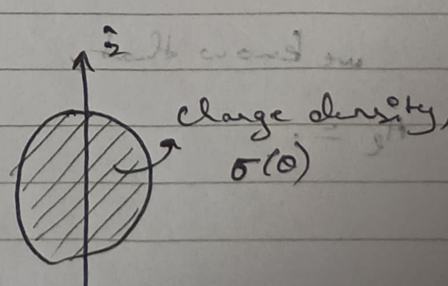
⇒ **More problems:**

How would we solve if charge density is given? $\sigma(\theta)$



$$\Rightarrow \left(\frac{\partial \Phi_{out}}{\partial r} - \frac{\partial \Phi_{in}}{\partial r}\right)\Bigg|_{r=R} = \frac{-\sigma(\theta)}{\varepsilon_0}.$$

Show that:

① $\nabla^2\left(\frac{1}{r}\right) = -4\pi\delta^3(\bar{r})$ ② $\nabla\cdot\left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(\bar{r})$

$\nabla\cdot\bar{E} = \rho/\varepsilon_0$ and $\nabla^2\Phi = -\frac{\rho}{\varepsilon_0}$

20/1/2026

Review Assignment

① Image charge Analysis:

(a) Point charge infront of grounded conducting plane ($z=0$)
(b) point charge in near grounded conducting spherical shell.

~~Potential & field due to point charge~~

~~Consider a~~

Green's Approach

21/1/26

⇒ Formal Reciepe for Green's function Approach:

⇒ For linear operators (∇^2),

$$L u(x) = f(x)$$

where $L \rightarrow$ linear operator & $u(x)$, $f(x)$ are func[ns]

then, $u(x) = L^{-1} f(x)$

⇒ Lets take the function, ⇒ $v(x,x') \equiv v(r,r')$

then,

$$\int L u(x) v(x,x')\, dx = \int f(x) v(x,x')\, dx.$$

⇒ LHS:

$$\int u(x) L^{+} v(x,x')\, dx = \dots$$

where L^{+} is the adjoint of the operator $L(x)$

$\Rightarrow$ choose $L^{+} v(x, x') = \delta(x - x')$

If this is true, then LHS:

$$\int u(x)\, \delta(x - x')\, dx = \int f(x)\, v(x, x')\, dx$$

we already know that RHS is calculable.

thus,

$$\boxed{U(x') = \int f(x)\, v(x, x')\, dx.}$$

Here, the function taken, $v(x, x')$ such that

$L^{+} v(x, x') = \delta(x - x')$, is called Green's function

$\Rightarrow$ For $\nabla^2 \Phi = -\rho/\varepsilon_0$

where $\nabla^2 = L$, $\Phi = U(\bar{r})$, $-\rho/\varepsilon_0 = f(\bar{r}')$

we know that $(\nabla^2)^{+} = \nabla^2$, thus,

satisfying the condition, $L^{+} v(x, x') = \delta(x - x')$

$\nabla^2 G(\bar{r}, \bar{r}') \equiv \delta(\bar{r} - \bar{r}')$ satisfied.

we know that, $\boxed{\nabla^2 \left(\frac{1}{|\bar{r} - \bar{r}'|} \right) = -4\pi \delta(\bar{r} - \bar{r}')}$

Generally, in electrostatics,

$G(\bar{r}, \bar{r}') = \frac{1}{|\bar{r} - \bar{r}'|}$ is the green's function.

thus,

$$\int_V \Phi(\bar{r})\, \nabla^2 G(\bar{r}, \bar{r}')\, dV = -\frac{1}{\varepsilon_0} \int_V \rho(\bar{r})\, G(\bar{r}, \bar{r}')\, dV$$

$$\Rightarrow \int_V \Phi(\bar{r}')\left(\delta(\bar{r}-\bar{r}')\right) dV = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\bar{r})}{|\bar{r}-\bar{r}'|} dV.$$

$$\boxed{\Phi(r) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\bar{r})}{|\bar{r}-\bar{r}'|} dV}$$

$\Rightarrow$ say, $G(r,r') = \dfrac{1}{|\bar{r}-\bar{r}'|} + F(\bar{r},\bar{r}')$

then $\nabla^2 G(r,r') = \delta(\bar{r}-\bar{r}')$ is still satisfied.

$\Rightarrow$ Physical interpretation of Green's function for electrostatics. $G(r,r') = \dfrac{1}{|\bar{r}-\bar{r}'|}$ is to say that it represents a unit charge that acts as a "probe" for the "shifting" delta-function.

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Green's Theorem.

Consider two ~~function~~ scalar functions (ϕ & ψ)

$$\boxed{\int_V (\phi\nabla^2\psi - \psi\nabla^2\phi)dV = \oint_S (\phi\nabla\psi - \psi\nabla\phi)\cdot d\bar{s}} \quad \text{--- } (1)$$

where $V \rightarrow$ volume & $S \rightarrow$ bounding surface.

$\phi = \Phi(\bar{r})$

$\psi = G(\bar{r},\bar{r}')$ where $G(\bar{r},\bar{r}') = \dfrac{1}{|\bar{r}-\bar{r}'|} + F(\bar{r},\bar{r}')$

substituting in (1) we get,

$$\int_V (\Phi(\bar{r})\nabla^2 G - G\nabla^2\Phi(\bar{r}))dV = \oint_S (\Phi(\bar{r})\nabla G - G\nabla\Phi(\bar{r}))\cdot d\bar{s}$$

we take $G(\bar{r}, \bar{r}')$ such that

$$\nabla^2 G = (-4\pi\delta(\bar{r}-\bar{r}')) \Rightarrow \nabla^2 F(\bar{r}, \bar{r}') = 0$$

then,

$$\Rightarrow \int_V \left\{ \Phi(\bar{r}) (-4\pi\delta(\bar{r}-\bar{r}')) - G\left(\frac{-\rho(\bar{r})}{\varepsilon_0}\right) \right\} dv$$

$$= \oint_S \left\{ \Phi(r) \frac{\partial G}{\partial n} - G \frac{\partial \Phi}{\partial n} \right\} ds.$$

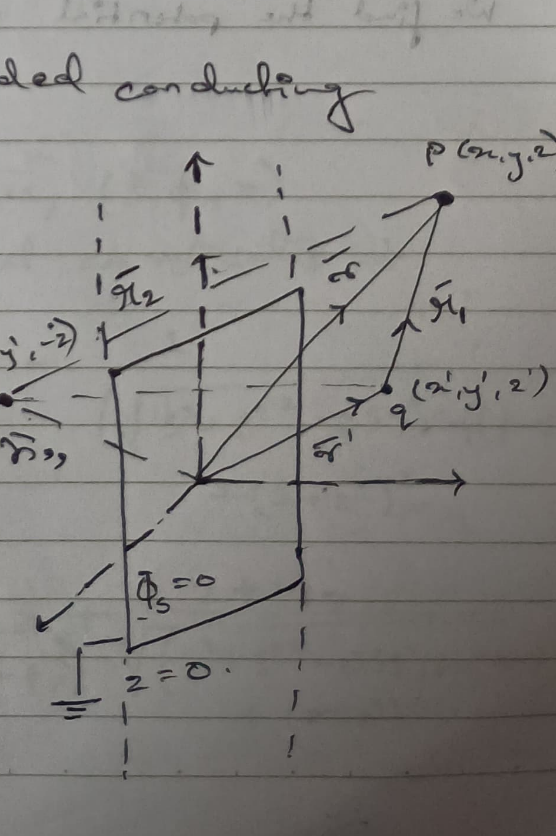
potential at the boundary

normal to the potential at the boundary.

$$\Rightarrow \boxed{\Phi(\bar{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \rho(\bar{r}) G(\bar{r}, \bar{r}') \, dv + \left(\frac{-1}{4\pi}\right) \oint_S \left\{ \Phi(r) \frac{\partial G}{\partial n} - G \frac{\partial \Phi}{\partial n} \right\}}$$

$\Rightarrow$ <u>Example:</u>

Point charge infront of a grounded conducting plane at $z = 0$.



$$\Phi_p = \frac{1}{4\pi\varepsilon_0} \left[\frac{q}{|\bar{r}-\bar{r}'|} + \frac{(-q)}{|\bar{r}-\bar{r}''|} \right]$$

where,

$$|\bar{r}-\bar{r}'| = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$|\bar{r}-\bar{r}''| = \sqrt{(x-x'')^2 + (y-y'')^2 + (z-z'')^2}$$

Consider,

$$G = \frac{1}{|\bar{r}-\bar{r}'|} - \underbrace{\left(\frac{1}{|\bar{r}-\bar{r}''|}\right)}_{F(\bar{r},\bar{r}')}$$

$$\nabla^2 F(\bar{r},\bar{r}') = 0 \Rightarrow -4\pi\delta(\bar{r}-\bar{r}'')$$

in the region of interest, $z>0$,

$\delta(\bar{r}-\bar{r}'') = 0 \;\forall z>0$, thus $\nabla^2 F(\bar{r},\bar{r}') = 0$

$$G = \frac{1}{\sqrt{(\;)^2+(\;)^2+(z-z')^2}} - \frac{1}{\sqrt{(\;)^2+(\;)^2+(z+z')^2}}$$

for boundary condition at $z=0$,

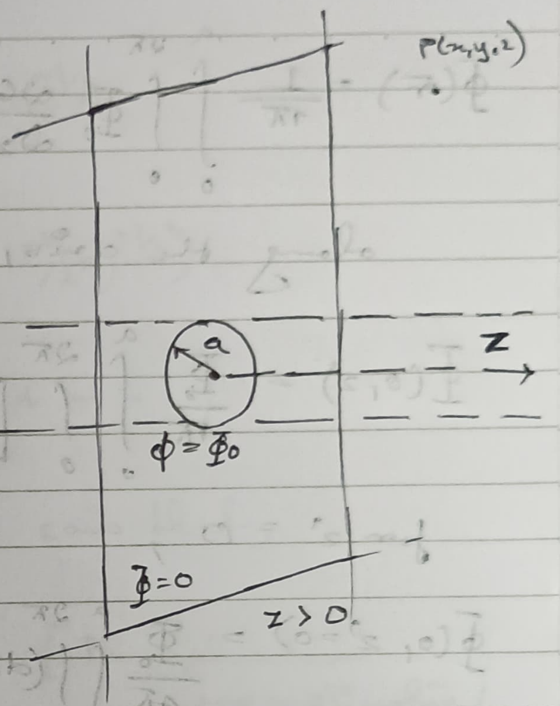
$$G = \frac{1}{\sqrt{(\;)^2+(\;)^2+(z')^2}} - \frac{1}{\sqrt{(\;)^2+(\;)^2+(z')^2}}$$

thus $G(z=0) = \boxed{0}$,

We find the potential at some $P(\bar{r})$

[Read from Jackson - Problem-2.7]

→ Example



find the potential at $P(x,y,z)$ given that the surface is at $\Phi = 0$ & a small circle inside it is at $\Phi = \Phi_0$ potential.

→ We know Green's function,

$$G(\bar{r}, \bar{r}') = \frac{1}{|\bar{r} - \bar{r}'|} - \frac{1}{|\bar{r} - \bar{r}''|}$$

$$\bar{r}' = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$\bar{r}'' = \sqrt{(x-x'')^2 + (y-y'')^2 + (z+z')^2}$$

$$\Phi(\bar{r}) = \left[\cancel{\frac{1}{4\pi\varepsilon_0} \int \rho(\bar{r}') G(\bar{r}, \bar{r}') \, dv'}^{\,0} - \frac{1}{4\pi} \oint \Phi(\bar{r}') \frac{\partial G}{\partial n'} ds' + \cancel{\frac{1}{4\pi} \oint_S G \frac{\partial \Phi}{\partial n'} ds'}^{\,0} \right]$$

We choose cylindrical coordinates, (ρ, ϕ, z)

our green's function in cylindrical coordinates ⇒

$$G(\bar{r}, \bar{r}') = \frac{1}{\left(\sqrt{\rho^2 + \rho'^2 - 2\rho\rho'\cos(\phi - \phi') + (z-z')^2}\right)} - \frac{1}{\left(\sqrt{\rho^2 + \rho''^2 - 2\rho\rho''\cos(\phi - \phi'') + (z+z')^2}\right)}$$

$$\frac{\partial G}{\partial n'} = \frac{\partial G}{\partial z'} = \left[\frac{(z-z')}{\sqrt{(\rho^2 + \rho'^2 - 2\rho\rho'\cos(\phi-\phi') + (z-z')^2)}^3} + \frac{z+z'}{\sqrt{(\rho^2 + \rho''^2 - 2\rho\rho''\cos(\phi-\phi'') + (z+z')^2)}^3}\right.$$

$$\Phi(\vec{r}) = \frac{1}{4\pi}\int_0^a\int_0^{2\pi} \Phi_0 \frac{\partial G}{\partial z'} \, \rho \, d\rho \, d\phi$$

along the axis, $\rho' = 0$,

$$\Phi(0,z) = \frac{\Phi_0}{4\pi}\int_0^a\int_0^{2\pi}\left[\frac{(z-z')}{(\rho^2+(z-z')^2)^{3/2}} + \frac{(+(z+z'))}{(\rho^2+(z+z')^2)^{3/2}}\right]$$

for $z' = 0$,

$$\Phi(0, z'=0) = \frac{\Phi_0}{4\pi}\int_0\int_0 (d\rho\, d\phi)\, \rho \left[\frac{2z}{(\rho^2+z^2)^{3/2}}\right]$$

$$= \frac{\Phi_0}{2\pi}\int_0^a\int_0^{2\pi}\left[\frac{z\rho}{(\rho^2+z^2)^{3/2}}\right] d\rho\, d\phi = \Phi_0\int_0^a\left[\frac{z\rho\, d\rho}{(\rho^2+z^2)^{3/2}}\right]$$

$$\Phi(0, z'=0) = \cancel{\Phi_0 \int_0 \frac{z\rho\, d\rho}{\rho^3\sqrt{1+\frac{z^2}{\rho^2}}^{\,3}}} = \Phi_0 z\int_0^a \frac{d\rho}{\frac{1}{\rho}\left(\sqrt{\rho^2+z^2}\right)^3}$$

$$\boxed{\Phi(0, z'=0) = \Phi_0\left(1 - \frac{z}{\sqrt{a^2+z^2}}\right)}$$

for $z \gg a$, then, $\Phi(0, z'=0) = 0$ as $z \to \infty$

for $z = 0$, then $\Phi(0, z'=0) = \Phi_0$

for $z > 0$,

$$\Phi(0, z'=0) = \Phi_0\left(1 - \frac{1}{\sqrt{1+a^2/z^2}}\right)$$ → we binomially expand this to find the potential

$$\cancel{\Phi(0, z'=a) = \Phi_0\left(1 - \frac{1}{\sqrt{1+a^2/z^2}}\right)^{-1/2}}$$

$$\Phi(0, z'=0) = \Phi_0\left(1 - \left(1 + a^2/z^2\right)^{-1/2}\right)$$

$$\Phi(z > a) = \Phi_0\left[\frac{1}{2}\frac{a^2}{z^2} - \frac{3}{8}\frac{a^4}{z^4} + \cdots\right]$$

from multipole expansion, we saw that,

$$\Phi_{out} = \sum_l \frac{B_l}{r^{l+1}} P_l(\cos\theta) \quad \text{[in spherical coordinates]}$$

since this was for azimuthal symmetry, we can write in cylindrical coordinates as,

$$\boxed{\Phi(z, \theta) = \Phi_0\left[\frac{1}{2}\frac{a^2}{z^2} P_1(\cos\theta) - \frac{3}{8}\frac{a^4}{z^4} P_3(\cos\theta) + \cdots\right]}$$

[Look into Properties of Green's Function]

Frozen in Polarization

→ induced / bound charge densities :

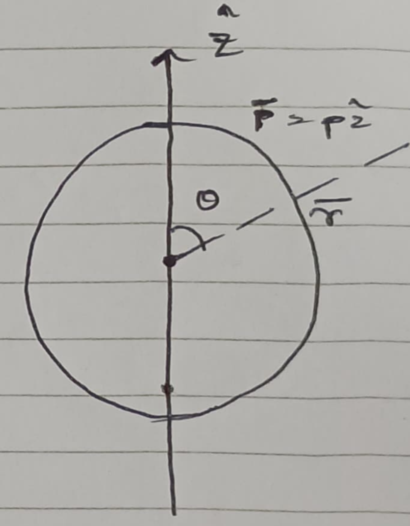
$$\sigma_b = \bar{P} \cdot \hat{n}$$

and $\rho_b = -\bar{\nabla} \cdot \bar{P}$

uniformly ~~charged~~ polarized sphere :

$\sigma_b = P\cos\theta \equiv \sigma_0 \cos\theta$

$\rho_b = 0$



we know the potential to be :

$$\Phi(r,\theta) = \begin{cases} \dfrac{Pr\cos\theta}{3\varepsilon_0} , & r<R \\[2ex] \dfrac{PR^3\cos\theta}{3\varepsilon_0 r^2} , & r>R. \end{cases}$$

to find the $\bar{E}$ for uniformly polarized sphere :

$$\bar{E}(r,\theta)\Big|_{r<R} = -\nabla\Phi = \left(\frac{-P}{3\varepsilon_0}\cos\theta\,\hat{r} + \frac{P}{3\varepsilon_0}\sin\theta\,\hat{\theta}\right)$$

$$\bar{E}(r,\theta)\Big|_{r>R} = .$$

Maxwell's Equations for Dielectrics

$\rho = \rho_{free} + \rho_b$ ← bounded charge distribution.

$$\Rightarrow \bar{\nabla}\cdot\bar{E} = \rho/\varepsilon_0 = \frac{(\rho_{free} + \rho_b)}{\varepsilon_0}$$

$$\Rightarrow \varepsilon_0(\bar{\nabla}\cdot\bar{E}) = \rho_{free} - \bar{\nabla}\cdot\bar{P}$$

$$\bar{\nabla}\cdot(\varepsilon_0\bar{E} + \bar{P}) = \rho_{free}$$

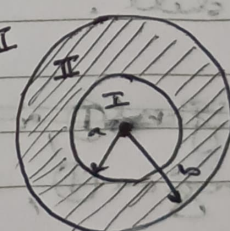
(1) $\boxed{\bar{\nabla}\cdot\bar{D} = \rho_{free}}$

where $\bar{D}$ = Electric displacement $= (\varepsilon_0\bar{E} + \bar{P})$

from Equation (1), we can write down the modified Gauss law to be,

$\boxed{\oint \bar{D}\cdot d\bar{s} = Q_{free}}$ ← modified Gauss law.

⇒ example: Consider a concentric spherical shell made of a particular dielectric.



$\bar{P} = \frac{k}{r}\hat{r}$ ← polarization.

in region (I) : $r < a$

$\oint \bar{D}\cdot d\bar{s} = 0$ & $\bar{P} = 0$

$\therefore \bar{D} = 0 \Rightarrow \varepsilon_0\bar{E} = 0 \Rightarrow \boxed{\bar{E} = 0}$

in region (II) : $a < r < b$

$\oint \bar{D}\cdot d\bar{s} = 0$ since $Q_{free} = 0$, $\bar{P} = \frac{k}{r}\hat{r}$

$\varepsilon_0\bar{E} + \bar{P} = \bar{D} \Rightarrow \varepsilon_0\bar{E} + \bar{P} = 0 \Rightarrow \boxed{\bar{E} = \frac{-k}{r\varepsilon_0}\hat{r}}$

in region (III) : $r > b$

$\oint \bar{D}\cdot d\bar{s} = 0$, & $\bar{P} = 0 \Rightarrow \boxed{\bar{E} = 0}$

Linear Dielectrics

$\Rightarrow \bar{P} = \alpha \bar{E}$ ← polarization.

$\bar{P} = (\varepsilon_0 \chi_e)\bar{E}$ where $\varepsilon_0 \rightarrow$ permissivity ε_1

$\chi_0 \rightarrow$ susceptibility.

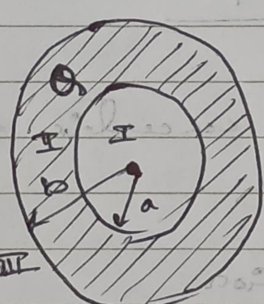
then,

$\bar{\nabla} \cdot \bar{D} = \rho_{free} \Rightarrow \bar{\nabla} \cdot (\varepsilon_0 \bar{E} + \varepsilon_0 \chi_e \bar{E}) = \rho_{free}$ #

$\therefore \boxed{(\bar{\nabla} \cdot \bar{E}) = \frac{\rho_{free}}{\varepsilon_0 (1+\chi_e)}} = \frac{\rho_{free}}{\varepsilon}$

relative permittivity of the material $\Rightarrow \varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = (1+\chi_e)$

$\Rightarrow$ Example:



for charge Q is uniformly distributed in the shell.

~~in region III, r > a,~~

in region ①, r<a,

$\oint \bar{D} \cdot ds = \rho_{free} = 0 \Rightarrow \bar{D} = 0 \;\; \varepsilon_1 \bar{E} = 0.$

$\therefore \boxed{\bar{E} = 0}$

in region II, $a < r < b$,

$\oint \bar{D} \cdot ds = \rho_{free} = Q \Rightarrow D \cdot (4\pi r^2) \cancel{= (\varepsilon_0 \bar{E} + \bar{P})} = Q$

$\bar{P} + \varepsilon_0 \bar{E} = \frac{Q}{4\pi r^2} \rightarrow \boxed{\bar{E} = \frac{Q}{4\pi \varepsilon_0 \varepsilon_r r^2}}$

in region III, $r > b$

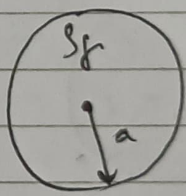
$$E = \frac{Q}{4\pi \varepsilon_0 r^2}$$

potential $\Phi(r=0)$

$\Longrightarrow$ <u>Example</u>:

Consider sphere with ρ_f, find,

$\bar{E}(r<a)$:



$\bar{E}(r>a)$:

$\Phi(r=0)$:

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→ We saw that,

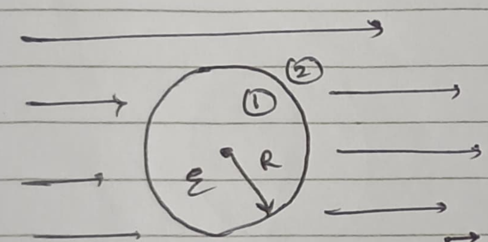
$$\boxed{\oint \vec{D}\cdot d\vec{s} = Q_{free}}$$

We get the new Boundary conditions for the material to be : $\boxed{\varepsilon_+ E^{\perp}_{above} - \varepsilon_- E^{\perp}_{below} = \sigma_{free}}$

and $\boxed{\Phi_+ = \Phi_-}$

→ Example:

Consider sphere of material with ε, find the potential due to this dielectric.



$\vec{E} = E_0 \hat{z}$

~~using multipole expansion~~ ⇒ for $\Phi(r >> R) = -E_0 z$

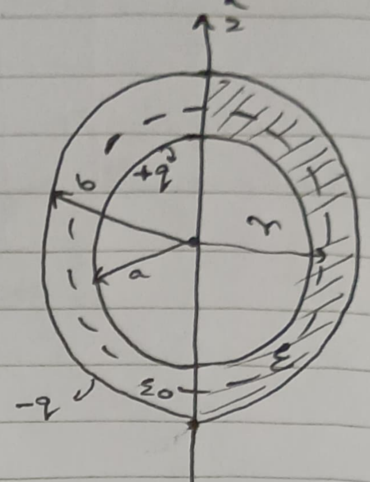
using boundary condition,

$$\varepsilon_0 \frac{\partial \Phi}{\partial n}\bigg|_{R+} - \varepsilon \frac{\partial \Phi}{\partial n}\bigg|_{R-} = \sigma_{free} = 0.$$

→ Example:

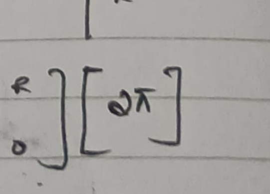
Consider concentric shell such that, half of it is filled with dielectric. What is the $\bar{E}$?

⇒



→ Example:

Give a cylinder with polarization $\bar{P} = (5z^2+7)\hat{k}$ then what is the q_{bound}



⇒ $-\bar{\nabla}\cdot\bar{P} = -10z \rightarrow \rho \Rightarrow$ (volume charge density)

$$q = \int \rho_b \, dv = -\int 10z \, s \, ds \, d\phi \, dz.$$

$$q = -\left[\int_0^L 10z\,dz \int_0^R s\,ds \int_0^{2\pi} d\phi\right] = -\left[\frac{10z^2}{2}\Big|_0^L\right]\left[\frac{s^2}{2}\Big|_0^R\right]\left[2\pi\right]$$

$$q = -\left[\frac{\cancel{10}^{5}L^2}{\cancel{2}}\frac{R^2}{\cancel{2}}\cancel{2}\pi\right] \Rightarrow \boxed{q = -5L^2R^2\pi}$$

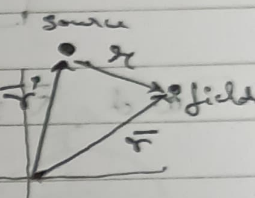
Magnetic forces do no work, they may alter the direction in which a particle moves but they cannot speed it up or slow it down.

Magnetostatics.

→ Magnetic field is produced by moving charges. and the force is given by the Lorentz force law.

$$\boxed{\bar F = q(\bar v \times \bar B)} \iff \boxed{\bar F = I\int (d\bar l \times \bar B)}$$

⟹ $$\boxed{B(\bar r) = \frac{\mu_0}{4\pi}\int \frac{\bar J(\bar r') \times \hat{\mathscr{r}}}{\mathscr{r}^2}\, dV}$$ where



Biot-Savart's law. = $\boxed{B(\bar r) = \frac{\mu_0}{4\pi}\left(\frac{q\bar v\times\hat{\mathscr{r}}}{\mathscr{r}^2}\right)}$ $\bar{\mathscr{r}} = \bar r - \bar r'$

→ for magnetostatics, we get the Maxwell's equations to be:

$$\boxed{\bar\nabla\times\bar B = \mu_0 \bar J} \quad ; \quad \boxed{\bar\nabla\cdot\bar B = 0}$$

from fundamental theorem of curl: $\int_A (\bar\nabla\times\bar B)\,da = \int_\Gamma \bar B\cdot d\bar l$

$\int_\Gamma \bar B\cdot d\bar l \neq \cancel{\text{Not}}\ 0$ & thus for a closed loop, the work done is path dependent

∴ $\bar B$ is not conservative field.

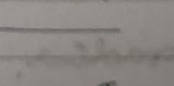
[Experimental phenomenon stating that $\bar B$ is non-conservative (?)

→ current densities:

$$\boxed{F = \int(\bar v\times\bar B)\rho\, dv} \iff \boxed{F = \int(\bar J\times\bar B)\,dv \text{ or } \int(\bar K\times\bar B)\,ds}$$

where, $\bar J = \rho\bar v$ — volume charge density

and $\bar J$ → current density ($\bar J$) volume. = $dI/da_\perp$



$\bar K$ → surface current density ($\bar K$) = $dI/dl_\perp$

Continuity Equation: $\nabla \cdot \bar{J} = -\frac{\partial \rho}{\partial t}$ where ρ = volume charge density

for magnetostatics i.e. steady currents $\boxed{\nabla \cdot \bar{J} = 0}$

Biot - Savart's Law

$$\boxed{\bar{B}(\bar{r}) = \frac{\mu_0}{4\pi} \int \frac{\bar{J}(\bar{r}') \times \hat{\mathscr{r}}}{\mathscr{r}^2} dv'}$$

or, $\mathscr{r} = |\bar{r} - \bar{r}'|$

$$\boxed{\bar{B}(\bar{r}) = \frac{\mu_0}{4\pi} \int \frac{\bar{K}(\bar{r}') \times \hat{\mathscr{r}}}{\mathscr{r}^2} dv'}$$

where $\bar{J}(\bar{r}')$ = volume current

$K(\bar{r}')$ = surface current

and $\bar{\nabla} \times \bar{B} = \mu_0 \bar{J}$ — ① ; $\bar{\nabla} \cdot \bar{B} = 0$ — ②

Ampere's law: $\boxed{\oint B \cdot dl = \mu_0 I_{enclosed}}$

Potential

$\Rightarrow$ We define a vector potential $\bar{A}$ such that,

$$\boxed{\bar{B} = \bar{\nabla} \times \bar{A}} \text{ — ③}$$

using ③ in equations ① & ②,

$$\bar{\nabla} \times (\bar{\nabla} \times \bar{A}) = \mu_0 \bar{J}$$

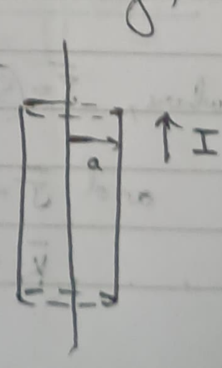
[using $\bar{A} \times (\bar{B} \times \bar{C})$ identity] $B(A \cdot C) - C(A \cdot B)$

$$\Rightarrow \boxed{-\nabla^2 \bar{A} = \mu_0 \bar{J}}$$

$$\Rightarrow \boxed{\bar{A}(\bar{r}) = \frac{\mu_0}{4\pi} \int \frac{\bar{J}(\bar{r}')}{\mathscr{r}} dv' \quad \text{or} \quad = \frac{\mu_0}{4\pi} \int \frac{\bar{K}(\bar{r}')}{\mathscr{r}} ds}$$

$\Rightarrow$ Example

Current I distributed uniformly over a wire of radius a, then find, (i) $\bar{J}$ / $\bar{K}$



$\Rightarrow \bar{J} = I/\pi a^2$; $\bar{K} = I/2\pi a$.

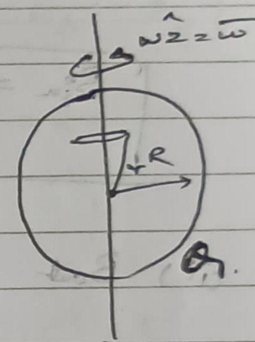
(ii) Say $\bar{J} = k r \hat{z}$, find I

$$\Rightarrow I = \int \bar{J}\, d\tau = \int_0^a \int_0^{2\pi} k r \hat{z}\, r\, dr\, d\phi = k\left(\frac{r^3}{3}\Big|_0^a\right)(2\pi)\, \cancel{\hat{z}}$$

$$\boxed{I = \frac{2\pi k a^3}{3}\hat{z}}$$

⇒ Example:

consider a sphere rotating with $\bar{\omega} = \omega\hat{z}$ then find current density. $\bar{J}$



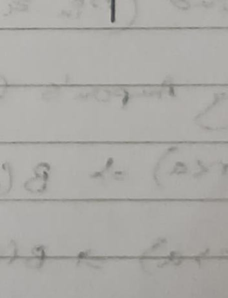
$$\Rightarrow J(\bar{r}, \theta) = \rho \omega r \sin\theta\, \hat{\phi} = \rho \bar{v}$$

$[\bar{v} = \omega r \sin\theta\, \hat{\phi}]$

⇒ Example:

consider a cylindrical wire of radius a with $\bar{k}$ and I. Find $\bar{B}(r<a)$ & $\bar{B}(r>a)$

from Analogous Gauss's law for magnetostatics



$$\oint \bar{B} \cdot d\bar{l} = \mu_0 I_{enclosed}$$

(i) for $r<a$,

$$\oint \bar{B} \cdot d\bar{l} = \mu_0 (0) \quad [I_{enclosed} = 0]$$

$$\boxed{\bar{B} = 0}$$

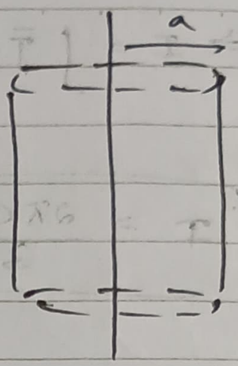
(ii) for $r>a$,

$$\oint \bar{B} \cdot d\bar{l} = \mu_0 I$$

$$\boxed{\bar{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}}$$

$\Rightarrow$ Example

consider cylindrical wire with rad = a, & $\bar{J}$. find B_{in} & B_{out}



$J = \frac{I}{\pi a^2}$

(i) B_{in} $(r<a) \Rightarrow \oint \bar{B}\cdot d\bar{l} = \mu_0 I_{enclosed}$

$$\oint \bar{B}\cdot d\bar{l} = \frac{\mu_0 I \pi r^2}{\pi a^2} \Rightarrow B(2\pi \cancel{r}) = \frac{\mu_0 I r^{\cancel{2}}}{a^2}$$

$$\boxed{B_{in} = \frac{\mu_0 I r}{2\pi a^2}}$$

(ii) B_{out} $(r>a) \Rightarrow \oint \bar{B}\cdot d\bar{l} = \mu_0 I_{enclosed}$

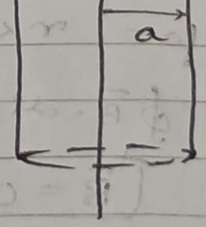
$$(B)(2\pi r) = \mu_0 I \Rightarrow \boxed{B_{out} = \frac{\mu_0 I}{2\pi r}\hat{\phi}}$$

~~(iii) Find~~

$\Rightarrow$ Example:

Given, $J = kr\hat{z}$

find (B_{in} & B_{out}) & (A_{in} & A_{out})



using Ampere's law: $\oint B\cdot dl = \mu_0 I_{enclosed}$.

$B(r<a) \Rightarrow B(2\pi r) = \mu_0 \int_0^r kr^2 2\pi dr \Rightarrow B = \frac{\mu_0 k r^2}{3}\hat{\phi}$

$B(r>a) \Rightarrow B(2\pi r) = \mu_0 \int_0^a kr^2 2\pi dr \Rightarrow B = \frac{\mu_0 k a^3}{3r}\hat{\phi}$

for potential A : Say $A = A(r)\hat{z}$ then $\nabla\times A = B$.

$$B = \begin{vmatrix} \hat{r} & r\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & 0 & A(r) \end{vmatrix} \frac{1}{r} = -\frac{\partial A(r)}{\partial r}\hat{\phi}$$

upon integrating for both cases

$A_{in} = \frac{-\mu_0 k r^3}{9} + C_1$; $A_{out} = \frac{-\mu_0 k a^3}{3} \ln(r) + C_2$, say $C_1 = 0$ for $A(0) = 0$

$A(r=a) = A_{in}(a) = A_{out}(a) \Rightarrow C_2 = \frac{\mu_0 k a^3}{3}\ln(a) - \frac{\mu_0 k a^3}{9}$ //

$\Rightarrow$ Example :

Consider, $\bar{A} = k\hat{\phi}$ what should be the current density $\bar{J}$?

$\Rightarrow \bar{A} = k\hat{\phi}$, we know that ~~$(\nabla \times \bar{A}) = \mu_0 \bar{J}$~~

~~$\bar{B} = \frac{\mu_0}{4\pi}\int \bar{J}$~~ ~~$(\nabla \times \bar{A}) = \bar{B}$~~ $\Rightarrow$ ~~$\bar{B} = \begin{vmatrix} \hat{s} & \hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_s & A_\phi & A_z \end{vmatrix}$~~

~~$\Rightarrow \bar{B} = \begin{vmatrix} \hat{s} & \hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & k & 0 \end{vmatrix} =$~~

We know that,

$\nabla^2 A = -\mu_0 J$; $A_\phi = k\hat{\phi}$

$$\bar{B} = \bar{\nabla}\times\bar{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{r} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\phi} + \frac{1}{r}\left(\frac{\partial (r A_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi}\right)\hat{z}$$

$$\bar{B} = \bar{\nabla}\times\bar{A} = (0)\hat{r} + (0)\hat{\phi} + \frac{1}{r}(k-0)\hat{z} = \frac{k}{r}\hat{z}$$

Now, we know that $\bar{\nabla}\times\bar{B} = \mu_0\bar{J}$; $B_z = \frac{k}{r}\hat{z}$

$$\bar{\nabla}\times\bar{B} = \left(\frac{1}{r}\frac{\partial B_z}{\partial \phi} - \frac{\partial B_\phi}{\partial z}\right)\hat{r} + \left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right)\hat{\phi} + \frac{1}{r}\left(\frac{\partial (r B_\phi)}{\partial r} - \frac{\partial B_r}{\partial \phi}\right)\hat{z}$$

$$= (0)\hat{r} + \left(0 + \frac{k}{r^2}\right)\hat{\phi} + (0)\hat{z} = \left(\frac{k}{r^2}\right)\hat{\phi}$$

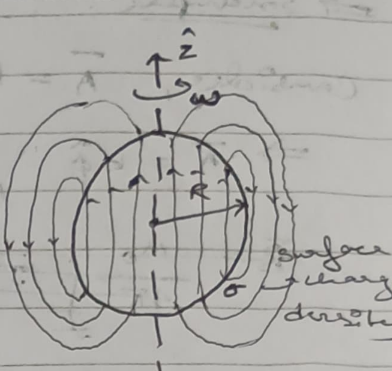
$$\mu_0\bar{J} = \left(\frac{k}{r^2}\right)\hat{\phi} \Rightarrow \boxed{\bar{J} = \left(\frac{k}{\mu_0 r^2}\right)\hat{\phi}}$$ //

3/2/2026

→ Vector potential due to a spinning charged spherical shell:

We can find the vector potential by finding the surface current density.

$$\boxed{\bar{A} = \frac{\mu_0}{4\pi}\int \frac{\bar{k}(\bar{r}')\,da'}{|\bar{r}-\bar{r}'|}}$$



↓ Has the same direction as the current density, Here, $\hat{\phi}$, thus,

$$A_\phi(r,\theta) = \frac{\mu_0 R\omega\sigma}{3}\, r\sin\theta\,\hat{\phi}, \text{ for } r<R.$$

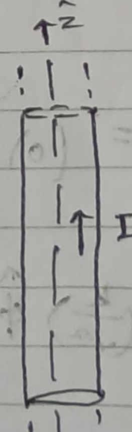
$$A_\phi(r,\theta) = \frac{\mu_0 R^4\omega\sigma}{3r^2}\sin\theta\,\hat{\phi}, \text{ for } r>R.$$

Calculating $\bar{B}$,

$$\bar{B} = \bar{\nabla}\times\bar{A} \Rightarrow \boxed{\bar{B}_z(r,\theta) = \frac{2}{3}\mu_0\sigma R\omega\,\hat{z}} \quad \text{for } r<R$$

& for $r>R$, $$\boxed{\bar{B}_z(r,\theta) = \frac{\mu_0 R^4\omega\sigma}{3}\left(\frac{2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}}{r^3}\right)}$$

→ Vector potential for a Hollow pipe:



Taking Amperian loop for $r<R$.

$$\oint B\cdot dl = (I_{enclosed})\mu_0$$

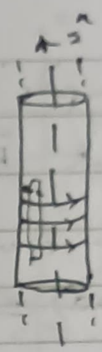
$\boxed{\bar{B}=0}$ since $I_{enc} = 0$, for $r<R$.

for $r>R$, $\oint B\cdot dl = (I_{en})\mu_0$

$$B(2\pi r) = I\mu_0 \Rightarrow \boxed{\bar{B} = \frac{I\mu_0}{2\pi r}\hat{\phi}} \quad \text{for } r>R$$

⇒ Example :

Consider, $\bar{K} = K_0 \hat{\phi}$ for Hollow pipe.



Taking a rectangular Amperian loop,

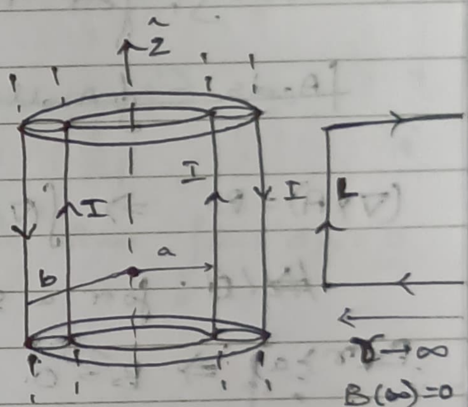
$\bar{B}$ (for $r > a$) $= \boxed{0}$

$\oint B \cdot dl = \mu_0 (I_{enclosed})$

$\bar{B}$ (for $r < a$) $= \boxed{\mu_0 K_0 \hat{z}}$

⇒ Example:

Consider two Hollow pipes, coaxial.



$\bar{B}(r < a) = 0$

$\bar{B}(a < r < b) = \mu_0 I / 2\pi r$

$\bar{B}(r > b) = 0$

Calculating the vector potentials,

We know that, $\bar{B} = \bar{\nabla} \times \bar{A}$

using, $\int (\bar{\nabla} \times \bar{A}) \cdot ds = \int A \cdot dl \Rightarrow \int \bar{B} \cdot ds = \oint A \cdot dl$

for $r > b \Rightarrow B = 0$ thus $\oint A \cdot dl = 0 \Rightarrow \boxed{A = 0}$

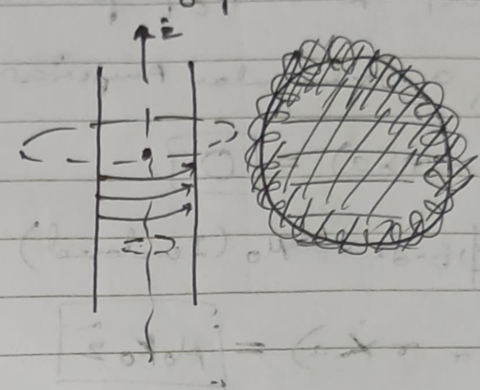
for $a < r < b \Rightarrow B = \frac{\mu_0 I}{2\pi r} \Rightarrow \int \left(\frac{\mu_0 I}{2\pi r}\right)(ds) =$

thus, $\boxed{A = \frac{\mu_0 I}{2\pi} \ln\left(\frac{b}{r}\right)}$

for $r < a \Rightarrow$ we get $\boxed{A = \frac{\mu_0 I}{2\pi} \ln\left(\frac{b}{a}\right)}$

⟹ Example:

Calculate $\bar{A}$ for solenoid, given $\bar{K} = k_0 \hat{\phi}$



We saw that,

~~$B(r>a) = 0$~~ ; $B(r<a) = \mu_0$

~~$\int A \cdot ds = \oint A \cdot dl = 0$~~

~~$A_{outside} = \frac{\mu_0 k_0 a^2}{2r} \hat{\phi}$~~

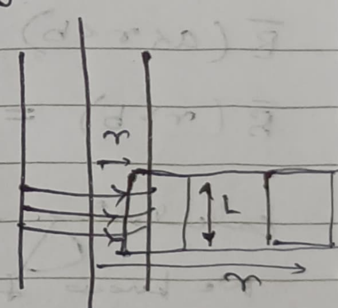
~~Similarly, for $r<a$,~~

~~$\int A \cdot ds = \oint A \cdot dl \Rightarrow A_{inside} = \frac{\mu_0 k_0 r}{2} \hat{\phi}$~~

$(\nabla \times A) = B \Rightarrow \int (\nabla \times A) ds = \oint A \cdot dl \Rightarrow \underbrace{\int B \cdot ds}_{\phi_B \to \text{mag flux.}} = \oint A \cdot dl.$

~~AA~~ for a solenoid,

$B(r>a) \Rightarrow B = 0.$ $\quad k_0 = nI.$



for $B(r<a) \Rightarrow \oint B \cdot dl = \mu_0 I_{enclosed}$

$B\cancel{L} = \mu_0 \cancel{L} n I \Rightarrow \mu_0 n I = B$

$B = \mu_0 k_0 \; ; (r<a)$

⟹ for $(s<a)$, $\int B \cdot ds = \oint A \cdot dl \Rightarrow (\mu_0 k_0)(\pi s^2) = (A) 2\pi s$

$$\boxed{A = \frac{\mu_0 k_0 s}{2} \hat{\phi}}$$

for $s>a$, $\int B \cdot ds = \oint A \cdot dl \Rightarrow (\mu_0 k_0)(\pi a^2) = A 2\pi s$

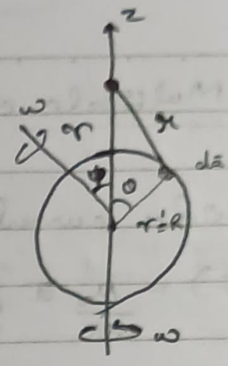
$$\boxed{A = \frac{\mu_0 k_0 a^2}{2s} \hat{\phi}}$$

$$\boxed{\int B \cdot ds = \oint A \cdot dl}$$

where $A \to$ magnetic potential

$\int B \cdot ds = \phi_B \Rightarrow$ magnetic flux.

→ Example: Vector potential for spinning charged spherical shell, with σ.



$$A = \frac{\mu_0}{4\pi}\int \frac{K(r')}{\mathscr{r}}\,da' \quad ; \quad \cancel{K = \sigma R\omega}$$

$da' = R^2 \sin\theta' d\theta' d\phi'$

$K = \sigma v$

$|\mathscr{r}| = \sqrt{R^2 + r^2 - 2Rr\cos\theta}$

$$v = \omega \times r' = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \omega\sin\psi & 0 & \omega\cos\psi \\ R\sin\theta'\cos\phi' & R\sin\theta'\sin\phi' & R\cos\theta' \end{vmatrix}$$

$$v = -\hat{x}\left(R\omega\cos\psi\sin\theta'\sin\phi'\right) - \hat{y}\left(R\omega\sin\psi\cos\theta' - R\omega\cos\psi\sin\theta'\cos\phi'\right) + \hat{z}\left(R\omega\sin\psi\sin\theta'\sin\phi'\right)$$

all terms that have $\sin\phi'$ or $\cos\phi'$ vanish since $\int_0^{2\pi}\sin\phi'\,d\phi' = 0$

and $\int_0^{2\pi}\cos\phi'\,d\phi' = 0$.

$$v = -\hat{y}\left(R\omega\sin\psi\cos\theta'\right)$$

$$A = -\frac{\mu_0}{4\pi}\iint\left(\frac{\sigma R\omega\sin\psi\cos\theta'\,\hat{y}}{\sqrt{R^2+r^2-2Rr\cos\theta'}}\right)\left(R^2\sin\theta' d\theta' d\phi'\right) = \frac{-\mu_0\sigma\omega R^3\sin\psi}{4\pi}(2\pi)\int I$$

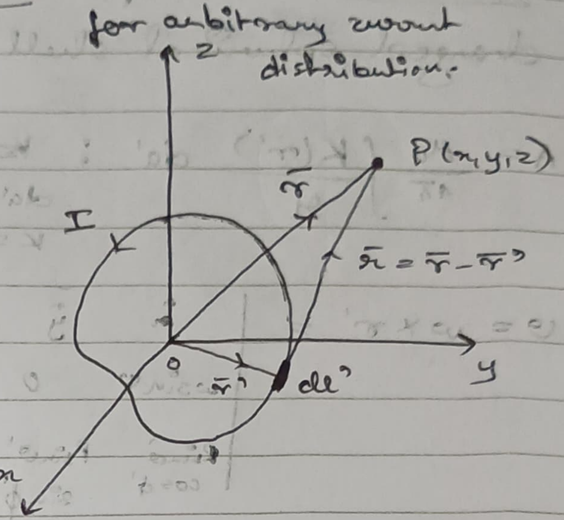
$$A = -\frac{\mu_0\sigma\omega R^3\sin\psi}{2}\left(\underbrace{\int_0^{\pi}\frac{\cos\theta'\sin\theta'}{\sqrt{R^2+r^2-2Rr\cos\theta'}}\,d\theta'}_{}\right)\hat{y}$$

~~taking~~ → Integrating $u = \cos\theta'$

and taking $\omega\times r = -(\omega r\sin\psi)\hat{y}$

$$\boxed{A = \begin{cases} \dfrac{\mu_0 R\sigma}{3}(\omega\times r) & \text{for } r < R \\[2ex] \dfrac{\mu_0 R^4\sigma}{3r^3}(\omega\times r) & \text{for } r > R. \end{cases}}$$

→ Multipole expansion of $\bar{A}$, $(r >> r')$:



Consider current loop,

$$\bar{A}(\bar{r}) = \frac{\mu_0}{4\pi} I \oint \frac{d\bar{l}}{|\bar{r} - \bar{r}'|} \quad \text{—(2)}$$

writing the multipole terms,

Substituting (1) in (2), we get,

$$\boxed{\bar{A}(\bar{r}) = \frac{\mu_0 I}{4\pi}\left(\frac{1}{r}\right)\sum_{l=0}^{\infty}\oint\left(\frac{r'}{r}\right)^l P_l(\cos\theta')\,dl'}$$

$$\bar{A}(\bar{r}) = \frac{\mu_0 I}{4\pi r}\left[\oint P_0(\cos\theta')\,dl' + \oint\left(\frac{r'}{r}\right)P_1(\cos\theta')\,dl' + \oint\left(\frac{r'}{r}\right)^2 P_2(\cos\theta')\,dl' + \ldots\right]$$

showing monopole term $= 0$,

we see that, from approximation,

$$\frac{1}{|\bar{r} - \bar{r}'|} = \frac{1}{r_>}\sum_{l=0}^{\infty}\left(\frac{r_<}{r_>}\right)^l P_l(\cos\theta) \quad \text{—(1)}$$

then,

outside: $r_> = r$ and $r_< = r'$

inside: $r_> = r'$ and $r_< = r$

dipole moment

the dipole term : $$\boxed{\bar{A}_{dip}(\bar{r}) = \frac{\mu_0}{4\pi} \frac{\bar{m} \times \hat{r}}{r^2}}$$

$$= \frac{\mu_0}{4\pi} \int \frac{\bar{M}(r') \times \hat{\imath}}{\imath^2} \, dv'$$, here $\bar{M}$ is

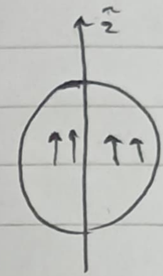
$$\bar{M} = \frac{\text{dipole moment}}{\text{volume}}.$$

$\Longrightarrow$ Drawing parallel relations between $\bar{E}$ & $\bar{B}$,

$\bar{J}_{mag}(\bar{r}) = \bar{\nabla} \times \bar{M}$ and $\bar{K}_{mag}(\bar{r}) = \bar{M} \times \hat{n}$

→ Example:

Consider a polarized sphere with polarization, $\bar{M} = M_0 \hat{z}$, find $\bar{J}$ & $\bar{K}$



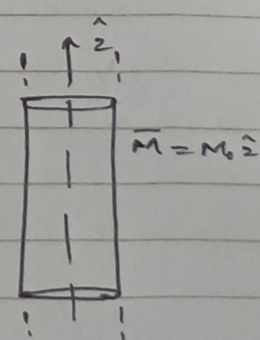
$$\bar{J} = \bar{\nabla} \times \bar{M} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & M_0 \end{vmatrix} = 0.$$

$$\bar{K} = \bar{M} \times \hat{r} = M_0 \hat{z} \times \hat{r} = M_0 \sin\theta \, \hat{\phi}$$

What would be the $\bar{B}$ for a spinning sphere?

→ Example:

Given infinitely long cylinder, Calculate the field & potential.



$$\bar{J} = \bar{\nabla} \times \bar{M} =$$

$$\bar{K} = \bar{M} \times \hat{r} = M_0 \hat{z} \times \hat{r} = \boxed{M_0 \hat{\phi}}$$

→ for $\bar{M} = M_0 \hat{\phi}$

$$\bar{J} = \bar{\nabla} \times \bar{M} =$$

$$\bar{K} = \bar{M} \times \hat{r} = M_0 \hat{\phi} \times \hat{r} =$$

→ Example:

Consider, $\bar{M} = kr^2 \hat{\phi}$, find the field & the potential.

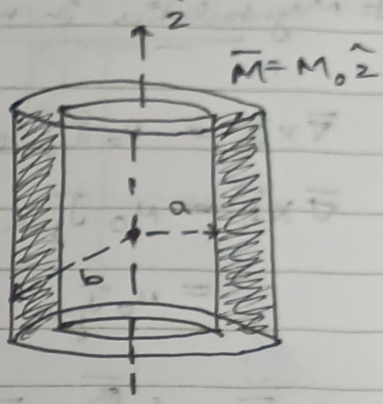
⟹ Example:

Given $\bar{M} = kr\hat{\phi}$ — ① and $\bar{M} = \frac{k}{r}\hat{\phi}$ — ②

find field & potential for both.

⟹ Example:

Consider two cylinders, find the field inside, outside and inbetween. and total enclosed current.



⟹

10/2/06

Magnetostatics in matter

$$\bar{\nabla} \times \bar{B} = \mu_0 \vec{J}_{total}$$

$$\bar{\nabla} \times \bar{B} = \mu_0 J_{free} + \mu_0 \bar{J}_{mag}$$

$$= \mu_0 \bar{J}_{free} + \mu_0 (\bar{\nabla} \times \bar{M})$$

$$\Rightarrow \bar{\nabla} \times \left(\frac{\bar{B}}{\mu_0} - \bar{M}\right) = J_{free}$$

$$\Rightarrow \boxed{\bar{\nabla} \times \bar{H} = \bar{J}_{free}}$$

where $\bar{H} = \dfrac{\bar{B}}{\mu_0} - \bar{M}$

Analogy between magnetostatics & electrostatics:

$$\frac{\bar{B}}{\mu_0} = \bar{H} + \bar{M} \qquad \text{Magnetostatics}$$

$$\Updownarrow \qquad \Updownarrow \qquad \Updownarrow \qquad \Updownarrow$$

$$\bar{D} = \varepsilon_0 \bar{E} + \bar{P} \qquad \text{Electrostatics.}$$

Time Varying Fields

$$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \qquad \text{[Faraday's Law]}$$

$$\oint_{\Gamma} \bar{E} \cdot d\bar{l} = -\frac{\partial}{\partial t} \int_S \bar{B} \cdot d\bar{s}$$

$$\bar{\nabla} \times \bar{E} = -\frac{\partial}{\partial t} (\bar{\nabla} \times \bar{A})$$

[since time variation is independent of space variation]

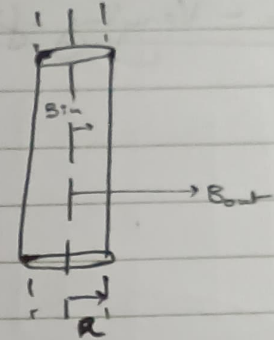
$$\boxed{\bar{E}_{induced} = -\frac{\partial \bar{A}}{\partial t}}$$

⇒ example:

Consider Hallow cylinder with $\bar{K}(t) = k_0 \sin\omega t \hat{z}$

thus $\bar{I} = I_0 \sin\omega t \hat{z}$

What is B_{in} & B_{out}?



→ ~~$\nabla \times \bar{B} =$~~ Compare with staticcase,

$\bar{K} = k_0 \hat{z}$.

$$\left.\begin{aligned} \Rightarrow B_{out} &= \mu_0 k_0 \sin\omega t \hat{\phi} \\ B_{in} &= 0. \end{aligned}\right\} \leftarrow \text{time dependent}$$

for staticcase,

$$\left.\begin{aligned} B_{out} &= \mu_0 k_0 \hat{\phi} \\ B_{in} &= 0. \end{aligned}\right\} \leftarrow \text{time independent.}$$

⇒ for staticcase, from $\bar{B} = \bar{\nabla} \times \bar{A}$

$A_{in} =$

$A_{out} =$

⇒ Example:

Consider solenoid with $\bar{K}(t) = k_0 \sin\omega t \hat{\phi}$. What is B_{in} & B_{out}

$\bar{K}(t) = k_0 \sin\omega t \hat{\phi}$, then,

$$\left.\begin{aligned} B_{in} &= \mu_0 k_0 \sin\omega t \hat{z} \\ B_{out} &= 0. \end{aligned}\right\} \text{time dependent}$$

Over one time period T, B_{in} is zero, ~~What will~~ How will it change for $\bar{A}$ & $\bar{E}$?

~~Mon~~ 18/02 (PHY 552)

Date:

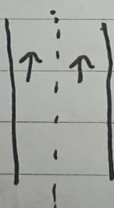
Quasi-static fields (time-varying fields)

$$\nabla^2 \bar{A} = \mu \sigma_c \frac{\partial \bar{A}}{\partial t}$$

$$\dim (t / L^2) = \dim (\mu \sigma_c)$$

characteristic length scale : $L \approx \dfrac{1}{\sqrt{\mu \sigma_c \omega}}$ → skin dept

eg: current flow in a wire



I

$$I = I_0 e^{-i\omega t} \hat{z}$$

$$\bar{E} = E_0(r) e^{-i\omega t} \hat{z}$$

$$\cancel{e^{-i\omega t}} \frac{\partial^2 E_0(r)}{\partial r^2} = \cancel{\mu \sigma_c \omega^2 e^{-i\omega t} E_0(r)}$$

$$-i\omega \cancel{\mu \sigma_c e^{-i\omega t} E_0(r)}$$

Date:

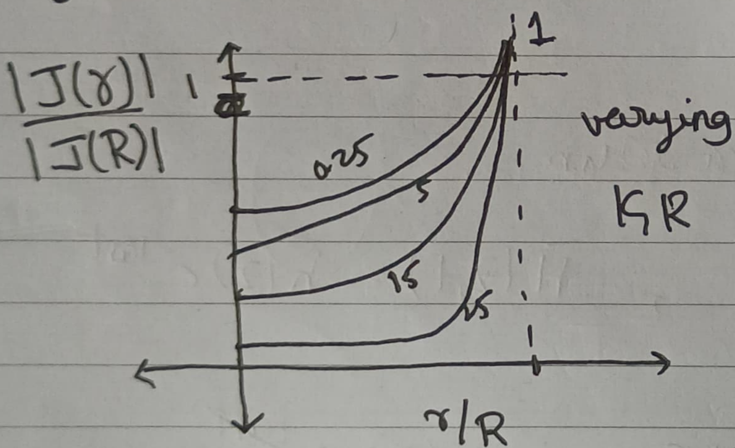
$$\Rightarrow \frac{\partial^2 E_0(r)}{\partial r^2} + \frac{1}{r}\frac{\partial E_0(r)}{\partial r} = -\mu_0 \sigma_c i \omega E_0(r)$$

$$\Rightarrow \frac{\partial^2 E_0(r)}{\partial r^2} + \frac{1}{r}\frac{\partial E_0(r)}{\partial r} = -K^2 E_0 \quad , \quad K^2 = \sqrt{\mu_0 \sigma_c \omega i}$$

sol^n are bessel functions

$$\Rightarrow K = (i+1)\sqrt{\frac{\mu \omega \sigma_c}{2}}$$

$$E_0(r) = A J^{\text{bessel}}(Kr)$$



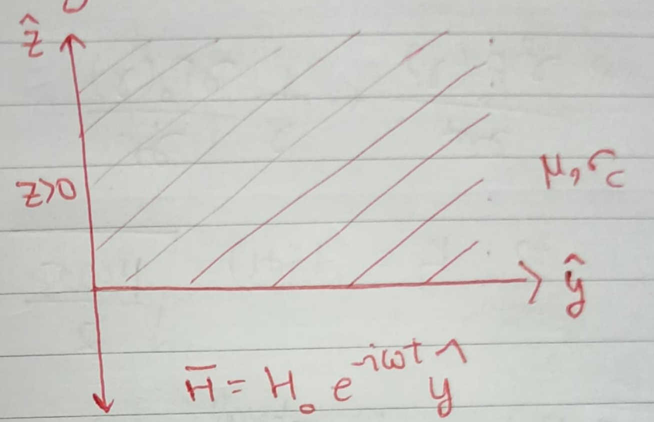
$R_{AC} > R_{DC}$

$\delta(\omega)$	ω (Hz)
10 nm	10^{15}
1 μm	10^{12}
0.1 mm	10^{6}
1 cm	10

(Cu)

PHY552 (19/02)

Magnetic field inside a conducting slab



$z>0$:

$$\bar{H}(z,t) = h(z) e^{-i\omega t}$$

magnetic diffusion eq^n:

$$\nabla^2 \bar{H} = \mu \sigma_c \frac{\partial \bar{H}}{\partial t}$$

problem is symmetric in cartesian

$$\Rightarrow \frac{\partial^2 \bar{H}}{\partial z^2} = \mu \sigma_c \frac{\partial \bar{H}}{\partial t}, \quad \bar{H}(z,t) = h(z) e^{-i\omega t}$$

$$\Rightarrow \frac{\partial^2 h(z)}{\partial z^2} \cancel{e^{-i\omega t}} = \mu \sigma_c \, (-i\omega) h(z) \cancel{e^{-i\omega t}}$$

$$\Rightarrow \frac{\partial^2 h(z)}{\partial z^2} = \underbrace{-i\omega \mu \sigma_c}_{k^2} h(z), \quad K = \frac{(1+i)}{\sqrt{2}} \sqrt{\mu \omega \sigma_c}$$

$$\Rightarrow K = (1+i) / \delta(\omega)$$

$$\Rightarrow h(z) = h_0 \exp(-iKz) + \cancel{h_0 \exp(}$$

$$\delta(\omega) = \sqrt{\frac{\mu \omega \sigma_c}{2}}$$

general sol^n:

$$\boxed{h(z) = h_1 \exp(iKz) + h_2 \exp(-iKz)}$$

Date:

~~For $z>0$: $H(z,t) = \{h_0 \exp^{-i\omega t}\{e^{-ikz} + e^{ikz}\}$~~

~~$H(z,t) = \exp^{-i\omega t}$~~

$$H(z,t) = e^{-i\omega t}\left[h_1 e^{ikz} + h_2 e^{-ikz}\right]$$

$$k = \frac{(1+i)}{\delta(\omega)}, \quad \delta(\omega) = \sqrt{\frac{\sigma_c \mu \omega}{2}}$$

$$\Rightarrow H(z,t) = e^{-i\omega t} e^{-z/\delta}\left[h_1 e^{z/\delta} e^{iz(1+i)/\delta} + h_2 e^{z/\delta} e^{-iz(1+i)/\delta}\right]$$

$$= e^{-i\omega t} e^{-z/\delta}\left[h_1 e^{iz/\delta} + h_2 \underbrace{e^{2z/\delta}}_{} e^{-iz/\delta}\right]$$

can not exist as for $z \to \infty$, it explodes

$$\Rightarrow H(z,t) = e^{-i\omega t} e^{-z/\delta} h_0 e^{iz/\delta}$$

$$\Rightarrow \boxed{H(z,t) = h_0 e^{-i(\omega t - z/\delta)} e^{-z/\delta}}$$

$$\text{Re}\{H(z,t)\} = h_0 e^{-z/\delta} \cos(\omega t - z/\delta)$$

$$\text{Im}\{H(z,t)\} = h_0 e^{-z/\delta} \sin(\omega t - z/\delta)$$

f(Hz)	$\delta(\omega)$ (μm)
1GHz	2.6
100 MHz	82
1 MHz	82

$\delta \propto \sqrt{\omega}$

$\rightarrow \text{Re}\{H\} \propto e^{-1/\sqrt{\omega}}$

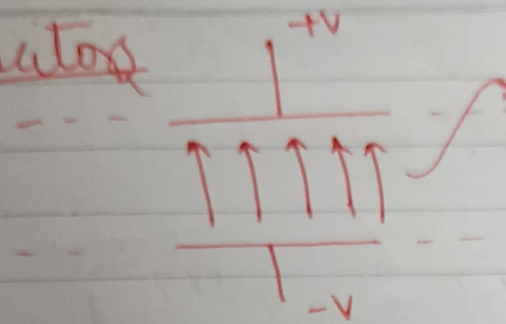
no decay

$\omega \gg 1$: $\text{Re}\{H\} \rightarrow h_0 \cos$

$\omega \to 0$: $\text{Re}\{H\} \to 0$

Date:

Capacitor



uniform field inside the capacitor

$$\bar{E}^{(0)} = E_0 \cos\omega t\,\hat{z}$$

$$\bar{\nabla}\times\bar{B} = \mu_0\varepsilon_0 \underbrace{\frac{\partial E}{\partial t}}_{-E_0\,\omega\sin\omega t\,\hat{z}}$$

$$\Rightarrow \bar{\nabla}\times\bar{B} = -\mu_0\varepsilon_0 E\omega\sin\omega t\,\hat{z}$$

$$\Rightarrow \bar{B}^{(1)}(r;t) = -\mu_0\varepsilon_0\,\omega\,E_0\sin\omega t\left(\frac{r}{2}\right)\hat{\phi}$$

$$\bar{E}^{(1)} = \frac{-\omega^2 r^2}{4c^2}E_0\cos\omega t$$

$$\bar{\nabla}\times\bar{E}^{(1)} = -\frac{\partial B^{(1)}}{\partial t} = -\mu_0\varepsilon_0\omega^2 E_0\cos\omega t\left(\frac{r}{2}\right)\hat{\phi}$$

$\hat{r}$ $\hat{z}$ $\hat{\phi}$

$E_r\hat{r}$ $E_z\hat{z}$ $\hat{\phi}E_\phi$

$$E = E_0\cos\omega t\left(1-\frac{\omega^2 r^2}{4c^2}+\frac{\omega^4 r^4}{16c^4}-\cdots\right)$$

PHY552 (20 Jan)

Fields in disc Capacitor

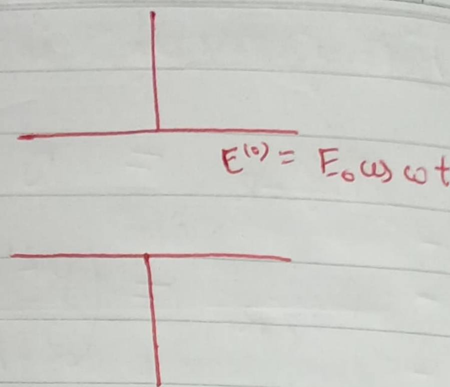
$\bar{E}^{(0)} = E_0 \cos\omega t\ \hat{z}$

$\bar{B}^{(1)} = \dfrac{-\omega r}{2c^2} E_0 \sin\omega t\ \hat{\phi}$

$\bar{E}^{(1)} = \dfrac{-\omega^2 r^2}{4c^2} E_0 \cos\omega t\ \hat{z}$

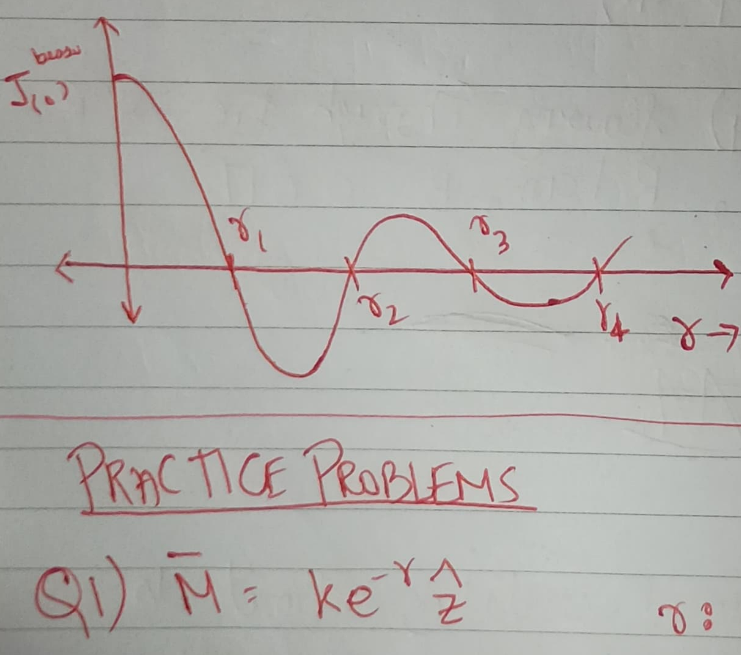
$\bar{B}^{(2)} = \dfrac{+\omega^3 r^3}{16c^4} E_0 \sin\omega t\ \hat{\phi}$

$\vdots$

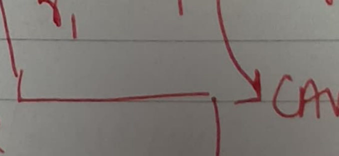


$\bar{E} = E_0 \cos\omega t \left(1 - \dfrac{\omega^2 r^2}{4c^2} + \cdots \right)$

$= E_0 \cos\omega t \ [J^{\text{bessel}}(\omega r)]$



Cavity → can sustain a given mode of oscillation



PRACTICE PROBLEMS

Q1) $\bar{M} = k e^{-r}\hat{z}$ $\quad r: 0 \to R$

Find magnetic field inside cylinder (infinitely long) ($\bar{B}_{r<R}$)

(a) Find current density

(b) from J find B

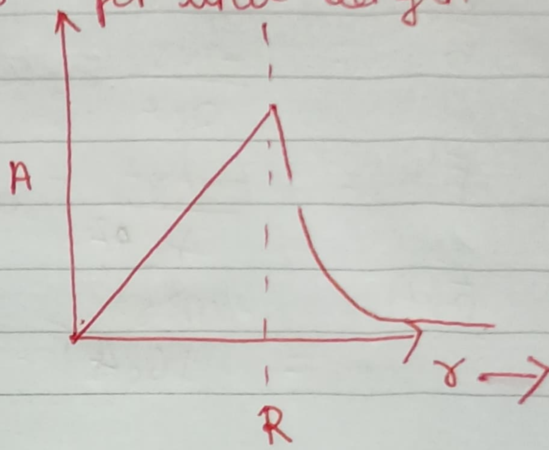
(c) from J, H B

Ans- $\mu_0 k(1 - e^{-r})\hat{z}$

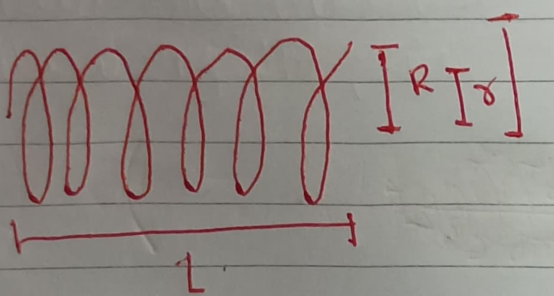
Q2) Long Solenoid $(R << L)$, n turns per unit length

$$A_{r<R} = r$$

$$A_{r>R} = \left(\frac{R^2}{2r}\right)$$



behaviour of solenoid:



③ Circular loop of radius 'R' placed $\perp^r$ in a magnetic field, $\vec{B} = B_0 \exp(-t/t)$

Find the total ~~&~~ charges (q) flowing through the loop ~~loop~~ (resistance = $2\,\Omega$, $R = 1$ cm, $B_0 = 0.017$, $t_0 = 1$ sec) $t = 2$ sec.

~~Ans~~ :

④ Solid cylinder of radius 'R' rotating about its axis (ω). charge Q distributed uniformly over its volume. Find the ~~magnet~~ magnetic ~~mom~~ ~~magnetic~~ ~~moment~~ moment.

$$m = QR^2\omega K, \quad K =$$

Date:

For a long solenoid:

(5) $\bar{A} = \frac{\mu_0 n I}{2} r \hat{\phi}$ $\cancel{\bar{A}_1 = \frac{\mu_0 n I}{2} r (\alpha \cos \beta \hat{\phi} + \beta \sin \phi)}$

$\bar{A}_1 = \frac{\mu_0 n I}{2} r \left(\alpha \cos \beta \hat{\phi} + \beta \sin \phi \, \hat{r} \right)$

$B_A = B_{A+A_1}$ Find relation b/w α and β

$\nabla \times \bar{A} = \nabla \times (\bar{A} + \bar{A}_1)$

(6) $\bar{A} = B_0 x \hat{y}$

$\bar{A} = -B_0 y \hat{x}$

$\bar{A} = \frac{B_0}{2} (x \hat{x} + y \hat{y})$

$\bar{A} = \frac{B_0 y}{2} (-\hat{x} - \hat{y})$

Out of these four, which one does not create a uniform $B\hat{z}$

(7) $\bar{A} = (y^2 \hat{x} + x^2 \hat{y})$

Find $\bar{J}$