11/09/19

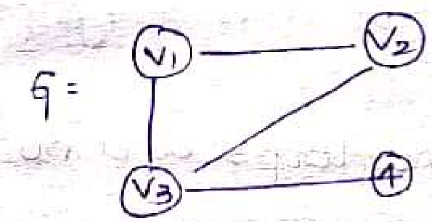
Graph Theory (Unit - 6)

→ A graph G is a pair of sets (V, E)
where V = A set of vertices (nodes) and
E = A set of edges (lines)

→ V(G) = set of vertices in G

→ ~~V(E)~~ E(G) = set of edges in G = V(E), E

→ |V(G)| = number of vertices in graph G
= order of G.



V(G) = {V₁, V₂, V₃, V₄}

|E(G)| = 4

|V(G)| = 4

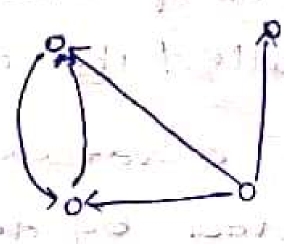
Types of graphs:

→ Non-Directed Graph (Undirected Graph):

The elements of E are unordered pairs (sets) of vertices. In this case an edge {u, v} is said to be join u and v or to be between u and v.

→ Directed Graph:

In a digraph the elements of E are ordered pairs of vertices. In this case an edge (u, v) is said to be from u to v

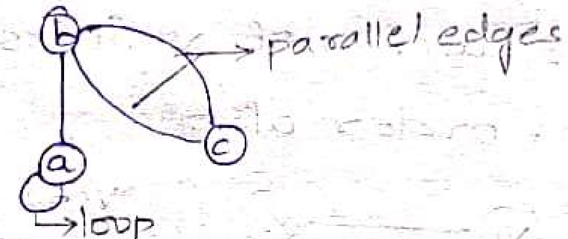


→ Loop:

An edge drawn from a vertex to itself

→ MultiGraph:

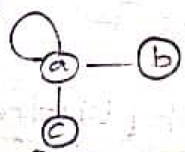
If one allows more than one edge to join a pair of vertices, the result is then called a multi graph.



→ Simple Graph:

A graph with no loops and no parallel edges.

Degree



Degree of a = 4
Degree of b,c = 1
self is considered as 2

Degree of a vertex is an undirected graph is the no. of edges incident with it, except that a loop at a vertex contribute twice to the degree of that vertex. The degree of the vertex 'v' is denoted by deg(v)

In-degree and Out-degree:

In a digraph, the no. of edges incident to a vertex is called the in-degree of the vertex and no. of vertexes incident from a vertex is called its out-degree.

→ The in-degree of a vertex 'v' in a graph G is denoted by deg+(v)

→ The out-degree of a vertex v is denoted by $deg^-(v)$.

→ A loop at a vertex in a digraph is counted as one edge for both in-degree and out-degree of that vertex.

Neighbours:

If there is an edge incident from u to v, or incident on u and v, then u and v are said to be adjacent (neighbors).

→ $\delta(G)$ - minimum of all the degrees of vertices in a graph G.

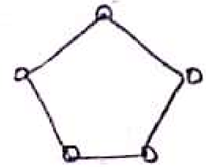
→ $\Delta(G)$ - maximum of all the degrees of vertices in a graph G.

Regular Graph:

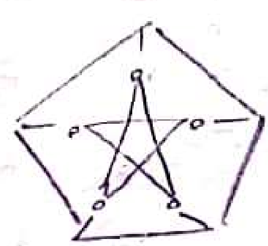
In a graph G, if $\delta(G) = \Delta(G) = k$ i.e., if each vertex of G has degree k, then G is said to be a regular graph of degree k (k-regular).

eg: Polygon is a 2-regular graph.

eg: A 3-regular graph is a cubic grap.



2-regular graph



3-regular graph

Complete Graph:

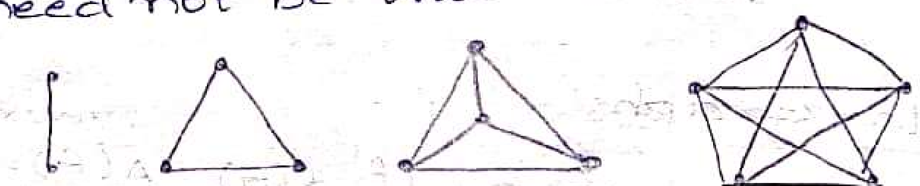
A simple non-directed graph with 'n' mutually adjacent vertices is called a completed graph on 'n' vertices & may be represented by k_n.

Note:

A complete graph on 'n'-vertices has $[\{n(n-1)\}/2]$ edges and each of its vertices has degree 'n-1'

→ Every complete graph is a regular graph.

→ The converse of the above statement need not be true.



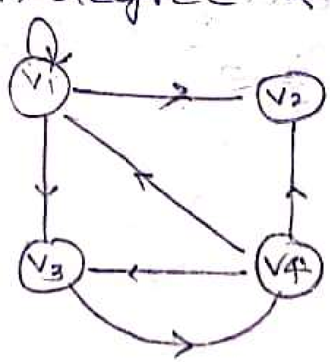
k_2 k_3 k_4 k_5

→ eg.

regular but not complete graph.

(Q) In-degree & out-degree.

→



	in-degree	outdegree
v_1	2	3
v_2	2	0
v_3	2	1
v_4	1	3

Scanned with CamScanner

16/9/19

Cycle graph (C_n):

A cycle graph of order 'n' is a connected graph whose edges form a cycle of length 'n'.

eg:

C_3 C_4 C_5

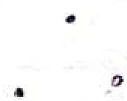
Note:

A cycle graph 'C_n' of order n has n vertices and n edges.

Null graph:

A null graph of order n has a graph with n vertices and no edges.

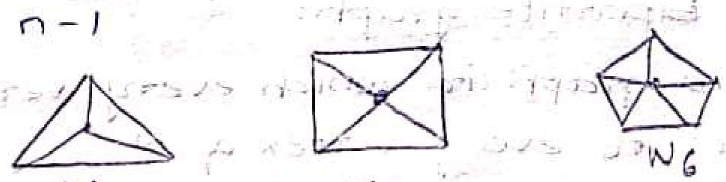
eg:



Wheel graph (W_n):

A wheel graph of order n is ~~order~~ obtained by adding a single new vertex x (the hub) to each vertex of a cycle graph of order n - 1

eg:



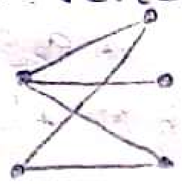
W_4 W_5 W_6

* → The no. of edges for n vertices of a graph is $2(n-1)$.

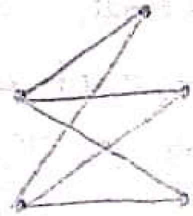
Bipartite graph ($K_{m,n}$):

A bipartite graph is a non-directed graph whose set of vertices can be participated into 2 sets M & N in such a way that each edge joins a vertex in M to a vertex in N.

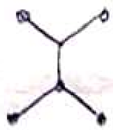
eg:



m n



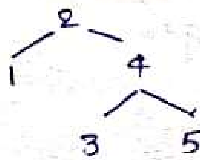
comple bipartite graph



Acyclic graph:

The graph without forming any cycle is called acyclic graph.

eg: Tree is a acyclic graph



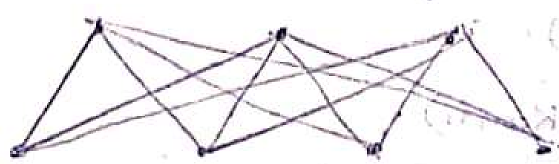
→ If n integers are present in an acyclic graph, then n-1 edges are possible.

*→ There are minimum of 2 vertices with degree 1 in an acycle graph

Complete Bipartite graph:

It is a graph in which every vertex of M is adjacent to every vertex of N.

→ If $|M| = m$, $|N| = n$, then complete Bipartite graph is denoted by $K_{m,n}$. It has 'mn' edges



Degree sequence:

If $v_1, v_2, v_3, \ldots v_n$ are vertices of a graph G, then the sequence $\{d_1, d_2, \ldots d_n\}$ where d_i = degree of v_i is called the degree sequence of G

→ Usually we order the degree sequence so that the degree sequence is monotonically decreasing.

17/9/15.

Sum of degrees Theorem:

If $V = \{v_1, v_2, \ldots v_n\}$ is the vertex set of non-directed graph G, then

$$\sum \deg(v_i) = 2|E|$$

eg.  $2+2+2+2 = 2|4|$.

$\Rightarrow 8 = 8$

Corrollories:

cor(1): sum of all the indegrees = sum of all outdegrees = total no of edges.

$$\sum \deg^+(v_i) = \sum \deg^-(v_i) = |E|$$

where (G) is is a digraph (directed graph)

cor(2): An undirected graph has an even no of vertices of odd degrees

eg: $\{1, 5, 5, 3\}$

no of vertices = 4 (even)

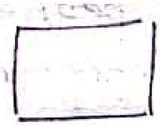
degree of each vertex is odd.

cor(3): If G is a k-regular graph then $k|V| = 2|E|$

$k \to$ degree of the vertex

$V \to$ no. of vertices

$E \to$ edges.

eg: 

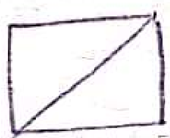
$2(4) = 2(4)$

$8 = 8$

2k

cor(4): In a non-directed graph; G if $k = \delta(G)$ and $m = \Delta(G)$ then

$$k|V| \leq 2|E| \leq m|V|$$

eg: 

In this $k = 2$ & $m = 3$.

$2|4| \leq 2|5| \leq 3|4|$

$8 \leq 10 \leq 12$.

Q. A non-directed graph contains 16 edges and all vertices are of degree 2. Find number of vertices in G.

Sol Given $|E| = 16$, $k = 2$

$k|V| = 2|E|$

$2|V| = 2(16)$

$|V| = 16$

$\therefore$ No. of vertices $= 16$.

Q. A simple non-directed graph G contains 21 edges, 3 vertices of degree 4 & the other other vertices are of degree 2. Find no. of vertices in the graph G.

Sol Given $|E| = 21$., 3 vertices of degree 4

$4(\cancel{n}\ 3) + 2(n-3) = 2|21|$

Let n be the total no. of vertices.

$$12 + 2n - 6 = 42$$

$$2n = 36$$

$$\boxed{n = 18}$$

$\therefore$ Total no. of vertices $= 18$

Q) What is the no. of vertices in a undirected connected graph with 27 edges, 6 vertices of degree 2, 3 vertices of degree 4 and remaining vertices of degree 3

sol Given $|E| = 27$

$$2(6) + 4(3) + (n-6-3)3 = 2(27)$$

$$12 + 12 + 3n - 27 = 2(27)$$

$$3n - 3 = 54$$

$$3n = 57$$

$$n = 19$$

Q) If a simple non-directed graph G contains 24 edges and all vertices are of same degree then find the no. of vertices.

sol We know that

$$k|v| \leq 2|E| \leq m|v|$$

$$k|v| \leq 2(24) \leq m|v|$$

$$k|v| \leq 48 \leq m|v|$$

$$|v| \leq \frac{48}{k}$$

If $K = 1, 2, 3, \ldots$

$$|v| = 48, 24, 16, \ldots$$

Q) What is the largest possible no. of vertices in a graph, with 35 edges and all vertices are of degree atleast 3?

Sol: $k = 3$, $E = 35$ (where k is the minimum degree)

$$k|V| \leq 2|E|$$

$$3|V| \leq 2(35)$$

$$|V| \leq \frac{70}{3} = 23.33$$

The largest possible no. of vertices with edges 35 and $k=3$ is 23.

Q Which of the following degree sequence represents a simple non-directed graph.

(a) {2, 3, 3, 4, 4, 5} (b) {2, 3, 4, 4, 5}

(c) {1, 3, 3, 4, 5, 6, 6} (d) {1, 3, 3, 3}.

→ (a) consider the degree sequence {2, 3, 3, 4, 4, 5}. We have 3 vertices with odd degree. But, by sum of degrees theorem an undirected graph should contain an even no. of vertices odd degree.

∴ Not a non-directed graph.

(b) We have a vertex with degree 5. A simple non-directed graph of order 5 cannot have a vertex with ~~order~~ degree 5.

∴ Not a non-directed graph.

(c) We have 2 vertex with ~~vertex~~ degree 6 then 2 vertices are adjacent to all vertices since order is 7

∴ A vertex with degree 1 doesn't exist

∴ Not a non-directed graph.

(d) Here order is 4 and we have 3 vertices with degree 3.

Three vertices are adjacent to all other vertices of graph. A vertex with degree 1 doesn't exist.

$\therefore$ Not a non-directed graph.

Q) Show that a degree sequence with all distinct elements cannot represent a simple non-directed graph.

Sol: Let $G = \{v_1, v_2, \ldots v_n\}$

Possible degree sequences are $\{0, 1, 2, \ldots n-1\}$ and $\{1, 2, \ldots n\}$.

In simple graph of order n, if there is a vertex with degree $n-1$ then a vertex with degree 0 does not exit.

→ A simple non-directed graph of order 'n' cannot have a vertex with degree n.

Hence, a degree sequence with all distinct elements cannot represent a simple non-directed graph.

Representation of Graphs:

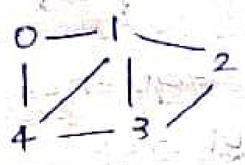
Adjacency list:

One way to represent a graph with no multiple edges is to use adjacency lists, which specify the vertices that are adjacent to each vertex of the graph.

Adjacency matrix:

The adjacency matrix of a graph is a matrix with rows & columns labeled by the vertices such that its entry in row i, column j is 1, if there is an edge between 2 vertices, else 0

eg:

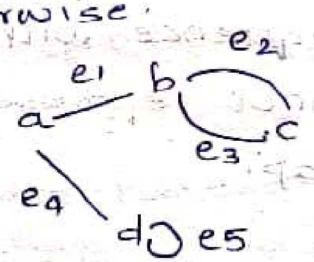


	0	1	2	3	4
0	0	1	0	0	1
1	1	0	1	1	1
2	0	1	0	1	0
3	0	1	1	0	1
4	1	1	0	1	0

Incidence matrix:

The incidence matrix of a graph is a matrix labeled by vertices & columns labeled by edges, so that entry for row v column e is 1 if e is incident on v, & 0 otherwise.

eg:



	e_1	e_2	e_3	e_4	e_5
a	1	0	0	1	0
b	1	1	1	0	0
c	0	1	1	0	0
d	0	0	0	1	1

Path Matrix:

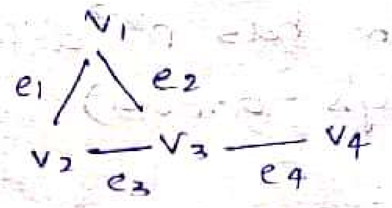
A path matrix is generally defined for a specific pair of vertices then the path matrix is denoted as $P(u,v) = P_{ij}$

$P_{ij} = 1$ if the jth edges lies in the ith path

$= 0$ otherwise.

Scanned with CamScanner

eg:



$$P(V_1, V_4) = \begin{array}{c|cccc} & e_1 & e_2 & e_3 & e_4 \\ \hline 1 & 0 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 & 1 \end{array}$$

Paths:

1: $\{e_2, e_4\}$

2: $\{e_1, e_3, e_4\}$

fig: path matrix b/w v_1, v_4 of Graph.

Spanning trees:

Tree:

A connected graph with no cycles is called tree

→ A tree with n vertices has (n-1) edges

→ A tree with n vertices (n>1) has atleast 2 vertices of degree 1.

→ A subgraph H of a graph G is called a spanning tree of G if

(i) H is a tree.

(ii) H contains all vertices of G.

Note:

In general, if G is a connected graph with n vertices of m edges, a spanning tree of G must have (n-1) edges.

The no. of edges that must be removed before a spanning tree is obtained must be m-(n-1). This number is called "circuit rank of G"

→ A non-directed graph G is connected if and only if G contains a spanning tree.

→ A complete graph, K_n has n^{n-2} different spanning trees (Caley's formula)

Minimal Spanning Tree:

Let G be a connected graph where each edge of G is labeled with a non-ve cost. A spanning tree T where the total cost c(T) is minimum is called a minimal spanning tree.

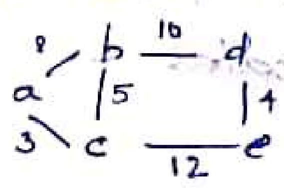
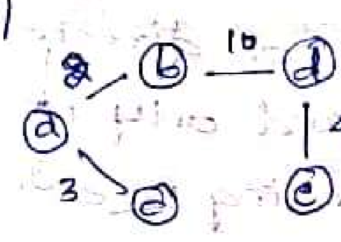
Kruskal's Algorithm: (For finding minimal spanning tree of a connected weighted graph

Input: A connected graph G with non -ve (non-negative) values assigned to each edge

Output: A minimal spanning tree for G

Method:

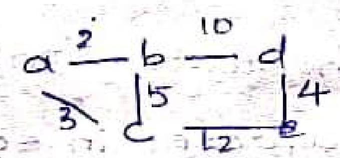
1. Select any edge of minimal value that is not a loop. This is the first edge of T (if there is more than one edge of minimal values, arbitary choose one of these edges
2. Select any remaining edges of G of having minimal value that doesn't form a circuit with the edges already included in T.
3. Continue step-2 until T contain (n-1) edges when $n = |V(G)|$

eg:  ⇒ 

Prim's Algorithm: (for finding a minimal spanning tree) (connectivity should be maintained)

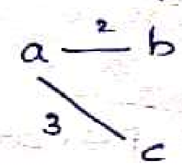
1. Let G be the connected graph with non-negative values assigned to each edges first let T be the tree consisting of any vertex v_1 of G.
2. Among all the edges not in T, that are incident on a vertex in T and do not form a circuit when added to T, select one of minimal cost & add it to T.
3. The process terminates after we have added $(n-1)$ edges where $n = |V(G)|$

eg.

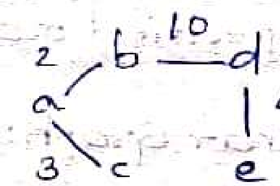


Prim's algorithm:

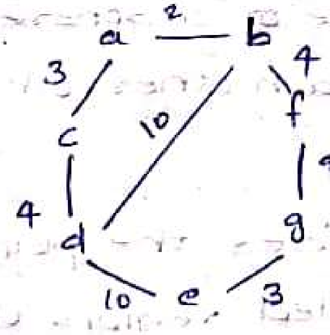
(i)

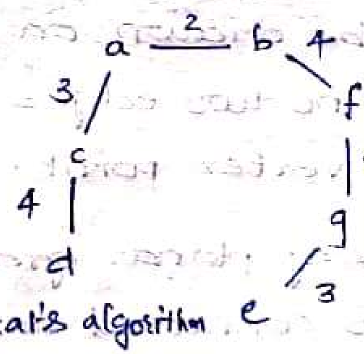


(ii)



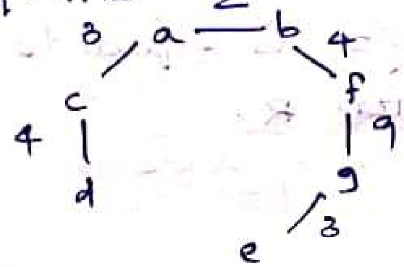
eg.



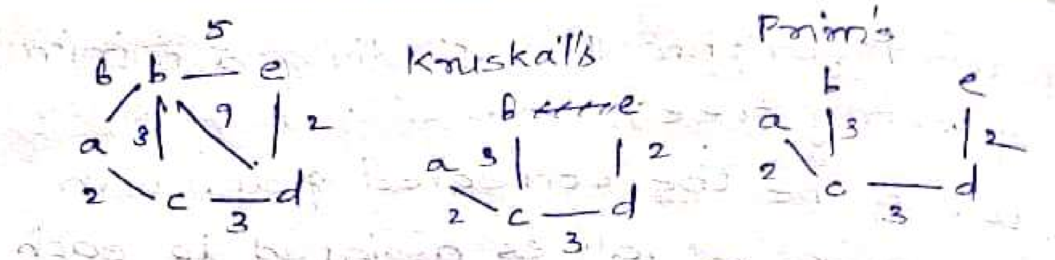


Kruskal's algorithm

prim's



eg:



Construction of Spanning Tree:

1. DFS (Depth First search)
   - → It uses stack.
   - → The principle of the algorithm is quite simple to go toward (in depth) while there is such possibility, otherwise to backtrack.
2. BFS (Breadth First Search)
   - → It uses queue.
   - → Bredth First search starts with given node
   - → Then visits nodes adjacent in some specified order.

* Planar Graph:

A graph G is said to be planar if it can be drawn on a plane or a sphere, so that no two edges cross each other, other than vertex point.

* Every planar graph divides the plane into connected areas called regions of the planar graph.
* Degree of interior region is equal to no of edges closing the region R.

* Degree of exterior region is equal to no. of edges exposed to the region 'D'.

r_1, r_2, r_3 - interior regions

r_4 - exterior region

Degree of r_1 : 3

Degree of r_2 = 3

Degree of r_3 = 3

Degree of r_4 = 3.

Properties of planar graph:

→ For any planar graph with 'n' - vertices, sum of degrees of all vertices is equal to 2(edges)

$$\sum_{i=1}^{n} dg(v_i) = 2(E)$$

→ Sum of degrees of regions:

For any planar graph with 'n' regions

$$\underbrace{\sum_{i=1}^{n} deg(r_i)}_{\text{sum of degrees of all regions}} = 2(E)$$

→ In the planar graph, if the degree in each region is 'k' then

$$k|R| = 2|E|$$

R - no. of regions

→ In the planar graph, if degree of each region is atleast 'k'.

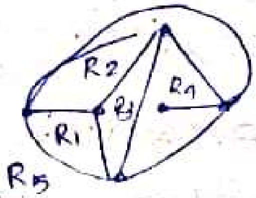
$$k|R| \leq 2|E|$$

→ In a planar graph if the degree of each region is atmost 'k'

$$k|R| \geq 2|E|$$

→ For simple planar graph, (graph free from loops and parallel edges.

$3|R| \le 2|E|$



$d(R_2) = 4$
$d(R_1) = 3$
$d(R_3) = 3$
$d(R_4) = 5$
$d(R_5) = 3$

(1) $3+3+4+4+3+1 = 2(9)$

$18 = 18$

(2) $4+3+3+5+3 = 2(9)$

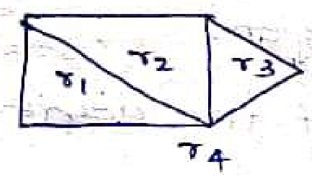
$18 = 18$

Euler Formula:

According to Euler formula, let G be the connected graph, then

$|V| + |R| = |E| + 2$

eg:



$5+4 = 7+2$

$9 = 9$

Hence proved.

here, $|V|$ = no. of vertices

$|R|$ = no. of regions

$|E|$ = no. of edges.

Polyhedral graph:

A simple connected planar graph in which degree of every vertex is greater than or equal to 3 is called polyhedral graph.

→ For polyhedral graph, the following properties hold good.

(i) $3|V| \leq 2|E|$

(ii) $3|R| \leq 2|E|$

(Q) Let G be the connected planar graph with 25 vertices & 60 edges. Find the no. of regions.

$\begin{array}{r} 62 \\ 25 \\ \hline 37 \end{array}$

Sol We know that,

$|V| + |R| = |E| + 2$

$25 + |R| = 60 + 2$

$|R| = 37$

(Q) Let G be the planar graph with its 10 vertices; 15 edges, 3 components. Find no. of regions.

(Q) Let G be the connected planar graph with 20 vertices, each of its degree is 3. Find no. of regions in planar graph.

(Q) Let G be the connected planar graph with 35 regions each of its degree is 6. Find the no. of vertices.

3 Sol $3|20| = 2|E|$

$60 = 2|E|$

$|E| = 30$

$20 + R = 30 + 2$

$R = 12$

4 Sol : $R = 35$, $k = 6$

$6|35| = 2|E|$

$|E| = 105$

$105 + 2 = 35 + |V|$

$|V| = 72$

23/9/19

Coloring:

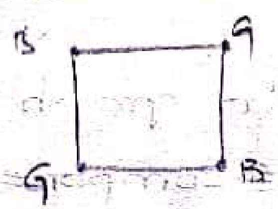
Vertex Coloring:

Assignment of colors to the vertices of the graph G so that no two vertices of G have same colour is called vertex coloring.

Chromatic Number of the Graph:

The minimum no. of colours needed for vertex coloaring of a graph G is called Chromatic Number of the Graph which is denoted as $\chi(G)$.

eg:



Chromatic number of the graph = 2

* → Chromatic Number of Null Graph is one. (since null graph has vertices but not edges)
→ The chromatic number of non-empty Graph G is greater than or equal "two".
→ A graph G is n-colorable if there exist a vertex coloring which uses atmost n-colour.

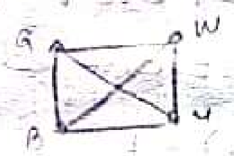
~~X~~ ~~G~~ Chromatic number of graph is less than or equal to n

Four colour Theorem:

Every planar graph G is four colourable i.e the chromatic number of any graph is less than or equal to four.

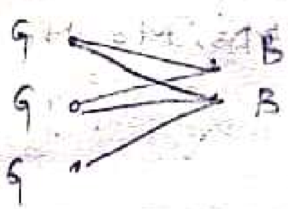
Q. Chromatic number of the complete vertex graph K_n.

(a) 4 (b) n-1 (c) n (d) None.

eg: 

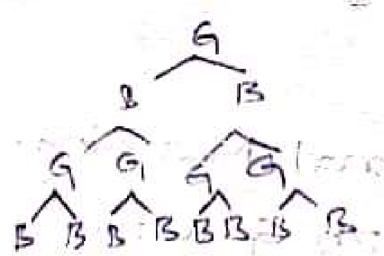
Q. The chromatic number of bi-parted graph.

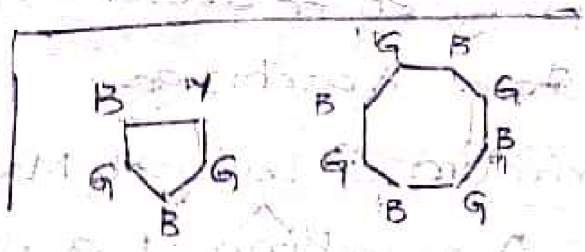
(a) $\lfloor \frac{n}{2} \rfloor$ (b) $\lceil \frac{n}{2} \rceil$ (c) 4 (d) 2.

eg. 

Q. Chromatic number of tree with n vertices where n is greater than or equal to 2

(a) $\lfloor \frac{n}{2} \rfloor$ (b) $\lceil \frac{n}{2} \rceil$ (c) 4 (d) 2.

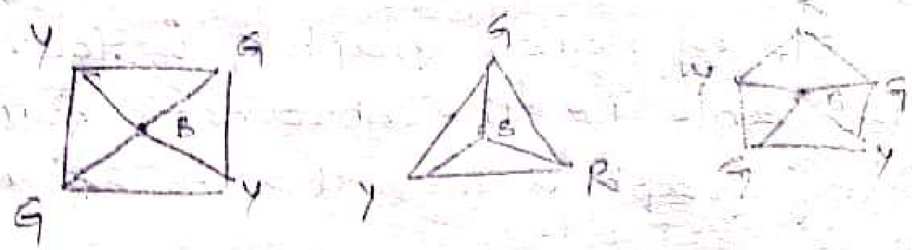




Q. Chromatic number of cyclic graph For odd - 3, even - 2.

Scanned with CamScanner

(5) Chromatic number of wheel graph.

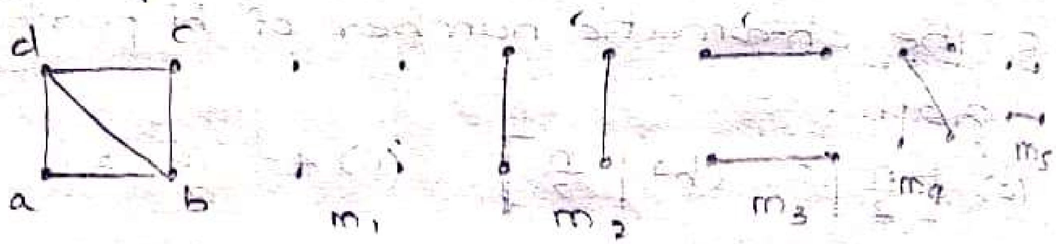


odd - 4   even - 3

24/9/19

Matchings:

Let $G = (V, E)$ be a graph, a sub-graph of G) is called Matching. If every vertex of G is incident with atmost one edge in m i.e degree of vertex. $\deg(v) \leq 1$



For a graph M; M_1, M_2, M_3, M_4, M_5 are possible matchings.

Maximal Matching:

A matching of a graph is said to be maximal if no other edges of the graph can be added to the matching for the graph given in above m_2, m_3 & m_4 are maximal matchings.

Maximum or Largest Maximal Matching:

A matching of a graph G with maximum no. of edges is called largest

Maximal matching or Maximum matching

→ The no. of edges Maximum matching no. of a graph.

→ For the graph given in the above m_2 & m_3 are maximum graph.

Matching no. of graph is 2.

Perfect Matching:

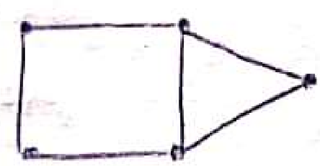
→ A matching of a graph is said to be perfect if every vertex of the graph is match i.e the degree of vertex is 1.

$deg(v) = 1 \ \forall \ v \in G$.

→ m_2 and m_3 are the perfect matching

→ A graph G has a perfect matching then no. of vertices in the graph is even.

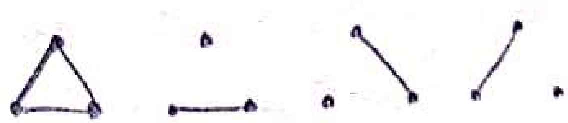
→ The converse of the statement need not be true.

(i) 

odd no. of vertices it is not possible for perfect matching.

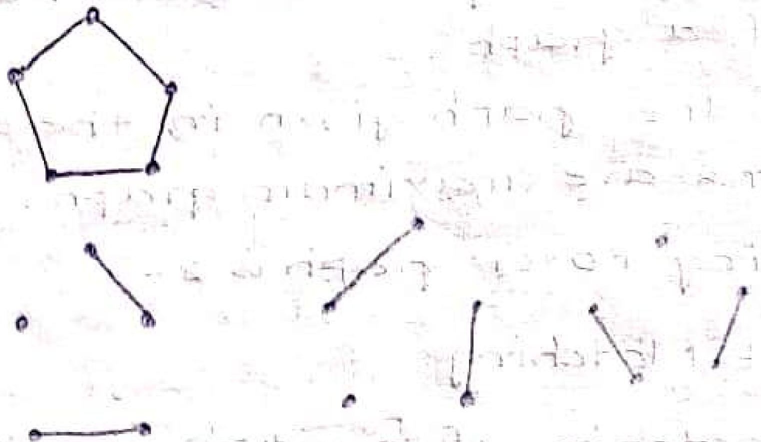
(ii) 

Eventhough for even no. of vertices it is not possible to have perfect matching

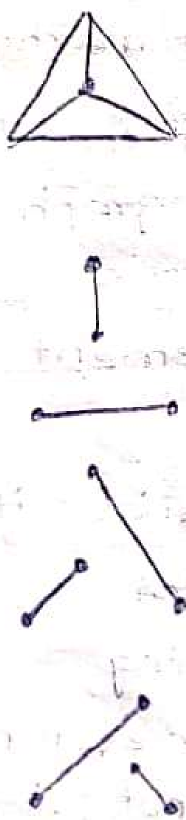
(iii) 

⇒ ①

(iv)



Cyclic Graph : $\lceil \frac{n}{2} \rceil$



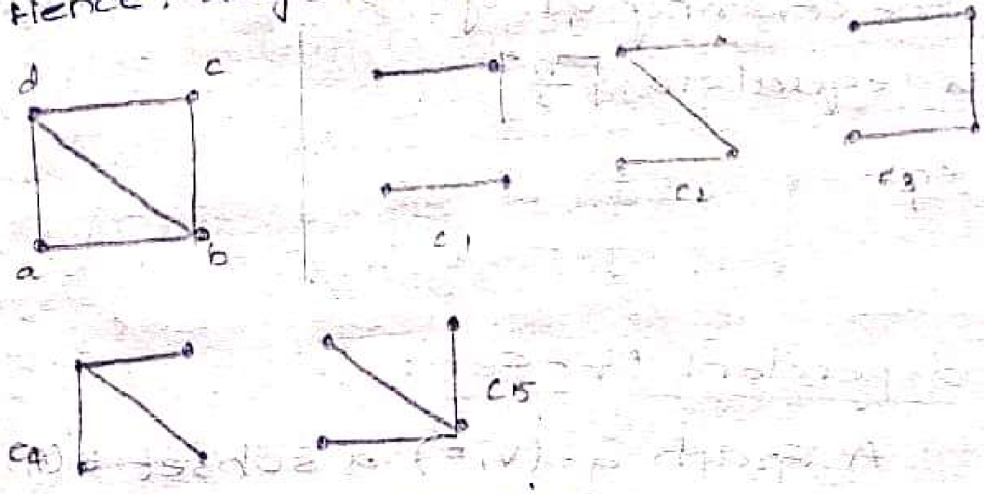
Coverings:

Let $G = (V, E)$ be a graph. a subsequent $C(G)$ is called line covering of 'G'.

→ If every vertex of G with atleast one edge in C.

Hence, $\deg(v) \geq 1 \quad \forall\, v \in G$.

eg.



Minimal line covering:

A line covering of a graph is said to be minimal, if no edge can be deleted it from the covering without destroying the ability to cover the graph.

From above eg, c1, c4, c5 are the minimal line covering.

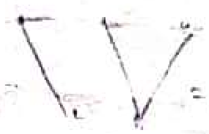
Minimum line covering or smallest minimal line covering.

A line covering of a graph with minimum no. of edges is called smallest minimal line covering or minimum line covering.

→ No. of edges in a smallest line covering is called "line covering number of the graph" (α_1)

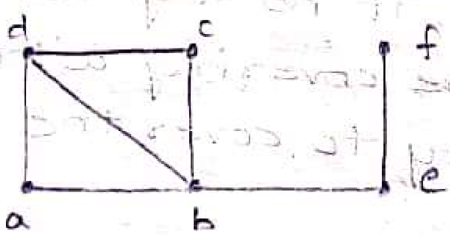
From above eg, minimum line covering is c_1 and line covering number is 2.

*→ line covering of graph with n vertices is equal to $\lceil \frac{n}{2} \rceil$

eg:  $\lceil \frac{6}{2} \rceil = 3$  $= 3$

Independent line set:

A Graph $G = (V, E)$ a subset $E'(G)$ is said to be an independent line set if no two edges of E' are adjacent.

eg: 

$L_1 = \{(a,b), (c,d) (e,f)\}$
$L_2 = \{(a,d), (b,c), (e,f)\}$
$L_3 = \{(a,d) (e,f)\}$
$L_4 = \{(a,d)\}$, $L_5 = \{(b,c)\}$, $L_6 = \{(a,b)\}$,
$L_7 = \{(b,d)\}$, $L_8 = \{(c,d)\}$, $L_9 = \{(b,e)\}$,
$L_{10} = \{(b,c), (e,f)\}$ etc.

→ Maximal Independent Line Set:

A independent line set no other edges of the graph can be added is called maximal independent line set.

· Largest Maximal Line Set or Maximum Independent Line set:

An independent line set with maximum no. of edges is called maximum independent line set

→ The no. of edges in maximum independent line set is called "line independent number". Line independent number is denoted by "β_1"

· From above eg, L_1, L_2 are maximal independent line set. And maximum line independent number is 3.

→ sum of Line independent number and line covering no. of graph is total no. of vertices.

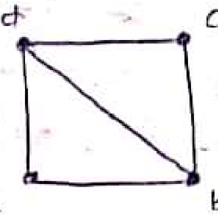
$$\boxed{\alpha_1 + \beta_1 = \text{no. of vertices}}$$

25/9/19

Vertex Covering:

Let $G = (V, E)$ be a graph with n vertices a subset ~~k(v)~~ k of V is called vertex ~~covin~~ covering if every edge of the graph is incident with a vertex in k.

eg:



$k_1 = \{a, b, c, d\}$

$k_2 = \{b, d\}$

$k_3 = \{a, b, c\}$

$k_4 = \{a, c\}$ × not a vertex covering

Minimal Vertex Covering:

A vertex covering is said to be minimal if no vertex can be deleted from covering without destroying in its ability to cover the edges of the graph.

For the graph given in the above eg., k_2 and k_3 are minimal vertex covering. k_1 is not minimal vertex covering because we can delete 'd'.

Minimum Vertex Covering:

A vertex covering with minimum no of vertices is called minimum vertex covering. *

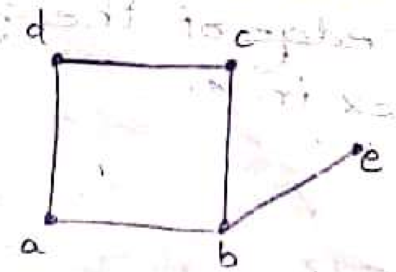
For the graph given in the above example, k_2 is the minimum vertex covering.

* Vertex covering number $(\alpha_2) = 2$

Independent Vertex Set:

A Graph, $G = (V, E)$; a subset S of V is said to be independent if no two vertices in S are adjacent.

eg:



$k_1 = \{a, c, e\}$ $k_5 = \{b, e\}$
$k_2 = \{b, d\}$ $k_6 = \{c, e\}$
$k_3 = \{a, c\}$ $k_7 = \{a\}$
$k_4 = \{a, e\}$ $k_8 = \{b\}$
$k_9 = \{e\}$ $k_{10} = \{c\}$ $k_{11} \{d\}$
$k_{12} = \{d, e\}$

Maximal Independent Vertex Set.

An independent set of vertices is maximal if no other vertices of the graph can be added to the set.

From above example, ~~K₀~~ K_1, K_2, K_{12} are the maximal independent vertex set.

Maximum Independent Vertex Set:

An independent set with maximum no. of vertices is called maximum independent vertex set.

From above example, K_1 is maximum independent vertex set.

Maximum Independent Vertex Number:

$(\beta_2) = 3$.

* $\alpha_2 + \beta_2$ = no. of vertices in the graph (n)

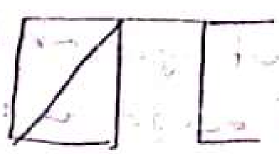
26/9/19

Connectivity:

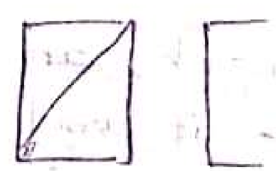
A graph G is said to be connected graph if there exist a path between every pair of vertices in G.

→ A graph which is not connected will have 2 or more connected components.

eg:



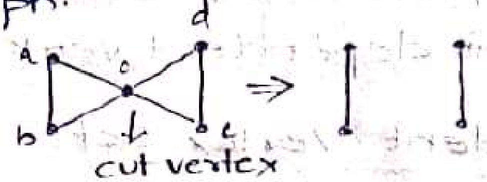
connected graph

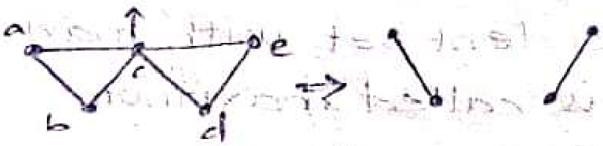


not connected graph.

Cut Vertex:

Let G be a connected graph, a vertex 'V' ∈ G is called a cut-vertex of G if G-{V} result in a dis-connected graph.

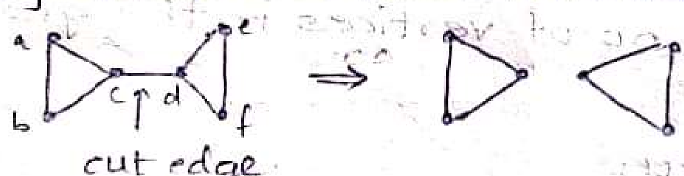
eg:


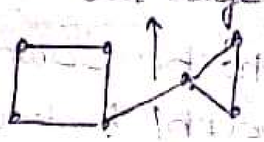
eg:


Here 'c' is cut vertex.

Cut Edge:

Let G be a connected graph, an edge 'E ∈ G is called a cut edge of G if G-{E} results a disconnected graph.

eg:


eg:


Note:

→ If ~~block~~ graph has a cut edge then atleast one vertex of the cut edge is a cut vertex.

→ In a graph, an edge of graph is a cut edge if and only if edge is not part any cycle in the graph.

Scanned with CamScanner

Vertex Connectivity:

Let G be a connected graph, then the minimum no. of vertices whose removal makes the graph disconnected is called vertex connectivity of the graph. It is denoted by G = K(G)

For the above eg, K(G) = 1 i.e either c or d

Edge Connectivity of the Graph:

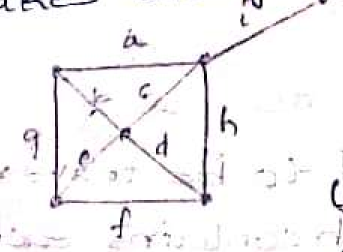
Let G be a connected graph, then the minimum no. of edges whose removal makes the graph disconnected is called edge connectivity of the graph. It is denoted by λ(G).

For the above eg, λ(G) = 1

Cut set:

Let G be a connected graph, a subset of edges E' of G is called a cut set of G if removal of all the edges of the E' from G makes G disconnected and removal of no proper subset of E' from the graph G makes the graph disconnected.

eg:



Which of the following is a cut set?

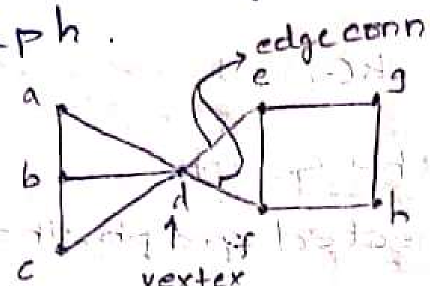
(a) {a, b, g} (b) {a, b, e, g}

(c) {a, c, h, d} (d) {b, d, h, a}

(e) {i}

{a, b, g, e} is not a proper set.
since {a, b, g} itself forms a disconnected graph.
Similarly {a, c, h, d} is not a cutset.
By removing {a, c, h} we get disconnected graph.

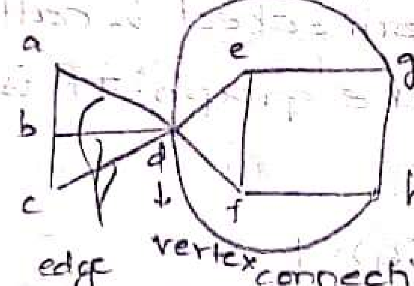
eg:



$\kappa(G) = 1$

$\lambda(G) = 2$

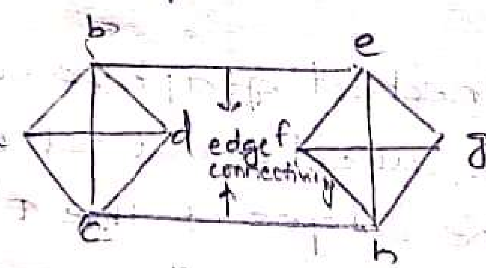
eg:



$\kappa(G) = 1$

$\lambda(G) = 3$.

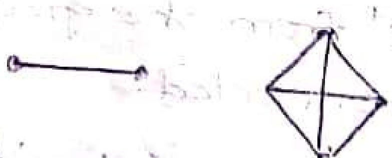
eg:



$\lambda(G) = 2$

$\kappa(G) = 2$

b, c - vertex connectivity
or
e, h



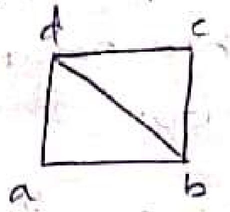
30/9/19

Traversability:

A graph G is said to be traversal if there exist a path which contains each edge of the graph exactly once and each vertex of the graph G atleast once.

→ This traversal is also called as "Euler traversal" and such path is called "Euler path".

eg:



Euler path:
d-a-b-d-c-b

Theorem:

A connected graph is traversal if and only if no·of vertices with odd degree is exactly two or zero.

case(i): A connected graph G, if no·of vertices with odd degree two, then only Euler path exist but not Euler circuit.

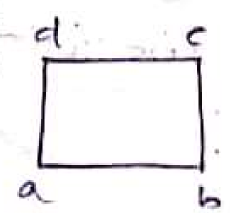
→ The Euler path begins a vertex of odd degree and ends with other vertex of odd degree.

case(ii): If the no·of vertices with odd degree is zero then Euler circuit exist in the graph

Euler Circuit:

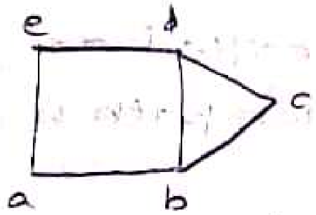
A Euler path in which the starting vertex is same as ending vertex is called Euler circuit.

eg:



Euler circuit:
a-b-c-d-a.

eg:



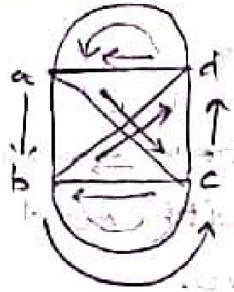
Only Euler path is possible.

~~a-b~~

d-e-a-b-d-c-b.

Euler circuit is not possible because odd degree vertices are two.

eg:



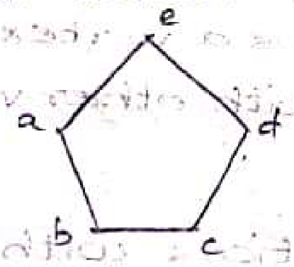
Both Euler path & Euler circuit is possible. Because odd degree vertices = 0.

d-a-c-b-d-a-b-c-d

Hamiltonian Graph:

A connected graph G is said to be Hamiltonian if there exist a cycle which contains each vertex of the graph exactly once.

eg:



Hamiltonian path:

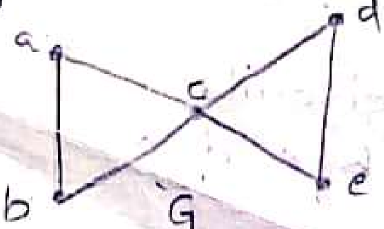
a-b-c-d-e-a.

→ The starting vertex can be repeated at the last

Hamiltonian Cycle:

It contains each vertex of the graph exactly once but may skip some edges is called Hamiltonian cycle.

eg:



For the following graph which of the following is false.

(a) 'G' Ko is traversable (Euler path)

(b) 'G' has Euler circuit.

(c) 'G' has Hamiltonian path

(d) 'G' has Hamiltonian cycle.

sol: Euler path & Euler circuit exist because odd degree of vertices is zero.

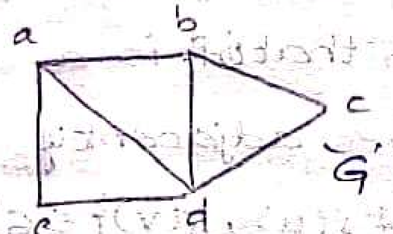
Hamiltonian path: a-b-c-d-e-c-a

vertex should not repeat.

Hamiltonian cycle: a-b-c-d-e

Since we can skip the edge ce.

(Q) Which of the



The Graph G has Euler path.

d-e-a-b-c-d-a.

The graph G donot have Euler circuit because odd vertices is 2.

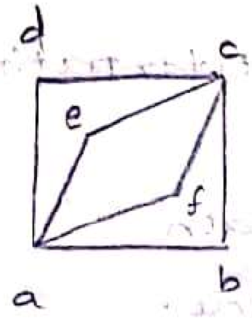
Hamiltonian path - a-b-c-d-e-a

vertices are repeated.

Hamiltonian cycle: d-e-a-b-c-d.

∴ Hamiltonian cycle, Euler path, Hamiltonian path is possible but not Euler circuit.

Scanned with CamScanner

(9) 

Euler path & Euler circuit
c-d-a-e-c-f-a-b-c.

Hamiltonian path:
d- c - b - a - e - c - f - c - d.

Vertices cannot be repeated

Hamiltonian cycle : d-c-b-a-e-c-f

∴ Graph does not contain Hamiltonian path and cycle.

Isomorphic Graph:

Two graphs G and G' are said to be isomorphic if there exist a function 'f' : $V(G) \longrightarrow V(G')$ such that (i) f is a bijection. (ii) f preserves adjacency

i.e $\{u, v\} \in G$ then $\{f(u), f(v)\} \in G'$

Therefore, G is isomorphic to G'.

Note:

If G is isomorphic to G' i.e $G \cong G'$ the following functions must hold good.

(i) $|V(G)| = |V(G')|$

(no. of vertices in G should be equal to no. of vertices in G')

(ii) (No. of edges in G equals to no. of edges in G')

$|E(G)| = |E(G')|$

(iii) The degree of sequence G and G' are same.

(iv) If there exist a cycle $\{v_1 - v_2 \cdots v_k - v_1\}$ in the G then there should be a similar cycle in G' $\{\delta(v_1) - \delta(v_2) \cdots \delta(v_k) - \delta(v_1)\}$

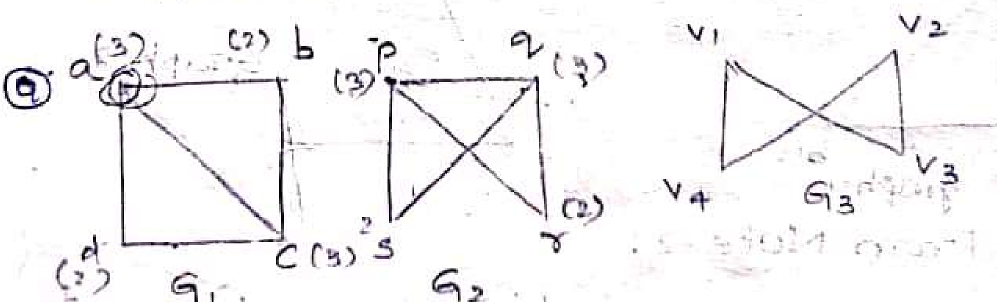
* The above conditions are necessary for two graphs to be isomorphic but not sufficient to prove that they are isomorphic.

* Note:

1. $G \cong G' \Leftrightarrow \bar{G} \cong \bar{G'}$

(If G is isomorphic to G' then G complement is isomorphic to G' complement.

2. G is ~~com.~~ isomorphic to G' if and only if the corresponding subgraph (obtained by deleting a vertex in G & its image in G') of G and G' are isomorphic.

Q 

$|V(G_1)| = 4$ $|V(G_2)| = 4$ $|V(G_3)| = 4$.

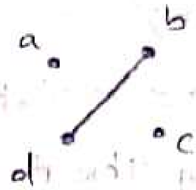
$|E(G_1)| = 5$ $|E(G_3)| = 5$ $|E(G_3)| = 4$.

Degree sequence $(G_1) = \{2, 2, 3, 3\}$.

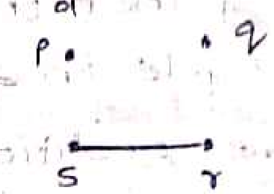
Degree of sequence $(G_2) = \{2, 2, 3, 3\}$.

acb is a cycle, prq is a cycle.

$\therefore$ Complement of G_1



complement of G_2



$|V(\bar{G_1})| = 4$ $\quad$ $|V(\bar{G_2})| = 4$

$|E(\bar{G_1})| = 1$ $\quad$ $|E(\bar{G_2})| = 1$

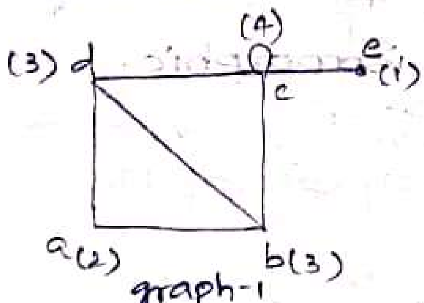
$\{0,1,0,1\}$ $\quad$ $\{0,0,1,1\}$

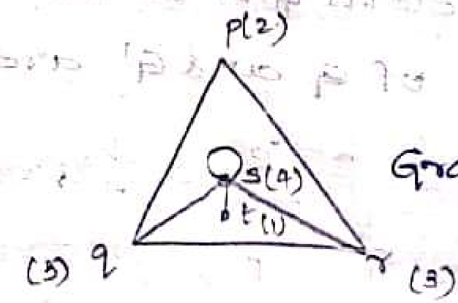
b, d. $\quad$ s, r.

$\therefore$ $\bar{G_1}$ and $\bar{G_2}$ are isomorphic.

1/10/19.

Q) Find whether the following graphs are isomorphic or not.





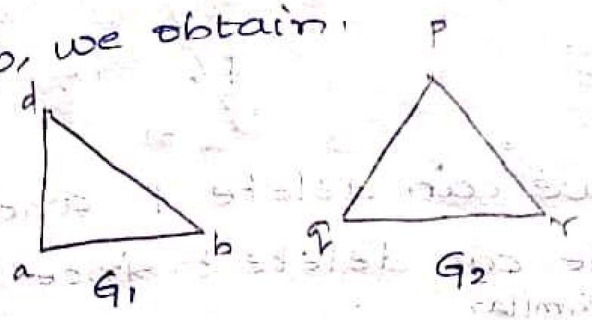
Graph-2

Sol. From Note-2:

vertex c and s of Graph-1 and Graph-2 respectively has similar degree & adjacency. i.e neighbours are same.

So, we can delete c from graph 1 and s from graph 2.

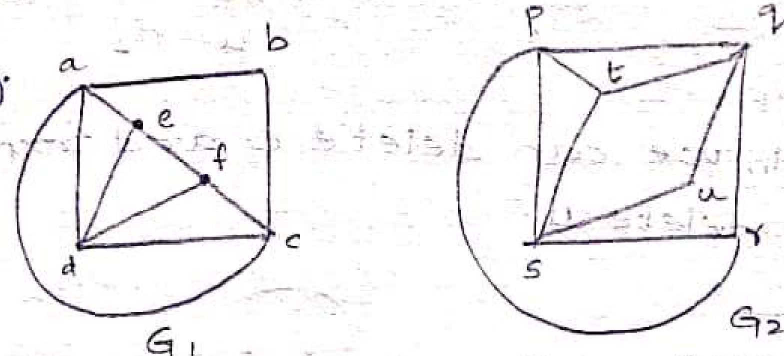
So, we obtain,



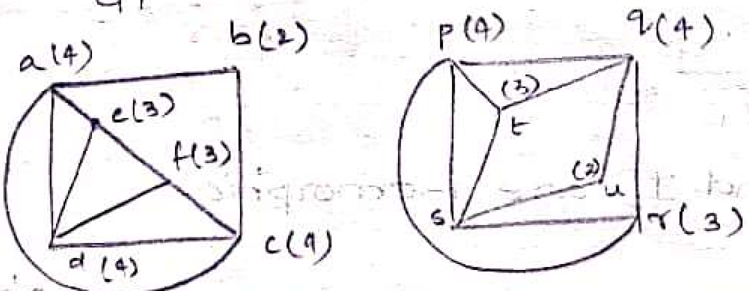
Graph G_1 and G_2 are isomorphic.

Therefore, Graph-1 and Graph-2 are also isomorphic.

Q.



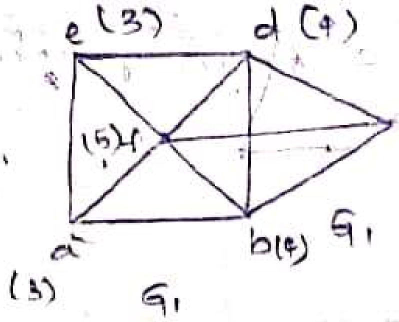
Sol:

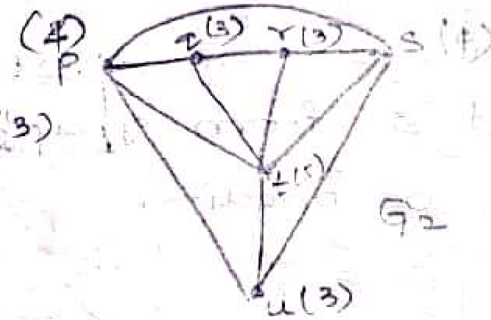


G_1 and G_2 are not isomorphic because in G_1, the two vertices of degree 3 'e' and 'f' are adjacent whereas in G_2 the two vertices of degree 3 i.e 't' and 'r' are not adjacent.

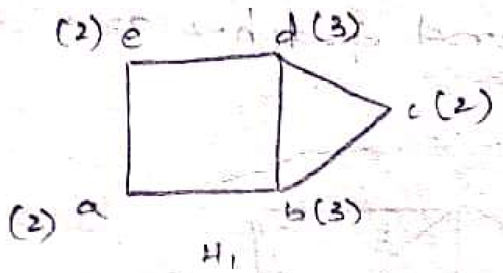
$\therefore$ G_1 & G_2 are not isomorphic.

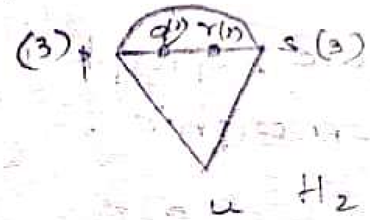
(5)



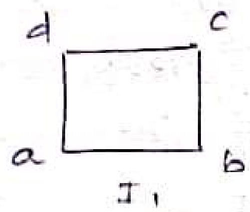


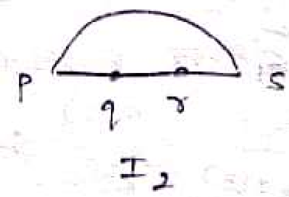
Sol: From G_1, we can delete f and from G_2, we can delete t since they have ~~same~~ similar neighbours.





Ily, From H_1, we can delete c and from H_2, we can delete u.





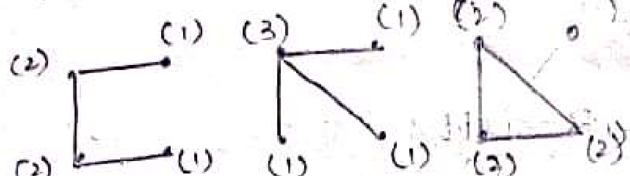
$\therefore$ I_1 and I_2 are isomorphic.

(6) How many simple non-isomorphic graphs are possible with 3 vertices.

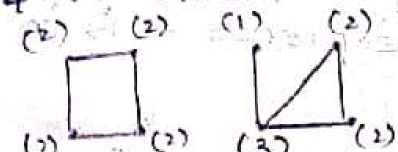
(a) 3 (b) 4 (c) 6 (d) 8.

Q) How many simple non-isomorphic graphs are possible with 4 vertices & 2-edges. — 2

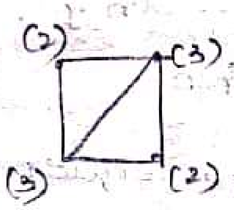
4 vertices 3-edges. — 3



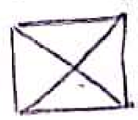
4-vertices 4-edges. — 2



4-vertices 5-edges — 1



4-vertices 6-edges — 1



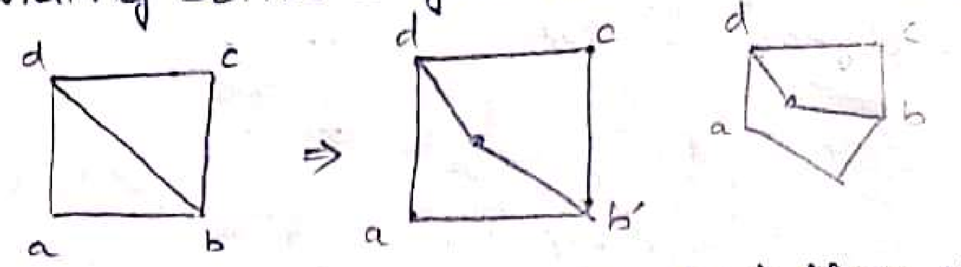
Kurfowskis Theorem:

A graph G is a non-planar graph if and only if G has a sub-graph K_5 (or) $K_{3,3}$ (homo-morphic to $K_{3,3}$).

Note:

→ Two graphs G_1 and G_2 are said to be homo-morphic, each of the graph can be obtained from the other graph by dividing some edges with more vertices.

eg:



→ If two graphs are isomorphic then they are homomorphic.

3/10/19 Havel-Hakmi- Result:

Consider the following degree sequences s1 and s2 in which the sequence s1 is in descending order.

$s1 = \{s1, t_1, t_2, \dots t_s, d_1, d_2, \dots d_n\}$.

$s2 = \{t_1 - 1, t_2 - 1, \dots t_s - 1, d_1, d_2, \dots d_n\}$.

s1 graphic $\Leftrightarrow$ s2 is graphic.

Q) Which of the following degree sequences represent a simple non-directed graph.

(i) $\{6, 6, 6, 6, 4, 3, 3, 0\}$.

(ii) $\{6, 5, 5, 4, 3, 3, 2, 2, 2\}$

Sol: (i) $s_1 = \{6, 6, 6, 6, 4, 3, 3, 0\}$.

Arrange the sequence in descending order. Remove first one and subtract one from next.

$s_2 = \{5, 5, 5, 3, 2, 2, 0\}$.

~~So~~ Again we need to repeat this process since we can't decide.

$s_3 = \{4, 4, 2, 1, 1, 0\}$.

Again repeat the same process

$s_4 = \{3, 1, \cancel{0}, 0, 0\}$.

s4 is not possible Because a ver

(ii) $s_1 = \{6, 5, 5, 4, 3, 3, 2, 2, 2\}$.

$s_2 = \{4, 4, 3, 2, 2, 1, 2, 2\}$.

~~$s_3 = \{3, 2, 1, 1, 1, 2, 2\}$~~

$s_4 = \{1, 0, 0, 1, 2, 2\}$.

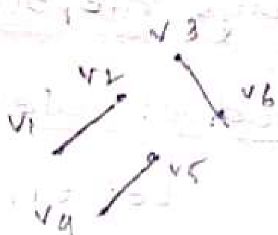
~~$s_5 = \{0$~~ Arranging in descending order

$s_2 = \{4, 4, 3, 2, 2, 2, 2, 1\}$.

$s_3 = \{3, 2, 1, 1, 2, 2, 1\}$

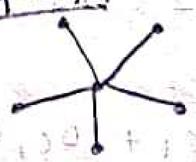
$s_3 = \{3, 2, 2, 2, 1, 1, 1\}$.

$s_4 = \{1, 1, 1, 1, 1, 1\}$. simple non directed graph is not possible.



s4 is not possible. Because all the 6 vertices can have degree 1.

Stargraph:



A vertex is connected to all other vertices.