3(a)

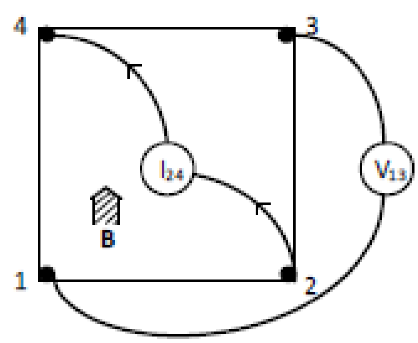


Fig. 3(b)

The error in this result may be estimated by using this value of R_5 on the L.H.S. of Eq. (4)

$$e^{-(\pi R_A \ln 2/\pi R)} + e^{-(\pi R_B \ln 2/\pi R)} = \Delta$$

Comparison with the R.H.S. of Eq. (4) shows that the error in the result is (Δ - 1).

BULK ELECTRICAL RESISTIVITY

The bulk electrical resistivity ρ can now be evaluated using the relation

$$\rho = dR_s \qquad \text{---------------- (7)}$$

If c is the length of contacts and l is the length of a side of the square/ rectangle, the accuracy of the results is less than $\sim \frac{c}{l}\%$.

HALL VOLTAGE MEASUREMENT

The Hall voltage measurement can be carried out with a constant current through the sample and a constant magnetic field applied perpendicular to the plane (X-Y) of the sample 1234. The Hall voltage measurement is carried out by voltage measurement perpendicular to the current and in the plane of this sample as shown in Figs. 3 (a) and 3 (b).

The average value $\left(\frac{V}{I}\right)$ of voltage to current ratio [Figs. 3 (a & b)] is obtained

$$\frac{V}{I} = \frac{1}{2}\left(\frac{V_{24}}{I_{13}} + \frac{V_{13}}{I_{24}}\right)$$

The Hall coefficient is given by

$$R_H = \frac{V\,d}{B\,I} \qquad \text{---------------- (8)}$$

The parameters like carrier mobility μ and carrier density n can now be calculated

Carrier mobility

$$\mu = \frac{R_H}{\rho} \qquad \text{---------------- (9)}$$

Carrier density

$$n = \frac{1}{R_H q} \qquad \text{---------------- (10)}$$

RESISTIVITY MEASUREMENT OBSERVATIONS

Thickness of the sample: d 0.05 cm

S. No.	Voltage (mV)		Current (mA)		Resistance (Ohm)	
1.	V_{43}	142.8	I_{12}	2.02	$R_A^{'}$	70.69
2.	V_{34}	142.9	I_{21}	2.02	$R_A^{''}$	70.74
3.	V_{12}	142.8	I_{34}	2.02	$R_A^{'''}$	70.69
4.	V_{21}	142.6	I_{43}	2.02	R_A^{iv}	70.59
5.	V_{23}	112.3	I_{14}	2.02	$R_B^{'}$	55.59
6.	V_{32}	112.3	I_{41}	2.02	$R_B^{''}$	55.59
7.	V_{14}	112.3	I_{23}	2.02	$R_B^{'''}$	55.59
8.	V_{41}	112.3	I_{32}	2.02	R_B^{iv}	55.59

CALCULATIONS

$$R_A = \frac{1}{4}(R_A^{'} + R_A^{''} + R_A^{'''} + R_A^{iv})$$

$$= \frac{1}{4}(70.69 + 70.74 + 70.69 + 70.59) = 70.68\Omega$$

$$R_B = \frac{1}{4}(R_B^{'} + R_B^{''} + R_B^{'''} + R_B^{iv})$$

$$= \frac{1}{4}(55.59 + 55.59 + 55.59 + 55.59) = 55.59\Omega$$

$$R = \frac{1}{2}(R_A + R_B)$$

$$= \frac{1}{2}(70.68 + 55.59) = 63.14\Omega$$

$$R_S = \frac{\pi R}{\ln 2}$$

$$= \frac{3.141 x 63.14}{0.69}$$

$$= 287.40$$

Bulk Resistivity

$$\rho = R_S \, x d$$

$$= 287.40 \text{ x } 0.05$$

$$= 14.37 \ \Omega.\text{cm}$$

HALL EFFECT OBSERVATIONS

1. Thickness of the sample d = 0.5mm
2. Magnetic Field B = 1 KG =1000 Gauss

S. No.	Voltage (mV)		Current (mA)			
1.	V_{24}	47.1	I_{13}	5.00	V_{24} / I_{13}	9.24
2.	V_{13}	48.8	I_{24}	5.00	V_{13} / I_{24}	9.56

CALCULATIONS

$$\text{Average}\left(\frac{V}{I}\right)=\frac{1}{2}\left(\frac{V_{24}}{I_{13}}+\frac{V_{13}}{I_{24}}\right)$$
$$=\frac{1}{2}(9.42+9.76)$$
$$=9.59$$

(1) Hall Coefficient (R)

$$R_H=\frac{V_h.z}{I\,H}$$

$$R=\frac{46.925x10^{-3}x5.0x10^{-2}}{5.02x10^{-3}x1.0x10^{3}}=46.74\times10^{-5}\text{ volt cm }A^{-1}\ G^{-1}$$

$$R=46.74\times10^{-5}\times10^{8}\text{ cm}^3\text{ coulomb}^{-1}=46.74\times10^{3}\text{cm}^3\text{ coulomb}^{-1}$$

(2) Carrier Density (n)

$$R_H=\frac{1}{nq}\Rightarrow n=\frac{1}{R_Hq}$$

$$n=\frac{1}{46.74x\ 10^{3}\ x\ 1.6\ x\ 10^{-19}}$$

$$n=1.34\ x\ 10^{14}\ cm^{-3}$$

(3) Carrier Mobility (μ)

$$\mu=R\sigma$$

$$\mu=\frac{46.74x\ 10^{3}}{14.06}$$

$$\mu=3324\ cm^2\ volt^{-1}\ sec^{-1}$$

The above observations and calculations are only sample. The actual values may not necessarily replicate.

REFERENCES

1. Van der Pauw, L.J. (1958). "A method of measuring specific resistivity and Hall Effect of discs of arbitrary shape" Philips Research Reports 13 : 1 – 9

APPENDIX

Iterative solution of Eq. (4)

$$e^{-(\pi R_A/R_S)} + e^{-(\pi R_B/R_S)} = 1$$

$\pi R_A = \pi \times 70.68$ $\quad$ $\pi R_B = \pi \times 55.59$

$= 222.048$ $\quad$ $= 174.641$

S. No.	Trial Rs	$\pi R_A / R_S$	$\pi R_B / R_S$	$e^{-(\pi R_A/R_S)}$ (a)	$e^{-(\pi R_B/R_S)}$ (b)	a+b
1	287.0	0.73686	0.608505	0.461309	0.544164	1.0054
2	286.0	0.776392	0.610633	0.460063	0.543007	1.0031
3	285.0	0.779116	0.612775	0.458812	0.541845	1.0019
4	284.0	0.781859	0.614933	0.457555	0.540677	0.99823
5	284.5	0.780485	0.613852	0.458184	0.541262	0.99945

Result $R_S = 284.5$

$\rho = R_S * d$

$\rho = 284.5 \times 0.05 = 14.23\Omega.cm$

6. Chaos Experiment

Object: - To study chaos in the diode- R-L circuit.

Theory:

Chaos typically refers to unpredictability or disorder mathematically, chaos means an a periodic deterministic behavior. Chaotic systems look random but actually they are deterministic systems, governed by non-linear equations. Hence such systems are very sensitive to the initial condition. All chaotic systems, exhibit self-similarity i.e. the chaotic behavior resembles itself at all scales.

The Logistic Map and Frequency Bifurcations:

We are used to the notion that physical systems are described by differential equations that can be initial condition. This is not true in complex systems governed by non-linear equations. A typical example is the flow of fluids. At low velocity one can identify individual "streamlines" and predict their evolution. However, when a particular combination of velocity, viscosity, and boundary dimensions is reached, turbulence sets in and eddies and vortices are formed. The motion becomes chaotic. Many chaotic systems exhibit self –similarity: that is when the flow breaks into eddies break into smaller eddies and so on. Such scaling is universal; it is observed in all chaotic systems.

A particularly simple case is that of systems that obey the logistic map introduced in connection with population growth. Designate by X_j the number of members of a group may be the population on an island, the bacteria in a colony, etc. The index j labels a population on an interval (such as a day or a year) or the successive "generations" of the population. If the reproduction rate in one generation is λ, then it would hold that

$$x_{j+1} = \lambda_{x_i} \qquad (1)$$

However the population will also decrease due to deaths.In particular if the food supply on the island is finite the death rate will be proportional to x^2_j. Thus the evolution[1] is governed by the map

$$x_{j+1} = \lambda_{x_i} - sx_j^2 \qquad (2)$$

We use the term map, because given x_j we can find x_{j+1} uniquely .Both λ and s are assumed nonnegative. We see immediately that if λ>1 and s=0 the population will grow exponentially, while if λ<1 the population will tend to 0. The map of Eq. (2) can be rescaled by introducing

$$y_j = \frac{sx_j}{\lambda} \qquad \text{for all j.}$$

Then y_j obeys the logistic map

$$y_{j+1} = \lambda y_j(1 - y_j) \tag{3}$$

The above map has the interesting property that if the reproduction rate for one generation is restricted in the range

$$0 < \lambda < 4,$$

Then y_j remains bounded between

$$0 < y_j < 1.$$

We are interested in the fate of the group after many generations, namely in the value of y_j as $j \rightarrow \infty$. We find, as already stated, that:

If $\lambda \leq 1$, as $j \rightarrow \infty$ $y_j \rightarrow 0$ the population decays to 0.

If $1 < \lambda < 3$, as $j \rightarrow \infty$ $y_j \rightarrow y \rightarrow y^*$ the population tends to a stable point y*, namely.

$$y* = \lambda y * (1 - y *) \tag{4}$$

With solutions

$$y^*=0, \qquad y^* = (1 - \tfrac{1}{\lambda})$$

In this case the solution y* = 0 is unstable, because if $y_0 = \varepsilon$

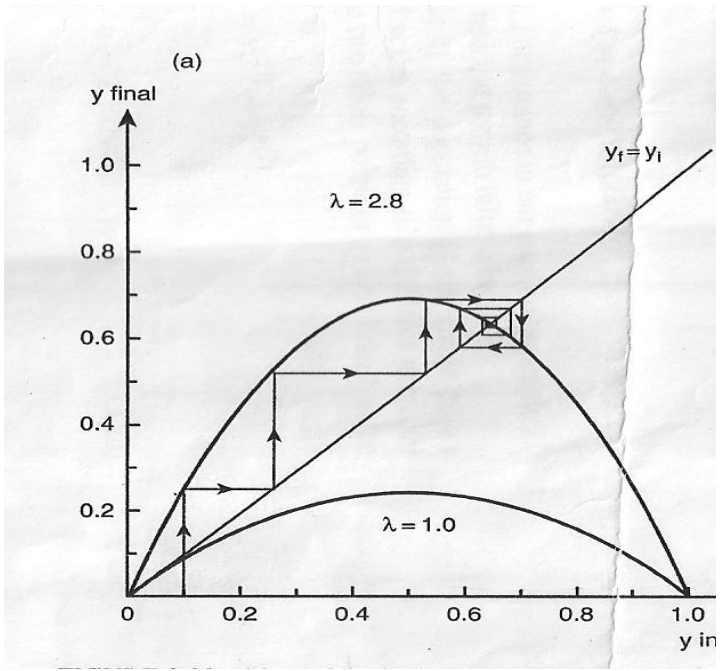
(ε infinitesimal) y_∞ will tend to (1- 1/λ).

When λ > 3 the system behaves in a very different manner. As soon as λ> 3 but λ< 3.4495…. the population alternate between 2 stable values. When λ> 3.4495…. the population alternates between 4 stable values until λ > 3.54…., where it alternates between 8 stable values; for λ > 3.56…. the population alternates between 16 stable values, and this continues at ever more closely spaced intervals of λ. We say that there is a bifurcation[2] at these specific values of λ. These results can be easily checked with a pocket calculator or a simple program. Table 1 gives some typical result for $\lambda = 2.8$, $\lambda = 3.2$, and $\lambda = 3.5$, and the stable points are shown in the graphical construction of Fig.1.

What is plotted in Fig.1. is y_{final} vs y_{inital}. The continuous curve is the equation of the logistic map $y_f = \lambda y_i (1-y_i)$. In Fig. 1a the cures

Table 1: Example of stable point of the Logistic Map

$\lambda = 2.8$	$y^* = 0.6429\ldots$..
$\lambda = 3.2$	$y^* = 0.5310\ldots$..
	$y^* = 0.7995\ldots$..
$\lambda = 3.5$	$y^* = 0.3828\ldots$.
	$= 0.5009\ldots$..
	$= 0.8269\ldots$..
	$= 0.8750\ldots$..



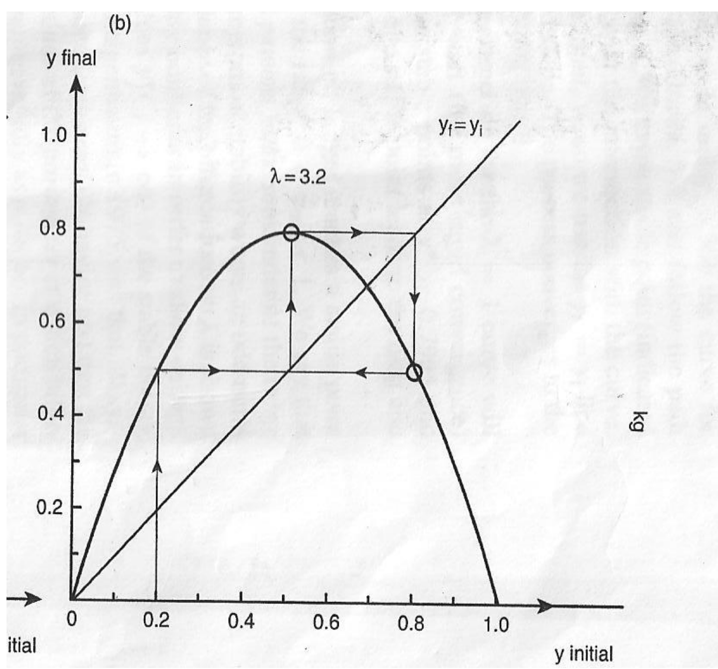


Figure1: Plot of the logistic map: for (a)λ=1.0, λ=2.8; λ=2.8 there is one stable point at y*=0.6429……..(b) for λ=3.2; there are two stable points at y*=0.7995….and y*=0.5130. see the text for details of the path leading to the stable points.

For λ =2.8 and λ =1.0 are shown, while in Fig.1b the curve for λ =3.2.The lines for $y_f = y_i$ are also drawn. We can follow the path from some initial value y_0 =0.1 in Fig.1a to the stable point (indicated by a circle). Given y_o we find $y_1 = y_f$ at the intersection with the curve. However, y_1

must now be used as an input, y_i, so we use the $y_f = y_i$ line to locate y_i and proceed to find y_2 and so on. The process converges to the circled point at $y^* = 0.6429$…..

It is also evident that the same construction for the $\lambda = 1$ curve will lead to $y^* = 0.0$. In Fig.1b we now find the two stable points at $y^* = 0.7995$ and $y^* = 0.5130$. The map requires that one stable point leads to the next and vice versa.

When $\lambda > 3.5699$… the population no longer reaches a stable point but takes on an infinity of values in the range $0 < y_\infty < 1$. We say that the system behaves chaotically. This persists in the remainder of the range $3.5699.... < \lambda < 4.0$, but one finds regions of stability where an odd number of stable points exist. The dependence of the bifurcations on λ is shown in Fig. 2. Where the λ –scale is highly nonlinear in order to shown enough detail; the vertical scale given values y_j^* ($j \rightarrow \infty$) of the stable point.

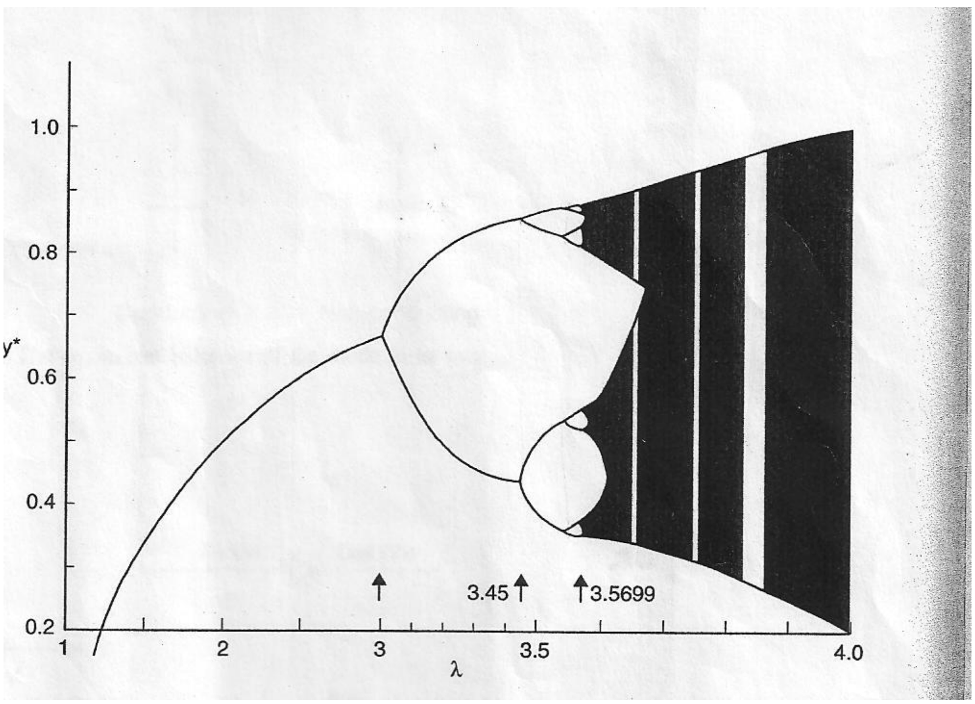


Fig.2. The stable points of the logistic map as a function of λ . The λ scale is highly nonlinear in order to clearly show the bifurcations. The black parts of the plot indicate the chaotic region. Note, however the thin white lines, which indicate islands of stability.

The remarkable discovery by M. Feigenbaum in 1975 was that all systems that exhibits chaos follow the same(universal) that the deference $\Delta_n = \lambda_{n+1} - \lambda_n$ of the values of the parameter at witch bifurcations(period doubling) occur converges rapidly as $n \to \infty$. In particular as $n \to \infty$ the ratio,

$$\frac{\lambda_{n+1}-\lambda_n}{\lambda_{n+2}-\lambda_{n+1}} = \delta = 4.669201660910 \ldots \qquad (5)$$

is a universal constant.

Also, the amplitude at the stable point (while bifurcating) exhibits universal behavior. If $y_n^{(1)}$ and $y_n^{(2)}$ are the two stable point of a given branch at the bifurcation value λ_n ,and

$$\Delta y^*_n = y^*_n{}^{(1)} - y^*_n{}^{(2)}$$

Then,

$$\lim_{n\to\infty} \Delta y^*_n / \Delta y_{n+1} \to \alpha = 2.5029078\ldots\ldots \qquad (6)$$

Here, δ and α are universal constant and are called Feigenbaum constant.
This indicates that as λ increases the system replicates itself after rescaling by a factor $1/\alpha$, as shown in Fig.2; typical intervals[3]. Δy^*_2 and Δy^*_3 are indicated.
In this experiment, we study the chaotic behavior in a diode-R-L circuit, driven at its resonant frequency. Here the diode is a non-linear device; hence bifurcation and chaos are natural for this system. The effect was first reported by Linsay[4] and was analyzed in details by Rollins and Hunts[5]. The circuit is shown below.

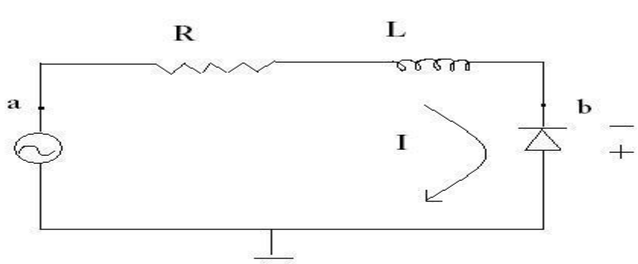


Fig.3: The diode-R-L CIRCUIT

Here , the source is assumed to be sinusoidal of amplitude V_0 ,hence,

$$V_a = V_0 \, Cos\omega t$$

Then, for half the cycle, there is appositive voltage across the diode, during it conduct and appears as an EMF.

$$I(t) = \left(\frac{V_0}{\sqrt{R^2 + L^2\omega^2}}\right)\cos(\omega t - \theta_a) + Ae^{\left(-\frac{R}{L}\right)t} + \frac{V_f}{R}$$

$$V_b(t) = -V_f$$

During the other half, the diode dose not conduct draws a charging current, and the voltage at b follows the frequency of the source. It can be calculated by

$$I(t) = \frac{V_0}{\sqrt{R^2 + \frac{L^2(\omega^2 - \omega_0^2)^2}{w^2\cos(\omega t - \theta_b) + Be^{-\left(\frac{2R}{L}\right)t}\cos(\omega_b t + \varphi)}}}$$

$$V_b(t) = V_0 cos\omega t - I(t)R - L\left(\frac{dI}{dt}\right)$$

With $\quad \omega_0 = \frac{1}{\sqrt{LC}} \qquad \omega_b^2 = \omega_0^2 - \left(\frac{R}{2L}\right)^2$

and A, B, φ are constants.

The behavior of current and voltage are shown in Fig.4.

Clearly, the amplitude of V_b will change with every cycle.

Then depending on $\lambda = V_0 / V_F$, V_b repeats with a period T_0,$2T_0$,$4T_0$ and so on fill it becomes chaotic.

Observation:

To perform the experiment you should use a 1N 40007 diode , a inductance L=……..mH and a resistance R=…………….Ω ,The source frequency F=………..kHz.

As you increase the V_0 slowly, you can observe the bifurcation (in V_b) by an oscilloscope. The Y-t and the X-Y traces of the bifurcations should be attach and the data obtained is given as following.

Bifurcation	V_0(in V)	V_b(inV)	λ_n

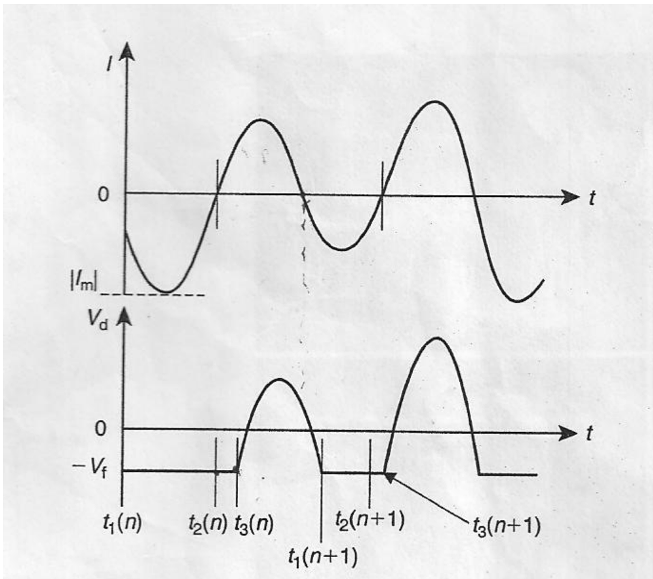


Fig.4: The current and voltage in the diode R-L circuit shown as a function of time.

Experimental Results

The circuit is set up as shown in Fig. 3. A function generator is used to drive the circuit. A fairly hefty variable inductance (L=10 mH) is used since the diode capacity is small. The series resistance was R≅ 50 ohm. The diode should not be too slow (such as are rectifier diodes) nor too fast. Good results were obtained with a 1N4007 diode; other diodes, namely 1N4001 and 1N5404, gave qualitatively similar (but quantitatively different) results. The first step is to tune the inductor to find the resonant frequency of the circuit. In this case it was found that $\omega_0/2\pi$ = 71.5 kHz ≅ $1/(2\pi\sqrt{LC})$. In Figs.5a-5d are shown the voltage across the diode V_b and the driving voltage V_0. For $V_0 < 0.875$V, V_b has the same periodicity as V_0. However just above V_0= 0.875 V , V_b alternates between two different values as shown in Fig. 5a. The effect is clear, but not very pronounced, because the data have been taken only slightly above the first bifurcation. Fig. 5b corresponds to V_0= 2.033V where the second bifurcation sets in. The period of V_b is now four times that of V_0. Again the difference between the two high level states is very small and that between the two low level states is not observable. The next scope traces, Fig. 5c, correspond to

V_0 = 2.280V and were taken right after the third bifurcation. The period of V_b is now eight times that of V_0 and similar comments apply as to the distinguishability of the different states. A fourth bifurcation was observed at V_0 = 2.340 V. Finally Fig.5 d shows V_b when $V_0 \geq$ 2.355 V where chaos was observed to set in.

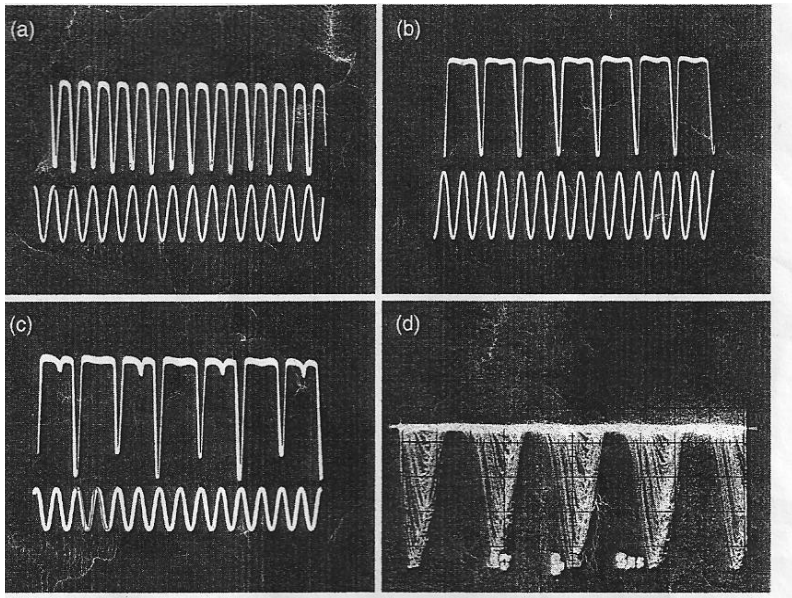


Fig. 5: Oscilloscope traces of the voltage, V_b, across the diode (upper trace) and of the driving voltage V_0 (lower trace) . The driving frequency is 71.5 kHz. (a) Immediately after the first bifurcation. Note that the upper trace is bimodal and has period 2 T_0. (b) Immediately after the second bifurcation. Note that the large peaks are bimodal; the period is 4 T_0. (c) Immediately after the third bifurcation; the period is now 8 T_0. (d) Chaotic behavior.

A plot of the bifurcations obtained for this diode is shown in Fig.6. The error in determining the exact bifurcation voltage[6] is + 5mV. We summarize the results in Table 2.

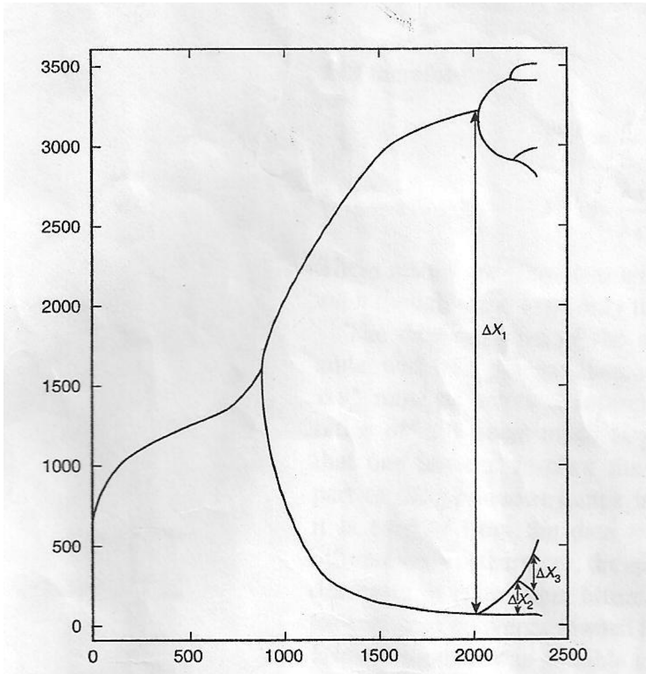


Fig.6: Plot of V_b Vs V_0 as measured for the 1N4007 diode . The bifurcations are clearly observed. Some ΔV_b spacing's are also indicated. Chaos sets in at $V_0 = 2.355$ V.

Table 2: Bifurcation Data from Measurements of Chaos

Bifurcation	V_0(mV)
1st	875
2nd	2033
3rd	2280
4th	2340
Chaos	2355

From these data we calculate the Feigenbaum number δ. We have,

$\lambda_2 - \lambda_1 = 1158 \pm 7$ mV

$\lambda_3 - \lambda_2 = 247 \pm 7$mV

$\lambda_4 - \lambda_3 = 60 \pm 7$mV

and therefore

$$\delta^{(1)} = \frac{\lambda_2 - \lambda_1}{\lambda_3 - \lambda_2} = 4.688 \pm 0.13$$

$$\delta^{(2)} = \frac{\lambda_3 - \lambda_2}{\lambda_4 - \lambda_3} = 4.117 \pm 0.49$$

These results are consistent with the asymptotic value given in eq. (5). Even though input from only the first four bifurcations was used.

The determination of the second Feigenbaum number α is not possible with the present data. As pointed out previously, the intervals Δy_n^* must be selected appropriately, but even then (see Fig.6) the ratios of Δy_n^* seem much larger than α . This is due in part to the fact that one has not reached the asymptotic regime of Eq. (6) and in part to discontinuous jumps in V_b at certain values[7] of V_0 . However, it is evident from the data that the system replicates itself after each bifurcation. Furthermore, the spacing between stable points in every branch decreases in subsequent bifurcations by a multiplicative factor ; this factor seems to converge toward feigenbaum's α. We also note that for the 1N4001 diode it was possible to observe islands of stability in the chaotic region.

Notes and References:

[1] The first study of these issues is due to the English sociologist T. R. Malthus (1766-1834)

[2] Henri Poincare in 1900 had noticed such behavior in mechanical systems and named it the "exchange of stability."

[3]The intervals Δy_n^*must be chosen appropriately as is also evident from Fig.2.

[4]P.S. Linsay , Phys. Rev. Lett. 47, 1349(1981).

[5] R. W. Rollins and E.R. Hunt , Phys. Rev. Lett. 49 , 1295 (1982); R.W. Rollins and E. R. Hunt , Phys. Rev. A29, 3327 (1984).

[6] A more precise determination of the voltage at which bifurcation occurs can be made when a signal analyzer (FFT) is available. In this case the onset of period doubling is evident from the appearance of sub harmonics in the frequency spectrum.

[7]Some diodes show marked hysteresis associated with these discontinuities.

[8]N. B. Tufillaro, T. Abbott and J. Reilly, An Experimental Approach to Nonlinear Dynamics and Chaos, Addison-Wesley, New York, 1992.

[9]M. Hasler, "Electrical circuits with chaotic behaviour," Proceedings of IEEE, Vol. 75, No. 8, pp. 1009–1021, August 1987.

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[11]C. W. Wu and N. F. Rulko, "Studying chaos via 1-d maps: a tutorial" IEEE

Trans. Circ. Syst. I, Vol. 40, No. 10, pp. 707–721, October 1993.
[12]M. J. Ogorzalek, Chaos and Complexity in Nonlinear Electronic Circuits, World Scientific Series on Nonlinear Science, Vol. 22, 1997.

7. Electronic Oscillator Circuits

Objective: - To study electronic *RC* oscillators using IC741 (OpAmp), IC311 (Schmidt Trigger), IC555 (Timer) and IC7400 (CMOS inverters).

Equipments: IC741 (OpAmp), IC311 (Schmidt Trigger), IC7400 (CMOS inverters), IC555 (Timer), Digital CRO, Function generator, ±15V DC power supply, resistors and capacitors

Introduction:

Oscillators are electronic circuits that generate a square wave (or any other desired waveform) pattern without any input signal. Square wave pattern at the output means that the circuit behaves like an astable multivibrator where both the output states are not stable. Such circuit use both positive and negative feedback to achieve this. In this lab, we are going to discuss oscillators (square wave pattern generator) using oscillators using IC741 (OpAmp), IC311 (Schmidt Trigger), IC555 (Timer) and IC7400 (CMOS inverters). In all four types of circuits, the output swings between two voltage states are decided by the power supply or the user.

Procedure:

(i) OpAmp RC oscillator

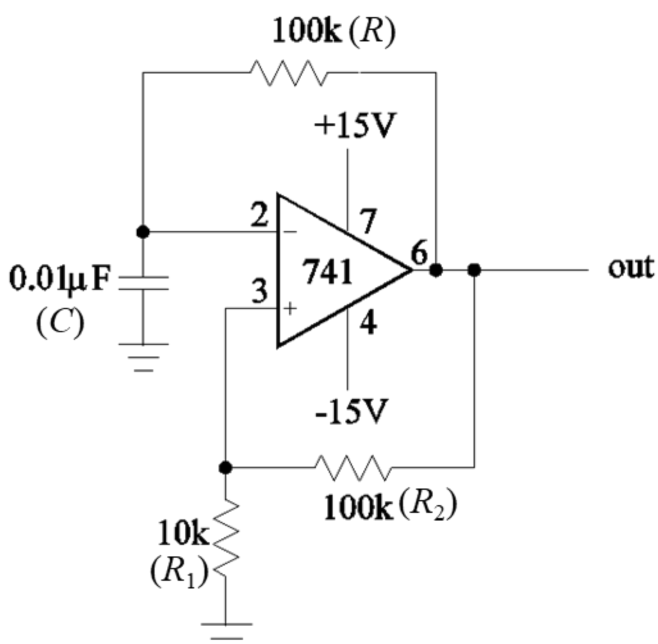


Figure 1: OpAmp based RC Oscillator involving both positive and negative feedback circuits.

Now we make a circuit which incorporates both positive and negative feedback simultaneously. This is the RC oscillator shown in Figure 1. This circuit has no external input but has a square wave output. The circuit output will be usually in saturation at about ±15V and so the voltage at the NI input will be about ±1.3V (verify this). In this circuit the voltage at the inverting input will always be trying to catch-up the voltage at the NI-input. The moment the inverting input voltage reaches the NI-input voltage the output voltage will switch from one saturation value to the other one and hence the NI-input voltage also switches. The inverting input again tries to chase the NI-input voltage and it goes on and on and we have a square wave at the output of the circuit.

$$T = 2RC \ln\left(\frac{2R_1 + R_2}{R_2}\right)$$

Predict what should be the frequency of oscillation. Measure the output frequency. How good is the agreement with the prediction? Now replace the 10k resistor by a 1k resistor and watch the waveform at the capacitor. It should be a triangle wave. Explain this observation.

(ii) Schmitt trigger RC oscillator

Make the Schmitt trigger circuit as shown in Figure 2 to work as an RC oscillator. Note that the pin 1 is now connected to a -15V supply. This circuit has no external input. We also have both positive and negative feedback in this circuit. Predict what should be the frequency of oscillation.

$$T = 2RC \ln\left(\frac{2R_1 + R_2}{R_2}\right)$$

Measure the output frequency. How good is the agreement? Now replace the 10k resistor by a 1k resistor and watch the waveform at the capacitor. It should be a triangle wave.

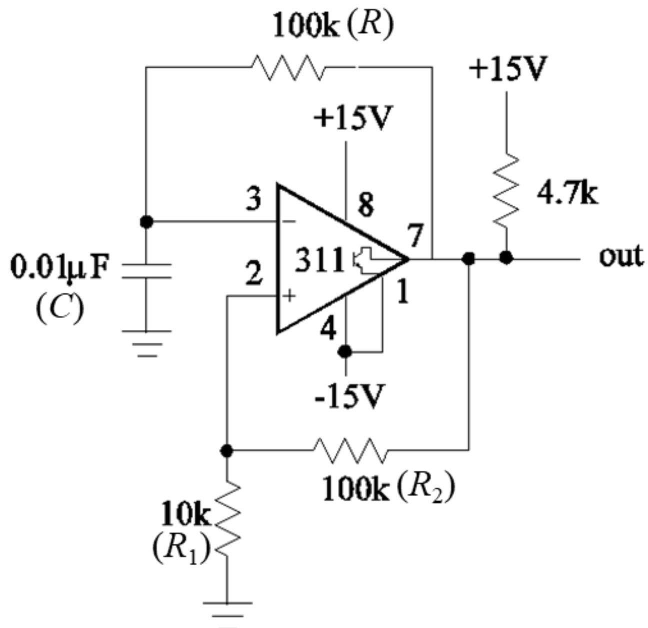


Figure 2: Schmitt Trigger (IC 311) based RC Oscillator involving both positive and negative feedback circuits.

(iii) CMOS inverter relaxation oscillator

This experiment is to be done with two CMOS inverters (NOT gates). If you do not have isolated CMOS inverters, then you can make them easily with IC 74HC00 quad two- input NANDs. Tie the two inputs of a NAND gate to make a NOT gate. A supply voltage in the range 4 to 5 volts is very safe for the IC. Here we have taken the 74HC series because of its higher output drive capability which is important in this experiment.

$$T = 2.2\, RC$$

Make the square wave oscillator shown in the Fig. 3. Choose R=10k and C=0.1μF. Here R_1 should be a large resistor (say at least 10 times R). The frequency of the oscillator is about 1/RC. Verify that this is the case by watching the output of the circuit on your scope.

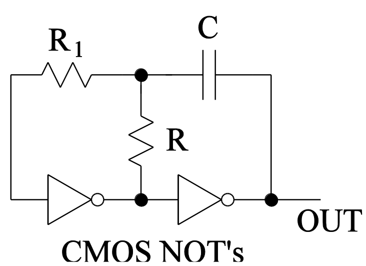


Figure 3: RC relaxation oscillator using CMOS inverters (NOT gates)

(iv) Timer IC 555 oscillator

Construct the 555 square wave oscillator shown in Figure 4. What is the frequency and duty cycle of the oscillator and compare it with theoretical values? Look at the output waveform and the waveform on the capacitor simultaneously. What is the swing for the voltage on the capacitor? Now replace the resistor between pins 6 and 7 by a short. Predict what you would see at the capacitor and the output. Check whether what you see agrees with the prediction.

Time period of the waveform:

$$\boldsymbol{T = \frac{(R_A + 2R_B)C}{1.44}}$$

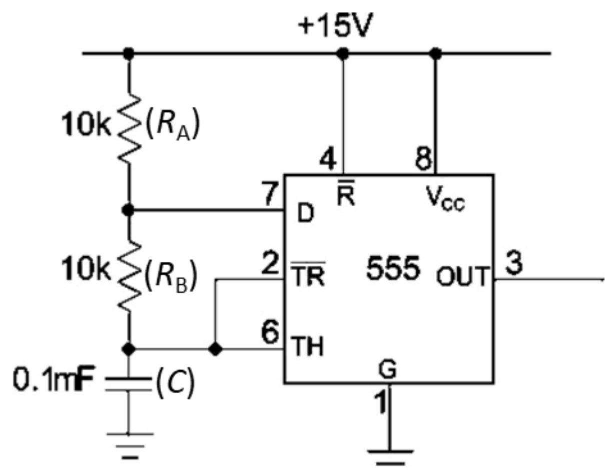
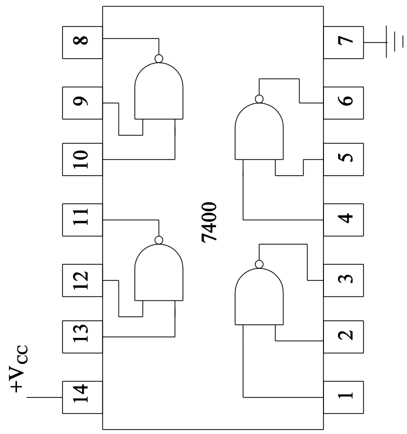


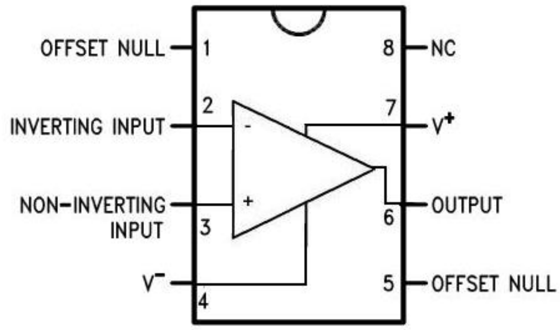
Figure 4: Square wave oscillator using the timer IC 555

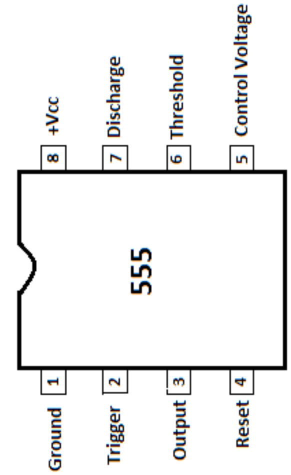
References:

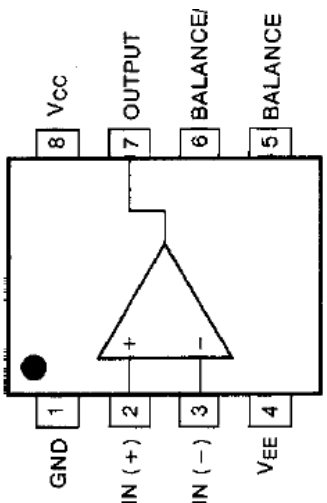
1. Microelectronic Circuits (The Oxford Series in Electrical and Computer Engineering), Seventh edition, Adel S. Sedra and Kenneth C. Smith
2. The Art of Electronics (Cambridge University Press), 2nd Edition, Paul Horowitz and Winfield Hill
3. Student Manual for The Art of Electronics (Cambridge University Press, Thomas C. Hayes and Paul Horowitz

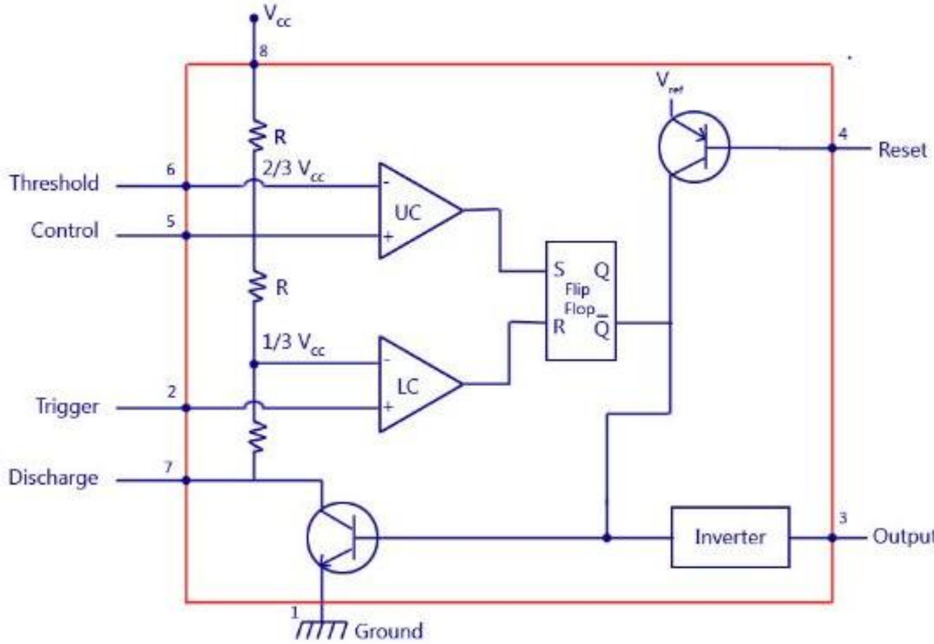
Pin diagrams of the ICs used in the experiment:











8. Speed of Light Experiment

Objective: Measuring the speed of light by the modulated laser method.

Equipments:

Diode Laser with power adapter, Component Carrier (2), Lens,+127mm(1), Laser Alignment Bench(1), Light Receiver(1), Stainless steel pads(4), Cable, coaxial, RCA male to BNC male(1),Cable, coaxial, 3.5mm phone plug to BNC male(10), Cable, coaxial, BNC to male(1), Concave Mirror Assemble(1.) and Function Generator (wide range), Oscilloscope (60MHz), Tape measure.

Introduction:

The velocity of light in free space is an important and intriguing constant of nature. Whether the light comes from a laser on a desktop or from a star that is hurling away at fantastic speeds, the velocity of light will yield the same constant value. In more precise terminology, the velocity of light is independent of the relative velocities of the light source and the observer.

As Einstein first presented in his Special Theory of Relativity, the speed of light is critically important in some surprising ways:

1. The velocity of light establishes an upper limit to the velocity that may be imparted to any object.
2. Objects moving near the velocity of light follow a set of physical laws drastically, not only from Newton's Lows, but from the basic assumptions of human intuition.

It is not surprising that a great deal of time and effort has been invested in measuring the speed of light. Some of the most accurate measurements were made by Albert Michelson between 1926 and 1929. Michelson measured the velocity of light in air to be 2.99712 x 10^8 m/sec. from this result; he deduced the velocity in free space tube 2.99796 x 10^8 m/sec.

Equipment Setup:

1. Stick four stainless steel strips to the front of the receiver.
2. Mount the laser on its L-shaped bracket with the bracket bent away from the laser.
3. Arrange the laser, lens, receiver and component carriers on the laser alignment bench.
4. Place the alignment bench on a horizontal surface. You will need 10 to 20m of space in the laser.
5. Mount the mirror on the tripod and it a few meters in the front of the laser.
6. Mark the position of the mirror on the floor with tape. (Attach a plumb bob to the tripod, so that you mark a point directly below the mirror.

7. With tape, mark the floor at regular intervals to about 10-20 meters from the laser. Allow for at least 10 different intervals within the allotted space.
8. Using the BNC male –to-male cable, connect the TTL output of the function generator to channel 1 of the oscilloscope.
9. Using the phone plug-to-BNC male cable, connect the power jack of the laser to the output of the function generator.
10. Using the RCA male-to-BNC male cable, connect the "video" output of the receiver to channel 2 of the oscilloscope.
11. Set the function generator for a square wave, and press the dc offset button. Turn the output and DC offset knobs completely counterclockwise.
12. Turn the laser switch to the "on" position.
13. On the function generator, turn up the DC offset knob until you see laser light. **Do not look directly into the laser light!**
14. Align the laser, mirror, lens and receiver so that the laser is focused onto the "video" sensing element of the receiver.
15. Set the scope to dual trace:

a) Set channel 1 to 1v/div., DC.
b) Set channel 2 to1 v/div., AC.
c) Set the trigger to channel 1.
d) Set the trigger level to about 2.5 volts.
e) Set the time base to 50 ns/div.

16. Adjust the alignment of the laser, mirror, lens and receiver to maximize the sine wave signal on channel 2
17. Adjust the DC offset and amplitude of the function generator to maximize the signal channel 2.

Diode laser- The diode laser emits an intense, narrowly-focused beam of light. In this experiment, the diode laser is powered by a function generator, which modulates the light intensity at approximately 3MHz. the laser is equipped with adjustment screws for precisely aiming the light at the mirror.

Concave mirror-The concave surface of the mirror help to focus the light as it is reflected. The mirror is also equipped with adjustment screws for aiming the light back to the receiver.

Light Receiver-The receiver is designed for receiving audio and video signal transmitted via modulated light. Since the light receiver is sensitive to very high-frequency modulation, it is ideally suited to the experiment, in this manual. There are two sensitive element on the receiver, in this experiment, you will use only will use only the one labeled "Video."

+127mm lens- the lens is used to focus the light onto the sensitive element of receiver.

Experiment 1: Modulated laser method for measuring the speed of light: In this experiment, you will measure the speed of light using a laser modulated at a very high frequency and an oscilloscope. You will measure the time, Δt that elapses while the light signal travels a known distance, Δd, and you will calculate the speed of light, which is defined as Δd/Δt.

The light signal, originating at the laser, will travel to mirror and back to the light receiver. You will vary Δd by moving the mirror and measuring the corresponding effect on Δt with the oscilloscope.

Δt changes as Δd varies.

Therefore, you will actually measure an elapsed time Δt′ relative to an arbitrary (but constant) baseline. This elapsed time can be expressed mathematically as:

$$\Delta t^{'} = \Delta t + t_k \quad (1)$$

Where, tk is an unknown constant. For the same reason, you can also measure d' instead of Δd where

$$\Delta d^{'} = \Delta d + d_k \text{ d} \quad (2)$$

The equation of a line fitted to a plot of Δd' vs. Δt'

$$\Delta d^{'} = c\Delta t' \quad (3)$$

Where, **c** represents the slope of the line. The combinations of equation 1, 2, and3 yields

$$\Delta d = c\, \Delta t + K \quad (4)$$

Where, K is another arbitrary constant. In equation 4, it is evident that the slope c, equals Δd/Δt, which is the speed of light.

Procedure:

1. Adjust the alignments of the laser and mirror and the positions of the lens and receiver to maximize the signal. (Adjust the receiver up, down, left, and right on the carrier, but do not change the position of the carrier on the bench.)
2. On the oscilloscope, adjust the scale and vertical position of the signal to maximize the signal trace. Do not change the horizontal position of the trace.

3. Record the position of mirror (relative to its initial position) and the phase of the signal in Table 1. If your oscilloscope is equipped with cursors, use them to measure the phase. Otherwise, estimate the phase to ½ of smallest division on the time scale.
4. Move the mirror back to the next mark and repeat steps 1 through 4.

Table 1

Mirror Position(m)	Phase(s)

Analysis:

Plot $\Delta t'$ vs. $\Delta d'$ (Remember that $\Delta d'$ is two times the mirror position. The slope of the best-fit line is the speed of light.

Note: You can plot your data and obtain the best-fit line using Data Studio. For instructions, see Appendix B of this manual.

References:

1. A. A. Tyapkin, Lett. Nuovo Cimento, 7, 15, pp. 760-4, (1973).
2. H. Lorentz, Lectures on Theoretical Physics, V.1 pp. 14-19 (1927).
3. J. Larmor, Aether and Matter, Cambridge University Press (1900).
4. A. Michelson, E. Morley, Am. J. Sci, 34, 203, pp. 333–345 (1887).
5. M. Ruderfer, Phys. Rev. Lett. , 5, 5, pp.191-2 (1960).
6. K. Turner, H. Hill, Bull Am. Phys. Soc., 8, pp. 28 (1963).
7. D. Champeney, et. al., Phys. Lett., 7,4 ,pp.241-3 (1963).
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Appendix A: Specifications

Component	Description
Diode Laser	1 mw, 650 nm
Component Carrier	6.3 cm (height) x 7.6 cm (length); 3.8 cm x 7.5 cm (base)
Lens	127 mm
Laser Alighment Bench	38.8 cm length
Metrologic Light Receiver	10.2 cm x 6.2 cm x 4.2 cm; two channel output (one audio, one video), dual light sensors, 9 VDC
Stainless steel pads	0.97 x 4.5 cm, rectangular, stainless steel
Cable, coaxial, RCA male to BNC male	183 +/- 2.5 cm length
Cable, coaxial, 3.5 mm phone plug to BNC male	183 +/- 2.5 cm length
Cable, coaxial, BNC to male	183 +/- 2.5 cm length
Concave Mirror Assembly	13.5 meters

Appendix B: Plotting Data in DataStudio

The following instructions are provided for new users or those unfamiliar with DataStudio. The following instructions explain how to create an x-y graph and/or calculate statistics on previously collected data.

1. Open DataStudio. When the Welcome to DataStudio window opens, select "Enter Data." An editable table and Graph display open.

2. On the main toolbar, click the **Summary** button.

3. In the Data list, double click on the data icon to open the Data Properties dialog.

4. In the Data Properties dialog, do the following:

a) Enter a name for your experiment in the **Name** box.

b) Click on the **X tab** and enter the variable name "Phase" to label the x axis. Enter the units for phase (ns or s).

c) Click on the **Y tab** and enter the variable name "Path Length" to label the y axis. Enter the units for length (m). Click **OK**.

5. In the Editable data table, enter your values for phase and path length. The data values automatically plot in the graph to the right.

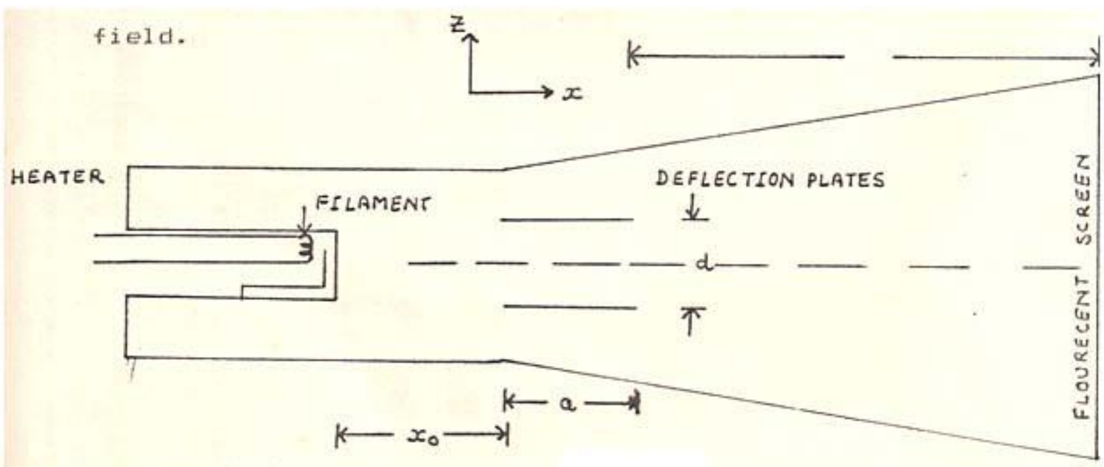
6. On the graph toolbar, click the **Curve Fit** button and select "Linear Fit." Slope, intercept, correlation, and standard deviation values appear in a box on your display.

9. e/m by Bar Magnet Experiment

Objective: Determination of e/m of the electron using cathode ray tube and bar magnets.

Apparatus: Cathode ray tube, power supply, deflection Magnetometer, two bar magnet, wooden stand with scale, voltmeter (0-100V).

Introduction: The cathode ray produces a beam of electrons of uniform velocity (along, say x direction), the magnitude of which depends on the accelerating potential by varying the plate voltage and grid voltage of the tube, one can change the intensity of the beam and focus the beam on the fluorescent screen. The beam deflects if it is subjected to an electric or magnetic field. Electric field is provided by applying voltage to a pair of deflecting plates in the tube (along Z direction). Details are given at the end of experiment. A pair of bar magnets placed on the arms of the wooden stand (y direction) causes the magnetic field.

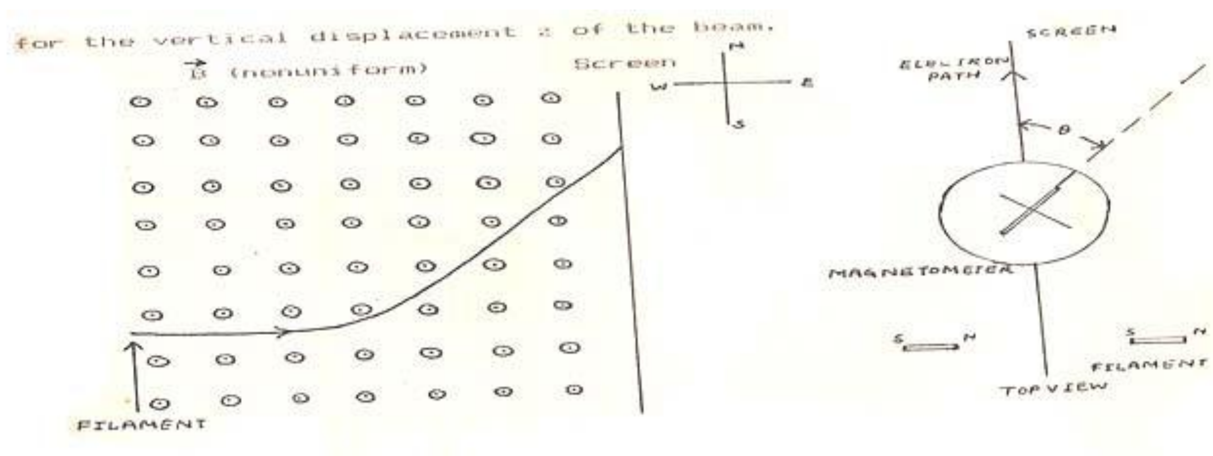


Procedure: Align the axis of the CRT along the magnetic meridian so that the earth's magnetic field does not cause a force on the beam of electrons. This is done by setting the arms of the power supply and focus the beam on the screen. Apply a deflecting voltage to obtain a vertical deflecting of about 1 cm. Note the deflection and measure the deflecting potential by a voltmeter connected across the terminals provided in the supply. Since the deflecting field acts only for a/v sec. (a= length of the deflecting plates and v= horizontal (x-direction) velocity of the electron) the vertical velocity of the beam at the edge of deflecting plates is (eV /md) (a/V) (V is the deflecting potential and d is the spacing of the deflecting plates). This is because eV/md is nothing but the vertical displacement caused after crossing the edge of the deflecting plates is simply vertical velocity multiplied by L/V. thus, the net displacement on the screen is,

$$Z = \frac{eV}{md}\, a\, (L + \frac{a}{2})\, \frac{1}{V^2} \qquad (1)$$

Where, L is the distance between the deflecting plates and the screen. Note the values of V and z, and switch off the deflecting electric field.

Next, place the bar magnets on the arms at such positions that the same vertical displacement is reproduced. In order to calculate the deflection due to magnetic field, one should measure the magnetic field using a deflection magnetometer. Note that the magnetic field due to bar magnets exerts a force on the electron all along the path and is not uniform across the length of the beam axis. The field B is measured all along the beam axis using a deflection magnetometer: $B = B_{earth}$ tan is the angle the magnetic needle makes the earth's field (fig 3). A plot of B vs x is made and this field distribution is responsible for the vertical displacement z of the beam.



The curvature of the trajectory due to B is given by

$$\frac{1}{R} = \frac{e}{m}\frac{B}{V} \qquad (2)$$

Since $\frac{1}{R} = \frac{d^2z}{dx^2}$ integrating twice,

$$Z = \frac{e}{m}\frac{1}{V}\int_0^{x_0+a+L} dx \int_0^{x} dx' B(x') \qquad (3)$$

Where x refers to the length from the filament to the edge of the deflection plate (see fig.2.1). evaluate the double integral graphically (How). Let its value be I, eliminating v in eqs .(1) and (3). We get

$$\frac{e}{m} = Z\frac{V}{d}a(L+\frac{a}{2})\frac{1}{I^2} \qquad (4)$$

Repeat the steps for several deflecting voltages (both +ve and –ve)

Useful Data:

Characteristic of CRT-**8SJ31J**

Spacing of Ist deflecting plate ((17.8+1.92)/2) =9.86 mm

Length of Ist deflecting plate = 30.0 mm

Spacing of IInd deflecting plate = ((11.18+1.92)/2)=6.6 mm

Length of of IInd deflecting plate= 30.0 mm

Distance of the filament from the edge of the Ist deflecting plate = 49.30 mm

Distance between filament and the edge of the IInd deflecting plate = 85.00 mm

Distance between filament to screen =195 mm

Horizontal component of Earth's magnetic field at Kanpur =0.363 Gauss

References:

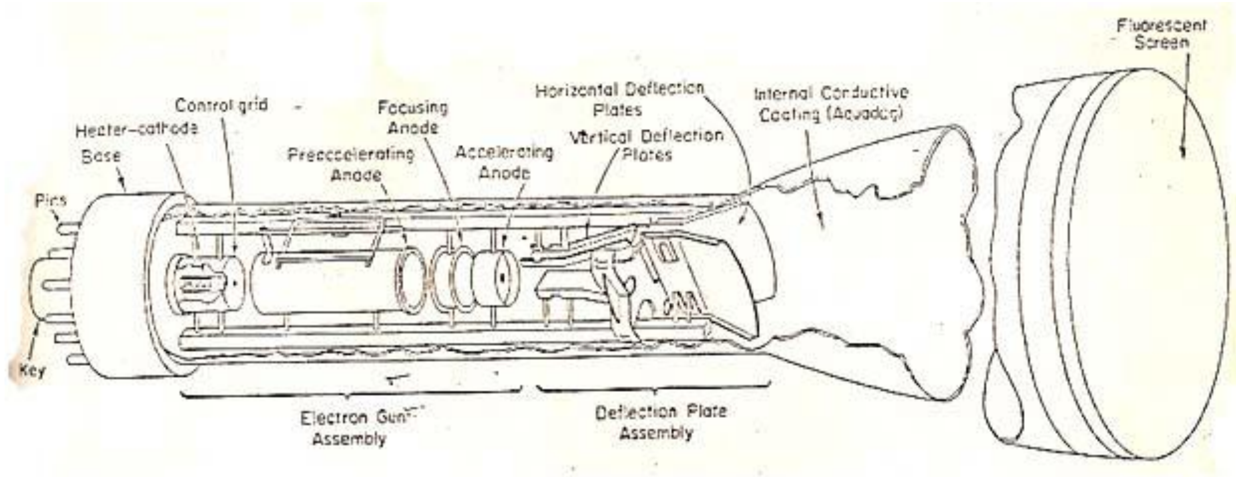
1. B.L. Worsnop and H.L.Flint," Advanced Practical Physics for students" , page 669.

<u>CATHODE RAY TUBE (CRT)</u>

A schematic view of a cathode ray tube is shown in fig.2.4

The main components of a general purpose CRT are

a. Electron gun assembly
b. Deflection plate assembly
c. Fluorescent screen
d. Glass envelope and base of the tube



Electron gun assembly:

The name electron gun is derived from the analogy between the motion of an electron emitted from the CRT gun structure and the travel path of a bullet fired from a gun. Electron gun assembly produces a narrow and sharply focused beam of electrons that leaves that gun with a very high velocity and travels toward the fluorescent screen.

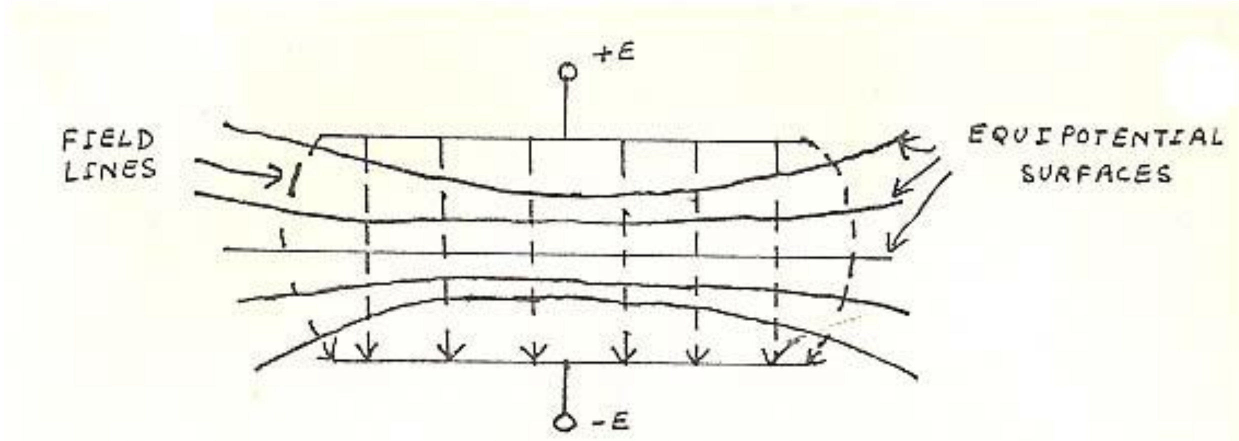
The electrons are emitted from an indirectly heated thermionic cathode. The cathode is completely surrounded by a control grid, which consists of nickel cylinder with a small centrally located hole, coaxial with the tube axis. The electrons that manage to pass through this small hole grid together make up the so called beam current.

The magnitude of the beam current can be adjusted by varying the negative voltage (bias) of the control grid with respect to cathode. An increase in control grid bias reduces the beam current,

and hence the intensity of the CRT image, while a decrease in grid bias increases the beam current. This action is identical to that of the control grid in a conventional vacuum control marked INTENSITY, this knob essentially varies the negative voltage (bias) of the control grid with respect to the cathode.

The electrons emitted by the cathode and passing through the small hole in the control grid are accelerated by the high positive potential applied to two accelerating anodes. These two anodes are separated by a focusing anode, which provides a method for focusing the electrons into a narrow and sharply defined beam, to be discussed later.

Electrostatic deflection method is used for deflecting the stream of electrons from the electron gun. In fig 2.5 we show a hypothetical electron situated at rest in an electric field.



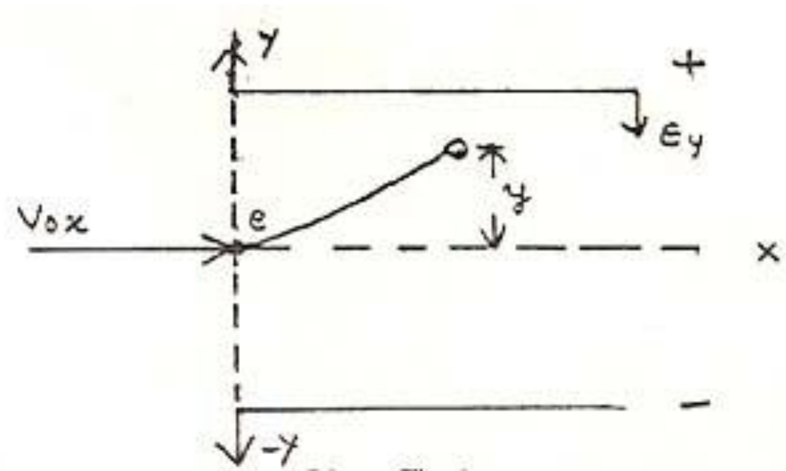
By definition of the electric field intensity, the force on the electron is f = -e Newton. This force will accelerate the electron in the direction of the positive electrode, along the lines of the field flux. The acceleration, acan be calculated using Newton's second low of motion,

$$F = ma$$

$$a = \frac{F}{m} = -\frac{eE}{m} \quad \text{m/s}^2 \qquad (5)$$

Where, m is the mass of the electron, in kg.

Consider now an electric field of constant intensity with the lines of force pointing in the negative Y direction as shown as in fig 2.6



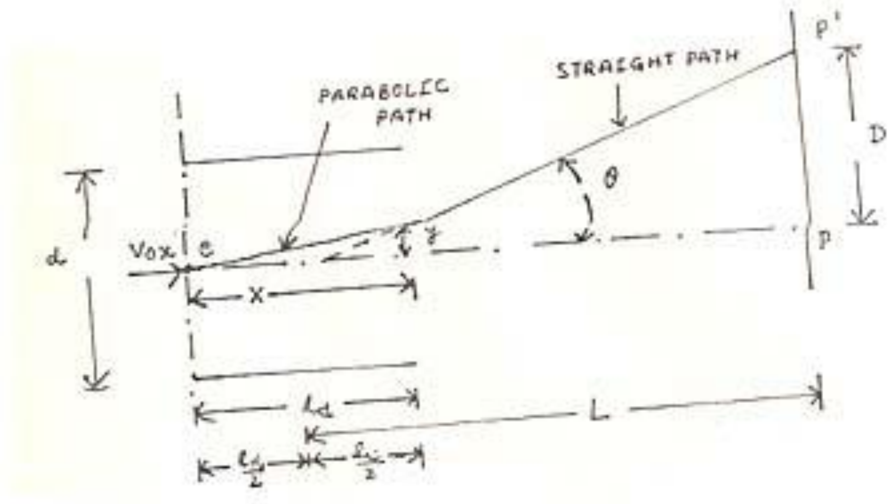
An electron entering this field in the X-direction with an initial velocity will experience a force. It is easy to derive an expression for vertical deflection as a function of the horizontal distance travelled by the electron;

$$Y = [-\frac{eE_y}{2v_{0x}^2 m}]x^2 \qquad (6)$$

(Derive the above result)

Eq. 6 show that the path of an electron, travelling through an e3lectric field of constant intensity and entering the field at right angles to the lines of flux, is parabolic in the X-Y plane.

Let, s use the idea described above to understand deflection plates. Fig.2.7 shows two such plates placed at a distance d apart and suppose that a potential difference E is applied.



Our aim is to calculate the deflection D on the screen; it is given by;

$$D = L\frac{eE_y l_d^2}{mv_{0x}^2} = \frac{Ll_d E_d}{2dE_a} \qquad (7)$$

Where D = deflection on the fluorescent screen (m)

L = distance from center of deflection plates to screen (m)

l_d = effective length of the deflection plates (m)

E_d = deflection voltage (volts)

E_a = accelerating voltage (volts)

(Derive the expression for D)

D is directly proportional to E_d. Usually, E_d, deflection voltage is a varying quantity and the image on the screen follows the variation of the deflection voltage in a linear manner. (Thus CRT may be used a linear voltage indicating device).

The deflection sensitivity S of a CRT is defined as the deflection on the screen (in meters) per volt of deflection voltage,

$$S = \frac{D}{E_d} = \frac{Ll_d}{2dE_a} \qquad (8)$$

The deflection factor G of a CRT is defined as the reciprocal of the sensitivity S and expressed as

$$G = \frac{1}{S} = \frac{2dE_a}{Ll_d} \qquad (9)$$

Note both S and G are independent of deflection voltage but very linearly with the accelerating potential. High accelerating voltages therefore produce an electron beam that requires a high deflection potential for a given excursion on the screen. A highly accelerated beam will obviously have more kinetic energy and therefore produces a brighter image on the CRT screen, at the same time this beam is more difficult to deflect and is sometimes referred to as hard beam. Typical values of deflection factors range from 10V /cm to 100V /cm, corresponding to sensitivities of 1.0mm /V to 0.1 mm /V respectively.

Screen for CRTs:

Screens for CRTs are made of a material called phosphor when the electron beam strikes the screens the screen of the CRT, a spot of light is produced. The phosphor(on the inner surface of the CRT). Absorbs the kinetic energy of the bombarding electrons and reemits energy at a lower frequency in the visible spectrum. This property is called fluorescence. The intensity of the light emitted from the CRT screen, called luminance, depends on

(1) Beam current, increasing the beam current increases the luminance.
(2) Accelerating potential, luminance increases with increase in accelerating potential.
(3) Time the beam strikes a given area of the phosphor, therefore, sweep speed affects the luminance.
(4) Physical characteristics of the phosphor itself.

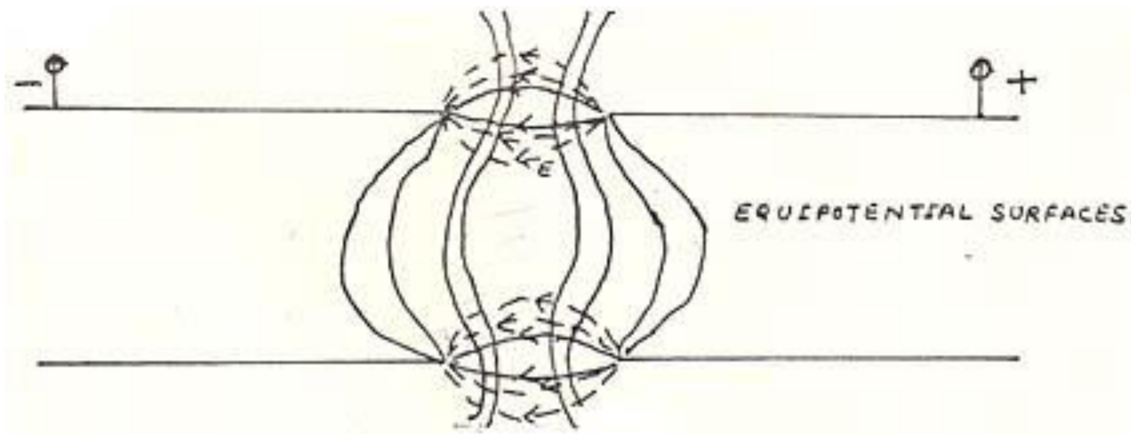
The bright available phosphor is P31 which fluoresces in green region is ageneral purpose phosphor.

N.B. CRT Screen is a delicate part of tube. Handle it carefully. It is possible to inflict serious damage to it by careless handling. When a phosphor is excited by an electron beam with excessive current density permanent damage of the phosphor may occur thorough burning, and the occurrence of burning are beam density and duration of excitation. Thus burning can be avoided by keeping the beam intensity low and the exposure time short. In an oscilloscope beam density is controlled by INTENSITY, FOCUS AND ASTIGMATISM controls the front panel.

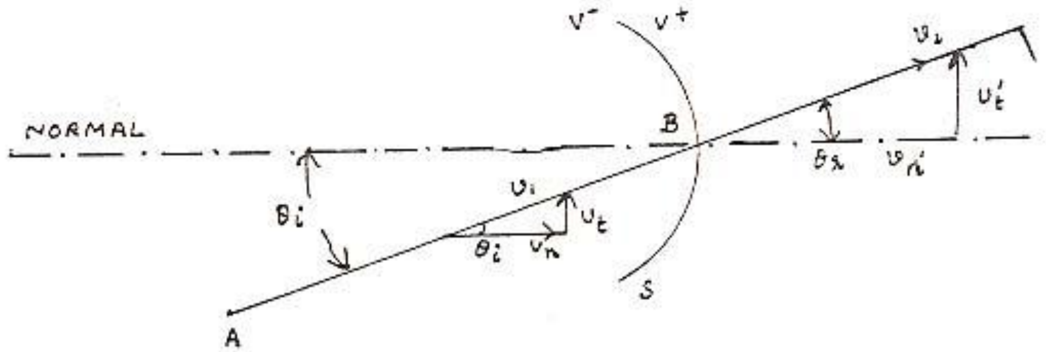
Electrostatic Focusing:

How do we get a focused beam on the screen? We have as yet not touched this point. This is achieved by electrostatic focusing. To understand the effect let's go back to fig.2.5 field lines will be straight only for the case when a charged particle is placed in a uniform intensity. Usually, the lateral repulsion of the electric field lines causes a spreading of the space between the lines, resulting in a curvature of the field at the ends of the plates. We get an equipotential surface if we connect points of equal potential on each of field lines.

When two cylinders are placed end and a potential difference is applied to them, the resulting electric field between the two cylinders is not of uniform density. lateral repulsion will cause spreading of the lines, resulting in a field as show in fig.2.8. the equipotential surfaces are curved (why?)



Consider, now the regions on both sides of an equipotential surface, S shown in fig.2.9. the potential to the left of S is V and to the right of s is V. now if we assume an electron moving in a direction AB at an angle with the normal to the equipotential surface and entering the area to the left of S with a velocity V . it experiences a force at the surface S. the force acts in a direction normal to the equipotential surface.



Because of this force, the velocity of the electron increases to a new value, after it has passed through S. the tangential component of velocity V_t , on both side of S remains the same, the normal component of the velocity, V_n increases by the force at the equipotential surface to V_n , from fig2.9. it follows.

$$v_t = v_1 \sin\theta_i = v_2 \sin\theta_r$$

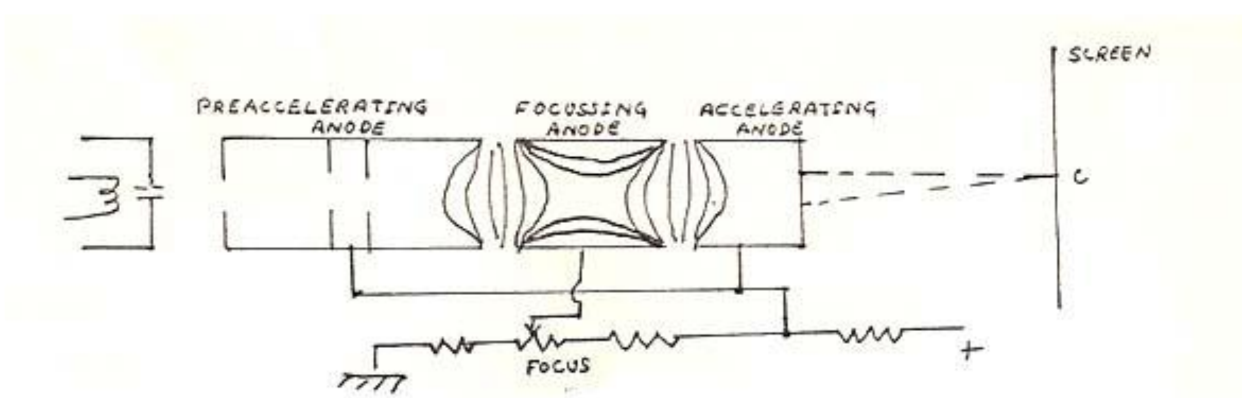
Or

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{v_2}{v_1}$$

Where Θ_i and Θ_r being the angle of incidence and refraction respectively of the electron ray.

The above relation states that the refraction of an electron beam at an equipotential surface follows the same laws as the refraction of light at a refracting surface. That is why, an electrostatic focusing system in a CRT is sometimes called an electron lens.

Consider now the system as shown in fig.2.4, the electrostatic focusing system is essentially a three elements component as shown in fig2.10



The first electrode of the system is the pre-accelerating a node, a metal cylinder containing several baffles to collimate the electron beam that enters through the small opening on the left hand side. The second electrode is the focusing anode. And the third electrode is the accelerating anode. The three electrodes are cylindrical in form and placed coaxial with the CRT axis.

The pre-accelerating anode and the accelerating anode are connected together to a high positive potential (say +1500V) supplied by the CRT HV supply. the focusing anode is connected to a lower positive potential (say500V).the potential difference between the focusing anode and the accelerating anode creates electric field between the cylindrical elements. Since the field lines are non uniformly spread, as shown in fig 2.10, the equipotential surfaces are curved to form a double concave lens system. Electrons are emitted by the cathode as a slightly divergent beam. Those electrons that enter the electric field between the preaccelerating anode the focusing anode at angles other than normal to the equpotential surface will be refracted toward the normal. The electron beam, therefore, tends to become parallel to the CRT axis, as shown. This approximately parallel beam then enters the second concave lens and is refracted once again to become slightly convergent and focused on the screen at the centre of the CRT axis. The focal length of this double concave lens system can be increased or decreased by varying the voltage on the focusing anode, so that the focal point of the beam is moving along the CRT axis. The potentiometer that provides adjustment of the voltage on the focusing anode is oscilloscope is front panel control marked Focus.

10. Optical Spectroscopy & Quantum Dots Experiment

Optical Spectroscopy Experiment

Objective: Determination of Rydberg constant.

Apparatus: Hydrogen discharge tube together with induction coil, spectrometer, prism, grating.

Introduction:

The energy levels of the hydrogen atom are obtain by solving the Schrodinger Equation; we get

$$E_n = \frac{-me^4}{8v_0^2 h^2 n^2} \qquad \text{(M.K.S.Unit)} \qquad (1)$$

The frequency v_0 of emitted light is given by

$$h\gamma_0 = E_i - E_f \qquad (2)$$

Different transitions can be arranged into series known after their corresponding discovers

n_f=1 ,	ni=2,3,4…………	Lyman series (ultraviolet)
n_f=2 ,	ni=3,4,5…………	Balmer Series (Visible)
n_f=3 ,	ni-=4,5,6………..	Paschen Series (Infrared)
n_f=4 ,	ni= 5,6,7………	Brackett Series (Infrad)
n_f=5 ,	ni=6,7,8……….	Pfund Series (Far infrared)

From Eq. (1) and (2) we get the inverse wavelength.

$$\lambda^{-1} = \frac{\gamma_0}{c} = \frac{me^4}{8\gamma_0^2 h^3 c}\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right] \qquad (3)$$

Rydberg constant R is defined as

$$\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right] \qquad (4)$$

So that its theoretical value is $R = \ldots\frac{me^4}{8\gamma_0^2 h^3 c}\ldots\ldots\ldots\ldots$ (5)

Procedure:

The object of the experiment is to determine R using Eq. (4). For Balmer series of lines .An electric discharge tube is used to excite hydrogen atoms to higher levels and a grating spectrometer is used to measure the wavelength of the emitted radiation.

Ensure the following standard adjustment in the spectrometer.

(1) The telescope is focused for infinity.

(2) The collimator is adjusted as to give a parallel beam.

(3) The turntable is leveled.

(4) The turntable is so placed as to make the plane of the grating normal to the incident beam.

(The last two adjustments are made by making sure that a particular image, say the red one in first order, is observed at the same angle and the same height on the left and right sides).

Use a narrow slit and illuminate it by Hydrogen discharge tube and measure the angular Positions of diffraction images on both sides of the direct image .in all visible orders.

(You should be able to see at least three colors in the first and second orders. Try also to locate at least one image in third order).Calculate the wavelength using the formula.

$$d \sin \theta = m\lambda \qquad (6)$$

Where d is the spacing of the grating (grating has 15,000 lines /inch), m the order of diffraction, and ɵ is the angle of diffraction which is measured .Calculate R from Eq. 4 and compare it with the theoretical value obtained from Eq. 5.

Note: To observe all diffraction images switch off all lights in the room.

References:

1. B. L. Worsnop And N.T.Flint : Advanced Practical Physics for students "(for adjustment of the spectrometer).
2. D. D. Halliday and R. Resnick:"Physics Part II (For grating),P.1123-1134.
3. A. Beiser:' Concepts of Modern Physics' (Ch.7) or R. B. Leighton :Principles of Modern Physics (For theory of Hydrogen atom).

Quantum Dots Experiment

Objectives:

1. Determining the size of a Quantum Dot.

Theory/derivation of formulas: The florescence is the emission of light from a substance after it has adsorbed energy. The absorbed energy can be in the form of electricity, light, heat or other forms of energy. In the case where the absorbed energy is provide in the form of light, the emitted wavelength is often of lower energy longer wavelength, red end of the spectrum) than the absorbed energy (shorter wavelength, blue end of the spectrum). Since violet has the highest energy (shortest wavelength) of the visible spectrum, it is used in this lesson to "excite" the quantum dots. Additionally, even thought the same light source is used to illuminate the different solutions of quantum dots, their physical size limits the energy each one re-emits. Small dots will florescence blue while large dots will florescence red.

Wave particle duality can be readily taught using this teaching aid to introduce students to the particle in a box concept. In its simplest form, wave particle duality can be taught be conceptualizing matter waves as skipping ropes or waves in pool. With this concept and the constant that the wave must be pinned at the wall of the box, we are able to illustrate the key feature of wave particle duality.

For quantum dots, mathematically this expressed as:

$$E = \frac{h^2 n^2 p^2}{2m_e R^2} + \frac{h^2 n^2 p^2}{2m_h R^2} + E_g$$

Where R is the radius of the Quantum Dot, m_e is the effective mass of the electron inside the semiconductor, m_h is the effective mass of a hole (the absence of an electron) inside the semiconductor and $E_g$is the bandgap energy of the semiconductor

$$E_g = 2.15 \times 10^{-19}$$

$$m_e = 7.29 \times 10^{-32}\ Kg$$

$$m_h = 5.47 \times 10^{-31} Kg$$

The first 2 terms of this formula illustrates the size scale at which these wave particle duality effects begin to dominate given that the 3rd term is a bulk material (or macroscopic world term) and illustrates how these effects become important when dealing with very small length scales.

Since we will use the wavelength (color) of the fluorescence form the Quantum Dots to measure the zero-point energy, we must also convert this wavelength information to energy using

$$E= h\upsilon \quad \text{and} \quad V = C/\lambda.$$

Where E, v and λ are the energy, frequency and wavelength of the emitted light, c is the speed of light (3×10^{-8} m/s) and h is Plank's Constant (6.62×10^{-34} j. s)

1 .Setup/ Procedure:

Each band of spectrometer will have slightly different operation instructions so please follow your user manual. The following is designed for the Ocean Optics Red Tide Spectrometer and Diva spectrometer.

Procedure:

1. Carefully remove one of the vials from the aluminum rack by using a hex key, loosening both hex bolts.
2. Set the vial upright on a flat surface free of other obstructions.
3. Set up the spectrometer probe so it is held stationary and pointed at the solution in the vial.
4. Turn of the lights or cover the apparatus to improve the quality of the readings.
5. Turn on the spectrometer and have the software ready to scan the wavelengths.
6. The 400 nm lights source needs to be illumination the vial but perpendicular to the spectrometer probe. If the light source is directly across from the probe you will not get an accurate reading of the wavelength radiated by the solution.
7. Turn on the 400 nm light source with it pointed at the solution.
8. Initiate the software to record the radiated wavelength.
9. Using the peak wavelength and the formula, calculate the size of the quantum dots.

Objectives:

2. Determining the absorption of Quantum Dot solution.

Setup /Procedure

Each brand of spectrometer will have slightly different operation instruction but overall the functions are the same. The following is design for the Spec 20 Spectrometer

Procedure

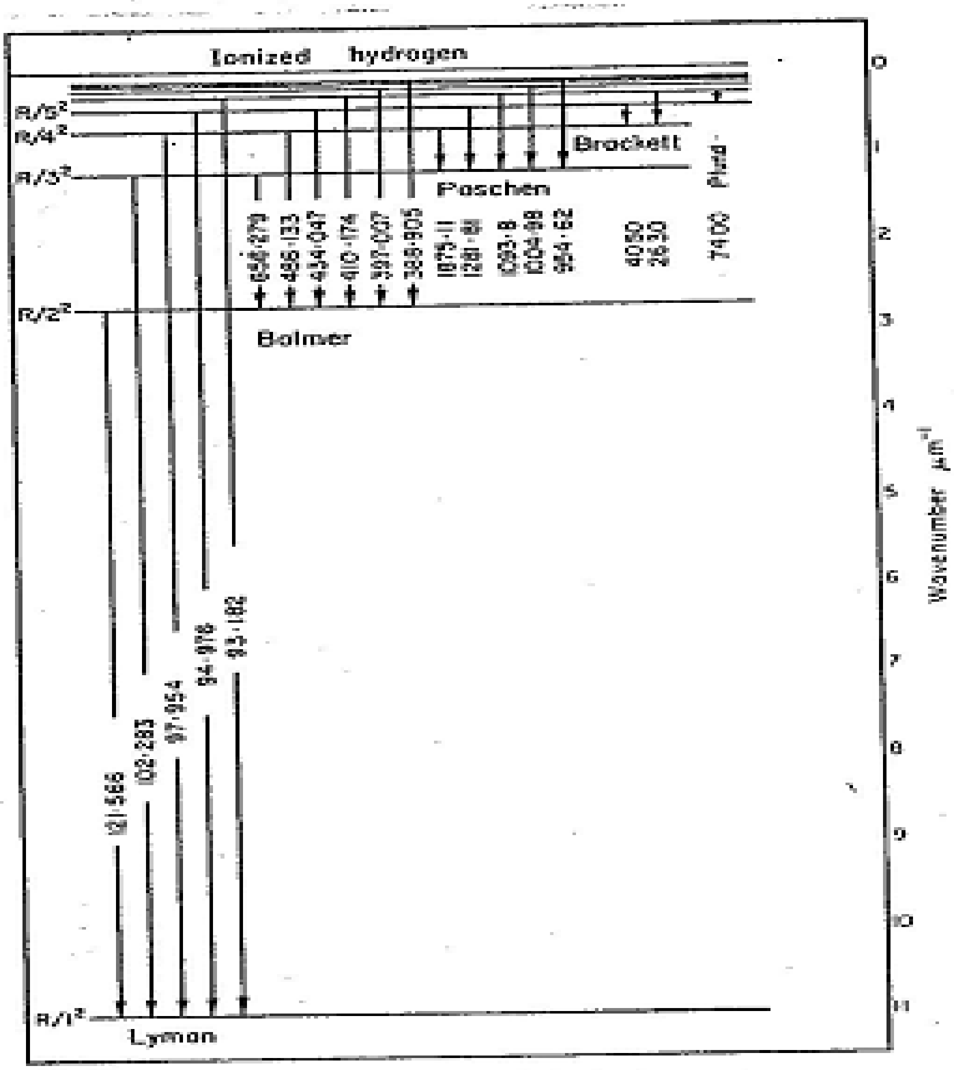
1. Since the vials are so small you need to use an adapter to fit the vial into the sample compartment. You will also need to place a 15 mm lifter in the bottom of the adapter as well to raise the solution to the level of the light source.

2. Starting at the lowest wavelength, zero out the unit and then insert the solution.
3. Record the data on the absorption of the solution, increasing the wavelength in increments of 10 nm.
4. Repeat steps 2 an 3 until you reach the upper limit of your spectrophotometer.
5. Repeat with each of the other three vials.

Question

1. What was the peak wavelength for each vial?
2. Using the formula from the Theory Section calculate the size of the Quantum Dots in each vial?
3. Find the percent e error between your measured wavelength and dot radius versus the actual values provided by your instructor?
4. Plot the quantum dot radius versus the emitted wavelength, what relationship do you see? Use graph paper.
5. What happens if the radius of the quantum dot gets very large(approaching infinity)?

The Balmer series for hydrogen.



Energy levels of the hydrogen atom.

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11. Peltier Heat Pump Experiment

Objectives:

1. To determine the cooling capacity P_c the pump as a function of the current and to calculate the efficiency rating η_c at maximum output.
2. To determine the heating capacity P_w of the pump and its efficiency rating η_w at constant current and constant temperature on the cold side.
3. To determine P_w, η_w and P_c, η_c from the relationship between temperature and time on the hot and cold sides.
4. To investigate the temperature behavior when the pump is used for cooling, with the hot side air-cooled.

Equipments:

Thermo generator(1), Flow-through heat exchanger(1), Air cooler(1), Heating coil with sockets, Distributor, Rheostat, 33 Ohm, 3.1 A, Connecting plug, 2 pcs., Power supply, universal, Digital Multimeter, Stopwatch, digital, 1/100 sec., Cold a. hot air blower , 1000 W, Lab thermometer, -10…+100C, Thermometer, -10…+ 50 C, Rubber tubing, i.d. 7 mm, Universal clamp,Tripod base –PASS-, Support rod, l 250 mm, Right angle clamp -PASS-, Heat conductive paste, 50 g, Connecting cords.

Theory:

When an electric current flows through a circuit composed of two different conductors, heat will be liberated at one junction and absorbed at the other) depending on the direction in which the current is flowing (Peltier effect). The quantity of heat Q liberated per unit time is proportional to the current I:

$$\frac{Q}{t} = P_p = \pi . I = \alpha . T . I$$

Where p is the Peltier coefficient, α is the Seebeck coefficient and T is the absolute temperature.

If an electric current I flow in a homogeneous conductor in the direction of a temperature gradient$\frac{dT}{dx}$. The heat will be absorbed or given out, depending on the material (Thomson effect):

$$P_t = \tau . I . \frac{dT}{dx}$$

Where, t is the Thomson coefficient.

The direction in which the heat flows depends on the sign of the Thomson coefficient, the direction in which the current flows and the direction of the temperature gradient.

If an electric current I flows in an isothermal conductor of resistance R, we have the Joule effect:

$P_J = R \cdot I^2$

Because of heat conduction, heat also flows from the hot side (temperature T_h) to the cold side (temperature T_c):

$$P_L = L\frac{A}{D}(T_h - T_c)$$

Where, L is the conductivity, A the cross-sectional area and d the thickness of the Peltier component.

Writing $\Delta T = T_h - T_c$, we obtain for the heat capacity of the pump on the cold side (the cooling capacity):

$$-P_c = \alpha T_c I \pm \frac{\tau I \Delta T}{2d} - \frac{1}{2} I^2 R - \frac{L A \Delta T}{d}$$

and, for the heat capacity of the pump on the hot side (the heating capacity):

$$+P_h = \alpha T_h I \pm \frac{\tau I \Delta T}{2d} + \frac{1}{2} I^2 R - \frac{L A \Delta T}{d}$$

The electric power supplied is

$$+P_{el} = \alpha I \Delta T + R I^2 + \frac{\tau I \Delta T}{d} = U_p . I_p$$

1. The pump cooling capacity P_c was found to be 49 W when I_p = 5 A and $P_H = P_c$

   The efficiency rating

   $\eta_c = \frac{P_c}{P_{el}}$

   becomes, for the measured values I_p = 5.0 A and U_p = 14.2 V,

   η_c = 0.69 $\upsilon_h = \upsilon_c$ = 20°C)

2. From the slope of the curve in Fig. 6 (where the curve starts off as a straight line) we can calculate the pump heating capacity

   $P_h = \frac{C_{tot} . \Delta T_h}{\Delta t}$

   And corresponding efficiency rating

$$\eta_h = \frac{P_h}{P_{el}}$$

Where, P_{el} =U_p-I_p, as follows,

m_w=0.194 Kg, Cw= 4182 J/Kg.K, m_{Br}=0.0983 Kg, C_{Br}= 381 J/Kg.K,

m_{cu}= 0.712 Kg, Ccu=383 J/Kg.K,

$C_{tot} = m_w . C_w . m_{Br} . C_{Br} + m_{cu} . C_{cu} = 1121$ J/Kg.K,

Where, m_w is the mass of water, Cw the specific heat capacity of water, m_{cu} the mass of copper block, Ccu The specific heat capacity of copper, m_{Br} the mass of the brass bath, C_{Br} the specific heat capacity of the brass, I_p the pump current and U_p the mean pump voltage. With the slope

$\frac{\Delta T_h}{\Delta T} = 6.7 \times 10^{-2}$ K/s

We obtain a value P_h of 75 W.

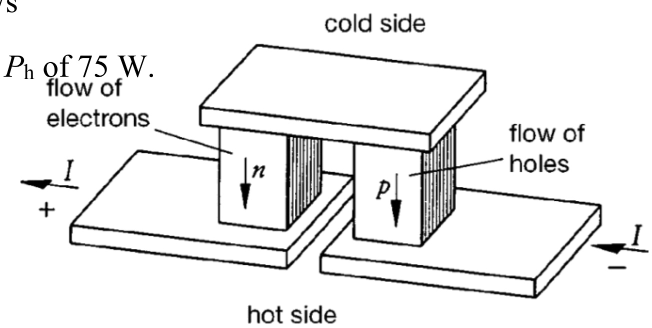

Fig. 1: The Construction of a Peltier semi-conductor element. In practice, several elements are generally connected in series (electrically) and in parallel (thermally).

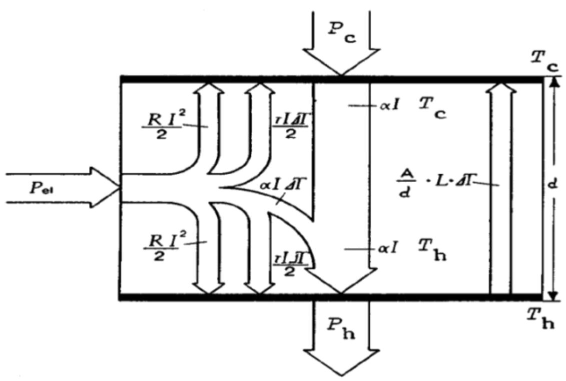

Fig. 2: Power balance flow chart in a Peltier component. (The example illustrated is for the case where $P_T > 0$).

3. P_h and P_c, and η_h and η_c, can be calculated from the slopes of the curves $\vartheta_h = f(t)$ and $\vartheta_c = f(t)$ and the relevant heat capacities.

With $\vartheta_h / \Delta T = 0.056$ K/s (start of curve) and

$\Delta\vartheta_{c/\Delta T} = -0.023$ K/s and with $C_{tot.} = 1121$ J/K, we obtain:

$P_h = 63$ W; $P_c = 26$ W.

In the range considered, the voltage U_p (average value) was

12.4 V, so that we obtain the efficiency ratings $\eta_h = 1.3$ and $\eta_c = 0.52$. ($I = 4$ A, $T = 22^\circ$C).

4. Water temperature can be calculated when the hot side was cooled with the air cooler. The temperature ϑ_h of the hot side was approx. 72°C after 20 minutes (no blower). The maximum

temperature difference $\vartheta_h - \vartheta_c = 60$ K is thus attained and the pump output of the Peltier component is zero. When the blower was used, T_h remained constant at approx. 45°C after 20 minutes

Setup and procedure:

1. Fit a water bath on the cold side and a heat exchanger through which tap water flows on the hot side. A heating coil (resistance approx.3 ohms), operated on AC, dips into the water-filled bath. For each current value I_p set the heating capacity $P_H = U_H \cdot I_H$ with the rheostat R so that the temperature difference between the hot and the cold side is approximately zero. The power supplied then exactly corresponds to the cooling capacity P_c. Measure the heater current I_H and voltage U_H, the operating current I_p and volt-age U_p and the temperatures of the hot side T_h and the cold side T_c .
2. Remove the heating coil as it is no longer required. Reverse the operating current so that the water in the bath now heats up. Measure the rise in the temperature of water T_w at constant current I_p. Measure also I_p , U_p and T_c. Calculate the heat capacities of a copper block C_{Cu}, of the water C_w and of the brass bath C_{Br} from their dimensions or by weighing.
3. Fit water baths to both sides of the heat pump and fill them with water of the same temperature. With the current I_p constant) measure the changes in the temperature of the
   Two water baths) i. e. $T_h = f(t)$, $T_c = f(t)$, I_p and U_p.
4. For this fourth experiment we have a water bath on the cold side, an air cooler on the hot. Measure the temperature of the cold side as a function of time, with the cooler

a) in static atmospheric air, and

b) Force-cooled with a blower.

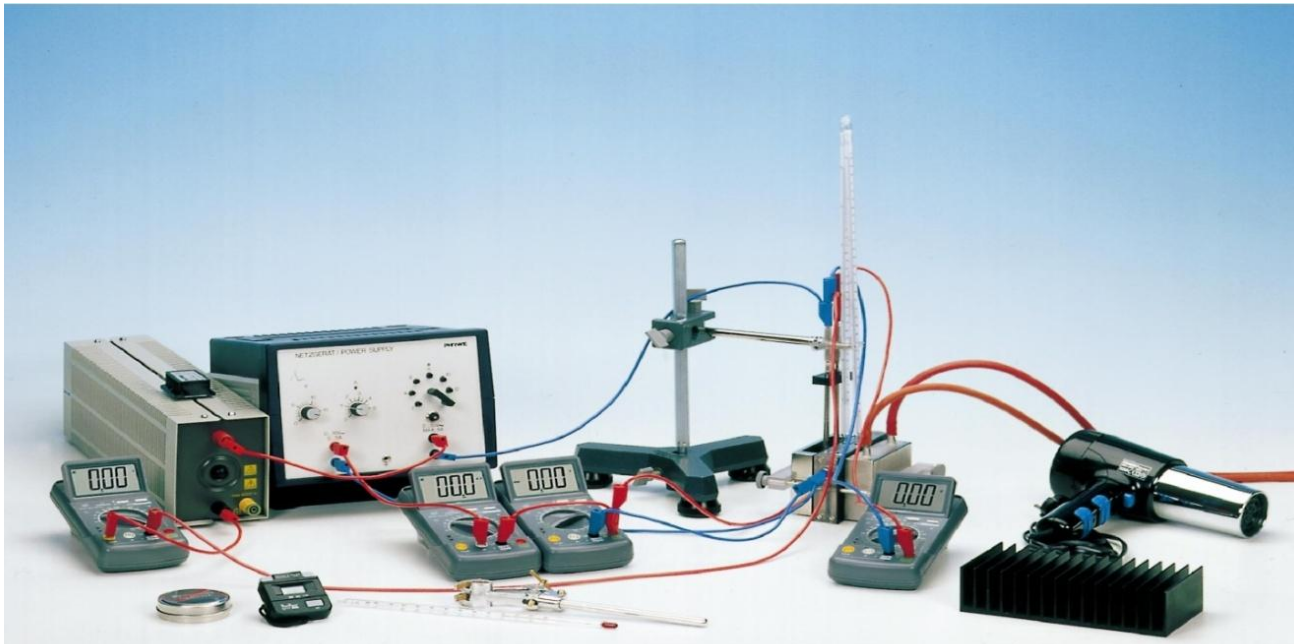

Fig. 3: Experimental set-up for measuring cooling capacity.

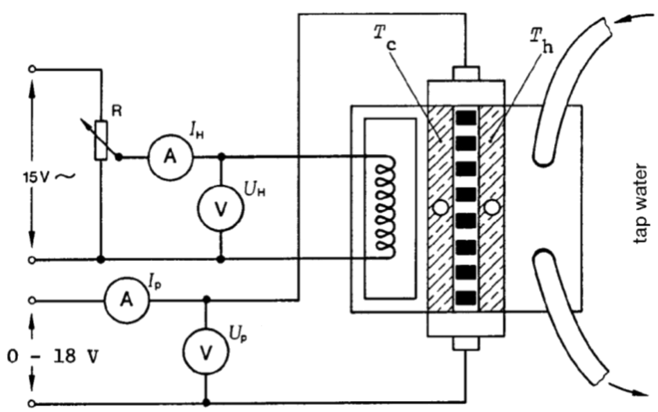

Fig. 4: Set-up for determining cooling capacity.

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2. Active and Passive Elec. Comp.22,165-174(1999)
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4. Pitts, D. R., and L. E. Sisson, 1977, Heat Transfer (Schaum's Outline Series), McGraw-Hill,Inc.
5. Pollock, D. D., 1982a, Physical Properties of Materials for Engineers, Vol. 1, 2 &3, 2nd Ed., CRC Press Inc., Boca Raton, FL.

12. Thin Film Deposition and Characterization

Objective: High Vacuum Deposition and characterization of the thin films of the desired material and thickness.

Operation Procedure:

1. Switch ON the power supply and 'main MCB' of the system. Close all valves (Hivac-Valve (HV) and Combination-Valve (CV) in closed position).
2. Close the chamber properly and switch on the Rotary pump.
3. Put CV to Roughing.
4. Wait till the vacuum reaches ~0.05 mbar in GH2.
5. Change CV to Backing.
6. Put water supply ON and switch ON the Diffusion Pump (DP).
7. Wait for around 20 mins, GH1 should show ~0.05 mbar now.
8. Change the CV to Roughing for 1-2 mins to get back ~0.05 mbar in GH2. Change the CV back to Backing (~ 0.05 at GH1).
9. Put LN_2 in the cold-trap.
10. Open the HV.
11. Wait for ~30 min. Expected vacuum is ~10^{-5} mbar.
12. Before deposition, attach the Digital Thickness Monitor (DTM) to the system.
13. For deposition select the particular source. Check that electrode's safety-shutter is closed. Switch ON the LT. Be careful, high current flows through the system now-on.
14. Increase the current slowly at a rate of 2 Amp/Min. till deposition temp is reached.
15. Monitor the rate of deposition on the DTM (Preferably use shutter).
16. After deposition, decrease current at a rate of 2 Amp/Min.
17. After reaching back 0 Amp, wait 10 min and close the HV. Switch OFF the DP.
18. After ~20 mins, as the DP heater cools down, Put CV to Close, switch OFF the Rotary pump and close water supply.
19. Deposition process is now complete. Vent the chamber to take the samples.
20. Leave the system back to vacuum using Rotary in Roughing mode.

Never do the Following:

1. Never put the CV on the Roughing when HV is Open (after step 10 above). This might spoil the system permanently.

2. Never close the water supply when the DP is ON.

3. Never handle the valves roughly. Operate the valves slowly.

<u>Special Reminders:</u>

1. The system should not be operated in any case till you get the required permission.

2. Clean the chamber properly during uploading the substrates.

3. Always keep the system in vacuum. Remind yourself to keep it in vacuum before leaving the lab.

3. Always close the water supply after all the processes.

4. Note the date, operation, task, sample positions, students actively present etc. in the Log-Book. Any unusual incidence MUST be written down in detail in the Log-Book.

(II) Motivation:

Most scientists work with their samples in a vacuum system. The reasons for this are several fold: first, many samples react with the gases in ordinary room air which means they must be kept in a clean environment; second, the experimental probes used to measure sample properties may depend on electron or other beams that simply could not exist outside a vacuum. Overall, the quality of the thin film grown and the measurements done are of extremely high quality under vacuum.

(III) The Instrument:

(IIIa) System Overview:

The system, illustrated in Figure 1, contains the essential elements typically required to obtain high vacuum. The most common and reliable systems utilize three pumping devices: The rotating mechanical pump, the diffusion pump, and the cold trap. Other system components, such as valves and baffles, aid or control the action of these pumps. The rotary pump serves as a primary source for creating vacuum. However, they can reach only upto ~ 10^{-3} mbar. The Diffusion pumps using hot oil have the advantage of reaching ~ 10^{-7} mbar and has no moving parts although they must be backed by a rotary pump. The cold trap reduces pressure by condensing, or freezing out, onto its cold surfaces, condensable vapors that may exist in the system. It also prevents oil vapor from the diffusion pump from diffusing back, or "back streaming", into the system. By removing "condensables" such as water vapor, a trap actually serves as a pump.

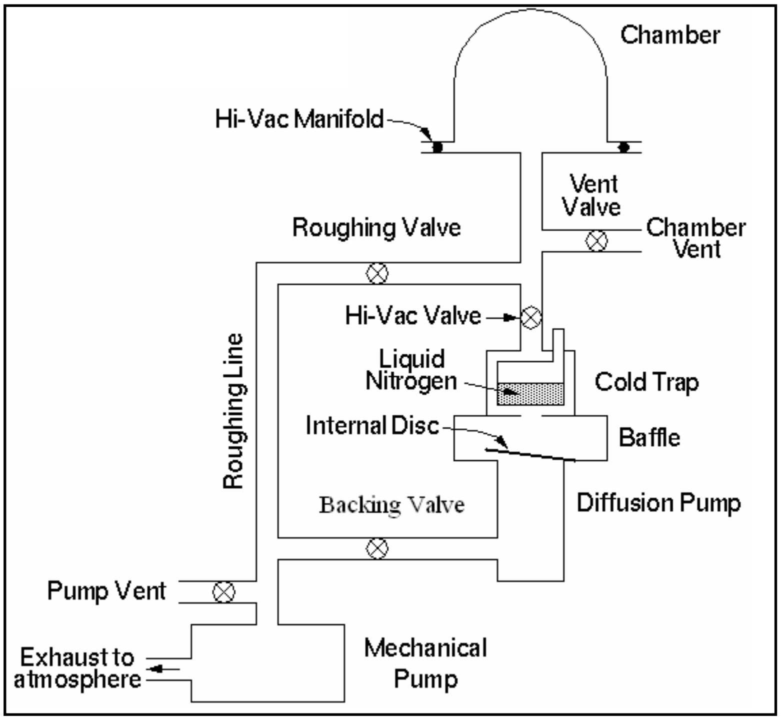

Figure.1: The vacuum system illustrating the essential elements.

(IIIb) The Two Modes of Operation (Roughing and Backing):

Exposure to atmosphere when at operating temperature will result in decomposition of diffusion pump oil. It is therefore necessary to employ a bypass line around a heated diffusion pump when evacuating a chamber from atmospheric pressure to a "rough" vacuum prior to connecting the chamber to the diffusion pump; hence, the terms "roughing line" and "roughing valve". The foreline valve and the hi-vac valve (Figure 2) serve to isolate the diffusion pump, the baffle and cold trap from the object being roughed. During roughing the roughing valve is open. When roughing has been completed (at ~20 mtorr), the roughing valve is closed before the foreline valve and the hi-vac valve are opened. The manifold vent valve admits air to the port manifold to "break" the vacuum and make possible the removal of objects after they have undergone vacuum processing. The roughing valve and the hi-vac valve must be closed during this operation if the pumps on the vacuum system are still in operation. The mechanical pump vent valve serves to admit atmosphere to the roughing line, thus bringing the mechanical pump to atmospheric pressure.

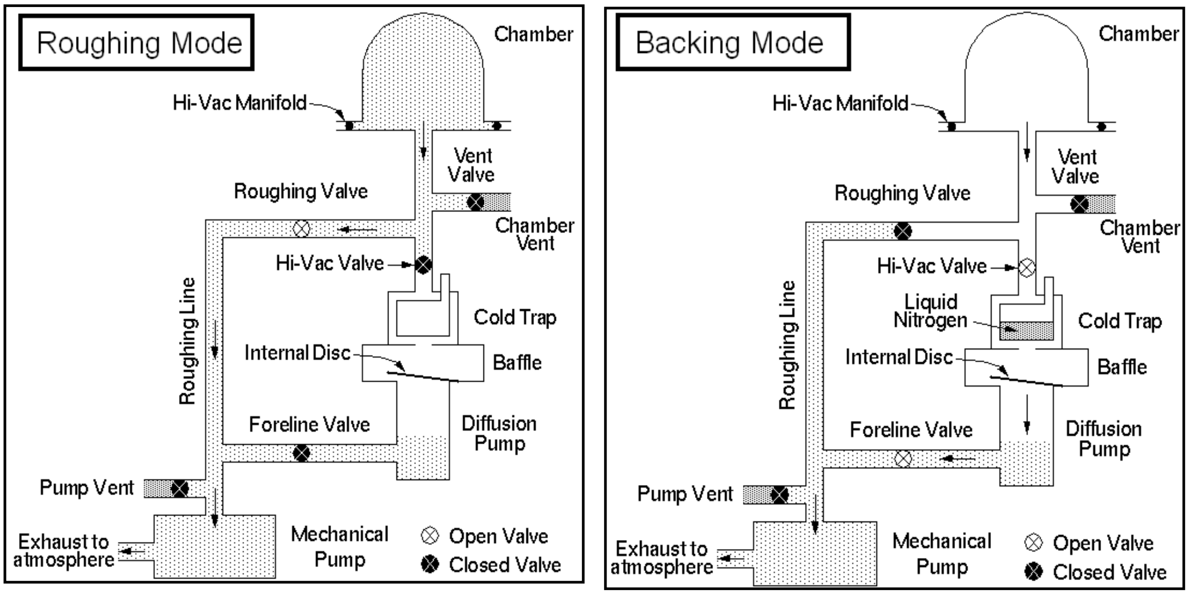

Figure.2: The two modes of operation of the system.

(IIIc) Details of Roughing Pump and Diffusion Pump:

*** Roughing (Rotary) Pump:**

It is capable of reducing pressure to about 10 millitorr. A typical mechanical pump is shown schematically in Figure 3(a). Mechanical pumps physically "sweep" the air from the system, usually with a rotary device as shown. The rotor is eccentric to the pump cavity. The rotating vane (or sweep) is kept in contact with the walls of the pump cavity by means of a compression spring. Rotating vane, positive displacement pumps have large gas handling capacities, but cannot achieve high vacuum. They are used for two purposes: to remove ("rough") the bulk of the air from a system which is initially at atmospheric pressure, and, once this is accomplished, to "back" the diffusion pump, (see below), since a diffusion pump cannot exhaust against atmospheric pressure. Hence, mechanical pumps are often called roughing pumps, backing pumps or forepumps.

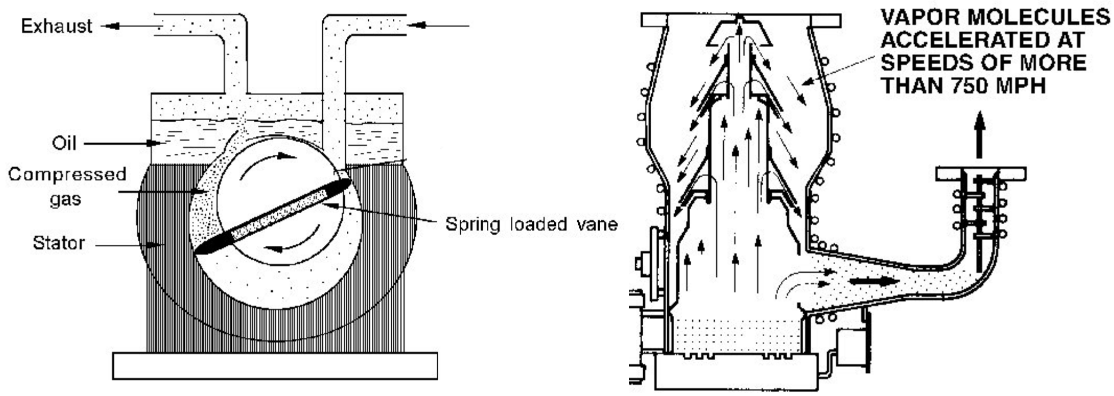

Figure.3. (a) the rotary vane pump and (b) the backing pump in action.

*** Diffusion Pump:**

It is capable of reducing system pressure to the region of 10-7 torr. A diffusion pump has a maximum pressure against which it can exhaust; this is usually in the mtorr region. The maximum exhaust pressure is also known as the "tolerable forepressure". The rotary pump noted above provides and maintains this exhaust pressure for the diffusion pump. Fast pumping action is achieved through the use of high speed jets of oil vapor which collide with gas molecules and compress them in the direction of the mechanical pump (see Figure 3(b)). The term "jet" is used to refer to both the vapor stream and to the nozzles from which the vapor issues. The oil pool at the bottom of the pump is heated, causing oil vapor to be forced up the jet stack. The vapor strikes the umbrellas, and is projected downward and outward through the nozzles of the jet stack. In passing through the narrow jets, the oil vapor flows at a very high velocity (near that of sound). The high speed vapor jet collides with gas molecules giving them a downward direction toward the fore line. The oil molecules condense on the walls of the pump which are cooled either by an air stream or by water, and flow back to the bottom pool. Thus, a continuous cycle of vaporization, condensation and re-evaporation takes place.

13. Cavendish Experiment

Objective: Determination of the Universal Gravitational constant by Cavendish method.

Introduction:

The Gravitational Torsion Balance consists of two 38.3 gram masses suspended from a highly sensitive torsion ribbon and two 1.5 kilogram masses that can be positioned as required. The Gravitational Torsion Balance is oriented so the force of gravity between the small balls and the earth is negated (the pendulum is nearly perfectly aligned vertically and horizontally). The large masses are brought near the smaller masses, and the gravitational force between the large and small masses is measured by observing the twist of the torsion ribbon.

An optical lever, produced by a laser light source and a mirror affixed to the torsion pendulum, is used to accurately measure the small twist of the ribbon. Three methods of measurement are possible: the final deflection method, the equilibrium method, and the acceleration method.

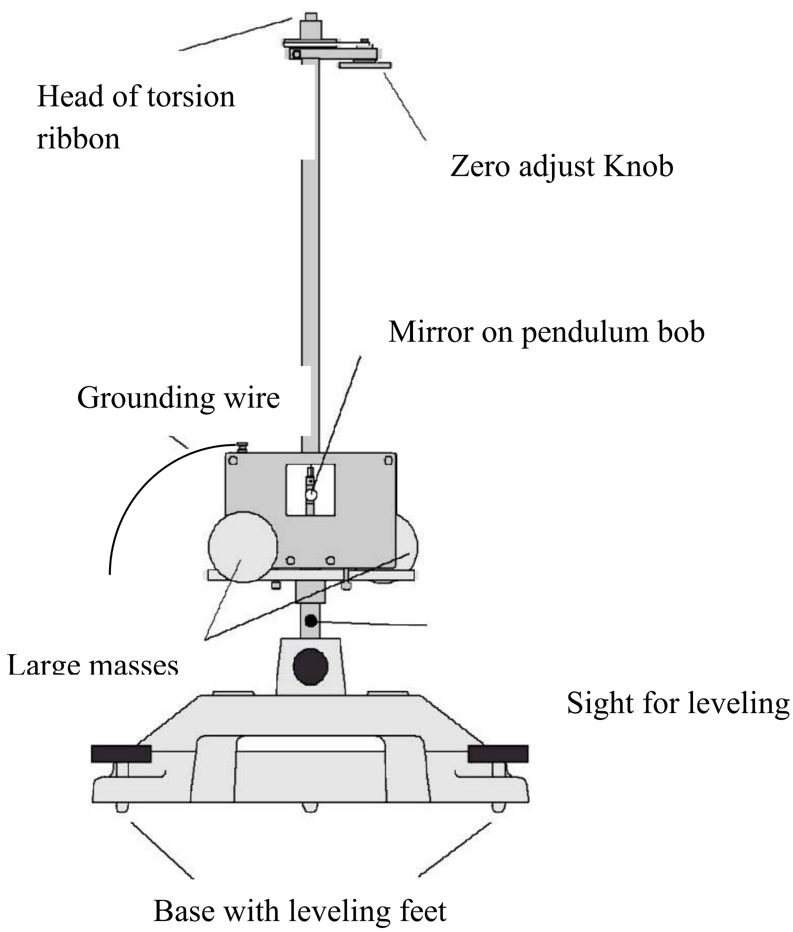

Figure 1: Assembled Gravitational Torsion Balance, ready to begin Henry Cavendish's classic experiment to determine the gravitational constant.

Background:

The gravitational attraction of all objects toward the Earth is obvious. The gravitational attraction of every object to every other object, however, is anything but obvious. Despite the lack of direct evidence for any such attraction between everyday objects, Isaac Newton was able to deduce his law of universal gravitation.

However, in Newton's time, every measurable example of this gravitational force included the Earth as one of the masses. It was therefore impossible to measure the constant, *G*, without first knowing the mass of the Earth (or vice versa).

The answer to this problem came from Henry Cavendish in 1798, when he performed experiments with a torsion balance, measuring the gravitational attraction between relatively small objects in the laboratory. The value he determined for *G* allowed the mass and density of the Earth to be determined. Cavendish's experiment was so well constructed that it was a hundred years before more accurate measurements were made.

Newton's law of universal gravitation:

$$F = G\frac{m_1 m_2}{r^2}$$

Where m_1 and m_2 are the masses of the objects, r is the distance between them and $G= 6.67\text{x } 10^{-11} Nm/kg^2$.

Equipment:

Included:

- Gravitational Torsion Balance
- Support base with leveling feet
- 1.5 kg lead ball (2)
- Plastic plate
- Replacement torsion ribbon
- 2-56 x 1/8 Phillips head screws (4)
- Phillips screwdriver (not shown)

Additional Required:

- laser light source (such as He-Ne Laser)
- meter stick

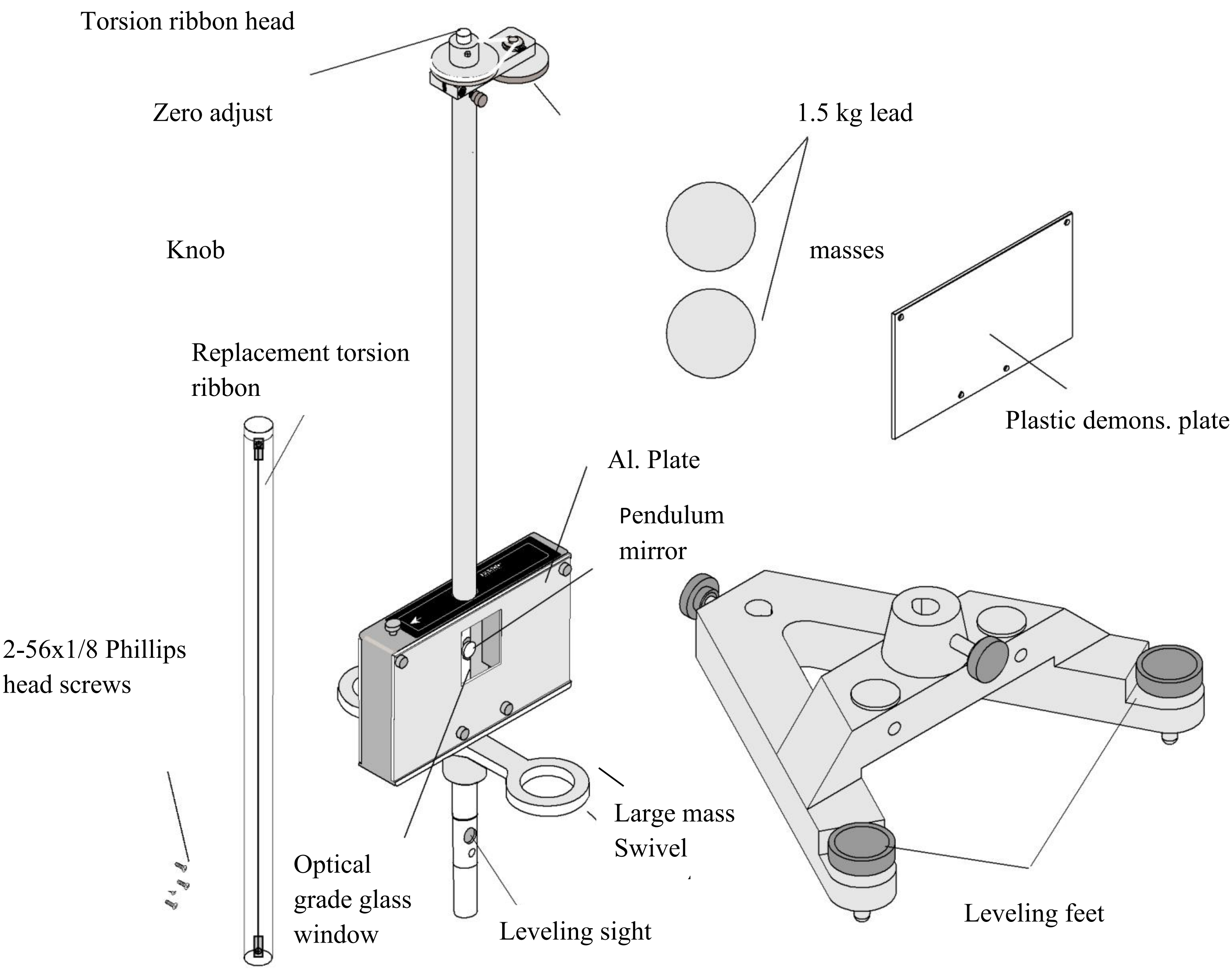

Figure-2: Equipment includes.

Equipment Parameters:

• Small lead balls

Mass: 38.3 g $\pm$ 0.2 g (m_2) Radius: 9.53 mm

Distance from ball center to torsion axis: d = 50 .0 mm

• Large lead balls

Mass: 1500 g $\pm$ 10 g (m_1) Radius: 31.9 mm

• Distance from the center of mass of the large ball to the center of mass of the small ball when the large ball is against the aluminum plate and the small ball is in the center position within the case: b = 46.5 mm (Tolerances will vary depending on the accuracy of the horizontal alignment of the pendulum.)

- Distance from the surface of the mirror to the outer surface of the glass window: 11.4 mm
- Torsion Ribbon Material: Beryllium Copper

Length: approx. 260 mm

Cross-section: .017 x .150 mm

Important Notes:

The Gravitational Torsion Balance is a delicate instrument. We recommend that you set it up in a relatively secure area where it is safe from accidents and from those who don't fully appreciate delicate instruments.

The first time you set up the torsion balance, do so in a place where you can leave it for at least one

day before attempting measurements, allowing time for the slight elongation of the torsion band that will occur initially.

Keep the pendulum bob secured in the locking mechanisms at all times, except while setting up and

conducting experiments.

Equipment Setup

Initial Setup

1. Place the support base on a flat, stable table that is located such that the Gravitational Torsion Balance will be at least 5 meters away from a wall or screen.

Note: For best results, use a very sturdy table, such as an optics table.

2. Carefully remove the Gravitational Torsion Balance from the box, and secure it in the base.

3. Remove the front plate by removing the thumbscrews , and carefully remove the packing foam from the pendulum chamber.

Note: Save the packing foam, and reinstall it each time the Gravitational Torsion Balance is transported.

4. Fasten the clear plastic plate to the case with the thumbscrews.

Do not touch the mirror on the pendulum.

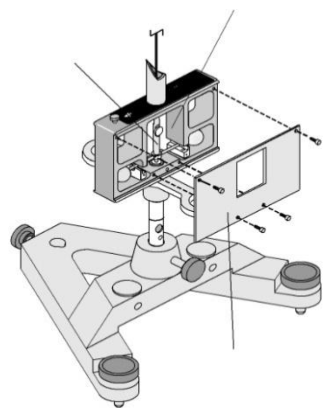

Figure 3: Removing a plate from the chamber box.

Leveling the Gravitational; Torsion Balance

1. Release the pendulum from the locking mechanism by unscrewing the locking screws on the case. Lowering the locking mechanisms to their lowest positions (fig.4).

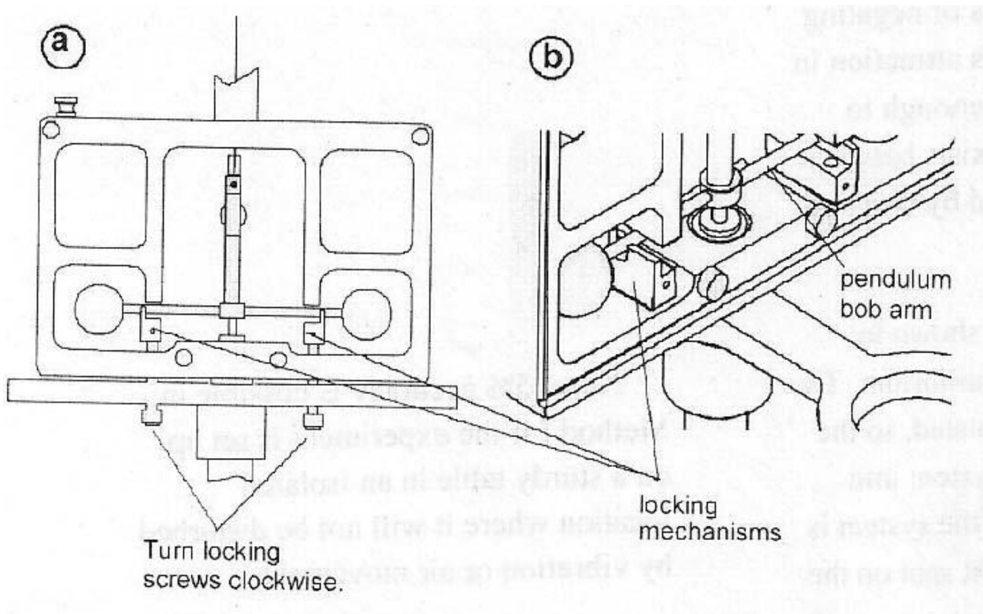

Figure-4: Lowering the locking mechanism to release the pendulum bob arms.

2. Adjust the feet of the base until the pendulum is centered in the leveling sight (figure -5). (The base of the pendulum will appear as a dark circle surrounded by a ring of light).
3. Orient the Gravitational Torsion Balance so the mirror on the pendulum bob faces a screen or wall that is at least 5 meters away.

Vertical Adjustment of the Pendulum

The base of the pendulum should be flush with the floor of the pendulum chamber. If it is not, adjust the height of the pendulum:

1. Grasp the torsion ribbon head and loosen the Phillips retaining screw (Figure 6a).
2. Adjust the height of the pendulum by moving the torsion ribbon head up or down so the base of the pendulum is flush with the floor of the pendulum chamber
(Figure 6b).
3. Tighten the retaining (Phillips head) screw.

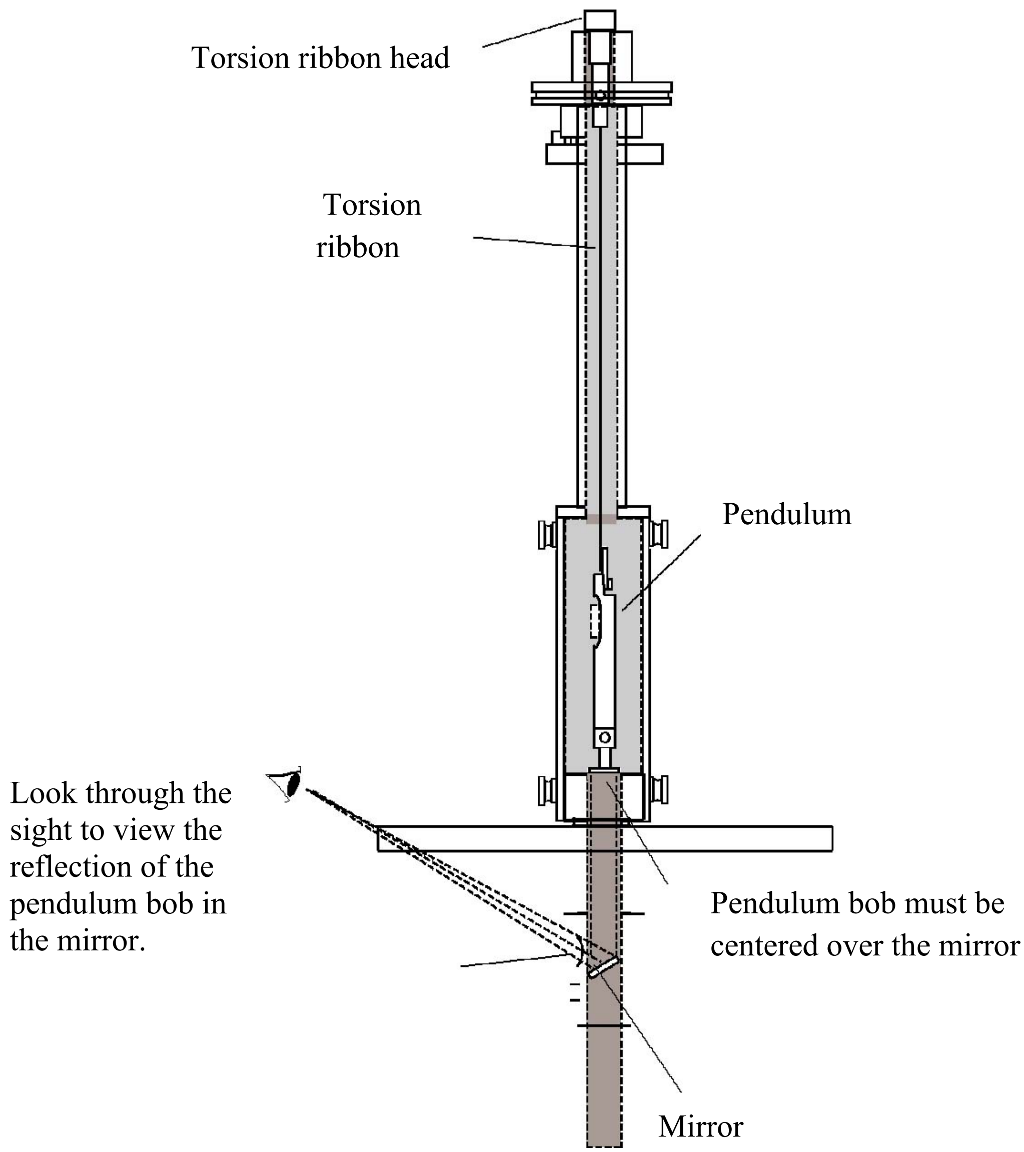

Figure -5: Using the leveling sight to level the Gravitational Torsion Balance.

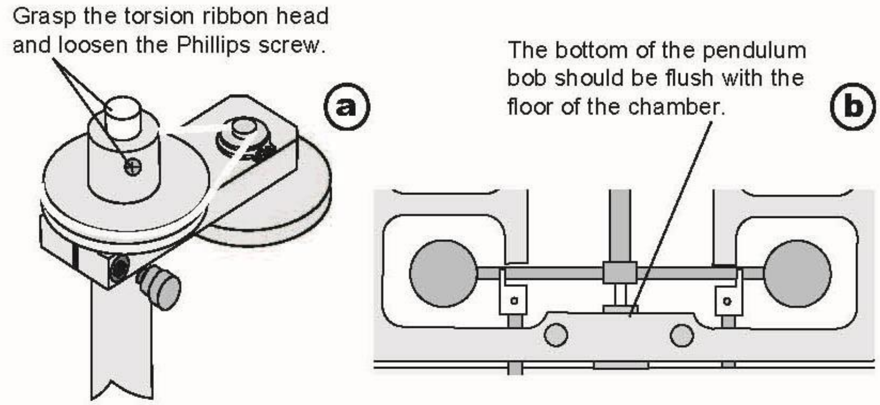

Figure 6
Adjusting the height of the pendulum bob

Note: Vertical adjustment is only necessary at initial setup and when you change the torsion ribbon or if someone has loosened the retaining screw by mistake; it is not normally done during each experimental setup.

Rotational Alignment of the Pendulum Bob Arms (Zeroing)

The pendulum bob arms must be centered rotationally in the case

— that is, equidistant from each side of the case (Figure 7). To adjust them:

1. Mount a metric scale on the wall or other projection surface that is at least 5 meters away from the mirror of the pendulum.

2. Replace the plastic cover with the aluminum cover.

3. Set up the laser so it will reflect from the mirror to the projection surface where you will take your measurements (approximately 5 meters from the mirror). You will need to point the laser so that it is tilted upward toward the mirror and so the reflected beam projects onto the projection surface (Figure 8). There will also be a fainter beam projected off the surface of the glass window.

4. Rotationally align the case by rotating it until the laser beam projected from the glass window is centered on the metric scale (Figure 9).

5. Rotationally align the pendulum arm:

a. Raise the locking mechanisms by turning the locking screws until both of the locking mechanisms barely touch the pendulum arm. Maintain this position for a few moments until the oscillating energy of the pendulum is dampened.

b. Carefully lower the locking mechanisms slightly so the pendulum can swing freely. If necessary, repeat the dampening exercise to calm any wild oscillations of the pendulum bob.

c. Observe the laser beam reflected from the mirror. In the optimally aligned system, the equilibrium point of the oscillations of the beam reflected from the mirror will be vertically aligned below the beam reflected from the glass surface of the case (Figure 9).

d. If the spots on the projection surface (the laser beam reflections) are not aligned vertically, loosen the zero adjust thumbscrew, turn the zero adjust knob slightly to refine the rotational alignment of the pendulum bob arms (Figure 10), and wait until the movement of the pendulum stops or nearly stops.

e. Repeat steps 4a – 4c as necessary until the spots are aligned vertically on the projection surface.

6. When the rotational alignment is complete, carefully tighten the zero adjust thumbscrew, being careful to avoid jarring the system.

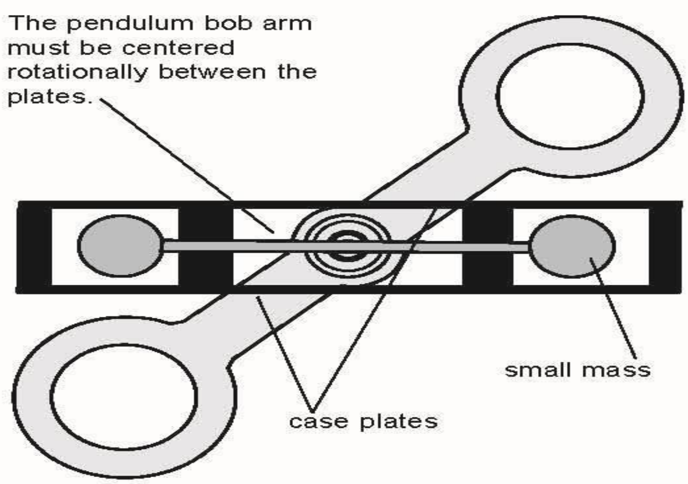

Figure 7
Aligning the pendulum bob rotationally

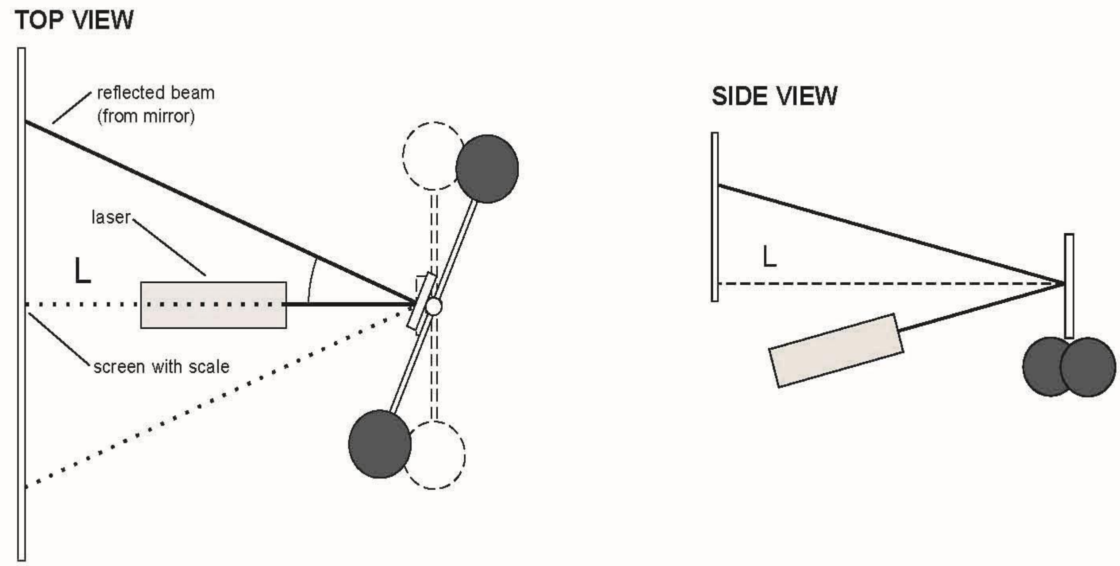

Figure 8
Setting up the optical lever

Hints for speedier rotational alignments:

- Dampen any wild oscillations of the pendulum bob with the locking mechanisms, as described;
- Adjust the rotational alignment of the pendulum bob using small, smooth adjustments of the zero adjust knob;
- Exercise patience and finesse in your movements

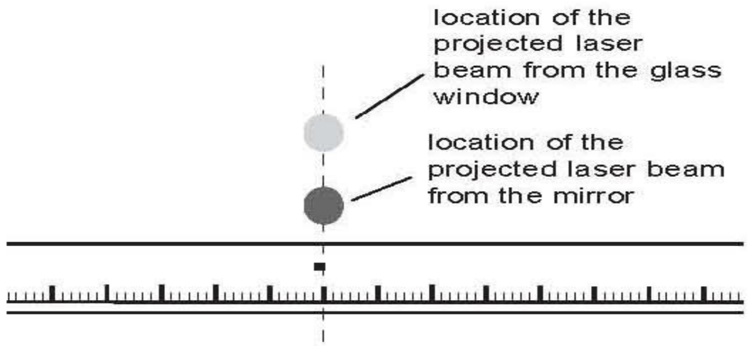

Figure 9
Ideal rotational alignment (zeroing) of the pendulum

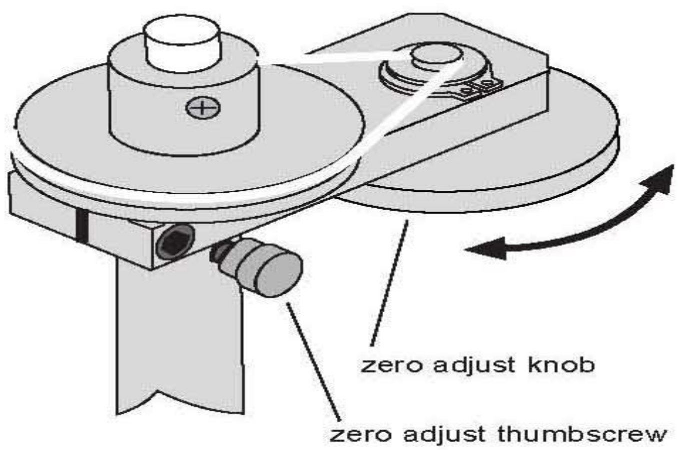

Figure 10
Refining the rotational alignment of the pendulum bob

Setting up for the Experiment

1. Take an accurate measurement of the distance from the mirror to the zero point on the scale on the projection surface (*L*) (Figure 8). (The distance from the mirror surface to the outside of the glass window is 11.4 mm.)

Note: Avoid jarring the apparatus during this setup procedure.

2. Attach copper wire to the grounding screw (Figure 11), and ground it to the earth.
3. Place the large lead masses on the support arm, and rotate the arm to Position I (Figure 12), taking care to avoid bumping the case with the masses.
4. Allow the pendulum to come to resting equilibrium.
5. You are now ready to make a measurement using one of three methods: the final deflection method, the equilibrium method, or the acceleration method.

   Note: The pendulum may require several hours to reach resting equilibrium. To shorten the time required, dampen the oscillation of the pendulum by smoothly raising the locking mechanisms up (by turning the locking screws) until they just touch the crossbar, holding for several seconds until the oscillations are dampened, and then carefully lowering the locking mechanisms slightly.

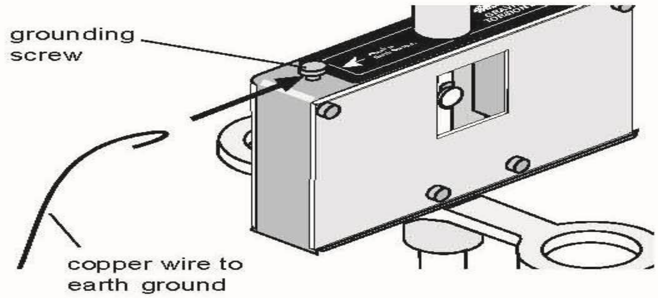

Figure 11
Attaching the grounding strap to the grounding screw

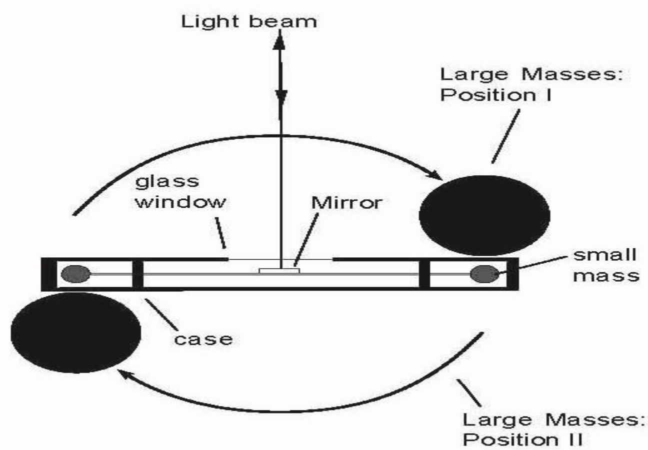

Figure 12
Moving the large masses into Position I

Measuring the Gravitational Constant

Over View of the Experiment

The gravitational attraction between a 15 gram mass and a 1.5 kg mass when their centers are separated by a distance of approximately 46.5 mm (a situation similar to that of the Gravitational Torsion Balance) is about 7 x 10^{-10} Newton's. If this doesn't seem like a small quantity to measure, consider that the weight of the small mass is more than two hundred million times this amount. Any of three methods can be used to determine the gravitational constant, *G*, from the motion of the small masses. In Method I, the final deflection method, the motion is allowed to come to resting equilibrium— a process that requires several hours — and the result is accurate to within approximately 5%. In method II, the equilibrium method, the experiment takes 90 minutes or more and produces an accuracy of approximately

5% when graphical analysis is used in the procedure. In Method III, the acceleration method, the motion is observed for only 5 minutes, and the result is accurate to within approximately 15%.

Note1: 5% accuracy is possible in Method I if the experiment is set up on a sturdy table in an isolated location where it will not be disturbed by vibration or air movement.

Note2: 5% accuracy is possible in Method II if the resting equilibrium points are determined using a graphical analysis program.

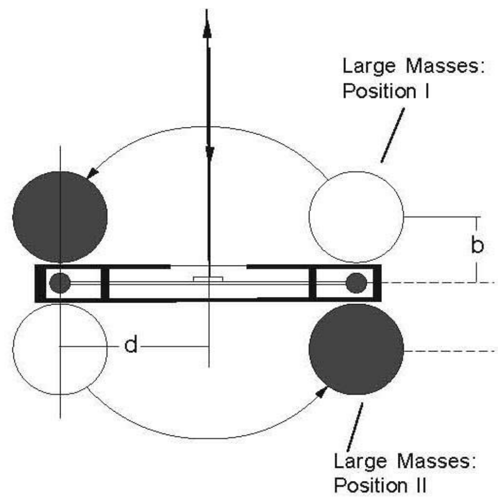

Figure 13
Origin of variables *b* and *d*

METHOD I: Measurement by Final Deflection

Setup Time – 45 minutes; Experiment Time: several hours

Accuracy: 5 %

Theory

With the large masses in Position I (Figure 13), the gravitational attraction, F between each small (m) and is neighboring large mass (m_1) is given by the law of universal gravitation:

$$F= G\, m_1\, m_2/b^2 \qquad (1.1)$$

Where b= the distance between the centers of the two masses.

The gravitational attraction between the two small masses and their neighboring large masses produces a net torque (τ_{grav}) on the system

$$\tau_{grav} = 2Fd \qquad (1.2)$$

Since the system is in equilibrium, the twisted torsion band must be supplying an equal and opposite torque. This torque(τ_{band}) is equal to the torsion constant for the band (k) times the angle through which it is twisted (θ), or

$$\tau_{band} = -k\theta \qquad (1.3)$$

Combining equations 1.1, 1.2, and 1.3, and taking into account that $\tau_{grav} = -\tau_{band}$, gives:

$$k\theta = 2dGm_1m_2/b^2$$

Rearranging this equation gives an expression for *G*:

$$G = \frac{k\theta b^2}{2dm_1m_2} \qquad (1.4)$$

To determine the values of *k* and θ -the only unknowns in equation 1.4 — it is necessary to observe the oscillations of the small mass system when the equilibrium is disturbed. To disturb the equilibrium (from S_1), the swivel support is rotated so the large masses are moved to Position II. The system will then oscillate until it finally slows down and comes to rest at a new equilibrium position (S_2) (Figure 14). At the new equilibrium position S_2, the torsion wire will still be twisted through an angle θ, but in the opposite direction of its twist in Position I, so the total change in angle is equal to 2θ. Taking into account that the angle is also doubled upon reflection from the mirror (Figure 15):

$$\Delta S = S_2 - S_1,$$

$$4\theta = \Delta S/L \qquad \text{or}$$

$$\theta = \Delta S/4L \qquad (1.5)$$

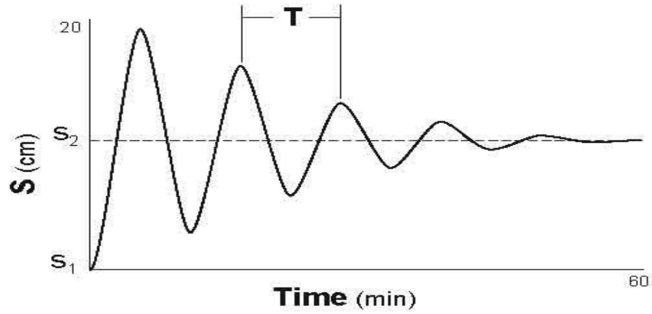

Figure 14
Graph of Small Mass Oscillations

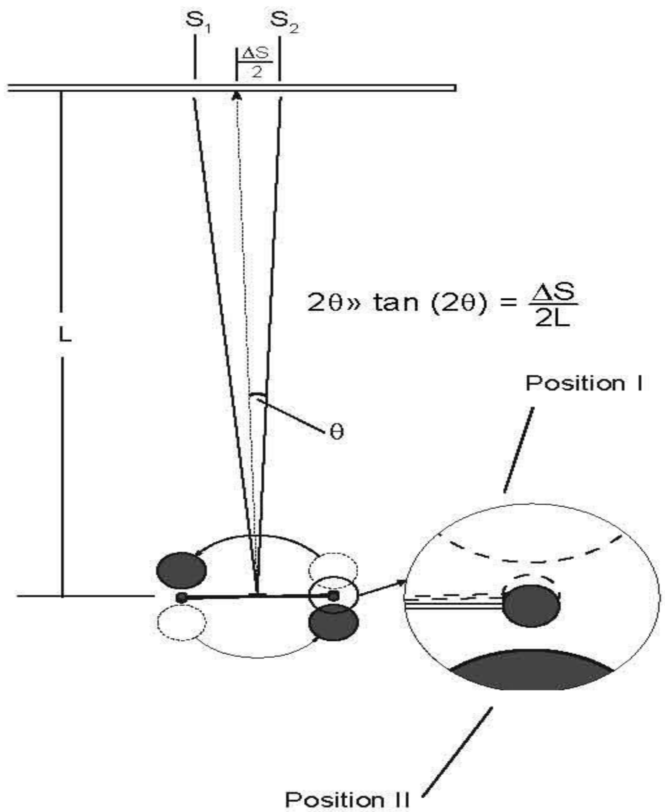

Figure 15
Diagram of the experiment showing the optical lever.

The torsion constant can be determined by observing the period (*T*) of the oscillations, and then using the equation:

$$T^2 = 4\Pi^2 I/K \tag{1.6}$$

Where, *I* is the moment of inertia of the small mass system.

The moment of inertia for the mirror and support system for the small masses is negligibly small compared to that of the masses themselves, so the total inertia can be expressed as:

$$I = 2m_2\left(d^2 + \frac{2}{5r^2}\right) \tag{1.7}$$

Therefore,

$$K = \frac{8\Pi^2 m_2\left(d^2+\frac{2}{5r^2}\right)}{T^2} \tag{1.8}$$

Substituting equations 1.5 and 1.8 into equation 1.4 gives:

$$G = \Pi^2 \Delta S b^2 \frac{\left(d^2+\frac{2}{5r^2}\right)}{T^2 m_1 L d} \tag{1.9}$$

All the variables on the right side of equation 1.9 are known or measurable:

r = 9.55 mm

d = 50 mm

b = 46.5 mm

m_1 = 1.5 kg

L = (Measure as in step 1 of the setup)

By measuring the total deflection of the light spot (ΔS) and the period of oscillation (T), the value of G can therefore be determined.

Procedure-

1. Once the steps for leveling, aligning, and setup have been completed (with the large masses in Position I), allow the pendulum to stop oscillating.
2. Turn on the laser and observe the Position I end point of the balance for several minutes to be sure the system is at equilibrium. Record the Position I end point (S_1) as accurately as possible, and indicate any variation over time as part of your margin of error in the measurement.

3. Carefully rotate the swivel support so that the large masses are moved to Position II. The spheres should be just touching the case, but take care to avoid knocking the case and disturbing the system.

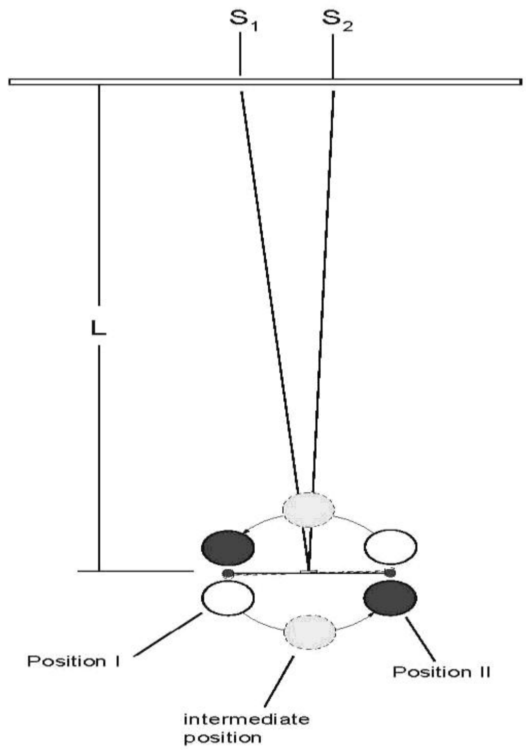

Figure 16
Two-step process of moving the large masses to reduce the time required to stop oscillating

Note: You can reduce the amount of time the pendulum requires to move to equilibrium by moving the large masses in a two-step process: first move the large masses and support to an intermediate position that is in the midpoint of the total arc (Figure 16), and wait until the light beam has moved as far as it will go in the period; then move the sphere across the second half of the arc until the large mass support just touches the case. Use a slow, smooth motion, and avoid hitting the case when moving the mass support.

4. Immediately after rotating the swivel support, observe the light spot and record its position (S_1).

5. Use a stop watch to determine the time required for one period of oscillation (T). For greater accuracy, include several periods, and then find the average time required for one period of oscillation.

Note: The accuracy of this period value (T) is very important, since the T is squared in the calculation of G.

6. Wait until the oscillations stop, and record the resting equilibrium point (S_2).

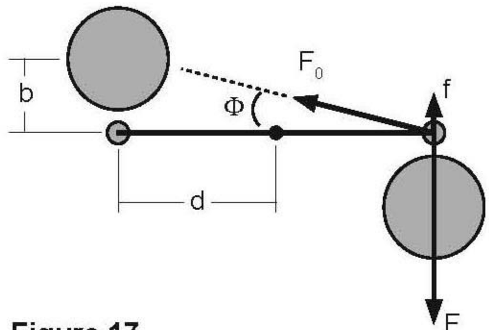

Figure 17
Correcting the measured value of *G*

Analysis:

1. Use your results and equation 1.9 to determine the value of *G*.
2. The value calculated in step 2 is subject to the following systematic error. The small sphere is attracted not only to its neighboring large sphere, but also to the more distant large sphere, though with a much smaller force. The geometry for this second force is shown in Figure 17 (the vector arrows shown are not proportional to the actual forces).

From figure 17-

$$f = \mathrm{F}_0 \sin \emptyset$$

$$sin\emptyset = \frac{b}{(b^2 + 4d^2)^{1/2})}$$

The force, F_0 is given by the gravitational law, which translates, in this case, to:

$$F = \frac{Gm_2m_1}{(b^2 + 4d^2)^{1/2}}$$

And has a component f that is opposite to the direction of the force F :

$$f = \frac{Gm_2m_1b}{(b^2 + 4d^2)(b^2 + 4d^2)^{\frac{1}{2}}} = \beta F$$

This equation defines a dimensionless parameter, b that is equal to the ratio of the magnitude of f to that of F. Using the equation $F = Gm_1\, m_2 /b^2$, it can be determined that:

$$b = \frac{b^3}{(b^2+4d^2)^{3/2}}$$

From figure 17,

Fnet = F - f = F - bF = F(1 - b)

Where F_{net} is the value of the force acting on each small sphere from *both* large masses, and *F* is the force of attraction to the nearest large mass only.

Similarly,

$$G = G_0(1 - b)$$

Where G is your experimentally determined value for the gravitational constant, and G_0 is corrected to account for the systematic error.

Finally,

$$G_0 = \frac{G}{(1 - b)}$$

Use this equation with equation 1.9 to adjust your measured values.

METHOD II: Measurement by Equilibrium Positions

Observation Time: ~ 90+ minutes

Accuracy: ~ 5 %

Theory:

When the large masses are placed on the swivel support and moved to either Position I or Position II, the torsion balance oscillates for a time before coming to rest at a new equilibrium position. This oscillation can be described by a damped sine wave with an offset, where the value of the offset represents the equilibrium point for the balance. By finding the equilibrium point for both Position I and Position II and taking the difference, the value of □ *S* can be obtained. The remainder of the theory is identical to that described in method I.

Procedure

1. Set up the experiment following steps 1–3 of Method I.
2. Immediately after rotating the swivel support to Position II, observe the light spot. Record the position of the light spot (*S*) and the time (*t*) every 15 seconds. Continue recording the position and time for about 45 minutes.
3. Rotate the swivel support to Position I. Repeat the procedure described in step 2.

Note: Although it is not imperative that step 3 be performed immediately after step 2, it is a good idea to proceed with it as soon as possible in order to minimize the risk that the system will be disturbed between the two measurements. Waiting more than a day to perform step 3 is not advised.

Analysis

1. Construct a graph of light spot position versus time for both Position I and Position II. You will now have a graph similar to Figure 18.
2. Find the equilibrium point for each configuration by analyzing the corresponding graphs using graphical analysis to extrapolate the resting equilibrium points S_1 and S_2 (the equilibrium point will be the center line about which the oscillation occurs). Find the difference between the two equilibrium positions and record the result as ΔS.
3. Determine the period of the oscillations of the small mass system by analyzing the two graphs. Each graph will produce a slightly different result. Average these results and record the answer as T.
4. Use your results and equation 1.9 to determine the value of *G*.
5. The value calculated in step 4 is subject to the same systematic error as described in Method I. Perform the correction procedure described in that section (*Analysis, step 3*) to find the value of G_0.

Note: To obtain an accuracy of 5% with this method, it is important to use graphical analysis of the position and time data to extrapolate the resting equilibrium positions, S_1 and S_2.

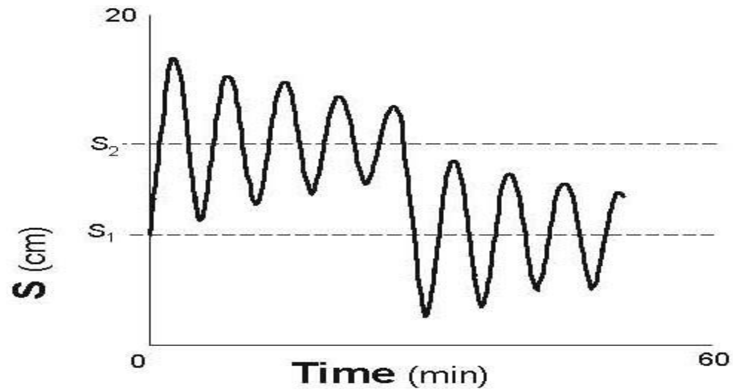

Figure 18
Typical pendulum oscillation pattern showing equilibrium positions

METHOD III:

Measurement by Acceleration

Observation Time: ~ 5 minutes

Accuracy: ~ 15%

Theory

With the large masses in Position I, the gravitational attraction, F, between each small mass (m_2) and its neighboring large mass (m_1) is given by the law of universal gravitation:

$$F = Gm_1m_2 /b^2 \qquad (3.1)$$

This force is balanced by a torque from the twisted torsion ribbon, so that the system is in equilibrium. The angle of twist, θ is measured by noting the position of the light spot where the reflected beam strikes the scale. This position is carefully noted, and then the large masses are moved to Position II. The position change of the large masses disturbs the equilibrium of the system, which will now oscillate until friction slows it down to a new equilibrium position.

Since the period of oscillation of the small masses is long (approximately 10 minutes), they do not move significantly when the large masses are first moved from Position I to Position II. Because of the symmetry of the setup, the large masses exert the same gravitational force on the small masses as they did in Position I, but now in the opposite direction. Since the equilibrating force from the torsion band has not changed, the total force (F_{total}) that is now acting to accelerate the small masses is equal to twice the original gravitational force from the large masses, or:

$$F_{total} = 2F = 2Gm_1m_2/b^2 \qquad (3.2)$$

Each small mass is therefore accelerated toward its neighboring large mass, with an initial acceleration (a_0) that is expressed in the equation:

$$m_2a_0 = 2Gm_1 m_2/b^2 \qquad (3.3)$$

Of course, as the small masses begin to move, the torsion ribbon becomes more and more relaxed so that the force decreases and their acceleration is reduced. If the system is observed over a relatively long period of time, as in Method I, it will be seen to oscillate. If, however, the acceleration of the small masses can be measured before the torque from the torsion ribbon changes appreciably, equation 3.3 can be used to determine *G*. Given the nature of the motion—damped harmonic—the initial acceleration is constant to within about 5% in the first one tenth of an oscillation. Reasonably good results can therefore be obtained if the acceleration is measured in the first minute after rearranging the large masses, and the following relationship is used:

$$G = b^2a_0/2m_1 \qquad (3.4)$$

The acceleration is measured by observing the displacement of the light spot on the screen. If, as is shown in Figure 19:

Δs = the linear displacement of the small masses,

d = the distance from the center of mass of the small masses to the axis of rotation of the torsion balance,

ΔS = the displacement of the light spot on the screen, and

L = the distance of the scale from the mirror of the balance,

Then, taking into account the doubling of the angle on reflection,

$$\Delta S = \Delta s(2L/d) \qquad (3.5)$$

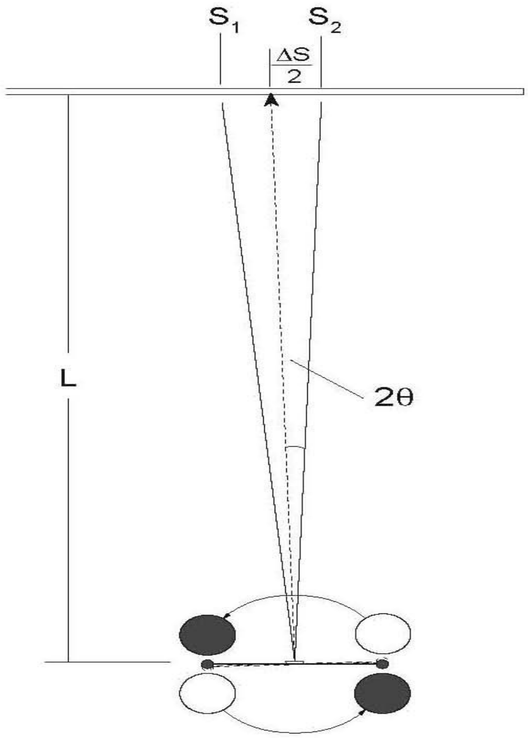

Figure 19
Source of data for calculations in Method III

Using the equation of motion for an object with a constant acceleration ($x = 1/2\ at^2$), the acceleration can be calculated:

$a_0 = 2\Delta s/t^2 = \Delta Sd/t^2L$ (3.6)

By monitoring the motion of the light spot over time, the acceleration can be determined using equation 3.6, and the gravitational constant can then be determined using equation 3.4.

Procedure

1. Begin the experiment by completing steps 1-3 of the procedure detailed in Method I.
2. Immediately after rotating the swivel support , observe the light spot. Record the position of the light spot (S) and the time (t) every 15 seconds for about two minutes.

Analysis

1. Construct a graph of light spot displacement ($\Delta S = S - S_1$) versus time squared (t^2), with t^2 on the horizontal axis (Figure 20). Draw a best –fit line through the observed data points over the

first minute of observation.

2. Determine the slope of your best –fit line.

3. Use equations 3.4 and 3.6 to determine the gravitational constant.

4. The value calculated in step 3 is subject to a systematic error. The small sphere is attracted not only to its neighboring large sphere, but also to the more distant sphere, although with a much smaller force. Use the procedure detailed in Method I (Analysis, step 3) to correct for this force.

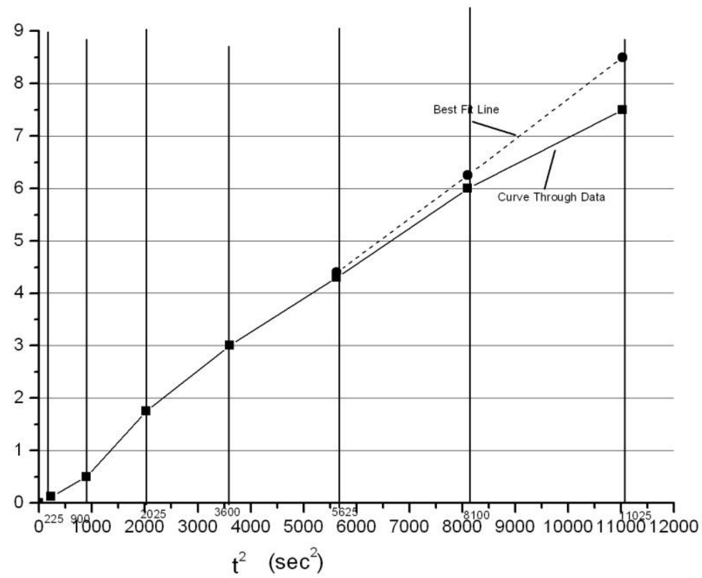

Figure 20: Sample data and best –fit line.

References:

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2. B.E. Clotfelter, The Cavendish experiment as Cavendish knew it, Am. J. Phys 55, 210, 1987.
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6. Boys, C.V. "The Newtonian Constant of Gravitation", *Nature*, 50, 571, Oct. 1894.

7. Cavendish, H. "Experiments to Determine the Density of the Earth" in *Scientific Memoirs The Laws of Gravitation* A.Stanley Mackenzie ed. 1900. Digitized by Google.

8. Clotfelter, B.E. "The Cavendish experiment as Cavendish knew it", *Am. J. Phys*. **55**, 210, March 1987

14. Thermoelectric Power Experiment

Objective: To calculate thermoelectric power (Q), Fermi Energy (E_f) and carrier concentration (n) of a given ferrite sample.

Apparatus: Thermoelectric power setup, Microvoltmeter, Thermocouple, Temperature indicator, Variac.

Theory:

Thermoelectric Power:

A temperature gradient between two ends of a ferrite material gives rise to an emf known as thermo emf (V_s). It is also called as seebeck voltage. It develops due to the majority of carriers diffused from a surface having high temperature (T_2) to a surface with relatively low temperature (T_1). It has been observed that the generated thermo emef is proportional to the temperature difference (T_2- T_1) between two surfaces of the ferrite and is given by the relation,

$$V_s = Q\,(T_2 - T_1)$$

Or $$V_s = Q\Delta T \qquad (1)$$

Where, Q is the seebeck coefficient or thermoelectric power and has the dimension of volt per degree.

Fermi Energy: When the conduction is only due to one type of charge carriers i.e. electrons, the Fermi energy of an n-type semiconductor is related to the seebeck coefficient (Q) by a relation,

$$QT = E_g - E_f + 2KT \qquad (2)$$

And $$QT = E_f - KT \qquad (3)$$

For a p-type semiconductor, where

E_g= Energy gap of ferrite semiconductor.

E_f = Height of Fermi energy level from the top of the filled valency band and

2KT= The term which accounts for the transfer of K.E. of the ferrite to a cold one.

It has been assumed that the conduction in materials like-NiO, CoO, MgO, α-Fe_2O_3 etc, may take place in exceedingly narrow bands or in localized levels. This assumption leads to the result that the kinetic energy term in equation (2) and (3) may be neglected. As ferrites are also supposed to be narrow band magnetic semiconductors we can write that,

$E_f = E_g - QT$ (n-type semiconductors) (4)

$E_f = QT$ (p-type semiconductors) (5)

Carrier concentration: In the case of low mobility semiconductors such as ferrites, the activation energy is often associated with the mobility of charge carriers are considered as localized at the ions or vacant sites and the conduction occurs via a hopping type process which implies a thermally activated electronic mobility. In such cases, it is appropriate to consider small polarons as charge carriers rather than electrons/holes. Further, it is known that the concentration (n) is given by

$$Q = -\frac{K}{e}\left[\frac{ln\beta\left(\frac{N}{n}\right)}{n} + \frac{S_T}{K}\right] \qquad (6)$$

S_T= Entropy transport term, which is negligible for ferrite materials. Therefore S_T/K is neglected.

N= density of states or number of available sites.

K= Boltzmann constant

e = electronic charge

β = degeneracy factor which includes both spin and orbital degeneracy and its value is taken as unity (β=1)

Considering that n<<N we can reduce the above formula to

$$n = N \exp(Qe/K) \qquad (7)$$

If V is the volume of the sample and the value of K/e is found to be 86.4 μV/K, the equation

$$n = N/V \exp(Q/86.4) \qquad (8)$$

Where, Q is in μV/K.

In the case of (low mobility semiconductor like) ferrites having exceedingly narrow bands or localized levels, value of N, the density of states can be taken as 10^{22} Cm^{-3}

A point hot probe is used here. Since ferrite samples are very good thermal conductors if a pointed probe is not used the upper and the lower surfaces of the samples will attain almost the same temperature and no temperature gradient will be maintained between them.

In the case of n-type semiconductor material the hot surface becomes positively charged, as it loses some of its electrons. The cold surface of the semiconductor becomes negatively charged due to the diffusion of free electrons from the hot portion. Conversely, in a p-type semiconducting material, the hot surface becomes negative, and the cold one positive. Thus the type of conduction in a given semi-conducting material can readily be determined from the sign of the thermo emf.

The values of the thermo emf have been noted while cooling because the sample will attain sufficient thermal stability while cooling rather than while heating. The sample is maintained at a given temperature for about 10 minutes. The temperature of two surfaces have been measured with the help of a thermocouple.

PROCEDURE:

The experimental set up to determine thermo emf of ferrite sample is shown

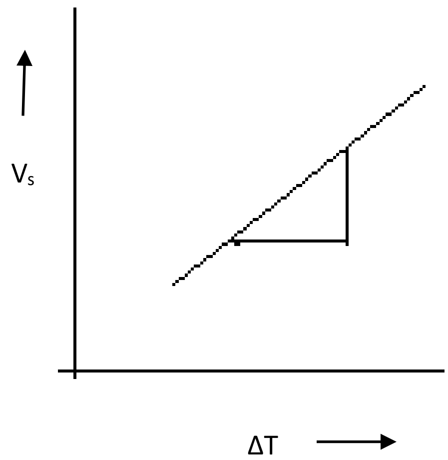
It consists of point contact probe (A) which acts as a hot junction and a base (D) which acts as cold junction. Between the two junction a ferrite sample (C) is kept. The ferrites of approximately 0.3X 0.3 Sq. Cm and 0.2X 0.3 Sq Cm thicknesses are used in the present investigation. The hot probe and cold junction (base) are connected to an electrometer (B) for measuring the thermo emf.

The temperature of the hot probe is raised with the help of an electric heater, which is wound round the hot probe. Slowly increase the voltage of the dimmerstat and see that hot junction temperature attains 250 ^{o}C.

Then record the temperatures of hot and cold junctions and the thermo emf at these temperatures.

Now decrease the voltage of the dimmer stat in step of 10 volts at regular intervals of 10 minutes and thus note the temperatures and thermo emf while cooling upto room temperature.

Draw the graph between difference of temperature ΔT of hot and cold junctions at different temperature and corresponding emf. Calculate the slope of the curve. This slope value gives thermoelectric power (Q) or Seebeck coefficient. Calculate the Fermi energy E_r using (4) and (5) and concentration of change carriers equation (7).



Observations:

Sl. No.	Temperature		ΔT	Vs
	Cold junction (T_1)	Hot junction (T_2)		

Draw a graph between ΔT (on X-axis) and Vs (on Y-axis) and determine the slope of the curve which gives the seebeck coefficient Q = Slope (Vs/ΔT)

Volume of specimen V =……………..cm^3.

Fermi energy E_f = QT = for p-type semiconductor

E_g -E_f=QT = for n-type semiconductor

Carrier Concentration (η) = (N/V) exp (Q/86.4) =……….cm^3

N= 10^{22} cm^3

Result: (a) Thermoelectric power (Q) =

(b) Fermi energy (E_f)

(c) Carrier concentration (η) =

Precaution:

a. Increase in temperature must be slow.
b. See that the temperature should remain constant at least for few minutes, while taking the readings.

Question:

1. What do you mean by Seebeck, Peltier and Thomson’s effect?
2. What is the basic principle involved in the above three effects?
3. What is the difference between photoelectric effect & thermoelectric effect?
4. Mention a few applications of thermoelectric power?

15. Measurement of Dielectric Constant

Objective: To measure the dielectric constant of a given sample.

Apparatus: dielectric constant measurement setup, Capacitance Meter and a Sample.

Theory: Research in the area of Ferroelectrics is driven by the market potential of next generation memories and transducers. Thin films of ferroelectrics and dielectrics are rapidly emerging in the field of MEMS applications. Ultrasonic micro-motors utilizing PZT thin films and pyroelectric sensors using micro-machined structures have been fabricated. MEMS are finding growing application in accelerometers for air bag deployment in cars, micro- motors and pumps, micro heart valves, which have reached the commercial level of exploitation in compact medical, automotive, and space applications. Extremely sensitive sensors and actuators based on thin film and bulk will revolutionize every walk of our life with Hi-Tech gadgets based on ferroelectrics. Wide spread use of such sensors and actuators have made Hubble telescope a great success story. New bulk ferroelectric and their composites are the key components for the defense of our air space, the long coastline and deep oceans. The quest of human beings for developing better and more efficient materials is never ending. Material Science has played a vital role in the development of society. Characterization is an important step in the development of different types of new materials. This experiment is aimed to expose the young students to Dielectric and Curie Temperature Measurement technique for Ferroelectric Ceramics. Dielectric or electrical insulating materials are understood as the materials in which electrostatic fields can persist for a long time. These materials offer a very high resisitance to the passage of electric current under the action of the applied *direct-current* voltage and therefore sharply differ in their basic electrical properties from conductive materials. Layers of such substances are commonly inserted into capacitors to improve their performance, and the term dielectric refers specifically to this application. The use of a dielectric in a capacitor presents several advantages. The simplest of these is that the conducting plates can be placed very close to one another without risk of contact. Also, if subjected to a very high electric field, any substance will ionize and become a conductor. Dielectrics are more resistant to ionization than air, so a capacitor containing a dielectric can be subjected to a higher voltage. Also, dielectrics increase the capacitance of the capacitor. An electric field polarizes the molecules of the dielectric (Figure-1), producing concentrations of charge on its surfaces that create an electric field opposed (antiparallel) to that of the capacitor. Thus, a given amount of charge produces a weaker field between the plates than it would without the dielectric, which reduces the electric potential. Considered in reverse, this argument means that, with a dielectric, a given electric potential causes the capacitor to accumulate a larger charge.

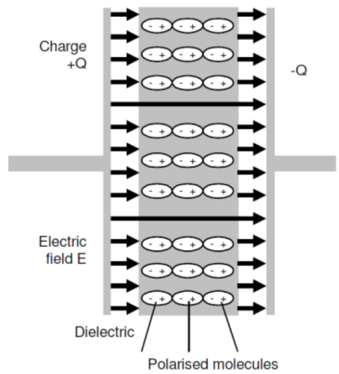


Figure 1- An electric field polarizes the molecules of the dielectric

The electrons in the molecules shift toward the positively charged left plate. The molecules then create a leftward electric field that partially annuls the field created by the plates. (The air gap is shown for clarity; in a real capacitor, the dielectric is in direct contact with the plates.)

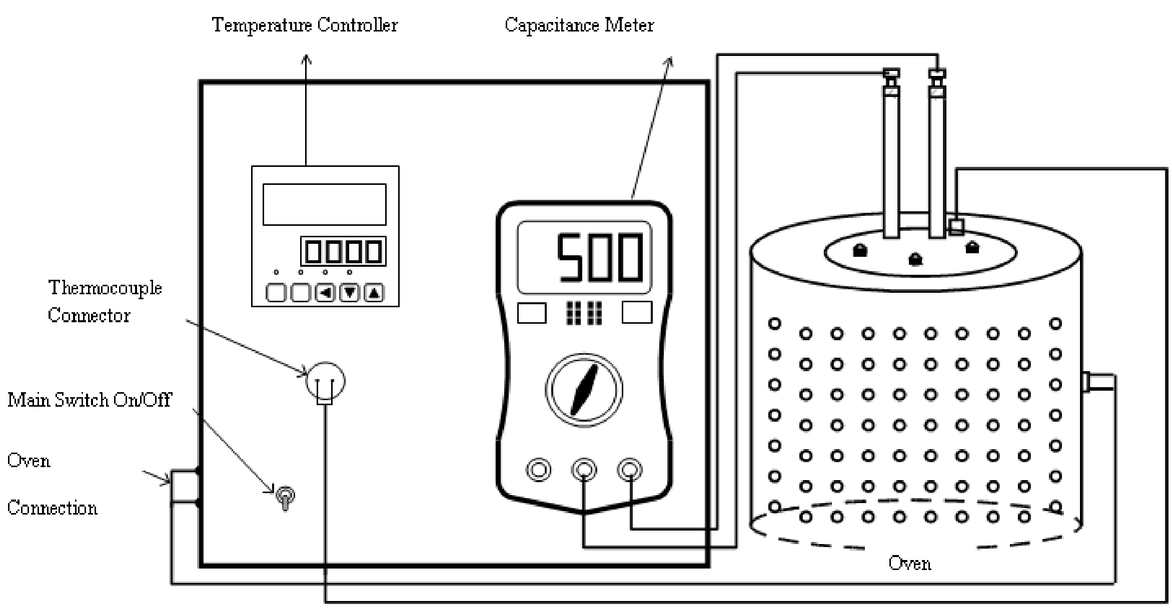


Figure 2- Panel Diagram of Measurement of Dielectric Constant Set-up

Required equipment/components:

1. Measurement of Dielectric Constant
2. Probe Arrangement with Thermocouple Sensor
3. Sample: PZT
4. Aluminum Foil
5. High Temperature Oven

Dielectric Constant:

The dielectric constant (ε) of a dielectric material can be defined as the ratio of the capacitance using that material as the dielectric in a capacitor to the capacitance using a vacuum as the dielectric. Typical values of ε for dielectrics are:

Material	Dielectric Constant (ε)
Vacuum	1.000
Dry Air	1.0059
Barium Titanate	100-1250
Glass	3.8-14.5
Quartz	5
Mica	4-9
Water distilled	34-78
Soil dry	2.4-2.9
Titanium dioxide	100

Dielectric constant (ε) is given by

$$\varepsilon = \frac{C}{C_o}, \qquad C_o = \frac{\varepsilon_o A}{t}$$

Where

C = capacitance using the material as the dielectric in the capacitor, C_0 = capacitance using vacuum as the dielectric, ε_0 = Permittivity of free space (8.85×10^{-12} F/m), A = Area of the plate/ sample cross section area, t = Thickness of the sample

Experimental Procedure:

1. Make the top and bottom surfaces of the sample pellet conducting by applying silver paste and wait till it dries.
2. Put a small piece of aluminum foil on the base plate. Pull the spring loaded probes upward, insert the aluminum foil and let them rest on it. Put the sample on the foil. Again pull the top of one of the probe and insert the sample below it and let it rest on it gently. Now one of the probes would be in contact with the upper surface of the sample, while the other would be in contact with the lower surface through aluminum foil.
3. Connect the probe leads to the capacitance meter.
4. Connect the oven to the main unit and keep it in OFF position.
5. Switch on the main unit and note the value of capacitance. It should be a stable reading and is obtained directly in pf.
6. Adjust the desired set temperature in the temperature controller as shown in Fig. 2. The controller will slowly reach the desired temperature. Take the capacitance reading and continue the experiment by adjusting different temperature.

Observations and Calculations:

Sample : PZT, **Dia** : 9.97 mm, **Area (A)** : 78.03 mm^2, **Thickness (t)** : 2.45 mm

Permittivity of Space (ε_0): 8.85 x 10^{-12} F/m or 8.85 x 10^{-3} pf/ mm

$$\varepsilon = \frac{C}{C_o}, \quad \text{where} \quad C_o = \frac{\varepsilon_0 A}{t} = \frac{8.85x10^{-3}x78.03}{2.45} = 281.9x10^{-3}pf$$

Typical Results:

1. A plot of temperature v/s dielectric constant of a typical sample (PZT) is shown in Fig. 3.
2. From the graph, **Curie Temperature** (T_C) = 370°C

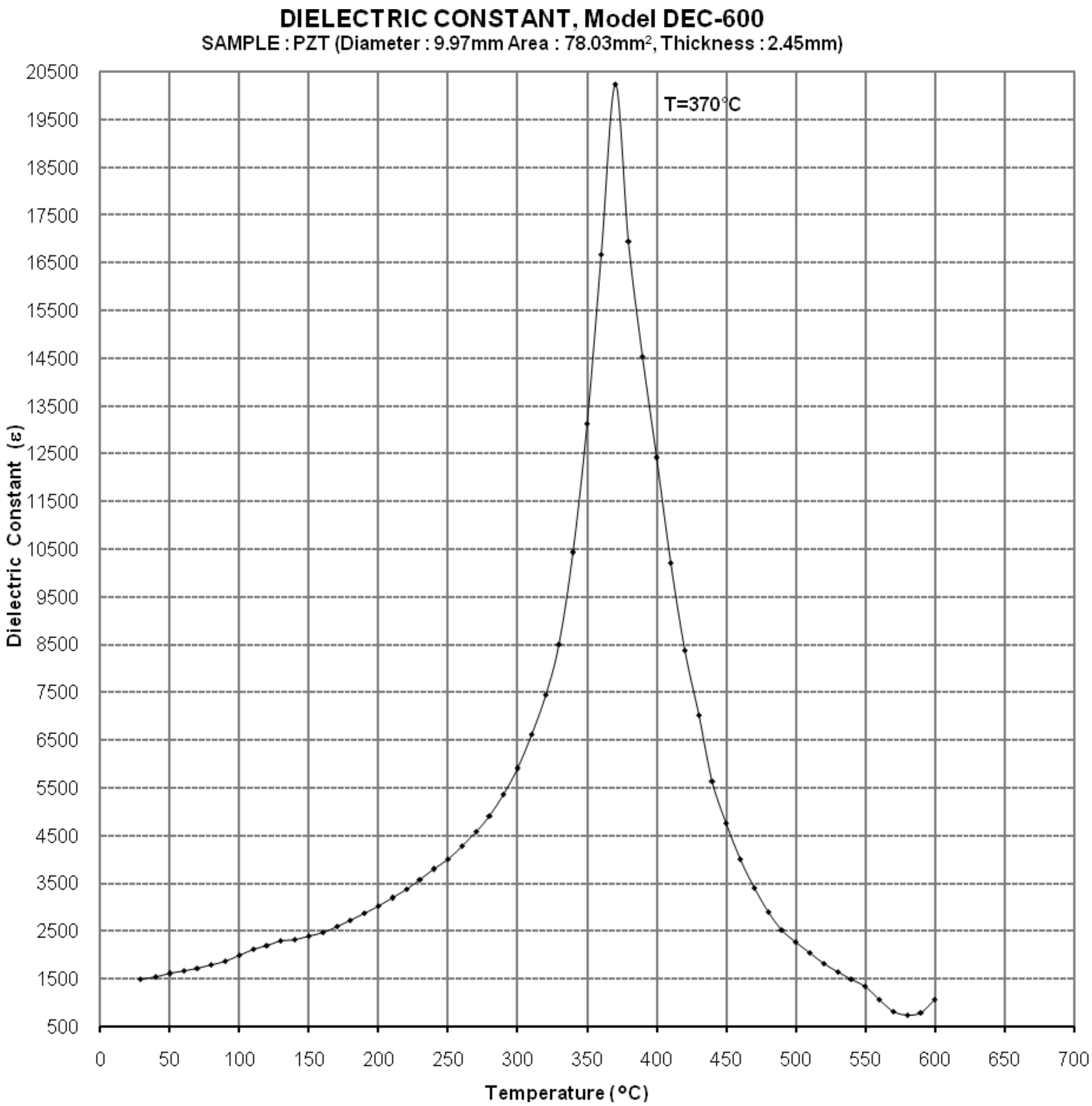


Figure 3- Variation of dielectric constant of PZT as a function of temperature.

Precautions:

The spring loaded probe should be allowed to rest on the sample very gently, other wise it may damage the conducting surface of the sample or even break the sample.

Reference:

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16. Johnson Noise Measurement

Objective: (1) To measure the Johnson Noise dependence on resistance at Room Temperature and fixed bandwidth.

(2) To measure the Johnson noise dependence on bandwidth at Room Temperature and fixed resistance.

(3) Obtain the amplifier Noise

(4) Calculate Johnson noise density and Boltzmann's constant.

Apparatus: High level electronics, Low level electronics, Digital Oscilloscope, Multimeter BNC cables, electronic components etc.

Introduction:

Just as a weed is an unwanted plant, a noise, *ordinarily speaking*, is an unwanted sound. In the fields of physics, electrical engineering, and many other places, we extend the definition of 'noise' beyond acoustics to the general field of information. Since almost any signal that's a function of time can be translated into a voltage, we will often use the concept of a voltage signal. We'll call it a 'noisy signal' if, in addition to the voltage we expect or wish to see, there is unwanted, typically (but not always) a randomly-fluctuating, voltage. Surprisingly, the noise signal is sometimes not only wanted, but is the essence of the measurement.

There are several kinds of noise. One of them is 'interference', which is the presence of an unwanted signal, added to the desired signal. It's easy to imagine that your neighbor's electronic apparatus is polluting your TV or radio signal with some sort of interference. The kind of interference students are likely to encounter in these experiments probably comes from three sources: electrostatic coupling to the apparatus from fluorescent lights in the laboratory, electromagnetic coupling due to nearby transformers or motors, and vibrational coupling due to microphonic components within the unit.

Another source of noise we will call 'technical noise' since it is the noise generated by the technique of the investigation, or that gets into the circuits due to *faulty* experimental techniques. For example, a student's failure to tighten the cover on the preamplifier section, or a poor electrical connection to the first-stage op-amp, can add extraneous noise to the signal path.

Of greatest interest to us is 'fundamental noise', noise that is intrinsic and inevitable because of the physical nature of an apparatus. We'll observe noise sources that arise from the Second Law of Thermodynamics, and from the quantization of electrical charge. Physicists and electrical engineers know these as Johnson and shot noise respectively. Noise sources like this display the characteristics of non-periodic, unpredictable, random waveforms, but nevertheless conforming, in their statistical properties, to universal laws.

Fundamental noise is especially worthy of study, for at least two reasons. The first reason is that fundamental noise presents us with a physics-based limit on the degree to which we can measure in a given experiment. In many cases in research and technology, it often defines what is possible within the limits of physical law. In particular, fundamental noise can and does set limits to the rate of data-transfer in a host of contexts in communication.

The second reason we care about noise is that it becomes possible to use noise to measure the values of some fundamental constants. Boltzmann's constant k_B can be determined from the voltage or Johnson noise of resistors; and the magnitude of the charge on the electron, e, can be determined from the current or shot noise of a photocurrent.

But measurement of 'fundamental noise' has its experimental challenges. There is a saying about noise measurements: 'you're either measuring too much or too little signal'. You will understand this quip better after you have had some experience with these measurements. Our advice here is to read some of the references and do your measurements carefully.

Johnson noise at room temperature:

It is known that $V = i R$, which really says that there's a potential difference ΔV across any resistor R which has a current i passing through it. This of course predicts a ΔV of *zero* for a resistor with no current. But for deep reasons, any actual resistor at any temperature above absolute zero, will display a 'noise voltage' $V_J(t)$ across its terminals, a potential difference that has all the character of an internal (a.c.) *emf* built into the resistor. The emf which the resistor generates is called 'Johnson noise', and it arises because of the deep thermodynamic connection between dissipation (which any resistor surely has) and fluctuations (which here show up as a fluctuating emf). The size of this emf is also predicted by fundamental theory, and it should not surprise you to learn that $V_J(t)$ is, *on average*, zero. But $V_J(t)$ exhibits fluctuations, positive and negative, about that average value of zero. To quantify these, we form the (always-positive) *square* of $V_J(t)$ and time-average that, giving a 'mean square' voltage which we denote as $\langle V_J^2(t)\rangle$. The predicted value for $\langle V^2(t)\rangle$ was first deduced by Nyquist, following

Johnson's empirical discovery of the noise, and it's given by the expression

$$<V_j^{\,2}(t)> = 4\, k_B\, R\;\, T\, \Delta f$$

Here k_B is Boltzmann's constant, T is the (absolute) temperature of the resistor, and Δf is the novel factor -- it is the 'bandwidth' used in the measurement electronics.

The involvement of bandwidth Δf is a first hint that 'noise' is quite distinct from 'signal'. Everyone starts with 'd.c. signals', which have nothing but a sign and a value, in Volts. Then there are 'a.c. signals', which have a magnitude (perhaps specified by amplitude, or rms value, or peak-to-peak excursion) but also a *frequency*, or a mixture of frequencies. But it is the essence of fundamental noise that it contains, or is composed of, *all frequencies*. In fact, the amount of energy we can get out of a 'noise source' depends on the *range* of frequencies to which we arrange to be sensitive, and this is the reason for the inclusion of the bandwidth-factor Δf in the expression above.

How large a Johnson-noise voltage should we expect from a typical resistor? Let's calculate this mean-square voltage for a 100 kΩ resistor at room temperature. Suppose that our electronics for detecting and measuring $V_J\,(t)$) are fully sensitive to all frequencies from 0 to 100 kHz, but entirely insensitive to higher frequencies. Then:

$T = 22\ ^{o}\text{C} = 295$ K

$k_B = 1.38 \times 10^{-23}$ J/K (textbook value)

$\Delta f\ = 100$ kHz $= 10^5$ Hz

$<V_j^{\,2}(t)> = 4\ (1.38\times 10^{-23}\ \text{J/K})\ (295\ \text{K})\ (10^5\ \Omega)\ (10^5 Hz)$

$= (1.63 \times 10^{-20}\ \text{J})\ (10^5\ \text{V/A})\ (10^5\ \text{/s})$

$= 1.63 \times 10^{-10}\ \text{V}^2$.

Not everyone is familiar with the curious unit of the square-of-a-Volt, so we often take the square root of this mean-square noise voltage, to give a 'root-mean-square' or 'rms' measure of the noise voltage,

$V_J(\text{rms}) = <V_j^{\,2}(t)>^{1/2} = 1.28\text{x}10^{-5}$ V

So if we have a room-temperature 100-kΩ resistor simply hooked up to an ideal voltmeter, and if that voltmeter responds to all (but only) frequencies under 100 kHz, then the voltmeter's instantaneous reading will *not* be zero volts, but instead will fluctuate (rapidly: in this case, on a microsecond time scale) around zero, with typical excursions of order ±10 μV. We further assert that this is an actual emf intrinsic to the resistor, and it will still be present, though typically unwanted, in addition to any iR-drop that the resistor may exhibit. It follows that measurement of any iR-drop to microVolt precision in such a case would require thinking about this effect.

There are many textbook derivations of Nyquist's prediction, and the best of them emphasize the connection to thermodynamics and to blackbody radiation. Here's a 'thought experiment' to help you see that some sort of Johnson noise must exist. First imagine a cubic meter of iron at room temperature and another cubic meter of cold iron (say, at temperature T = 4 K), spaced 10 meters apart in empty space. (If you like, think of them as located at the two focal points of a large evacuated ellipsoidal reflecting cavity which surrounds them both, and isolates them from the external universe.) It should be clear to you that each iron block is giving off blackbody radiation, with a range of frequencies and in all directions -- but that the warm block is giving off lots more. Since the blackbody radiation of each block will run into the other block, there will be a net flow of (radiant) energy from the warmer block to the colder one, and their temperatures will therefore start to equilibrate.

Now imagine a 50-Ω resistor at room temperature, connected to nothing but a lossless coaxial cable of 50-Ω impedance; and imagine there's another 50-Ω resistor, but down in a Dewar at T = 4 K, connected to the far end of this cable. Even if there is no thermal conductivity in the cable, there is still electrical conductivity. It's the 'Johnson emf' in each resistor which still acts like a black-body source, here generating travelling waves of (confined) radiation along the one-dimensional cable structure, and that 'radiation' is caught and dissipated in the far end's resistor. This is the mechanism by which the two resistors will tend toward thermal equilibrium, as the hotter resistor will experience a net outflow, and the colder a net inflow, of electrical energy.

'Seeing' Johnson noise

This exercise will let you see, directly on an oscilloscope, a time-dependent waveform which can be traced all the way back to the Johnson noise generated in a resistor. You'll need to ensure that you've restored the 'default condition' of the system for this to work.

You need to plug, into your 100-to-240-V outlet, the line cord of the universal power supply which supplies power to the high-level electronics (HLE). You should see a green LED on the transformer unit light up. Now connect the output of this supply to the receptacle on the back of the HLE. You should see a green LED on the front panel of the HLE light up. (Note there is no power switch in the HLE box; instead, it gets powered up as soon as you establish the power-supply connections.) Now find the power cable emerging from the LLE box, and plug it into the connector on the front panel of the HLE box. You should see a green LED light up on the front panel of the LLE. Once you have three green LEDs lit, everything in your system is being powered.

Set the switch to select a 'source resistor' of R_{in} = 100 kΩ in the pre-amplifier module installed in the LLE box. This resistor is connected only to the high-impedance input of the first stage of amplification in the pre-amp. That first stage is wired to give a 'gain', or amplification factor, of 6.00, *provided* you set the feedback resistor, Rf, to its 1-kΩ setting. (The feedback capacitance C_f is not connected in the default mode, so its setting is irrelevant.) Read the graphics on the panel of the pre-amp to see that there is an additional amplification stage, with gain 100., following this first stage. Now you can connect the pre-amp's output, by a coaxial cable, to an oscilloscope, to see if there is any signal present. Use a rather sensitive vertical scale on your 'scope (of perhaps 10 mV/division sensitivity), a sweep speed of 5 µs/div on the horizontal axis, and trigger near zero volts.

Below are the schematic, and the wiring, diagrams of the circuit you're using.

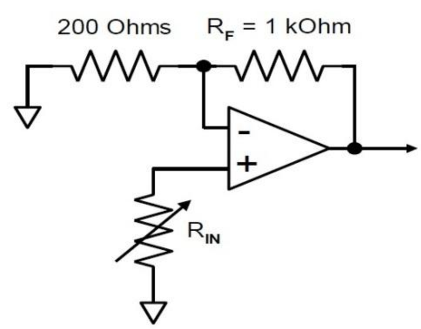


Fig 1: Johnson noise preamplifier schematic

The wiring diagram for this configuration is shown in Fig. 2. The connections indicated in grey-scale printing are those you need to check, or establish. (By contrast, connections shown in thin solid lines are already established for you on the printed- circuit boards.)

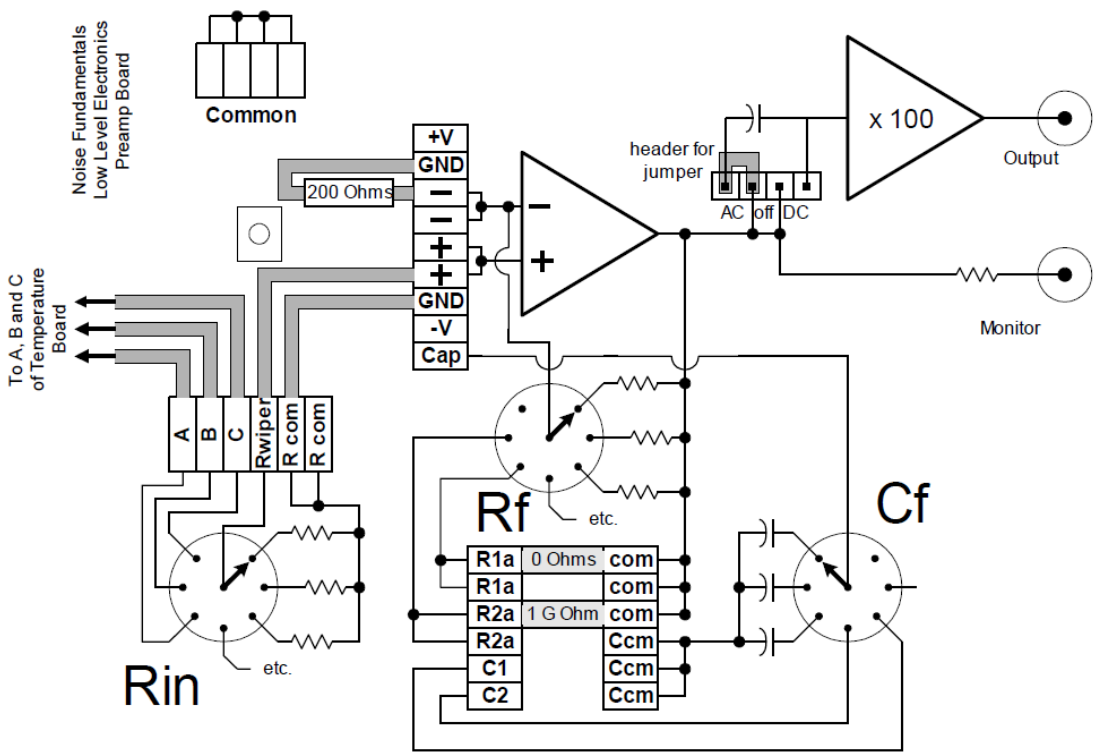


Fig. 2: Wiring diagram of the default condition of the interior of the low-level electronics.

The signals you've seen emerging from the pre-amp are rather small. So next use a BNC cable to convey the pre-amp output to the HLE box instead, where you can filter and amplify the still-small noise signals. If you use the settings and the cabling shown in Fig.3, you will be selecting a frequency band, extending from about 100 Hz to about 100 kHz, to pass along to the main amplification stages. The first filter shown has its high-pass output in use; you may think of this as passing frequencies on the high side of 100 Hz, or equivalently as blocking frequencies below 100 Hz. The second stage is used as a low-pass filter, here passing all frequencies on the low side a chosen 100 kHz. So after the output of the two filters, you have Johnson noise, pre-amplified by factor 600, and then filtered to pass only the 0.1 - 100 kHz frequency band.

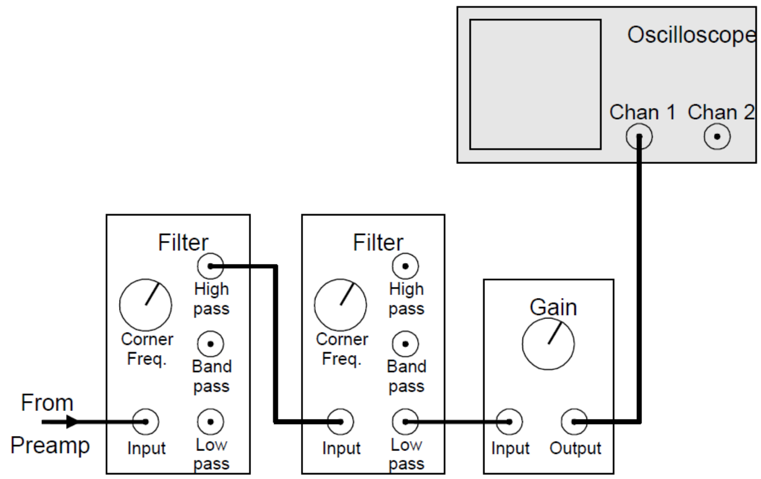


Fig. 3: Cabling diagram for first use of the high-level electronics. (Left) Filter: selector to .1k (for 0.1 kHz), switch to AC (for a.c. coupling) (right) Filter: selector to 100k (for 100 kHz), switch to AC (for a.c. coupling) Gain Fine Adjust 30, toggle x1, toggle x10

Notice the figure shows more cabling, now to amplify this signal by a further factor of 300. You achieve this by a setting of gain x1 and x10 at two toggle-switch settings, and a further gain of x30 on the rotary switch setting. (Here too you can switch to AC for a.c. coupling at the input.) Finally, at the output of this main amplifier, you'll have a signal large enough to see easily on a 'scope. A view of it, using a 2 V/div vertical sensitivity, and a 10 μs/div horizontal scale, is shown in Fig.4.

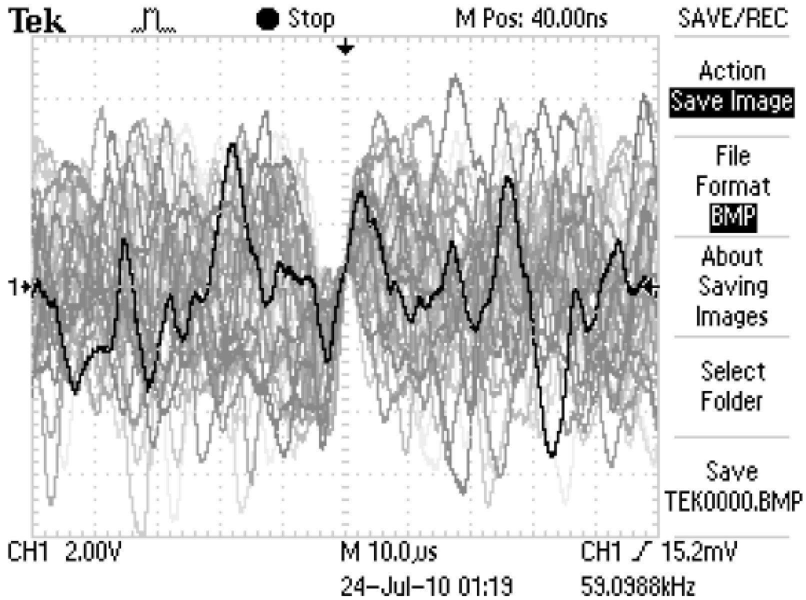


Fig. 4: Samples of amplified Johnson noise from a 100-kΩ resistor, using pre-amp gain 600, filtering to 0.1 - 100 kHz bandwidth, and main-amp gain 300.Vertical scale 2 V/div, horizontal scale 10 μs/div, triggering on positive-going zero-crossings.

To get a first, qualitative, indication that this 'noise signal' has something to do with the original source resistor at the front end of this pre-amp/filter/main-amp chain go back to the pre-amp, and change the source resistor from 100 kΩ to 10 kΩ. You should see the size of the noise signal on your 'scope *change* -- it should decrease, and by a factor of about three.

For a first rough understanding of the *size* of these 'scope signals' consider our claimed 13 μV (rms measure, in the 0-100 kHz band) Johnson-noise signal emerging from a 100-kΩ source resistor. The pre-amp gain of 600 ought to raise this to about 8 mV (rms), and further main-amp gain of 300 ought to raise this to about 2.5 V (rms). (The intervening filter stages enforce the limitation to the 0.1 – 100 kHz band, and they provide a gain very near 1.00 within that band.) We'll see later a good way to measure the rms value of signals such as shown in Fig. 4, but you can now see why those voltage excursions fall (mostly) in the ± 5 V range.

If your signals differ dramatically from those shown here, something is amiss. It's certainly possible for the signal you see to be smaller, say if you've made wrong connections or wrong settings. It's also possible for the signal to be 'too large', particularly if there are unwanted (interference) signals present. But the apparatus you're using, in the configuration you've set up, ought to be displaying a noise almost wholly due to nothing else than the Johnson noise of your source resistors. It is the universality of Johnson noise that lets us be sure that your signals should match, in rms measure, those shown here, certainly to within a factor smaller than two!

Quantifying Johnson noise

If you've done above part, you've seen a rapidly-fluctuating signal on an oscilloscope, which we claim is due mostly to Johnson noise, and which you now want to quantify. The method we'll describe here executes quite directly, in analog electronics, the very operation built into the mean-square definition of noise. You need one more cable to convey the filtered-and-amplified noise signal to the Multiplier module, configured as a 'squarer' as shown in the Fig.5. Conduct the noise signal to the 'A' input, and choose the A x A on the toggle switch. The multiplier circuit delivers at the MONITOR point, a real-time output voltage

$$V_{out}\,(t) = [V_{IN}\,(t)]^{2} / (10\text{ V}),$$

Which still has dimensions Volts (due to the fixed 'scale factor' of 10 Volts in the denominator above). Take a look at $V_{out}\,(t)$ on your 'scope, and notice that it is always positive, *un*like your input noise signal $V_{IN}\,(t)$, which is as often negative as positive.

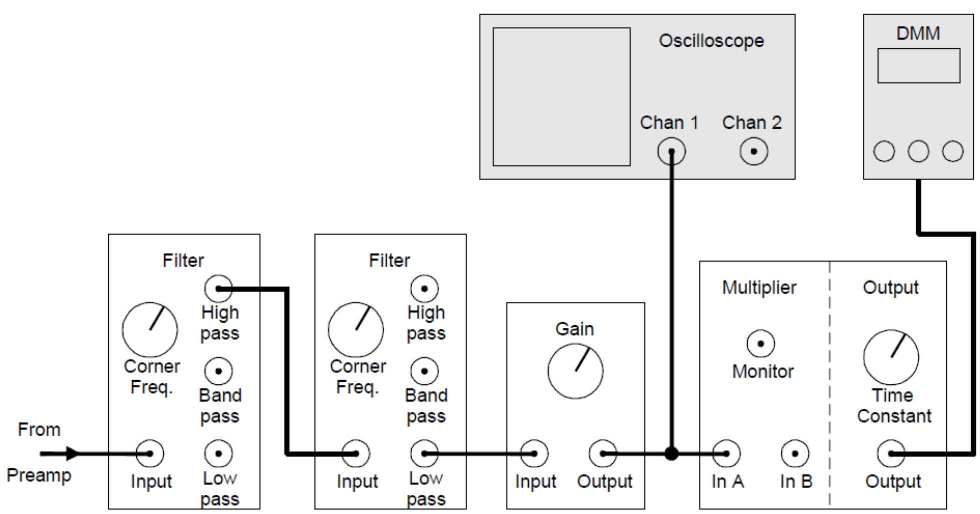


Fig. 5: Cabling diagram for using the multiplier as squarer. High-pass filter 0.1 kHz, a.c. coupling; Low-pass filter 100kHz, a.c .coupling; Gain 400, a.c. coupling ; multiplier AxA, a.c. coupling.

In fact, to persuade yourself that the squarer is working, use the XY-display capability on your 'scope. Convey the squarer's input V_{in} (t), both to the squarer and to the X-channel of your 'scope, and convey V_{out} (t) to the Y-channel, and have a look at a real-time XY- display. You should see a parabola emerge. See to it that you understand the origin of your XY-coordinate system, and then try changing some things: What are the right sensitivities to choose on the two axes? What would happen to your parabola if you raised the gain in the main-amplifier module of the HLE? Why does your data lie on a parabola, after all?

Now without the need for a further cable, the output of the squarer is already being sent internally to the Meter module of your HLE. What this module does is to take the time- average of V_{out} (t), averaged over a time interval you can select (by switch) to 1.0 second. This time average will *not* be zero, since V_{out} (t), though fluctuating, is always and only on the *positive* side of zero. (Recall that the multiplier's squaring function ensures that V_{out} (t) is proportional to the *square* of V_{IN} (t).) The meter will display that time-average, either on its 0-10 V or its 0-2 V scale. We suggest the use of the 0-2 V scale, and also suggest you go back and change the main-amp gain until the meter reaches a value near mid- scale, about 1 Volt on the 0-2 V scale.

What can you infer from this? Start with V_J (t), the actual instantaneous Johnson-noise voltage generated by the source resistor. At the output of the pre-amp, you have a signal:

$$(6.00)\ (100) \bullet V_J\ (t)$$

After the filter stages, you have the 0.1-100 kHz bandwidth-selected, or filtered, part of this signal. After the main amp, you have a signal

$$G_2 \bullet (600) \bullet V_J\ (t),$$

where G_2 is the main-amp gain, perhaps 300. Then after the squarer, you have a signal

$$[(300) \bullet (600) \bullet V_J\ (t)]^{2}\ /\ (10\text{ V}).$$

Finally, using the <...> brackets to indicate a time average, what you have displayed on your meter is the signal

$$V_{\text{meter}} = \langle V_J{}^2(t)\rangle \bullet (600 \bullet 300)^2\ /\ (10\text{ V})$$

From this result and the meter reading, you can work all the way backwards to find $\langle V_J{}^2(t)\rangle$, the mean-square voltage present (within your chosen bandwidth) across the source resistor.

Now use a cable to carry this time-averaged positive voltage to a digital multimeter. You should see a number consistent with your analog-meter indication, and you should see it fluctuate. Note that with the use of a 1-second time constant, you'll have to wait rather *longer* than one second for results to stabilize to any new value, especially if you're waiting for the 3rd or 4th digit of a multimeter display to settle down. Once the reading *has* settled, you'll notice the residual fluctuations, but go ahead and write down multiple readings from the multimeter, taking a new reading every second or so. See if you can persuade yourself that the readings display fluctuations about a mean value, and compute that mean value. It is connected, by a known chain of amplification and filtering, to the mean-square Johnson-noise voltage at the source.

Observing and Correcting for Amplifier Noise

You have now seen how all-analog electronics can take you all the way from a Johnson- noise source voltage $V_J(t)$ to a time-averaged d.c. voltage which is a traceable measure of $\langle V_J^2(t)\rangle$. This section teaches you how to

a) make that measurement optimally, and

b) correct that measurement for amplifier noise.

a) The noise measurements you perform all depend on the linear operation of the amplifiers, and they (like all analog electronics) have only a ***finite range of output voltages over which they remain linear***. For the high level electronic amplifiers, that range is (-10 V, +10 V). If you were to put a simple sinusoid through the amplifiers, you could use the full ± 10 V excursions . But since you are amplifying *noise*, you have to ensure that even the rare large fluctuations of the noise stay within the ± 10 V 'span' of the amplifier. In practice, a maximum ***average noise signal*** of 3 Volts (rms) is a safe choice. This should avoid serious distortion of the signal, called 'clipping', like that shown in Fig.6. For an average noise signal of 3 Volts rms, an excursion beyond ± 10 V is so rare as not to spoil the accuracy of your measurement.

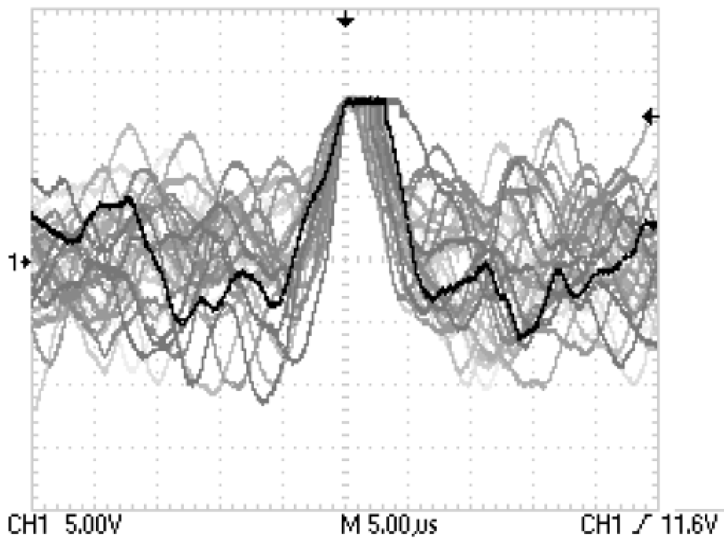


Fig.6: Clipped signal from HLE – notice the clipping level is near +12 Volts.

Now if the rms measure of the signal at the A-input of the squarer, $V_A\ (t)$, is 3 V, then (by definition) its mean-square value is

$$<V^2_A\ (t)> = (3\text{ V})^2 = 9\text{ V}^2,$$

and under these circumstances, the squarer's MONITOR output will give

$$V_{sq}\,(t) = [V_A\,(t)]^2 / (10\text{ V})$$

So that the time-average at the OUTPUT will be

$<V^2_{sq}\,(t)> = <V^2_A\ (t)>/\,(10\text{ V}^2) = (9\text{ V})/\,(10\text{ V}) = 0.9\text{ V}.$

You could use a smaller rms size for the input $V_A(t)$, but you'd be getting an even smaller output from the squarer, and your readings might be affected by zero-offsets in the squarer's output.

So from here onwards, whenever you measure a noise voltage, you should check the main-amp output to see that it fits easily into the ± 10-V range. If it exceeds these limits, reduce the gain. And you should look at the squarer's output on the panel meter, to see a time-averaged output near, or a bit below, 1 Volt. Again, if it's much larger, you want to reduce the gain, or if much smaller, raise the gain. Whenever you do take a reading of the time-average of the squarer's output, be sure to record also the net gain you've used to attain that reading, since this is your ticket to tracing the meter reading

back to the desired mean-square noise $<V_J{}^2$ (t).

a) Now back to Johnson noise. The problem you're now going to address is tracing noise back to a source, because here you have to consider the possibility that some of the noise you're seeing is *not* due to the Johnson noise of the of source resistor, but instead due to the amplifier chain which follows it. Since this 'amplifier noise' is just as featureless and random as the resistor's Johnson noise, there's apparently no way to separate the two waveforms *once they're added*. But there *is* a way to separate their effects, if we can assume that the amplifier noise does not depend on the source resistor's value. Here's the demonstration: let $V_J(t)$ be the instantaneous noise voltage from the source resistor, and let $V_N(t)$ be the instantaneous noise voltage apparently present at the input of the amplifier. That is to say, $V_N(t)$ is a model for a noise emf which, applied to the input of an ideal *noiseless* amplifier, would match the noise actually observed at the output of the real amplifier, driven only by its internal noise. If the gain of the amplifier is G, its output will be

$V_{out}(t) = G\,[V_J(t) + V_N(t)]$,

and the mean-square of this output will be

$<V_{out}^2(t)> = G^2 < [V_J(t) + V_N(t)]^2$

$= G^2 \{<V^2_J(t)> + 2 <V_J(t) \bullet V_N(t)> + <V^2_N(t)>\}$.

There's a 'cross term' in this expression, the time average of the product $V_J(t) \bullet V_N(t)$, but this time average is <u>zero</u>. The reason is that $V_J(t)$ and $V_N(t)$ can be safely assumed to be **<u>un</u>correlated**, arising as they do from distinct physical mechanisms in two different objects. So when $V_J(t)$ happens to be positive, the amplifier noise $V_N(t)$ is just as likely to be negative as it is positive; thus the product of the two factors is also as likely to be negative as positive. That's why the absence of correlation enforces a zero for the time- average of the product. But that fact leaves

$<V_{out}^2(t)> = G^2 \{<V^2_J(t)> + 0 + <V^2_N(t)>\}$,

which says that ***mean-square voltages from uncorrelated sources are simply additive***. In particular, it gives us a way to measure the amplifier noise -- we just change temporarily to a configuration in which the Johnson-noise term in this sum is negligible. Theory says that a choice of $R = 0$ for source resistance would give $<V^2_J(t)> = 0$, but in practice, it suffices to use the R = 1-Ω or 10-Ω settings for giving a $<V^2_J(t)>$ which is small enough that the result is a good measure of the amplifier noise, $<V^2_N(t)>$.

Once that latter value is measured***, it can be assumed to be present, and unchanged, in any use of the amplifier***. So for any source resistor R_{in} > 10 Ω, the amplifier noise contribution previously established can be subtracted off, leaving $<V_J{}^2(t)>$ isolated by itself.

Here's a concrete illustration: we have the values R_{in} = 1 Ω, 10 Ω, etc. We pick the 0.1 - 100 kHz bandwidth as before, and we pick gains to give good results at the squarer. In a particular example, the time-averaged outputs of the squarer we find are the $<V_{sq}>$ values below:

R_{in} chosen	**GainG_2**	**$<V_{sq}>$Read**	**$<V^2_J+V^2_N>$infrared**	**$<V^2_J>$Derived**
1 Ω	1500	0.6353 V	7.843×10^{-12} V^2	≈ 0.002×10^{-12} V^2
10 Ω	1500	0.6372	7.867	0.026
100 Ω	1500	0.6516	8.044	0.203
1 kΩ	1500	0.7911	9.767	1.926
10 kΩ	1000	0.9801	27.225	19.384

Now we expect, for the time-averaged output of the squarer,

$<V\text{sq}\ (t)> = <V_{\text{in}}^{2}\ (t)> / (10\ \text{V})$

$= \quad \{(G_1\ G_2)^2 / (10\ \text{V})\} <V_J{}^2 + V_N{}^2>,$

So, we can use $G_1 = 600$ and G_2 as-listed value to compute the column with $<V_J^2+V_N^2>$ values. We can eyeball extrapolate to the $R \rightarrow 0$ limit and deduce contribution of $7.841 \times 10^{-12}\ \text{V}^2$ for $<V_N{}^2>$ alone, the amplifier noise contribution (for this particular amplifier chip, at this particular bandwidth -- your number will vary!).

Subtracting this contribution from all the entries gives the rightmost column for $<V_J{}^2>$, our estimate of the mean-square Johnson noise of the source resistor, corrected for the effects of amplifier noise. Notice that the amplifier-noise corrections are large, even dominant, for small values of source resistance! You'll find (for the present choice of pre- amp input stage) that Johnson noise surpasses amplifier noise only when the source resistance has risen to about 3 k Ω.

Johnson noise dependence on resistance

The previous sections have discussed about how to configure the pre-amp/filter/main-amp combination, and how to select a gain for optimal use of the squarer. The results can also be corrected for amplifier noise, and traced back to an inferred mean-square measure of Johnson noise $<V_J{}^2(t)>$, for any source resistor from $R = 10\Omega$ upwards.

You should now investigate systematically the dependence of $<V_J{}^2(t)>$, upon source resistance R. To do so, you can use the R=10Ω through 10 MΩ choices built into the pre-amp module. (These internal source resistors have tolerances of 0.1% to 1 MΩ, and 1% thereafter.) But the selector switch also gives you access to three more test positions, A_{ext}, B_{ext}, and C_{ext}, which you are free to 'populate' with devices of your choice behind the pre-amp panel.

Here's how to do so: You can 'flip' the pre-amp panel to expose the back (component) side of the pre-amp's circuit board. You can also find the pre-amp power switch (near the internal power-on red LED inside the low-level electronics), and **turn OFF the pre-amp power** before making any changes to the board. Now use the diagram below to find the screw-connect terminal strips, and find also the location of the two endpoints for the components you're putting into the A, B, and C positions.

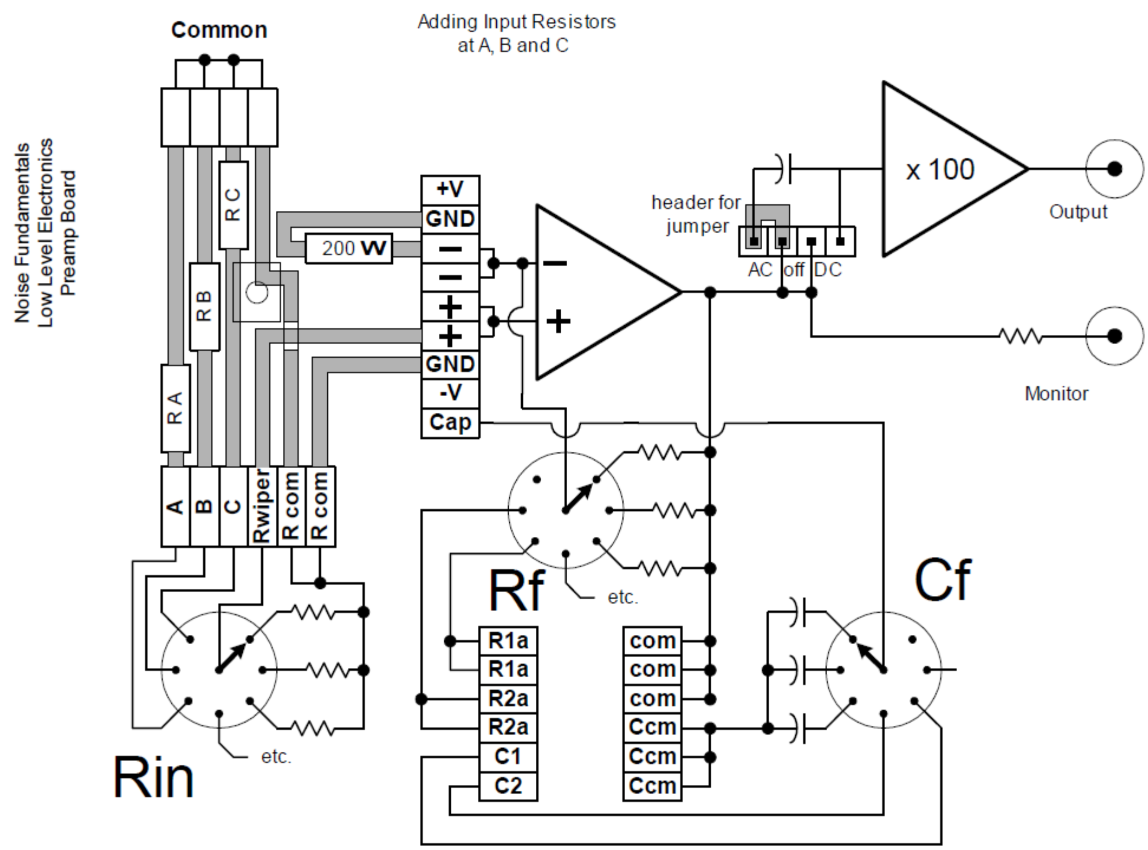


Fig. 7: Wiring diagram for adding components at the A, B, C, positions of the pre-amp's input. Note all input resistors have a common ground.

You can choose resistors of any value in the 20Ω to 5MΩ range ; you can even choose different kinds of resistors. (Most resistors sold nowadays are of metal-film construction, but ask around for some carbon-composition or wirewound resistors -- and look up what kinds of resistors Johnson himself used.) You can clearly use resistors of any power capability you like -- their internal Johnson-noise emf is *not* going to overheat them! If you wish, you can have a comrade *hide* from you the resistance values, so you'll be measuring some actual unknowns. Don't forget to turn the pre-amp power back ON before you re-flip the front panel and close up the box.

Now you can take noise data for your own resistors, as well as for the built-in source resistors. Once you have values for $\langle V_J^2(t)\rangle$ each corrected for amplifier noise, you can plot those values as a function of R. Since both axes will vary over many orders of magnitude, a log-log plot is appropriate. The vertical axis has units of Volts-squared, the horizontal axis has units of Ohms. Nyquist's theory predicts a first-power power-law dependence on resistance R, namely

$$\langle V_J^2(t)\rangle = (4k_B T\Delta f)\, R$$

and you might see this confirmed. There will be deviations from this behavior at the high-R end of the plot. At the *low*-resistance end of the plot, you'll see the amplifier-noise-corrected values enable you to follow Johnson noise to a regime well below the apparent limit set by amplifier noise. You will be able to establish value of $\langle V_J^2(t)\rangle$ which are less than 1% of the amplifier noise $\langle V_N^2(t)\rangle$ that overlays them. Of course, the corrected value of Johnson noise will be the difference between two nearly equal quantities, so the results will be subject to larger uncertainties than other data points.

Johnson noise dependence on bandwidth

Thus far you've learned how to observe and quantify Johnson noise, and you've seen how to isolate its mean-square value from amplifier noise. You have also seen its dependence on source resistance R. But Nyquist's formula claims that $\langle V_J^2(t)\rangle$ also depends on the bandwidth Δf; ie. on the range of frequencies to which your system is sensitive.

So for now you should stay at room temperature, and stay at a fixed R-value; we suggest a starting value of R_{in} = 10 kΩ. The goal is to see how the choice of bandwidth matters. The method is to imagine a 'white noise spectrum', ie. noise power uniformly spread in frequency at its origin, but subsequently modified by the high-pass and low-pass filter sections as depicted below

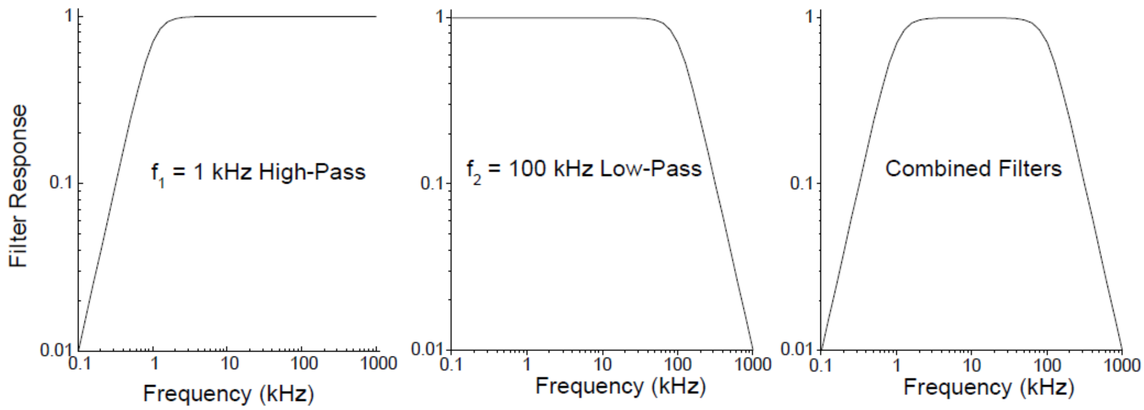


Fig. 8: Representation (left) of the transmission of a high-pass filter, of corner frequency f_1; (center) of a low-pass filter, of corner frequency f_2; (right) the combined effect of both filters. The graph's scales, horizontal and vertical, are all logarithmic.

You have a range of choices for the 'lower corner' frequency f_1 or high-pass filter setting, and a separate range of choices for the 'upper corner' frequency f_2 or low-pass filter setting. You might first think that the bandwidth Δf should be given by $|f_2 - f_1|$, which is a decent approximation, but subject to significant corrections. Here, we present you with the generic corrections which are the result of a model calculation. The model predicts the effective bandwidth Δf for each combination of f_1 and f_2, and gives the results shown in Table below.

	f_2 = 0.33 kHz	1 kHz	3.3 kHz	10 kHz	33 kHz	100 kHz
f_1 = 10 Hz	355	1,100	3,654	11,096	36,643	111,061
30 Hz	333	1,077	3,632	11,074	36,620	111,039
100 Hz	258	1,000	3,554	10,996	36,543	110,961
300 Hz	105	784	3,332	10,774	36,321	110,739
1000 Hz	9	278	2,576	9,997	35,543	109,961
3000 Hz	0.4	28	1,051	7,839	33,324	107,740

Table1: Effective noise bandwidths, Δf, given in Hertz, computed for model filter responses These computed values are all subject to uncertainties of order 4%; They are all computed for ideal filter responses, ignoring systematic effects. Inclusion of those effects may raise values in

the rightmost column by (3±1)%, and may raise values in the next-to-rightmost column by (1±1)%. There are further corrections to effective noise bandwidths for large f_2-values, in the case of large source resistance, due to capacitive effects.

Your goal is to measure the mean-square Johnson noise of the resistor, $<V_J{}^2(t)>$ for as many (f_1, f_2) combinations as you wish. Recall that for each choice of filter settings, you'll want to adjust the gain so as to use the squarer optimally. Recall that each mean- square value you measure needs to be corrected for amplifier noise (measured at *that* bandwidth setting: the amplifier-noise contribution to the mean-square depends, as does the Johnson-noise contribution, on the bandwidth you use.

You can plot your data for $<V_J{}^2(t)>$ in various ways: as a function of the f_1-value used to obtain it; as a function of the f_2-value used to obtain it; as a function of the difference $|f_2 - f_1|$ of the f_1- and f_2-values used to obtain it; or as a function of the equivalent noise bandwidth, from the table above.

Which plot is the most nearly linear? Try again using log-log scales, to be able to see all you data points, spread as they are over many orders of magnitude.

If your plot is consistent with $<V_J{}^2(t)> \alpha\ \Delta f$, then the coefficient of this proportionality tells you a 'noise power spectral density', as you'll see in the next Section. Its units are V^2/Hz, and it's usually denoted by *S*.

Johnson noise density, and Boltzmann's constant

Previous sections have shown you how to measure noise, and have tested its dependence on source resistance *R* and on measurement bandwidth Δf. This section introduces you to noise *density*, and then relates your measured values, via Nyquist's formula, to Boltzmann's constant.

If you have shown that measured mean-square noise $<V_J{}^2(t)>$ has a linear dependence on the bandwidth Δf used, you are entitled to infer the existence of a 'noise density' that's uniform in frequency. Here's an analogy to mass density that should make this clear -- we'll use a one-dimensional example. Suppose you have a string, of unknown composition, laid out on an *x*-

axis, and that you can make clean cuts at arbitrary locations $x1$ and $x2$, and then weigh the piece of string you've extracted. If (and only if) you find that the observed mass M is always proportional to $|x2 - x1|$, you may conclude the string is of uniform density. You can also see that the quotient

$$(\text{mass } M) / |x2 - x1|$$

gives the value for this density, given in units of mass per unit length.

Similarly, if you've shown that mean-square noise $<V_J{}^2(t)>$ *is* always proportional to the bandwidth Δf you used to obtain it, then you can define the 'noise power density'

$$<V_J{}^2(t)> /\Delta f$$

in this case with units of Volts-squared per Hertz, or V^2/Hz. [Strictly speaking, this is not a power density -- but if a voltage $V(t)$ is applied across a resistance R then the quotient $V^2(t)/R$ is a power. So the quotient above is just a factor-of-R away from being an actual power density, with units Watts per Hertz.]

Your data for a single source resistance R = 10 kΩ has given you a noise power density; you can go back to your data and convert that data to noise power density as well, to check the dependence-on-R of this density. The motivation for all of this is that Nyquist's formula can be written as

$$\text{noise density } S = <V_J{}^2(t)>/\Delta f = 4k_BTR$$

So you should plot all of your data thus far and perhaps more data that you now take for various R- and Δf-values, to see if you can further establish the linear-in-R claim of the prediction above. If you establish a regime of linear dependence on R, your plot, or fit, will give you a value for a slope, ($4\ k_B\ T$). What *units* should it have? (Answer: rise over run, so V^2/Hz per Ohm -- and what unit is *that*?) What *value* does it have? Hardest: what *uncertainty* can you assign to your value? (Do so before you look up any 'book values', because the uncertainty intrinsic to your experiment is conceptually a matter quite separate from any discrepancy between your value and anyone else's.)

Finally, if you know you room's temperature T (and express it in absolute, ie. Kelvin, units), you can now conclude by finding a value (and uncertainty!) for Boltzmann's constant k_B.

Procedure:

1. Make the connection in the back panel as per figure 2, 3, 5 and your desired objective.

2. Keeping the gain of the amplifier fixed, vary the resistance (R_{in}) (between 1 Ω to 10 kΩ) and calculate the average square voltage $<V_{sq}(t)>$.
3. Keeping the gain of the amplifier fixed, calculate the average square voltage $<V_{sq}(t)>$ as a function of the bandwidth (changing the f_1 and f_2).
4. Calculate the noise density and Boltzmann's Constant for different resistance values and Find out the noise density dependence on resistance and bandwidth via making a plot of noise density as a function of resistance and Δf (as explained in the introduction part).
5. Fit the Data with help of Origin/Matlab or any other software as you convenient and do the error calculation for each set of experiment.

Result and Observations: (1) in your Results You should show that how Johnson noise dependent on resistance and bandwidth?

(2)You have to find out the value of Boltzmann's Constant-

(a) For fixed Δf and varying R

(b) For Fixed R and varying Δf

(c) Calculate the Noise density for both (a) and (b)

(3) Discuss the various aspect of change in Johnson noise and what would be the behavior of Johnson noise at very large and low values of R & Δf

Precautions: (1) you should always choose a correct voltage measurement range in the Voltmeter and adjust the gain of amplifier accordingly to avoid clipping of the noise.

(2) be careful while making change in back panel of Low level electronics panel and don't touch the rest of the circuit other than suggested.

(3) Before making your connection check you BNC cables.

(4) After finishing your experiment take out the component that you installed and put back to the component box.

17. Shot Noise Measurement

Objective: (1.) Measure the shot noise in photo current at constant value of resistances and bandwidth.

(2.) Measure the Shot noise at different values of resistances, bandwidth and gain.

(3.) Measure the Shot-noise using a transimpedance amplifier.

Introduction:

The reason for shot noise, and its predicted size

Shot noise is generated because of the discrete nature of charge and random arrival times of electrons. Shot noise results from the fact that the current is not a continuous flow but the sum of discrete pulses in time, each corresponding to the transfer of an electron through the conductor. Its spectral density is proportional to the average current, I, and is characterized by a white noise spectrum up to a certain cut-off frequency, which is related to the time taken for an electron to travel through the conductor. It's quite remarkable that certain macroscopic electric currents will display noise, ie. random fluctuations about their average d.c. value, and that this noise is directly attributable to the microscopic 'graininess' of electric current. While shot noise might be a nuisance, or a limiting factor in certain measurements, its existence makes possible the determination of the magnitude of the quantum of charge, just by measuring noise. So, shot noise provides a route to a tabletop measurement of the value of 'e', the fundamental charge.

Here's some reasoning about why there *might be* fluctuations about the mean value of a current. Let's think about a d.c. current of average value i_{dc}, created by a flow of electrons in a wire. In a time interval τ, the (average) amount of charge arriving is

$Q = i_{dc}\,\tau$,

and because that charge arrives in charge packets, each of size (-)e, the number of electrons arriving is

$n = Q/e = i_{dc}\,\tau/e$.

The next step in the argument depends entirely on physical characteristics of the source of the electron current. If the electrons were arriving with perfect regularity, then in any time interval of duration τ, you'd expect only a ± 1-count uncertainty in the number n. At the opposite extreme from regular, periodic, and predictable arrivals, think of an electron current consisting of the statistically-independent arrivals of entirely *uncorrelated* electrons. In such a case, the average number of electrons arriving in time τ would be n, but the actual number on any particular trial would be subject to the same $n \pm \sqrt{n}$ 'counting statistics' that you'd get in radioactive decay or any other Poisson process.

It is *not* obvious which limiting case you'll achieve with actual electric currents! In fact, there are simple room-temperature ways to generate electric currents that show 'full shot noise', but there are also ways to produce currents which are well below the 'shot-noise limit'. (In fact, if you deliver electrons in bursts, you can get *above*-shot-noise fluctuations -- think about the ns-long, 10^6-electron pulses arriving at the anode of a photomultiplier tube.)

Here are the consequences of those statistical fluctuations: Suppose we have in a time τ the arrival of n electrons on average, delivering an average charge of $Q = i_{dc}\,\tau$. Now there will be fluctuations in charge about this mean, with standard deviation

$\sigma_Q = e\,\sqrt{n} = e\,(i_{dc}\,\tau/e)^{1/2}$.

Since current is charge per unit time, the instantaneous current $i(t)$ will also show statistical fluctuations about its mean i_{dc} , with standard deviation $\sigma_i = \sigma_Q/\tau = (i_{dc}\,e/\tau)^{1/2}$.

Standard deviation is defined as $< [i(t) - i_{dc}]^2 >^{1/2}$, we see that the deviation from the average, $\delta i(t) = i(t) - i_{dc}$, has a mean-square value given by

$< [\delta i(t)]^2 > = i_{dc}\,e/\tau$,

Which is, in fact, a result for the mean-square noise in the current $i(t)$. That we can measure! The only complication in this simple derivation is to relate the effective measurement time τ to the bandwidth Δf of the measurement system, as previously defined. A low-pass filter of corner frequency f_c, and a bandwidth approximately given by $\Delta f = f_c - 0$, defines a time-scale of $(2\pi f_c)^{-1}$, which is approximately the time scale on which statistically-independent readings can be made. So we might expect a result near

$<\delta i^2(t)> \approx i_{dc}\,e/\tau = i_{dc}\,e\,(2\pi f_c)^{+1}$.

In fact, the correct result has exactly of this character, but with different numerical constants; Schottky's prediction for the mean-square noise in a current of uncorrelated electrons is

$\langle \delta i^2(t) \rangle = i_{dc}\, e\, (2\, \Delta f) = 2\, e\, i_{dc}\, \Delta f$,

where Δf is the noise bandwidth'.

First views of noise on a photocurrent

The previous section has explained how to confirm that a d.c. photocurrent from an illuminated photodiode has been generated. Now it's time to look for the fluctuations, ie. the noise, in such a photocurrent. Those fluctuations will display 'full shot noise' if the electrons flowing to constitute the photocurrent are uncorrelated, statistically-independent arrivals of charge.

The most persuasively independent electrons are photo-electrons, the more so if the light producing them is 'thermal' light. In your first shot-noise experiment, the light involved will be produced by an incandescent bulb, which has negligible spatial and temporal coherence. So it's appropriate to think of the bulb as shedding a rain of independent photons down onto any surface. (The word 'shot' in shot-noise is meant to remind you of the sound of pellets of birdshot, or perhaps raindrops, falling onto a sheet-metal roof.) When those photons fall onto a photo-diode, it is a fair picture to think of each photon absorbed in the p-n junction as producing an electron-hole pair. The internal electric field of the junction separates that pair, and drives the electron through an external circuit as part of an electric current. It's easy to think of the photons as statistically-independent (since they're independently produced, and thereafter non-interacting); what's not quite so obvious is that the photo-electrons thereby produced will create a current of still-statistically-independent electrons (given that electrons in a wire certainly *can* interact through their electric field with other electrons).

Use the same circuit with the use of the incandescent bulb to illuminate the photodiode, adjusted so as to produce 10 μA (but not more, to avoid complications) of photocurrent. The evidence for this will be a d.c. voltage of $i_{dc}\, R_{in}$ = (-)(10 μA)(10 kΩ) = (-)10^{-1} V = (-)100 mV at the MONITOR point in the pre-amplifier of the LLE.

Now here's a calculation of the expected size of the shot-noise fluctuations in that current, and their detectable consequences. Recall that for i_{dc} = 10 μA, and for an equivalent noise bandwidth of Δf = 100 kHz in the processing chain downstream, Schottky's formula predicts the rms measure of current fluctuations will be

$\delta i_{rms} = < [i(t) - i_{dc}]^2 >^{1/2} = (2\, e\, i_{dc}\, \Delta f)^{1/2}$

$= (2 \cdot 1.6 \times 10^{-19}\, C \cdot 10^{-5}\, A \cdot 10^5\, Hz)^{1/2} = 5.7 \times 10^{-10}\, A = 0.57\, nA.$

So the 10 μA, or 10,000 nA, photocurrent is predicted to exhibit fluctuations of only 0.57 nA (rms).

How will these fluctuations be made visible? The load resistor R_{in} = 10 kΩ not only 'maps' i_{dc} to a voltage $V_{mon} = i_{dc} R_{in}$, it also maps a fluctuation δi to a fluctuation $\delta V = \delta i\, R_{in}$. In rms measure, we map 0.57 nA to (0.57 nA)(10 kΩ) = 5.7 μV. Such voltage fluctuations are still too small to see directly. But as suggested in Fig.1, this signal is sent, by a.c. coupling, to the further gain stage (G_1 = 100) in the pre-amplifier. So the output of the pre-amp ought to exhibit fluctuations of rms measure 570 μV (in a 100-kHz bandwidth). These sub-mV fluctuations might still be too small to be shown directly on an oscilloscope.

So, to see if these amplified fluctuations are present, set up the high-level electronics (HLE) to include a 100-kHz low-pass filter, and a gain G_2 of 10 x 10 x 60 = 6000. This ought to give a noise signal of rms measure 3.4 Volts (in a 0-100 kHz bandwidth). Applied in the usual way to the squarer, you can expect an output with non-zero d.c. average value

$< V_{sq} > = < V_{in}\, (t)^2 > / \, 10\, V = (3.42\, V)^2 / \, 10\, V = 1.17$ Volts .

Your value will *differ* from this, because of complications and deficiencies in this circuit. But you can now reduce the illumination on your photodiode, to show that part of this $<V_{sq}>$ is attributable to the light.

Shot-noise measurement using a transimpedance amplifier

The previous sections showed you the existence, and approximate size, of shot-noise fluctuations in a photocurrent. In this section, we describe a circuit that has been optimized for measuring shot noise, and thus accurately determines the electronic charge, *e*. The novelty of this section is a method for operating an illuminated photodiode in a reverse-biased configuration, and converting its photocurrent to a voltage by a precisely known coefficient.

You will create the circuit by changing the default wiring of the pre-amp's first stage, and reconfiguring it to act as a current-to-voltage converter. Since the non-inverting input is grounded, feedback through R_f will actively hold the inverting input at near-zero potential as well. This ensures that the voltage drop across the photodiode always has the full value that is set on the biasing supply.

For this section circuit (as in Fig. 4 & 5 of the Procedure), the photodiode current i_{dc} passes entirely through R_f (since the inverting input of the op-amp draws negligible current). This ensures that

$0 - i_{dc} R_f = V_{out}$, ie. $V_{out} = - R_f i_{dc}$.

So, the photocurrent has been mapped to an output voltage, which is measurable by a d.c.-coupled path at the pre-amp's MONITOR output. An a.c.-coupled path passes the a.c. components of this signal (including all the noise components with $f \geq 16$ Hz) to the subsequent gain stages of the pre-amp.

Now if the photodiode current $i(t) = i_{dc} + \delta i(t)$ shows the sum of a d.c. average current plus the current fluctuations representing shot noise, then the i-to-V converter will give an output

$V_{out}(t) = (-1)\, i_{dc} R_f + (-1)\, \delta i(t) R_f$.

Pre-amp, what follows in the a.c.-coupled path is 100-fold voltage amplification, so the cable connecting the pre-amp output to the high-level electronics will be conveying the voltage signal

$V_{pre}(t) = 100 \text{ x } (-1)\, R_f\, \delta i(t)$.

Inside the high-level box, use high- and low-pass filters just as before to create some chosen bandwidth Δf, and then use the main amp to provide further gain G_2 to bring the fluctuating signal up to the size suitable for the squarer. The output of the squarer will thus be

$V_{sq}(t) = [- G_2\, 100\, R_f\, \delta i(t)]^2 / (10 \text{ V})$,

and the familiar time-average of the squarer's output will give

$< V_{sq}(t) > = < \delta i^2(t) > (100\, G_2\, R_f)^2 / (10 \text{ V})$.

If Schottky's formula is correct, then this gives

$< V_{sq}(t) > = 2\, e\, i_{dc}\, \Delta f\, (100\, G_2\, R_f)^2 / (10 \text{ V})$,

So, after noting the squarer's output in the presence of, and then in the absence of, the average photocurrent i_{dc}, and correcting for 'amplifier noise' by subtraction, you can infer a value for the mean-square fluctuation of the photocurrent:

$< \delta i^2(t) > = [< V_{sq}(t) > \cdot 10 \text{ V}] / (100\, G_2\, R_f)^2$.

From these quantified mean-square fluctuations in the photocurrent, you can solve for the electron charge e; you will of course also need to know the d.c. photocurrent i_{dc} you measured indirectly, and the equivalent bandwidth Δf you used

You can now investigate shot noise systematically. With a fixed bandwidth Δf, you might first check the dependence of current noise $<\delta i^2(t)>$ on the average photocurrent i_{dc}. As was the case with Johnson noise, you can see changes in the noise which are much smaller than the amplifier noise. You should be able to see the noise increase slightly with only 0.1μA of photocurrent. (That's 1 mV/10 kΩ measured at the MONITOR on the preamp module; to see a 1-mV level here, you'll need to note the d.c. offset at this MONITOR point.).

You will find it profitable to compute the values of mean-square current fluctuation per unit bandwidth, or 'current noise power density', $<\delta i^2(t)>/\Delta f$, with units of A^2/Hz. That's because the Schottky formula predicts this quotient ought to have a simple dependence on $i$dc.

Thus far we have assumed that the shot noise is 'white', ie. spectrally uniform. To test this, temporarily fix i_{dc} and R_f at some suitable values, and test the effect of changing the choice of filter bandwidth in the HLE. As you lower the bandwidth, the amount of noise emerging should drop, and as usual, you'll increase the main-amplifier gain to keep the squarer in its optimum regime. But you should test to see if the quotient $<\delta i^2(t)> / \Delta f$ stays fixed -- this is a test of the claim that the shot noise is 'white'. (Because of the effects of capacitance, this uniformity may fail at choices of largest bandwidth -- the spectral uniformity of the response of the electronics chain is hardest to maintain at the high-frequency end of the large-bandwidth coverage.)

If you can confirm that the current noise power spectral density' $<\delta i^2(t)>/\Delta f$ is in fact independent of your bandwidth choices, but that it does depend on the average photocurrent i_{dc}, then you can try a log-log plot of $<\delta i^2(t)>/\Delta f$ as a function of $i$dc. Your plot will have x-axis values in Amperes, and y-axis values in A^2/Hz. If you get a linear variation, that line will have slope with units (rise over run) of $(A^2/Hz)/A = A/Hz = A\cdot s = C$, Coulombs. In fact, Schottky's theory predicts a power-law fit, with power-law exponent 1 and coefficient ($2e$), since

$$<\delta i^2(t)> / \Delta f = (2e)\, i_{dc}$$

expresses the theory. Is your plot consistent with a power-law exponent of 1.00? If so, you can read off (2 *e*), in Coulombs, as the coefficient of your fit to the data! As is often the case, estimating the <u>uncertainty</u> in your value for *e* might be the hardest part of your experiment. (Remember this is *not* the same as the discrepancy, if any, between your value and the 'book value'.)

Despite its apparent simplicity, there can be experimental and conceptual pitfalls to this experiment. The most annoying is that some light bulbs, under some conditions, give unstable light output, despite stable voltage input. The effect, in changing the photocurrent in the photodiode, looks just like excess noise, which can falsify the derived value of *e*. The cause is (apparently) the intermittent shorting of adjacent turns of the finely-coiled tungsten wire in the filament of these bulbs. It may help to isolate the low-level electronics box from mechanical vibration.

Procedure: (1) Install the light source and photodiode in the back panel of the LLE as given in Fig.1.

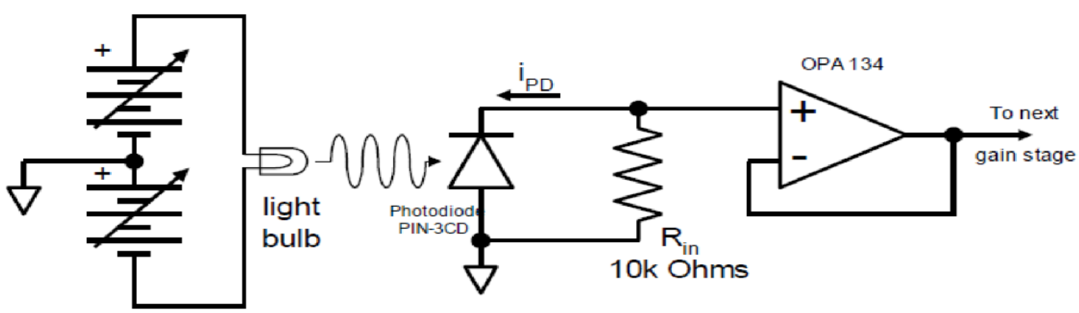


Fig. 1: Schematic diagram for the connection of a photodiode and a load resistor to a pre-amp input stage configured as a voltage follower.

(2) First use the tiny incandescent bulb to illuminate the photodiode to confirm the operation, you may also use LED as an alternative to lighten up the photodiode. The properties of different light source are given in table below:

Wavelength	Package	typical bias	Absolute max.	Pulsed	Photo-Current
Red LED ≈ 650 nm	clear plastic	20 mA	30 mA 50 mA with possible damage	100mA	0.25 mA
IR LED ≈ 930 nm	metal can with lens	100mA	100mA	2A for 0.1 us	1.2 mA
Light Bulb	glass bulb	24 Volts	28 Volts		0.4 mA

(3.) Follow Fig.2 for other wiring in the back panel of the LLE and fixed the R_f =10KΩ and R_{in} = R_1 or 10Ω.

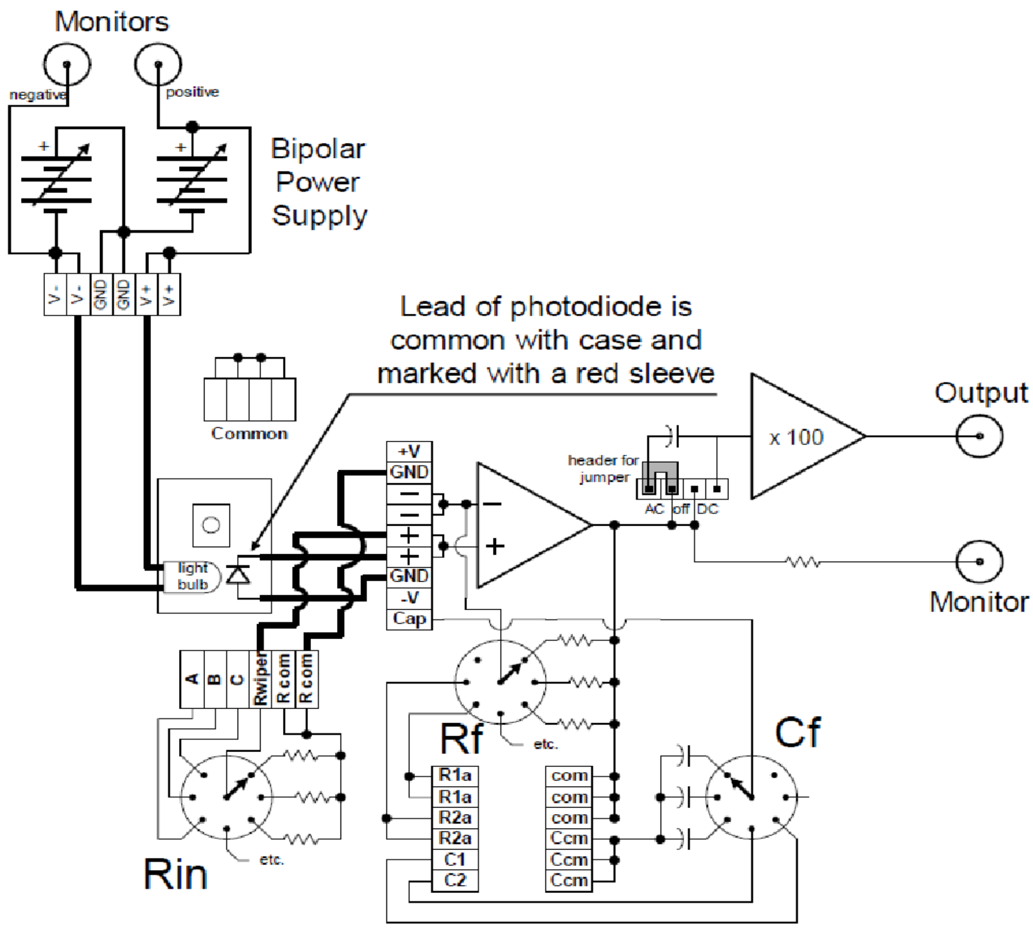


Fig. 2: Wiring diagram for the lamp and photodiode combination, and the voltage-follower topology of the input stage of the pre-amp.

(4.) To light the bulb, you can use the positive and negative outputs of the 0 - 11 V bipolar power supply built into the LLE.

(5.) Before installing LLE in the plastic block confirm that bulb is lighting up, and can vary its brightness over a wide range.

(6.) Now arrange for the bulb to be glowing dimly, and mount it into the black plastic block

(7.) To study the short noise, connect the LLE, HLE, Oscilloscope and Multimeter as shown in Fig.3 given below:

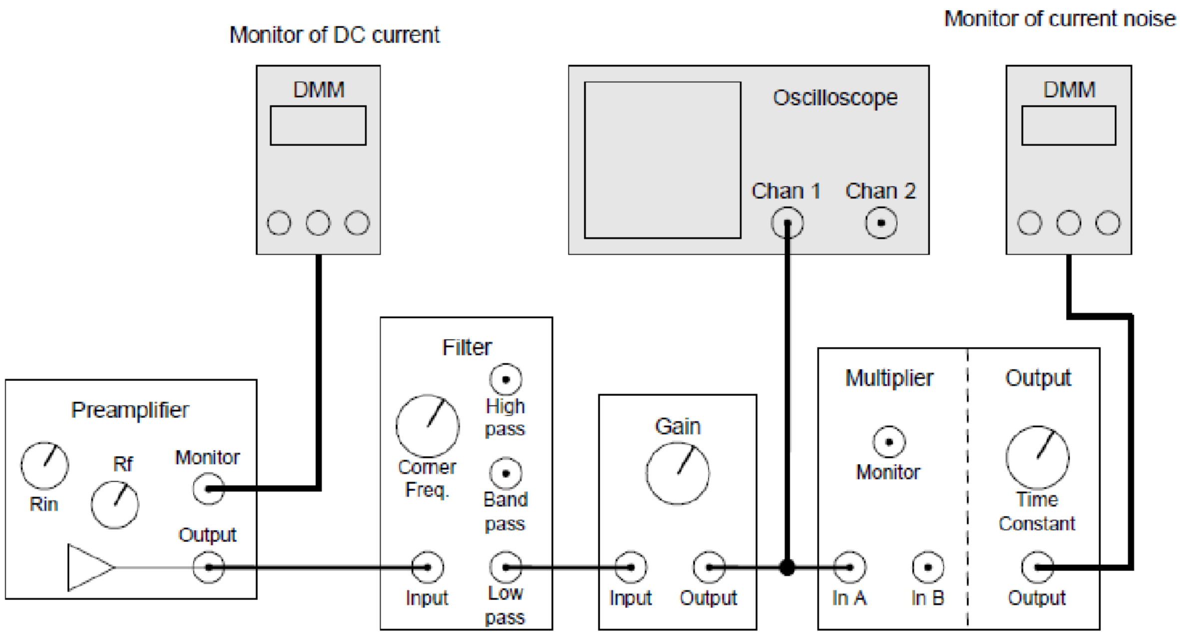


Fig. 3: Cabling diagram for LLE and HLE interconnections for first studies of shot noise

(8.) Measured the output at the pre-amp's MONITOR by using a multimeter and check that you are getting a negative voltage, which grows *more* negative as you dial up the voltage you're supplying to the incandescent bulb.

(9.) To measure the amplifier noise, turn down the supply to the incandescent bulb to its minimum value, which ought to reduce the photocurrent i_{dc} to zero as well. Check a surrogate for i_{dc} at the MONITOR point on the pre-amp. But the noise is predicted to go to zero as well. In practice, it will go down, but not to zero, since there remains noise from the pre-amplifier itself.

(10.) To measure the shot noise through transimpedance amplifier modify the circuit connection as shown in Fig.4 and Fig.5.

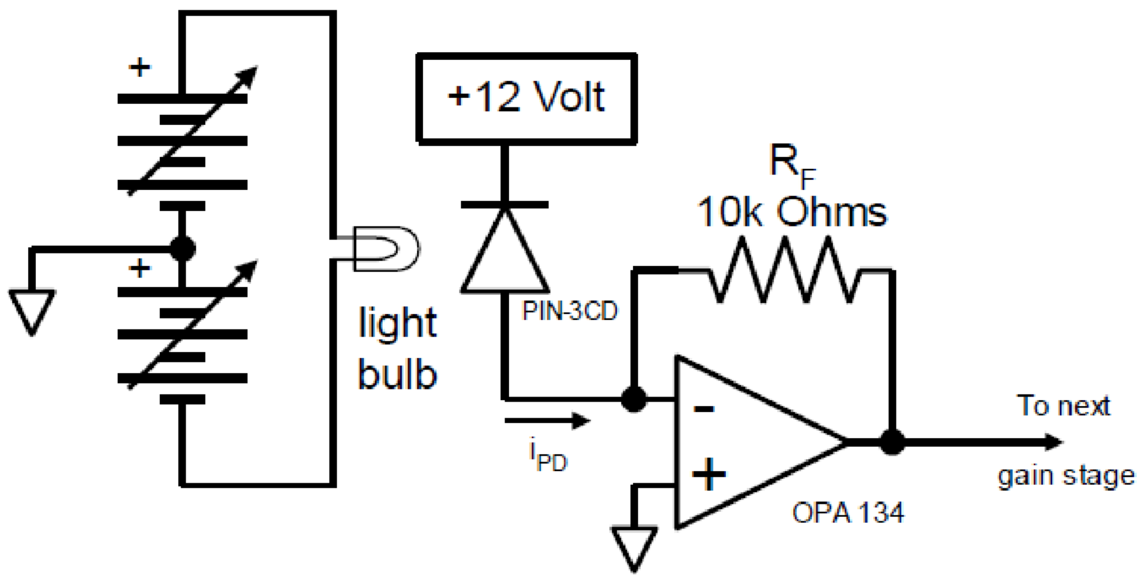


Fig. 4: Schematic diagram for the connection of a reverse-biased photodiode to a pre-amp input stage configured as a current-to-voltage converter, also called a **transimpedance amplifier (TIA)**.

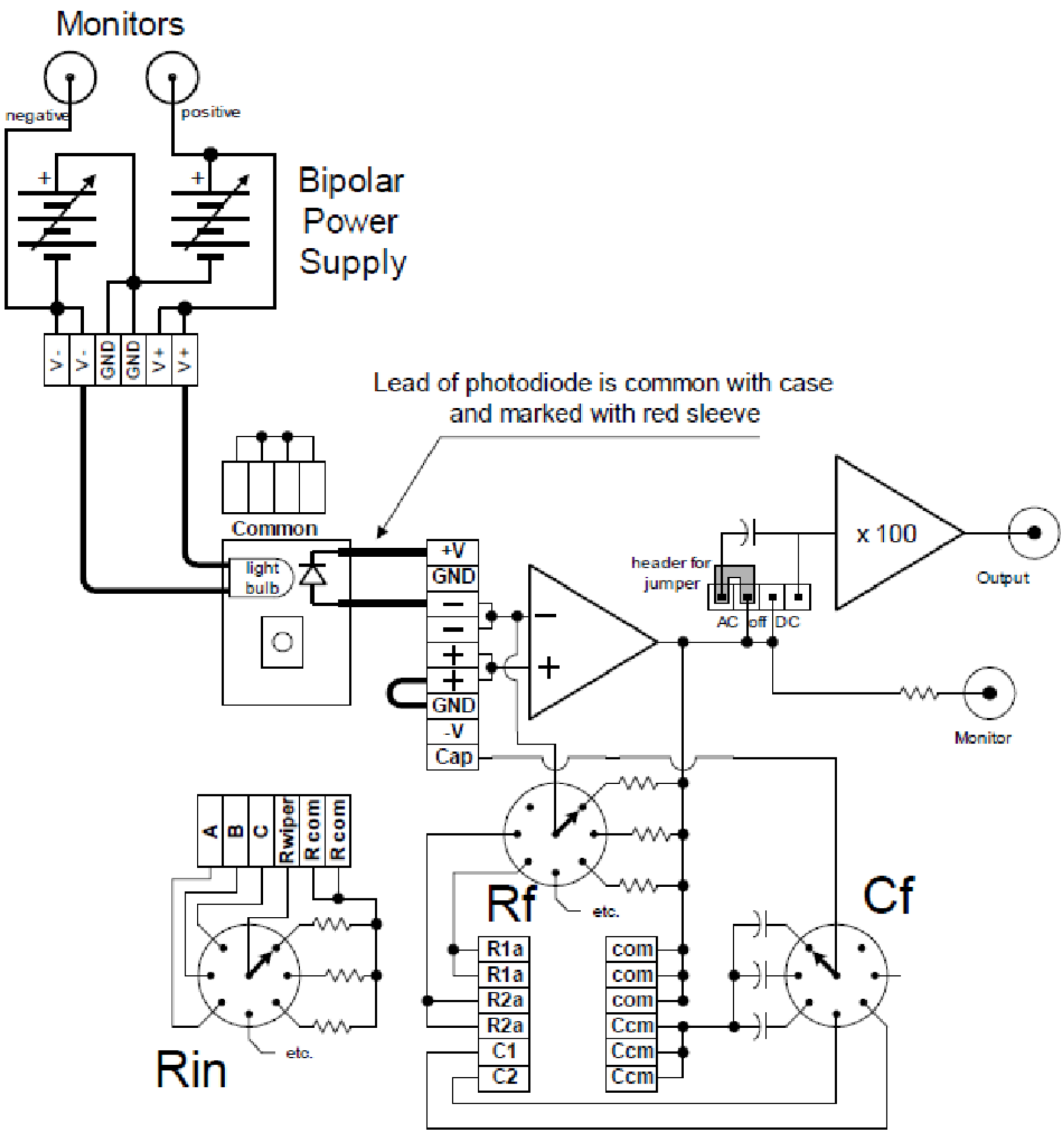


Fig. 5: Wiring diagram for the input stage configured as an i-to-V converter

(11.) To confirm that the bulb-photodiode combination is working, connect a multimeter (set to d.c. Volts) to the MONITOR point on the panel of the pre-amp. This will display a voltage $V_{mon} = (-)\, i_{dc}\, R_f$ (where R_f is the value of the feedback resistor you choose). This will be your way to measure the value of i_{dc}.

(12.) Adjust the HLE gain G_2 to keep the multiplier output in its ‘good’ range (0.6 to 1.2 Volts).

Now turn the voltage to the light bulb to its minimum value. You should observe a decrease in the noise signal, and also notice, at the separate monitor point, that the d.c. current goes to zero.

Precautions: 1. While installing the lightsource and photodiode in the back panel of LLE, you should only finger-tighten the light bulb and photodiode and do not over-tighten the plastic screw holding the light bulb in place otherwise it will crack the glass of the bulb and destroy it.

2. For using the bulb, polarity of connections didn't matter, and a 0 - 22-V potential difference as a voltage source would drive the bulb. By contrast, in using the LEDs, polarity *does* matter -- attach the red lead (of either LED) to a positive polarity and Aa *current-limited* source is recommended for driving the LED.

3. A 100-mA current may ***NOT*** be safely passed through the 100-Ω, 1/4-Watt resistor provided by the Series-Resistor switch! To measure currents as large as this, mount a suitable resistor into the terminal blocks provided, and access it via the A_{ext} or B_{ext} position of the selector switch.

18. Quantum Analogs Experiment

(*Acoustic Experiments Modeling Quantum Phenomena*)

Objective:

Part 1: Standing Sound Waves in a Tube –Analog to a Quantum Mechanical Particle in a Box

(a) Measure a spectrum in the tube using an oscilloscope

(b) Measure a spectrum with the computer and compare it to the spectrum found with the oscilloscope

Part 2: Modeling a Hydrogen Atom with a Spherical Resonator

(a) Measure resonances in the spherical resonator and determine their quantum numbers

(b) Measure spectra and wave functions in the spherical resonator using the computer

Equipments Required:

Quantum Analog System: Controller, V-Channel & Aluminum Cylinders, Sine wave generator capable of producing 1-50 kHz with a peak-to-peak voltage of 0.50 V Two-Channel Oscilloscope.

Part 1: Standing Sound Waves in a Tube –Analog to a Quantum Mechanical Particle in a Box

Theory:

"Quantum Analogs" is the teaching of wave mechanics. The idea at the heart of this apparatus is the analogy between the mathematics of the Schrödinger wave equation, and the wave equations that describe the behavior of ordinary sound waves in air. Parts of this acoustic apparatus will allow you to explore acoustic analogs to quantum-mechanical systems in one, and three, dimensions. One of the advantages of the 'acoustic analog' is that sound phenomena occur on a very human scale of length and time.

A resonance occurs when a standing sound wave has developed in the tube. The sound emitted by the speaker is reflected back and forth between the two hard end-walls of the tube. The resonance develops when, after a round trip in the tube, the sound wave is in phase with the wave emitted by the speaker. In this case, the emitted sound interferes with the reflected sound constructively. The condition for resonance is fulfilled when:

$$2L = n\frac{c}{f} = n\lambda$$

with the length of the tube L, the speed of sound c, the frequency f, the wavelength λ and an integer number n=1,2,...∞. Resonances are observed when the tube length is an integer multiple of λ/2.

Differential equation for sound and boundary conditions:

The propagation of sound waves in air can be described by differential equations.

On one hand, there is the linearized Euler's equation

$$\frac{\delta \vec{u}}{\delta t} = -\frac{1}{\rho}\, grad\, p \qquad (1.1)$$

with the velocity of the air $\vec{u}$, the mass density of the air ρ and the pressure p. On the other hand, the continuity equation has to be fulfilled.

$$\frac{\delta \rho}{\delta t} = -\rho\, div\, \vec{u} \qquad (1.2)$$

Additionally, representing compressibility as κ, the density and the pressure of the air are connected by

$$\frac{\delta p}{\delta \rho} = \frac{1}{\kappa \rho} \qquad (1.3)$$

These equations can be combined to a wave equation for the pressure

$$\frac{\delta^2 p}{\delta t^2} = \frac{1}{\rho \kappa} \Delta p \qquad (1.4)$$

with the Laplace operator Δ. In this wave equation, however, the phase relation between velocity and pressure of the wave is lost, since the velocity has been eliminated. We need to refer to the velocity again, since the boundary conditions at the hard wall can be formulated best with the velocity. It is obvious that, at the surface of the wall, the velocity perpendicular to the wall has to be zero. (The air can not move into or out of the wall.) From eqn. (1.1), it also follows that, at the surface of the wall, the derivative of the pressure in the direction perpendicular to the wall is zero. This combination of boundary conditions is called a "Neumann boundary condition".

For frequencies lower than about 16 kHz, the air is not moving perpendicular to the symmetry-axis (x-axis) of the tube. Thus,

$$u_y(\vec{r}) = 0,\ u_z(\vec{r}) = 0,\ u_x(\vec{r}) = u_x(x) \text{ and } p(\vec{r}) = p(x)$$

The problem has now been reduced to a quasi one-dimensional problem and we can make a one-dimensional ansatz for the solution in the form:

$$p(x) = p_0 \cos(kx - \omega t + \alpha) \qquad (1.5)$$

Here, p_0 represents the amplitude of the wave and must not be confused with the background air pressure of about 1000 mbar. $\omega = 2\pi f$ is the angular frequency and $k = 2\pi/\lambda$ is the wave vector.

This function describes a wave propagating in the positive x-direction. In the tube we find a superposition of right and left (positive and negative x-direction) propagating waves, since the waves are reflected at the ends of the tube. The wave function is therefore given by

$$p(x) = \frac{1}{2} p_0 \cos(kx - \omega t + \alpha) + \frac{1}{2} p_0 \cos(-kx - \omega t - \alpha) \qquad (1.6)$$

This can be rewritten as-

$$p(x) = p_0 \cos(kx + \alpha) \cos(\omega t) \qquad (1.7)$$

Solutions of the differential equation are those wave functions $p(x)$ that fulfill the boundary conditions for a certain tube length L at all times. From the boundary conditions $dp/dx(0) = 0$ and $dp/dx(L) = 0$, we can easily derive the parameters to be $\alpha = 0$ and $k = n\,\pi/L$.

Dispersion of sound waves:

Redraw your graph of frequency as function of resonance-index (*fn vs. n)* to show angular frequency as function of wave vector $\omega(k)$. This new graph shows the dispersion relation of sound waves.

Analogy to a quantum mechanical particle in a box:

The sound wave in the tube can serve as an analog for a quantum mechanical particle in a one-dimensional square potential well. The differential equation that describes the particle is Schrödinger's equation:

$$i\hbar\frac{\delta}{\delta t}\,\psi(\vec{r},t) = -\frac{\hbar^2}{2m}\Delta\,\psi(\vec{r},t) + V(\vec{r})\psi(\vec{r},t) \qquad (1.8)$$

with the wave function $\psi(\vec{r},t)$, the particle mass m, and a scalar potential $V(r)$. In the case of a one-dimensional square potential well with infinitely high potential barriers at both ends, and $V = 0$ in the space between the ends, the equation reduces to

$$i\hbar\frac{\delta}{\delta t}\,\psi(x,t) = -\frac{\hbar^2}{2m}\Delta\,\psi(x,t) \qquad (1.9)$$

This differential equation has as a solution complex waves that are scattered back and forth between the ends of the well. The probability of finding the particle at a certain position x in the well is given by the probability density $|\psi(x,t)|^2$. When multiplied by the elementary charge e, it represents the charge density inside the well.

Most of the solutions of eqn. (1.9) result in time-dependent charge densities. These, however, would emit electromagnetic waves, since charge is moving. On the other hand, there are certain solutions that have a time independent charge density. They can be found by solving the time- independent Schrödinger equation

$$E\psi(\vec{r}) = -\frac{\hbar^2}{2m}\Delta\,\psi(\vec{r}) + V(\vec{r})\psi(\vec{r}) \qquad (1.10)$$

In our case, for the one-dimensional square potential well, the equation simplifies to

$$E\psi(x) = -\frac{\hbar^2}{2m}\Delta\,\psi(x) \qquad (1.11)$$

This equation can be solved for certain eigenvalues of energy E. We make an ansatz with standing waves of the form

$$\psi(x)=A\sin(kx+\propto) \qquad (1.12)$$

At the ends of the box, where the potential is infinitely high, the wave function has to be zero (Dirichlet boundary condition). These boundary conditions, $\psi(0) = 0$ and $\psi(L) = 0$, are fulfilled if $\alpha = 0$ and $k = n\,\pi/L$ where n is an integer. The total probability of finding the particle

Anywhere in the box has to be one. This determines that the amplitude of the wave function is $A=\sqrt{2/L}$.

The solution of Schrödinger's time-dependent equation (1.9) is obtained from the solution (1.12) by multiplying it with a time dependent phase factor

$$\psi(x) = A\sin(kx + \propto)\, e^{-i\omega t} \qquad (1.13)$$

You can convince yourself that, for this solution, $|\psi(x,t)|^2$ is indeed time-independent. The angular frequency in this expression is given by $\omega = E/\hbar$. Note that in quantum mechanics the energy is in general connected with the frequency by

$$E = hf = \hbar\omega \qquad (1.14)$$

We can now calculate the eigenvalues of energy that are given by

$$E(k) = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 n^2 \pi^2}{2mL^2} \qquad (1.15)$$

This is the dispersion relation of the quantum mechanical particle in a box.

What is analogous, what is different?

The classical sound wave in a tube and the quantum mechanical electron in a square potential well are similar in many respects, but some details are different. Both the sound wave and the wave- function of the electron are solutions of a wave equation describing a delocalized object. The particular aspect being described, however, is different. In the classical case, $p(x,t)$ is the amplitude of the signal picked up by a microphone located at this position. In the quantum mechanical case, the squared amplitude $|\psi(x,t)|^2$ at a certain position gives the probability of finding the electron at this position.

Both of the differential equations have the Laplace operator on the right side (second derivatives with respect to space). However, with respect to time they are different. In the classical case, we have a second derivative with respect to time that leads to wave-solutions. In the quantum mechanical case, the combination of the complex number i and a first-order derivative with respect to time leads to wave solutions. But these wave-solutions are complex due to this special form. It is also the first-order time-derivative that results in a parabolic dispersion $E(k)$ of the electron. In contrast, the sound wave has a linear dispersion due to the second-order time- derivative. Schrödinger's equation includes, in addition, a potential $V(r)$ that can not be simulated by the sound wave experiment. However, the reflection at a hard wall can be used to function as an analog to an infinitely high potential barrier. In later experiments, we will use irises as an analog for finite potential barriers with certain reflection and transmission probability.

In both cases, eigenstates are found in a well. For certain wavelengths, standing waves are found, and in both cases the wavevector of these waves is given by $k = n\ \pi/L$. However, the position of the nodes is different, because the boundary conditions are not the same. In the quantum mechanical case, the wave function must be zero at the boundary. In the case of sound waves, we have physical quantities that we use to describe the wave. One is the pressure and the other is the air-velocity. Like the quantum mechanical wave function, the velocity has a node at the boundary, but the velocity is a vector. The pressure has a local maximum at the boundary and is a scalar quantity. As an analog to the *scalar* quantum mechanical wave function, we therefore prefer the *scalar* pressure, even though it has an opposite boundary condition. A scalar "velocity potential" could also be used to describe the wave, but it does not help much, since its nodes are at the same position as those for the pressure. You should be aware of this difference.

To each eigenstate, an eigenfrequency, ω, is assigned. In quantum mechanics, it is found in time dependent phase$e^{i\omega t}$ In the case of sound waves, the eigenfrequency is factor

simply the frequency of the sound itself, $\omega=2\pi f$. In quantum mechanics, the frequency is directly related to an energy by the equation $E=h\omega$.

This has no direct analog in the sound experiments. When working with sound, we look at the frequency of the sound and not at an energy. We therefore consider energy-levels in quantum mechanics as being analogous to the "frequency-levels" in the sound experiments that are given by the sharp resonance frequencies. The dispersion $E(k)$, discussed in quantum mechanics, can be compared with $\varpi(k)$ in classical mechanics.

Another little difference is related to the absolute phase. The microphone can measure the phase of the sound wave, but in quantum mechanics the absolute phase of a state can not be measured. Relative phases between two wavefunctions can be measured in quantum mechanics and we can measure the phase of an acoustic wave function at different locations and determine the relative phase to compare with a quantum mechanical system. You should be aware that the sound experiments provide an experimentalist with more information about the system than can be extracted from an analogous quantum mechanical system.

Procedure:

1. Standing sound waves in a tube – an analog to a quantum mechanical particle in a box

Make a tube using the tube-pieces. Put the end-piece with the speaker on one end and the end- piece with the microphone on the other. Attach a BNC splitter to *SINE WAVE INPUT* on the Controller. Connect the output of your sine wave generator to one side of the splitter. Use a BNC cable to send the sound signal to the Channel 1 input of your oscilloscope. Plug the lead from the speaker end of your experimental tube to *SPEAKER OUTPUT* on the Controller. The same sine wave now goes to both the speaker and Channel 1. Connect the microphone output of the tube array to *MICROPHONE INPUT.* Connect *AC MONITOR* on the Controller to Channel 2 of the oscilloscope. Channel 2 will display the sound signal received by the microphone. Trigger the oscilloscope on Channel 1. Use the *ATTENUATOR* dial on the Controller to keep the signal on Channel 2 from going off scale. (Appendix 1 describes the function of each part of the Controller.)

Start at low frequency (100 Hz or less), and slowly increase the frequency.

What are you observing? How can you tell that you are at a resonance? Did you notice the phase- shift when going through a resonance? (Note that, due to unknown phase shifts in the speaker, microphone, and electronics, the absolute phase between input and output channel can not be interpreted.)

Change the length of the tube and repeat the experiment.

Do the resonance frequencies change? Are they higher/lower when the tube is longer/shorter?

Take a full set of data for one tube length:

Measure and record the length of the tube. Measure the first 20 resonance frequencies. Assign the lowest resonance frequency the index number n = 1, and plot the resonance frequency f_n as function of its index number, n.

From the resonance frequencies plotted as function of their index *n,* you can calculate the speed of sound *c*. Make a linear fit for your data. Calculate *c* from the slope and determine the uncertainty of your measurement.

(a) Measure a spectrum in the tube using an oscilloscope

Objective: In this experiment, the independent variable is the frequency provided by the generator, and the dependent variable is the amplitude of the sound wave reaching the microphone. First, we will examine the amplitude of the sound-wave received at the microphone as a function of the frequency of the sound. Then, we will determine how the spectrum (the pattern) observed depends on the length of the tube conducting the sound.

Setup:

With the tube, speaker and microphone arranged as before, connect the output of the sine wave generator to *SINE WAVE INPUT* on the Controller and the wire from the speaker to *SPEAKER OUPUT*. Connect the microphone on the experimental tube to *MICROPHONE INPUT*.

Locate the *FREQUENCY-TO-VOLTAGE CONVERTER* module on the Controller and set the toggle switch to *ON*. With the oscilloscope in the xy-mode, connect the *DC-OUTPUT* of the converter module to Channel 1, the x-axis. The converter provides a voltage proportional to the instantaneous frequency. The calibration is 1 V per 1 kHz and it can be used for frequencies up to 10 kHz (or, with offsets, up to 20 kHz).

Connect *DETECTOR OUTPUT* to Channel 2, the y-axis of the oscilloscope. The *DETECTOR OUTPUT* connection provides a dc signal that is proportional to the amplitude of the sound wave at the microphone.

You have now set up the oscilloscope to plot the amplitude of the sound at the microphone as a function of the frequency of the sound.

Set the image persistence time on the oscilloscope to infinite.

Now, sweep the frequency by hand. As you change the frequency, the oscilloscope will plot a spectrum with peaks.

You can use the *DC-OFFSET* knob to center the image on the oscilloscope screen.

Use the *ATTENUATOR* dial on the Controller to keep the signal from going off scale. (With an attenuator, a higher reading on the dial gives a smaller signal.

Experiment:

Take spectra for different tube lengths and compare them with the results you found in section one.

(b) Measure a spectrum with the computer and compare it to the spectrum found with the oscilloscope.

Objective: This experiment uses a computer sound card both to generate the sound wave and to sweep its frequency. We will use the oscilloscope to observe the actual sine wave signals both going into the speaker and coming from the microphone. Simultaneously, we will use the computer to display a spectrum which shows the amplitude of the signal from the microphone as a function of the frequency of the sound.

Equipment Required:

Quantum Analog System: Controller, V-Channel & Aluminum Cylinders Two-Channel Oscilloscope, Two adapter cables (BNC - 3.5 mm plug), Computer with sound card installed and Quantum Analogs "SpectrumSLC.exe" running.

Setup:

Using the tube-pieces, make a tube with the end-piece containing the speaker on one end and the end-piece with the microphone on the other.

Now, using connectors on the Controller, you will send the sound card signal to both the speaker and Channel 1 of the oscilloscope, and the microphone signal to both the microphone input of the computer and to Channel 2 of the oscilloscope.

First, make sure that the *ATTENUATOR* knob on the Controller is set at 10.0 (out of 10) turns.

Let's start with the sound signal. Attach a BNC splitter or "tee" to *SINE WAVE INPUT* on the Controller. Using the adapter cable, connect the output of the sound card to one arm of the splitter. With a BNC cable, convey the sound card signal from the splitter to Channel 1 of your oscilloscope. Plug the lead from the speaker end of your experimental tube to *SPEAKER OUTPUT* on the Controller. The sound card signal is now going to both the speaker and Channel 1. The microphone signal will also be sent two different places. Connect the microphone on your experimental tube to *MICROPHONE INPUT* on the Controller. Put a BNC splitter on the Controller connector labeled *AC-MONITOR*. From the splitter, use an adapter cable to send the microphone signal to the microphone input on the computer sound card. Use a BNC cable to send the same signal to Channel 2 of the oscilloscope to show the actual signal coming from the microphone. The computer will plot the instantaneous frequency generated by the sound card on the x-axis and the amplitude of the microphone input signal on the y-axis.

The next job is to adjust the magnitude of both the speaker and microphone signals so that you will have maximum signal while keeping the microphone input to the computer from saturating. Peak-to-peak signals to the microphone input can range from 0.50 to 2.0 volts depending upon your sound card. Once the program, SpectrumSLC.exe., is running, you can configure the computer. Go to the menu at the top of the screen and choose Configure > Input Channel/Volume At this point, choose *Line In*, if it is available; otherwise choose *Microphone*. On this screen, set the microphone volume slider to the middle of its range.

To set the speaker volume, use the *Amplitude Output Signal* on the lower left of the computer screen. That slider should also be set to middle range.

The microphone signal coming from the apparatus first passes through a built-in amplifier, and then through the *ATTENUATOR,* before reaching the *AC-MONITOR* connector. The ten-turn knob on the attenuator *decreases* the incoming signal by a factor ranging from zero to 100. For example, a setting of 9.0 turns (out of the 10 turns possible) stands for an attenuation of 9/10 or 90% attenuation of the signal. (A higher setting means a smaller signal.)

After taking an initial wide range spectrum, choose a section that includes the highest peak and a smaller one next to it. Readjust the scan to cover just this portion. Using the option that allows you to keep successive spectra visible, take Spectrum 1, 2, 3, etc. with the attenuator knob set at 9.9, 9.8, 9.7, . . . turns (out of ten). The nesting heights of the peaks will tell you whether or not the system is behaving in a linear fashion. Continue to go lower on the 10-turn dial setting until the computer program flashes ‘saturation’. You will also have visual evidence of saturation – a flat section on the tallest peak or a smaller “nesting” spacing.

Once you have reached saturation, drop back into the linear range. Now you can operate with confidence that the signals you see really are proportional to the amplitude of the sound wave you are studying.

Experiment:

Now you can use the computer to collect an overview spectrum from about 100 to 10,000 Hz. You can use coarse steps (~10 Hz) and a short time per step (~50 ms) for this investigation. As the frequency is changing, watch the trace on the oscilloscope. How is the oscilloscope showing the change in frequency? What is happening to the amplitude of the signal? How is this related to the trace being created on the computer?

Compare the spectrum recorded on the computer to the results you found using the oscilloscope in the first experiment.

Part 2. Modeling a Hydrogen Atom with a Spherical Resonator

Theory:

The hydrogen atom, with a single electron in the Coulomb potential of the nucleus, is an ideal object for studying the basic principles of atomic physics. As the simplest of all atoms, without any electron correlations, it can be solved analytically.

The spherical symmetry of the three-dimensional problem makes it possible to separate the angular and radial variables for the solution of Schrödinger's equation. The acoustic analog uses a spherical resonator that allows a separation of variables for the solution of the Helmholtz equation in the same way as is done for the hydrogen atom. We will see that the eigenfunctions with respect to the angular variables – the spherical harmonics $Y^m(\theta,\varphi)$ – are exactly the same for both problems. The radial eigenfunctions, however, are different.

The three-dimensional Schrödinger equation

$$E\psi(\vec{r}) = -\frac{\hbar^2}{2m}\Delta\psi(\vec{r}) - \frac{e^2}{r}\psi(\vec{r})$$

expressed in polar coordinates

$$E\psi = \frac{\hbar^2}{2mr^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{\hbar^2}{2mr^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{\hbar^2}{2mr^2\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2}-\frac{e^2}{r}\psi$$

can be separated in two differential equations with the ansatz

$$\psi(r,\theta,\varphi) = Y_l^m(\theta,\varphi)\ \chi_l(r)$$

The spherical harmonics are solutions of the differential equation

$$-\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]Y_l^m(\theta,\varphi) = l(l+1)Y_l^m(\theta,\varphi)$$

and $\chi_l(r)$ is a solution of the so called radial equation

$$-\frac{\hbar^2}{2mr}\frac{\partial^2}{\partial r^2}r\chi(r)-\frac{l(l+1)\hbar^2}{2mr^2}\chi(r)-\frac{e^2}{r}\chi(r) = E\chi(r)$$

In the case of the spherical acoustic resonator we transform eqn. 1.4

$$\frac{\partial^2 p}{\partial t^2} = \frac{1}{\rho\kappa}\Delta p$$

with the ansatz $p(r,t) = p(r)\cos(\omega t)$ into the time independent Helmholtz equation

$$\omega^2 p(\vec{r}) = -\frac{1}{\rho\kappa}\Delta p(\vec{r})$$

Using c as the speed of sound, equation 2.6 can be written as

$$-\frac{\omega^2}{c^2}p(\vec{r}) = \Delta p(\vec{r})$$

The Helmholtz equation in polar coordinates is given by

$$-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial p}{\partial r}\right)-\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial p}{\partial\theta}\right)-\frac{1}{r^2\sin^2\theta}\frac{\partial^2 p}{\partial\varphi^2} = \frac{\omega^2}{c^2}p$$

It separates into a radial-function *f(r)* and the spherical harmonics $Y^m(\theta,\varphi)$.

$$p(r,\theta,\varphi) = Y_l^m(\theta,\varphi)\ f(r)$$

With this ansatz the Helmholtz equation is separated in one differential equation for the spherical harmonics

$$-\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]Y_l^m(\theta,\varphi)=l(l+1)\,Y_l^m(\theta,\varphi)$$

and another for the radial function

$$-\frac{\partial^2 f}{\partial r^2}-\frac{2}{r}\frac{\partial f}{\partial r}+\frac{l(l+1)}{r^2}f(r)=\frac{\omega^2}{c^2}f(r)$$

You see immediately that eqn. 2.3 and eqn. 2.9 are exactly the same and have the same eigenfunctions and eigenvalues for the quantum numbers l (angular momentum or azimuthal quantum number) and m (magnetic quantum number). The radial equations are different, which, of course, results in different solutions. The Coulomb potential only appears in the radial equation (eqn. 2.4). Therefore, it does not affect the spherical harmonics. The eigenvalues of the radial equations are numerated by the quantum number n (radial quantum number).

The energy levels E_{nl} of the hydrogen atom are the eigenvalues of the radial equation (2.4) and the eigen frequencies of the spherical acoustic resonator ω_{nl} are eigenvalues of the radial equationSince the two differential equations are of different form, the resonance frequencies in the resonator can not be compared quantitatively with the energy levels of the hydrogen atom. However, the resonances can be classified with the same quantum numbers n (radial quantum number), l (azimuthal quantum number) and m (magnetic quantum number). The quantum numbers are integers and

$$n \geq 0 \qquad l \geq 0 \qquad l \leq m \leq l$$

In the non-relativistic description of the hydrogen atom, many energy levels are degenerate, due to the special form of the Coulomb potential. The energies can be written in the form

$$E_{n'l}=-\left(\frac{e^2}{\hbar c}\right)^2\frac{mc^2}{2(l+1+n')^2}$$

All levels with the same value for $(l+1+n)$ are degenerate. Therefore, a new quantum number is introduced that is called the "principal quantum number" n. It is given by

$$n = l + 1 + n$$

For a given principal quantum number n the azimuthal quantum number l can take the values

$$0 \le l \le n - 1$$

even though it runs to infinity for a given radial quantum number.

In the diagrams of the hydrogen atom spectrum shown below, the energy levels are labeled in two different ways. In the left figure they are labeled in the ordinary manner, using the principal quantum number. The right figure shows the energy levels labeled using the radial quantum number.

(4,0)=4s (4,1)=4p (4,2)=4d (4,3)=4f

(3,0)=3s (3,1)=3p (3,2)=3d

(2,0)=2s (2,1)=2p

(1,0)=1s

Energy levels of the hydrogen atom labeled with the principal quantum number in the ordinary way (n,l).

(3,0)=4s (2,1)=4p (1,2)=4d (0,3)=4f

(2,0)=3s (1,1)=3p (0,2)=3d

(1,0)=2s (0,1)=2p

(0,0)=1s

Energy levels of the hydrogen atom labeled with the radial quantum number (n',l).

The degeneracy of levels with the same principal quantum number does not have an analog in the spherical acoustic resonator, since the radial equation is different.

In the spherically symmetric case, the eigenvalues for different magnetic quantum numbers m are degenerate for any form of the radial equation. This is true for both the hydrogen atom and the spherical acoustic resonator. In general, the eigenvalues numbered by the quantum numbers (n,l) or by (n¢,l) are (2l +1)-fold degenerate. This degeneracy is lifted when the spherical symmetry is broken.

Procedure:

(a) Measure resonances in the spherical resonator and determine their quantum numbers

Objective:

Determine the resonance frequencies for the spherical resonator and gather data to determine their angular-momentum quantum numbers.

Equipment Required:

TeachSpin Quantum Analog System: Controller, Hemispheres, Accessories

Sine wave generator capable of producing 1-50 kHz with a peak-to-peak voltage of 0.50 V Two-Channel Oscilloscope

Setup:

Assemble two of the hemispheres so that the speaker is in the lower hemisphere and a microphone is in the upper hemisphere. (Looking carefully at the photo, you will see the speaker wire at the lower right.) Adjust the position of the upper hemisphere so that □□=180° on the scale is at the reference mark. In this position, the speaker and the upper microphone are at opposite ends of a diameter. (The microphones will be one above the other.) Attach a BNC splitter of "tee"to SINE WAVE INPUT on the Controller. Connect the output of your sine wave generator to one side of the splitter. Use a BNC cable to send the sound signal to Channel 1 of the oscilloscope. Plug the lead from the speaker on the lower hemisphere to SPEAKER OUTPUT on the Controller. The same sine wave now goes to both the speaker and Channel 1. Use a BNC cable to connect the microphone output from the upper hemisphere to MICROPHONE INPUT on the Controller. Connect AC MONITOR on the Controller to Channel 2 of the oscilloscope to display the sound signal received by the microphone. Trigger the oscilloscope on Channel 1. Use the ATTENUATOR dial on the Controller to keep the signal on Channel 2 from going off scale. Remember, with an attenuator, a higher reading on the dial gives a smaller signal. (Appendix 1 describes the function of each part of the Controller.)

Experiment:

Start at a low frequency and sweep the frequency up to about 8 kHz (8,000 Hz).

Write down all the resonance frequencies you observe. (If you listen carefully, you may actually hear some of them.)

Objective:

Observe, qualitatively, the way the amplitude of the resonance signal depends on the location of the microphone.

Experiment:

We will now gather data that will allow us to infer the angular quantum numbers of the resonances. Go to the second resonance, at about 3680 Hz. Fine-tune the frequency until it is as close as possible to the peak of the resonance. Shift the curves on the oscilloscope horizontally so that a maximum of the microphone signal (Channel 2) is located in the center of the image and marked by a vertical line. Now, watching the signal on the oscilloscope, slowly rotate the upper hemisphere, with respect to the lower one, from α = 180° to α = 0°.

Questions:

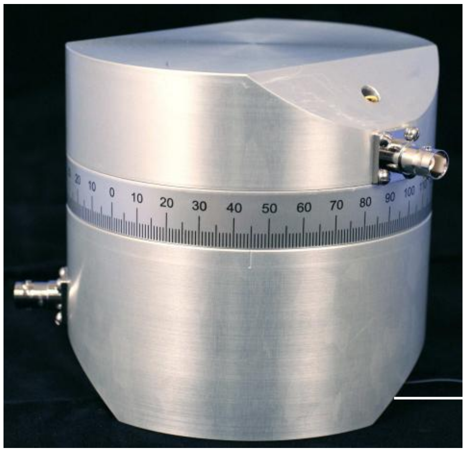
How did the amplitude change? Did the signal change its sign? Determine the angle where the amplitude is zero. At which angles is the signal maximal? Do both extrema have the same amplitude?

Note: Do not warm the aluminum parts too much by touching them with your hands. The speed of sound is temperature-dependent, and, in consequence, the resonance frequency would shift with temperature. While analyzing the angular dependence, the chosen generator frequency should remain on top of the resonance.

Analyze the data:

The angle α read on the scale is not a suitable angle for comparison with theory. Notice that the scale reading, α, running from 0 – 180°, tells you the rotation of the upper hemisphere about a vertical axis. The symmetry axis for this system, however, is determined by the speaker. The angle of interest, therefore, is measured using the speaker location as zero. To analyze the data, you must first use α to calculate the polar angle, θ. It is this angle which we use for polar coordinates. To clarify how this works try the following.

Assemble the sphere with the upper hemi- sphere set so that α = 180°. Temporarily open the resonator and notice that at this setting the speaker and the upper microphone are 180 degrees apart in space; θ= 180°. Reassemble the spheres, turn the upper sphere to α = 0°, and open the resonator again. You will see that the spatial separation of the speaker and microphone, the polar angle of interest, is now θ = 90°.



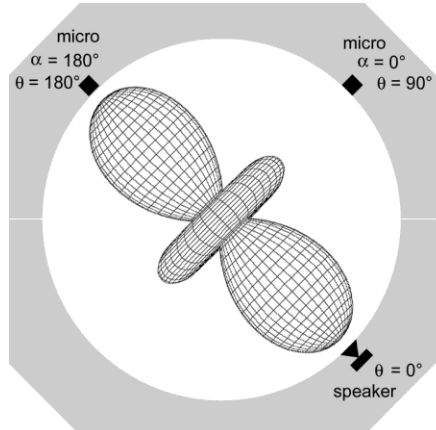


Figure 1: Left: Atom Analog. Microphones are imbedded under BNC connectors. Speaker is at lower right. Right: Sample sound amplitude pattern.

Both the speaker and microphone are at an angle of 45° with respect to the horizontal plane between the hemispheres. By rotating the hemispheres with respect to each other, the angle θ can be changed from $\theta = 90°$ (at $\alpha = 0°$) to $\theta = 180°$ (at $\alpha = 180°$). Intermediate angles can be calculated using the formula

$$\theta = \arccos(\tfrac{1}{2}\cos\alpha - \tfrac{1}{2})$$

You have measured the θ-dependence of the spherical harmonic function $Y_l^m(\theta, \varphi)$ with $l = 2$ and $m = 0$. Now we need to learn more about the spherical harmonics to compare the experiment with theory.

Objective: We will trace out the angular dependence of the amplitude of the wave function.

Additional Apparatus: dc voltmeter

Setup:

As in the first part of this experiment, attach a BNC splitter to SINE WAVE INPUT on the Controller. Connect the output of your sine wave generator to one side of the splitter. Use a BNC cable to send the sound signal to Channel 1 of the oscilloscope. Plug the lead from the speaker on the lower hemisphere to SPEAKER OUTPUT on the Controller. The same sine wave now goes to both the speaker and Channel 1.

Use a BNC cable to connect the microphone output from the upper hemisphere to

MICROPHONE INPUT. Connect AC MONITOR on the Controller to Channel 2 of the oscilloscope to display the sound signal received by the microphone. Trigger the oscilloscope on Channel 1.

This time put the upper hemisphere in the position □□= 0° on the scale. In this position the microphone is directly above the speaker which means angle θ will be 90°.

To observe the amplitude of the sound signal at the microphone, connect a voltmeter to DETECTOR- OUTPUT. You should also observe the sound signal itself by connecting the AC-MONITOR on the Controller with Channel 2 of the oscilloscope. Trigger the oscilloscope to Channel 1.

Experiment:

For a couple of major resonances, measure the amplitude as function of the angle □. You can read the absolute value of the amplitude on the voltmeter and use the oscilloscope to determine the sign.

Record the nodes (angle at which the amplitude is zero) for the same resonances.

Analyze the data:

Plot your data as function of the polar angle □□and fit the data with the Legendre polynomial that is the best match. Do this for all the resonances you have measured.

Compare the nodes you have measured with the nodes of the corresponding Legendre polynomial given in table 2.1.

Note:

Some of the resonances are very close to each other so that the peaks are overlapping. This will result in a superposition of two wavefunctions with different quantum numbers. In this case, the angular dependence you have measured does not fit to a single Legendre polynomial. We will analyze these cases in more detail by taking spectra with the computer.

(b) Measure spectra and wavefunctions in the spherical resonator with the computer

Objective:

In this experiment, you will use a computer sound card both to generate the sound wave and to sweep its frequency. You will use the oscilloscope to observe the actual sine wave signals both going into the speaker and coming from the microphone. Simultaneously, you will use the computer to display a spectrum which shows the amplitude of the signal from the microphone as a function of the frequency of the sound.

Equipment Required:

TeachSpin Quantum Analog System: Controller, Hemispheres, Accessories Two-Channel Oscilloscope

Two adapter cables (BNC - 3.5 mm plug)

Computer with sound card installed and Quantum Analogs "SpectrumSLC.exe" running.

Setup:

Now, using connectors on the Controller, you will send the sound card signal to both the speaker and Channel 1 of the oscilloscope, and the microphone signal to both the microphone input of the computer and to Channel 2 of the oscilloscope.

First, make sure that the ATTENUATOR knob on the Controller is set at 10 (out of 10) turns.

Let's start with the sound signal. Attach a BNC splitter or "tee" to SINE WAVE INPUT on the Controller. Using the adapter cable, connect the output of the sound card to one arm of the splitter. With a BNC cable, convey the sound card signal from the splitter to Channel 1 of your oscilloscope. Plug the lead from the speaker on the lower hemisphere to SPEAKER OUTPUT on the Controller. The sound card signal is now going to both the speaker and Channel 1.

The microphone signal will also be sent two different places. Connect the microphone on the upper hemisphere to MICROPHONE INPUT on the Controller. Put a BNC splitter on the Controller connector labeled AC-MONITOR. From the splitter, use an adapter cable to send the microphone signal to the microphone input on the computer sound card. Use a BNC cable to send the same signal to Channel 2 of the oscilloscope to show the actual signal coming from the microphone.

The computer will plot the instantaneous frequency generated by the sound card on the x-axis and the amplitude of the microphone input signal on the y-axis.

The next job is to adjust the magnitude of both the speaker and microphone signals so that you will have maximum signal while keeping the microphone input to the computer from saturating. Peak-to-peak signals to the microphone input can range from 0.50 to 2.0 volts depending upon your sound card.

Once the program, SpectrumSLC.exe., is running, you can configure the computer. Go to the menu at the top of the screen and choose Configure > Input Channel/Volume At this point, choose Line In, if it is available; otherwise choose Microphone. On this screen, set the microphone volume slider to the middle of its range.

To set the speaker volume, use the Amplitude Output Signal on the lower left of the computer screen. That slider should also be set to middle range.

The microphone signal coming from the apparatus first passes through a built-in amplifier, and then through the ATTENUATOR, before reaching the AC-MONITOR connector. The ten-turn knob on the attenuator decreases the incoming signal by a factor ranging from zero to 100. For example, a setting of 9.0 turns (out of the 10 turns possible) stands for an attenuation of 9/10 or 90% attenuation of the signal. (A higher setting means a smaller signal.)

After taking an initial wide range spectrum, choose a section that includes the highest peak and a smaller one next to it. Readjust the scan to cover just this portion. Using the option that allows you to keep successive spectra visible, take Spectrum 1, 2, 3, etc. with the attenuator knob set at 9.9, 9.8, 9.7, . . . turns (out of ten). The nesting heights of the peaks will tell you whether or not the system is behaving in a linear fashion. Continue to go lower on the 10-turn dial setting until the computer program flashes 'saturation'. You will also have visual evidence of saturation – a flat section on the tallest peak or a smaller "nesting" spacing. (See Appendix 2 or 3 for details.)

Once you have reached saturation, drop back into the linear range. Now you can operate with confidence that the signals you see really are proportional to the amplitude of the sound wave you are studying.

Experiment:

Set the hemispheres so that the scale angle □□= 180°.

Start the program SpectrumSLC.exe and measure an overview spectrum. You can use coarse steps such as 10 Hz and a short time per step such as 50 ms.

Change the angle between the upper and the lower hemisphere several times and observe the how the spectrum changes. Be sure to look at the spectrum for □□= 0°.

Question: What changes do you notice?

Experiment:

Go back to $\alpha = 0°$ and look in more detail at the peak near 5000 Hz. Actually, there are two peaks close to each other. Take a spectrum that measures slow enough and with sufficiently small steps to show the details of these two peaks. Also, take spectra for this range at $\alpha = 20°$ and $\alpha = 40°$.

Question: What do you notice?

Objective: Create polar plots for a series of resonances and use the plots to identify the angular momentum number and spherical harmonic function of each resonance.

Experiment:

Now we will measure the wavefunctions of the different resonances and visualize them by a polar plot of the amplitude $A(\theta)$. The computer calculates the polar angle θ from the angle α and it plots the absolute value of the amplitude as function of θ in a polar plot. This diagram makes it easy to identify the angular quantum number and the spherical harmonic function.

Take a spectrum with $\alpha = 180°$ from 2000 Hz to 7000 Hz sufficiently slowly. If you click with the left mouse button on a peak, the output frequency is adjusted to the value at which you clicked. Look at the oscilloscope and convince yourself that you are at a resonance. In the computer menu, go to "Windows" > "Measure Wave Function".

Adjust the hemispheres to $\alpha = 0°$, and measure the amplitude in steps of 10°. The program converts the angle α automatically to the polar angle θ and plots the absolute of the amplitude in a polar plot. Use the function "complete by symmetry" to complete the figure.

Create polar-plots for the prominent peaks and identify the quantum numbers.

Analyze data:

Compare the polar plots you have generated with polar-plots of the Legendre polynomials. Some of them are given below, the others you can visualize with the program PlotYlm.exe.

In case of overlapping peaks, you will find distorted figures, since there are contributions to the wave functions from two different eigenstates with different quantum numbers and symmetries.

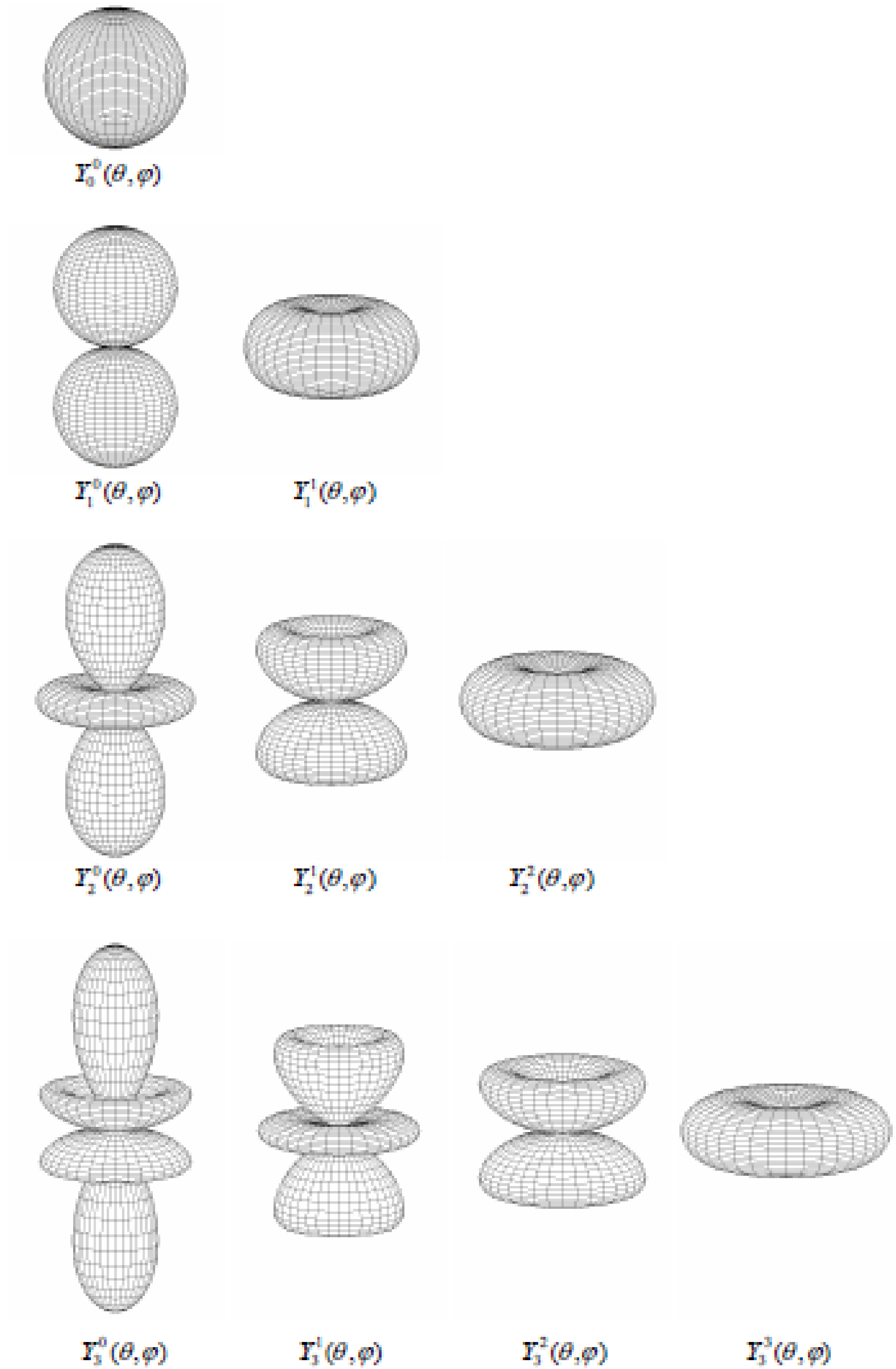


Figure 2: Plots of the spherical hamonics

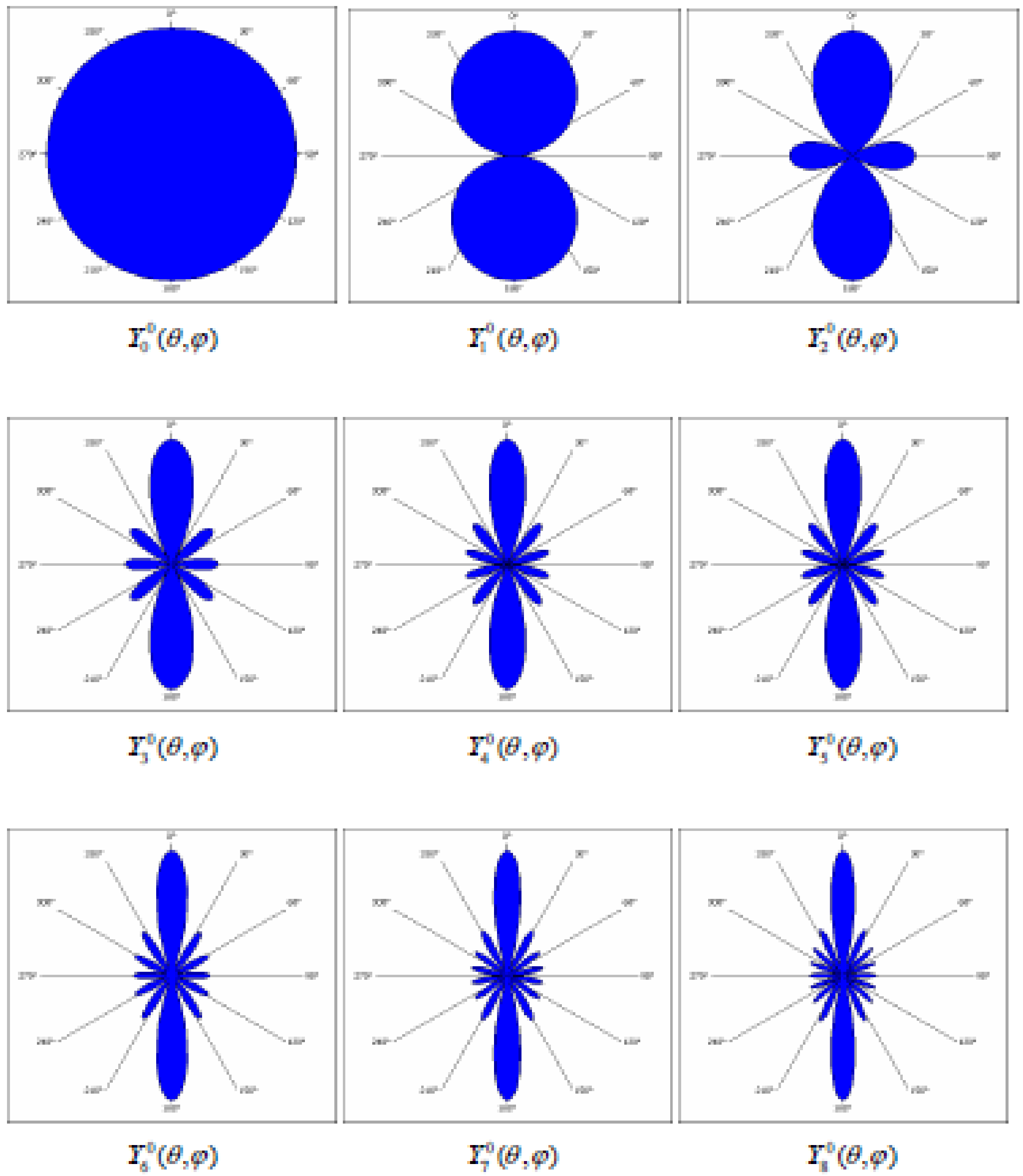


Figure 2: Cut through the spherical harmonics with magnetic quantum number m = 0.

WARNING: The BNC-to-3.5-mm adapter cables are provided as a convenient way to couple signals between the Controller and sound card. Unfortunately, they could also provide a way for excessive external voltage sources to damage a sound card. Most sound cards are somewhat protected against excessive inputs, but it is the user's responsibility to ensure that adapter cable voltages are kept BELOW 5 Volts peak-to-peak.

The maximum peak-to-peak value for optimum performance of the Quantum Analogs system depends on your sound card and can vary from 500 mV to 2 V.