Indian Institute of Technology Kanpur

Department of Physics

PHY 421: Mathematical Methods- I

Second Mid-semester Examination 2008-2009-I

Date: 30/9/2008 Max Marks: 40 Time: 9:30-10:30 am

[Note: (i) A result, done in class, can directly be used (unless otherwise stated).

(ii) Write answer to *a new question* on a ***new page***. *Parts of the same question* must appear *together*.]

1. **A** is a unitary operator on a 3-dimensional linear vector space V, with $\{|\mathbf{e}_1\rangle, |\mathbf{e}_2\rangle, |\mathbf{e}_3\rangle\}$ as an orthonormal basis on V.

   (i) Find $\langle \beta_i | \beta_j \rangle$, *for all* $i, j = 1,2,3$; *where* $|\beta_j\rangle \equiv \mathbf{A}|\mathbf{e}_j\rangle$; $j = 1,2,3$,

   (ii) **A** satisfies: $(i)\ \mathbf{A}|\mathbf{e}_1\rangle = \alpha|\mathbf{e}_2\rangle + \alpha|\mathbf{e}_3\rangle$; $(ii)\ \mathbf{A}|\mathbf{e}_3\rangle = \alpha|\mathbf{e}_2\rangle + \gamma|\mathbf{e}_3\rangle$. Using (i) or otherwise, find the values of α (assumed real positive) and γ (assumed real).

   (iii) Determine $\mathbf{A}|\mathbf{e}_2\rangle$ as completely as possible.

   (iv) With the given information, write the matrix representation of **A** in this basis as completely as possible. (2+4+4+4=14)

2. Using *only* the general results, what can you say about eigenvalues and eigenvectors of the following matrix:

$$A = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & 2 \\ -1 & -2 & 0 \end{bmatrix}.$$

   [Hint: You need not attempt to solve for λ's. Look at the symmetry of A.] (8)

3. Let V be a 2-dimensional linear vector space and A be an operator on it. A has two distinct eigenvalues λ_1 and λ_2 . Show how V decomposes as = $V_{\lambda 1} \oplus V_{\lambda 2}$ where every non-null vector in a subspace V_λ is an eigenvector of A with eigenvalue λ. (7)

4. Consider the set S of all real functions of a real variable *x* on [0,1] which are continuous on (0, ½) and on (½,1); but there can be a finite discontinuity at *x*= ½. Addition of $f_1(x)+f_2(x)=f(x)$, is given by algebraic addition and $\alpha\circ f(x)$ is given by algebraic multiplication. *Assuming* that the addition of vectors and multiplication by a scalar satisfy the usual requirements, answer the following questions: (i) If $f_1(x), f_2(x) \in S$, is $f_1(x)+f_2(x) \in S$?; (ii) Is S a vector space?;

   (iii)Does $\langle g|f\rangle \equiv \int_0^1 dx\, g^*(x) f(x)$ serve as a scalar product? (3+4+4=11)