

CL249: ASSIGNMENT 7

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Problem statement

We have to solve the given ODEs

$$\frac{dy_1}{dx} = -2y_1 + 4e^{-x}$$

$$y_1(x=0) = 2$$

$$\frac{dy_2}{dx} = \frac{-y_1 y_2^2}{3}$$

$$y_2(x=0) = 4$$

using Euler's Explicit Method for initial values given.

And plot  $y_1$  vs  $x$  and  $y_2$  vs  $x$  for different values of  $h$ .

Description of Method

Euler's Explicit method.

We divide ~~into~~ the interval into parts ( $N$ ) and find the function using the formula.

$$x_{i+1} = x_i + h \quad h = \frac{b-a}{N}$$

$$y_{i+1} = y_i + h \left. \frac{dy}{dx} \right|_{x=x_i}$$

$$\frac{dy}{dx} = f(x, y) \text{ is the differential eq}^n$$

We are given initial values of  $y$ s at  $x=0$ .

PSEUDO CODE

main.m

Initialize Interval  $[0, 4]$

loop for  $N$

$$N = 2^{i-1}$$

get  $x_1, y_1, y_2$  from solver

loop in  $1: \text{length}(y_{\text{pre}})$

$$\text{error}_1 = \left| \frac{y_1(i) - y_{\text{pre}}(i)}{y_i} \right|$$

loop in  $1: 1/\text{pre}$

$$\text{error}_2 = \left| \frac{y_2(i) - y_{\text{pre}}(i)}{y_{\text{pre}}(i)} \right|$$

plot  $(x, y_1)$

plot  $(x, y_2)$

solver.m

get arguments  $a, b, N$

$$h = (b-a)/N$$

$$y_1(1) = 2, y_2(1) = 4;$$

loop from 1 to  $(N-1)$

$$x_{i+1} = x_i + h$$

$$y_1(i+1) = y_1(i) + h(\text{derivative}_1)$$

$$y_2(i+1) = y_2(i) + h(\text{derivative}_2)$$

return

derivative.m

def arguments x, y1, y2

$$y_1' = f_1(x, y_1)$$

$$y_2' = f_2(x, y_1, y_2)$$

function f1(x, y1)

$$\text{return } -2y + 4e^{-x}$$

function f2(x, y1, y2)

$$\text{return } \frac{-y_1 y_2^2}{3}$$



## Convergence

The two plots  $y_1$  and  $y_2$ , the graphs are getting close as we are decreasing step size.

- $h = h/2$  is operating in each iteration as  $n = 2n$
- we have also found the max. error between  $y_1$  and  $y_2$  from previous iteration.