PSO201/PHY204: Assignment 3
Jan-Apr 2025

1. A particle of mass m moving in one dimension is confined to the region $0 < x < L$ by an infinite square well potential. In addition, the particle experiences a delta function potential of strength V_0 located at the center of the well.
   a) Find a transcendental equation for the energy eigenvalues E in terms of the mass m, the potential strength V_0, and the size L of the system.
   b) Sketch the ground state wave function and state whether the energy increases or decreases compared to when $V_0 = 0$.
   c) If it was originally E_0, what is the energy eigenvalue when V_0 is infinite?

2. Consider the scattering of particles of mass m moving from left to right off a step of height V_0, starting at $x = 0$ and extending along $x > 0$. Compute the transmission probability T and reflection probability R for both $E > V_0$ and $E < V_0$. Sketch $T(E)$.

3. Consider the scattering of particles of mass m moving from right to left off a step of height V_0, starting at $x = 0$ and extending along $x > 0$. What will happen when the particles move downhill from the right to left with energy $E > V_0$? Find the reflection and transmission coefficients.

4. Consider a finite potential square well of width $2a$ and depth $-V_0$. Find the transmission probability for a particle of energy $E > 0$, incident on the well. Write the expression in terms of dimensionless parameters. Find out the energies of the particle for which there will be perfect transmission. Plot the transmission probability as a function of the dimensionless parameter $\mathcal{E} = \frac{E}{V_0}$.

5. Consider a potential barrier of width d and height V_0. Find the transmission probability for a particle of energy $E < V_0$, incident on the barrier. Find the transmission probability when $E > V_0$.

6. The transmission probability T for a particle of mass m incident on a barrier of width d and height V_0. For $E < V_0$, T is given by
   $T = \frac{1}{1+\frac{{V_0}^2}{4E(V_0-E)}\sin^2 \kappa d}$, where $\kappa^2 = \frac{2m(Vo-E)}{\hbar^2}$,
   For $E > V_0$ it is given by,

   $T = \frac{1}{1+\frac{{V_0}^2}{4E(V_0-E)}\sin^2 kd}$, where $k^2 = \frac{2m(Vo-E)}{\hbar^2}$.
   The dimensionless parameter $z_0 = \frac{2mV_0 d^2}{\hbar^2}$ measures the strength of the barrier.
   a) Sketch T as a function of the dimensionless parameter $\mathcal{E} = \frac{E}{V_0}$ for a very wide barrier ($z_0 = 20$), a medium barrier ($z_0 = 2$), and a thin barrier ($z_0 = 0.2$).
   b) What is the limit of the transmission probability as the energy approaches the barrier height ($E \to V_0$) in each case?

7. The transmission probability for a particle of mass m incident on a potential well of depth $-V_0$ is given by,

$$T = \frac{1}{1 + \frac{V_0{}^2}{4E(E+V_0)}\sin^2 kd}$$

where $k^2 = \frac{2m(Vo+E)}{\hbar^2}$.

a) Show that you can derive this result from that of the barrier of height $+V_0$ by taking V_0 negative and keeping track of resulting signs and factors of i. Why does this work?

b) Sketch T as a function of $\mathcal{E} = E/V_0$ for a very wide well ($z_0 = 20$), a medium well ($z_0 = 2$), and a thin well ($z_0 = 0.2$).

8. Consider the scattering of particles of mass m off an attractive delta function potential of strength V_0 located at the origin,

   $V(x) = -V_0\,\delta(x)$, with $V_0 > 0$.

   Suppose a beam of particles is incident from the left with typical energy E. Calculate the fraction of particles in the incident beam that are reflected by this potential (i.e. find the reflection coefficient R). Write your answer in terms of E, m, V_0 and any fundamental constants needed. Check if your result is reasonable in the limits $V_0 \to 0$ for constant E, and $E \to 0$ for constant V_0.

9. Consider a particle beam approximated by a plane wave directed along the x-axis from the left and incident upon a potential $V(x) = V_0\delta(x),\ V_0 > 0$
   a) Give the form of the wave function for x < 0.
   b) Give the form of the wave function for x > 0.
   c) Give the conditions on the wave function at the boundary between the regions.
   d) Calculate the probability of transmission.

10. Consider a one-dimensional square-well potential (see Fig. 1.24):

$$V(x) = \infty, \quad x < 0$$
$$V(x) = -V_0, \quad 0 < x < a$$
$$V(x) = 0, \quad x > a$$

   Suppose a particle with energy $E > 0$ is incident upon this potential. Find the phase relation between the incident and the outgoing wave.