INTRODUCTION

- **Generally in the area of computer science we discuss few concepts of mathematics which are applicable.**

- **The trend is to select the various topics in mathematics that are essential to the study of computer science areas**

- **Mathematical topics that are discussed are logic, set theory, algebraic structures, graph theory.**

- **These topics will support many areas of computer science such as automata, artificial intelligence, syntactic analysis, switching theory, programming languages**

- **Mathematical logic is used to read and understand technical articles and books in computer science.**
- **Set theory, relations, recursive functions are mostly used in programming languages.**
- **Algebraic structures are used for syntactic analysis, error detecting and correcting codes.**
- **Graph theory is used in minimal-path problems, fault detection and diagnosis in computers**

Applications

Used to Design Digital Circuits

Example:

Used in aircrafts, washing machines

MATHEMATICAL LOGIC

- **It is branch of mathematics which is concerned with reasoning.**
- **Main aim is it provides rules and techniques for determining whether a given argument is valid.**

USES OF LOGIC REASONING

Mathematics	**to prove theorems.**
Computer Science	**to verify the correctness of programs and to prove theorems.**
Natural and Physical Sciences	**to draw conclusions from experiments.**
Social Sciences, and everyday lives	**to solve a multitude of problems.**

STATEMENTS/PROPOSITIONS

Statement: A declarative sentence which is either true or false but not both.

Example:

Delhi is capital of the India	-	True
2+3=5	-	True
SNIST is located at Miyapur	-	False

NOT A STATEMENT – NEITHER TRUE NOR FALSE

- **Is Canada a Country : it is a question.**
- **Please close the door : it is a request.**
- **X+3=7 : truth value depends upon the value of X**
- **1+101=110 : truth value depends upon the context**
- **Get out of the class : it is a command**
- **Jungle book is greatest movie of all the time : it is an opinion**

TYPES OF STATEMENTS

1.Primitive/Primary/Atomic/Simple statements

- Cannot be further broken down or analyzed into simpler sentences.
- A proposition that consists of one subject and one predicate is called a simple statement

Example:

subject
- **John** is a bachelor
- **MFCS** is easy

predicate

NOTATIONS

Statements have only one of the two possible truth-values.

true (T or 1)

false (F or 0)

Denoted by distinct symbols A, B, C, …, P, Q, ….

Ex: P : John is a bachelor.

Q : It is raining.

R : It is snowing.

2. Compound/molecular statement:

Statements that involve one or more of the connectives are compound statements (otherwise they are simple statements).

Example:

- **If you finish your homework then you can watch T.V.**
- **This is a question if and only if this is an answer.**
- **I have read this and I understand the concept.**

3. Quantified statement:

- **The words all, some, no or none are called quantifiers**
- **Statements containing quantifiers are called quantified statements.**

Example:

- **All poets are writers.**
- **Some people are narrow minded.**

TRUTH TABLE

- **Truth table displays the relationship among the truth-values of propositions.**

- **The truth value of a proposition ‘p’ is made up off individual propositions $p_1, p_2, \ldots\ldots p_n$.**

CONNECTIVES

- **New statements are obtained by the given statements with the help of words / phrases like 'not', 'and', 'or', 'if…then' and 'if and only if'. Such words or phrases are called connectives.**

- **The statements which do not contain connectives are called simple statements.**

CONNECTIVES

Negation
Conjunction
Disjunction
Conditional/Implication
BiConditional/Bi-implication

Negation (¬, ~, not, --)

- Formed by introducing the word "not" at a proper place in the statement or by prefixing the statement with the phrase "It is not the case that".
- ~P or not P.

Example:

P: London is a city

¬p: London is not a city

Truth Table

P	~P
T	F
F	T

Conjunction ($\wedge$)

$P \wedge Q$ (P and Q) has truth value T whenever both P and Q have the truth-values T; otherwise truth-value F.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Example: 1

P : The weather is cloudy.

Q : It is raining today.

$P \wedge Q$: The weather is cloudy and it is raining today.

Example: 2

Jack and Jill went up the hill

Jack went up the hill and Jill went up the hill

P : Jack went up the hill

Q : Jill went up the hill

Disjunction / inclusive or ($\vee$)

$P \vee Q$ (P or Q) has Truth value F only when both P and Q have the truth value F; otherwise Truth-value T.

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Example

P : The weather is cloudy.

Q : It is raining today.

$P \vee Q$: The weather is cloudy or it is raining today.

<u>Implication</u> <u>or</u> <u>Conditional</u> $P \to Q$

P: premise, hypothesis, or antecedent of the implication.

Q: conclusion or consequent of the implication.

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

1. If Jerry takes Calculus or Ken takes Sociology, then Larry will take English

- **J: Jerry takes Calculus**
- **K: Ken takes Sociology** $(J \lor K) \rightarrow L$
- **L: Larry takes English**

2. The crop will be destroyed if there is a flood

- **C: The crop will be destroyed**
- **F: There is a flood**
  - $F \rightarrow C$

Biconditional $P \leftrightarrow Q$

Conjunction of the conditionals $P \rightarrow Q$ and $Q \rightarrow P$.

True : when P and Q have the same truth-values.

False : otherwise.

$\leftrightarrow$ P if and only if Q - P iff Q

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Precedence of logical connectives:

~ highest

Λ second highest

$\vee$ third highest

$\rightarrow$ fourth highest

$\leftrightarrow$ fifth highest

WELL-FORMED FORMULAS (WFF)

Expression consisting of

- **Statements (Propositions/variables)**
- **Parentheses**
- **Connecting symbols.**

1. **A statement variable alone is a WFF**
2. **If A is a WFF, then ~A is a WFF.**
3. **If A and B are WFFs, then $(A \wedge B)$, $(A \vee B)$, $(A \rightarrow B)$, and $(A \leftrightarrow B)$ are WFF.**
4. **A string of symbols containing the statement variables, connectives, and parentheses is a WFF, iff it can be obtained by finitely many applications of 1, 2, and 3 above.**

Examples of Well-Formed Formulas

- P
- $\neg P$
- $\neg(P \land Q)$
- $(P \rightarrow (P \lor Q))$
- $((\ (P \rightarrow Q) \land\ (Q \rightarrow R)\ ) \leftrightarrow (P \rightarrow R)\ )$

WFF

- $\neg P \lor Q$
- $(P \rightarrow Q$

Not WFF

TAUTOLOGIES

- **A statement formula which is true regardless of the truth values of the statements which replace the variables in it is called a tautology or a logical truth.**

Method I:

Prove $(P \land Q) \rightarrow P$ as tautology by using truth table

P	Q	$P \land Q$	$(P \land Q) \rightarrow P$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

- **Converse of $P \to Q$ is $Q \to P$**
- **Opposite/Inverse of $P \to Q$ is $(\sim P) \to (\sim Q)$**
- **Contra positive of $P \to Q$ is $\sim Q \to \sim P$**

P Q	$P \to Q$	$\sim P$	$\sim P \vee Q$	$\sim Q$	Contra positive $\sim Q \to \sim P$	Converse $Q \to P$	Opposite $\sim P \to \sim Q$
T T	T	F	T	F	T	T	T
T F	F	F	F	T	F	T	T
F T	T	T	T	F	T	F	F
F F	T	T	T	T	T	T	T

Contradiction / Absurdity / Identically False

Propositional function whose truth-value is always false.

Ex: $P \wedge \sim P$

P	$\sim P$	$P \wedge \sim P$
T	F	F
F	T	F

Contingency

Propositional function that is neither a tautology nor a contradiction.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

EQUIVALENCE OF FORMULAS / LOGICAL EQUIVALENCE

- **Two well-formed formulas A and B are said to be equivalent, if the truth value of A is equal to the truth value of B for every one of the 2^n possible sets of truth values assigned.**
- **The Statement formulas A and B are equivalent provided $A \leftrightarrow B$ is a tautology.**
- **It is represented by $A \Leftarrow\Rightarrow B$ or $A \equiv B$.**

EQUIVALENT FORMULAS:

Commutative Properties

$$P \vee Q \equiv Q \vee P$$

$$P \wedge Q \equiv Q \wedge P$$

P	Q	PVQ	QVP	(PVQ) $\leftrightarrow$ (QVP)
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

Associative Properties

$$P \vee (Q \vee R) \equiv (P \vee Q) \vee R$$

$$P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$$

<=> or $\equiv$

Distributive Properties

$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$$

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$$

Idempotent Properties

$$P \vee P \equiv P$$

$$P \wedge P \equiv P$$

Properties of Negation (De Morgan's Law)

$$\sim(\sim P) \equiv P$$

$$\sim(P \lor Q) \equiv (\sim P) \land (\sim Q)$$

$$\sim(P \land Q) \equiv (\sim P) \lor (\sim Q)$$

Identity Law

$$P \land T \equiv P$$

$$P \lor F \equiv P$$

Domination Law

$$P \lor T \equiv T$$

$$P \land F \equiv F$$

Absorption Law

$$P \lor (P \land Q) \equiv P$$

$$P \land (P \lor Q) \equiv P$$

Law of Complementation

$$P \lor \sim P \equiv T$$

$$P \land \sim P \equiv F$$

DeMorgan's laws

$\sim(P \vee Q) \equiv (\sim P) \wedge (\sim Q)$

$\sim(P \wedge Q) \equiv (\sim P) \vee (\sim Q)$

Law of Double Negation

$P \equiv \sim(\sim P)$.

$(P \rightarrow Q) \equiv (\sim P) \vee Q$ **(Law of implication)**

$(P \rightarrow Q) \equiv (\sim Q \rightarrow \sim P)$ **(Law of contrapositive)**

Properties of operations on equivalence

- $(P \to Q) \equiv ((\sim P) \vee Q)$
- $(P \to Q) \equiv (\sim Q \to \sim P)$
- $(P \leftrightarrow Q) \equiv ((P \to Q) \wedge (Q \to P))$
- $\sim(P \to Q) \equiv (P \wedge \sim Q)$
- $\sim(P \leftrightarrow Q) \equiv ((P \wedge \sim Q) \vee (Q \wedge \sim P))$
- $P \leftrightarrow Q \equiv (P \wedge Q) \vee (\sim P \wedge \sim Q)$

Method II

Prove that ((P∧Q) →P) is a tautology with out using truth tables

(P∧Q) →P <=> ~(P∧Q) V P [Law of Implication]

<=> ~P V ~Q V P [Demorgan's law]

<=> (~P V P) V ~Q [Associative law]

<=> T V ~Q [Law of Complementation]

<=> T

Verify whether it is Tautology or not?

1. $(P \wedge Q) \rightarrow P$
2. $(P \wedge Q) \rightarrow Q$
3. $P \rightarrow (P \vee Q)$
4. $Q \rightarrow (P \vee Q)$
5. $\sim P \rightarrow (P \rightarrow Q)$
6. $\sim(P \rightarrow Q) \rightarrow P$
7. $(P \wedge (P \rightarrow Q)) \rightarrow Q$
8. $(\sim P \wedge (P \vee Q)) \rightarrow Q$
9. $(\sim Q \wedge (P \rightarrow Q)) \rightarrow \sim P$
10. $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$

$\{(P \rightarrow Q) \wedge (Q \rightarrow R)\} \rightarrow (P \rightarrow R)$	
$\sim\{(\sim P \vee Q) \wedge (\sim Q \vee R)\} \vee (\sim P \vee R)$	Law of Implication
$\{(P \wedge \sim Q) \vee (Q \wedge \sim R)\} \vee (\sim P \vee R)$	Demorgans law
$(P \wedge \sim Q) \vee (Q \wedge \sim R) \vee (\sim P \vee R)$	
$\underset{a}{\underline{(\sim P \vee R)}} \vee (\underset{b}{\underline{P}} \wedge \underset{c}{\underline{\sim Q}}) \vee (Q \wedge \sim R)$	Associative law
$((\sim P \vee R) \vee P) \wedge ((\sim P \vee R) \vee \sim Q) \vee (Q \wedge \sim R)$	Distributive law
$((P \vee \sim P) \vee R) \wedge ((\sim P \vee R) \vee \sim Q) \vee (Q \wedge \sim R)$	Associative law
$(T \vee R) \wedge ((\sim P \vee R) \vee \sim Q) \vee (Q \wedge \sim R)$	Law of Complementation
$T \wedge ((\sim P \vee R) \vee \sim Q) \vee (Q \wedge \sim R)$	Domination law
$\underset{a}{\underline{((\sim P \vee R) \vee \sim Q)}} \vee (\underset{b}{\underline{Q}} \wedge \underset{c}{\underline{\sim R}})$	Identity law
$(((\sim P \vee R) \vee \sim Q) \vee Q) \wedge (((\sim P \vee R) \vee \sim Q) \vee \sim R)$	Distributive law
$((\sim P \vee R) \vee Q \vee \sim Q)) \wedge (\underline{\sim R \vee R} \vee \sim P \vee \sim Q))$	Associative law
$((\sim P \vee R) \vee T) \wedge (T \vee (\sim P \vee \sim Q))$	Law of Complementation
$T \wedge T$	Domination law
T	

Prove $P \vee \{P \wedge (P \vee Q)\} \Leftrightarrow$ **P**

$\Leftrightarrow P \vee P$ By absorption law

$\Leftrightarrow P$ By idempotent law

Given truth values of P and Q as 'T' and
R and S as 'F'.

Find the truth values of the following

1. $(\sim(P \wedge Q) \vee \sim R) \vee \{(Q \leftrightarrow \sim P) \rightarrow (R \vee \sim S)\}$
2. $(P \leftrightarrow R) \wedge (\sim Q \rightarrow S)$
3. $\{P \vee (Q \rightarrow (R \wedge \sim P))\} \leftrightarrow (Q \vee \sim S)$
4. $P \vee (Q \wedge R)$
5. $(P \wedge (Q \wedge R)) \vee \sim((P \vee Q) \wedge (R \vee S))$
6. $(\sim(P \wedge Q) \vee \sim R) \vee (((\sim P \wedge Q) \vee \sim R) \wedge S))$

TAUTOLOGICAL IMPLICATIONS

- **A Statement P is said to <u>tautologically imply</u> a Statement Q if and only if $P \rightarrow Q$ is a tautology. We shall denote this as P =>Q.**
- **Here, P and Q are related to the extent that, Whenever P has the truth value T then so does Q.**

Example:

Show that the implication of P==>(Q→P) is a Tautological implication

P	Q	Q→P	P→(Q→P)
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

- **$P \implies P \vee Q$**
- **$P \implies (Q \rightarrow P)$**
- **$(P \rightarrow Q) \wedge (Q \rightarrow R) \implies P \rightarrow R$**
- **$\sim P \implies P \rightarrow Q$**
- **$Q \implies P \rightarrow Q$**
- **$(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow R) \implies R$**

Duality Law

- **Two formulas A and A* are said to be duals of each other if either one can be obtained from the other by replacing $\wedge$ by v and v by $\wedge$.**
- **A(P,Q,R) : P v (Q $\wedge$ R)**
- **A*(P,Q,R) : P $\wedge$ (Q v R)**
- **If the formula A contains special variables T or F then its dual A* is obtained by replacing T by F and F by T.**
- **The connectives $\wedge$ and v are called duals of each other.**

Example

Given $\sim P \wedge \sim(Q \vee R)$

Show that $\sim A(P,Q,R) <=> A^{*}(\sim P,\sim Q,\sim R)$

Sol: $\sim A(P,Q,R)$: $\sim(\sim P \wedge \sim(Q \vee R))$

: $P V (Q V R)$

$A^{*}(\sim P,\sim Q,\sim R)$: $\sim(\sim P)\ V \sim(\sim Q \wedge \sim R)$

: $P V (Q V R)$

Therefore $\sim A(P,Q,R) <=> A^{*}(\sim P,\sim Q,\sim R)$

NORMAL FORMS

- **It is always not possible to construct truth table for practical purposes, especially when the number of variables is large.**
- **We therefore consider other procedures known as Normal Forms**
- **In our present discussion, we shall use the term '<u>product</u>' in the place of conjunction and '<u>sum</u>' in the place of disjunction.**

- **Let $A(P_1, P_2, \ldots\ldots, P_n)$ be a statement formula where $P_1, P_2, \ldots\ldots, P_n$ are the atomic variables.**

- **If we consider all possible assignments of the truth values to $P_1, P_2, \ldots\ldots, P_n$ and obtain the resulting truth values of the formula A. Such a truth table contains 2^n rows.**

- **If A has the truth value T for at least one combination of truth values assigned to $P_1, P_2, \ldots\ldots, P_n$ then A is said to be satisfiable.**
- **The problem of determining , in a finite number of steps, whether a given statement formula is a tautology or a contradiction or at least satisfiable is known as a decision problem.**

ELEMENTARY PRODUCT

- **Product of the variables and their negations in a formula.**

Example: P

Q

$\sim P \wedge Q$

$\sim Q \wedge P \wedge \sim P$

$P \wedge \sim P$

$Q \wedge \sim P$

ELEMENTARY SUM

- **Sum of the variables and their negations in a formula.**

Example: P

$\sim P \vee Q$

$\sim Q \vee P \vee \sim P$

$P \vee \sim P$

$Q \vee \sim P$

Factors of The Elementary Sum or Product

- Any part of an elementary sum or product, which is itself is an elementary sum or product.

Example: Factors of $\sim Q \wedge P \wedge \sim P$ are:

$$\sim Q, \ \sim P,$$

$$P \wedge \sim P$$

$$\sim Q \wedge P$$

Disjunctive Normal Forms

A formula equivalent to a given formula and consists of a sum of elementary products of the given formula.

Examples

1. Obtain Disjunctive Normal Form of $P \wedge (P \to Q)$.

$P \wedge (P \to Q) \Leftrightarrow P \wedge (\sim P \vee Q)$ [since $P \to Q \equiv (\sim P \vee Q)$]

$\Leftrightarrow (P \wedge \sim P) \vee (P \wedge Q)$ [Distributive law]

2.Obtain Disjunctive Normal Form of $\sim(P \vee Q) \leftrightarrow (P \wedge Q)$.

$\sim(P \vee Q) \leftrightarrow (P \wedge Q)$

$$\Leftrightarrow (\sim(P \vee Q) \wedge (P \wedge Q)) \vee (\sim(\sim(P \vee Q)) \wedge \sim(P \wedge Q))$$

since $[R \leftrightarrow S \Leftrightarrow (R \wedge S) \vee (\sim R \wedge \sim S)]$

$$\Leftrightarrow (\sim P \wedge \sim Q \wedge P \wedge Q) \vee \{\underbrace{(P \vee Q)}_{a} \wedge (\underbrace{\sim P}_{b} \vee \underbrace{\sim Q}_{c})\}$$

[Distributive law]

$$\Leftrightarrow (\sim P \wedge \sim Q \wedge P \wedge Q) \vee \{((P \vee Q) \wedge \sim P) \vee ((P \vee Q) \wedge \sim Q)\}$$

$$\Leftrightarrow (\sim P \wedge \sim Q \wedge P \wedge Q) \vee (P \wedge \sim P) \vee (Q \wedge \sim P) \vee (P \wedge \sim Q) \vee (Q \wedge \sim Q)$$

Conjunctive Normal Forms

A formula equivalent to a given formula and consists of a product of elementary sums of the given formula.

Examples:

1. Obtain Conjunctive Normal Form of $P \wedge (P \to Q)$.

$$P \wedge (P \to Q) \Leftrightarrow P \wedge (\sim P \vee Q)$$

[law of implication]

2.Obtain Conjunctive Normal Form of $\sim(P \vee Q) \leftrightarrow (P \wedge Q)$.

$\Leftrightarrow (\sim(P \vee Q) \rightarrow (P \wedge Q)) \wedge ((P \wedge Q) \rightarrow \sim(P \vee Q))$

since $[R \leftrightarrow S \Leftrightarrow (R \rightarrow S) \wedge (S \rightarrow R)]$

$\Leftrightarrow ((P \vee Q) \vee (P \wedge Q)) \wedge (\sim(P \wedge Q) \vee (\sim P \wedge \sim Q)$

$[\text{since } P \rightarrow Q \equiv (\sim P \vee Q)]$

$\Leftrightarrow \{(P \vee Q \vee P) \wedge (P \vee Q \vee Q)\} \wedge \{(\sim P \vee \sim Q) \vee (\sim P \wedge Q)\}$

[Distributive law]

$\Leftrightarrow (P \vee Q \vee P) \wedge (P \vee Q \vee Q) \wedge (\sim P \vee \sim Q \vee \sim P) \wedge (\sim P \vee \sim Q \vee \sim Q)$

PRINCIPAL DISJUNCTIVE NORMAL FORM

- **Min terms: A minterm consists of conjunction in which each statement variable or its negation should appear only once and should not be repeated.**
- **Example: For two variables P & Q minterms are:**

$$(P \wedge Q),\quad (\sim P \wedge Q),\quad (\sim P \wedge \sim Q),\quad (P \wedge \sim Q)$$

- **For a given formula , an equivalent formula consisting of disjunction of minterms only is known as its principal disjunctive normal form.**
- **Also called sum-of –products canonical form.**

Procedure to obtain PDNF:

Method 1: For every truth value T of given formula in truth table obtain its corresponding minterm and have their sum to obtain PDNF.

Example: Obtain PDNF of (PVQ)

P	Q	PVQ	
T	T	T	→ $(P \wedge Q)$
T	F	T	→ $(P \wedge \sim Q)$
F	T	T	→ $(\sim P \wedge Q)$
F	F	F	

Therefore PDNF obtained is $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

Method II:

- **Replace →, ↔ by their equivalent terms in ~,V, ∧.**
- **Apply negation to the statement variables only using demorgans law & apply distributive law**
- **Remove the elementary products which are always false**
- **Obtain the minterms by introducing missing factors.**
- **Delete identical minterms**

Example: Obtain the PDNF of $P \rightarrow Q$

$P \rightarrow Q$

$\Leftrightarrow \sim P \vee Q$ [law of implication]

$\Leftrightarrow [\sim P \wedge (Q \vee \sim Q)] \vee [Q \wedge (P \vee \sim P)]$ [since $(Q \vee \sim Q) \Leftrightarrow T$]

$\Leftrightarrow (\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (Q \wedge P) \vee (Q \wedge \sim P)$

[Distributive law]

$\Leftrightarrow (\sim P \wedge Q) \vee (Q \wedge P) \vee (Q \wedge \sim P)$

PRINCIPAL CONJUNCTIVE NORMAL FORM

Maxterms: A maxterm consists of disjunction in which statement variable or its negation should appear only once and should not be repeated.

- **Example: For two variables P & Q maxterms are:**

$$(P \vee Q),\quad (\sim P \vee Q),\quad (\sim P \vee \sim Q),\quad (P \vee \sim Q)$$

Note: Maxterms are duals of Minterms

- **For a given formula , an equivalent formula consisting of conjunctions of maxterms only is known as its principal conjunctive normal form.**
- **Also called products-of-sums canonical form.**

Obtain the PCNF for $(P \rightarrow R) \wedge (Q \leftrightarrow P)$

$(P \rightarrow R) \wedge (Q \leftrightarrow P)$

$\Leftrightarrow (\sim P \vee R) \wedge \{ (\sim Q \vee P) \vee (\sim P \vee Q) \}$

$\Leftrightarrow \{ \underbrace{(\sim P \vee R)}_{a} \vee (\underbrace{Q}_{b} \wedge \underbrace{\sim Q}_{c}) \} \wedge \{ (\sim Q \vee P) \vee (R \wedge \sim R) \} \wedge$
$\quad \{ (\sim P \vee Q) \vee (R \wedge \sim R) \}$

$\Leftrightarrow (\sim P \vee R \vee Q) \wedge (\sim P \vee R \vee \sim Q) \wedge (\sim Q \vee P \vee R) \wedge (\sim Q \vee P \vee \sim R)$
$\quad \wedge \quad (\sim P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R)$ [Distributive law]

$\Leftrightarrow (\sim P \vee Q \vee R) \wedge (\sim P \vee R \vee \sim Q) \wedge (\sim Q \vee P \vee R) \wedge (\sim Q \vee P \vee \sim R)$
$\quad \wedge \quad (\sim P \vee Q \vee \sim R)$

RULES OF INFERENCE

- **Used mainly to draw conclusions from assertions**
- **Criteria for determining the validity of an argument.**
- **These rules are stated in the form of the statements (premises and conclusions) involved.**
- **Therefore rules are given in terms of statement formulas rather than in terms of any specific statements.**

VALIDITY USING TRUTH TABLES

Let A & B be two statement formulas. " B logically follows A" (or) "B is a valid conclusion of the premise A"

Iff A $\longrightarrow$ B is a tautology, that is A =>B.

Now, we extend to a set of formulas rather than a single formula. Say a set of premises

{H1,H2,H3…Hn} , conclusion C follows logically iff

{H1∧H2 ∧H3,…Hn} => C ….. (1)

To determine whether a conclusion logically follows from the given premises we construct truth table.

Let P1,P2,P3…Pn be the atomic variables appears in the formulas H1,H2,H3….Hn and C be conclusion

If for all possible combinations of truth values are assigned to P1,P2,P3…Pn and truth values of H1,H2,H3….Hn & C are entered in the table

Then by seeing the table we can say whether (1) holds is true or not.

- **We look for rows in which all H1,H2,H3….Hn have truth value 'T'. If for every such row C also have a truth value 'T', then (1) holds.**

- **Alternatively, we may look for a row in C, which has value 'F'. If in every such row atleast one of such values of H1,H2,H3….Hn is 'F' then (1) also holds.**

- **We call such a method as "Truth table validation technique" for determination of the validity of a conclusion.**

Examples:

H1: $P \longrightarrow Q$ H2 :P   C: Q

P	Q	P→Q
T	T	T
T	F	F
F	T	T
F	F	T
H2	C	H1

- Only the first row is the one in which both premises have value T.
- The conclusion C also have the value T in that row
- Hence C is a valid conclusion
- H1 Λ H2 => C
- (P→Q) Λ P => Q

RULES OF INFERENCE

There are two rules of inferences

Rule P

A premise may be introduced at any point in the derivation

Rule T

A formula S may be introduced in a derivation, if S is tautologically implied by any one or more of the preceding formulas in the derivation

IMPLICATIONS

Inference Rule	Tautology	Name
$\dfrac{p}{\therefore\ p \vee q}$	$p \rightarrow (p \vee q)$	addition
$\dfrac{p \wedge q}{\therefore\ p}$	$(p \wedge q) \rightarrow p$	simplification
$\dfrac{\begin{matrix} p \\ q \end{matrix}}{\therefore\ p \wedge q}$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	conjunction
$\dfrac{\begin{matrix} p \vee q \\ \neg p \vee r \end{matrix}}{\therefore\ q \vee r}$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	resolution

inference rule	tautology	name
$\begin{array}{rl} & p \\ & p \to q \\ \hline \therefore & q \end{array}$	$(p \wedge (p \to q)) \to q$	Modus ponens (mode that affirms)
$\begin{array}{rl} & \neg q \\ & p \to q \\ \hline \therefore & \neg p \end{array}$	$(\neg q \wedge (p \to q)) \to \neg p$	Modus tollens (mode that denies)
$\begin{array}{rl} & p \to q \\ & q \to r \\ \hline \therefore & p \to r \end{array}$	$((p \to q) \wedge (q \to r)) \to (p \to r)$	hypothetical syllogism
$\begin{array}{rl} & p \vee q \\ & \neg p \\ \hline \therefore & q \end{array}$	$((p \vee q) \wedge (\neg p)) \to q$	disjunctive syllogism

- **Column 1 specifies the set of numbers, which shows the premises on which the formula in the line depends**
- **Column 2 designates the formula as well as line of derivation in which it occurs**
- **Column 3 indicates rule P or T to obtain a formula or tautology**

Example:

Demonstrate that R is a valid inference from the premises P $\rightarrow$Q, Q$\rightarrow$R and P

Column 1	Column 2	Column 3
{1}	(1)P$\rightarrow$Q	Rule P
{2}	(2)P	Rule P
{1,2}	(3)Q	Rule T[Modus Ponens]
{4}	(4)Q$\rightarrow$R	Rule P
{1,2,4}	(5) R	Rule T[Modus Ponens]

Arguments

Example

-Gary is intelligent or a good actor.
-If Gary is intelligent, then he can count from 1 to 10.
-Gary can only count from 1 to 2.
-Therefore, Gary is a good actor.

-i: “Gary is intelligent.”
-a: “Gary is a good actor.”
-c: “Gary can count from 1 to 10.”

$$\begin{array}{l} a \lor i \\ i \rightarrow c \\ \neg c \\ \hline \therefore \quad a \end{array}$$

Arguments

$$\begin{array}{l} a \vee i \\ i \rightarrow c \\ \neg c \\ \hline \therefore \quad a \end{array}$$

-i: "Gary is intelligent."
a: "Gary is a good actor."
c: "Gary can count from 1 to 10."

Column 1	Column 2	Column 3
{1}	(1) $\neg c$	Rule P
{2}	(2) $i \rightarrow c$	Rule P
{1,2}	(3) $\neg i$	Rule T Modus tollens Steps 1 & 2
{4}	(4) $a \vee i$	Rule P
{1,2,4}	(5) a	Rule T Disjunctive Syllogism

-Conclusion: **a** ("Gary is a good actor.")

- Example:
- If I study,I will not fail in the examination.
- If I donot watch TV in the evenings,I will study
- I failed in the examination.
- Therefore,I must have watched TV in the Evening.

- Example: Verify whether the following arguments are valid or not:

1)P->Q, R->S, PvR => QvS

2)P->Q, R->S, ~Qv~S => ~(P^R)

3)P->Q, Q->R, P =>R

4) ~Q, P->Q =>~P

5)S.T SvR IS TAUTOLOGICALLY IMPLIED BY (PvQ)^(P->R)^(Q->S)

6) S.T R^(PvQ) is valid conclusion from the premises PvQ, Q->R, P->M and ~M.

7)P->Q,Q->~R, R, Pv(J^S) => J^S

8) ~J->(MvN),(HvG)->~J, HvG =>MvN

Show that $P \vee Q$ follows logically from the premises $C \vee D$, $(C \vee D) \rightarrow \urcorner H$,

$\urcorner H \rightarrow (A \wedge \urcorner B)$ and $(A \wedge \urcorner B) \rightarrow (P \vee Q)$.

Rule CP (CP: Conditional Proof)

- **If we can derive S from R and a set of premises, then we can derive $R \rightarrow S$ from the set of premises alone.**
- **Rule CP is also called deduction theorem and is generally used if the conclusion is in the form $R \rightarrow S$.**
- **In such cases R is taken as an additional premise and S is derived from the given premises and R.**

Example:

Show that R→S can be derived from the premises P→(Q→S), ~RVP and Q

Column 1	Column 2	Column 3
{1}	(1)~RVP	Rule P
{2}	(2)R	Rule P (assumed premise)
{1,2}	(3)P	Rule T (Disjunctive syllogism)
{4}	(4)P→(Q→S)	Rule P
{1,2,4}	(5)Q→S	Rule T(Modus Ponens)
{6}	(6)Q	Rule P
{1,2,4,6}	(7)S	Rule T(Modus Ponens)
{1,4,6}	(8)R→S	Rule CP

Consistency of premises:

- A set of formulas H1,H2,H3….Hm is said to be consistent if their conjunction has truth value T for some assignment of the truth values to the variables appearing in H1,H2,H3….Hn

Inconsistency of premises:

- If for every assignment of the truth values to the atomic variable, atleast one of the formulas H1,H2,H3….Hm is false, so that their conjunction is identically false, then the formulas H1,H2,H3….Hm are called inconsistent.
- Alternatively, a set of premises H1,H2,H3….Hm is inconsistent if their conjunction implies a contradiction i.e,

$$H1,H2,H3\ldots.Hm \Rightarrow R \land \sim R$$

Show that the fallowing premises are inconsistent $P \rightarrow Q$, $P \rightarrow R$, $Q \rightarrow \sim R$, P

Column 1	Column 2	Column 3
{1}	(1)$P \rightarrow Q$	Rule P
{2}	(2)$Q \rightarrow \sim R$	Rule P
{1,2}	(3)$P \rightarrow \sim R$	Rule T (Hypothetical syllogism)
{4}	(4)P	Rule P
{1,2,4}	(5)$\sim R$	Rule T(Modus Ponens)
{6}	(6)$P \rightarrow R$	Rule P
{1,2,4,6}	(7)$\sim P$	Rule T(Modus Tollenss)
{1,2,4,6}	(8)$P \land \sim P$	Rule T(Conjunction)

PROOF BY CONTRADICTION

- **This is also known as indirect method of proof. In this we use inconsistency.**
- **In order to show that C logically fallows from the premises H1,H2,H3….Hm, we assume that C is false and take ~C as an additional premise.**
- **If the new set of premises are inconsistent, so that they imply a contradiction, then the assumption that ~C is true does not hold.**
- **Therefore C is true when ever H1,H2,H3….Hm is true.**

Using indirect method show $R \rightarrow \sim Q$, $R \vee S$, $S \rightarrow \sim Q$, $P \rightarrow Q \Rightarrow \sim p$

Assume $\sim(\sim P) = P$ as an additional premise

Column 1	Column 2	Column 3
{1}	(1)$P \rightarrow Q$	Rule P
{2}	(2)P	Rule P (Assumed premise)
{1,2}	(3)Q	Rule T(Modus Ponens)
{4}	(4)$R \rightarrow \sim Q$	Rule P
{1,2,4}	(5)$\sim R$	Rule T(Modus Tollens)
{6}	(6)$R \vee S$	Rule P
{1,2,4,6}	(7)S	Rule T(Disjunctive Syllogism)
{8}	(8)$S \rightarrow \sim Q$	Rule P
{1,2,4,6,8}	(9)$\sim Q$	Rule T(Modus Ponens)
{1,2,4,6,8}	(10)$Q \wedge \sim Q$	Rule T(Conjunction)

Therefore ~P logically fallows from the given set of premises