Probability and Statistics

Assignment 3

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Question 1

Roll 12 dice simultaneously, and let X denotes the number of 6's that appear. Calculate the probability of getting 7, 8 or 9 6's using R.

```r
# Number of dice
n = 12

# Probability of getting a 6
p = 1/6

# Probability of getting 7, 8 or 9 6's
prob = sum(dbinom(7:9, n, p))

print(prob)
```

Question 2

Assume that the test scores of a college entrance exam fits a normal distribution. Furthermore, the mean test score is 72, and the standard deviation is 15.2. What is percentage of students scoring 84 or more in the exam?

```r
# Mean test score
mean = 72

# Standard deviation
sd = 15.2

# Percentage of students scoring 84 or more
prob = 1 - pnorm(84, mean, sd)

print(prob)
```

Question 3

On the average, five cars arrive at a particular car wash every hour. Let X count the number of cars that arrive from 10AM to 11AM, then X ☐Poisson(λ = 5). What is probability that no car arrives during this time. Next, suppose the car wash above is in operation from 8AM to 6PM, and we let Y be the number of customers that appear in this period. Since this period covers a total of 10 hours, we get that Y ☐ Poisson(λ = 5×10 = 50). What is the probability that there are between 48 and 50 customers, inclusive?

```
# Probability that no car arrives
prob1 = dpois(0, 5)

# Probability that there are between 48 and 50 customers
prob2 = sum(dpois(48:50, 50))

print(prob1)
print(prob2)
```

Question 4

Suppose in a certain shipment of 250 Pentium processors there are 17 defective processors. A quality control consultant randomly collects 5 processors for inspection to determine whether or not they are defective. Let X denote the number of defectives in the sample. Find the probability of exactly 3 defectives in the sample, that is, find P(X = 3).

```
# Number of processors
n = 250

# Number of defective processors
defective = 17

# Probability of getting a defective processor
p = defective/n

# Probability of getting exactly 3 defectives
prob = dbinom(3, 5, p)

print(prob)
```

Question 5

A recent national study showed that approximately 44.7% of college students have used Wikipedia as a source in at least one of their term papers. Let X equal the number of students in a random sample of size n = 31 who have used Wikipedia as a source.

- How is X distributed?

- Sketch the probability mass function.
- Sketch the cumulative distribution function.
- Find mean, variance and standard deviation of X.

```
# Number of students
n = 31

# Probability of using Wikipedia as a source
p = 0.447

# X is distributed as Binomial(n, p)

# Sketch the probability mass function
x = 0:n
y = dbinom(x, n, p)
plot(x, y, type = "h", xlab = "Number of students", ylab = "Probability",
  ↪  main = "Probability Mass Function")

# Sketch the cumulative distribution function
y = pbinom(x, n, p)
plot(x, y, type = "s", xlab = "Number of students", ylab = "Probability",
  ↪  main = "Cumulative Distribution Function")

# Mean
mean = n*p

# Variance
var = n*p*(1-p)

# Standard deviation
sd = sqrt(var)

print(mean)
print(var)
print(sd)
```