UNIT-III 22/10/24

* Test of Hypothesis for large Samples *

Law of Samples:-

i) Large sample:-
If the size of the sample $n \geq 30$ then the sample is called large sample.

ii) small sample:-
If the size of the sample $n \leq 30$ then the sample is called small sample

Hypothesis:-

The statement about the population parameter is called a Hypothesis

Test of Hypothesis:-

The procedure which involves, to decide on the basis of sample result whether the hypothesis is true or not (accept or reject) is called test of hypothesis.

Ex: The average height of a solider is 167 cm.

There are 2 types of hypothesis:-

i) Null-hypothesis
ii) Alternative-hypothesis

1) NULL Hypothesis:-

The hypothesis which is to be actually tested for acceptance or rejection. is called NULL hypothesis

. It is denoted by (H_0)

2) Alternative Hypothesis:-

An hypothesis which is contridiction to NULL-hypothesis is called Alternative Hypothesis.

It is Denoted by (H_1)

Ex: The average weight of a student in a class is 50

H_0 $\mu = 50$

H_1 $\mu \neq 50$ (Two tailed test) |zcal| >

H_1 : $\mu > 50$ (Right " ")

H_1 : $\mu < 50$ (left " ")

Note :— The normal is used for to test large sample

Classification of Errors :—

There are 2 types of Errors.

1) Type-I Error :—

Reject H_0 when it is true

The probability of Type-I error = α

2) Type-II error :— Accept H_0 when it is false.

The probability of Type-II error = β

Level of significance :— [LOS]

The probability of comiting type-I Error is called level of significance.

procedure :— of testing of hypothesis :—

1. Setup the null hypothesis (H_0)
2. Setup the alternative hypothesis (H_1)
3. Choose the LOS (α)
4. Calculate $\boxed{z = \frac{t - E(t)}{SE(t)}}$
5. Conclusion. Now we compare calculated value of z with table value z_α.

→ That is if $|z| < z_\alpha$ then accept H_0

$|z| > z_\alpha$ " reject H_0

we have the following 4 important tests:-

1) Test of Hypothesis for single mean.
2) Test of " " difference of means.
3) " " " " single proportion.
4) " " " for difference of proportion.

1) Test of Hypothesis for single mean:-

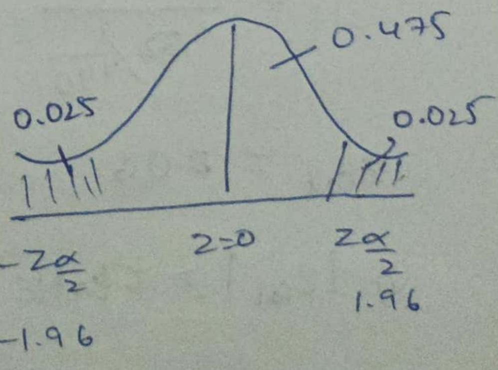
suppose we want to test whether the given sample as been taken from population with mean (μ) and variance σ^2 and sample mean $\bar{x}$ then the test statistic is $\boxed{Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}}$

Problems:-

① A sample of 400 items whose SD $\sigma = 10$ taken from the population the mean of sample is 40. Test whether the sample has been taken from the population with mean 38. at $\alpha = 0.05$ also calculate the confidence interval at 95% level.

Sol:- given $n = 400$, $\sigma = 10$, $\mu = 38$, $\alpha = 0.05$
$\bar{x} = 40$

1) H_0 $\mu = 38$
2) H_1 $\mu \neq 38$ (two tailed test)



3) LOS $\alpha = 0.05 \Rightarrow \frac{\alpha}{2} = 0.025$

$P(0 \le Z \le Z_{\alpha/2}) = 0.475$

$Z_{\frac{\alpha}{2}} = 1.96$

4) $Z_{cal} = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$= \dfrac{40 - 38}{10/\sqrt{400}} = 4$

$\therefore |Z_{cal}| = |4| > Z_{\alpha/2}$

$\therefore$ Ho is rejected.

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② The oceanographer wants to check whether the depth of the ocean is in a certain region is 57.4 fathons. As had been recorded previously. what can conclude that at the 0.05 level of significance if the reading taken at 40 random locations in the given region. A mean of 59.1 fathons with S.D of 5.2 fathons.

Sol:- $\mu = 57.4$ fathons $n = 40$

sample mean $\bar{x} = 59.1$ $S.D = 5.2$

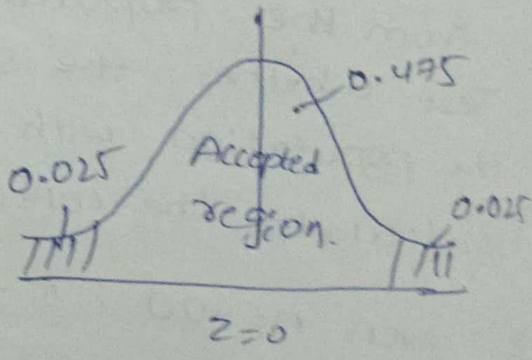
(i) Null hypothesis (H_0) $\mu = 57.4$

(ii) Alternative " (H_1) $\mu \neq 57.4$ (two tailed test)

(iii) LOS $\alpha = 0.05 \Rightarrow \frac{\alpha}{2} = 0.025$

we have $P(0 \leq z \leq z_{\alpha/2}) = 0.475$

$z_{\frac{\alpha}{2}} = 1.96$



(iv) $z_{cal} = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

$= \dfrac{59.1 - 57.4}{5.2 / \sqrt{40}}$

$z_{cal} = 2.06$

v) $|z_{cal}| > z$ table

$\therefore$ H_0 is rejected

③ An ambulance service claims that it takes avg less than 10 min to reach its destination and emergency calls. A sample of 36 calls has mean 11 min and variance 16 min, test the claim at $\alpha = 0.05$

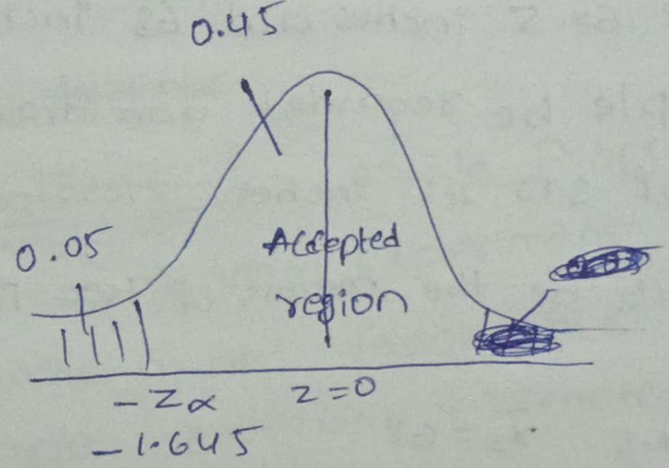
Sol:- $\mu = 10$, $n = 36$

$\bar{x} = 11$ $\sigma^2 = 16$

(i) Null hypothesis (H_0) $\mu = 10$

(ii) Alternative " (H_1) $\mu < 10$ (Left tailed left)

(iii) LOS $\alpha = 0.05$



we have $P(0 \le z \le z_\alpha) = 0.45$

$Z_{table} = Z_\alpha = 1.645$

(iv) $Z_{cal} = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$Z_{cal} = \dfrac{11-10}{4/\sqrt{36}}$

$Z_{cal} = 1.5$

$|Z_{cal}| < Z_{table}$

∴ Accepted.

Test of difference :-

Let $\bar{x}_1$ be the mean of sample size n_1 from a population with mean (μ_1) and variance σ_1^2 and $\bar{x}_2$ with mean of Sample size n_2 from a population with mean (μ_2) and variance (σ_2^2)

Then to test whether tere is any significance difference b/w $\bar{x}_1$ and $\bar{x}_2$ we use the test

test Static $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

Q) The mean of two large samples of size 1000 and 2000 members are 67.5 inches and 68 inches repectively. Can the sample be regarded are drawn from same population of S.D 2.5 inches.

Sol:- Let μ_1, μ_2 be the means of two populations

given.

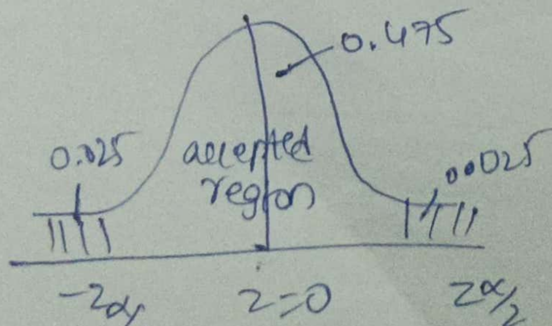
$\bar{x}_1 = 67.5 \quad \bar{x}_2 = 68$

$n_1 = 1000 \quad n_2 = 2000 \quad \sigma = 2.5 \quad \sigma_1^2 = \sigma_2^2$

(i) Null hypothesis (H_0) $\mu_1 = \mu_2$

(ii) Alternative " (H_1) $\mu_1 \neq \mu_2$ (two failed)

(iii) L.O.S $\alpha = 0.05$ $\alpha/2 = 0.025$



$\Rightarrow z_{\frac{\alpha}{2}} = 1.96$

Given $\sigma_1^2 = \sigma_2^2 = \sigma^2$

(iv) $$Z_{cal} = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{67.5 - 68}{2.5\sqrt{\frac{1}{1000} + \frac{1}{2000}}}$$

$$= \frac{-0.5}{2.5\sqrt{\frac{3}{2000}}}$$

$$Z_{cal} = -5.1639$$

(v) $|Z_{cal}| = 5.1639$ $Z_{tab} = 1.96$

$|Z_{cal}| > Z_{tab}$

$\therefore$ H_0 is rejected.

→ Test of Hypothesis for single Proportion

Proportion: In a random experiment the total no. of events are n and favourable events are x then the sample proportion is denoted by p and it is defined as $p = \frac{x}{n}$

Ex: In a manufactoring company out of 200 goods 18 were defective then the sample proportion for defective $= \frac{18}{200}$

Note:—

If the population proportion P which is unknown generally we take $P = 0.5$

$P + Q = 1$

Test statistic for single proportion

Suppose sample of size n is taken from population proportion P with sample proportion p then the test statistic for sample proportion is

$$Z = \frac{p-P}{\sqrt{\frac{PQ}{n}}}$$ } standard error.

Test of hypothesis for difference b/w two portions

Let P_1, P_2 be the sample proportions taken from two populations having the proportions P_1, P_2 of two large sample sizes n_1, n_2 then to test the significant difference b/w the sample proportions P_1, P_2 is

$$Z = \frac{P_1 - P_2}{\sqrt{\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}}}$$

$Q_1 = 1-P_1$

$Q_2 = 1-P_2$

Note:

When the population proportions P_1 & P_2 are not given but sample proportions p_1 & p_2 are given then

$$Z = \frac{p_1 - p_2}{\sqrt{\frac{p_1q_1}{n_1} + \frac{p_2q_2}{n_2}}}$$

2) we can convert two sample proportions p_1 & p_2 into single proportion P by

$$\boxed{P = \frac{n_1p_1 + n_2p_2}{n_1 + n_2}}$$

Then test statistic

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

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Q) A manufacture claimed that atleast 95% of equipment which is supplied to a factory confirmed to specifications. An examination of sample of 200 pieces of equipment out of that 18 were faulty. Test his claim at 5% LOS.

Sol: $n = 200$

no. of pieces conforming to specification $200 - 18 = 182$

$\therefore p = \frac{182}{200}$

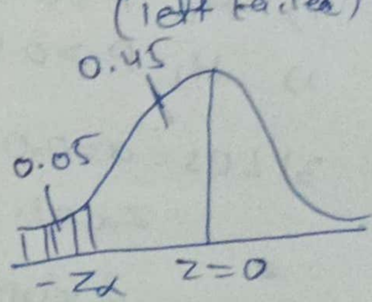
$\boxed{p = 0.91}$

given $P = 95\% = 0.95$

$Q = 1 - P = 0.05$

1) NULL Hypothesis H_0 $P = 0.95$

2) Alternate " H_1 $P < 0.95$



3) LOS $\alpha = 5\% = 0.05$

$P(-Z_\alpha \leq Z \leq 0) = 0.45$

$\boxed{Z_\alpha = 1.645}$

4) we have $Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.91 - 0.95}{\sqrt{\frac{0.95 \times 0.05}{200}}}$

$Z = -2.595$

$\therefore |Z| = 2.595$

5) $|Z| > Z_\alpha$

$\therefore$ H_0 is rejected.

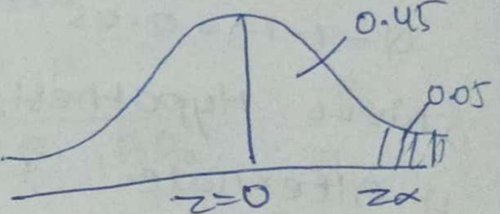
~~Find a) $P(t<2.365)$ when $\gamma=7$~~
b) $P(-1.356<t<3.179)$ when $\gamma=12$
b) $P(-1.356<t<3.179)$ when $\gamma=17$
c) $P(t>-2.567)$ when $\gamma=17$

2) In a city 325 men out of 600 men are found to be smokers. Does this information support the conclusion that the majority of men in the city are smokers.

$n=600$

sample proportion for smokers $p=\frac{325}{600}=0.5417$

1) $H_0 \; P=0.5$
2) $H_1 \; P>0.5$ (Right tailed test)



3) LOS $\alpha=0.05$
$Z\alpha=1.645$

4) $$z=\frac{0.5417-0.5}{\sqrt{\frac{0.5\times0.5}{600}}}$$

$z=2.085$

5) $|z|>Z\alpha$

$\therefore H_0$ is rejected

3) Random sample of 400 men and 600 women are asked whether they would like to have a flyover near their residency 200 men & 325 women were in favour of this proposal. Test the hypothesis that proportions of men & women in favour of the proposal are same at 5% LOS.

$n_1 = 400 \quad n_2 = 600$

proportion of men $P_1 = \frac{200}{400} = 0.5$

" of women $P_2 = \frac{325}{600} = 0.541$

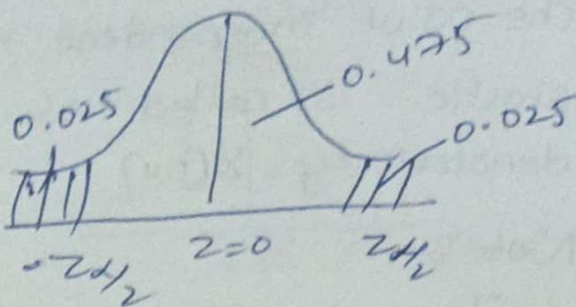
1) $H_0 \quad P_1 = P_2$

2) $H_1 \quad P_1 \neq P_2$ (two tailed test)

3) $LOS = \alpha = 0.05$

$\alpha/2 = 0.025$

$z_{\alpha/2} = 1.96$



4) $$z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$P = \frac{n_1P_1 + n_2P_2}{n_1+n_2} = \frac{400\times0.5 + 600\times0.541}{400+600}$$

$P = 0.525 \quad q = 1-P = 0.475$

$$z = \frac{0.5-0.541}{\sqrt{0.525\times0.475\left(\frac{1}{400}+\frac{1}{600}\right)}} = -1.27$$

5) $|z| = 1.27 \quad z_{\alpha/2} = 1.96$

$\therefore |z| < z_{\alpha/2}$

$\therefore H_0$ is accepted