Probability and Events

1. For any two events E and F of a sample space, given P(E|F) = a, P(E) = b, and P(F|E) = y, find the relations between a, b, and y if events E and F are mutually exclusive.
2. For any two events E and F of a sample space, given P(E|F) = a, P(E) = b, and P(F|E) = y, find the relations between a, b, and y if E is a sub-event of F.
3. For any two events E and F of a sample space, given P(E|F) = a, P(E) = b, and P(F|E) = y, find the relations between a, b, and y if E and F are mutually exclusive and collectively exhaustive.
4. Two events A and B are independent with P(A) > P(B), P(A u B) = 0.626 and P(A n B) = 0.144. Determine the values of P(A) and P(B).
5. Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys, and 1 girl and 3 boys. One child is selected at random from each group. Find the probability that the three selected children consist of 1 girl and 2 boys.
6. A manager has two assistants and he bases his decision on information supplied independently by each of them. The probability that he makes a mistake in his thinking is 0.005. The probability that an assistant gives wrong information is 0.3. Assuming that the mistakes made by the manager are independent of the information given by assistants, find the probability that he reaches a wrong decision.
7. A and B take turns alternately rolling a pair of fair dice. A wins if he rolls a sum of 6 before B rolls a sum of 7, while B wins if he rolls a sum of 7 before A rolls a sum of 6. If A starts the game, find the probability that A wins.
8. Two fair dice are rolled together. Let A be the event that the upper face of the first die is "4", and B be the event that the upper face of the second die is "6". Are A and B independent events?
9. A chemical plant has an emergency alarm system. When an emergency situation exists on a particular day, the alarm sounds with probability 0.95. When an emergency situation does not exist, the alarm sounds with probability 0.02. A real emergency situation is an event having the probability 0.004. Find the probability that on a particular day the alarm will sound.
10. Given that the alarm has just sounded, find the probability that a real emergency situation exists.
11. Police plan to enforce speed limit by using radar traps at four different locations within the city limits. The radar traps at each of the locations L1, L2, L3, and L4 will be operated 40%, 30%, 20%, and 10% of the time. If a person who is speeding on his/her way to work has probabilities of 0.2, 0.1, 0.5, and 0.2 respectively of passing through these locations, what is the probability that he/she will receive a speeding ticket?
12. Police plan to enforce speed limit by using radar traps at four different locations within the city limits. The radar traps at each of the locations L1, L2, L3, and L4 will be operated 40%, 30%, 20%, and 10% of the time. If a person who is speeding on his/her way to work has probabilities of 0.2, 0.1, 0.5, and 0.2 respectively of passing through these locations, and the person received a speeding ticket on the way to work, what is the probability that he/she passed through the radar trap located at L2?
13. Two unbiased dice are thrown. Find the expected value and the variance of the sum of number of points on both.
14. Suppose urn one has 3 red and 2 blue balls, and urn two has 4 red and 7 blue balls. One of the two urns is selected at random, and then one of the balls within that urn is picked randomly. What is the probability that urn two is selected at the first stage and a blue ball is selected at the second stage?
15. Suppose urn one has 3 red and 2 blue balls, and urn two has 4 red and 7 blue balls. One of the two urns is selected at random, and then one of the balls within that urn is picked randomly. What is the probability that a blue ball is obtained.

16. Suppose urn one has 3 red and 2 blue balls, and urn two has 4 red and 7 blue balls. One of the two urns is selected at random, and then one of the balls within that urn is picked randomly. Given the information that the ball picked is blue, find the conditional probability that we had selected urn two.
17. A producer of a certain type of electronic component ships to suppliers in lots of twenty. Suppose that 60% of all such lots contain no defective components, 30% contain one defective component, and 10% contain two defective components. A lot is picked, two components from the lot are randomly selected and tested, and neither is defective. What is the probability that zero defective components exist in the lot? What is the probability that one defective exists in the lot?
18. If A and B are two events such that P(Ac) = 0.6, P(B) = 0.7, P(A U B) = 0.6, then find P(A n B) and P(Bc|Ac).
19. Suppose that the probability that any person will not believe a rumour is 0.3. What is the probability that the fifth person to hear the tale is the second one to believe it?
20. Suppose that the probability that any person will not believe a rumour is 0.3. What is the probability that the sixth person to hear this tale is the first one to believe it?
21. In a competitive examination, an examinee either guesses, copies, or knows the answer to a multiple-choice question with four choices. The probability that he guesses is 0.35 and the probability that he copies the answer is 0.20. The probability that the answer is correct, given that he copied it is 0.15. Find the probability that he guesses the answer to the question, given that he correctly answered it.
22. In a competitive examination, an examinee either guesses, copies, or knows the answer to a multiple-choice question with four choices. The probability that he guesses is 0.35 and the probability that he copies the answer is 0.20. The probability that the answer is correct, given that he copied it is 0.15. Find the probability that he copies the answer to the question, given that he correctly answered it.
23. Three distinct integers are chosen at random from the first 20 positive integers. Compute the probability that their product is even.

Random Variables and Probability Distributions

1. Suppose that the life in hours of a certain part of radio tube is a continuous random variable X with density function given by f(x) = 10000/x^2 for x > 100 and 0 otherwise. What is the probability that all of three such tubes in a given radio set will have to be replaced during the first 150 hours of operation?
2. Suppose that the life in hours of a certain part of radio tube is a continuous random variable X with density function given by f(x) = 10000/x^2 for x > 100 and 0 otherwise. What is the probability that a tube will last less than 200 hours if it is known that the tube is still functioning after 150 hours of service?
3. Fifty percentage of all American who travel by car look for gas station and food outlet that are close to or visible from the highway. Suppose a random sample of 35 Indian who travel by car are asked how they determine where to stop for food and gas. Let x be the number in the sample who respond that they look for gas stations and food outlets that are close or visible from the highways. Calculate the mean and variance of x.
4. Let X and Y be jointly distributed with density function f(x, y) = 1/(4(1 + xy)) for |x| < 1, |y| < 1, and 0 otherwise. Check whether X and Y are independent random variables?
5. Let X and Y be jointly distributed with density function f(x, y) = 1/(4(1 + xy)) for |x| < 1, |y| < 1, and 0 otherwise. Find the probability density function of Y^2.

6. Let X and Y be jointly distributed with density function $f(x, y) = 1/(4(1 + xy))$ for $|x| < 1$, $|y| < 1$, and 0 otherwise. Find the conditional density function of X given Y = y.
7. Consider writing onto a computer disk and then sending it through a certifier that counts the number of missing pulses. Suppose this number X has a Poisson distribution with parameter 0.2. What is the probability that disk has exactly one missing pulse?
8. Consider writing onto a computer disk and then sending it through a certifier that counts the number of missing pulses. Suppose this number X has a Poisson distribution with parameter 0.2. What is the probability that disk has at least two missing pulses?
9. Consider writing onto a computer disk and then sending it through a certifier that counts the number of missing pulses. Suppose this number X has a Poisson distribution with parameter 0.2. If two disks are independently selected, what is the probability that neither contains a missing pulse?
10. Following is the cumulative distribution function (CDF) of a discrete random variable X: X: -3 -1 0 1 2 3 5 8 FX(x): 0.10 0.30 0.45 0.5 0.75 0.90 0.95 1.0. Find the probability mass function of X.
11. Following is the cumulative distribution function (CDF) of a discrete random variable X: X: -3 -1 0 1 2 3 5 8 FX(x): 0.10 0.30 0.45 0.5 0.75 0.90 0.95 1.0. Find P(X = Even), $P(1 \leq X \leq 8)$ and $P(X \geq 3|X > 0)$.
12. The probability density function of a random variable X is given by $f(x) = A(1 + x)$, $-1 < x \leq 0$, $A(1 - x)$, $0 < x < 1$, 0, elsewhere. Find the value of A and plot f(x).
13. The probability density function of a random variable X is given by $f(x) = A(1 + x)$, $-1 < x \leq 0$, $A(1 - x)$, $0 < x < 1$, 0, elsewhere. Find the cumulative distribution function (CDF) FX(x).
14. The probability density function of a random variable X is given by $f(x) = A(1 + x)$, $-1 < x \leq 0$, $A(1 - x)$, $0 < x < 1$, 0, elsewhere. Find the point c such that $P(X > c) = 1/2 * P(X < c)$.
15. A fair die is thrown, and an outcome of 4 or 5 is considered a success. If the die is thrown 9 times and X denotes the number of successes, then find the mean and variance of X.
16. A fair die is thrown, and an outcome of 4 or 5 is considered a success. If the die is thrown 9 times and X denotes the number of successes, then find P(X = 2) and $P(2X - 3 = 2)$.
17. A fair die is thrown, and an outcome of 4 or 5 is considered a success. If the die is thrown 9 times and X denotes the number of successes, then find $P(X \leq 2)$.
18. Let X be a continuous random variable with probability density function given by $f(x) = 2x$, $0 < x < 1$, and 0 otherwise. Find the probability density function and hence the cumulative density function of $Y = 3X + 1$.
19. If X1 and X2 are independent random variables having normal distribution with means μ1, μ2 and variances σ1^2, σ2^2 respectively, then show that the random variable Y = a1X1 + a2X2 has a normal distribution with mean μy = a1μ1 + a2μ2 and variance σy^2 = a1^2σ1^2 + a2^2σ2^2.
20. In a certain city, the daily consumption of electric power, in millions of Kilowatt-hours, is a random variable X having a gamma distribution with parameters α and β, and mean μ = 6 and variance σ^2 = 12. Find the value of α and β. Find the probability that on any given day the daily power consumption will exceed 12 million Kilowatt-hours.
21. Let X and Y be two independent random variables representing the lifetimes (in hours) of two electronic components. The joint probability density function (PDF) of X and Y is given by f(x,y) = λ^2 * exp(-λ(x + y)) for x > 0, y > 0, and 0 otherwise, where λ > 0 is a constant. Define two new random variables: U = X + Y and V = X / (X + Y). Find the joint PDF of U and V.
22. For a randomly chosen policyholder, let X represent the number of months between successive premium payments. The cumulative distribution function (CDF) of X is given by: $F(x) = 0$, $x < 1$; 0.15, $1 \leq x < 2$; 0.45, $2 \leq x < 3$; 0.60, $3 \leq x < 4$; 0.85, $4 \leq x < 5$; 1, $x \geq 5$. Find the probability mass function, $f(x) = P(X = x)$.

23. For a randomly chosen policyholder, let X represent the number of months between successive premium payments. The cumulative distribution function (CDF) of X is given by: $F(x) = 0, x < 1$; $0.15, 1 \le x < 2$; $0.45, 2 \le x < 3$; $0.60, 3 \le x < 4$; $0.85, 4 \le x < 5$; $1, x \ge 5$. Find the variance of $Y = X^2$.
24. If X and Y are two random variables having joint density function given by $f(x, y) = A(6 - x - y)$, whenever $0 < x < 2$, $2 < y < 4$ and 0 otherwise. Find the value of A.
25. If X and Y are two random variables having joint density function given by $f(x, y) = A(6 - x - y)$, whenever $0 < x < 2$, $2 < y < 4$ and 0 otherwise. Find the marginal density functions of both X and Y.
26. If X and Y are two random variables having joint density function given by $f(x, y) = A(6 - x - y)$, whenever $0 < x < 2$, $2 < y < 4$ and 0 otherwise. Find $P(X < 1, Y < 3)$.
27. A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets. If X is the number of defective sets purchased by the hotel, find the probability distribution of the random variable X.
28. The population of people with blood groups O, A, B, and AB in a particular population are in the ratio 45:38:14:3. Determine the probability that a sample of 25 people from the population contains exactly 12 with blood group O.
29. The population of people with blood groups O, A, B, and AB in a particular population are in the ratio 45:38:14:3. Determine the probability that a sample of 25 people from the population contains at most 2 with blood group AB.
30. A box contains 5 colored pens, 2 black and 3 white. Pens are drawn successively without replacement. If the random variable X is the number of draws until a black pen is obtained, find the probability mass function for the random variable X.
31. The probability distribution function of a random variable X is given by: x: 0, 1, 2; p(x): $3k^2$, $4k - 10k^2$, $5k - 1$. Where $k > 0$. Find k and also find $P(X < 2)$.
32. If a random variable X is defined such that $E[(X - 1)^2] = 10$ and $E[(X - 2)^2] = 6$, find the mean and variance of X.
33. If the joint probability density function of X and Y is given by $f(x,y) = 24xy$ for $0 < x < 1$, $0 < y < 1$, $x + y < 1$, find $P(X+Y<1/2)$.
34. If the probability that a child exposed to certain viral fever will be infected is 0.3, find the probability that the eighth child exposed to the disease will be the fourth to be infected.

Moment Generating Functions

1. Find the moment generating function (M.G.F) of the random variable X, following the Poisson's distribution function with probability mass function given by $f(x) = e^{-\lambda} \cdot \lambda^x / x!$, for $\lambda > 0$, $x = 0, 1, 2, \ldots$
2. Using the moment generating function, find the variance of the random variable X, following the Poisson's distribution function with probability mass function given by $f(x) = e^{-\lambda} \cdot \lambda^x / x!$, for $\lambda > 0$, $x = 0, 1, 2, \ldots$
3. Let X be a random variable having probability density function $f(x) = 2(1-x)$, $0 < x < 1$, and 0 otherwise. Find the moment generating function $M_X(t)$ of X at $t = 1$. Also find the mean of random variable X.
4. Find the moment generating function (MGF) of the random variable X having cumulative distribution function (CDF) $F_X(x) = 0, x < 0$; $x^2/2, 0 \le x < 1$; $2x - x^2/2 - 1, 1 \le x < 2$; $1, x \ge 2$.
5. If the moment generating function of X is given by $M_X(t) = 1 / (4-t)$, where $|t| < 1$, find the moment generating function of $Y = (X-4) / 4$.

Expectation, Variance, and Covariance

1. For random variables X and Y with joint probability distribution f(x, y) and constants a, b, and c, show that var(aX + bY + c) = a^2 * var(X) + b^2 * var(Y) + 2ab * cov(X, Y), where var(Z) is the variance of the random variable Z and cov(X, Y) is the covariance of X and Y.
2. Show that the minima of E[(X - a)^2] is the variance of a continuous random variable X.
3. For random variables X and Y, the joint probability density function is given by f(x,y) = 4xy, 0 < x < 1, 0 < y < 1, and 0 otherwise. Find the covariance of X and Y.

Joint Probability Distributions and Conditional Probability

1. If X denotes the number of kings and Y denotes the number of aces when two cards are drawn at random without replacement from a well-shuffled deck of 52 cards, find the joint probability distribution of (X, Y).
2. If X denotes the number of kings and Y denotes the number of aces when two cards are drawn at random without replacement from a well-shuffled deck of 52 cards, find the marginal distributions of X and Y.
3. If X denotes the number of kings and Y denotes the number of aces when two cards are drawn at random without replacement from a well-shuffled deck of 52 cards, find P(X = 2 | Y = 1).
4. If X denotes the number of kings and Y denotes the number of aces when two cards are drawn at random without replacement from a well-shuffled deck of 52 cards, find P(X < 2 | 0 < Y < 2).
5. Let the joint density function of random variables X and Y be given by f(x, y) = kxy, 0 < x < y < 1, x + y < 1, and 0 otherwise. Find the value of k and the conditional probability density function of y given x, i.e., p(y|x).

Discrete Probability Distributions

1. A biased die is thrown thirty times and the number of sixes seen is eight. If the die is thrown twelve times, find the probability that six will occur exactly twice.
2. A biased die is thrown thirty times and the number of sixes seen is eight. If the die is thrown twelve times, find the mean and variance of the number of sixes.

Binomial Distribution

1. Define a binomial random variable along with its probability distribution formula. Further prove that if X is a binomial random variable with parameters n and p, then E(X) = np and var(X) = np(1 − p).
2. According to Chemical Engineering Progress, approximately 30% of all pipework failures in chemical plants are caused by operator error. What is the probability that out of the next 20 pipework failures at least 10 are due to operator error? What is the probability that no less than 5 out of 20 such failures are due to operator error?

Poisson and Binomial Distributions

1. Prove that poisson distribution is the limiting case of binomial distribution under the following conditions: n $\longrightarrow\infty$, p $\longrightarrow 0$, np = λ (finite).

Poisson Distribution

1. If X is a Poisson's random variable such that P(X = 1) = P(X = 2), then find the value of P(X = 4). Also find P(X > μ), where μ is the mean of X.

2. If X is a Poisson variate such that $P(X = 2) = 3 - 2P(X = 1)$, find variance and moment generating function of a random variable X.

Geometric Distribution

1. Let the random variable X follow a geometric distribution. Find the moment generating function (MGF) of X, and use it to determine the mean and variance.

Uniform Distribution

1. If X is uniformly distributed over the interval $[-1, 1]$, find the probabilities $P(|X| > 1)$ and $P(|X - 1| \geq 1/2)$ using CDF approach.
2. If the random variable X is uniformly distributed over the interval $[-\sqrt{2}, \sqrt{2}]$, find the exact probabilities for $P(|X - \mu| \geq \sqrt{(3/2)}\sigma)$ and $P(|X - \mu| < \sqrt{(3/2)}\sigma)$. Additionally, compare these probabilities with the upper and lower bounds provided by Chebyshev's inequality, respectively.

Normal Distribution

1. In a normal distribution, 7% of the items are under 35 and 89% are under 63. Find mean and variance of the distribution.
2. Find the value of c such that $P(|X-25|<c) = 0.9544$ where X follows a Normal distribution with mean 25 and variance 36. Given that $P(Z<-2) = 0.0228$ and $P(Z<-1.69) = 0.0456$, where Z is a standard normal variate.

Chebyshev's Inequality

1. State and prove Chebyshev's inequality.
2. Let X be a continuous random variable with finite mean μ and finite variance σ^2. For any real number $k > 0$, prove that $P(|X - \mu| \geq k\sigma) \leq 1 / k^2$.
3. For the probability mass function given by $f(x) = 2^{-x}$ for $x = 1, 2, 3, \ldots$, using Chebyshev's inequality, estimate the probability in the interval $0 < x < 4$. Compare with the exact probability.
4. A random variable X has a mean $\mu = 10$ and a variance $\sigma^2 = 4$. Using Chebyshev's theorem, find $P(|X - 10| > 3)$ and the value of the constant c such that $P(|X - 10| > c) < 0.04$.
5. A symmetrical dice is thrown 600 times. Use Chebyshev's inequality to find the lower bound for the probability of getting 80 to 120 sixes.

Weak Law of Large Numbers

1. If the variable Xp assumes the values 2p-2log p with probability 2-P, p = 1,2,3, ... Examine if the weak law of large number holds for the sequence X1, i = 1,2, ... of identical and independent random variables.

Descriptive Statistics

1. Plot a stem leaf graph for the data: 22, 22, 23, 23, 23, 23, 23, 5, 7, 10, 15, 19, 21, 21, 24, 24, 24, 24, 25. Calculate the range, interquartile range and standard deviation.
2. Given the following data representing the length of life in years of 30 similar fuel pumps: 2.0, 3.0, 0.3, 3.3, 1.3, 0.4, 0.2, 6.0, 5.5, 6.5, 0.2, 2.3, 1.0, 6.0, 5.6, 1.5, 4.0, 5.9, 1.8, 4.7, 0.7, 4.5, 0.3, 1.5, 0.5, 2.5, 5.0, 6.0, 1.2, 0.2. Construct a stem-and-leaf plot for the life in years of the fuel pumps. Compute the sample mean and sample range.

Correlation and Regression

1. In a partially destroyed laboratory record of an analysis of correlation data, the following results only are legible: Variance of X = 9, regression equations: 8x - 10y = 66 and 40x - 18y = 214. What are (i) mean values X and Y (ii) correlation coefficient between X and Y, and (iii) standard deviation of Y.

Chi-Square Test

1. A study conducted in northwest England made an assessment of long-term care facilities that have residents with dementia. The homes included those that provided specialized services for elderly people with mental illness/health problem, known as "EMI homes", as well as others classified as "non-EMI homes". It was expected that EMI homes would score higher on several measures of service quality for people with dementia. One measure includes the structure of home and the service provided. The following are the figures obtained from this experiment: Care Type, Home Type, EMI, Non-EMI. Nursing Care: 54, 22. Residential Care: 59, 77. Dual-Registered: 49, 26. Do these data indicate that the type of care provided varies by the home types?

Hypothesis Testing

1. To find out whether a new serum will arrest leukemia, 9 mice, all with an advanced stage of the disease, are selected. Five mice receive the treatment, and four do not. Survival times, in years, from the time the experiment commenced are given as follows: Treatment: 2.1, 5.3, 1.4, 4.6, 0.9. No Treatment: 1.9, 0.5, 2.8, 3.1. At the 0.05 level of significance, can the serum be said to be effective?
2. A coin is tossed 400 times. Use the normal curve approximation to find the probability of obtaining between 185 and 210 heads inclusive. Use the normal curve approximation to find the probability of obtaining fewer than 205 heads.
3. For a binomial distribution with 4 trials, it is desired to test Ho: p = 1/3 against H1: p = 1/2 by agreeing to accept HO if x ≤ 2 and to reject otherwise. What are the probabilities of committing: type I error, type II error.
4. As the population ages, there is increasing concern about accident-related injuries to the elderly. A survey reported on an experiment in which maximum lean angle - the furthest a subject is able to lean and still recover in one step - was determined for both a sample of younger females (21-29 years) and a sample of older females (67-81 years). The following observations are consistent with the following data: Younger female: 29, 34, 33, 27, 28,32. Older female: 18, 15, 23, 13, 12, 21. Does the data suggest that true average maximum lean angle for older female is more than 10 degrees smaller than it is for younger females?

Estimation and Unbiased Estimators

1. For i = 1, 2, ..., n, if each Xi is a random variable having a normal distribution with mean μ and variance 1, then show that the following is an unbiased estimator of μ^2: estimator = (ΣXi)^2 - n / n^2.
2. For the sample x1, x2, ..., xn drawn on X which takes the values 1 and 0 with respective probabilities p and 1 - p, show that Σxi (Σxi -1) / n(n-1) is an unbiased estimator of p^2.

Estimation and Confidence Intervals

1. Suppose a random variable X represents the execution time (in milliseconds) of a machine learning algorithm on a dataset, which follows a normal distribution with a mean of 250

milliseconds and a standard deviation of 20 milliseconds. Determine the minimum sample size required to ensure that the margin of error for a 95% confidence interval does not exceed 2 milliseconds. If a random sample of 25 execution times is taken, what is the probability that the sample mean execution time, X, lies between 245 milliseconds and 255 milliseconds?

Bayes' Theorem

1. State and prove Bayes' theorem for probability.