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CL249: ASSIGNMENT 6

PROBLEM: Numerical Integration

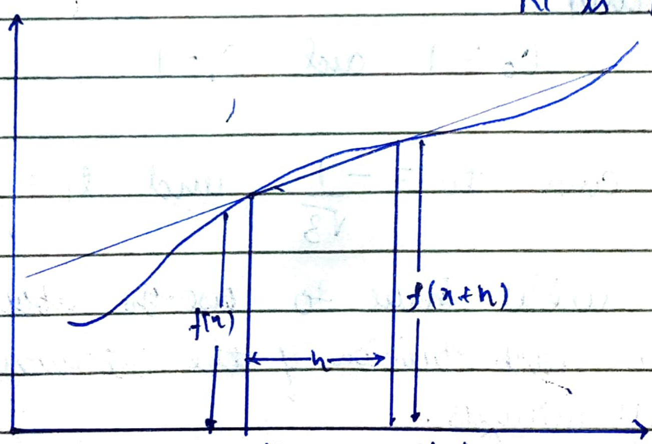
We have to find the value of integration

$$I = \int_0^{30} \left(\frac{250x}{x+6}\right) e^{-\frac{x}{10}}\, dx$$ using two different methods: Trapezoidal rule and Gauss Quadrature rule.

Description of Method.

(1) **Trapezoidal Rule**: $I = \int_a^b f(x)\,dx$ here $h = \frac{b-a}{N}$

N is the step size



Area of a single element (trapezium) $= \frac{1}{2}\left(f(x+h)+f(x)\right)(h)$

Area of whole f^n $= \sum_{x=a}^{x=b-h} \frac{1}{2}\left(f(x+h)+f(x)\right) h$

$$\therefore \int_a^b f(x)\,dx \simeq \sum_{x=a}^{b-h} \frac{1}{2}\left(f(x+h)-f(x)\right) h$$

② Gauss-Quadrature

$$I = \int_a^b f(x)\,dx = \int_{-1}^{1} g(t)\,dt$$

In this method, we try to transform it into $\int_{-1}^{1} g(t)\,dt$ by substitution

Then $g(t)$ comes out to be

$$\left(\frac{b-a}{2}\right)\left(f\left(\frac{b-a}{2}\right)t + \frac{(b+a)}{2}\right)$$

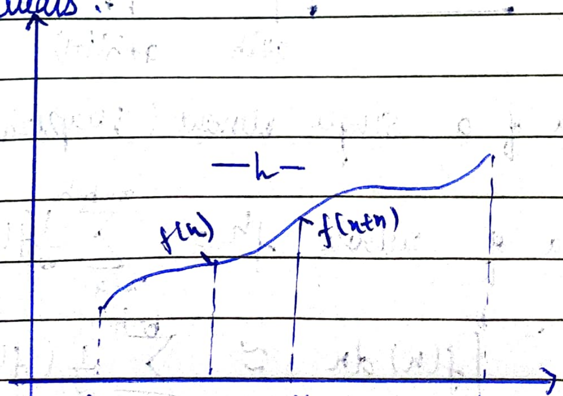
$$\therefore \int_{-1}^{1} g(t) = c_0\, g(t_0) + c_1\, g(t_1)$$

and

$c_0 = 1$ and $c_1 = 1$

and $t_0 = -\frac{1}{\sqrt{3}}$ and $t_1 = \frac{1}{\sqrt{3}}$

So, we'll have to use the above result in each division of the interval, and sum over all intervals.



PSEUDO - CODE

Trapezoidal.m

define $h = \frac{b-a}{2}$

↓

let $I = 0$

↓

loop : $a \le b-h$ → $I = I + \frac{h}{2}\left(f(a+h) - f(a)\right)$

$a = a+h$

NO ↓

return I

Gauss-quad.m

define $I = 0$

and $x_1 = \frac{-1}{\sqrt{3}}$ $x_2 = \frac{1}{\sqrt{3}}$

and $h = \frac{b-a}{N}$

let i iterate from 1-N :

let $A = h/2$ $\left(a + ih + (a+(i-1)h)\right)/2$

and $B = a + ih - \frac{h}{2}$ $\left(a+ih + (a+(i-1)h)\right)/2$

$I = I + A\left(f(Ax_1+B) + f(Ax_2+B)\right)$

return I

main.m

take N as input

calculate I

from trapezoidal.m

and Gauss-quad.m

declare empty matrices

trapezoids & gauss

let N = 1: 500

calculat h and Integrals

for every N

plot