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CL249: ASSIGNMENT 8

PROBLEM

We have to solve a set of differential equations using RK4 and RK5 technique, which represent one of the simple models to describe spreading of COVID 19.

$$\frac{dy_1}{dx} = -Cy_1y_2 \qquad \frac{dy_2}{dx} = Cy_1y_2 - dy_2 \qquad \frac{dy_3}{dx} = dy_2$$

y_1 = Healthy people
y_2 = Infected people
y_3 = People under quarantine

$$y_1(x=0) = 95 \qquad y_2(x=0) = 5 \qquad y_3(x=0) = 0$$

And plot the graphs

Description of Method

RK4 Method

we have $\frac{dy}{dx} = f(x,y)$

and we define h represents stepsize

$$K_1 = f(x_i, y_i)$$

$$K_2 = f\left(x_i + \frac{1}{2}h,\ y_i + \frac{1}{2}K_1 h\right)$$

$$K_3 = f\left(x_i + \frac{1}{2}h,\ y_i + \frac{1}{2}K_2 h\right)$$

$$K_4 = f(x+h,\ y + K_3 h)$$

and

$$y_{i+1} = y_i + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)h$$

where y is the required function

RK5 Method

we have $\frac{dy}{dx} = f(x, y)$

define h = step size

$$K_1 = f(x_i, y_i)$$

$$K_2 = f\left(x_i + \frac{1}{4}h, y_i + \frac{1}{4}K_1 h\right)$$

$$K_3 = f\left(x_i + \frac{h}{4}, y_i + \frac{K_1 h}{8} + \frac{K_2 h}{8}\right)$$

$$K_4 = f\left(x_i + \frac{h}{2}, y_i - \frac{K_2 h}{2} + K_3 h\right)$$

$$K_5 = f\left(x_i + \frac{3h}{4}, y_i + \frac{3K_1 h}{16} + \frac{9K_4 h}{16}\right)$$

$$K_6 = f\left(x_i + h, y_i - \frac{3K_1 h}{7} + \frac{2K_2 h}{7} + \frac{12}{7}K_3 h - \frac{12}{7}K_4 h + \frac{8K_5 h}{7}\right)$$

and

$$y_{i+1} = y_i + \frac{1}{90}(7K_1 + 32K_3 + 12K_4 + 32K_5 + 7K_6)h$$

where y is the required f^h

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PSEUDO CODE

RK4.m to calculate using RK4 method.

let x, y_1, y_2, y_3 be empty array
and $c = 1$ $d = 5$ be given cost.

Initial values
$y_1(1) = 95$; $y_2(1) = 5$;
Iterate through 1 to N-1
$x_{i+1} = x_i + h$
$(K_1y_1, K_2y_1, K_3y_1) = \text{derivative}(y_1(i), y_2(i))$
$(K_2y_1, K_2y_2, K_2y_3) = \text{der}(y_i + \frac{K_1y_1h}{2})$
$(K_3y_1, K_3y_2, K_3y_3) = \text{der}(y_i + \frac{K_2y_1h}{2})$
$(K_4y_1, K_4y_2, K_4y_3) = \text{der}(y_i + \frac{K_3y_1h}{2})$

update values y_1 y_2 y_3 as
$y_{i+1} = y_i + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

return y_1, y_2, y_3;

derivative.m

get y_1 and y_2 and c, d
$y_1' = -cy_1y_2$
$y_2' = cy_1y_2 - dy_2$
$y_3' = dy_2$
return all values.

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RK5.m

get all the arrays and constants.
assign initial values to y_1 y_2 y_3

Iterate through 1 to N-1
Increase x_{i+1} as $x_i + h$
&
Calculate for y_1 y_2 y_3

(K_1y, K_2y, K_3y) = derivative (x_i, y_i)

K_2y ~~K_1~~

K_2 = derivative $\left(y_i + \frac{K_1 h}{4}\right)$

K_3 = derivative $\left[y_i + \frac{K_1 h}{8} + \frac{K_2 h}{8}\right)$

K_4 = derivative $\left(y_i - \frac{K_2 h}{2} + K_3 h\right)$

K_5 = derivative $\left(y_i + \frac{3K_1 h}{16} + \frac{9K_4 h}{16}\right)$

K_6 = derivative $\left(y_i - \frac{3K_1 h}{7} + \frac{2K_2 h}{7} + \frac{12K_5 h}{7} - \frac{12K_4 h}{7} + \frac{8K_5 h}{7}\right)$

update each y as ~~$y_{i+1} = y_i + 4$~~

$y_{i+1} = y_i + \frac{h}{90}(7K_1 + 32K_3 + 12K_4 + 32K_5 + 7K_6)$

Main.m

get all y_1 y_2 y_3 from RK4 and RK5
plot ~~*~~ y_1 y_2 y_3 vs x

end