Problem Sheet 8, PHY401

1) Consider the anharmonic oscillator given by the equation $\ddot{x} + \omega_0^2 x + \beta x^3 = 0$. Find the lowest order correction to the natural frequency ω_0, which results form the condition that a perturbative expansion $x = x_0 + \beta x_1$ does not have resonances, where $\ddot{x}_0 + \omega_0^2 x_0 = 0$

2)Consider the anharmonic oscillator given by the equation $\ddot{x} + \omega_0^2 x + \alpha x^2 = 0$. Find the lowest order correction to the natural frequency ω_0 which results from the condition that a perturbative expansion $x = x_0 + \alpha x_1$ does not have resonances, where $\ddot{x}_0 + \omega_0^2 x_0 = 0$.

3) Consider the equation $\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = C_0 \cos \omega t - \beta x^3$ where C_0 can be taken to be real. Obtain a cubic equation for the real part of the square of the resulting amplitude along with the non-linear correction.

4) Consider the equation $\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = C_0 \cos \omega t - \alpha x^2$ where C_0 can be taken to be real. Obtain an equation for the real part of the square of the resulting amplitude along with the non-linear correction.