(1) $\frac{\partial S}{\partial t} + \frac{1}{2m}\left(\frac{\partial S}{\partial q}\right)^2 = 0$; $S = f(q) - Et$

$\Rightarrow \frac{\partial S}{\partial q} = \sqrt{2mE} = \sqrt{2}$.

(2) $F_2 = S = \sqrt{2}\,q - Et = \sqrt{2} - 1$.

(3) $\frac{\partial S}{\partial t} + \frac{1}{2m}\left(\frac{\partial S}{\partial x}\right)^2 + x = 0 \Rightarrow \frac{\partial S}{\partial x} = \sqrt{2m}\sqrt{E-x}$

$\Rightarrow S = f(t) - \frac{2}{3}(E-x)^{3/2} \times \sqrt{2m} = -Et - \frac{4}{3}(E-x)^{3/2}$.

$\therefore S = -1 - \frac{4}{3}\left(\frac{1}{2}\right)^{3/2}$.

(4) $\frac{\partial S}{\partial x} = \sqrt{2m}\sqrt{1-x^2/2} = \sqrt{4\times(1-1/8)} = \sqrt{7/2}$.

(5) $E = \frac{1}{2}kA^2 = 1$; $\omega = \sqrt{2}$; $\oint p\,dq = 2\pi\frac{E}{\omega} = 2\pi\frac{1}{\sqrt{2}} = \pi\sqrt{2}$.

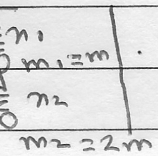
(6) $E = \frac{1}{2m}\left(\frac{\partial F_2}{\partial q}\right)^2 + V(q) \Rightarrow \left(\frac{\partial F_2}{\partial q}\right)^2 = 2\times\left[1 - \frac{1}{2}\times 2\times\frac{1}{4}\right]$

$\Rightarrow \frac{\partial F_2}{\partial q} = \sqrt{3/2}$.

(7) $F_2 = \int_0^q \sqrt{2m(E-V)}\,dq$; $f(q) = \sqrt{2\times 1} = \sqrt{2}$.

(8) $\oint p\,dq = \oint I\,d\theta$ (canonical trans^n) $\Rightarrow 2\pi \oint I\,d\theta = 4\pi$.

(9) $m_1 = m$, $m_2 = 2m$
$$\begin{cases} m_1\ddot{x}_1 = -kx_1 + kx_2 + m_1 g \\ m_2\ddot{x}_2 = -kx_2 + m_2 g \end{cases}$$



$x_2' = x_2 - \frac{2mg}{k}$ ($m_2 = 2m$)

$\Rightarrow m_2\ddot{x}_2' = -kx_2'$

$m\ddot{x}_1 = -kx_1 + kx_2' + 3mg$; $x_1' = x_1 - \frac{3mg}{k}$

$$\Rightarrow \left.\begin{matrix} m_1\ddot{x}_1' = -kx_1' + kx_2' \\ m_2\ddot{x}_2' = -kx_2' \end{matrix}\right]$$

$\alpha = \sqrt{\frac{k}{m}}$ and $\frac{1}{\sqrt{2}}\sqrt{\frac{k}{m}}$;

$\nu_1 = 1$, $\nu_2 = \frac{1}{\sqrt{2}}$.

(10) $\pm\sqrt{3}$.

(11) $m\ddot{x} = -x + y$, $m\ddot{y} = -y + x$. $\Big|$ $\ddot{x} + \ddot{y} = 0$, $\ddot{x} - \ddot{y} = -\frac{2}{m}(x-y) \Rightarrow$ normal freq: 1.

(12) $\pm\frac{1}{2}\sqrt{5 - \sqrt{17}}$.

(13) $\ddot{x} + \dot{x} + 4x = e^{i\omega t}$, $x_p = Ae^{i\omega t} \Rightarrow A = \frac{1}{(4-\omega^2) + i\omega} = ae^{i\delta}$

$\therefore \frac{(4-\omega^2) - i\omega}{(4-\omega^2)^2 + \omega^2} = a\cos\delta + ia\sin\delta$

$\Rightarrow \tan\delta = 3/5$

(14) $a = \sqrt{a^2\cos^2\delta + a^2\sin^2\delta} = \frac{1}{\sqrt{10}}$

(15)

(15) $a = \frac{3}{\sqrt{(1-w^2)^2 + 4r^2w^2}} = \frac{3}{\sqrt{4\epsilon^2 + 4r^2}}$; $a_{max} = \frac{3}{2r}$; $n = \frac{3}{2}$

Page No :
Date :

(16). $V = -\frac{x^2}{2} + \frac{x^4}{4} = \epsilon$; in $\lim \epsilon \to 0$, $x_{fp} = \pm\sqrt{2}$.

(17) $x^2 + \frac{\beta x^4}{2} = 2 \Rightarrow y^2 + \frac{2}{\beta}y - \frac{4}{\beta} = 0$ where $y = x^2$.

$y = -\frac{1}{\beta} + \frac{1}{\beta}\sqrt{1+4\beta} = \frac{1}{\beta}\left[-1 + \sqrt{1+4\beta}\right]$

$y^{1/2} = \frac{1}{\sqrt{\beta}}\left[-1 + 1 + 2\beta - 2\beta^2\right] = x_{fp}$

$\therefore x_{\phi fp} \cong \sqrt{2}(1 - \beta/2)$; $n = 1/2$.

(18) $\ddot{x}_1 + w_0^2 x_1 + x_0^2 = 0 \Rightarrow \ddot{x}_1 + w_0^2 x_1 = -\frac{a^2}{2}(1 + \cos 2t)$ $\left[w_0 = 1\right.$ $\left.a = 1\right]$ given

at $t = \pi/4$; $x_p = -\frac{a^2}{2} = -\frac{1}{2}$.

(19) $3^{t/T} g(t) = 1 \Rightarrow g(t) = \frac{1}{3}$ at $t = T$.

(20) $1/2$

(21) $\frac{d\theta_1}{d\theta_2} = 2/3 \Rightarrow \theta_1 = \frac{2}{3}\theta_2 + c$; $\frac{\pi}{2} = \frac{\pi}{3} + c \Rightarrow c = \pi/6$

$\therefore$ $4\pi + \pi/3 = \frac{2}{3}\theta_2 + \pi/6$; $\theta_2 = \frac{2}{3}\theta_2 = \frac{\pi}{6} \Rightarrow \theta_2 = \frac{\pi}{4}$.

(22) $\dot{\theta}_1 - \dot{\theta}_2 = (w_1 - w_2) + \frac{1}{2}\sin(\theta_1 - \theta_2)$

$\dot{\theta}_1 + \dot{\theta}_2 = (w_1 + w_2) + \frac{1}{2}\sin(\theta_1 + \theta_2)$.

$\therefore$ $w_1 + w_2 \le \frac{1}{2}$; $w_1 = \frac{1}{3} \Rightarrow w_2 = \frac{1}{6}$.

(23) $\frac{\dot{\theta}_1 - 2}{3} = -\frac{\dot{\theta}_2 - 5}{2} \Rightarrow 2\dot{\theta}_1 + 3\dot{\theta}_2 = 19$ (conserved).

(24) $\dot{\theta}_1 + \dot{\theta}_2 = 1 + 2\sin\frac{\theta_2 - \theta_1}{2}\cos\frac{\theta_2 + \theta_1}{2}$

$\dot{\theta}_1 - \dot{\theta}_2 = -2\sin\frac{\theta_1 + \theta_2}{2}\cos\frac{\theta_1 - \theta_2}{2} + 2k\sin(\theta_2 - \theta_1)$

F.P: $\theta_1 - \theta_2 = \pi$

Then $1 - 2\cos\frac{\theta_2 + \theta_1}{2} = 0 \Rightarrow \cos\frac{\theta_1 + \theta_2}{2} = \frac{1}{2}$.

(25) $L = \frac{m}{2}(\dot{x}^2 + a^2t^2) - mg\left(\frac{at^2}{2}\right)$

$H = \frac{m}{2}\dot{x}^2 - \frac{m}{2}a^2t^2 + mg(at^2/2)$

$E = \frac{m}{2}\dot{x}^2 + \frac{m}{2}a^2t^2 + mg(at^2/2)$

$E - H = m a^2 t^2 = 4$.

(~~26~~)

(26) $L = mr^2\dot{\theta} \Rightarrow \dot{\theta} = \frac{5}{9}$

$\ddot{r} = r\dot{\theta}^2 - \frac{k}{r^3} = \frac{8}{9}$ (E-L eqn.)

(27) $\ddot{r} = r\dot{\theta}^2 - \frac{1}{r^2} + \frac{1}{r^3}$. $\dot{\theta} = \frac{5}{4}$.

$\therefore \ddot{r} = 2 \times \frac{25}{16} - \frac{1}{4} + \frac{1}{8} = 3$.

(28) $L = \frac{m}{2} r^2\dot{\theta}^2 + \frac{m}{2}(r\sin\theta)^2\omega^2 + mgr\cos\theta$.

$\frac{\partial L}{\partial \theta} = \omega^2 \cancel{m} r \sin\theta\cos\theta - \cancel{m}g\cancel{r}\sin\theta = 0$

$\Rightarrow \cos\theta = \frac{g}{\omega^2 r} = \frac{5}{8}$.

(29) $L = \frac{m}{2}\left(\dot{x}^2 + r^2\dot{\theta}^2 + 2r\dot{x}\dot{\theta}\cos\theta\right) + \frac{1}{2}M\dot{x}^2 + \frac{1}{2}Mr^2\left(\frac{\dot{x}}{r}\right)^2 - mgr(1+\cos\theta)$.

$L = \frac{m}{2}\left(\dot{x}^2 + r^2\dot{\theta}^2 + 2r\dot{x}\dot{\theta}\cos\theta\right) + M\dot{x}^2 - mgr(1+\cos\theta)$

$\frac{d}{dt}\left(\dot{x} + r\dot{\theta}\cos\theta + 4\dot{x}\right) = 0$.

$\Rightarrow 5\dot{x} + r\dot{\theta}\cos\theta = C$; $\theta = 0 \Rightarrow r\dot{\theta} = v_0$, $\dot{x} = 0$.

Then $C = v_0$

Then $5\dot{x} + r\dot{\theta}\cos\theta = v_0$; $\theta = \pi/2 \Rightarrow \dot{x} = \frac{v_0}{5}$

$\therefore n = \frac{1}{5}$.

(30) Invariance: $x \to x + \epsilon$; $y \to y + 2\epsilon$.

Conserved: $\frac{\partial L}{\partial \dot{x}} \times 1 + \frac{\partial L}{\partial \dot{y}} \times 2 = (6\dot{x} + 7\dot{y}) + 2(7\dot{x} + 4\dot{y})$

$= 20\dot{x} + 15\dot{y}$

$\therefore$ at $\dot{x} = 1$, $\dot{y} = 2$ conserved qty $= 50$.