(a) Gauss Siedel

$$x_i^{(k+1)} = b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}$$

$$x_1^{(2)} = \frac{b_1 - \sum_{j=2,3} a_{ij} x_j^{(1)}}{a_{11}}$$

$$x_1^{(2)} = \frac{41 - 0}{0.8} = 51.25$$

$$x_2^{(2)} = \frac{b_2 - a_{21} x_1^{(2)} - a_{13} x_3^{(1)}}{a_{22}}$$

$$x_2^{(2)} = \frac{25 - (-0.4)(51.25)}{0.8} - 0 = 56.875$$

$$x_3^{(2)} = \frac{105 - a_{31} x_1^{(2)} - a_{32} x_2^{(2)}}{a_{33}}$$

$$x_3^{(2)} = \frac{105 - (-0.4)(56.875)}{0.8} = 159.68$$

$$\boxed{\begin{aligned} x_1 &= 51.25 \\ x_2 &= 56.875 \\ x_3 &= 159.68 \end{aligned}}$$

SOR $(\omega = 1.2)$

$$x_i^{(k)} = (1-\omega) X_i^{(k-1)} + \omega X_{i,GS}^{k}$$

$$x_1^{(2)} = (1-\omega) x_1^{(1)} + \omega X_{1,GS}^{(2)}$$

$$x_1^{(2)} = (1-1.2)(0) + (1.2) 51.25$$

$$x_1 = 61.5$$

$$x_2^{(2)} = (1-\omega) x_2^{(1)} + \omega\left(\frac{b_2 - a_{21}x_1^{(2)} - a_{23}x_3^{(1)}}{a_{22}}\right)$$

$$x_2^{(2)} = 1.2\left(\frac{25 - (-0.4)(61.5) - (-0.4)0}{0.8}\right)$$

$$x_2^{(2)} = 74.4$$

$$x_3^{(2)} = (1-\omega) x_2^{(1)} + \omega\left(\frac{b_3 - a_{31}x_1 - a_{32}x_2}{a_{33}}\right)$$

$$x_3^{(2)} = (1-\omega)0 + 1.2\left(\frac{105 - (0) - (-0.4)(74.4)}{0.8}\right)$$

$$x_3^{(2)} = 202.14$$

$$\boxed{\begin{aligned} x_1^{(2)} &= 61.5 \\ x_2^{(2)} &= 74.4 \\ x_3^{(2)} &= 202.14 \end{aligned}}$$

(b) Gauss Siedel

$$\begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & & \vdots \\ \vdots & & \ddots & 0 \\ a_{n1} & \cdots & & a_{nn} \end{bmatrix} \begin{bmatrix} x_1^{(k+1)} \\ x_2^{(k+1)} \\ \vdots \\ x_n^{(k+1)} \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} - \begin{bmatrix} 0 & a_{12} & \cdots & a_{1n} \\ 0 & 0 & & \vdots \\ \vdots & & \ddots & a_{n-1\,n} \\ 0 & \cdots & & 0 \end{bmatrix} \begin{bmatrix} x_1^{(k)} \\ x_2^{(k)} \\ \vdots \\ x_n^{(k)} \end{bmatrix}$$

$$\begin{bmatrix} 0.8 & 0 & 0 \\ -0.4 & 0.8 & 0 \\ 0 & -0.4 & 0.8 \end{bmatrix} X^{(2)} = \begin{bmatrix} 41 \\ 25 \\ 105 \end{bmatrix} - \begin{bmatrix} 0 & -0.4 & 0 \\ 0 & 0 & -0.4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(2)} \\ x_2^{(2)} \\ x_3^{(2)} \end{bmatrix} = (L+U)^{-1} B = \begin{bmatrix} 51.25 \\ 56.875 \\ 159.68 \end{bmatrix}$$

$$(L+U)(X^{(3)}) = B - U X^{(2)}$$

$$\begin{bmatrix} 0.8 & 0 & 0 \\ -0.4 & 0.8 & 0 \\ 0 & -0.4 & 0.8 \end{bmatrix} X^{(3)} = \begin{bmatrix} 41 \\ 25 \\ 105 \end{bmatrix} - \begin{bmatrix} 0 & -0.4 & 0 \\ 0 & 0 & -0.4 \\ 0 & 0 & 0 \end{bmatrix} X^{(2)}$$

$$\begin{bmatrix} x_1^{(3)} \\ x_2^{(3)} \\ x_3^{(3)} \end{bmatrix} = (L+U)^{-1} \begin{bmatrix} 63.75 \\ 88.75 \\ 105 \end{bmatrix} = \begin{bmatrix} 79.6875 \\ 150.9375 \\ 206.7188 \end{bmatrix}$$

SOR

$$(D + \omega L) X^{(k)} = ((1-\omega) D - \omega U) X^{(k-1)} + \omega b$$

$$D = \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.8 \end{bmatrix} \quad L = \begin{bmatrix} 0 & 0 & 0 \\ -0.4 & 0 & 0 \\ 0 & -0.4 & 0 \end{bmatrix} \quad \omega = 1.2$$

$$U = \begin{bmatrix} 0 & -0.4 & 0 \\ 0 & 0 & -0.4 \\ 0 & 0 & 0 \end{bmatrix} \quad b = \begin{bmatrix} 41 \\ 25 \\ 105 \end{bmatrix}$$

$$D + \omega L = \begin{bmatrix} 0.8 & 0 & 0 \\ -0.48 & 0.8 & 0 \\ 0 & -0.48 & 0.8 \end{bmatrix}$$

$$(D + \omega L) X^{(2)} = ((1 - 1.2) D - 1.2 U) X^{(1)} + \omega b$$

$$(D + \omega L) X^{(2)} = (-0.2 D - 1.2 U) \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \omega b$$

$$(D + \omega L) X^{(2)} = 1.2 \times \begin{bmatrix} 41 \\ 25 \\ 105 \end{bmatrix}$$

$$X^{(2)} = (D + \omega L)^{-1} \cdot 1.2 \cdot \begin{bmatrix} 41 \\ 25 \\ 105 \end{bmatrix}$$

$$X^{(2)} = \begin{bmatrix} 61.5 \\ 74.4 \\ 202.14 \end{bmatrix}$$

$$(D+\omega L)\,x^{(3)} = (-0.2D - 1.2U)\,x^{(2)} + \omega b$$

$$(D+\omega L)\,x^{(3)} = \begin{bmatrix} -0.16 & 0.48 & 0 \\ 0 & -0.16 & 0.48 \\ 0 & 0 & -0.16 \end{bmatrix} \begin{bmatrix} 61.5 \\ 74.4 \\ 202.14 \end{bmatrix} + 1.2 \begin{bmatrix} 41 \\ 25 \\ 105 \end{bmatrix}$$

$$= \begin{bmatrix} 25.872 \\ 85.1232 \\ -32.3424 \end{bmatrix} + 1.2 \begin{bmatrix} 41 \\ 25 \\ 105 \end{bmatrix}$$

$$x^{(3)} = (D+\omega L)^{-1} \begin{bmatrix} 25.872 \\ 85.1232 \\ -32.3424 \end{bmatrix}$$

$$x^{(3)} = \begin{bmatrix} 93.84 \\ 200.208 \\ 237.1968 \end{bmatrix}$$