

$$\begin{bmatrix} 0.8 & -0.4 & 0 \\ 0.4 & 0.8 & -0.4 \\ 0 & -0.4 & 0.8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 41 \\ 25 \\ 105 \end{bmatrix}$$

① Jacobi vs Gauss Siedel

(a) Jacobi

$$x_i^k = b_i - \sum_{\substack{j=1 \\ j \neq i}}^n a_{ij} x_j^{(k-1)}$$

$$x_1^{(1)} = 0 \quad x_2^{(1)} = 0 \quad x_3^{(1)} = 0$$

$$x_1^{(2)} = \frac{b_1 - \sum_{j=2}^3 a_{1j} x_j^{(1)}}{a_{11}}$$

$$x_1^{(2)} = \frac{41 - 0}{0.8} = 51.25$$

$$x_2^{(2)} = \frac{b_2 - \sum_{j=1,3} a_{2j} x_j^{(1)}}{a_{22}}$$

$$x_2^{(2)} = \frac{25 - 0}{0.8} = 31.25$$

$$x_3^{(2)} = \frac{105 - 0}{0.8} = 131.25$$

$$\begin{aligned} x_1 &= 51.25 \\ x_2 &= 31.25 \\ x_3 &= 131.25 \end{aligned}$$

(b) Gauss Siedel

$$x_i^{(k+1)} = b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^n a_{ij} x_j^{(k)}$$

$$x_1^{(2)} = \frac{b_1 - \sum_{j=2,3} a_{1j} x_j^{(1)}}{a_{11}}$$

$$x_1^{(2)} = \frac{41 - 0}{0.8} = 51.25$$

$$x_2^{(2)} = \frac{b_2 - a_{21} x_1^{(2)} - a_{23} x_3^{(1)}}{a_{22}}$$

$$x_2^{(2)} = \frac{25 - (-0.4)(51.25) - 0}{0.8} = 56.875$$

$$x_3^{(2)} = \frac{105 - a_{31} x_1^{(2)} - a_{32} x_2^{(2)}}{a_{33}}$$

$$x_3^{(2)} = \frac{105 - (-0.4)(56.875)}{0.8} = 159.68$$

$$x_1 = 51.25$$

$$x_2 = 56.875$$

$$x_3 = 159.68$$

(b) Jacobi

$$\begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 41 \\ 125 \\ 105 \end{bmatrix} - \begin{bmatrix} 0 & -0.4 & 0 \\ -0.4 & 0 & -0.4 \\ 0 & -0.4 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(2)} \\ x_2^{(2)} \\ x_3^{(2)} \end{bmatrix} = \frac{1}{0.8} \begin{bmatrix} 41 \\ 125 \\ 105 \end{bmatrix} = \begin{bmatrix} 51.25 \\ 31.25 \\ 131.25 \end{bmatrix} \quad \text{--- (1)}$$

$$\begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.8 \end{bmatrix} \begin{bmatrix} x_1^{(3)} \\ x_2^{(3)} \\ x_3^{(3)} \end{bmatrix} = \begin{bmatrix} 41 \\ 25 \\ 105 \end{bmatrix} - \begin{bmatrix} 0 & -0.4 & 0 \\ -0.4 & 0 & 0.4 \\ 0 & -0.4 & 0 \end{bmatrix} \begin{bmatrix} 51.25 \\ 31.25 \\ 131.25 \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(3)} \\ x_2^{(3)} \\ x_3^{(3)} \end{bmatrix} = \begin{bmatrix} 41 \\ 25 \\ 105 \end{bmatrix} - \begin{bmatrix} -12.5 \\ -73 \\ -12.5 \end{bmatrix} = \begin{bmatrix} 53.5 \\ 98.0 \\ 117.5 \end{bmatrix} \quad \text{--- (2)}$$

Gauss Siedel

$$\begin{bmatrix} a_{11} & 0 & \dots & 0 \\ a_{21} & a_{22} & & \\ \vdots & & \ddots & \\ a_{n1} & \dots & & a_{nn} \end{bmatrix} \begin{bmatrix} x_1^{(k+1)} \\ x_2^{(k+1)} \\ \vdots \\ x_n^{(k+1)} \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} - \begin{bmatrix} 0 & a_{12} & \dots & a_{1n} \\ 0 & 0 & & \\ \vdots & & \ddots & a_{nn} \\ 0 & \dots & & 0 \end{bmatrix} \begin{bmatrix} x_1^{(k)} \\ x_2^{(k)} \\ \vdots \\ x_n^{(k)} \end{bmatrix}$$

$$\begin{bmatrix} 0.8 & 0 & 0 \\ -0.4 & 0.8 & 0 \\ 0 & -0.4 & 0.8 \end{bmatrix} x^{(2)} = \begin{bmatrix} 41 \\ 25 \\ 105 \end{bmatrix} - \begin{bmatrix} 0 & -0.4 & 0 \\ 0 & 0 & -0.4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(2)} \\ x_2^{(2)} \\ x_3^{(2)} \end{bmatrix} = (L+U)^{-1} B = \begin{bmatrix} 51.25 \\ 56.875 \\ 159.68 \end{bmatrix}$$

$$(L+U)(x^{(3)}) = B - Ux^{(2)}$$

$$\begin{bmatrix} 0.8 & 0 & 0 \\ -0.4 & 0.8 & 0 \\ 0 & -0.4 & 0.8 \end{bmatrix} x^{(3)} = \begin{bmatrix} 41 \\ 25 \\ 105 \end{bmatrix} - \begin{bmatrix} 0 & -0.4 & 0 \\ 0 & 0 & -0.4 \\ 0 & 0 & 0 \end{bmatrix} x^{(2)}$$

$$\begin{bmatrix} x_1^{(3)} \\ x_2^{(3)} \\ x_3^{(3)} \end{bmatrix} = (L+U)^{-1} \begin{bmatrix} 63.75 \\ 86.75 \\ 105 \end{bmatrix} = \begin{bmatrix} 79.6875 \\ 150.9375 \\ 206.7188 \end{bmatrix}$$