S.O. + functional (only component A)

Let us go back to the information given by the initial condition. Given that the *complexity* is given by $\sim \frac{1}{q}$, the equation is:

$$k = n(1 - H(q))$$

and we may rewrite n in the form of α and n_0 as:

$$n = \frac{n_0}{1 - \alpha}$$

Now going back to the image of the Self Organised plus the functional Component.

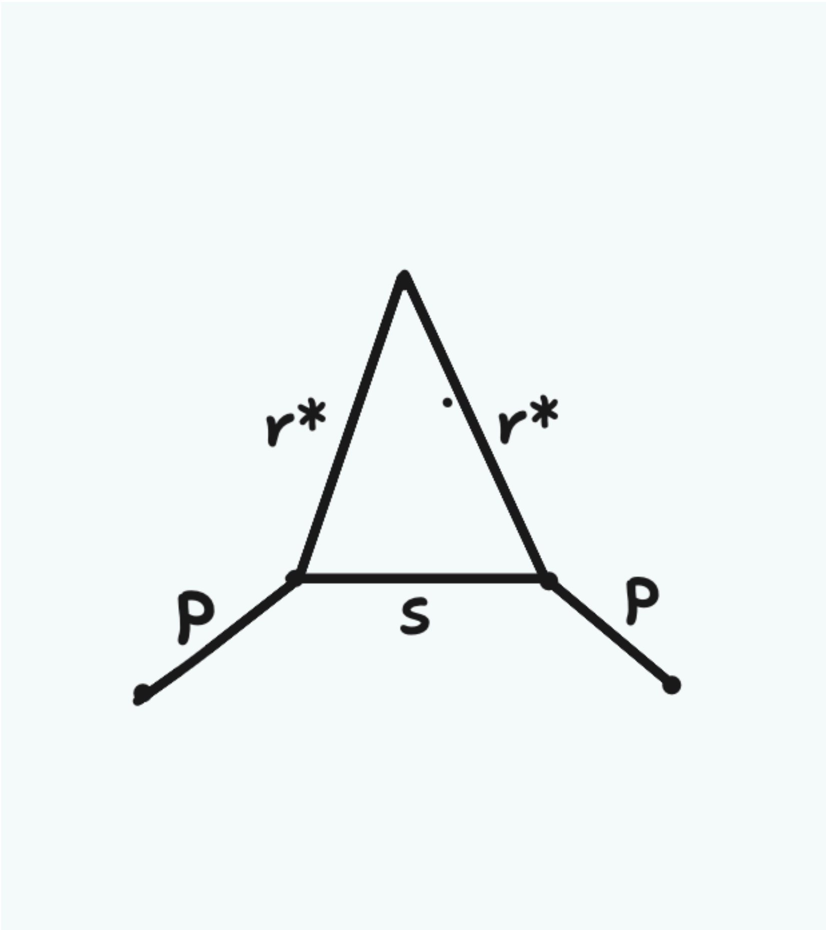

Figure 1: The constraint diagram for self organised copying with a functional constraint, note that the r^* in the figure is just $\hat{r}$. I didn't have the functionality to add hat on top of a variable in the paint software that was used to create the image.

We define r to be the folllowing (note that $r^* = \hat{r}$).

$$r = \hat{r} \oplus p$$
$$\hat{r} = \left(\frac{1}{2}\right) \frac{r - p}{\frac{1}{2} - p}$$

if the strings are generated iid with bernoulli trials, of length l_s each then the probability of finding a string with some empirical probaility distribution of the element as $\{q\}$ when the probability of the underlying thing is given by $\{p\}$ is:

$$\mathcal{P} = 2^{-l_s D_{KL}(\tilde{q} \| p)}$$

Thus the probability of finding two strings with the relative distance s with each other while both mutually being at a distance $\hat{r}$ from some other string (calculation can be done for a string of all zeros).

from the normalisation condition,

$$\tilde{q}_1 + \tilde{q}_2 + \tilde{q}_3 + \tilde{q}_4 = 1$$

and the conditions on relative distances will give three more equations,

$$\hat{r} = \tilde{q}_2 + \tilde{q}_3$$

$$\hat{r} = \tilde{q}_3 + \tilde{q}_4$$

$$s = \tilde{q}_2 + \tilde{q}_3$$

one can solve for $\tilde{q}$'s and then get the following,

$$\begin{aligned}\tilde{q}_1 &= 1 - \frac{1}{2}(2\hat{r} + s),\\ \tilde{q}_2 = \tilde{q}_3 &= \frac{s}{2},\\ \tilde{q}_4 &= \hat{r} - \frac{s}{2}\end{aligned}$$

for a randomly drawn string with bernulli trial and $p_0 = p_1 = \frac{1}{2}$ then, we can compute the $D_{KL}(q\|p)$ easily,

$$\begin{aligned}D(q\|p) &= \sum q_i \log\left(\frac{q_i}{p_i}\right)\\ D(q\|p) &= \sum q_i \log(4q_i)\\ D(q\|p) &= 2 + \sum q_i \log(q_i)\\ D(q\|p) &= 2 - H(\{q\})\end{aligned}$$

thus the probability of finding these strings is:

$$\mathcal{P} = 2^{-\alpha n(2 - H(\{\tilde{q}\}))}$$

$$\log\left(\frac{1}{\mathcal{P}}\right) = \alpha n(2 - H(\tilde{q}))$$

and the number of initial condition and genomic information bits:

$$\begin{aligned}k + l &= \alpha n(2 - H(\{\tilde{q}_i\}))\\ l &= \alpha n(2 - H(\{\tilde{q}_i\})) - k\\ l &= n(\alpha(2 - H(\{\tilde{q}_i\})) - (1 - H(q)))\\ l &= \left(\frac{n_0}{1-\alpha}\right)[\alpha[2 - H(\{\tilde{q}_i\})] - [1 - H(q)]]\end{aligned}$$