UNIT - I Assignment

Expected value and Variance

1) Expected value and Variance

x :	-10	-20	30	75	80
$p(x)$:	$1/5$	$3/10$	$1/2$	$1/10$	$1/20$

Sol:- Expected value : $E(x) = \sum x\, p(x)$

$$\Rightarrow (-10)\times\left(\frac{1}{5}\right) + (-20)\left(\frac{3}{20}\right) + 30\left(\frac{1}{2}\right) + 75\left(\frac{1}{10}\right) + 80\left(\frac{1}{20}\right)$$

$$\Rightarrow -2-3+15+7.5+4$$

$$\Rightarrow 14+7.5$$

$$\Rightarrow \underline{21.50}$$

Variance : $Var(x) = E(x^2) - [E(x)]^2$

$$E(x^2) = \sum x^2 p(x)$$

$$E(x^2) = (-10)^2\left(\frac{1}{5}\right) + (-20)^2\left(\frac{3}{20}\right) + (30)^2\left(\frac{1}{2}\right) + (75)^2\left(\frac{1}{10}\right) + (80)^2\left(\frac{1}{20}\right)$$

$$\Rightarrow 100\left(\frac{1}{5}\right) + 400\left(\frac{3}{20}\right) + 900\left(\frac{1}{2}\right) + 5625\left(\frac{1}{10}\right) + 6400\left(\frac{1}{20}\right)$$

$$\Rightarrow 20+60+450+562.5+320$$

$$\Rightarrow 1412.5$$

$$Var(x) = E(x^2) - [E(x)]^2$$

$$= 1412.5 - (21.5)^2$$

$$= 1412.5 - 462.25$$

$$= \underline{950.25}$$

2) A random variable X has the following probability function

(i) Find the value of k

(ii) Mean (iii) Variance

X	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Sol: (i) $\Sigma P(x) = 1$

$$k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$k(1+2+3+2+1) + (1+2+7)k^2 = 1$$

$$9k + 10k^2 = 1$$

$$10k^2 + 9k - 1 = 0$$

$$k = \frac{-9 \pm \sqrt{9^2 + 40}}{20}$$

$$k = \frac{-9 \pm 11}{20}$$

$$\boxed{k = 0.1, -1}$$

(or)

$$\boxed{k = 0.1}$$

(ii) Mean of x

$$E(x) = \Sigma x\, p(x)$$

$$= \cancel{0(k) + 1(2k) + 2(2k) + 3(3k) + 4}$$

$$\Rightarrow 1(k) + 2(2k) + 3(2k) + 4(3k) + 5(k^2) + 6(2k^2) + 7(7k^2 + k)$$

$$\Rightarrow k(1+4+6+12+7) + k^2(5+12+49)$$

$$\Rightarrow 30k + k^2(66)$$

$$\Rightarrow 30(0.1) + (0.1)^2(66)$$

$$\Rightarrow 3 + 0.66$$

$$\Rightarrow \underline{\underline{3.66}}$$

iii) Find the variance of x:

$Var(x) = E(x^2) - [E(x)]^2$

$E(x^2) = \sum x^2 p(x=x)$

$\Rightarrow 1^2(0.1) + 2^2 \cdot 2(0.1) + 3^2 \cdot 2(0.1) + 4^2 \cdot 3(0.1) + 5^2 \cdot (0.1)^2 + 6^2 \cdot 2(0.1)^2 + 7^2 (7(0.1)^2 + (0.1))$

$\Rightarrow \cancel{0.1 + 8.0 +}$

$\Rightarrow 0.1 + 0.8 + 1.8 + 4.8 + 0.25 + 0.72 + 8.33$

$\Rightarrow 16.8$

$Var(x) = E(x^2) - [E(x)]^2$

$= 16.8 - (3.66)^2$

$= 16.8 - 13.39$

$= \underline{\underline{3.41}}$

3) A continuous random variable has the probability density function

$$E(x) = \begin{cases} k \cdot x e^{-\lambda x} & \text{for } x \geq 0, \lambda = 0 \\ 0 & \text{otherwise.} \end{cases}$$

Determine (i) k (ii) Mean (iii) Variance

Sol: ~~Mean~~ k, Mean $\mu = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$

We know that

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\Rightarrow \underbrace{\int_{-\infty}^{0} f(x)\, dx}_{0} + \int_{0}^{\infty} f(x)\, dx = 1$$

$$\Rightarrow \int_0^\infty k \cdot x \cdot e^{-\lambda x}\, dx = 1$$

$$\Rightarrow k \int_0^\infty x \cdot e^{-\lambda x}\, dx = 1$$

$$\Rightarrow k\left[\frac{x e^{-\lambda x}}{-\lambda} - \int_0^\infty (1)\, \frac{e^{-\lambda x}}{-\lambda}\right] = 1$$

$$\Rightarrow k\left[\frac{x e^{-\lambda x}}{-\lambda} - \left(\frac{1}{-\lambda}\right)\left[\frac{e^{-\lambda x}}{-\lambda}\right]\right]_0^\infty = 1$$

$$\Rightarrow k\left[\frac{x e^{-\lambda x}}{-\lambda} - \frac{e^{-\lambda x}}{\lambda^2}\right]_0^\infty = 1$$

$$\Rightarrow k\left[\frac{1}{\lambda^2}\right] = 1 \quad \Rightarrow \boxed{k = \lambda^2}$$

<u>Mean</u>

$$\mu = \int_0^\infty x \cdot f(x)\, dx$$

$$\Rightarrow \int_0^\infty x\, \lambda^2 x \cdot e^{-\lambda x}\, dx$$

$$\Rightarrow \lambda^2 \int_0^\infty x^2 e^{-\lambda x}\, dx$$

$$\Rightarrow \lambda^2\left[x^2\left(\frac{e^{-\lambda x}}{-\lambda}\right) - \int_0^\infty \left(2x\left(\frac{e^{-\lambda x}}{\lambda}\right) dx\right) dx\right]$$

$$\Rightarrow \lambda^2\left[x^2\left(\frac{e^{-\lambda x}}{-\lambda}\right) - 2\left[x\left(\frac{e^{-\lambda x}}{\lambda^2}\right) - \int_0^\infty \left(\frac{e^{-\lambda x}}{\lambda^2}\right) dx\right]\right]$$

$$\Rightarrow \lambda^2\left[x^2\left(\frac{e^{-\lambda x}}{-\lambda}\right) - 2\left[x\left(\frac{e^{-\lambda x}}{\lambda^2}\right) - \left(\frac{e^{-\lambda x}}{-\lambda^3}\right)\right]\right]_0^\infty$$

$$\Rightarrow \lambda^2 \left[\frac{2}{\lambda^3} \right]$$

$$\boxed{\mu = \frac{2}{\lambda}}$$

Variance

$$Var(x) = E(x^2) - [E(x)]^2$$

$$= E(x^2) - \mu^2$$

$$E(x^2) = \lambda^2 \int_0^\infty x^2 . x e^{-\lambda x} dx$$

$$= \lambda^2 \int_0^\infty x^3 . e^{-\lambda x} dx$$

$$= \lambda^2 \left(\frac{6}{\lambda^4} \right)$$

$$= \frac{6}{\lambda^2}$$

$$Var(x) = \frac{6}{\lambda^2} - \left(\frac{2}{\lambda} \right)^2$$

$$= \frac{6}{\lambda^2} - \frac{4}{\lambda^2} \Rightarrow \frac{2}{\lambda^2}$$

$$\boxed{S.D. = \frac{\sqrt{2}}{\lambda}}$$

Qs. If X is a poisson variate such that $3P(X=4) = \frac{1}{2} P(X=2) + P(X=0)$, find

(a) The mean of X (b) $P(X \leq 2)$

Sol: x is a poisson variate

If $(x=4) = \frac{1}{2} P(x=2) + P(x=0)$

$$\boxed{P(x=k) = \frac{e^{-\lambda}\lambda^k}{k!}}$$

$$3\frac{e^{-\lambda}\lambda^4}{4!} = \frac{1}{2}\left(\frac{e^{-\lambda}\lambda^2}{2!}\right) + \frac{e^{-\lambda}\lambda^0}{0!}$$

$$\Rightarrow \frac{3\lambda^4}{4!} = \frac{\lambda^2}{4} + 1$$

$$\Rightarrow \frac{3\lambda^4}{24} = \frac{\lambda^2}{24} + 1 \quad \Rightarrow 12\lambda^2 + 24 = 3\lambda^4$$

$$\Rightarrow \lambda^4 - 4\lambda^2 - 8 = 0$$

take $y = x^2$

$$y^2 - 4y - 8 = 0$$

$$y = \frac{4 \pm 4\sqrt{3}}{2}$$

$$\lambda^2 = 2 + 2\sqrt{3}$$

$$\boxed{\lambda = 2.37}$$

(a) Mean $x = \lambda = \sqrt{2+2\sqrt{3}} \cong 2.37$

(b) $P(x \le 2) = P(x=0) + P(x=1) + P(x=2)$

$$= e^{-\lambda} + \frac{e^{-\lambda}\lambda^1}{1!} + \frac{e^{-\lambda}\lambda^2}{2!}$$

$$= e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{2}\right]$$

$$= e^{-2.37}\left[1 + 2.37 + \frac{(2.37)^2}{2}\right]$$

$$= \underline{0.578}$$

If x is a normal variable with mean 30 and Standard deviation 5. Find the probability that.

(i) $26 \leq x \leq 40$ (ii) $x \geq 45$

Sol:- (i) $P(26 \leq x \leq 40)$

$$Z = \frac{x-\mu}{\sigma}$$

for $x = 26$

$$Z_1 = \frac{26-30}{5} = \frac{-4}{5} = -0.8$$

for $x = 40$

$$Z_2 = \frac{40-30}{5} = \frac{10}{5} = 2$$

Cummulative probability $\cdot\ Z_1 = -0.8$

$Z_1 = 0.2113$

$Z_2 = 0.9772$

$$P(26 \leq x \leq 40) = P(z \leq 2) - P(z \leq -0.8)$$

$$= 0.9772 - 0.2113$$

$$= 0.7659$$

(ii) $P(x \geq 45)$:

$$z = \frac{45-30}{5} = \frac{15}{5} = 3$$

The Cummulative probability for $z = 3$ is 0.9987

$$P(x \geq 45) = 1 - P(z \leq 3)$$

$$= 1 - 0.9987$$

$$= \underline{0.0013}$$

3. In a certain examination 10% of the students appeared for an exam and got less than 30 marks and 97% of students got less than 62. Assuming the distribution to be normal then find mean (μ), standard deviation

Sol: Given that,

10% of students scored less than 30

97% of students scored less than 62

$$\boxed{z = \frac{x-\mu}{\sigma}}$$

(i) for 10%.

Cummulative probability of 0.10 is

$$z_1 = -1.28$$

(ii) for 97%.

Cummulative probability of 0.97 is

$$z_2 = 1.88$$

for $x = 30$

$$z_1 = \frac{30-\mu}{\sigma} = -1.28$$

$$30 - \mu = -1.28\sigma$$

$$\mu - 30 = 1.28\sigma \quad - ①$$

for $x = 62$

$$z_2 = \frac{62-\mu}{\sigma} = 1.88$$

$$62 - \mu = 1.88\sigma$$

$$\mu - 62 = -1.88\sigma$$

Solving ① & ②

$$\sigma = \frac{32}{3.16} \cong 10.13 \quad \text{sub in } ①$$

$\mu - 30 = 1.28 \times 10.13$

$\mu = 30 + 12.96$

$\boxed{\mu = 42.96}$

Mean $(\mu) = \underline{\underline{42.96}}$

$S.D = \underline{\underline{10.13}}$

8: The marks obtained in mathematics by 1000 students in normally distributed with mean 78% and standard deviation. Determine How many students got marks above 90%. ?

Sol:- Given, mean $(\mu) = 78\%$.

$S.D(\sigma) = 11\%$.

$$Z = \frac{x - \mu}{\sigma}$$

$$\Rightarrow Z = \frac{90 - 78}{11} = \frac{12}{11} \cong 1.09$$

$$P(Z \geq 1.09) = 1 - P(Z < 1.09)$$

$$= 1 - 0.8621$$

$$= 0.1379$$

Percentage of students who scored above 90% is 13.79%.

no. of students $= 0.1379 \times 1000$

$= \underline{\underline{13.8}}$.

Assignment - 2. (Unit - 2)

1. Samples of size 2 are taken from the population 4, 8, 12, 16, 20, 24 without re-placement. Find
(a) mean of population
(b) S.D of population
(c) Construct sampling distribution of means
(d) mean of sampling distribution of mean and
(e) S.D of sampling distribution of mean.

Sol:- (a) $\mu = \frac{4+8+12+\cdots+24}{6} = \frac{84}{6} = 14$

(b) $\sigma = \sqrt{\frac{\Sigma(x_i-\mu)^2}{n}}$

$= \sqrt{\frac{(4-14)^2+(8-14)^2+(12-14)^2+(16-14)^2+(20-14)^2+(24-14)^2}{6}}$

$= \sqrt{\frac{100+36+4+4+36+100}{6}} = \sqrt{\frac{280}{6}} = \sqrt{46.67}$

$= 6.83$

(c)

Sample	mean
(4,8)	6
(4,12)	8
(4,16)	10
(4,20)	12
(4,24)	14
(8,12)	10
(8,16)	12
(8,20)	14
(8,24)	16
(12,16)	14
(12,20)	16
(16,20)	18
(16,24)	20
(20,24)	22
(12,24)	13

(d) $\mu_{\bar{x}} = \frac{\Sigma \text{ sample mean}}{\text{total Samples}}$

$= \frac{6+8+\cdots+22}{15}$

$= \frac{210}{15} = 14$

(e) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6.83}{\sqrt{2}}$

$= \frac{6.83}{1.41}$

$= 4.84$

2) Standard deviation of Sampling distribution of means. Calculate the mean and S.D of Sampling distribution of mean of 80 samples each of size 25 by Sampling (a) with replacement (b) without replacement from a normal population of 3000 with mean 68 and S.D = 3.

Sol: Given $\mu = 68$, $\sigma = 3$, $n = 25$, No. of Samples = 80,
$N = 3000$, $\mu_{\bar{x}} = \mu = 68$

(a) with replacement

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{25}} = \frac{3}{5} = \underline{0.6}$$

(b) Without replacement

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}} = 0.6 \times \sqrt{\frac{3000-25}{3000-1}} \simeq \underline{0.594}$$

3) Determine the probability that the sample mean area covered by sample of 40 one liter paint boxes will be b/w 510 to 520 square feet given that a one liter of such paint box covers on the average 513.3 square feet with s.d of 31.5

Sol:- Given mean(μ) = 513.3, $\sigma = 31.5$, $n = 40$

for $x = 510$

$$\Rightarrow z = \frac{x-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{510-513.3}{\frac{31.5}{\sqrt{40}}} = \frac{-3.3}{\frac{31.5}{6.32}} \Rightarrow -0.66$$

for $x = 520$

$$z = \frac{x-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{520-513.3}{\frac{31.5}{\sqrt{40}}} = \frac{6.7}{4.99} = 1.34$$

using Z-table:

$P(Z \le -0.66) = 0.2546$

$P(Z \le 1.34) = 0.9099$

$P(510 \le x \le 510) = 0.9099 - 0.2546$

$= 0.6553$

4) Given $A = \{2, 7, 9, 11\}$, $B = \{3, 5, 6\}$

Then find (a) Mean of A (b) Mean of B
(c) Mean of A+B (d) Variance of A
(e) Variance of B (f) Variance of A+B.

Sol:- (a) $\mu_A = \frac{2+7+9+11}{4} = \frac{29}{4} = 7.25$

(b) $\mu_B = \frac{3+5+6}{3} = \frac{14}{3} = 4.67$

(c) $\mu_{A+B} = \mu_A + \mu_B = 7.25 + 4.67 = \underline{11.92}$

(d) $\sigma_A^2 = \frac{\sum(x_i - \mu_A)^2}{N} = \frac{(2-7.25)^2 + (7-7.25)^2 + \cdots + (11-7.25)^2}{4}$

$= \frac{27.56}{4} = 6.89$

(e) $\sigma_B^2 = \frac{\sum(x_i - \mu_B)^2}{N} = \frac{(3-4.67)^2 + \cdots + (6-4.67)^2}{3}$

$= \frac{2.67}{3} = \underline{\underline{0.89}}$

(f) $\sigma_{A+B}^2 = \sigma_A^2 + \sigma_B^2 = 6.89 + 0.89 = \underline{\underline{7.78}}$

5) The mean of life time at light bulbs produces by Company is 1500 hrs and S.D at 150 hrs. Find the probability that Lighting will take place for (i) Atleast 5000 hrs (ii) Atmost 4200 hrs. If three bulbs are Connected such that when one bulb burns out, another bulb will go on. Assume that life times are normally distribution.

Sol:- Mean lifetime of bulbs = 1500 hrs
Standard deviation = 150 hrs.

for $x = 5000$

$$Z = \frac{5000 - 1500}{150}$$

$$= \frac{3500}{150}$$

$$= 23.33$$

$P(Z > 23.33) \simeq 0$. (almost impossible event)

for $x = 4200$

$$Z = \frac{4200 - 1500}{150}$$

$$= \frac{2700}{150}$$

$$= 18.0$$

$P(Z \leq 18.0) \simeq 1$ (almost certain event)

6) A random sample of size 64 is taken from a normal population with $\mu = 51.4$ and $\sigma = 6.8$. What is the probability that the mean of sample will (a) exceed 52.9
(b) fall between 50.5 and 52.3
(c) be less than 50.6

Sol:- Given $\mu = 51.4$, $\sigma = 6.8$, $n = 64$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6.8}{\sqrt{64}} = \frac{6.8}{8} = 8.5$$

(a) for $P(\bar{x} > 52.9)$

$$z = \frac{52.9 - 51.4}{8.5} = \frac{1.5}{8.5} \Rightarrow \underline{0.176}$$

from z-table

$$P(z > 0.176) = 1 - 0.5699 = \underline{\underline{0.4301}}$$

(b) for $P(50.5 \leq \bar{x} \leq 52.3)$

$$z_1 = \frac{50.5 - 51.4}{8.5} = -0.106$$

$$z_2 = \frac{52.3 - 51.4}{8.5} = 0.106$$

from z-table

$$P(-0.106 \leq z \leq 0.106)$$
$$= 0.5423 - 0.4577$$
$$= \underline{\underline{0.0846}}.$$

(c) for $P(\bar{x} < 50.6)$

$$z = \frac{50.6 - 51.4}{8.5}$$

$$z = -0.094$$

from z-table:

$$P(z < -0.094) = 0.4625$$

7) The pulse rate of 50 yoga practitioners decreased on the average by 20.2 beats/minute with S.D of 3.5. If mean of sample 20.2 is used as a point estimate to the true average decrease in pulse rate.

(a) What can we ~~used~~ assert with 99% confidence about the max. error E

(b) Construct 95% confidence intervals for the true average decrease in pulse rate

(c) How large a sample should be chosen to assert with 99% confidence that the sample mean is off by atmost 0.50?

Sol:- Given $n = 50$, mean decrease in pulse = 20.2 beats/min, Standard dev = 3.5

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3.5}{\sqrt{50}} = \frac{3.5}{7.07} = 0.495$$

(a) Z-table

$$Z_{0.01} = 2.576$$

$$E = 2.576 \times 0.495 = 1.276$$

(b) Z-table

$$Z_{0.05} = 1.96$$

$$E = 1.96 \times 0.495$$
$$= 0.978$$

$$\text{Interval} = (20.2 - 0.970, 20.2 + 0.970)$$
$$= (19.23, 21.17)$$

(c) $n = \left(\frac{Z_{\frac{\alpha}{2}} \times 6}{E} \right)^2$

$n = \left(\frac{2.576 \times 7.5}{0.60} \right)^2 = 324.57$

$\therefore \boxed{n \simeq 325}$

8) A random sample of 300 items is taken from a population whose standard deviation is 18. The mean of sample is 82. How large a sample should be taken to assert with 95% confidence that the sample mean is off by atmost 0.45?

Sol:- Given,

Sample size $n = 300$

Sample mean $\bar{x} = 82$

population standard deviation $\sigma = 18$

Confidence level $= 95\%$

We know that,

$$n = \left(\frac{Z_{\frac{\alpha}{2}} \times \sigma}{E} \right)^2$$

for 95% confidence, $Z_{0.05} = 1.96$

$$n = \left(\frac{1.96 \times 18}{0.45} \right)^2 = 6146.56$$

$\therefore \boxed{n = 6147}$

9) A random sample of height of 81 students from a large population of students in a University having a S.D of 0.72. It has an average height of 5.5 ft. Find 95%, 99% confidence limits for the average height of all students of University.

Sol:. Given,

Sample Size $n = 81$

Sample mean $\bar{x} = 5.5$

population S.D $\sigma = 0.72$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.72}{\sqrt{81}} = \frac{0.72}{9} = 0.08$$

for 95% confidence ($Z_{0.05} = 1.96$):

$E = 1.96 \times 0.08 = 0.1568$

Interval $= (\bar{x} - E, \bar{x} + E)$

$= (5.5 - 0.1568, 5.5 + 0.1568)$

$= (5.34, 5.66)$

for 99% Confidence ($Z_{0.01} = 2.576$)

$E = 2.576 \times 0.08 = 0.2061$

Interval $= (\bar{x} - E, \bar{x} + E)$

$= (5.5 - 0.2061, 5.5 + 0.2061)$

$= (5.29, 5.71)$

10) A random sample of size 100 taken from a population of size of 1000, the mean and S.D of sample characteristic are found to be 4.8 and 1.1 respectively, what can we assert with 95% confidence about the max error ϵ.

Sol: Given,

Sample size $(n) = 100$

population size $(N) = 1000$

Sample mean $(\bar{x}) = 4.8$

Sample standard deviation $(s) = 1.1$

Confidence level $= 95\%$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}} = \frac{1.1}{\sqrt{100}} \times \sqrt{\frac{1000-100}{1000-1}}$$

$$\sigma_{\bar{x}} = 0.11 \times 0.949 \Rightarrow \underline{0.104}$$

for 95% of confidence,

$$Z_{0.05} = 1.96$$

$$\epsilon = 1.96 \times 0.104$$

$$= \underline{0.204}$$

UNIT-3 (Assignment)

1) Mice with an average lifespan of 32 months will live upto 40 months when fed by a certain nutritious food. If 64 mice fed on this diet have an average lifespan of 38 months and standard deviation of 5.8 months is there any reason to believe that average life span is less than 40 months.

Sol:- Given $\mu = 40$, $n = 64$, $\bar{x} = 38$, $\sigma = 5.8$

i) N.H. H_0 : $\mu = 40$

ii) A.H. H_1 : $\mu < 40$ (L.O.T.T)

iii) L.O.S. $\alpha = 5\% = \frac{5}{100} = 0.05$

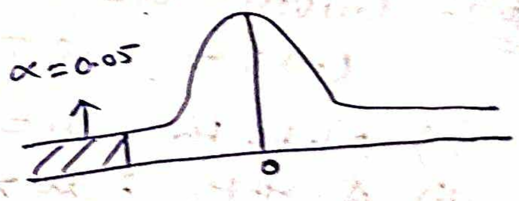
iv) Tabulated Z

As A.H is of < type, So use L.O.T.T

Since $\alpha = 5\%$

$= \frac{5}{100}$

$\alpha = 0.05$



from fig. Area from 0 to Z_α

$= 0.5 - 0.05$

$= 0.45$

Now, for the area 0.45, from table

$\boxed{Z_\alpha = -1.65}$

v) Calculated z

$$z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} = \frac{38 - 40}{\frac{5.8}{\sqrt{64}}} = \boxed{z = -2.75}$$

vi) Decision

Since $|Cal\ z| < tab\ z$

$|-2.75| < 1.65$

$2.75 < 1.65$ ×

$2.75 > 1.65$

Hence it is Rejected.

Q2:) A machine runs on an avg of 125 hrs/yr. A random sample of 49 machines has an annual avg use of 126.9 hrs with S.D 8.4 hrs. Does this suggest to believe that machines are used on the avg more than 125 hrs annually at 0.05 level of significance?

Sol:- Given $\mu = 125$, $n = 49$, $\bar{x} = 126.9$, $\sigma = 8.4$

L.O.S = 0.05

i) N.H. H_0 : $\mu = 125$

ii) A.H. H_1 : $\mu > 125$

iii) L.O.S $\alpha = 0.05$

iv) tabulated z:

As A.H is of > type, use R.O.T.T

Since $\alpha = 0.05$

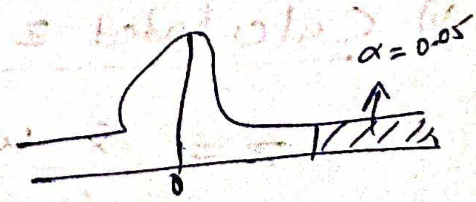
from fig, Area from 0 to z_α

$\Rightarrow 0.5 - 0.05$

$\Rightarrow 0.45$

for Area 0.45

$\boxed{z_\alpha = 1.65}$



v) Cal z

$$z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} \Rightarrow \frac{126.9 - 125}{\left(\frac{8.4}{\sqrt{49}}\right)} = \boxed{z = 1.58}$$

vi) Decision

Since $|\text{cal } z| < \text{tab } z$

$1.58 < 1.65$

Hence, N.H. H_0 is accepted.

3) In a random sample of 100 tubelight produced by company A. The mean life tym of tubelight is 1190 hrs. with S.D of 90 hrs. Also in a random sample of 75 tubelights from company B. The mean life time is 1230 hrs with S.D of 120 hrs. Is there a difference b/w a mean life tyme of the two brands of tubelight a significance level of 0.05?

Sol:- Given

$n_1 = 100$, $\bar{x}_1 = 1190$, $S_1 = 90 = \sigma_1$

$n_2 = 75$, $\bar{x}_2 = 1230$, $S_2 = 120 = \sigma_2$

i) N.H. H_0: $\mu_1 = \mu_2$ (T.T.T)

ii) A.H. H_1: $\mu_1 \neq \mu_2$

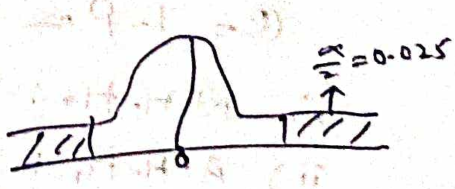
iii) L.O.S $\alpha = 0.05$

iv) Tab Z

As A.H is $\neq$ type so use T.T.T

Since $\alpha = 0.05$

$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$



from fig,

Area from 0 to $Z_\alpha = 0.5 - 0.025$

$= \underline{0.475}$

for Area, 0.475 $\boxed{Z_\alpha = 1.96}$

v) Cal z

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{1190-1230}{\sqrt{\frac{90^2}{100} + \frac{120^2}{75}}} = \boxed{Z = -2.42}$$

vi) Decision

$|\text{Cal} z| < \text{tab} z$

$2.42 < 1.96$ ×

Hence, it is Rejected (N.H. H_0)

9.) Random Samples of 400 men & 600 women were asked whether they would like to have a flyover near the resident. 200 men & 325 women were in favour of the proposal are same at 5% level.

Sol:- Given $n_1 = 400$

$n_2 = 600$

let $p_1 = \frac{200}{400} \Rightarrow 0.5$

$p_2 = \frac{325}{600} \Rightarrow 0.54$

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{400 \times 0.5 + 600 \times 0.54}{400 + 600}$$

$P = 0.52$

$Q = 1 - P = 1 - 0.52 \Rightarrow 0.48$

i). N.H. H_0 : $p_1 = p_2$

ii) A.H. H_1 : $p_1 \neq p_2$

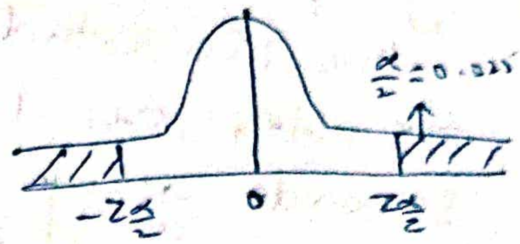
iii) L.O.S $\alpha = 5\% = \frac{5}{100} = 0.05$

iv) tabulated z

A.H ~~from~~ is of ≠ type, use T.T.T

Since $\alpha = 0.05$

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$



Area from 0 to z_α

$$= 0.5 - 0.025$$
$$= 0.475$$

Now, for the area 0.475, from table

$$\boxed{z_{\frac{\alpha}{2}} = 1.96}$$

v) Cal z

Test Statistics for two proportion is given by

$$z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.5 - 0.54}{\sqrt{(0.524 \times 0.476)\left(\frac{1}{400} + \frac{1}{600}\right)}}$$

$$= \frac{-0.04}{\sqrt{0.000984}} = \boxed{z = -1.29}$$

vi) Decision:-

Since, $|\text{Cal } z| < \text{tab } z$

$$1.29 < 1.96$$

Hence, N.H. H_0 is Accepted.

4.) A researcher wants to know the intelligence of students in a school. He selected two groups of students. In the first group there 150 students having mean IQ of 75 with standard deviation of 15 in the second group there 150 students having mean IQ of 70 with standard deviation of 20 (1 % LOS).

Sol:- Given

$n_1 = 150$, $\bar{x}_1 = 75$, $\sigma_1 = 15$

$n_2 = 150$, $\bar{x}_2 = 70$, $\sigma_2 = 20$

L.O.S. $\alpha = 1\% = \frac{1}{100} = 0.01$

i) N.H. H_0 : $\mu_1 = \mu_2$

ii) Al.H H_1 : $\mu_1 \neq \mu_2$ (T.T.T)

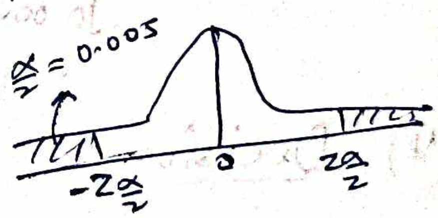
iii) L.O.S $\alpha = 0.01$

iv) tabulated Z,

A.H is of $\neq$ type, use T.T.T

Since $\alpha = 0.01$

$\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$



from fig, Area from 0 to Z_α

$\Rightarrow 0.5 - 0.005$

$\Rightarrow 0.495$

$\boxed{Z_\alpha = 2.58}$ from table

v) Cal Z

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{75-70}{\sqrt{\frac{15^2}{150} + \frac{20^2}{150}}}$$

$$= \frac{75-70}{\sqrt{\frac{15^2+20^2}{150}}}$$

$$= \frac{5}{\sqrt{\frac{625}{150}}} = \frac{5}{\sqrt{4.167}}$$

$$\Rightarrow \frac{5}{2.042} = \boxed{Z = 2.45}$$

vi) Decision

|Cal z| < tab z

2.45 < 2.58

Hence, N.H. Ho is Accepted.

6) If in a random sample of 600 cars making a right turn at a certain traffic junction 157 drove in to the wrong line, test whether actually 30% of all drivers make this mistake or not at this given junction, Use (a) 0.05 (b) 0.01 L.O.S

Sol Given $P = 30\% = \frac{30}{100} = 0.30$

$Q = 1 - P = 1 - 0.30 = 0.70$

$n = 600$

let $p = \frac{157}{600} = 0.26$

(a)

i) N.H. H_0: $P = 30\%$

ii) A.H. H_1: $P \neq 30\%$ (T.T.T)

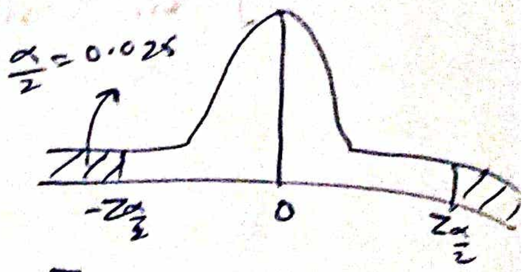
iii) L.O.S $\alpha = 0.05$

iv) Tabulated Z

As A.H. is of ≠ type, so use T.T.T

Since $\alpha = 0.05$

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$



from fig, Area from 0 to Z_α

$= 0.5 - 0.025$

$= \underline{0.475}$

Now, for the Area 0.475, from table

$$\boxed{Z_\alpha = 1.96}$$

v) Cal Z

Test Statistics for one proportion

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.26 - 0.30}{\sqrt{\frac{0.30 \times 0.70}{600}}} = \frac{-0.04}{\sqrt{\frac{0.21}{600}}}$$

$$\Rightarrow \frac{-0.04}{\sqrt{0.00035}} = \frac{-0.04}{0.187} = \boxed{-0.21}$$

vi) Decision

$|Cal z| < tab z$

$|-0.21| < 1.96$

$0.21 < 1.96$ ✓

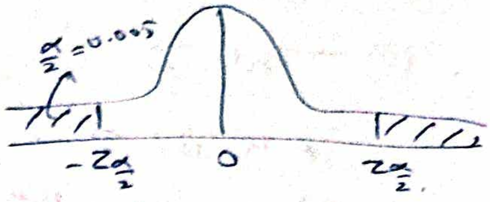
Hence, N.H.H. is accepted

(b) (iv) Tab z

As A.H is of ≠ type, so use T.T.T

Since $\alpha = 0.01$

$$\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$$



from fig. Area from 0 to z_α

$\Rightarrow 0.5 - 0.005$

$\Rightarrow 0.495$

from table,

$$\boxed{z_\alpha = 2.58}$$

vi) Decision

$|cal z| < tab z$

$|-0.211| < 2.58$

$0.21 < 2.58$

Hence, N.H. H_0 is Accepted.

Q. If 57 of 150 patients suffering with certain disease are used by Allopathic medicine and 33 out of 100 patients of same disease are cured by homeopathy. Is there any reason to believe that allopathic is greater than homeopathy? (use $\alpha = 0.05$)

Sol:- $n_1 = 150$, $n_2 = 100$

$p_1 = \frac{57}{150} = 0.38$, $p_2 = \frac{33}{100} = 0.33$

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{150 \times 0.38 + 100 \times 0.33}{150 + 100}$$

$$= \frac{90}{250} = \boxed{P = 0.36}$$

$Q = 1 - P = 1 - 0.36 = 0.64$

i) N.H H_0 : $p_1 = p_2$

ii) A.H H_1 : $p_1 > p_2$

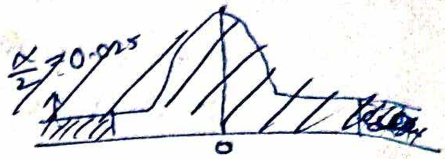
iii) L.O.S $\alpha = 0.05$

iv) Tab Z

A.H is of > type, use T.T.T

Since $\alpha = 0.05$

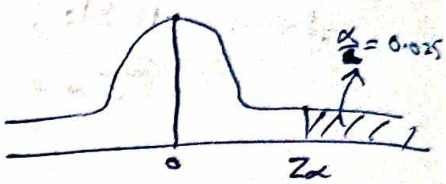
$\cancel{\frac{\alpha}{2} = \frac{0.05}{2} = 0.025}$



from fig, Area from 0 to Z_α

$\Rightarrow 0.5 - 0.05$

$\Rightarrow 0.45$



from table, 0.45

$\boxed{Z_\alpha = 1.65}$

v) Cal Z

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.38 - 0.33}{\sqrt{(0.36 \times 0.64)\left(\frac{1}{150} + \frac{1}{100}\right)}}$$

$$= \frac{0.05}{\sqrt{0.23 \times 0.0167}} = \frac{0.05}{\sqrt{0.0038}}$$

$$\Rightarrow \frac{0.05}{0.0616} = \boxed{Z = 0.811}$$

vi) Decision

$|\text{Cal } z| < \text{tab } Z$

$0.811 < 1.65$

Hence, N.H H_0 is Accepted.

8. Out of two welding machines at a super market, the first machine fails to work 13 times in 250 trials and second fails to work 7 times in 250 trials. Test whether the difference between corresponding population proportion is significant? (use $\alpha = 0.05$)

Sol:- Given,

$$n_1 = 250, \quad n_2 = 250$$

$$p_1 = \frac{13}{250} = 0.052 \qquad p_2 = \frac{7}{250} = 0.028$$

$$P = \frac{0.052 \times 250 + 0.028 \times 250}{250 + 250} = \underline{0.04}$$

$$Q = 1 - P = 1 - 0.04 = \underline{0.96}$$

i) N.H. $H_0 : p_1 = p_2$

ii) A.H. $H_1 : p_1 \neq p_2$

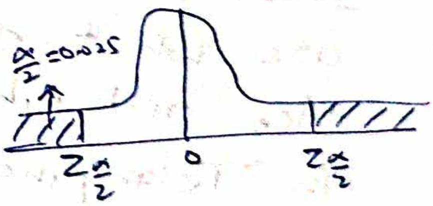
iii) L.O.S $\alpha = 0.05$

iv) Tab Z

A.H is of $\neq$ type, use T.T.T

Since $\alpha = 0.05$

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$



from fig, Area from 0 to Z_α

$$\Rightarrow 0.5 - 0.025$$

$$\Rightarrow 0.475$$

$$\boxed{Z_\alpha = 1.96}$$

v) Cal Z

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\Rightarrow \frac{0.052 - 0.028}{\sqrt{(0.04 \times 0.96)\left(\frac{1}{250} + \frac{1}{250}\right)}}$$

$$\Rightarrow \frac{0.027}{\sqrt{(0.0384) \times 0.008}} = \frac{0.027}{\sqrt{0.0003072}}$$

$$= \frac{0.027}{0.0175}$$

$$\boxed{Z = 1.54}$$

vi) Decision

|Cal z| < tab z

1.54 < 1.96 (✓)

Hence, N.H.Ho is accepted.

5. In a sample of 1000 population in Telangana State, 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat eaters are equally popular in Telangana State with L.O.S 1%.

Sol: Given, $n = 1000$.

$$p = \frac{540}{1000} = 0.54$$

P = (Probability of rice eaters) = 50% $= \frac{50}{100} = 0.5$

$Q = 1 - 0.5 = 0.5$

i) N.H. H_0 : $P = 50\%$

ii) A.H. H_1 : $P \neq 50\%$ (T.T.T)

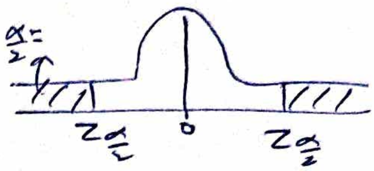
iii) L.O.S $\alpha = 1\% = \frac{1}{100} = 0.01$

iv) Tab Z

As A.H is of $\neq$ type, So, use T.T.T

Since $\alpha = 0.01$

$$\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$$



from fig, Area from 0 to Z_α

$= 0.5 - 0.005$

$= 0.495$

Now, for the Area 0.495, from table

$$\boxed{Z_\alpha = 2.58}$$

v) Cal Z

Test Statistics for one proportion

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.54 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{1000}}} = \frac{0.04}{\sqrt{0.00025}}$$

$$Z_\alpha = \frac{0.04}{0.0159} = 2.51$$

vi) Decision

Since, $|\text{Cal } z| < \text{tab } z$

$|2.51| < 2.58$

$2.51 < 2.58$ ($\checkmark$)

Hence, N.H. H_0 is Accepted.