

# Probability and Statistics

## Assignment 3

By Himanshu Sardana

### Question 1

The probability distribution of  $X$ , the number of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width, is given as

$X$	0	1	2	3	4
$P(X)$	0.41	0.37	0.16	0.05	0.01

Find the average number of imperfections per 10 meters of fabric.

```
# Imperfections
X = c(0, 1, 2, 3, 4)

# Probability of imperfections
P = c(0.41, 0.37, 0.16, 0.05, 0.01)

# Average number of imperfections
mean = sum(X * P)
# OR
mean = weighted.mean(X, P)

print(mean)
```

### Question 2

The time  $T$ , in days, required for the completion of a contracted project is a random variable with probability density function  $f(t) = 0.1 e^{(-0.1t)}$  for  $t > 0$  and 0 otherwise. Find the expected value of  $T$ .

```
# Probability density function
f = function(t) {
  if (t > 0) {
    return(0.1 * exp(-0.1 * t))
  } else {
    return(0)
  }
}
```

```

    }
}

# Expected value of T
mean = integrate(function(t) {t * f(t)}, lower = 0, upper = Inf)$value

print(mean)

```

### Question 3

A bookstore purchases three copies of a book at \$6.00 each and sells them for \$12.00 each. Unsold copies are returned for \$2.00 each. Let  $X = \{\text{number of copies sold}\}$  and  $Y = \{\text{net revenue}\}$ . If the probability mass function of  $X$  is

X	0	1	2	3
P(X)	0.1	0.2	0.2	0.5

Find the expected value of  $Y$ .

```

# Number of copies sold
X = c(0, 1, 2, 3)

# Probability of copies sold
P = c(0.1, 0.2, 0.2, 0.5)

# Net revenue
Y = 12 * X - 6 * 3 + 2 * (3 - X)

# Expected value of Y
mean = sum(Y * P)

print(mean)

```

### Question 4

Find the first and second moments about the origin of the random variable  $X$  with probability density function  $f(x) = 0.5e^{-|x|}$  for  $1 < x < 10$  and  $f(x) = 0$  otherwise. Further use the results to find Mean and Variance.

```

# Probability density function
f = function(x) {
  if (x > 1 && x < 10) {
    return(0.5 * exp(-abs(x)))
  }
}

```

```

    } else {
      return(0)
    }
  }
}

# First moment about the origin
mean = integrate(function(x) {x * f(x)}, lower = -Inf, upper = Inf)$value

# Second moment about the origin
second_moment = integrate(function(x) {x^2 * f(x)}, lower = -Inf, upper =
  ↪ Inf)$value

# Variance
variance = second_moment - mean^2

print(mean)
print(variance)

```

## Question 5

Let  $X$  be a geometric random variable with probability distribution

$$f(x) = \frac{3}{4} \left(\frac{1}{4}\right)^{x-1} \text{ for } x = 1, 2, 3, \dots$$

Write a function to find the probability distribution of the random variable  $Y = X^2$  and find probability of  $Y$  for  $X = 3$ . Further, use it to find the expected value and variance of  $Y$  for  $X = 1, 2, 3, 4, 5$ .

```

# Probability distribution of X
f = function(x) {
  return(3/4 * (1/4)^(x - 1))
}

# Probability distribution of Y
g = function(y) {
  if (y == 1) {
    return(f(1))
  } else {
    return(f(sqrt(y)) + f(-sqrt(y)))
  }
}

# Probability of Y for X = 3
print(g(9))

# Expected value and variance of Y for X = 1, 2, 3, 4, 5
for (x in 1:5) {
  mean = sum(sapply(1:100, function(y) {y * g(y)}))

```

```
second_moment = sum(sapply(1:100, function(y) {y^2 * g(y)}))  
variance = second_moment - mean^2  
print(paste("X =", x, "Mean =", mean, "Variance =", variance))  
}
```