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CL249 : ASSIGNMENT 6

PROBLEM: Numerical Integration

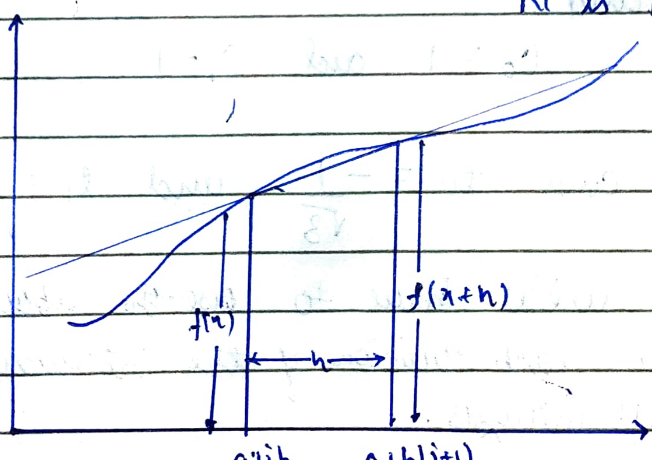
We have to find the value of integration

$$I = \int_0^{30} \left(\frac{250x}{x+6}\right) e^{-\frac{x}{10}}\, dx$$ using two different methods: Trapezoidal rule and Gauss Quadrature rule.

Description of Method.

(1) **Trapezoidal Rule:** $I = \int_a^b f(x)\,dx$ here $h = \frac{b-a}{N}$

N is the step size



Area of a single element (trapezium) $= \frac{1}{2}(f(x+h)+f(x))\,(h)$

Area of whole $f^n = \sum_{x=a}^{x=b-h} \frac{1}{2}(f(x+h)+f(x))\,h$

$$\therefore \int_a^b f(x)\,dx \simeq \sum_{x=a}^{b-h} \frac{1}{2}(f(x+h)-f(x))\,h$$

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(2) Gauss- Quadrature

$$I = \int_a^b f(x)\,dx = \int_{-1}^{1} g(t)\,dt$$

In this method, we try to transform it into $\int_{-1}^{1} g(t)\,dt$ by substitution

Then $g(t)$ comes out to be

$$\left(\frac{b-a}{2}\right)\left(f\left(\frac{b-a}{2}\right)t + \frac{(b+a)}{2}\right)$$

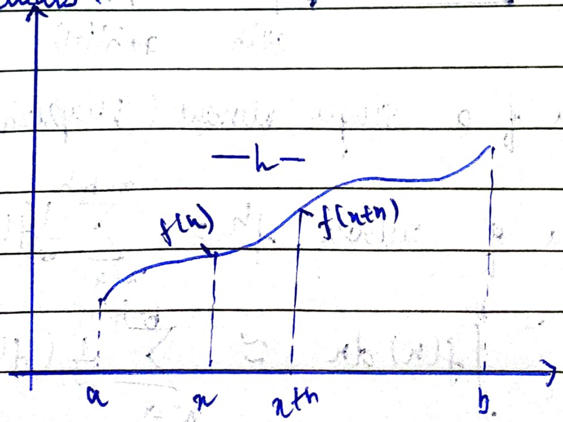
$$\therefore \int_{-1}^{1} g(t) = C_0\, g(t_0) + C_1\, g(t_1)$$

and

$C_0 = 1$ and $C_1 = 1$

and $t_0 = -\frac{1}{\sqrt{3}}$ and $t_1 = \frac{1}{\sqrt{3}}$

So, we'll have to use the above result in each division of the interval, and sum over all intervals.



PSEUDO-CODE

Trapezoidal.m

define $h = \frac{b-a}{2}$

let $I = 0$

loop: $a \le b-h$ → $I = I + \frac{h}{2}\left(f(a+h) - f(a)\right)$

$a = a + h$

NO

return I

Gauss_quad.m

define $I = 0$

and $x_1 = \frac{-1}{\sqrt{3}}$ $x_2 = \frac{1}{\sqrt{3}}$

and $h = \frac{b-a}{N}$

let i iterate from 1–N:

let $A = h/2$ $(a + ih + (a+(i-1)h)/2$

and $B = a + ih - \frac{h}{2}$ $(a + ih + (a+(i-1)h)/2$

$I = I + A\left(f(Ax_1 + B) + f(Ax_2 + B)\right)$

return I

main.m

take N as input

calculate I

from Trapezoidal.m

and Gauss_quad.m

declare empty matrices

trapezoids & gauss

let N = 1:500

calculate h and Integrals

for every N

plot

```
% defining N (number of steps)
N = input('Enter Number of Steps (N) >> ');

% upper and lower limits
a = 0;
b = 30;

% Answer by Trapeziodal Method
I_trapez = trapeziodal_int(@f, a, b, N);
disp('Integral by Trapezuim Method - ')
disp(I_trapez)

% Answer by Gauss Quadrature Method
I_gauss = gauss_quad(@f, 0, 30, N);
disp('Integral by Gauss Quadrature Method - ')
disp(I_gauss)

% Vector for N's
Ns = 1:500;

% Step Size for each N
hs = (b-a)./Ns;

% Matrices for integrals for each N
trap_int = zeros(1,length(Ns));
gauss_int = zeros(1, length(N));

% Calculating integration for different N's
for i = 1:500
    trap_int(1,i) = trapeziodal_int(@f, a, b, Ns(i));
    gauss_int(1,i) = gauss_quad(@f, a, b, Ns(i));
end

% Plotting h vs I
plot(hs, trap_int, 'LineWidth', 1.25, 'DisplayName', 'Trapeziodal Method')
hold on
plot(hs, gauss_int, 'LineWidth', 1.25, 'DisplayName', 'Gauss Quadrature Method')
title('Step Size vs Integral')
xlabel('Number of Steps')
ylabel('Calculated Integral')
legend

% given integrand function
function y = f(x)
        y = ((250*x)/(x+6)) * exp((-1)*x/10);
        return
end
```

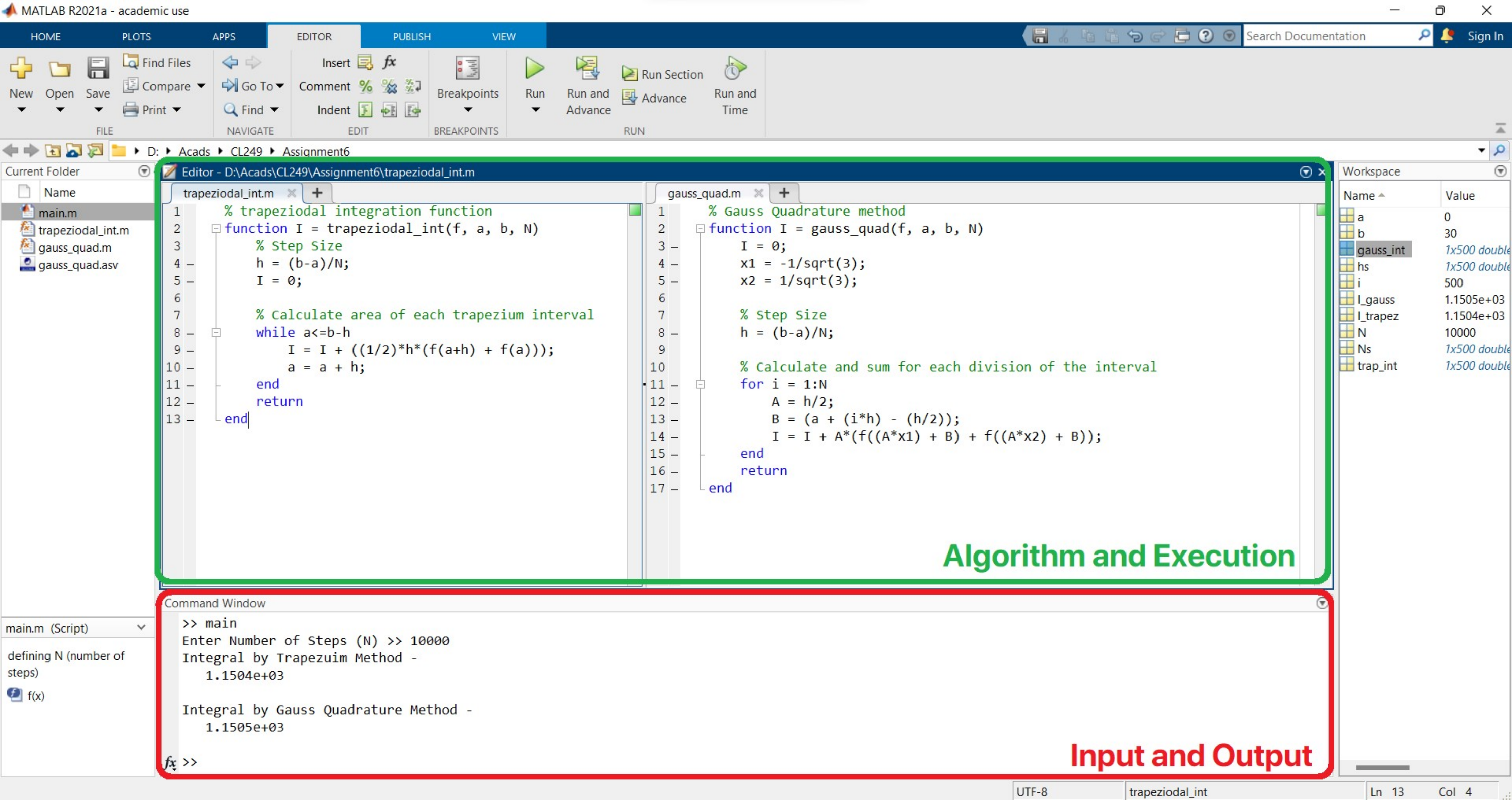
```
% trapeziodal integration function
function I = trapeziodal_int(f, a, b, N)
    % Step Size
    h = (b-a)/N;
    I = 0;

    % Calculate area of each trapezium interval
    while a<=b-h
        I = I + ((1/2)*h*(f(a+h) + f(a)));
        a = a + h;
    end
    return
end
```

```
% Gauss Quadrature method
function I = gauss_quad(f, a, b, N)
    I = 0;
    x1 = -1/sqrt(3);
    x2 = 1/sqrt(3);

    % Step Size
    h = (b-a)/N;

    % Calculate and sum for each division of the interval
    for i = 1:N
        A = h/2;
        B = (a + (i*h) - (h/2));
        I = I + A*(f((A*x1) + B) + f((A*x2) + B));
    end
    return
end
```



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Current Folder
main.m
trapeziodal_int.m
gauss_quad.m
gauss_quad.asv
Editor - D:\Acads\CL249\Assignment6\trapeziodal_int.m
trapeziodal_int.m
% trapeziodal integration function
function I = trapeziodal_int(f, a, b, N)
% Step Size
h = (b-a)/N;
I = 0;
% Calculate area of each trapezium interval
while a<=b-h
I = I + ((1/2)*h*(f(a+h) + f(a)));
a = a + h;
end
return
end
gauss_quad.m
% Gauss Quadrature method
function I = gauss_quad(f, a, b, N)
I = 0;
x1 = -1/sqrt(3);
x2 = 1/sqrt(3);
% Step Size
h = (b-a)/N;
% Calculate and sum for each division of the interval
for i = 1:N
A = h/2;
B = (a + (i*h) - (h/2));
I = I + A*(f((A*x1) + B) + f((A*x2) + B));
end
return
end
Algorithm and Execution
Workspace
Name Value
a 0
b 30
gauss_int 1x500 double
hs 1x500 double
i 500
I_gauss 1.1505e+03
I_trapez 1.1504e+03
N 10000
Ns 1x500 double
trap_int 1x500 double
Command Window
>> main
Enter Number of Steps (N) >> 10000
Integral by Trapezuim Method -
1.1504e+03
Integral by Gauss Quadrature Method -
1.1505e+03
>>
Input and Output
main.m (Script)
defining N (number of steps)
f(x)
UTF-8
trapeziodal_int
Ln 13 Col 4

```
>> main
Enter Number of Steps (N) >> 10000
Integral by Trapezuim Method -
   1.1504e+03

Integral by Gauss Quadrature Method -
   1.1505e+03

>>
```

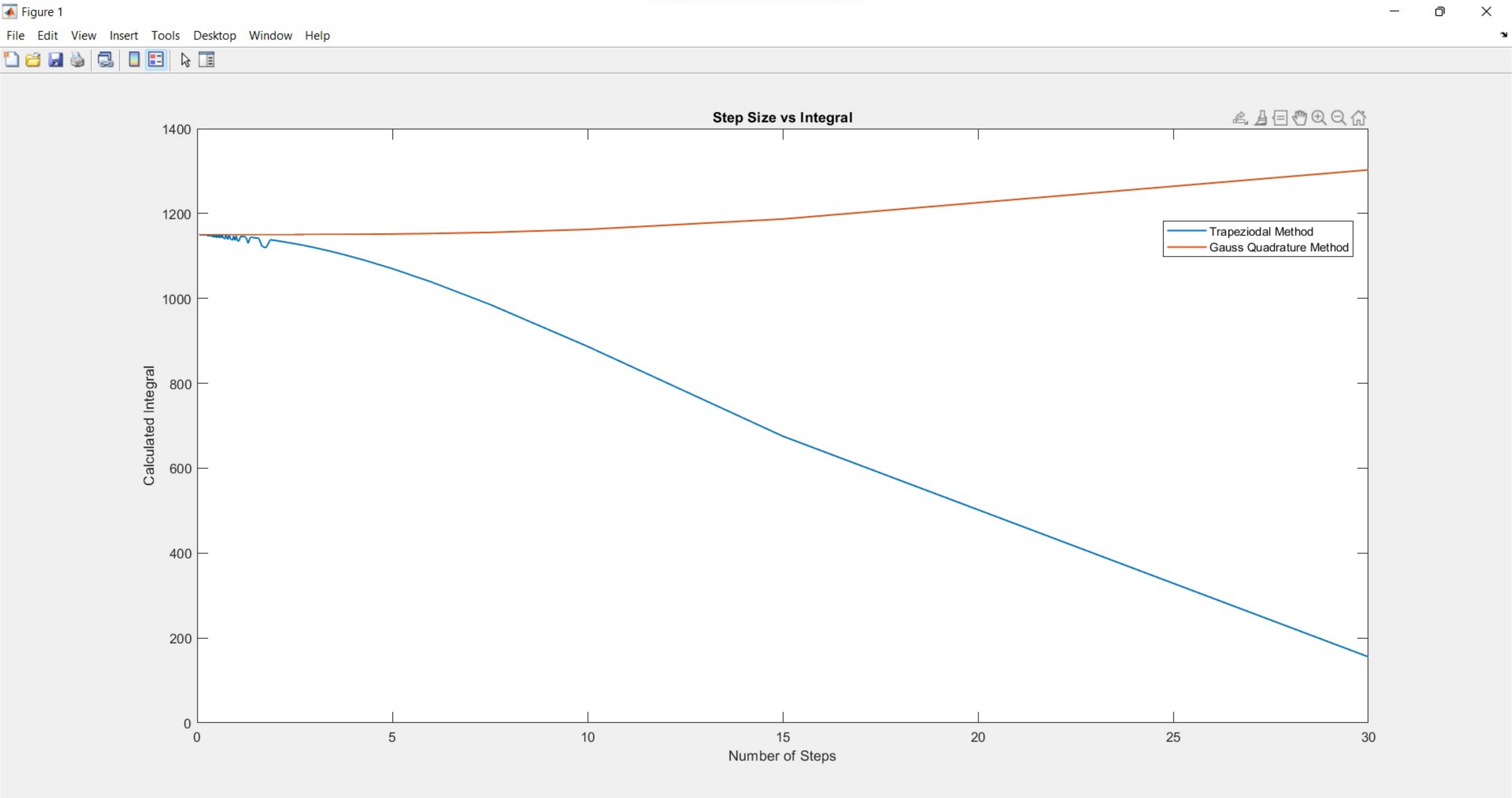
Figure 1
File Edit View Insert Tools Desktop Window Help
Step Size vs Integral
Calculated Integral
Number of Steps
Trapeziodal Method
Gauss Quadrature Method
0
200
400
600
800
1000
1200
1400
5
10
15
20
25
30