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CL249: ASSIGNMENT 7

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Problem statement

We have to solve the given ODES

$$\frac{dy_1}{dx} = -2y_1 + 4e^{-x} \qquad y_1(x=0) = 2$$

$$\frac{dy_2}{dx} = \frac{-y_1 y_2^2}{3} \qquad y_2(x=0) = 4$$

using Euler's Explicit Method for initial values given.

And plot y_1 vs x and y_2 vs x for different values of h.

Description of Method

Euler's Explicit method.

We divide ~~into~~ the interval into parts (N) and find the function using the formula.

$$x_{i+1} = x_i + h \qquad h = \frac{b-a}{N}$$

$$y_{i+1} = y_i + h^* \left.\frac{dy}{dx}\right|_{x=x_i}$$

$\frac{dy}{dx} = f'(x,y)$ is the differential eq^n

We are given initial values of ys at $x = 0$.

PSEUDO CODE

main.m

Initialize interval [0, 4)

loop for N

$N = 2^{i-1}$

get x, y_1, y_2 from solver

loop in 1: length (y_{pre})

$error_1 = \left| \frac{y_1(i) - y_{1pre}(i)}{y_i} \right|$

loop in 1 : lpre

$error_2 = \left| \frac{y_2(i) - y_{2pre}(i)}{y_2(i)} \right|$

plot (x, y_1)

plot (x, y_2)

solver.m

get arguments a, b, N

$h = b-a/N$

$y(1) = 2, y2(1) = 4;$

loop from 1 to (N-1)

$x_{i+1} = x_i + h$

$y_1(i+1) = y1(i) + h$ (derivative$_1$)

$y_2(i+1) = y_2(i) + h$ (derivative 2)

return

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derivative.m

get arguments x, y_1, y_2

$y_1' = f1(x, y_1)$
$y_2' = f2(y_1, y_2)$

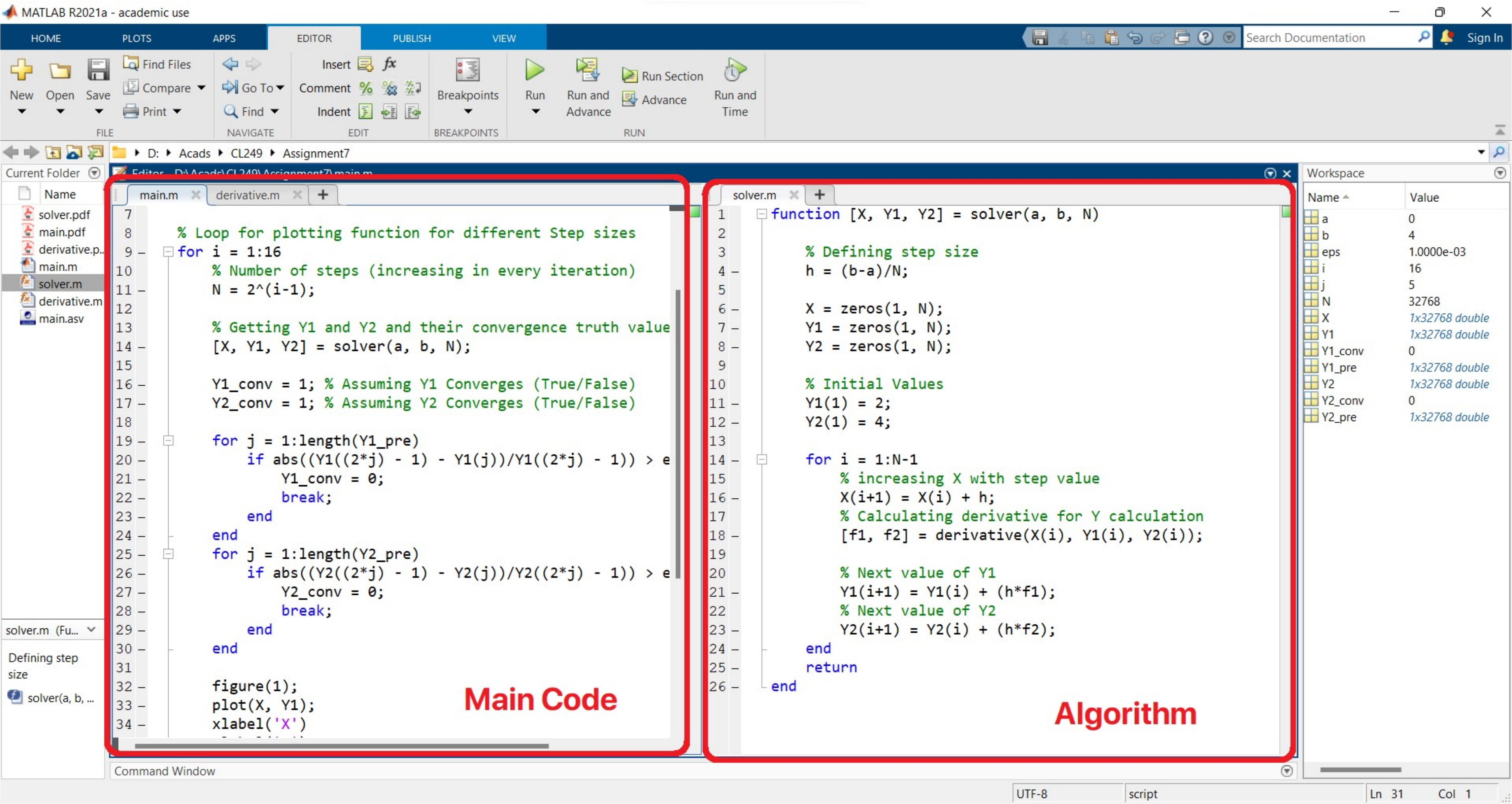
function f1 (x, y)
return $-2y + 4e^{-x}$

function f2 (y, y2)
return $-\frac{y_1 y_2^2}{3}$

```matlab
% Defining Interval
a = 0;
b = 4;
Y1_pre = 0;
Y2_pre = 0;
eps = 10^(-3);

% Loop for plotting function for different Step sizes
for i = 1:16
    % Number of steps (increasing in every iteration)
    N = 2^(i-1);

    % Getting Y1 and Y2 and their convergence truth values from solver
    [X, Y1, Y2] = solver(a, b, N);

    Y1_err = 0; % Assuming Y1 Converges (True/False)
    Y2_err = 0; % Assuming Y2 Converges (True/False)

    for j = 1:length(Y1_pre)
        temp = abs((Y1((2*j) - 1) - Y1(j))/Y1((2*j) - 1));
        if temp > Y1_err
            Y1_err = temp;
        end
    end
    for j = 1:length(Y2_pre)
        temp = abs((Y2((2*j) - 1) - Y2(j))/Y2((2*j) - 1));
        if temp > Y2_err
            Y2_err = temp;
            break;
        end
    end

    figure(1);
    plot(X, Y1);
    xlabel('X')
    ylabel('Y')
    title('Function Y1')
    hold on;

    figure(2);
    plot(X,Y2);
    xlabel('X')
    ylabel('Y2')
    title('Function Y2')
    hold on;

    Y1_pre = Y1;
    Y2_pre = Y2;
end
```

```
function [X, Y1, Y2] = solver(a, b, N)

    % Defining step size
    h = (b-a)/N;

    X = zeros(1, N);
    Y1 = zeros(1, N);
    Y2 = zeros(1, N);

    % Initial Values
    Y1(1) = 2;
    Y2(1) = 4;

    for i = 1:N-1
        % increasing X with step value
        X(i+1) = X(i) + h;
        % Calculating derivative for Y calculation
        [f1, f2] = derivative(X(i), Y1(i), Y2(i));

        % Next value of Y1
        Y1(i+1) = Y1(i) + (h*f1);
        % Next value of Y2
        Y2(i+1) = Y2(i) + (h*f2);
    end
    return
end
```

```
% Function to calculate derivatives of Y1 and Y2
function [y1_dash, y2_dash] = derivative(x, y1, y2)
    y1_dash = f1(x, y1);
    y2_dash = f2(y1, y2);
    return
end

% Given function Y1 slope
function t = f1(x, y)
    t = -(2*y) + (4*exp(-1*x));
    return
end

% Given function Y2 slope
function t = f2(y1, y2)
    t = (-1)*y1*(y2^2)/3;
    return
end
```


MATLAB R2021a - academic use
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Defining step size
solver(a, b, ...
main.m
derivative.m
% Loop for plotting function for different Step sizes
for i = 1:16
% Number of steps (increasing in every iteration)
N = 2^(i-1);
% Getting Y1 and Y2 and their convergence truth value
[X, Y1, Y2] = solver(a, b, N);
Y1_conv = 1; % Assuming Y1 Converges (True/False)
Y2_conv = 1; % Assuming Y2 Converges (True/False)
for j = 1:length(Y1_pre)
if abs((Y1((2*j) - 1) - Y1(j))/Y1((2*j) - 1)) > e
Y1_conv = 0;
break;
end
end
for j = 1:length(Y2_pre)
if abs((Y2((2*j) - 1) - Y2(j))/Y2((2*j) - 1)) > e
Y2_conv = 0;
break;
end
end
figure(1);
plot(X, Y1);
xlabel('X')
Main Code
solver.m
function [X, Y1, Y2] = solver(a, b, N)
% Defining step size
h = (b-a)/N;
X = zeros(1, N);
Y1 = zeros(1, N);
Y2 = zeros(1, N);
% Initial Values
Y1(1) = 2;
Y2(1) = 4;
for i = 1:N-1
% increasing X with step value
X(i+1) = X(i) + h;
% Calculating derivative for Y calculation
[f1, f2] = derivative(X(i), Y1(i), Y2(i));
% Next value of Y1
Y1(i+1) = Y1(i) + (h*f1);
% Next value of Y2
Y2(i+1) = Y2(i) + (h*f2);
end
return
end
Algorithm
Workspace
Name
Value
a 0
b 4
eps 1.0000e-03
i 16
j 5
N 32768
X 1x32768 double
Y1 1x32768 double
Y1_conv 0
Y1_pre 1x32768 double
Y2 1x32768 double
Y2_conv 0
Y2_pre 1x32768 double
Command Window
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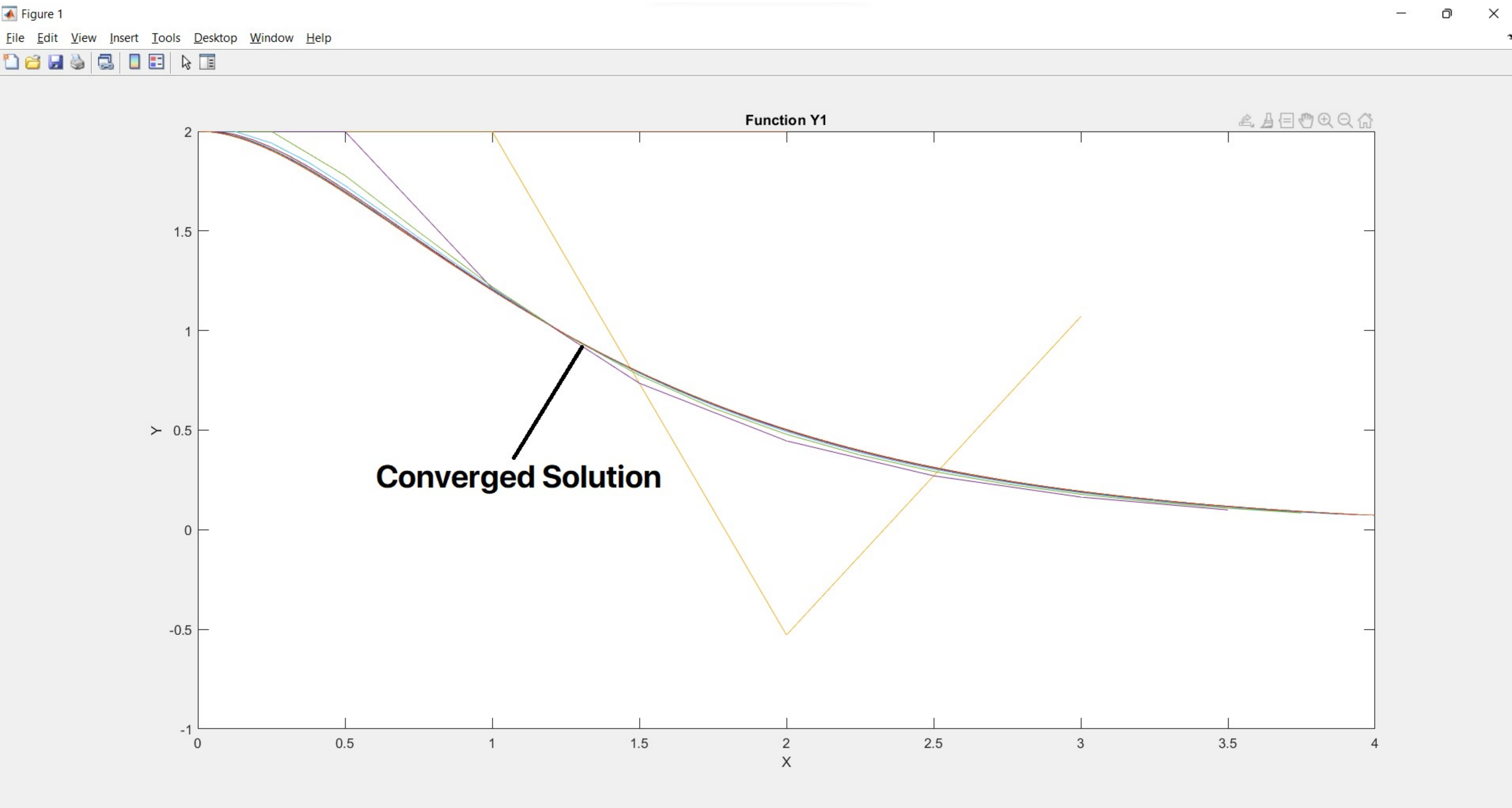
Figure 1
File Edit View Insert Tools Desktop Window Help
Function Y1
Y
X
Converged Solution
2
1.5
1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
2.5
3
3.5
4

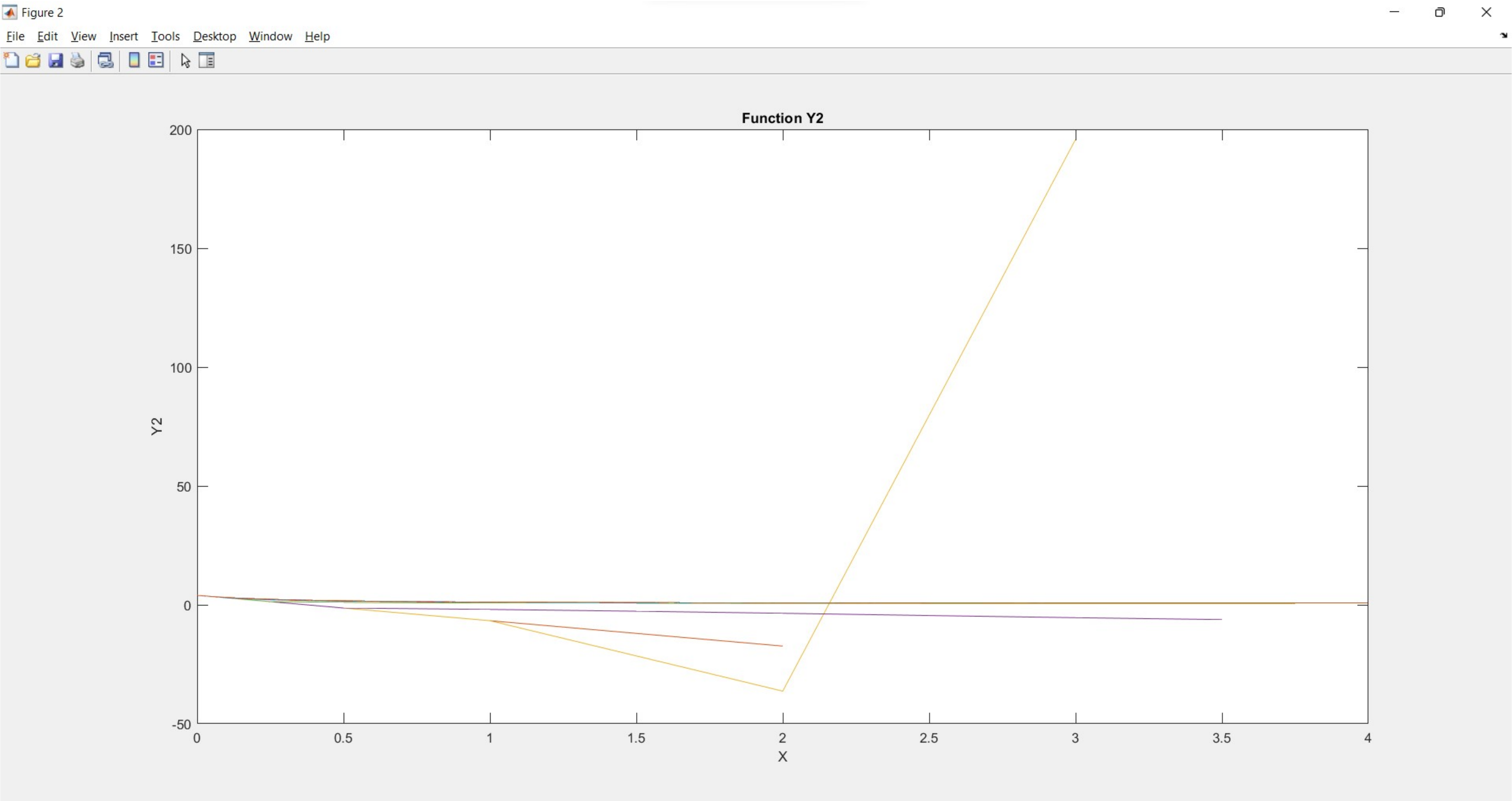
Figure 2
File Edit View Insert Tools Desktop Window Help
Function Y2
200
150
100
50
0
-50
Y2
0
0.5
1
1.5
2
2.5
3
3.5
4
X

Convergence

The two plots y_1 and y_2, the graphs are getting close as we are decreasing step size.

- $h = h/2$ is operating in each iteration as $n = 2n$

- we have also found the max. error between y_1 and y_1 from previous iteration.