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CL 249: ASSIGNMENT 8

PROBLEM

We have to solve a set of differential equations using RK4 and RK5 technique, which represent one of the simple models to describe spreading of COVID 19.

$$\frac{dy_1}{dx} = -cy_1y_2 \quad \frac{dy_2}{dx} = cy_1y_2 - dy_2 \quad \frac{dy_3}{dx} = dy_2$$

y_1 = Healthy people
y_2 = Infected people
y_3 = People under quarantine

$y_1(x=0) = 95 \quad y_2(x=0) = 5 \quad y_3(x=0) = 0$

And plot the graphs

Description of Method

RK4 Method

we have $\frac{dy}{dx} = f(x,y)$

and we define   h represents stepsize

$K_1 = f(x_i, y_i)$

$K_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_1h)$

$K_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_2h)$

$K_4 = f(x+h, y+K_3h)$

and

$$y_{i+1} = y_i + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)h$$

where y is the required function

RK5 Method

we have $\frac{dy}{dx} = f(x, y)$

define $\qquad\qquad h =$ stepsize

$$K_1 = f(x_i, y_i)$$

$$K_2 = f\left(x_i + \frac{1}{4}h, \; y_i + \frac{1}{4}K_1 h\right)$$

$$K_3 = f\left(x_i + \frac{h}{4}, \; y_i + \frac{K_1 h}{8} + \frac{K_2 h}{8}\right)$$

$$K_4 = f\left(x_i + \frac{h}{2}, \; y_i - \frac{K_2 h}{2} + K_3 h\right)$$

$$K_5 = f\left(x_i + \frac{3h}{4}, \; y_i + \frac{3K_1 h}{16} + \frac{9K_4 h}{16}\right)$$

$$K_6 = f\left(x_i + h, \; y_i - \frac{3K_1 h}{7} + \frac{2K_2 h}{7} + \frac{12K_3 h}{7} - \frac{12K_4 h}{7} + \frac{8K_5 h}{7}\right)$$

and

$$y_{i+1} = y_i + \frac{1}{90}(7K_1 + 32K_3 + 12K_4 + 32K_5 + 7K_6)h$$

where y is the required f^n

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PSEUDO CODE

RK4.m to calculate using RK4 method.

let X, Y_1, Y_2, Y_3 be empty array
and $C = 1$ $d = 5$ be given const.

Initial values
$Y_1(1) = 95$; $Y2(1) = 5$;
Iterate through 1 to N-1
$x_{i+1} = x_i + h$
$(K_1y_1; K_2y_2, K_3y_3) =$ derivative $(y1(i), y2(i))$
$(K_2y_1\ K_2y_2\ K_2y_3) =$ der $(y_i + \frac{K_1y_ih}{2})$

$(K_3y_1, K_3y_2, K_3y_3) =$ der $(y_{ij} + \frac{K_2y_ih}{2})$

$(K_4y_1, K_4y_2, K_4y_3) =$ der $(y_i + \frac{K_3yh}{2})$

update values $Y_1\ y_2\ y_3$ as
$y_{i+1} = y_i + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

return Y_1, Y_2, Y_3;

derivative.m

get y_1 and y_2 and c, d
$y_1' = -Cy_1y_2$
$y_2' = Cy_1y_2 - dy_2$
$y_3' = dy_2$
return all values.

RK5.m

get all the arrays and constants.
assign initial values to y_1 y_2 y_3

Iterate through 1 to N-1
Increase x_{i+1} as $x_i + h$
~~fr~~
Calculate for y_1 y_2 y_3

(K_1y, K_2y, K_3y) = derivative (x_i, y_i)
~~K_2y K~~
$K_2 = \text{derivative}\left(y_i + \frac{K_1h}{4}\right)$
$K_3 = \text{derivative}\left(y_i + \frac{K_1h}{8} + \frac{K_2h}{8}\right)$

$K_4 = \text{derivative}\left(y_i - \frac{K_2h}{2} + K_3h\right)$

$K_5 = \text{derivative}\left(y_i + \frac{3K_1h}{16} + \frac{9K_4h}{16}\right)$

$K_6 = \text{derivative}\left(y_i - \frac{3K_1h}{7} + \frac{2K_2h}{7} + \frac{12K_3h}{7} - \frac{12K_4h}{7} + \frac{8K_5h}{7}\right)$

update each y as ~~$y_{i+1} = y_i + 4$~~
$y_{i+1} = y_i + \frac{h}{90}(7K_1 + 32K_3 + 12K_4 + 32K_5 + 7K_6)$

Main.m

get all y_1 y_2 y_3 from RK4 and RK5
plot y_1 y_2 y_3 vs x

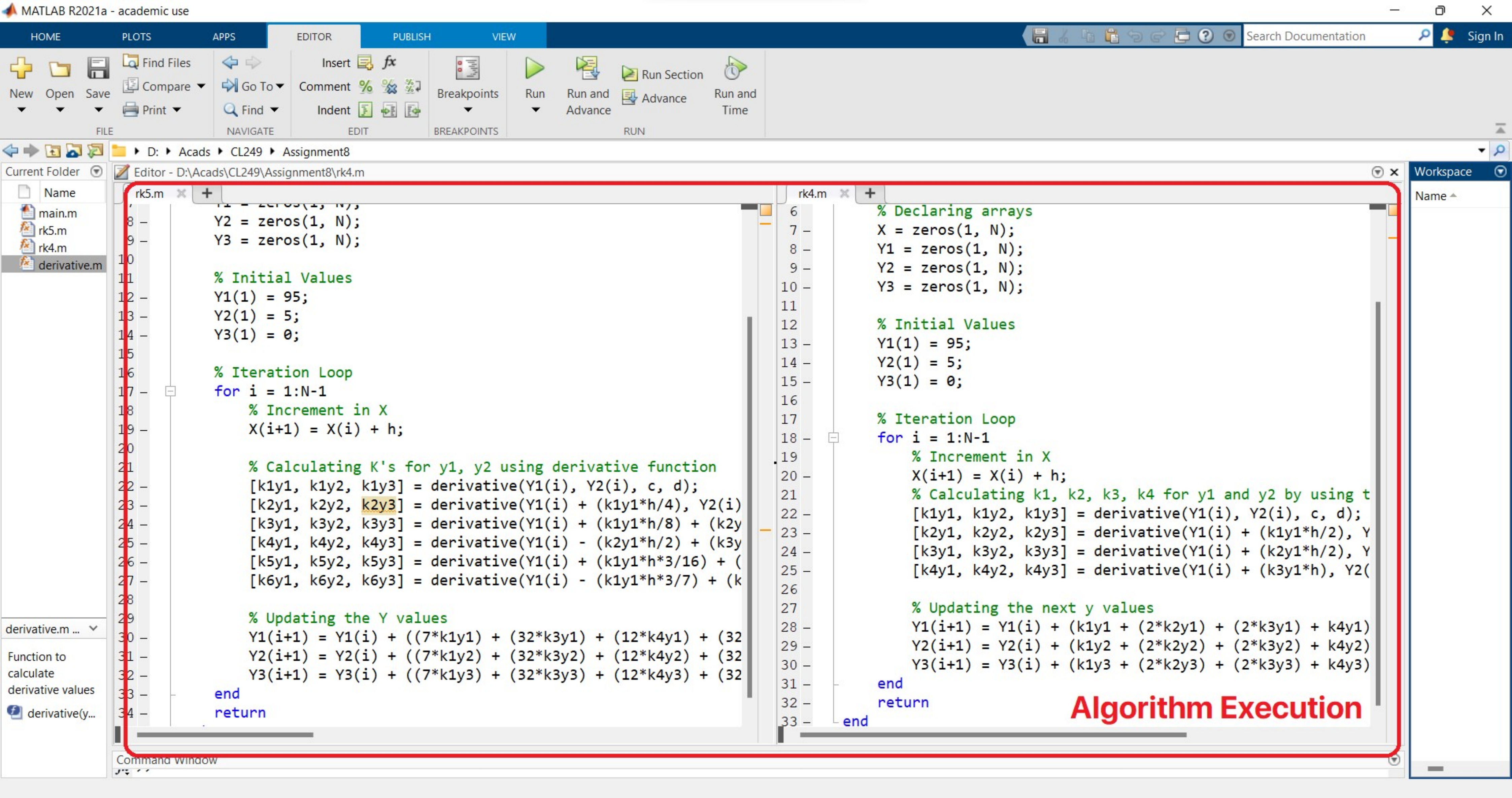
end

```
% Interval of X
a = 0;
b = 4;
% Number of steps
N = 500;
% Step size
h = (b-a)/N;

% Calculating Y1, Y2, Y3, by RK4 Method
[X1, Y1_RK4, Y2_RK4, Y3_RK4] = rk4(a, b, h, N);
% Calculating Y1, Y2, Y3 by RK5 Method
[X2, Y1_RK5, Y2_RK5, Y3_RK5] = rk5(a, b, h, N);

% Plotting the RK4 Method Solution
figure(1)
plot(X1, Y1_RK4, 'DisplayName', 'Y1 (Healthy People)', 'LineWidth', 1.25);
hold on
plot(X1, Y2_RK4,'DisplayName', 'Y2 (Infected Prople)', 'LineWidth', 1.25);
hold on
plot(X1, Y3_RK4, 'DisplayName', 'Y3 (People Under Quarantine)', 'LineWidth',↙
1.25);
title('Solution by RK1 Method')
xlabel('X')
ylabel('Y')
legend

% Plotting the RK5 Method Solution
figure(2)
plot(X2, Y1_RK5, 'DisplayName', 'Y1 (Healthy People)', 'LineWidth', 1.25);
hold on
plot(X2, Y2_RK5,'DisplayName', 'Y2 (Infected Prople)', 'LineWidth', 1.25);
hold on
plot(X2, Y3_RK5, 'DisplayName', 'Y3 (People Under Quarantine)', 'LineWidth',↙
1.25);
title('Solution by RK2 Method')
xlabel('X')
ylabel('Y')
legend
```

```
% Function to calculate solution by RK4 Method
function [X, Y1, Y2, Y3] = rk4(a, b, h, N)
    % Given Constants
    c = 1;
    d = 5;
    % Declaring arrays
    X = zeros(1, N);
    Y1 = zeros(1, N);
    Y2 = zeros(1, N);
    Y3 = zeros(1, N);

    % Initial Values
    Y1(1) = 95;
    Y2(1) = 5;
    Y3(1) = 0;

    % Iteration Loop
    for i = 1:N-1
        % Increment in X
        X(i+1) = X(i) + h;
        % Calculating k1, k2, k3, k4 for y1 and y2 by using the derivative
        [k1y1, k1y2, k1y3] = derivative(Y1(i), Y2(i), c, d);
        [k2y1, k2y2, k2y3] = derivative(Y1(i) + (k1y1*h/2), Y2(i) +
(k1y2*h/2), c, d);
        [k3y1, k3y2, k3y3] = derivative(Y1(i) + (k2y1*h/2), Y2(i) +
(k2y2*h/2), c, d);
        [k4y1, k4y2, k4y3] = derivative(Y1(i) + (k3y1*h), Y2(i) + (k3y2*h),
c, d);

        % Updating the next y values
        Y1(i+1) = Y1(i) + (k1y1 + (2*k2y1) + (2*k3y1) + k4y1)*h/6;
        Y2(i+1) = Y2(i) + (k1y2 + (2*k2y2) + (2*k3y2) + k4y2)*h/6;
        Y3(i+1) = Y3(i) + (k1y3 + (2*k2y3) + (2*k3y3) + k4y3)*h/6;
    end
    return
end
```

```
function [X, Y1, Y2, Y3] = rk5(a, b, h, N)
    % Given Constants
    c = 1;
    d = 5;
    % Declaring arrays
    X = zeros(1, N);
    Y1 = zeros(1, N);
    Y2 = zeros(1, N);
    Y3 = zeros(1, N);

    % Initial Values
    Y1(1) = 95;
    Y2(1) = 5;
    Y3(1) = 0;

    % Iteration Loop
    for i = 1:N-1
        % Increment in X
        X(i+1) = X(i) + h;

        % Calculating K's for y1, y2 using derivative function
        [k1y1, k1y2, k1y3] = derivative(Y1(i), Y2(i), c, d);
        [k2y1, k2y2, k2y3] = derivative(Y1(i) + (k1y1*h/4), Y2(i) +↙
(k1y2*h/4), c, d);
        [k3y1, k3y2, k3y3] = derivative(Y1(i) + (k1y1*h/8) + (k2y1*h/8), Y2↙
(i) + (k1y2*h/8) + (k2y2*h/8), c, d);
        [k4y1, k4y2, k4y3] = derivative(Y1(i) - (k2y1*h/2) + (k3y1*h), Y2(i)↙
- (k2y2*h/2) + (k3y2*h), c, d);
        [k5y1, k5y2, k5y3] = derivative(Y1(i) + (k1y1*h*3/16) +↙
(k4y1*h*9/16), Y2(i) + (k1y2*h*3/16) + (k4y2*h*9/16), c, d);
        [k6y1, k6y2, k6y3] = derivative(Y1(i) - (k1y1*h*3/7) + (k2y1*h*2/7)↙
+ (k3y1*h*12/7) - (k4y1*h*12/7) + (k5y1*h*8/7), Y2(i) - (k1y2*h*3/7) +↙
(k2y2*h*2/7) + (k3y2*h*12/7) - (k4y2*h*12/7) + (k5y2*h*8/7) , c, d);

        % Updating the Y values
        Y1(i+1) = Y1(i) + ((7*k1y1) + (32*k3y1) + (12*k4y1) + (32*k5y1) +↙
(7*k6y1))*h/90;
        Y2(i+1) = Y2(i) + ((7*k1y2) + (32*k3y2) + (12*k4y2) + (32*k5y2) +↙
(7*k6y2))*h/90;
        Y3(i+1) = Y3(i) + ((7*k1y3) + (32*k3y3) + (12*k4y3) + (32*k5y3) +↙
(7*k6y3))*h/90;
    end
    return
end
```

```
% Function to calculate derivative values
function [Y1_dash, Y2_dash, Y3_dash] = derivative(y1, y2, c, d)
    Y1_dash = -c*y1*y2;
    Y2_dash = (c*y1*y2) - (d*y2);
    Y3_dash = d*y2;
    return
end
```


MATLAB R2021a - academic use
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rk5.m
Y2 = zeros(1, N);
Y3 = zeros(1, N);
% Initial Values
Y1(1) = 95;
Y2(1) = 5;
Y3(1) = 0;
% Iteration Loop
for i = 1:N-1
% Increment in X
X(i+1) = X(i) + h;
% Calculating K's for y1, y2 using derivative function
[k1y1, k1y2, k1y3] = derivative(Y1(i), Y2(i), c, d);
[k2y1, k2y2, k2y3] = derivative(Y1(i) + (k1y1*h/4), Y2(i)
[k3y1, k3y2, k3y3] = derivative(Y1(i) + (k1y1*h/8) + (k2y
[k4y1, k4y2, k4y3] = derivative(Y1(i) - (k2y1*h/2) + (k3y
[k5y1, k5y2, k5y3] = derivative(Y1(i) + (k1y1*h*3/16) + (
[k6y1, k6y2, k6y3] = derivative(Y1(i) - (k1y1*h*3/7) + (k
% Updating the Y values
Y1(i+1) = Y1(i) + ((7*k1y1) + (32*k3y1) + (12*k4y1) + (32
Y2(i+1) = Y2(i) + ((7*k1y2) + (32*k3y2) + (12*k4y2) + (32
Y3(i+1) = Y3(i) + ((7*k1y3) + (32*k3y3) + (12*k4y3) + (32
end
return
rk4.m
% Declaring arrays
X = zeros(1, N);
Y1 = zeros(1, N);
Y2 = zeros(1, N);
Y3 = zeros(1, N);
% Initial Values
Y1(1) = 95;
Y2(1) = 5;
Y3(1) = 0;
% Iteration Loop
for i = 1:N-1
% Increment in X
X(i+1) = X(i) + h;
% Calculating k1, k2, k3, k4 for y1 and y2 by using t
[k1y1, k1y2, k1y3] = derivative(Y1(i), Y2(i), c, d);
[k2y1, k2y2, k2y3] = derivative(Y1(i) + (k1y1*h/2), Y
[k3y1, k3y2, k3y3] = derivative(Y1(i) + (k2y1*h/2), Y
[k4y1, k4y2, k4y3] = derivative(Y1(i) + (k3y1*h), Y2(
% Updating the next y values
Y1(i+1) = Y1(i) + (k1y1 + (2*k2y1) + (2*k3y1) + k4y1)
Y2(i+1) = Y2(i) + (k1y2 + (2*k2y2) + (2*k3y2) + k4y2)
Y3(i+1) = Y3(i) + (k1y3 + (2*k2y3) + (2*k3y3) + k4y3)
end
return
end
Algorithm Execution
Workspace
Name
Command Window

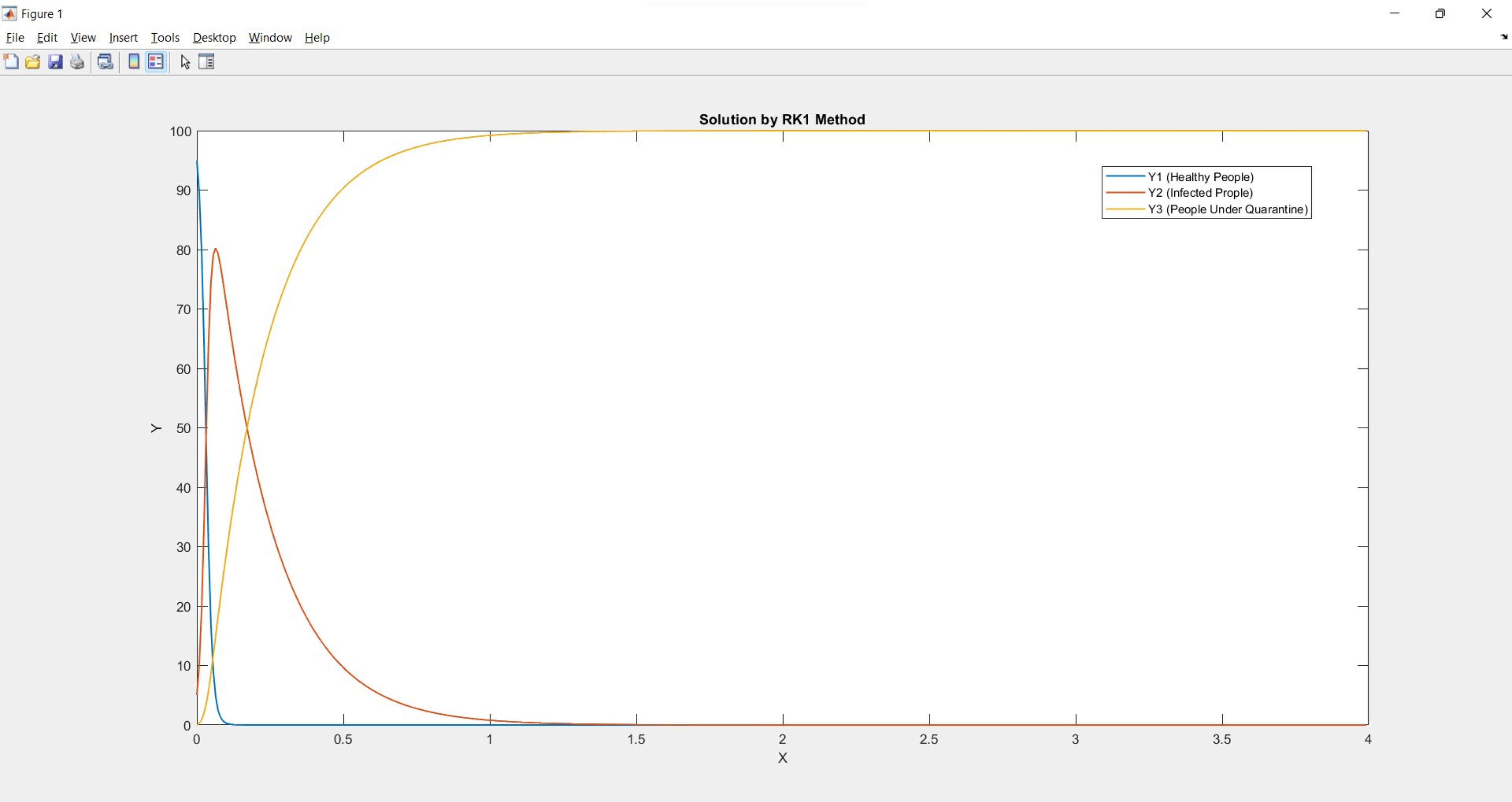
Figure 1
File Edit View Insert Tools Desktop Window Help
Solution by RK1 Method
Y1 (Healthy People)
Y2 (Infected Prople)
Y3 (People Under Quarantine)
Y
X
0
10
20
30
40
50
60
70
80
90
100
0.5
1
1.5
2
2.5
3
3.5
4

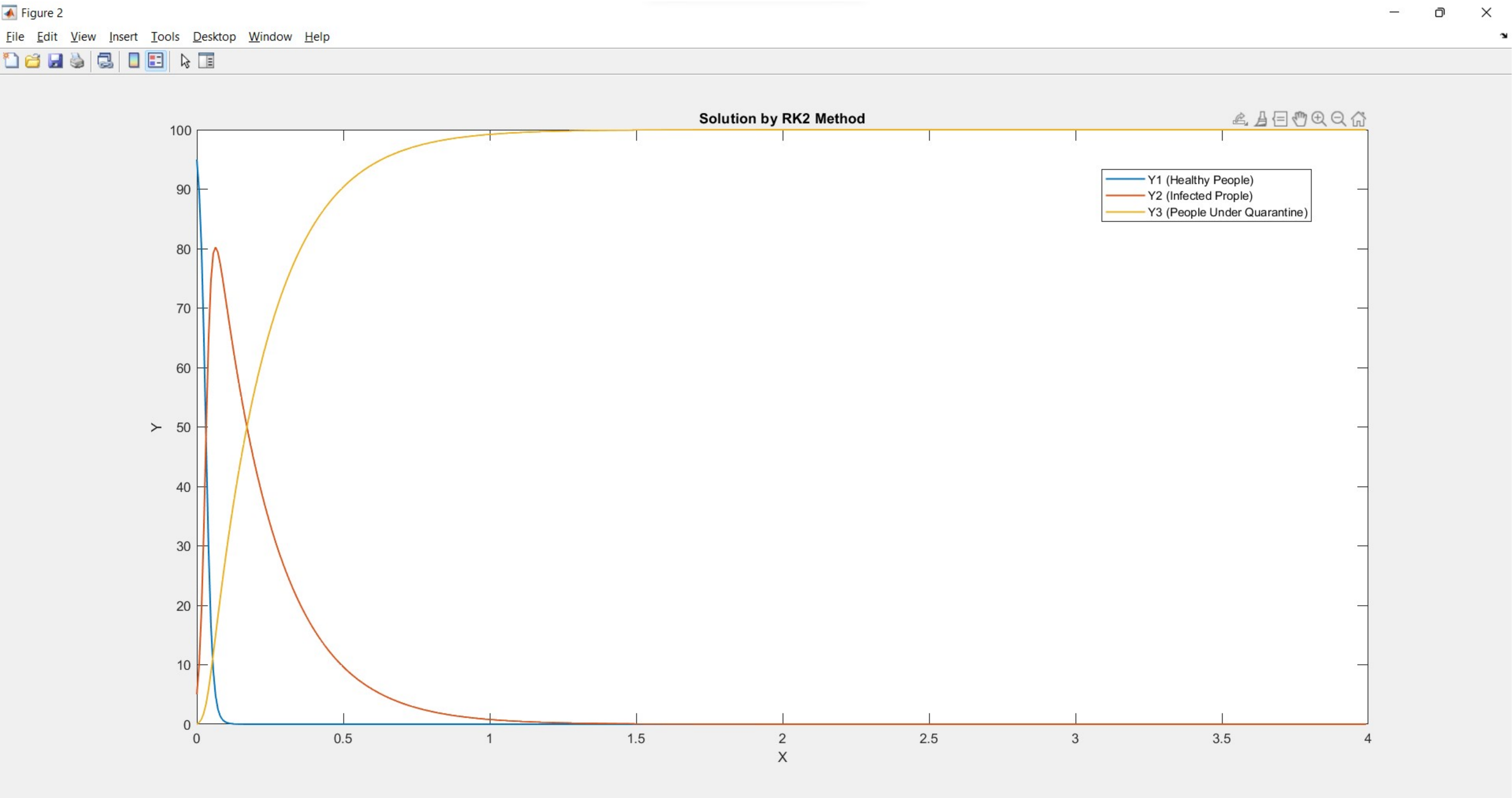
Figure 2
File Edit View Insert Tools Desktop Window Help
Solution by RK2 Method
Y1 (Healthy People)
Y2 (Infected Prople)
Y3 (People Under Quarantine)
Y
X
0
10
20
30
40
50
60
70
80
90
100
0.5
1
1.5
2
2.5
3
3.5
4

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Conclusions

- RK4 ~~And~~ method is accurate enough and takes less time (less calculations), so it is more reliable.

- RK5 method is accurate but takes more calculations and hence computational time.