Question/Response Sheet. End Sem PHY401. 16/12/2024 Time : 180 Minutes. Marks : 30

Name : .... Roll number : ....

- This paper has 4 back to back printed pages and 30 short questions in total. Each question carries 1 mark. There is no part marking.
- Steps leading to your answer must be shown clearly and systematically **in the copy provided separately**. **Any answer that has been entered but working has not been shown in the copy will attract penalty of 2 marks per incidence.**
- For each question, please give the answer **only in the corresponding answer box provided with each question. You can leave your answer in terms of simple fractions, square root, π etc.**
- If you think that a certain question is incorrect, you MUST show the working and write your obtained answer in the corresponding box to be considered for marking. **Blank boxes will not be considered for marking in these cases**.
- Answer to a given question written anywhere else or in a wrong box will not be considered.
- **Assume that dimensions have been taken care of. If needed, assume $g = 10$.**

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1) The Hamilton-Jacobi equation is given by $\frac{\partial S}{\partial t} + H = 0$ where the symbols have their usual meanings. The action admits a separation of variables $S(q_i, t) = f(q_i) - Et$ where $f(q_i)$ is a smooth function of the coordinates and E is the fixed energy. Consider a free particle in one dimension with mass $m = 1$ and $E = 1$. What is the numerical value of $\frac{\partial S}{\partial q}$ at $t = 1$ and $x = 1$?

Ans 1: $\boxed{}$

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2) For the free particle with $m = 1$ and $E = 1$, if the canonical transformation leading to the Hamilton-Jacobi equation is generated by an $F_2(q, P, t)$ type generating function what is the value of F_2 at $t = 1$ and $x = 1$? (Ignore any additive arbitrary constant. $p = \frac{\partial F_2}{\partial q}$, $Q = \frac{\partial F_2}{\partial P}$).

Ans 2: $\boxed{}$

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3) Consider a particle of mass $m = 2$ moving in one dimension in a potential $V = x$. Its total energy is $E = 1$. What is the value of S (defined in problem 1) at $t = 1$ and $x = 1/2$? Ignore any arbitrary additive constant.

Ans 3: $\boxed{}$

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4) In the same setup as problem (1), consider a particle of mass $m = 2$ moving in one dimension in a potential $V = x^2/2$. Its total energy is $E = 1$. What is the value of $\frac{\partial S}{\partial x}$ at $t = 0$ and $x = 1/2$?

Ans 4: $\boxed{}$

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5) A 1-D spring-mass simple harmonic oscillator has Hamiltonian $H = \frac{p^2}{2m} + \frac{kq^2}{2}$. It is given that $m = 1$, $k = 2$ and the amplitude of oscillation is 1. What is the value of $\oint p\,dq$ where the integration is taken over one complete cycle in phase space ?

Ans 5: $\boxed{}$

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6) Let $F_2(q, P)$ be the generator of the canonical transformation that transforms the Hamiltonian of problem (5) to the action-angle variables. Use the same values of k, m and the amplitude as in problem (5) to calculate $\frac{\partial F_2}{\partial q}$ at $q = \frac{1}{2}$.

Ans 6: $\boxed{}$

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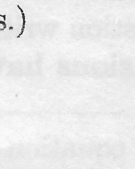
7) For a particle of mass $m = 1$ moving in a 1D potential $V = q$, the generator of canonical transformation F_2 that makes the coordinate q cyclic can be written as $\int^q f(q)dq$. If the total energy of the particle is $E = 2$, what is the value of $f(q)$ at $q = 1$?

Ans 7:

8) For a simple harmonic oscillator of mass $m = 2$ and $k = \frac{1}{2}$ i.e., potential $V = \frac{q^2}{4}$, it is given that the area $\oint pdq = 2$ on the phase plane. If I and θ denote the action and the angle variables respectively, what is the value of $2\pi \oint I d\theta$?

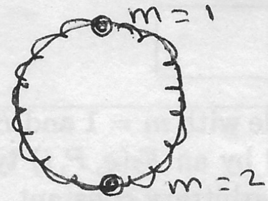
Ans 8:

9) In the setup shown in the figure, the unstretched lengths of both springs can be taken to be zero. The spring constants of both are k and $\omega^2 = k/m$. Motion takes place only along the vertical. The normal mode frequencies are $n_1\omega$ and $n_2\omega$. What are the values of n_1 and n_2? (Hint : Define $x_2' = x_2 - 2mg/k$ and a similar transformation for x_1. You should then get back familiar equations.)



Ans 9:

10) Two masses $m = 1$ and $m = 2$ are constrained to move on a horizontal circle as shown. Two identical massless springs with spring constant $k = 1$ wrap around the circle and connect the masses. What is the value of the non-zero normal mode frequency ?

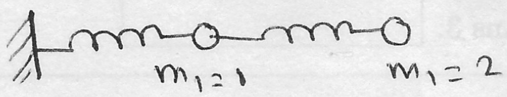


Ans 10:

11) A particle of mass $m = 2$ moves in the potential $V = (x - y)^2$. Find the non-zero normal mode frequency.

Ans 11:

12) Two identical springs of spring constant $k = 1$ are attached to two masses $m_1 = 1$ and $m_2 = 2$, and one wall as shown. The system is horizontal and there is no friction. Motion takes place only along the common direction of the springs. What are the normal mode frequencies?



Ans 12:

13) Consider the oscillator equation $\ddot{x} + \dot{x} + 4x = \cos\omega t$. The amplitude of the particular solution is A. If $A = ae^{i\delta}$ where δ is a constant, then what is the value of $\tan\delta$ when $\omega = 3$?

Ans 13:

14) In the same setup as problem (13), what is the value of a when $\omega = 1$?

Ans 14:

15) The equation of motion of a particle with mass $m = 1$ is given by $\ddot{x} + 2\gamma\dot{x} + x = 3\cos\omega t$. Find the amplitude of the particular solution and then set $\omega = 1 + \epsilon$. Both $\epsilon, \gamma \ll 1$. Work up to lowest order in ϵ and γ. If, as a function of ϵ, the maximum value of the (real) amplitude is $\frac{n}{\gamma}$, what is the value of n?

Ans 15:

16) The equation of motion of a particle with mass $m = 1$ is given by $\ddot{x} - x + x^3 = 0$. Assume that the total energy of the particle is $E = \epsilon$, where $\epsilon \ll 1$. Then in the limit $\epsilon \to 0$, what is the value of $|x_{tp}|$ where $x_{tp} \neq 0$ denotes the turning points of the motion?

Ans 16:

17) The equation of motion of a particle with mass $m = 1$ is given by $\ddot{x} + x + \beta x^3 = 0$, where $\beta \ll 1$. The total energy of the particle is $E = 1$. If $|x_{\rm tp}| = \sqrt{2}(1 - \frac{\beta}{n})$ denotes the magnitude of the turning points of the motion up to first order in β, then what is the value of n? (You will need the formula $\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \cdots$ for $x \ll 1$).

Ans 17:

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18) Consider the equation of motion $\ddot{x} + x + \alpha x^2 = 0$, where $\alpha \ll 1$. Taking the perturbative solution $x = x_0 + \alpha x_1$, where $x_0 = \cos t$, what is the magnitude of the *particular solution* of x_1 at $t = \pi/4$?

Ans 18:

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19) Consider the equation of motion $\ddot{x} + f(t)x = 0$ where $f(t)$ is a real function of t with period T. This equation has 2 independent solutions with the property $x_1(t+T) = n_1 x_1(t)$ and $x_2(t+T) = n_2 x_2(t)$, where n_1 and n_2 are real constants. If it is given that $x_1(t) = 3^{t/T}\cos t$ and $x_2(t) = g(t)\sin t$, with $g(t)$ a real function of t, what is the value of $g(t)$ at $t = T$?

Ans 19:

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20) You are given the set of equations $\dot{a} + b(\frac{\epsilon}{2} + \frac{1}{4}) = 0$ and $\dot{b} + a(-\frac{\epsilon}{2} + \frac{1}{4}) = 0$. Assuming that a and b are real and of the form $a = a_1 e^{st}$, $b = b_1 e^{st}$ with a_1, b_1 and s real, the range of ϵ is $-n < \epsilon < n$. What is the value of n?

Ans 20:

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21) A flow on a torus is described by $\dot{\theta}_1 = \frac{2}{3}$, $\dot{\theta}_2 = 1$. At $t = 0$, $\theta_1 = \theta_2 = \frac{\pi}{2}$. On the $\theta_1 - \theta_2$ diagram where θ_1 and θ_2 go from 0 to 2π, what is the value of θ_2 when $\theta_1 = 4\pi + \frac{\pi}{3}$?

Ans 21:

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22) A flow on a torus is described by the equations $\dot{\theta}_1 = \omega_1 + \frac{1}{2}\sin\theta_1\cos\theta_2$ and $\dot{\theta}_2 = \omega_2 + \frac{1}{2}\sin\theta_2\cos\theta_2$, with $\omega_1, \omega_2 > 0$. If $\omega_1 = \frac{1}{3}$, what is the maximum value of ω_2 for which there is a fixed point at which there is no oscillation?

Ans 22:

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23) For the flow on the torus given by the equations $\dot{\theta}_1 = 2 + 3\sin(\theta_2 - \theta_1)$ and $\dot{\theta}_2 = 5 + 2\sin(\theta_1 - \theta_2)$, what is the value of the conserved quantity?

Ans 23:

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24) A flow on the torus is given by the equations $\dot{\theta}_1 = \frac{1}{2} - \sin\theta_1 + K\sin(\theta_2 - \theta_1)$ and $\dot{\theta}_2 = \frac{1}{2} + \sin\theta_2 + K\sin(\theta_1 - \theta_2)$, where K is a real constant. For the system to have a fixed point at $\theta_1 - \theta_2 = \pi$, what is the magnitude of $\cos\frac{(\theta_1+\theta_2)}{2}$? ($\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$, $\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$)

Ans 24:

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25) A particle of mass $m = 1$ is constrained to move on a massless frictionless horizontal rod. The rod is accelerated vertically with constant acceleration 1, starting from rest at $t = 0$, when it coincides with the horizontal x-axis. Potential energy is measured from the x axis. If E is its energy and H the Hamiltonian, what is the magnitude of of $E - H$ at $t = 2$?

Ans 25:

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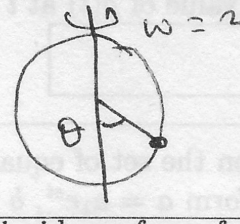
26) A particle of mass $m = 1$ is subject to an attractive force $\vec{F} = -\frac{\hat{r}}{r^3}$. There are no other forces. At an instant when its angular momentum about the origin equals 5, the distance from the center is 3. What is the value of $\frac{d^2r}{dt^2}$ at that instant? (Hint : the particle travels on a plane where the components of its velocity are $\dot{r}$ and $r\dot{\theta}$).

Ans 26:

27) A particle of mass $m = 1$ is subject to an attractive force $\vec{F} = -\frac{\hat{r}}{r^2} + \frac{\hat{r}}{r^3}$. There are no other forces. At an instant when its angular momentum about the origin equals 5, the distance from the center is 2. What is the value of $\frac{d^2r}{dt^2}$ at that instant?

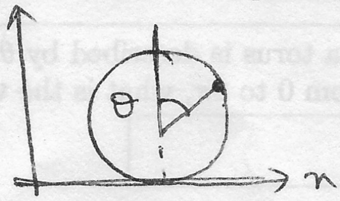
Ans 27:

28) A bead of mass 1 Kg slides without friction on a rotating wire hoop of radius 2 whose axis of rotation is through a vertical diameter. The constant angular velocity of the hoop is 2. The angle θ is as shown. When $\ddot{\theta} = 0$, what is the value of $\cos\theta$ where $\theta \neq 0, \pi$ $(g = 10)$.



Ans 28:

29) A particle of mass $m = 1$ is constrained to move frictionlessly on the inside surface of a circular wire of mass $M = 2$ and radius $R = 1$. The wire can roll without slipping on horizontal surface. At time $t = 0$, the system is at rest and the mass is at $\theta = 0$. At that time, the mass is given a horizontal speed v_0. When $\theta = \frac{\pi}{2}$, the horizontal speed of the mass is nv_0. What is the value of n?



Ans 29:

30) The Lagrangian of a system is given as $L = (3\dot{x}^2 + 7\dot{x}\dot{y} + 2\dot{y}^2) + 8(2x - y)$. If at an instant it is found that $\dot{x} = 1$ and $\dot{y} = 2$, what is the value of the conserved momentum (up to an overall multiplicative constant)?

Ans 30: