

CL249: ASSIGNMENT 4PROBLEM

We are given matrices  $A$  and  $B$  of size  $15 \times 15$  and  $15 \times 1$  respectively. We have to solve this equation  $Ax = b$  using Jacobi and Gauss Seidel Iteration Techniques, and have to compare the number of operations of Jacobi, Gauss-Seidel and Gauss Elimination.

Description of Method

- Jacobi Iteration method,

In Jacobi method, we have  $x^{(0)} = \text{initial guess}$

$$Ax = B$$

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & & & \\ \vdots & & & \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

that is

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$x_1^{(1)} = \frac{b_1 - \sum_{j=2}^n a_{1j}x_j^{(0)}}{a_{11}}$$

In general,

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1, j \neq i}^n a_{ij}x_j^{(k)}}{a_{ii}}$$

we basically update values of  $x^{(k+1)}$  ( $k+1$ <sup>th</sup> iteration) using  $k$ <sup>th</sup> iteration.

### • Gauss Seidel

we have  $Ax = b$

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & & & \\ \vdots & & & \\ a_{n1} & \dots & & a_{nn} \end{bmatrix} \begin{bmatrix} x_1^{(0)} \\ x_2^{(0)} \\ \vdots \\ x_n^{(0)} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$a_{11}x_1^{(0)} + a_{12}x_2^{(0)} + \dots + a_{1n}x_n^{(0)} = b_1$$

$$x_1^{(1)} = \frac{b_1 - \sum_{j=2}^n a_{1j}x_j^{(0)}}{a_{11}}$$

$$x_k^{(1)} = \frac{b_i - \sum_{j=k+1}^n a_{ij}x_j^{(0)} - \sum_{j=1}^{k-1} a_{ij}x_j^{(1)}}{a_{ii}}$$

in general

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^n a_{ij}x_j^{(k)}}{a_{ii}}$$

Converging condition

$$\left| \frac{x_j - x_i}{x_j} \right| < \text{tolerance}$$

## PSEUDO-CODE

### jacobi.m

declare  $X_g$  and  $X_f$  and error

iteration loop  $\rightarrow$

loop through rows of A

$$X_f(i) = B(i) - \sum_{j=1, j \neq i}^n a_{ij} X_j$$

add operations

$$X_i = X_f$$

iterate through rows of  $X_f$

if any element  $- X_i < \text{error}$   
continue

else break

Main file just  
declare matrices  
and uses these  
functions.

### Gauss-stedel.m

declare  $X_g$  and  $X_f$

iterating loop

iterate through A rows (i)

iterate through 1 to i-1

$$\text{sum} += A_{ij} X_f(i)$$

iterate through i+1, n

$$\text{sum} += A_{ij} X_g(i)$$

$$X_f(i) = \frac{B(i) - \text{sum}}{A_{ii}}$$

iterate through  $X_f$  rows

if  $\frac{X_f(i) - X_i(i)}{X_f}$   $< \text{error}$

continue

else break