Probability and Statistics

Assignment 3

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Question 1

The probability distribution of X, the number of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width, is given as

X	**0**	**1**	**2**	**3**	**4**
P(X)	0.41	0.37	0.16	0.05	0.01

Find the average number of imperfections per 10 meters of fabric.

```r
# Imperfections
X = c(0, 1, 2, 3, 4)

# Probability of imperfections
P = c(0.41, 0.37, 0.16, 0.05, 0.01)

# Average number of imperfections
mean = sum(X * P)
# OR
mean = weighted.mean(X, P)

print(mean)
```

Question 2

The time T, in days, required for the completion of a contracted project is a random variable with probability density function f(t) = 0.1 e(-0.1t) for t > 0 and 0 otherwise. Find the expected value of T.

```r
# Probability density function
f = function(t) {
  if (t > 0) {
    return(0.1 * exp(-0.1 * t))
  } else {
    return(0)
```

```
  }
}

# Expected value of T
mean = integrate(function(t) {t * f(t)}, lower = 0, upper = Inf)$value

print(mean)
```

Question 3

A bookstore purchases three copies of a book at $6.00 each and sells them for $12.00 each. Unsold copies are returned for $2.00 each. Let X = {number of copies sold} and Y = {net revenue}. If the probability mass function of X is

X	0	1	2	3
P(X)	0.1	0.2	0.2	0.5

Find the expected value of Y.

```
# Number of copies sold
X = c(0, 1, 2, 3)

# Probability of copies sold
P = c(0.1, 0.2, 0.2, 0.5)

# Net revenue
Y = 12 * X - 6 * 3 + 2 * (3 - X)

# Expected value of Y
mean = sum(Y * P)

print(mean)
```

Question 4

Find the first and second moments about the origin of the random variable X with probability density function $f(x) = 0.5e^{-|x|}$ **for** $1 < x < 10$ **and** $f(x) = 0$ **otherwise. Further use the results to find Mean and Variance.**

```
# Probability density function
f = function(x) {
  if (x > 1 && x < 10) {
    return(0.5 * exp(-abs(x)))
```

```
  } else {
    return(0)
  }
}

# First moment about the origin
mean = integrate(function(x) {x * f(x)}, lower = -Inf, upper = Inf)$value

# Second moment about the origin
second_moment = integrate(function(x) {x^2 * f(x)}, lower = -Inf, upper =
 ↪  Inf)$value

# Variance
variance = second_moment - mean^2

print(mean)
print(variance)
```

Question 5

Let X be a geometric random variable with probability distribution

$$f(x) = \frac{3}{4}(\frac{1}{4})^{x-1} \text{ for } x = 1, 2, 3, ...$$

Write a function to find the probability distribution of the random variable $Y = X^2$ and find probability of Y for X = 3. Further, use it to find the expected value and variance of Y for X = 1,2,3,4,5.

```
# Probability distribution of X
f = function(x) {
  return(3/4 * (1/4)^(x - 1))
}

# Probability distribution of Y
g = function(y) {
  if (y == 1) {
    return(f(1))
  } else {
    return(f(sqrt(y)) + f(-sqrt(y)))
  }
}

# Probability of Y for X = 3
print(g(9))

# Expected value and variance of Y for X = 1,2,3,4,5
for (x in 1:5) {
  mean = sum(sapply(1:100, function(y) {y * g(y)}))
```

```
  second_moment = sum(sapply(1:100, function(y) {y^2 * g(y)}))
  variance = second_moment - mean^2
  print(paste("X =", x, "Mean =", mean, "Variance =", variance))
}
```