200020059

HIMANSHU CHOUDHARY

CL249: ASSIGNMENT 4

PROBLEM

We are given matrices A and B of size 15x15 and 15x1 respectively. We have to solve this equation Ax=b using Jacobi and Gauss Siedel Iteration Techniques, and have to compare the number of operations of Jacobi, Gauss-Siedel and Gauss Elimination.

Description of Method

- Jacobi Iteration Method,

In Jacobi method, we have $x^{(0)}$ = initial guess

$$Ax = B$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & & & \vdots \\ \vdots & & \ddots & \vdots \\ a_{n1} & & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

that is

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

$$x_1^{(1)} = \frac{b_1 - \sum_{j=2}^{n} a_{1j}x_j^{(0)}}{a_{11}}$$

In general,

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1,\, j\neq i}^{n} a_{ij}x_j^{(k)}}{a_{ii}}$$

we basically update values of $x^{(k+1)}$ ($k+1^{th}$ iteration) using k^{th} iteration.

- Gauss Siedel

we have $Ax = b$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & & & \vdots \\ \vdots & & \ddots & \\ a_{n1} & \cdots & & a_{nn} \end{bmatrix} \begin{bmatrix} x_1^{(0)} \\ x_2^{(0)} \\ \vdots \\ x_n^{(0)} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$a_{11} x_1^{(0)} + a_{12}(x_2^{(0)}) + \cdots + a_{1n} x_n^{(0)} = b_1$$

$$x_1^{(1)} = \frac{b_1 - \sum_{j=2}^{n} a_{ij} x_j^{(0)}}{a_{11}}$$

$\vdots$

$$x_k^{(1)} = \frac{b_1 - \sum_{j=k+1}^{n} a_{ij} x_j^{(0)} - \sum_{j=1}^{k-1} a_{ij} x_j^{(1)}}{a_{ii}}$$

in general

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}$$

Converging condition

$$\left| \frac{X_f - X_i}{X_f} \right| < \text{tolerance}$$

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PSEUDO-CODE

jacobi.m

declare X_g and X_f and err
iteration loop {
  loop through rows of A
    $X_f(i) = B(i) - \text{sum}\sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij} x_j$

    add operations
  $X_i = X_f$
iterate through rows of X_f
  if any element $- X_i$ < error
    continue
  else break

main file just declare matrices and uses these functions.

gauss_seidel.m

declare X_g and X_f
iterating loop
  iterate through A rows (i) {
  iterate through 1 to i-1
    sum += $A_{ij} X_f(j)$
  iterate through i+1, n
    sum += $A_{ij} X_g(j)$
  $X_f(i) = \dfrac{B(i) - \text{sum}}{A_{ii}}$

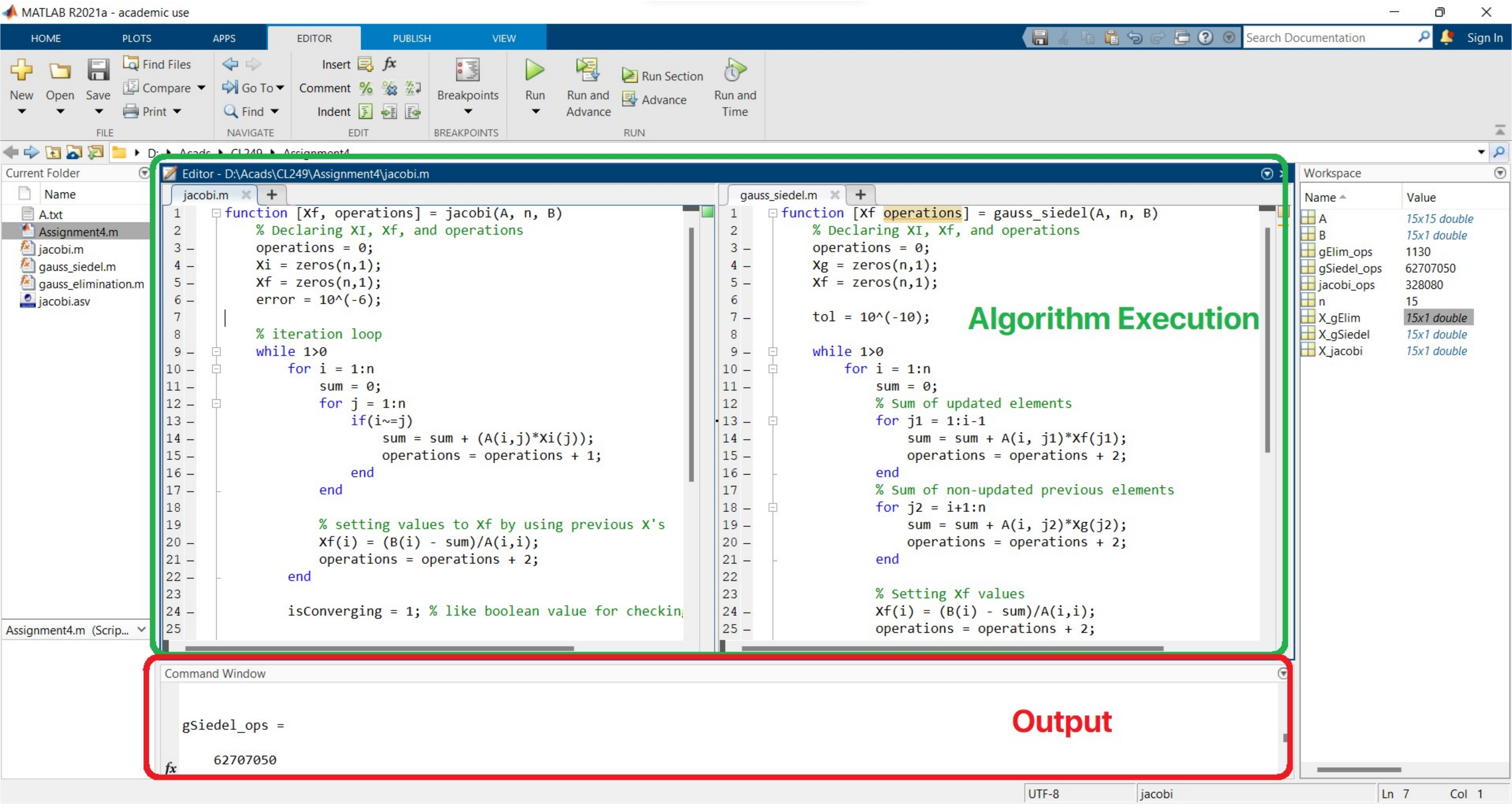
iterate through X_f rows
} if $\dfrac{X_f(i) - X_i(i)}{X_f}$ < tol
    continue
  else break

```
A = load('A.txt');
n = length(A);
B = zeros(n, 1) + 61;

[X_jacobi jacobi_ops] = jacobi(A, n, B);  % Solving through Jacobi method
[X_gSiedel gSiedel_ops] = gauss_siedel(A, n, B);  % Solving through Gauss Siedel ↙
method
[X_gElim gElim_ops] = gauss_elimination(A, B, n, n);  % Solving through Gauss ↙
Elimination

X_jacobi
jacobi_ops

X_gSiedel
gSiedel_ops

X_gElim
gElim_ops
```

```
function [Xf, operations] = jacobi(A, n, B)
    % Declaring XI, Xf, and operations
    operations = 0;
    Xi = zeros(n,1);
    Xf = zeros(n,1);
    error = 10^(-6);

    % iteration loop
    while 1>0
        for i = 1:n
            sum = 0;
            for j = 1:n
                if(i~=j)
                    sum = sum + (A(i,j)*Xi(j));
                    operations = operations + 1;
                end
            end

            % setting values to Xf by using previous X's
            Xf(i) = (B(i) - sum)/A(i,i);
            operations = operations + 2;
        end

        isConverging = 1; % like boolean value for checking convergence

        % checking convergence for every element
        for k = 1:n
            if abs((Xf(k) - Xi(k))/Xf(k)) > error
                isConverging = 0;
            end
        end

        if(isConverging == 1) % if converging, then break
            break;
        end
        Xi = Xf;
    end
    return
end
```

```
function [Xf operations] = gauss_siedel(A, n, B)
    % Declaring XI, Xf, and operations
    operations = 0;
    Xg = zeros(n,1);
    Xf = zeros(n,1);

    tol = 10^(-10);

    while 1>0
        for i = 1:n
            sum = 0;
            % Sum of updated elements
            for j1 = 1:i-1
                sum = sum + A(i, j1)*Xf(j1);
                operations = operations + 2;
            end
            % Sum of non-updated previous elements
            for j2 = i+1:n
                sum = sum + A(i, j2)*Xg(j2);
                operations = operations + 2;
            end

            % Setting Xf values
            Xf(i) = (B(i) - sum)/A(i,i);
            operations = operations + 2;
        end

        isConverging = 1; % like boolean value for checking convergence

        % checking convergence for every element
        for k = 1:n
            if abs((Xf(k) - Xg(k))/Xf(k)) > tol
                isConverging = 0;
            end
        end

        if(isConverging == 1) % break if every element is converging
            break;
        end

        Xg = Xf;
    end
    return
end
```

```matlab
function [Xf operations] = gauss_siedel(A, n, B)
    % Declaring XI, Xf, and operations
    operations = 0;
    Xg = zeros(n,1);
    Xf = zeros(n,1);

    tol = 10^(-10);

    while 1>0
        for i = 1:n
            sum = 0;
            % Sum of updated elements
            for j1 = 1:i-1
                sum = sum + A(i, j1)*Xf(j1);
                operations = operations + 2;
            end
            % Sum of non-updated previous elements
            for j2 = i+1:n
                sum = sum + A(i, j2)*Xg(j2);
                operations = operations + 2;
            end

            % Setting Xf values
            Xf(i) = (B(i) - sum)/A(i,i);
            operations = operations + 2;
        end

        isConverging = 1; % like boolean value for checking convergence

        % checking convergence for every element
        for k = 1:n
            if abs((Xf(k) - Xg(k))/Xf(k)) > tol
                isConverging = 0;
            end
        end

        if(isConverging == 1) % break if every element is converging
            break;
        end

        Xg = Xf;
    end
    return
end
```


MATLAB R2021a - academic use
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Comment
Indent
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BREAKPOINTS
Run
Run and Advance
Run Section
Advance
Run and Time
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A.txt
Assignment4.m
jacobi.m
gauss_siedel.m
gauss_elimination.m
jacobi.asv
Assignment4.m (Scrip...
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jacobi.m
function [Xf, operations] = jacobi(A, n, B)
% Declaring XI, Xf, and operations
operations = 0;
Xi = zeros(n,1);
Xf = zeros(n,1);
error = 10^(-6);
% iteration loop
while 1>0
for i = 1:n
sum = 0;
for j = 1:n
if(i~=j)
sum = sum + (A(i,j)*Xi(j));
operations = operations + 1;
end
end
% setting values to Xf by using previous X's
Xf(i) = (B(i) - sum)/A(i,i);
operations = operations + 2;
end
isConverging = 1; % like boolean value for checkin
gauss_siedel.m
function [Xf operations] = gauss_siedel(A, n, B)
% Declaring XI, Xf, and operations
operations = 0;
Xg = zeros(n,1);
Xf = zeros(n,1);
tol = 10^(-10);
while 1>0
for i = 1:n
sum = 0;
% Sum of updated elements
for j1 = 1:i-1
sum = sum + A(i, j1)*Xf(j1);
operations = operations + 2;
end
% Sum of non-updated previous elements
for j2 = i+1:n
sum = sum + A(i, j2)*Xg(j2);
operations = operations + 2;
end
% Setting Xf values
Xf(i) = (B(i) - sum)/A(i,i);
operations = operations + 2;
Algorithm Execution
Workspace
Name
Value
A
15x15 double
B
15x1 double
gElim_ops
1130
gSiedel_ops
62707050
jacobi_ops
328080
n
15
X_gElim
15x1 double
X_gSiedel
15x1 double
X_jacobi
15x1 double
Command Window
gSiedel_ops =
62707050
Output
UTF-8
jacobi
Ln 7
Col 1

```
>> Assignment4

X_jacobi =

   NaN
   NaN
   NaN
   NaN
   NaN
   NaN
   NaN
   NaN
   NaN
   NaN
   NaN
  -Inf
   NaN
   NaN
   NaN


jacobi_ops =

      328080


X_gSiedel =

   1.0e+05 *

    0.0183
    0.1006
    0.2385
    0.4239
    0.6496
    0.9089
    1.1956
    1.5043
    1.8300
    2.1685
    2.5162
    2.8700
    3.2275
    3.5868
    3.9467


gSiedel_ops =

    62707050


X_gElim =

   1.0e+05 *
```

```
    0.0183
    0.1006
    0.2385
    0.4239
    0.6496
    0.9089
    1.1956
    1.5043
    1.8300
    2.1685
    2.5162
    2.8700
    3.2275
    3.5868
    3.9467


gElim_ops =

        1130

>>
```

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Answer

Operations from Jacobi = 328080
But vector not converging to a real value

Operations from Gauss Siedel : 62707050
Converged to a value

Operations from Gauss Elimination : 1130

Gauss elimination is working best for this given matrix