1) A cylinder of mass M, radius R and length L rolls without slipping on a horizontal plank. The plank itself is moving perpendicular to the axis of the cylinder, with acceleration a, taken towards the right, say. Write the Lagrangian. Using the Euler-Lagrange equation, find the angular acceleration of the cylinder. Calculate the acceleration of the centre of the cylinder with respect to the ground.

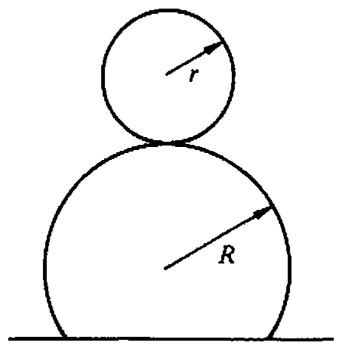
2) At time $t = 0$, an inclined plane with angle θ starts moving to the right from rest, with constant acceleration a. At the same time, a thin disc of radius R and mass M starts to roll down the plane from rest. Use appropriate generalized coordinates. Write the Lagrangian of the system. Using the Euler-Lagrange equation, obtain the position of the CM of the disc from the top of the plane, as a function of time.

$$L = -\frac{m}{2} q\ddot{q} - \frac{k}{2} q^2.$$

Do you recognize the equations of motion?

13. A heavy particle is placed at the top of a vertical hoop. Calculate the reaction of the hoop on the particle by means of the Lagrange's undetermined multipliers and Lagrange's equations. Find the height at which the particle falls off.

14. A uniform hoop of mass m and radius r rolls without slipping on a fixed cylinder of radius R as shown in the figure. The only external force is that of gravity. If the smaller cylinder starts rolling from rest on top of the bigger cylinder, use the method of Lagrange mulipliers to find the point at which the hoop falls off the cylinder.



15. A form of the Wheatstone impedance bridge has, in addition to the usual four resistances, an inductance in one arm and a capacitance in the opposite arm. Set up L and $\mathcal{F}$ for the unbalanced bridge, with the charges in the elements as coordinates. Using the Kirchhoff junction conditions as constraints on the currents, obtain the Lagrange equations of motion, and show that eliminating the λ's reduces these to the usual network equations.

16. In certain situations, particularly one-dimensional systems, it is possible to incorporate frictional effects without introducing the dissipation function. As an example, find the equations of motion for the Lagrangian

$$L = e^{\gamma t}\left(\frac{m\dot{q}^2}{2} - \frac{kq^2}{2}\right).$$

How would you describe the system? Are there any constants of motion? Suppose a point transformation is made of the form

$$s = e^{\gamma t} q.$$

What is the effective Lagrangian in terms of s? Find the equation of motion for s. What do these results say about the conserved quantities for the system?

17. It sometimes occurs that the generalized coordinates appear separately in the kinetic energy and the potential energy in such a manner that T and V may be written in the form

$$T = \sum_i f_i(q_i)\dot{q}_i^2 \qquad \text{and} \qquad V = \sum_i V_i(q_i).$$

Show that Lagrange's equations then separate, and that the problem can always be reduced to quadratures.

18. A point mass is constrained to move on a massless hoop of radius a fixed in a vertical plane that rotates about its vertical symmetry axis with constant angular speed ω. Obtain the Lagrange equations of motion assuming the only external forces arise from gravity. What are the constants of motion? Show that if ω is greater than a critical value ω_0, there can be a solution in which the particle remains stationary on the hoop at a point other than at the bottom, but that if $\omega < \omega_0$, the only stationary point for the particle is at the bottom of the hoop. What is the value of ω_0?

19. A particle moves without friction in a conservative field of force produced by various mass distributions. In each instance, the force generated by a volume element of the distribution is derived from a potential that is proportional to the mass of the volume element and is a function only of the scalar distance from the volume element. For the following fixed, homogeneous mass distributions, state the conserved quantities in the motion of the particle:

(a) The mass is uniformly distributed in the plane $z = 0$.

(b) The mass is uniformly distributed in the half-plane $z = 0, y > 0$.

(c) The mass is uniformly distributed in a circular cylinder of infinite length, with axis along the z axis.

(d) The mass is uniformly distributed in a circular cylinder of finite length, with axis along the z axis.

(e) The mass is uniformly distributed in a right cylinder of elliptical cross section and infinite length, with axis along the z axis.

(f) The mass is uniformly distributed in a dumbbell whose axis is oriented along the z axis.

(g) The mass is in the form of a uniform wire wound in the geometry of an infinite helical solenoid, with axis along the z axis.

20. A particle of mass m slides without friction on a wedge of angle α and mass M that can move without friction on a smooth horizontal surface, as shown in the figure. Treating the constraint of the particle on the wedge by the method of Lagrange multipliers, find the equations of motion for the particle and wedge. Also obtain an expression for the forces of constraint. Calculate the work done in time t by the forces of constraint acting on the particle and on the wedge. What are the constants of motion for the system? Contrast the results you have found with the situation when the wedge is fixed. [*Suggestion*: For the particle you may either use a Cartesian coordinate system with y vertical, or one with y normal to the wedge or, even more instructively, do it in both systems.]