Set Theory and Notations Particulars

* **Set :-** A set is any well defined collection of distinct or distinguisable elements by a well defined collection means that there exists a rule with the help of which we should be able to say whether any given object belongs to or not to the collection under some specific rule.

The set of students in a class.
The set of months in a year.
$A = \{1, 2, 3\}$.

Generally capital letters A, B, C, X, Y, Z etc are used to denote sets and its elements by lowercase letters a, b, c, x, y, z - - etc.

* **Elements of a set :-**

If $A = \{1, 2, 3\}$ the objects in a set are called elements or members.
The distinct elements means no element is repeated.
The Distinguishable means that given any object or element is either in the set or not in the set. so the elements of a set must be distinct and distinguishable.

The symbol $\in$ (epsilon) is used to indicate "belongs to" if 1 is the element of set A then symbolically we write

$$1 \in A$$

The symbol $\notin$ is used to indicate "does not belongs to" if 4 is not an element of set A then symbolically we write -

$$4 \notin A$$

* **Symbols :-**

N = Natural Numbers $= \{1, 2, 3, - - - - \}$
I or Z = set of Integers $= \{ - - - -2, -1, 0, 1, 2, - - - \}$
Z^+ = set of Positive Integers $= \{ 0, 1, 2, - - \}$
Z^- = set of Negative Integers $= \{ - - - -, -2, -1 \}$.
R = set of Real Numbers.
Q = Set of Rational Numbers (where $\frac{p}{q}$, $q \neq 0$)
C = set of Complex Numbers.

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Set Representation :-

Set are represented by two forms:-

- Roster or Enumeration or Tabular form.
- Set builder form.

- **Roster form :-** In this method the elements of a set are described by listing or written them within braces seperated by comma (,).

E.g. The set of vowel in English alphabet can be written as-

$V = \{a, e, i, o, u\}$

The set of Natural number.

$N = \{1, 2, 3, ----\}$.

- **Set builder form :-** In this method the elements of a set are represented by stating a property that uniquely characterised them.

E.g. Let consider the sets

$A = \{1, 2, 3\}$ $B = \{2, 4, 6, 8 ----\}$ $C = \{a, e, i, o, u\}$

$A = \{x : x \in N \ \& \ x < 4\}$.

$B = \{x : x = 2n \ \& \ n \in N\}$

OR

$= \{x : x$ is an even natural number$\}$.

OR

$= \{x : x$ is an even positive integer$\}$.

$C = \{x : x$ is a vowel in English alphabet$\}$.

* Types of Set :-

◉ Cardinality of set (OR) Cardinal No. of set :-

The number of distinct elements in a set is called cardinal no. of set.

The number of distinct elements in set A is called cardinal no. of set it is represented as -

$O(A)$ (OR) $n(A)$ (OR) $|A|$.

$A = \{1, 2, 3\}$

$\boxed{n(A) = 3.}$

- **Finite Set :-** A set is finite if it contains finite no. of

The set of rivers in a India.

- Infinite Set :- A set which is not finite is called Infinite set.

  e.g. Set of Natural Numbers.

- Singelton Set :- A set which contains only 1 element is called singelton set.

  e.g. $A = \{ x : 4 < x < 6 \ \& \ x \in N \}$

  $A = \{ \phi \}$.

- Null set (OR) Empty (OR) Void set :- A set which contain no or zero elements is called Null set.

  This set is denoted by ϕ (OR) $\{ \}$.

  Cardinality of null set is always zero.

- Equality of sets (OR) Equal Sets :- Two sets A and B are said to be equal if every element of set A is an element of B and also every element of set B is an element of A. The equality of two sets A and B are denoted by -

  $\boxed{A = B}$.

  symbolically, $A = B$ if and only if $\boxed{x \in A \iff x \in B}$

- Equivalence set :- If the elements of 1 set can be put in one to one maping with the elements of another set then two sets are called equivalence set.

  It is denoted by $\sim$ (OR) $\equiv$ (OR) $\cong$.

  If the elements of sets are same then it is known as equivalent set

* Subset :- Let A and B be <u>two non-empty sets</u>. The set A is subset of B if and only if every element of set A is an element of set B.

  subset is denoted by "$\subseteq$".

  symbolically, $A \subseteq B$ iff $x \in A \implies x \in B$.

  e.g. $A = \{1, 2\}$

  $B = \{1, 2, 3\}$.

  Here, $A \subseteq B$

  A is contained in B.

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* **Superset** :- It is denoted by "$\supseteq$"
If A is subset of B then, B is superset of A.
$B \supseteq A$.

* **Properties of Subset** :-
- ϕ is the subset of every set.
- If $A \subseteq B$ & $B \subseteq A$, then $A = B$.
- If $A \subseteq B$ & $B \subseteq C$ then $A \subseteq C$. (A is also subset of C).

* **Power Set** :- The set of all possible subsets of set A is called Power set of set A. Denoted by "P(A)".

e.g. $A = \{1, 2\}$

$P(A) = \{\{\phi, \{1\}, \{2\}, \{1,2\}\}$.

$A = \{1, 2, 3\}$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}; \{1,2\}, \{1,3\}; \{2,3\}, \{1,2,3\}\}$.

NOTE :- If a set has 'n' elements then '2^n' subsets of that set are possible

* **Proof** :- Let Set A has n elements
So, no. of subsets taking one element $= {}^nC_1$
No. of subsets taking two element $= {}^nC_2$
No. of subsets taking three element $= {}^nC_3$.
" " " " " "
" " " " " " "
No. of subsets taking n element $= {}^nC_n$.

We know that ϕ is subset of every set.
So, Total no. of subsets

$$= 1 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n.$$
$$= {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n$$
$$= (1+1)^n$$
$$= (2)^n.$$

Prob. Find P(A) for the following sets.

1) $A = \{a\}$. $P(A) = \{\phi, \{a\}\}$
2) $A = \{\phi, \{\phi\}\}$. $P(A) = \{\phi, \{\phi\}, \{\phi, \{\phi\}\}, \{\{\phi\}\}\}$
3) $A = \{\phi\}$. $P(A) = \{\phi, \{\phi\}\}$
4) $A = \{\{a\}\}$ $P(A) = \{\phi, \{\{a\}\}\}$
5) $A = \phi$ $P(A) =$ [illegible]

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- **Union :-** Let A and B be two sets non-empty sets the union of A and B is the set of all elements which are either in A or in B or in both A and B. and the union of A and B is denoted by 'A ∪ B'. It is also known as Joint or logical sum of A or B.

Symbolically,

$$A \cup B = \{ x : x \in A \text{ or } x \in B \}.$$

e.g. $A = \{1, 2, 3\}$ $\quad B = \{1, 2, 4, 5\}$.

$A \cup B = \{1, 2, 3, 4, 5\}$.

- **Properties of Union of Set :-**

★ **Commutative Property :-** Union of sets is commutative.
i.e. $A \cup B = B \cup A$.

<u>Proof</u> :- Let $x \in (A \cup B)$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in B$ or $x \in A$

$\Rightarrow x \in (B \cup A)$

So, $(A \cup B) \subseteq (B \cup A)$ —— (1).

Let $x \in (B \cup A)$

$\Rightarrow x \in B$ or $x \in A$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in (A \cup B)$.

So, $(B \cup A) \subseteq (A \cup B)$ —— (2)

from eqn (1) & (2)

$$\boxed{A \cup B = B \cup A}$$

★ **Associative Property :-** Union of sets is Associative.
ie $A \cup (B \cup C) = (A \cup B) \cup C$.

<u>Proof</u> :- Let $x \in (A \cup (B \cup C))$

$\Rightarrow x \in A$ or $x \in (B \cup C)$

$\Rightarrow x \in A$ or $(x \in B$ or $x \in C)$

$\Rightarrow (x \in A$ or $x \in B)$ or $x \in C$.

$\Rightarrow x \in (A \cup B)$ or $x \in C$.

$\Rightarrow x \in ((A \cup B) \cup C)$

So, $A \cup (B \cup C) \subseteq (A \cup B) \cup C$ —— (1)

Let $x \in ((A \cup B) \cup C)$

$\Rightarrow x \in (A \cup B)$ or $x \in C$.

$\Rightarrow (x \in A$ or $x \in B)$ or $x \in C$

$\Rightarrow x \in A$ or $(x \in B$ or $x \in C)$

$\Rightarrow x \in A$ or $x \in B \cup C$

$\Rightarrow x \in A \cup (B \cup C)$

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So, $(A \cup B) \cup C \subseteq A \cup (B \cup C)$ —— (2)

From eqn (1) & (2),

$$\boxed{A \cup (B \cup C) = (A \cup B) \cup C}$$

* Idempotent Property :- Union of sets is idempotent.
i.e. $A \cup A = A$.

Proof :- Let, $x \in A \cup A$

$\Rightarrow x \in A$ or $x \in A$

$\Rightarrow x \in A$.

So, $(A \cup A) \subseteq A$ —— (1)

Let $x \in A$.

$\Rightarrow x \in A$ or $x \in A$.

$\Rightarrow x \in (A \cup A)$

So, $A \subseteq (A \cup A)$ —— (2)

From eqn (1) & (2) $\boxed{A \cup A = A}$

* Let A and B be two non-empty sets.

$A = \{1, 2, 3\}$

$B = \{1, 2, 4, 5\}$

$A \cup B = \{1, 2, 3, 4, 5\}$.

$\Rightarrow A \subseteq (A \cup B)$

$\Rightarrow B \subseteq (A \cup B)$

* $A \cup \phi = A$

$A \cup U = U$

* 1) $A \subseteq B$ then $A \cup B = B$.

Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$.

$\Rightarrow x \in B$ or $x \in B$ ($\because A \subseteq B$ then $x \in A \Rightarrow x \in B$)

$\Rightarrow x \in B$.

So, $A \cup B \subseteq B$ —— (1)

We know that, $B \subseteq A \cup B$. —— (2)

From (1) & (2)

$$\boxed{A \cup B = B}$$

e.g. $A = \{1, 2\}$

$B = \{1, 2, 3\}$

$A \cup B = \{1, 2, 3\}$

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is the set of elements which belongs to both A and B.
(common to both A and B).
The intersection of A and B is denoted by '$A \cap B$.'
Symbolically,
$$A \cap B = \{x : x \in A \text{ and } x \in B\}.$$

e.g. $A = \{1, 2, 3\}$
$B = \{1, 3, 5\}$.
$A \cap B = \{1, 3\}$.

Properties of Intersection of Sets :-

Commutative Property :- Intersection of sets is commutative.
i.e. $A \cap B = B \cap A$.

Proof :- Let $x \in A \cap B$
$\Rightarrow x \in A$ and $x \in B$.
$\Rightarrow x \in B$ and $x \in A$.
$\Rightarrow x \in (B \cap A)$
So, $A \cap B \subseteq B \cap A$. —— (1)

Let $x \in B \cap A$
$\Rightarrow x \in B$ and $x \in A$
$\Rightarrow x \in A$ and $x \in B$
$\Rightarrow x \in (A \cap B)$
So, $B \cap A \subseteq A \cap B$ —— (2)

From eqn (1) & (2) -
$$\boxed{A \cap B = B \cap A}$$

Associative Property :- Intersection of sets is Associative.
i.e. $A \cap (B \cap C) = (A \cap B) \cap C$.

Proof :- Let $x \in A \cap (B \cap C)$
$\Rightarrow x \in A$ and $x \in B \cap C$
$\Rightarrow x \in A$ and ($x \in B$ and $x \in C$)
$\Rightarrow x \in A \cap B$ and $x \in C$.
$\Rightarrow x \in (A \cap B) \cap C$.
So, $A \cap (B \cap C) \subseteq (A \cap B) \cap C$. —— (1)

Let $x \in (A \cap B) \cap C$.
$\Rightarrow x \in (A \cap B)$ and $x \in C$.
$\Rightarrow$ ($x \in A$ and $x \in B$) and $x \in C$.
$\Rightarrow x \in A$ and $x \in (B \cap C)$
$\Rightarrow x \in A \cap (B \cap C)$
So, $(A \cap B) \cap C \subseteq A \cap (B \cap C)$ —— (2)

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from eqn ① & ② -

$$\boxed{A \cap (B \cap C) = (A \cap B) \cap C}$$

Property :- Intersection of sets is Idempotent
i.e. $A \cap A = A$.

Proof :-

Let $x \in A \cap A$

$\Rightarrow x \in A$ and $x \in A$.

$\Rightarrow x \in A$.

So, $A \cap A \subseteq A$ —— ①

Let $x \in A$.

$\Rightarrow x \in A$ and $x \in A$.

$\Rightarrow x \in (A \cap A)$

So, $A \subseteq A \cap A$ —— ②

From eqn ① & ②

$$\boxed{A \cap A = A}$$

$A = \{1, 2, 3\}$
$B = \{1, 3, 5\}$.

$A \cap B = \{1, 3\}$.

$\Rightarrow A \cap B \subseteq A$

$\Rightarrow A \cap B \subseteq B$.

$A \cap \phi = \phi$

$A \cap U = A$.

NOTE: [illegible]

- Compliment of sets :- Let U be the universal set and A be any subset of universal set the compliment of A is a set containing elements of universal set which do not belong to A.
The compliment of A is denoted by A' (OR) A^c (OR) $\bar{A}$.

e.g. $U = \{1, 2, 3, 4, 5, 6\}$
$A = \{1, 4, 6\}$.
$A' = \{2, 3, 5\}$.

Symbolically,
$A' = \{x : x \in U$ and $x \notin A\}$.

NOTE: $x \in A^c$ (OR) $x \notin A$ [illegible] same.

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difference of [illegible]
those elements which belongs to A but does not belongs to B.
It is denoted by 'A − B' (or) 'A / B'.
Symbolically,

$$A - B = \{ x : x \in A \text{ and } x \notin B \}$$

e.g. $A = \{1, 2, 3\}$
$B = \{2, 3, 4, 5\}$.

$A - B = \{1\}$
$B - A = \{4, 5\}$.

NOTE:- A B : A − [illegible]
B − A : B − [illegible]

$\boxed{A - B \neq B - A}$

* Symmetric Difference of Sets :- Let A and B be two sets then symmetric difference of A and B is a set containing all the elements that belong to A or B but not both.
It is denoted by 'A ⊕ B'.

e.g. $A = \{1, 2, 3, 4, 5\}$
$B = \{4, 5, 6\}$

$A \oplus B = \{1, 2, 3, 6\}$.

$A \oplus B = (A \cup B) - (A \cap B)$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$
$A \cap B = \{4, 5\}$

$A \oplus B = (A \cup B) - (A \cap B) = \{1, 2, 3, 6\}$.

* Disjoint Sets :- Let A and B be two sets then, if there is no common element b/w A and B then, they are said to be disjoint sets.

e.g. $A = \{1, 2, 3\}$
$B = \{4, 5, 6\}$.

If $A \cap B = \Phi$ Then A and B are disjoint sets.

* Proper Subset :- Let A and B be two non-empty sets and $A \subseteq B$ if there is atleast one element in B which does not belong to A then, A is called proper subset of B and it is denoted by '⊂' ($A \subset B$)

* Venn Diagram :- Venn Diagram is a pictorial representation of sets which are used to show relationships ~~b/w~~ among sets.
The universal set is represented by interior of a rectangl and its subsets are represented by circular areas within the rectangle.

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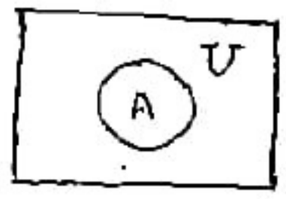
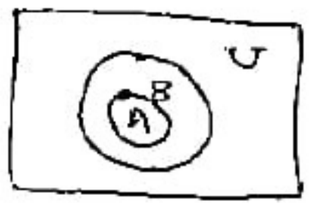
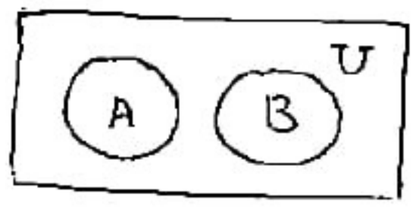
$A \subseteq U$



$A \subseteq B$

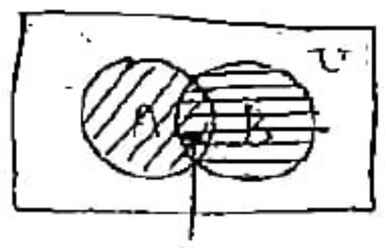


Disjoint sets

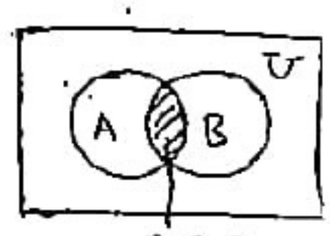


$A \cap B \neq \phi$

$A \cup B$
$A \cap B$.

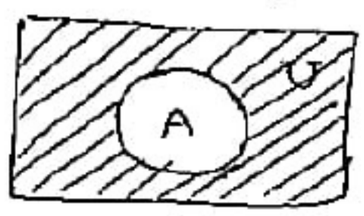


$A \cap B$



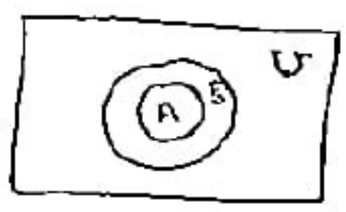
$A \cap B$

A^c



$A - B$

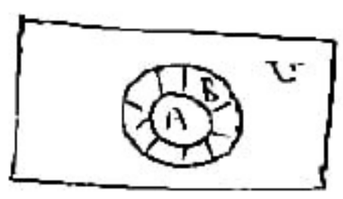
$A \subseteq B$



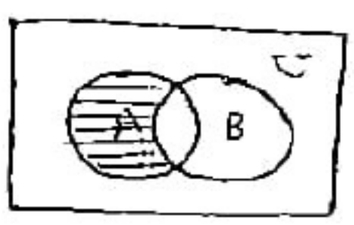
No shaded area means
So, $A - B = \phi$.

$B - A$.

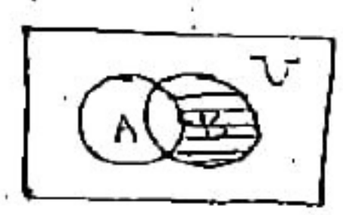
$A \subseteq B$



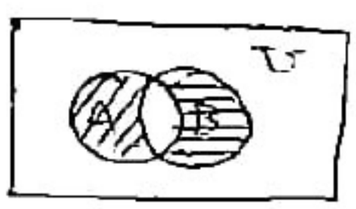
$A - B$



$B - A$



$A \oplus B$

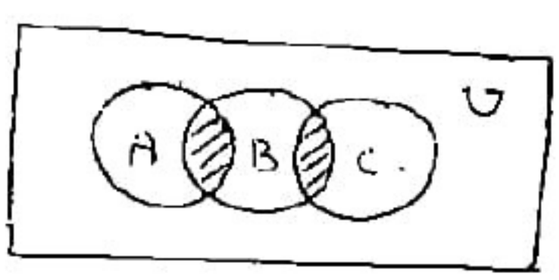


NOTE:

$A \oplus B = (A - B) \cup (B - A)$

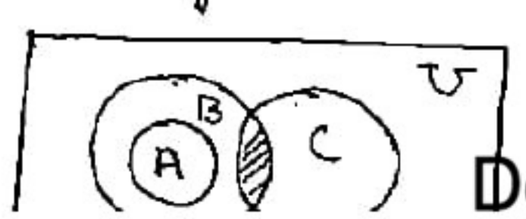
Q. Draw a venn diagram of sets A, B & C where - A and B have common elements; B and C have common elements but A and C are disjoint.

Soln



Q $A \subseteq B$, set A and C are disjoint but B and C have elements in common.

Soln.



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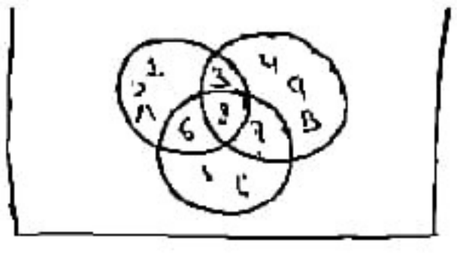


1) $A - (B \cap C)$
2) $(A \cup B) - C$
3) $B \oplus C$
4) $A - (B - C)$
5) $A - (B \oplus C)$
6) $A \cap (B \oplus C)$

1) $A - (B \cap C) = \{1, 2, 3, 8, 6\}$

2) $(A \cup B) - C = \{1, 2, 3, 4, 9\}$

3) $(B \oplus C) = \{6, 3, 4, 5, 9\}$

4) $A - (B - C) = \{1, 2, 6, 8\}$.

5) $A - (B \oplus C) = \{1, 2, 8\}$.

6) $A \cap (B \oplus C) = \{6, 3\}$.

$A = \{1, 2, 6, 3, 8$

$B = \{3, 4, 7, 8, 9$

$C = \{5, 6, 7, 8\}$.

$B \cap C = \{7, 8\}$

$A \cup B = \{1, 2, 3, 4, 6$

$B \oplus C = (B \cup C) - (B$

or $(B - C) \cup (C$

$B - C = \{3, 4, 9\}$

* Algebra of Sets :-

- Idempotent Law :- 1) $A \cup A = A$
  2) $A \cap A = A$.
- Commutative Law :- 1) $A \cup B = B \cup A$
  2) $A \cap B = B \cap A$.
- Associative Law :- 1) $A \cup (B \cup C) = (A \cup B) \cup C$.
  2) $A \cap (B \cap C) = (A \cap B) \cap C$.
- Identity Law :- 1) $A \cup \phi = A$
  2) $A \cap U = A$.
- Bound Law :- 1) $A \cup U = U$ (Universal)
  2) $A \cap \phi = \phi$. (Null Law).
- Compliment Law :- 1) $U^c = \phi$
  2) $\phi^c = U$
  3) $A \cup A^c = U$
  4) $A \cap A^c = \phi$.
- Involution Law :- $(A^c)^c = A$.
- Distributive Law :-

Q Prove that union of sets is distributive over intersect of sets and vice-versa.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

<u>Prove</u> :- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

<u>Proof</u> :- Let $x \in (A \cup (B \cap C))$

$\Rightarrow x \in A$ or $x \in (B \cap C)$

$\Rightarrow x \in A$ or $x \in B$ and $x \in C$.

Downloaded from TwoWaits

$\Rightarrow (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C)$

$\Rightarrow (x \in A \cup B) \text{ and } (x \in A \cup C)$.

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$ —— (1)

So, $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

Let, $x \in (A \cup B) \cap (A \cup C)$

$\Rightarrow x \in (A \cup B) \text{ and } x \in (A \cup C)$

$\Rightarrow (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C)$

$\Rightarrow x \in A \text{ or } x \in B \text{ and } x \in C$.

$\Rightarrow x \in A \text{ or } x \in B \cap C$.

$\Rightarrow x \in A \cup (B \cap C)$

So, $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$ —— (2)

from eqn (1) & (2)

$\Rightarrow \boxed{A \cup (B \cap C) = (A \cup B) \cap (A \cup C).}$

<u>Prove</u> $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

<u>Proof</u>:

Let $x \in A \cap (B \cup C)$.

$\Rightarrow x \in A \text{ and } x \in (B \cup C)$

$\Rightarrow x \in A \text{ and } x \in B \text{ or } x \in C$.

$\Rightarrow (x \in A \text{ and } x \in B) \text{ or } (x \in A \text{ and } x \in C)$

$\Rightarrow x \in (A \cap B) \text{ or } x \in (A \cap C)$.

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

So, $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ —— (1)

Let, $(A \cap B) \cup (A \cap C) \ni x$.

$\Rightarrow x \in (A \cap B) \text{ or } x \in (A \cap C)$

$\Rightarrow (x \in A \text{ and } x \in B) \text{ or } (x \in A \text{ and } x \in C)$

$\Rightarrow x \in A \text{ and } (x \in B \text{ or } x \in C)$

$\Rightarrow x \in A \text{ and } x \in (B \cup C)$

$\Rightarrow x \in A \cap (B \cup C)$

So, $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ —— (2)

From eqn (1) & (2)

$\Rightarrow \boxed{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$

Downloaded from TwoWaits

Demorgan's law :- 1) $(A \cup B)^c = A^c \cap B^c$.

2) $(A \cap B)^c = A^c \cup B^c$.

<u>Prove</u>:- $(A \cup B)^c = A^c \cap B^c$.

<u>Proof</u>:- Let $x \in (A \cup B)^c$.

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$.

$\Rightarrow x \in A^c$ and $x \in B^c$.

$\Rightarrow x \in (A^c \cap B^c)$

So, $(A \cup B)^c \subseteq (A^c \cap B^c)$ —— (1).

$\{ x \in A^c$
$x \notin A. \}$.

{ In negation for union we use and & for intersection we use or }

Let, $x \in A^c \cap B^c$

$\Rightarrow x \in A^c$ and $x \in B^c$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \in (A \cup B)^c$.

So, $(A^c \cap B^c) \subseteq (A \cup B)^c$ —— (2).

from eqn (1) & (2) —

$\Rightarrow \boxed{(A \cup B)^c = (A^c \cap B^c)}$

<u>Prove</u>: $(A \cap B)^c = A^c \cup B^c$.

<u>Proof</u>: Let $x \in (A \cap B)^c$

$\Rightarrow x \notin (A \cap B)$

$\Rightarrow x \notin A$ and $x \notin B$.

$\Rightarrow x \in A^c$ or $x \in B^c$.

$\Rightarrow x \in A^c \cup B^c$.

So, $(A \cap B)^c \subseteq (A^c \cup B^c)$ —— (1)

Let, $x \in A^c \cup B^c$.

$\Rightarrow x \in A^c$ or $x \in B^c$.

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \notin (A \cap B)$

$\Rightarrow x \in (A \cap B)^c$

So, $(A^c \cup B^c) \subseteq (A \cap B)^c$ —— (2)

From eqn (1) & (2)

$\Rightarrow \boxed{(A \cap B)^c = A^c \cup B^c}$

Downloaded from TwoWaits

Q ~~Show~~ Prove that $(A-B)-C = A-(B\cup C)$.

Let $x \in (A-B)-C$

$\Rightarrow x \in (A-B)$ and $x \notin C$.

$\Rightarrow (x \in A$ and $x \notin B)$ and $x \notin C$.

$\Rightarrow x \in A$ and $(x \notin B$ and $x \notin C)$

$\Rightarrow x \in A$ and $(x \notin (B\cup C))$

$\Rightarrow x \in A-(B\cup C)$

So, $(A-B)-C \subseteq A-(B\cup C)$ —— (1)

Let, $x \in A-(B\cup C)$

$\Rightarrow x \in A$ and $x \notin (B\cup C)$

$\Rightarrow x \in A$ and $(x \notin B$ and $x \notin C)$

$\Rightarrow (x \in A$ and $x \notin B)$ and $x \notin C$

$\Rightarrow x \in A-B$ and $x \notin C$

$\Rightarrow x \in (A-B)-C$.

So, $A-(B\cup C) \subseteq (A-B)-C$. —— (2)

From eqn (1) & (2)

$\Rightarrow \boxed{(A-B)-C = A-(B\cup C)}$

Q Prove that $(A-B)\cap(B-A) = \phi$.

Let, $x \in (A-B)\cap(B-A)$

$\Rightarrow x \in (A-B)$ and $x \in (B-A)$.

$\Rightarrow (x \in A$ and $x \notin B)$ and $(x \in B$ and $x \notin A)$

$\Rightarrow (x \in A$ and $x \notin A)$ and $(x \in B$ and $x \notin B)$.

$\Rightarrow x \in \phi$ and $x \in \phi$.

$\Rightarrow x \in \phi$.

So, $(A-B)\cap(B-A) \subseteq \phi$. —— (1)

Let, We know that ϕ is subset of every set.

$\phi \subseteq (A-B)\cap(B-A)$ —— (2)

From (1) & (2)

$\Rightarrow \boxed{(A-B)\cap(B-A) = \phi.}$

Q Prove that $A-(A\cap B) = (A-B)$

Let, $x \in A-(A\cap B)$

$\Rightarrow x \in A$ and $x \notin (A\cap B)$

$\Rightarrow x \in A$ and $(x \notin A$ or $x \notin B)$

$\Rightarrow (x \in A$ and $x \notin A)$ or $(x \in A$ and $x \notin B)$

$\Rightarrow x \in \phi$ or $x \in (A-B)$

$\Rightarrow x \in \phi \cup (A-B)$

$\Rightarrow x \in (A-B)$

So, $A-(A\cap B) \subseteq (A-B)$ —— (1)

Downloaded from TwoWaits

$\Rightarrow$ $x \in A$ and $x \notin B$

$\Rightarrow$ $x \in \phi \cup (A-B)$

$\Rightarrow$ $x \in \phi$ or $x \in (A-B)$.

$\Rightarrow$ $x \in \phi$ or $x \in A$ and $x \notin B$.

$\Rightarrow$ $(x \in \phi$ and $x \notin A)$ or $(x \in A$ and $x \notin B)$.

$\Rightarrow$ $x \in A$ and $(x \notin A$ or $x \notin B)$

$\Rightarrow$ $x \in A$ and $x \notin (A \cap B)$

$\Rightarrow$ $x \in A - (A \cap B)$.

So, $(A-B) \subseteq A - (A \cap B)$ —— (2)

$\Rightarrow$ $\boxed{A - (A \cap B) \subseteq A - B}$ From (1) & (2).

Q If A and B are two sets then $(A \cap B) \cup (A \cap \sim B)$ and $A \cap (\sim A \cup B)$ are equal to ?

$\sim B$ = compliment

$\sim (A \cup B) = (\sim A \cap \sim B)$

$(A \cap B) \cup (A \cap \sim B)$

$\Rightarrow$ $A \cap (B \cup \sim B)$ [By Distributive Law].

$\Rightarrow$ $A \cap U$ [By compliment Law].

$\Rightarrow$ A [By Identity Law].

$A \cap (\sim A \cup B)$

$\Rightarrow$ $(A \cap \sim A) \cup (A \cap B)$ [Distributive Law]

$\Rightarrow$ $\phi \cup (A \cap B)$ [By compliment Law]

$\Rightarrow$ $A \cap B$ [By Identity Law].

Q If A and B are two subsets of universal set then prove the following -

1) $A - B = B - A$ if & only if $A = B$.

2) $A - B = A$ iff $A \cap B = \phi$.

1) (i) If $A = B$ then $A - B = B - A$.

Let $x \in A - B$

$\Rightarrow$ $x \in A$ and $x \notin B$.

$\Rightarrow$ $x \in B$ and $x \notin A$.

$\Rightarrow$ $x \in (B - A)$

So, $(A-B) \subseteq B-A$ —— (1).

Let $x \in (B-A)$

$\Rightarrow$ $x \in B$ and $x \notin A$.

$\Rightarrow$ $x \in A$ and $x \notin B$.

$\Rightarrow$ $x \in (A-B)$

So, $(B-A) \subseteq (A-B)$ —— (2)

From (1) & (2).

$\Rightarrow$ $\boxed{(A-B) = (B-A)}$

Downloaded from TwoWaits

(1) If $(A-B) = (B-A)$ then $A = B$.

$x \in (B-A)$

$\Rightarrow x \in B$ and $x \notin A$. —— (1)

$x \in (A-B)$

$\Rightarrow x \in A$ and $x \notin B$. —— (2)

Eq^n (1) & (2) can be equal only when $A = B$.

(2) (i) ~~If $A \cap B = \phi$ then~~ If $A - B = A$ then $A \cap B = \phi$

~~$x \in A - B$.~~

~~$\Rightarrow x \in A$ and $x \notin B$.~~

Let, $A \cap B \neq \phi$

$\Rightarrow x \in (A \cap B)$

$\Rightarrow x \in (A-B) \cap B$. $\because [A - B = A]$

$\Rightarrow x \in A - B$ and $x \in B$.

$\Rightarrow (x \in A$ and $x \notin B)$, and $x \in B$.

$\Rightarrow x \in A$ and $(x \notin B$ and $x \in B)$

$\Rightarrow x \in A$ and $x \notin \phi$.

$\Rightarrow x \in A \cap \phi$.

$\Rightarrow x \in \phi$.

So, $A \cap B \subseteq \phi$. —— (1) But this contradicts our assumption $A \cap B = \phi$.

But, we know that ϕ is subset of every set ~~—— (2)~~

$\phi \subseteq A \cap B$. —— (2)

from eq^n (1) & (2)

$\boxed{A \cap B = \phi}$

(ii) If $A \cap B = \phi$ then $A - B = A$.

Let, $x \in A - B$.

$\Rightarrow x \in A$ and $x \notin B$.

$\Rightarrow x \in A$ $(\because A \cap B = \phi)$.

So, $A - B \subseteq A$ —— (1)

Let, $x \in A$

$\Rightarrow x \in A$ and $x \notin B$. $[\because A \cap B = \phi]$.

$\Rightarrow x \in (A-B)$

So, $A \subseteq A - B$ —— (2)

From (1) & (2)

$\boxed{A - B = A}$

(OR)

If $(A \cap B) = \phi$ then $A - B = A$

Downloaded from TwoWaits

Let, $x \in (A-B)$

$\Rightarrow x \in (A-B) \cup \phi$

$\Rightarrow x \in (A-B)$ or $x \in (A \cap B)$

$\Rightarrow (x \in A$ and $x \notin B)$ or $(x \in A$ and $x \in B)$

$\Rightarrow (x \in A$ and $x \in B')$ or $(x \in A$ and $x \in B)$.

$\Rightarrow x \in A$ and $(x \in B'$ ~~and~~ or $x \in B)$

$\Rightarrow x \in A$ and $x \in (B \cup B')$

$\Rightarrow x \in A$ and $x \in U$

$\Rightarrow x \in (A \cap U)$

$\Rightarrow x \in A$

So, $A - B \subseteq A$ —— ①.

* Cartesian Product :-

$A \times B = \{(x, y) : x \in A$ and $y \in B\}$.

e.g. Ordered Pair - $A = \{1, 2, 3\}$

$B = \{a, b\}$.

$A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$.

$B \times A = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

Ordered Pair :- It is a pair of objects formed by using the two components in specified order

In the ordered pair (x, y), x is the first component and y is the second component.

Consider two sets A and B, then cartesian product of A and B is denoted by '$A \times B$' and it is the set of all possible ordered pairs (x, y) with $x \in A$ and $y \in B$.

Symbolically,

$A \times B = \{(x, y) : x \in A$ and $y \in B\}$.

<u>NOTE</u>: $A \times B \neq B \times A$.

* Multiset :- Multisets are sets where an element appea more than ones.

e.g. $A = \{1, 1, 1, 2, 2, 3\}$.

multiset A can also be written as-

$A = \{3 \cdot 1, 2 \cdot 2, 1 \cdot 3\}$.

Multiplicity :- The multiplicity of an element in a multiset is defined to be the no. of times an element appears in the multiset.

3 is the multiplicity of 1.

2 " " " " 2.

1 " " " " 3.

Downloaded from TwoWaits

NOTE: It is a special type of multiset in which the multiplicity of every element is one or zero

- **Cardinality of Multiset :-** Cardinality of Multiset is equal to the cardinality of corresponding set

(OR)

Cardinality of Multiset is defined as the cardinality of its corresponding set. Assuming that all the elements are distinct in the ~~set~~ multiset.

So cardinality of multiset A = Cardinality of corresponding set A.

e.g. $A = \{1, 1, 1, 2, 2, 3\}$

$A = \{1, 2, 3\}$.

Cardinality of Multiset $A = 3$.

- **Operation in Multisets :-**

Let, $A = \{3 \cdot a, 2 \cdot b, 1 \cdot c\}$.

$B = \{2 \cdot a, 1 \cdot b, 1 \cdot d\}$.

Union of multisets (∪) :- $(A \cup B) = \{3 \cdot a, 2 \cdot b, 1 \cdot c, 1 \cdot d\}$

Intersection of multisets (∩) :- $A \cap B = \{2 \cdot a, 1 \cdot b\}$

Difference (−) :- $A - B = \{1 \cdot a, 1 \cdot b, 1 \cdot c\}$

Sum (+) :- $B - A = \{1 \cdot d\}$.

$A + B = \{5 \cdot a, 3 \cdot b, 1 \cdot c, 1 \cdot d\}$.

Union :- $A \cup B$ is the multiset where the multiplicity of an element is maximum of its multiplicities in A and B.

e.g. $A \cup B = \{3 \cdot a, 2 \cdot b, 1 \cdot c, 1 \cdot d\}$.

Intersection :- $A \cap B$ is the multiset where the multiplicity of an common element is minimum of its multiplicities in A and B.

e.g. $A \cap B = \{2 \cdot a, 1 \cdot b\}$.

Difference :- $A - B$ is the multiset where the multiplicity of an element is equal to multiplicity of element in A minus multiplicity of element in B, if the difference is positive but it is equal to zero if the difference is zero and negative.

e.g. $A - B = \{1 \cdot a, 1 \cdot b, 1 \cdot c\}$

$B - A = \{1 \cdot d\}$.

Sum :- $A + B$ is the multiset where multiplicity of an element is equal to the sum of multiplicities of an element in both multisets A and B.

e.g. $A + B = \{5 \cdot a, 3 \cdot b, 1 \cdot c, 1 \cdot d\}$.

Downloaded from TwoWaits

$Q = \{3\cdot a, 3\cdot b, 2\cdot d\}$.

find 1) $P \cup Q$, 2) $P \cap Q$, 3) $P - Q$, 4) $Q - P$, 5) $P +$

1) $P \cup Q = \{4\cdot a, 3\cdot b, 1\cdot c, 2\cdot d\}$.

2) $P \cap Q = \{3\cdot a, 3\cdot b\}$

3) $P - Q = \{1\cdot a, 1\cdot c\}$.

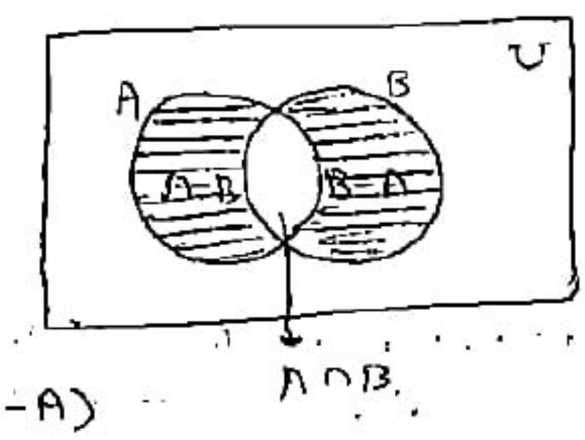
4) $Q - P = \{2\cdot d\}$.

5) $P + Q = \{7\cdot a, 6\cdot b, 1\cdot c, 2\cdot d\}$.

* Set Inclusion Exclusion Principle :-

(OR)

Counting Principle :-



$n(A) = n(A-B) + n(A \cap B)$ — (1)

$n(B) = n(B-A) + n(A \cap B)$ — (2)

$n(A \cup B) = n(A-B) + n(B-A) + n(A \cap B)$ — (3)

Put the values of $n(A-B)$ & $n(B-A)$ from eqn (1) & (2) in eqn (3).

$n(A \cup B) = n(A) - n(A \cap B) + n(B) - n(A \cap B) + n(A \cap B)$

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

Corollary :-

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Proof: Let $B \cup C = D$.

$$\begin{aligned}
n(A \cup D) &= n(A) + n(D) - n(A \cap D) \\
&= n(A) + n(B \cup C) - n[A \cap (B \cup C)] \\
&= n(A) + n(B) + n(C) - n(B \cap C) - n[(A \cap B) \cup (A \cap C)] \\
&= n(A) + n(B) + n(C) - n(B \cap C) - n(A \cap B) - n(A \cap C) \\
&\quad + n[(A \cap B) \cap (A \cap C)] \\
&= n(A) + n(B) + n(C) - n(B \cap C) - n(A \cap B) - n(A \cap C) \\
&\quad + n(A \cap B \cap C).
\end{aligned}$$

Downloaded from TwoWaits

Q 40 lecturers were interviewed for a job, 25 were mathematicians, 28 were physicist and 7 were niether. How many lecturers were mathematicians and physicists.

→ Let, M be the set of mathematicians and
P be the set of Physicists.

Given :- $n(M) = 25$
$n(P) = 28$.

Find :- $n(M \cap P) = ?$

Given :- $n(M \cup P) = 40 - 7 = 33$

$n(M \cup P) = n(M) + n(P) - n(M \cap P)$ {By set incl. ex. principle}

$n(M \cap P) = n(M) + n(P) - n(M \cup P)$

$= 25 + 28 - 33$

$= 53 - 33$

$\boxed{n(M \cap P) = 20}$

So, 20 lecturers were both mathematicians and physicists.

Q In a survey of 600 TV viewers given the following information.
1) 385 watch cricket matches.
2) 295 watch hockey matches.
3) 215 watch football matches.
4) 145 watch cricket & football matches both.
5) 170 watch cricket & hockey matches both.
6) 150 watch hockey & football matches both.
7) 150 does not watch any of the three games.

Find 1) How many people watch all three kind of matches.
2) How many people watch exactly one sport.

→ Let, C be the set of cricket viewers.
H be the set of Hockey viewers.
F be the set of football viewers.

Given :- $n(C) = 385$
$n(H) = 295$
$n(F) = 215$
$n(C \cap F) = 145$
$n(C \cap H) = 170$
$n(H \cap F) = 150$.
$n(C \cup H \cup F) = 600 - 150 = 450$

Now, $n(C \cup H \cup F) = n(C) + n(H) + n(F) - n(C \cap H) - n(C \cap F) - n(H \cap F) + n(C \cap H \cap F)$.

$450 = 385 + 295 + 215 - 170 - 145 - 150 + n(C \cap H \cap F)$

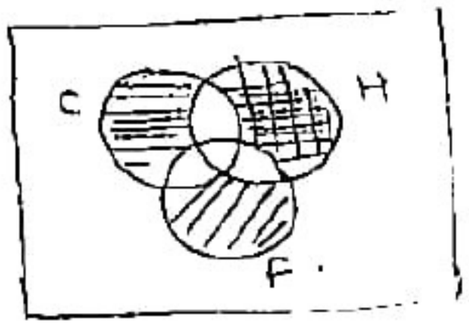
$450 = 895 - 465 + n(C \cap H \cap F)$

$450 = 430 + n(C \cap H \cap F)$

$\boxed{n(C \cap H \cap F) = 20}$. (By set Inc. Exc. Principle)

Downloaded from TwoWaits

2) Only cricket viewers.



$n(C) - n(C \cap H) - n(C \cap H \cap F) - n(C \cap F)$

$= 385 - 170 + 20 - 145$

$= 90.$

Only Hockey viewers.

$n(H) - n(C \cap H) + n(C \cap H \cap F) - n(H \cap F)$

$= 295 - 170 + 20 - 150.$

$= -5$

Only Football viewers.

$n(F) - n(C \cap F) + n(C \cap H \cap F) - n(H \cap F)$

$= 215 - 145 + 20 - 150.$

$= -60.$

No. of viewers exactly watch = n(exactly cricket) + n(exactly Hockey) + n(exactly Football).

$= 90 - 5 - 60.$

$= 25$

Q Among first 500 +ve integers. Find.

1) The no. of integers which are not divisible by 2 nor by 3 nor by 5.

2) The no. of integers which are exactly divisible by one of them.

→ Let ; A be the set of integers divisible by 2

B be the set of integers divisible by 3

C be the set of integers divisible by 5.

$n(A) = \left|\frac{500}{2}\right| = 250.$

$n(B) = \left|\frac{500}{3}\right| = 166.$

$n(C) = \left|\frac{500}{5}\right| = 100.$

$n(A \cap B) = \left|\frac{500}{2 \times 3}\right| = 83$

$n(B \cap C) = \left|\frac{500}{3 \times 5}\right| = 33.$

$n(A \cap C) = \left|\frac{500}{2 \times 5}\right| = 50.$

$n(A \cap B \cap C) = \left|\frac{500}{2 \times 3 \times 5}\right| = 16.$

1) $n(A' \cap B' \cap C') = ?$

$n(A' \cap B' \cap C') = n(A \cup B \cup C)'$

$= U - n(A \cup B \cup C)$

Downloaded from TwoWaits

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$= 250 + 166 + 100 - 83 - 33 - 50 + 16$

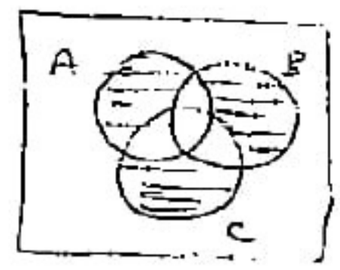
$= 366.$

$n(A' \cap B' \cap C') = n(U) - n(A \cup B \cup C)$

$= 500 - 366$

$= 134$

(2) Only divisible by 2.



$n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

$= 250 - 83 - 33 + 16$

$= 150$

Only divisible by 3.

$n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$

$= 166 - 83 - 50 + 16$

$= 49.$

Only divisible by 5

$n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$= 100 - 33 - 50 + 16.$

$= 33$

No. of Integers exactly divisible by one of them

$= n(\text{exactly divisible by } 2) + n(3) + n(5)$

$= 150 + 49 + 33$

$= 232$

Q 75 children went to a circus where they attend a magic show, comedy show and animal show. 20 of them attended all three shows and 55 attended atleast two shows. Each show cost 5 Rs. The total money collected 700 ₹. Find the no. of children who didnot attend any of three shows.

→ Let, M be the set of Magic show.
C be the set of comedy show.
A be the set of Animal show.

Total money collected = ₹ 700.

Cost per show = ₹ 5

Total no. of shows attended $= \frac{700}{5} = 140$

$n(M \cap C \cap A) = 20.$

No. of children attended atleast two shows = 55.

⇒ No. of children attended exactly two shows + No. of childrens attended all shows = 55.

⇒ No. of children attended Exactly 2 shows + 20 = 55.

⇒ No. of children attended exactly 2 shows = 35

Downloaded from TwoWaits

No. of children attended exactly 1 show $= 140 - (35 + 20 + 10)$

$= 140 - (70 + 60)$

$= 140 - 130$

$= 10$

No. of children did not attend any of 3 shows

$= 75 - (35 + 20 + 10)$.

$= 75 - (65)$

$= 10$.

* Countable and Uncountable Sets :-

Countable Infinite Set :- An infinite set A is said to be countably infinite or denumerable if it is equivalent to set N (set of Natural No.'s) i.e. if there exist a one to one maping

$f : N \to A$.

Countable set :- A set A is called countable when it is either

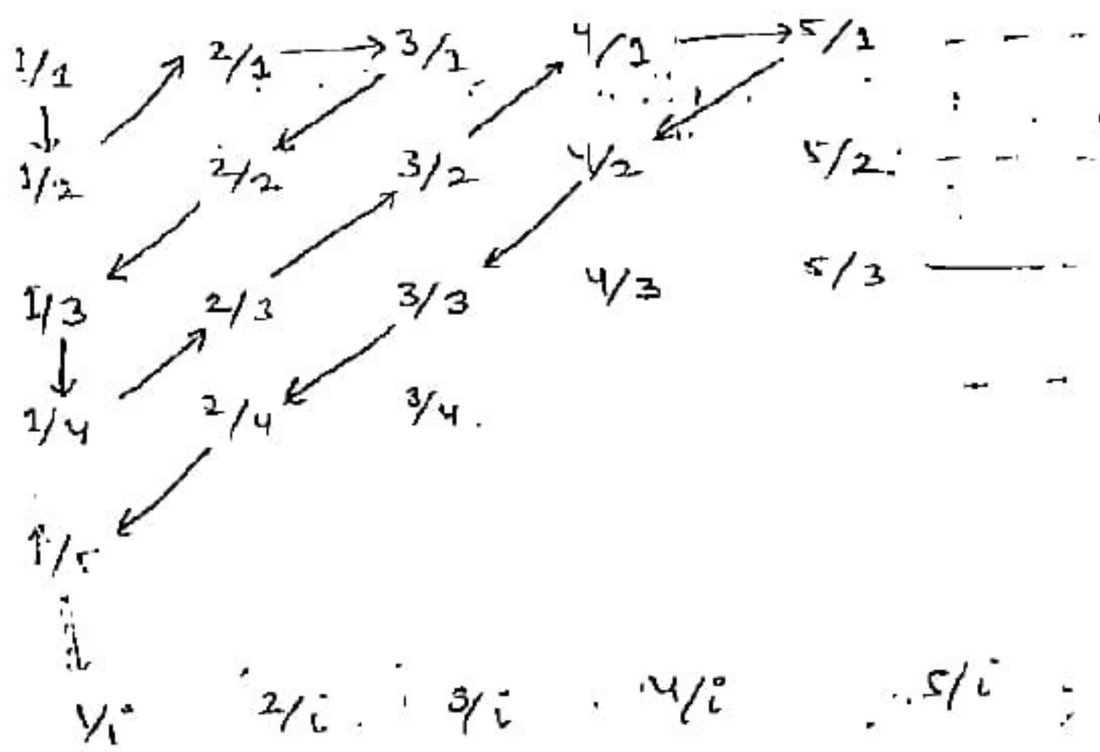
- Empty or Null set i.e. $A = \phi$. or
- Finite set or
- Countable Infinite set.

Natural No. : 1 2 3 4 5 6 7 8 ...

Even Natural No. : 2 4 6 8 10 12 14 16 ...

Q Show that the set of +ve rational no.'s is countable

$p + q = 2$, $p + q = 3$, $p + q = 4$.



1	2	3	4	5	6	7	8	...
↓	↓	↓	↓	↓	↓	↓	↓	
1	1/2	2	3	1/3	1/4	2/3	3/2	...

Q $A = \{ n^7 : n \in N \}$. Cardinality.

$A = \{ 1^7, 2^7, 3^7, 4^7 - \text{———} \}$

1	2	3	4	...
↓	↓	↓	↓	
1^7	2^7	3^7	4^7	...

$n(A)$ = This set is countable infinite.

Downloaded from TwoWaits

* Relation :-

Let A and B be two non-empty sets then R is a relation from A to B if $R \subseteq A \times B$. and it is the set of ordered pairs (a, b) where $a \in A$ and $b \in B$. It is denoted by $a R b$ or $(a,b) \in R$. and it is read as "a is related to b".

If there is no relation b/w a and b then $a \not R b$. or $(a,b) \notin R$. and it is read is "a is not related to b".

Symbolically,

$R = \{ (a,b) : a \in A \ \& \ b \in B$ and $a R b$ or $(a,b) \in R \}$.

Q e.g. $A = \{1, 2, 3, 4\}$, $B = \{1, 2\}$. A relation R is defined from A to B ie $a R b$ iff $a \times b$ = even no. Find relation R.

→ $R \subseteq A \times B$.

$A = \{1, 2, 3, 4\}$, $B = \{1, 2\}$.

$A \times B = \{(1,1), (1,2), (2,1), (2,2), (3,1), (3,2), (4,1), (4,2)\}$.

$R = \{(1,2), (2,1), (2,2), (3,2), (4,1), (4,2)\}$.

Q $A = \{2, 4, 6, 8\}$, $B = \{1, 2, 3\}$. Find the relation R such that $a R b$ iff $a \geq b$.

→ $A = \{2, 4, 6, 8\}$, $B = \{1, 2, 3\}$.

$A \times B = \{(2,1), (2,2), (2,3), (4,1), (4,2), (4,3), (6,1), (6,2), (6,3), (8,1), (8,2), (8,3)\}$

$R = \{(2,1), (2,2), (4,1), (4,2), (4,3), (6,1), (6,2), (8,1), (6,3), (8,2), (8,3)\}$.

Q $A = \{1, 2, 3, 4\}$ A relation R is defined on set A such that $a R b$ iff a is divisible by b.

→ $A = \{1, 2, 3, 4\}$

$A = \{1, 2, 3, 4\}$.

$A \times A = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$.

$R = \{(1,1), (2,1), (2,2), (3,1), (3,3), (4,1), (4,2), (4,4)\}$.

* Domain and Range of a Relation :-

Domain of a Relation :- It is the set consisting of all first elements of ordered pairs of relation R. It is denoted by $D(R)$ or $Dom(R)$.

Symbolically,

$D(R) = \{a : a \in A$ and $a R b\}$.

NOTE: $D(R) \subseteq A$.

Downloaded from TwoWaits

second elements of ordered pairs of relation R. It is denoted by R(R) or Ran(R).
Symbolically,

$$R(R) = \{ b : b \in B \text{ and } aRb \}.$$

NOTE: $R(R) \subseteq B$.

e.g. $A = \{1, 2, 3, 4\}$, $B = \{1, 2\}$.

Domain → $D(R) = \{1, 2, 3, 4\}$

Range → $R(R) = \{1, 2\}$.

$A = \{2, 4, 6, 8\}$, $B = \{1, 2, 3\}$.

Domain → $D(R) = \{2, 4, 6, 8\}$.

Range → $R(R) = \{1, 2, 3\}$

$A = \{1, 2, 3, 4\}$. $\{(s$

Domain → $D(R) = \{1, 2, 3, 4\}$.

Range → $R(R) = \{1, 2, 3, 4\}$.

Q Let $A = \{a, b, c\}$, $B = \{a, b, c, d\}$. R and S are two relatio defined from A to B such that $R = \{(a,b), (b,c), (c, \ldots$ $(a,a)\}$ and $S = \{(a,a), (a, d), (b, d)\}$. Find domain and range $(R \cup S)$, $(R \cap S)$ and $(R - S)$.

→ $R \cup S = \{(a,a), (a,b), (b,c), (c,d), (a,d), (b,d)\}$

$R \cap S = \{(a,a)\}$

$R - S = \{(a,b), (b,c), (c,d)\}$.

$D(R \cup S) = \{\cancel{(a,a), (a,b), (b,c), (c,d), (a,d), (b,d)}\}$.

$= \{a, b, c\}$

$R(R \cup S) = \{a, b, c, d\}$.

$D(R \cap S) = \{a\}$.

$R(R \cap S) = \{a\}$

$D(R - S) = \{a, b, c\}$

$R(R - S) = \{b, c, d\}$.

* Types of Relation :-

- **Universal relation** :- A relation R is called universal relation from A to B if $R = A \times B$.

For one set:

A relation R is called universal relation defined on set A. if $R = A \times A$.

e.g. $A = \{1, 2\}$

$R = \{(1,1), (1,2), (2,1), (2,2)\}$

Downloaded from TwoWaits

- **Identity Relation (OR) Diagonal Relation** :- A relation R is called identity relation on set A if every element of set is related to itself only.

It is denoted by 'I_A' or 'Δ_A' or 'Δ'.

$$I_A = \{(a, a) : \forall\ a \in A \text{ and } aRa\}.$$

e.g. $A = \{1, 2, 3\}$

$$I_A = \{(1,1), (2,2), (3,3)\}.$$

- **Inverse Relation** :- A relation defined from B to A is called Inverse relation if a relation R is defined from A to B. It is denoted by 'R^{-1}'.

Symbolically,

$$R^{-1} = \{(b, a) : b \in B,\ a \in A \text{ and } aRb\}.$$

e.g. $R = \{(1,1), (1,2), (1,3), (3,2)\}$

$$R^{-1} = \{(1,1), (2,1), (3,1), (2,3)\}.$$

- **Empty Relation** :- (OR) NULL (OR) VOID Relation :- A relation R is called empty relation if $R = \phi$.

e.g. $A = \{1, 2, 3\}$.

and aRb iff $a + b > 6$.

Here, $R = \phi$.

- **Compliment of Relation** :- If a relation is defined from A to B then compliment of R is set of ordered pairs such that

$$R^c = \{(a, b) : (a, b) \notin R\}.$$

$R^c = (A \times A) - R$. [Defined on one set]

$R^c = (A \times B) - R$. [Defined on two set].

It is denoted by R^c or R'.

e.g. $A = \{1, 2, 3\}$

$B = \{4, 5\}$

$R = \{(1,4), (2,4), (3,5)\}$.

$A \times B = \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)\}$.

$R^c = (A \times B) - R$.

$R^c = \{(1,5), (2,4), (2,5)\}$

Downloaded from TwoWaits

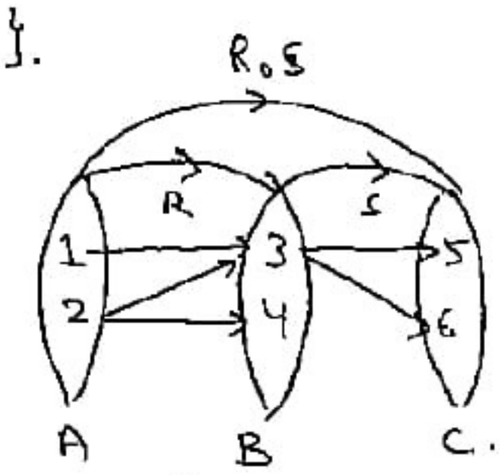
$R : A \to B$ $\qquad R = \{ (a,b) : aRb \}$

$S : B \to C$ $\qquad S = \{ (b,c) : bSc \}$

Let, R be the relation from A to B and S be the relation from B to C then composition of relation R and S is a relation consisting of odered pairs (a, c) where $a \in A$ and $c \in C$. provided that there exists some $b \in B$. such that aRb and bRc. It is denoted by '$R \circ S$'

Symbolically,

$$R \circ S = \{ (a,c) : \exists b \in B, aRb \text{ and } bSc \}.$$



e.g. $A = \{1,2\}$, $B = \{3,4\}$, $C = \{5,6\}$.

$R = \{ (1,3), (2,3), (2,4) \}$

$S = \{ (3,5), (3,6) \}$.

$R \circ S = \{ (1,5), (1,6), (2,5), (2,6) \}$.

Q. R and S relations are defined on set $A = \{1,2,3,4\}$ and

$R = \{ (1,2), (2,2), (2,3), (2,4), (3,2), (4,3) \}$.

$S = \{ (2,2), (2,3), (3,2), (3,3), (3,4), (4,2) \}$. Find $R \circ S$ & $S \circ R$

→ $R \circ S = \{ (1,2), (1,3), (2,2), (2,3), (2,4), (3,3), (3,2), (4$
$(4,3), (4,4) \}$.

$S \circ R = \{ (2,2), (2,3), (2,4), (3,3), (3,4), (3,2), (4,2), (4,3), (4,4$

Q. Prove that inverse of $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$.

→ Let, $(a,b) \in (R \cup S)^{-1}$

$\Rightarrow (b,a) \in (R \cup S)$.

$\Rightarrow (b,a) \in R$ or $(b,a) \in S$.

$\Rightarrow (a,b) \in R^{-1}$ or $(a,b) \in S^{-1}$.

$\Rightarrow (a,b) \in (R^{-1} \cup S^{-1})$. —— (1)

So, $(R \cup S)^{-1} \subseteq (R^{-1} \cup S^{-1})$

Let, $(a,b) \in (R^{-1} \cup S^{-1})$

$\Rightarrow (a,b) \in R^{-1}$ or $(a,b) \in S^{-1}$

$\Rightarrow (b,a) \in R$ or $(b,a) \in S$.

$\Rightarrow (b,a) \in (R \cup S)$

$\Rightarrow (a,b) \in (R \cup S)^{-1}$

So, $(R^{-1} \cup S^{-1}) \subseteq (R \cup S)^{-1}$ —— (2)

from eqn (1) & (2).

$\Rightarrow \boxed{(R \cup S)^{-1} = R^{-1} \cup S^{-1}}$

Downloaded from TwoWaits

Q Prove that $(R \cap S)^{-1} = (R^{-1} \cap S^{-1})$.

Let $(a, b) \in (R \cap S)^{-1}$

$\Rightarrow (b, a) \in (R \cap S)$

$\Rightarrow (b, a) \in R$ and $(b, a) \in S$.

$\Rightarrow (a, b) \in R^{-1}$ and $(a, b) \in S^{-1}$

$\Rightarrow (a, b) \in R^{-1} \cap S^{-1}$.

So, $(R \cap S)^{-1} \subseteq R^{-1} \cap S^{-1}$. — ①

Let, $(a, b) \in (R^{-1} \cap S^{-1})$

$\Rightarrow (a, b) \in R^{-1}$ and $(a, b) \in S^{-1}$

$\Rightarrow (b, a) \in R$ and $(b, a) \in S$.

$\Rightarrow (b, a) \in (R \cap S)$

$\Rightarrow (a, b) \in (R \cap S)^{-1}$

So, $(R^{-1} \cap S^{-1}) \subseteq (R \cap S)^{-1}$ — ②

From eqn ① & ②

$\boxed{(R \cap S)^{-1} = (R^{-1} \cap S^{-1})}$

Q If $R \subseteq S$ then, $R^{-1} \subseteq S^{-1}$.

Given, $R \subseteq S$.

Let, $(a, b) \in R^{-1}$

$\Rightarrow (b, a) \in R$.

$\Rightarrow (b, a) \in S$ $(\because R \subseteq S)$

$\Rightarrow (a, b) \in S^{-1}$

So, $R^{-1} \subseteq S^{-1}$.

Q Prove that composition of relation follows the associative law.

$\boxed{(R_o S)_o T = R_o (S_o T)}$

Proof :- Let, R be a relation from A to B. S be a relation from B to C. and T be a relation from C to D.

Let $(a, d) \in (R_o S)_o T$.

$\Rightarrow$ (such that) $\exists c \in C$, $(a, c) \in (R_o S)$ and $(c, d) \in T$. — ①

Now, $(a, c) \in R_o S$.

$\Rightarrow \exists b \in B$, $(a, b) \in R$ and $(b, c) \in S$ — ②

From eqn ① & ②,

$(b, c) \in S$ and $(c, d) \in T$.

$\Rightarrow (b, d) \in (S_o T)$ — ③.

From eqn ② & ③.

$(a, b) \in R$ and $(b, d) \in (S_o T)$

$\Rightarrow (a, d) \in R_o (S_o T)$ — ④

So, $(R_o S)_o T \subseteq R_o (S_o T)$ — ⑤

Downloaded from TwoWaits

Conversely, Let, $(a,d) \in R \circ (S \circ T)$

$\Rightarrow \exists\, b \in B$ such that $(a,b) \in R$ and $(b,d) \in (S \circ T)$ — (6)

Now, $(b,d) \in S \circ T$.

$\Rightarrow \exists\, c \in C$, $(b,c) \in S$ and $(c,d) \in T$ — (7)

from eqn (6) & (7)

$(a,b) \in R$ and $(b,c) \in S$

$\Rightarrow (a,c) \in R \circ S$ — (8)

From eqn (7) & (8)

$(a,c) \in R \circ S$ and $(c,d) \in T$

$\Rightarrow (a,d) \in (R \circ S) \circ T$ — (9)

so, $R \circ (S \circ T) \subseteq (R \circ S) \circ T$ — (10)

from eqn (5) and (10)

$$\boxed{[(R \circ S) \circ T] = R \circ (S \circ T)}$$

Q. Prove that $(R \circ S)^{-1} = (S^{-1} \circ R^{-1})$

→ Let, R be a relation from A to B.
and S be a relation from B to C.

Let, $(c,a) \in (R \circ S)^{-1}$

$\Rightarrow (a,c) \in (R \circ S)$

$\Rightarrow \exists\, b \in B$, $(a,b) \in R$ and $(b,c) \in S$ — (1)

from eqn (1).

$(a,b) \in R$.

$\Rightarrow (b,a) \in R^{-1}$ — (2)

from eqn (1)

$(b,c) \in S$.

$\Rightarrow (c,b) \in S^{-1}$ — (3)

from eqn (3) and (2)

$(c,b) \in S^{-1}$ and $(b,a) \in R^{-1}$

$\Rightarrow (c,a) \in S^{-1} \circ R^{-1}$ — (4)

so, $(R \circ S)^{-1} \subseteq (S^{-1} \circ R^{-1})$ — (5)

Conversely, Let, $(c,a) \in (S^{-1} \circ R^{-1})$

$\Rightarrow \exists\, b \in B$, $(c,b) \in S^{-1}$ and $(b,a) \in R^{-1}$ — (6)

from eqn (6)

$(c,b) \in S^{-1}$.

$\Rightarrow (b,c) \in S$ — (7)

from eqn (6)

$(b,a) \in R^{-1}$

$\Rightarrow \quad (a,b) \in R$ — (8)

from eqn (7) & (8)

$(a,b) \in R$ and $(b,c) \in S$

$\Rightarrow \quad (a,c) \in R \circ S.$

$\Rightarrow \quad (c,a) \in (R \circ S)^{-1}$ — (9)

So, $S^{-1} \circ R^{-1} \subseteq (R \circ S)^{-1}$ — (10)

From eqn (5) and (10).

$$\boxed{(R \circ S)^{-1} = S^{-1} \circ R^{-1}.}$$

Q/ Let S, R and T are relations on set $A = \{0,1,2,3\}$ defined by $R = \{(a,b) : a+b = 3\}$, $S = \{(a,b) : 3 \text{ is divisor of } a+b\}$ and $T = \{(a,b) : \max(a,b) = 3\}$. Find.

i) R^2. $(R \circ R)$. ii) $R \circ S$ iii) $R \circ T$ iv) $S \circ T$, v) $(R \circ S) \circ T$

$[R^n = R^{n-1} \circ R]$

$A = \{0,1,2,3\}$.

$R = \{(0,3), (3,0), (1,2), (2,1)\}$.

$S = \{(0,3), (3,0), (1,2), (2,1), (3,3)\}$.

$T = \{(0,3), (3,0), (1,3), (3,1), (2,3), (3,2), (3,3)\}$.

i) $R^2 \; (R \circ R) = \{(0,0), (3,3), (1,1), (2,2)\}$

ii) $R \circ S = \{(0,0), (0,3), (3,3), (1,1), (2,2)\}$

iii) $R \circ T = \{(0,0), (0,1), (0,2), (0,3), (3,3), (1,3), (2,3)\}$

iv) $S \circ T = \{(0,0), (0,1), (0,2), (0,3), (3,3), (1,3), (2,3), (3,0), (3,1), (3,2)\}$.

v) $(R \circ S) \circ T = \{(0,3), (0,0), (3,0), (3,3), (1,3), (2,3), (3,2), (3,1), (0,1), (0,2)\}$.

* Properties of Relation :-

- **Reflexive Relation** :- A binary relation R on set A is said to be reflexive relation if every element of set A is related to itself. i.e. $[\forall a \in A, aRa \text{ or } (a,a) \in R]$

e.g. $A = \{1,2,3\}$.

$R = \{(1,1), (1,2), (2,2), (2,3), (3,3)\}$.

R is reflexive relation since, $(1,1) \in R$. $(2,2) \in R$ $(3,3) \in R$.

$|A| = n$. (cardinality of set A)

Total binary relations $= 2^{n^2}$

Total reflexive relations $= 2^{n(n-1)}$

Total symmetric relations $= 2^{n(n+1)/2}$

NOTE: Every identity relation is a reflexive relation but every reflexive relation is not identity relation.

- **Irreflexive Relation** :- A relation R is defined on set A is said to be irreflexive if there is no element in A which is related to itself.

Downloaded from TwoWaits

e.g. $A = \{1, 2, 3\}$.

$R = \{(1,2), (2,3)\}$. , R is irreflexive relation

since, $(1,1) \notin R$.

$(2,2) \notin R$.

$(3,3) \notin R$.

- Non Reflexive Relation :- A relation R is defined on set A is said to be non-reflexive relation if it is neither reflexive nor irreflexive i.e. some elements are related to itself but their exist atleast one element which is not related to itself.

e.g. $A = \{1, 2, 3\}$.

$R = \{(1,1), (2,2), (1,3), (2,3), (3,1)\}$.

R is Non-Reflexive

since. $(3,3) \notin R$.

- Symmetric Relation :- A relation R is defined on set A is said to be symmetric relation if whenever aRb then bRa.

i.e. $\boxed{(a,b) \in R \Longrightarrow (b,a) \in R}$

e.g. $A = \{1, 2, 3\}$.

$R = \{(1,1), (1,2), (2,1), (2,3), (3,2)\}$.

R is symmetric relation.

since, $(1,1) \in R$.

$(1,2) \in R \Rightarrow (2,1) \in R$.

$(2,3) \in R \Rightarrow (3,2) \in R$.

Therefore, it is symmetric relation.

NOTE: A relation R is symmetric if '$R = R^{-1}$'.

- Asymmetric Relation :- A relation R on set A is said to be asymmetric relation if $(a,b) \in R \Rightarrow (b,a) \notin R$.

i.e. $\boxed{(a,b) \in R \Rightarrow (b,a) \notin R}$

{ $p \rightarrow q$: if then - sign.

e.g. $A = \{1, 2, 3\}$

$R = \{(1,2), (3,2)\}$.

R is assymmetric relation.

since, $(1,2) \in R \Rightarrow (2,1) \notin R$.

$(3,2) \in R \Rightarrow (3,2) \notin R$.

- Anti Symmetric Relation :- A relation R is defined on set A is said to be anti-symmetric relation if $\boxed{(a,b) \in R \text{ and } (b,a) \in R \Rightarrow a = b}$

e.g. contrapositive definition of Anti-symmetric Relation :-

$\boxed{\text{If } (a,b) \in R \text{ and } (b,a) \notin R \Rightarrow a \neq b}$

Downloaded from TwoWaits

e.g. $A = \{1, 2, 3\}$.
$R = \{(1,1), (1,2)\}$.

{Divide is always a antisymmetric relation}

R is antisymmetric relation.
since, $(1,1) \in R$.

R is contrapositive anti-symmetric relation.
since, $(1,2) \in R$
$(2,1) \notin R$.
and also, $1 \neq 2$.

- Transitive Relation :- A relation R is defined on set A is said to be transitive relation if

$$\boxed{(a,b) \in R \text{ and } (b,c) \in R \Rightarrow (a,c) \in R.}$$

e.g. $A = \{1, 2, 3\}$.
$R = \{(1,2), (2,3), (1,3)\}$.
R is transitive relation
since, $(1,2) \in R$. and $(2,3) \in R$.
$\Rightarrow (1,3) \in R$.

- Equivalence Relation :- A relation R is defined on set A is said to be equivalence relation if it is reflexive, symmetric and transitive. i.e. equivalence relation satisfies following properties.

1) $(a,a) \in R$, $\forall a \in A$. (Reflexive)
2) $(a,b) \in R \Rightarrow (b,a) \in R$. (symmetric)
3) $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$. (Transitive).

Q Determine whether the relation R on set $A = \{-3, -2, -1, 0, 1, 2, 3\}$ is Reflexive, symmetric, irreflexive, antisymmetric, asymmetric and transitive. where $(a,b) \in R$ iff i) $a = 1$, ii) $a = 2b$, iii) $a + b$ = even.

→ $A = \{-3, -2, -1, 0, 1, 2, 3\}$.

$R_1 = \{(1,-3), (1,-2), (1,-1), (1,0), (1,1), (1,2), (1,3)\}$.

1) Every element of A is not related to itself therefore, it is not reflexive relation.
$\because (2,2) \notin R$, $(3,3) \notin R$. etc.

2) $(1,-3) \in R \Rightarrow (-3,1) \notin R$. Therefore, it is not ~~sy~~ symmetric relation.

3) since $(1,1) \notin R$ - Therefore it is not irreflexive relation.

4) $(1,1) \in R$ and $1 = 1$. $(1,-3) \in R \Rightarrow (-3,1) \notin R$
$1 \neq -3$.

Therefore, it is antisymmetric relation.

5) $(1,1) \in R$ - Therefore it is not asymmetric relation.

6) $(1,1) \in R$, $(1,2) \in R$ etc. Therefore it is transitive relation.

Downloaded from TwoWalts

$R_2 = \{(-2,-1), (2,1), (0,0)\}$. $[a-b]$

1) Every element is not related to itself. Therefore, it is not a reflexive relation.
   $(-2,-2)$ and $(2,2) \in R$ so, not a reflexive relation.

2) $(-2,-1) \in R \Rightarrow (-1,-2) \notin R$. Therefore, it is not symmetric relation.

3) $(0,0) \in R$. Therefore, it is not irreflexive relation

4) $(0,0) \in R$, $(2,1) \in R$, $(1,2) \notin R \Rightarrow 1 \neq 2$. so, it is antisymmetric relation.

5) Since, $(0,0) \in R$. so, it is not asymmetric relation.

6) Not transitive relation.

$[a+b = \text{even}]$

$R_3 = \{(1,1), (2,2), (3,3), (-1,-1), (-2,-2), (-3,-3), (-1,-3), (1,3), (0,2), (0,-2), (1,-1), (2,-2), (3,-3), (-3,-1), (-2,2), (2,0), (-2,0), (-1,1), (-3,3), (3,1), (1,-3), (-3,1), (-1,3), (3,-1)\}$.

1) It is not reflexive.
2) symmetric relation.
3) Not reflexive relation.
4) Not antisymmetric.
5) Not Asymmetric
6) Transitive relation.

Q Prove that the relatio R "$(a-b)$ is divisible by 6" $\forall a, b \in Z^+$. is an equivalence relation.

→ Imp congurent Relation:-

$\boxed{a \equiv b \bmod m.}$

$\Rightarrow a-b$ divisible by m.

Equivalence relation satisfies when reflexive symmetric and transitive relation satisfies.

1) 0 is divisible by 6.
   $\Rightarrow (a-a)$ is divisible by 6, $\forall a \in Z^+$. $\Rightarrow (0,a) \in R$.
   So, it follows reflexive relation.

2) Let $(a,b) \in R$.
   $\Rightarrow (a-b)$ is divisible by 6.
   $\Rightarrow -(a-b)$ is divisible by 6.
   $\Rightarrow (b-a)$ is divisible by 6.
   $\Rightarrow (b,a) \in R$.
   It follows symmetric property.

3) Let. $(a,b) \in R$ and $(b,c) \in R$.
   $\Rightarrow (a-b)$ is divisible by 6 and $(b-c)$ is divisible by 6.
   $\Rightarrow (a-b)+(b-c)$ is divisible by 6.
   $\Rightarrow (a-c)$ is divisible by 6.
   $\Rightarrow (a,c) \in R$.
   So, It follows transitive relation. So, R is an equivalen relatio

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Q State whether following relations are equivalent or not. on the set of lines in a planes.

1) $x \parallel y$

2) $x \perp y$.

For Equivalence relation reflexive, symmetric and transitive relation must satisfy.

1) $x \parallel y$.

Reflexive :- Every line is parallel to itself. ie. $x \parallel x$.
So, it is reflexive.

Symmetric: $x \parallel y \Rightarrow y \parallel x$. It follows symmetric relation.

Transitive: $x \parallel y$ and $y \parallel z \Rightarrow x \parallel z$. It follows transitive relation. Hence, it is an equivalence relation.

2) $x \perp y$.

Reflexive :- No line is $\perp$ to itself. so, it does not follow reflexive property.

Hence, it is not an equivalence relation.

Q i) Show that $R = \{(a,b) : a \equiv b \bmod m\}$ is an equivalence relation on Z^+.

ii) Show also if $x_1 \equiv y_1$ and $x_2 \equiv y_2$ then $(x_1 + x_2) \equiv (y_1 + y_2)$.

i) $x_1 \equiv y_1 \Rightarrow (x_1 - y_1)$ is divisible by m — (1)

and $x_2 \equiv y_2 \Rightarrow (x_2 - y_2)$ is divisible by m — (2)

Adding eqn (1) & (2) -

$\Rightarrow ((x_1 - y_1) + (x_2 - y_2))$ is divisible by m.

$\Rightarrow (x_1 + x_2) - (y_1 + y_2)$ is divisible by m.

$\Rightarrow (x_1 + x_2) \equiv (y_1 + y_2) \bmod m$.

$\Rightarrow (x_1 + x_2) \equiv (y_1 + y_2)$.

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Q Show that subset relation on collection of sets.

→ For equivalence relation reflexive, symmetric and transitive relation must satisfy.

For Reflexive Property – If $(a,a) \in R$.
then, $a \subseteq a \Rightarrow a \subseteq a$.
So, it is reflexive.

For symmetric property – If $(a,b) \in R$, then $a \subseteq b$.
then. $b \subseteq a$ ie $(b,a) \notin R$.

It does not follow symmetric property.

So, it is not an equivalence relation.

Also, it is not an equivalence relation on collection of sets.

Q Check whether $R = \{(a,b) : a \geq b\}$ on set of real numbers is an equivalence relation or not.

→ $R = \{(a,b) : a \geq b\}$.

For Reflexive Property :-
If $(a,a) \in R$, ie, $a \geq a$.
$\Rightarrow a \geq a$.
so, it is reflexive.

For symmetric Property :-
If $(a,b) \in R$. $\Rightarrow$ ie. $a \geq b$.
but, $b \ngeq a \Rightarrow (b,a) \notin R$.
So, it is not an symmetric property.
Therefore, it is not an equivalence relation.

Q Prove that if R is an equivalence relation then R^{-1} is also an equivalence on set A.

→ Given R is an equivalence relation.

1) for Reflexive Property :- If R is reflexive relation.
then if $(a,a) \in R$.
$\Rightarrow (a,a) \in R^{-1}$
R^{-1} is an eq. reflexive relation.

2) for symmetric Property :- If R is symmetric relation.
then if $(a,b) \in R$ and $(b,a) \in R$.
then $(b,a) \in R^{-1}$ and $(a,b) \in R^{-1}$.

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So, R^{-1} is a symmetric relation.

3) for transitive property :- since R is transitive relation.
then $(a,b) \in R$, $(b,c) \in R \Rightarrow (a,c) \in R$.

So, then $(b,a) \in R^{-1}$ and $(c,b) \in R^{-1} \Rightarrow (c,a) \in R^{-1}$

So, R^{-1} is a transitive relation.

Therefore, R^{-1} is an equivalence relation.

Q If R and S are two equivalence relations then $R \cap S$ is also an equivalence relation.

→ Given R and S is an equivalence relation.

1) for Reflexive relation :- If $(a,a) \in R$.
and $(a,a) \in S$.
then, $(a,a) \in R \cap S$.

So, it is reflexive.

2) for symmetric property :- $(a,b) \in R$ & $(b,a) \in R$.
$(a,b) \in S$ & $(b,a) \in S$

For $(R \cap S)$: $(a,b) \in R \cap S$ & $(b,a) \in R \cap S$.

So, it is symmetric relation.

3) for transitive property :- $(a,b) \in R$ & $(b,c) \in R$ & $(a,c) \in R$.
$\Rightarrow (a,b) \in S$ & $(b,c) \in S$ & $(a,c) \in S$.

then, $(a,b) \in R \cap S$ & $(b,c) \in R \cap S \Rightarrow (a,c) \in R \cap S$.

So, it is transitive relation.

Therefore, it is an equivalence relation.

* **Equivalence Class** :- Let R be an equivalence relation defined on set A the set of all elements that are related to an element $a \in A$ is called equivalence class of that element a.

If $a \in A$

then, $\boxed{[a] = \{ b : aRb \}}$

It is represented by $[a]_R$ or $[a]$.

e.g. $A = \{1,2,3\}$.

$R = \{(1,1), (1,2), (2,1), (2,2), (3,3)\}$.

$[1] = \{1,2\}$.

$[2] = \{1,2\}$.

$[3] = \{3\}$.

Properties :-

- $a \in [a]_R$, $\forall a \in A$.
- $[a] = [b]$ iff $(a,b) \in R$.
- $[a] \neq [b]$ or $[a] \cap [b] = \phi$.

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elements of set A under equivalence relation

R are is called <u>quotient set of A by R</u>:
It is denoted by $\boxed{A/R}$ (A is induced by R).

$A = \{1, 2, 3\}$.

$R = \{(1,1), (1,2), (2,1), (2,2), (3,3)\}$.

$[1] = \{1, 2\}$.

$[2] = \{1, 2\}$

$[3] = \{3\}$

$A/R = \{[2]_R, [3]_R\}$.

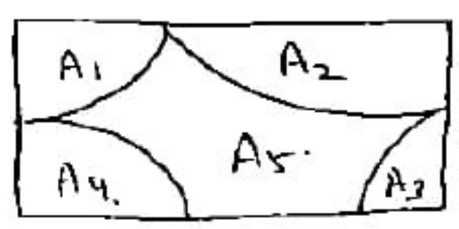
* Partition :- The partition of A set is a collection of mutuall disjoint, non-empty subsets of A whose union is A.

Partition so, P is called partition of A if -

- P contains non-empty subset of A i.e. $A_1, A_2, A_3, \ldots A_n$ where, $\boxed{A_i \subseteq A}$ for $i = \{1, 2, 3, \ldots n\}$.
- Union of all subsets under partition P gives set A i.e. $A_1 \cup A_2 \cup A_3 \cup A_4 \cup \ldots \cup A_n = A$.

  <u>or</u> $\boxed{\bigcup_{i=1}^{n} A_i = A}$
- The elements of partition P are pair wise disjoint i.e. $\boxed{A_i \cap A_j \neq \phi}$ or $\boxed{A_i = A_j}$.

Theorem :- Every equivalence relation on a set generates a unique partition of set. The blocks of this partition form equivalence classes.



Q Let R be defined on set $A = \{1, 2, 3, 4\}$ as $R = \{(a,b) : a - b = c\}$. Find all the equivalence classes.

→ $A = \{1, 2, 3, 4\}$.

$R = \{(1,1), (1,3), (2,2), (2,4), (3,1), (3,3), (4,2), (4,4)\}$.

$[1] = \{1, 3\}$.

$[2] = \{2, 4\}$.

$[3] = \{1, 3\}$.

$[4] = \{2, 4\}$.

Imp Q Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and Let R be the relation of $A \times A$ defined by $(a, b) R (c, d)$ iff $a + d = b + c$.

1) Prove that R is an equivalence relation.
2) Find $[2, 5]_R$

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1) (b) Reflexive :- $(a+b) = (a+b)$

$\Rightarrow (a,b) \; R \; (a,b)$.

So, it is reflexive.

(2) Symmetric :- Let $(a,b) \; R \; (c,d)$.

$\Rightarrow a+d = b+c$.

$\Rightarrow b+c = a+d$.

$\Rightarrow c+b = d+a$.

$\Rightarrow (c,d) \; R \; (a,b)$.

So, it is symmetric.

(3) Transitive :- Let, $(a,b) \; R \; (c,d)$.

$\Rightarrow a+d = b+c$. —(1).

and, $(c,d) \; R \; (e,f)$.

$\Rightarrow (c+f) = (d+e)$ —(2).

Adding eqn (1) & (2).

$a+\cancel{d}+\cancel{c}+f = e+\cancel{d}+b+\cancel{c}$.

$a+f = b+e$.

$\Rightarrow (a,b) \; R \; (e,f)$

So, it is transitive.

$\Rightarrow$ It is an equivalence relation.

(2) $[2,5]_R = ?$

$(2,5) \; R \; (x,y)$

$\Rightarrow 2+y = x+5$

$\Rightarrow y = x+3$.

$x=1$, $y=4$

$x=2$, $y=5$

$x=3$, $y=6$.

$x=4$, $y=7$.

$x=5$, $y=8$.

$x=6$, $y=9$

$[2,5]_R = \{(1,4), (2,5), (3,6), (4,7) \; (5,8), (6,9)\}$.

* Closure of Relation :-

Reflexive Closure :- Let R be a relation on set A then $R \cup I_A$ is called reflexive closure of R.

where, $I_A = \{(a,a) ; \forall \; a \in A\}$

$$\boxed{R_f = R \cup I_A}$$

eg. Let, $A = \{1,2,3\}$

$R = \{(1,1), (1,2), (2,3)\}$

$R_f = \{(1,1), (1,2), (2,3), (2,2), (3,3)\}$.

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$R \cup R^{-1}$ is called symmetric closure of R

where, R^{-1} is the inverse of relation R.

$$\boxed{R_S = R \cup R^{-1}}$$

e.g. $A = \{1, 2, 3\}$.

$R = \{(1,1), (1,2), (2,3)\}$.

$R^{-1} = \{(1,1), (2,1), (3,2)\}$.

$R_S = \{(1,1), (1,2), (2,3), (2,1), (3,2)\}$.

Transitive closure :- Let R be a relation on set A then transitive closure R^* or R^+ is equal to

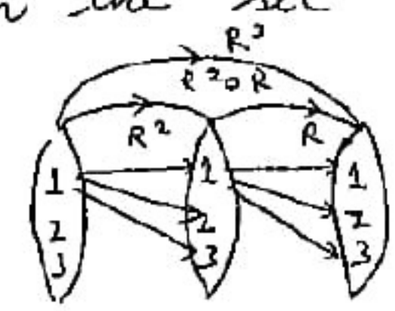
$$R^* = R \cup R^2 \cup R^3 \cup R^4 \cdots \cup R^n.$$

or $$\boxed{R^* = \bigcup_{i=1}^{n} R^i}.$$

where, 'n' is the number of elements in the set.

e.g. $A = \{1, 2, 3\}$.

$R = \{(1,1), (1,2), (2,3)\}$.



$R \circ R = \{(1,2), (1,3), (1,1)\}$.

$R^2 \circ R = \{(1,1), (1,2), (1,3)\}$.

$R^3 = \{(1,1), (1,2), (1,3)\}$.

* Pictorial Representation of Relation :-

A relation can be expressed in following ways-

- Relation as a Matrix.
- Relation as a Directed Graph (Di-graph).
- Relation as an Arrow Diagram.
- Relation as a Table.

• Relation as a Matrix :-

Let, $A = \{a_1, a_2, \cdots a_i, \cdots a_m\}$.

$B = \{b_1, b_2, \cdots b_j, \cdots b_n\}$.

are finite sets & contain m and n elements respectively.

Let, R be a relation from set A to set B then the relation matrix of R is a $m \times n$ matrix defined by -

$$M_R = [m_{ij}] = \begin{cases} 1, & \text{if } (a_i, b_j) \in R \\ 0, & \text{if } (a_i, b_j) \notin R \end{cases}$$

e.g. $A = \{1, 2, 3\}$.

$R = \{(1,1), (1,2), (2,3), (3,3), (2,1)\}$.

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$$M_R = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 3 & 0 & 0 & 1 \end{array}$$

This matrix is also called <u>zero one</u> matrix or <u>Boolean</u> matrix or <u>adjacency</u> matrix of relation of R.

*(1) If $m_{ii} = 1$ then relation is Reflexive i.e. main diagonal of matrix contain 1.

*(2) If $m_{ij} = m_{ji}$ then relation is symmetric.

<u>or</u> $M_R = M_R^T$ (T = Transpose of Matrix).

Q Let $N = \{1, 2, 3, ---\}$ and the relation R is defined in $N \times N$ as follows. $(a,b) R (c,d)$ iff $ad = bc$. Then show that R is an equivalence relation or not.

→ 1) <u>Reflexive</u> :- $ab = ba$.

$\Rightarrow (a,b) R (a,b)$

So, it is reflexive.

2) <u>Symmetric</u> :- Let, $(a,b) R (c,d)$

$ad = bc$.

$bc = ad$.

$cb = da$

$\Rightarrow (c,d) R (a,b)$

So, it is symmetric.

3) <u>Transitive</u> : Let $(a,b) R (c,d)$

$ad = bc$. — ①

& $(c,d) R (e,f)$

$cf = de$. — ②

multiply eqn ① & ②

$a\cancel{d} \cdot \cancel{c}f = b\cancel{c} \cdot \cancel{d}e$

$a \cdot f = b \cdot e$.

$a \cdot f = e \cdot f$.

$(a,b) R (e,f)$

So, it is transitive.

Therefore, it is an equivalence relation.

*(3) A relation R is transitive if

$\boxed{M_R^2 + M_R = M_R}$

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Join and Meet of Relation Matrix :-

Join ($\vee$) $\Rightarrow$ OR

Meet ($\wedge$) $\Rightarrow$ AND.

Join :- It is denoted by '$\vee$'.

$1 \vee 1 = 1$

$1 \vee 0 = 1$

$0 \vee 1 = 1$

$0 \vee 0 = 0$.

Join of two matrices can be obtained by performing join of corresponding elements of matrices.

Union of two Matrices:- $\boxed{M_{R \cup S} = M_R \vee R_S.}$

Meet :- It is denoted by '$\wedge$'.

$1 \wedge 1 = 1$.

$1 \wedge 0 = 0$

$0 \wedge 1 = 0$

$0 \wedge 0 = 0$.

Meet ~~Join~~ of two matrices can be obtained by performing meet of correspondi elements of matrices.

Intersection of Relation :- $\boxed{M_{R \cap S} = M_R \wedge M_S}$

eg
$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

e.g
$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \wedge \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Transpose of a Relation :- $\boxed{M_R^{-1} = M_R^T.}$

Q. Set $A = \{a, b, c\}$ has two relations R & S represented by following matrices.

$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ & $M_S = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Find $M_{R \cup S}$ or, $M_{R \cap S}$, $M_{R \oplus S}$, $M_{R \oplus S}$.

$\rightarrow$

$$M_{R \cup S} = M_R \vee M_S. = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M_{R \cap S} = M_R \wedge M_S = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \wedge \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$M_{R \circ S} = M_R \cdot M_S$ (Boolean product element wise add is 1.) $= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0+0 & 1+0+0 & 1+0+1 \\ 0+0+0 & 0+0+0 & 0+0+1 \\ 1+1+0 & 1+1+0 & 0+1+1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

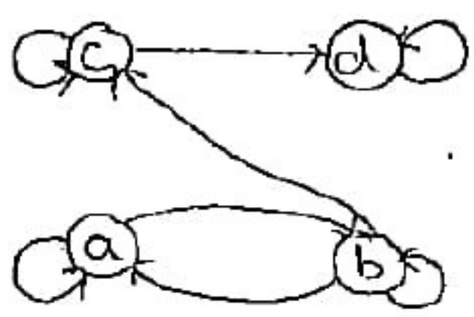
$M_{R \oplus S} = M_{R \cup S} - M_{R \cap S}$ (XOR) (Same input = 0, diff input = 1.) $= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

- Relation as a Directed Graph (Di-Graph) :-

e.g. $A = \{a, b, c, d\}$

$R = \{(a,a), (a,b), (b,a), (b,b), (b,c), (c,c), (c,d), (d,d)\}$



* Partial order relation (POR) :- A relation R defined on set A is said to be POR if it is -

- Reflexive $\Rightarrow$ $[(a,a) \in R, \forall a \in A]$
- Antisymmetric $\Rightarrow$ $[(a,b) \in R$ and $(b,a) \in R \Rightarrow a = b]$
- Transitive $\Rightarrow$ $[(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R]$

Any POR relation can be represented as - '$\preccurlyeq$'

* Poset :- The set A along with POR is called Poset or Partial ordered set.

It is represented by. $(A, \leq)$

e.g. The relation "$\leq$" is a POR on the set of integers 'Z'.

$(Z, \leq)$.

1) Reflexive - $a \leq a$, $\forall a \in Z$.
2) Antisymmetric - $a \leq b$ and $b \leq a \Rightarrow a = b$.
3) Transitive - $a \leq b$ and $b \leq c \Rightarrow a \leq c$.

$\leq$ is a POR
$\geq$ is a POR
$\subset$ is a POR

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A total order relation defined on set A is said to be total order relation if and only if it is.

- Partial order Relation.
- Either $(a,b) \in R$ or $(b,a) \in R$, $\forall\ a, b \in A$. (It is called comparability).

e.g. $R = \{(a,b) : a \le b\}$ over the set of Integers is a TOR

Reflexive $\rightarrow$ $a \le a \in R$.

Antisymmetric $\rightarrow$ $a \le b$ and $b \le a \Rightarrow a = b$.

Transitive $\rightarrow$ $a \le b$ and $b \le c \Rightarrow a \le c$.

It is POR.

* Function :-

Let X and Y be two non-empty sets. A function f from X to Y is a rule that assigns to each elements $x \in X$, a unique element $y \in Y$.

It is denoted by $\boxed{f : X \rightarrow Y}$

function is also called <u>Mapping</u>, <u>transformation</u> or <u>correspondence</u>.

If $x \in X$, $y \in Y$ and $f : X \rightarrow Y$ then we write $f(x) = Y$.

If $\boxed{X}$ has $\boxed{m}$ elements and $\boxed{Y}$ has $\boxed{n}$ elements.

$f : X \rightarrow Y$	n^m	(No. of function)
$f : Y \rightarrow X$	m^n	
$f : X \rightarrow X$	m^m	

Note: Total no. of relation from one set to another set $(X \rightarrow Y)$

Total no. of relations from one set to one set $(X \rightarrow X)$

No. of relations > No. of functions. (Always).

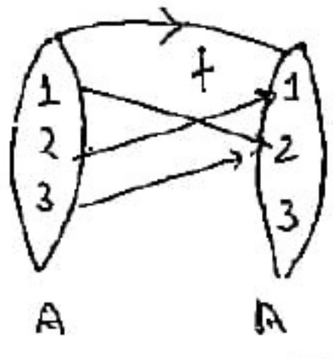
$\boxed{2^{m \times n} > n^m}$

* Domain and Codomain of a function :-

If $f : X \rightarrow Y$

X should be domain & Y should be codomain

$A = \{1, 2, 3\}$.



$f : A \rightarrow A$.

Domain = $A = \{1, 2, 3\}$

Co-domain = $A = \{1, 2, 3\}$.

y is image of x.

x is preimage of y.

$f(1) = 2$.

$f(2) = 1$

$f(3) = 2$.

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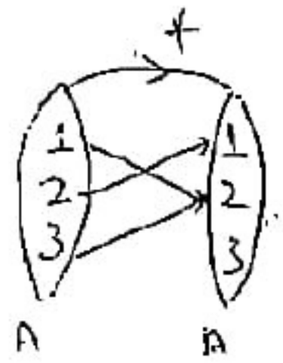
* Range of a Function :- Range is subset of Codomain.

Range $\subseteq$ Codomain.

e.g



Codomain $= \{1, 2, 3\} = A$.

Range $= \{1, 2\}$.

y is image of x.

x is preimage of y.

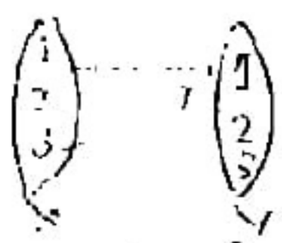
Q. Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d\}$, & $f : A \to B$ such that $f = \{(1,a), (2,c), (3,d), (4,d)\}$. Find domain, codomain and range.

Domain $= A = \{1, 2, 3, 4\}$.

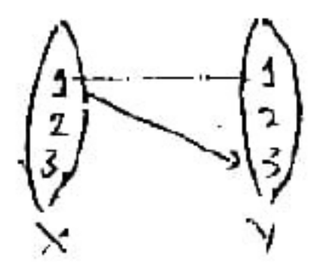
Codomain $= B = \{a, b, c, d\}$.

Range $= \{a, c, d\}$.

NOTE:



Not a function. (No element should be left in X)

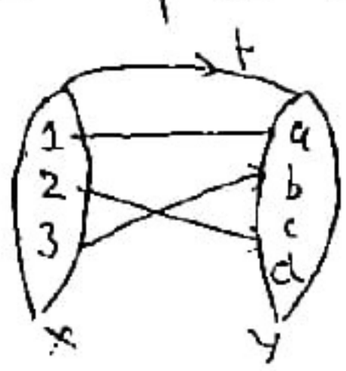


Not a function (No two images of single element is allowed)

* Types of Function :-

• One to One (Injective) Function :- Let $f : X \to Y$ then f is called one to one function if for distinct elements of X there are distinct images in Y. i.e. f is one to one iff $\boxed{f(x_1) = f(x_2) \Rightarrow x_1 = x_2}$ $\forall\ x_1, x_2 \in X$.

e.g



$\{(1,a), (2,c), (3,b)\}$

e.g Let $f(x) = 5x + 1$. on set of integer.

Let $f(x_1) = f(x_2)$.

$\Rightarrow 5x_1 + \cancel{1} = 5x_2 + \cancel{1}$

$\Rightarrow \cancel{5}x_1 = \cancel{5}x_2$

$\Rightarrow \boxed{x_1 = x_2}$.

• Onto (Surjective) Function :- Let $f : X \to Y$ then f is called onto function if for every element $y \in Y$ there is a element $x \in X$ such that $[f(x) = y]$ (or) function f is onto if $\boxed{\text{codomain} = \text{Range}}$.

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eg. $f(x) = 5x+1$ over the set of real numbers. $f: R \to R$.

$\Rightarrow f(x) = 5x+1$

$\Rightarrow y = 5x+1$

$\Rightarrow \boxed{x = \frac{y-1}{5}}$ $\forall\ y \in R \Rightarrow x \in R$. (there exist $\exists\, x \in R$

$f(x) = 5x+1$ over the set of integer no's $f: Z \to Z$.

$\Rightarrow f(x) = 5x+1$

$\Rightarrow y = 5x+1$

$\Rightarrow \boxed{x = \frac{y-1}{5}}$ $\forall\ y \in Z \Rightarrow x \notin Z$. (there does not exist $\exists\, x \in Z$).

- One to one onto (Bijective) function :- A function reaches one to one and onto both is called bijective function.

Let $f(x) = 5x+1$ on set of real no's.

$\Rightarrow$ Let $f(x_1) = f(x_2)$

$\Rightarrow 5x_1 + \cancel{1} = 5x_2 + \cancel{1}$

$\Rightarrow \cancel{5}x_1 = \cancel{5}x_2$

$\Rightarrow \boxed{x_1 = x_2}$

Let. $f(x) = 5x+1$.

$\Rightarrow y = 5x+1$

$\boxed{x = \frac{y-1}{5}}$ $\forall\ y \in R \Rightarrow$ there exist $\exists\, x \in R$.

- Into function :- A function which is not onto is called into function.

- Identity function :- Let for $f: X \to X$ function f is call identity function if $\boxed{f(x) = x}$ $\forall\ x \in X$.

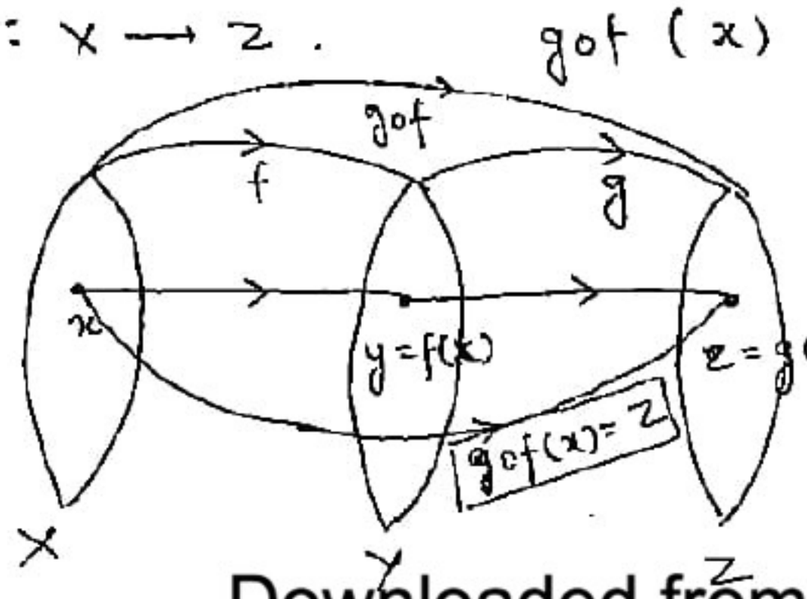
If image of every element is element itself then it is called Identity function.
It is denoted by I_X.

* Composition of function :-

$f: X \to Y$. $f(x) = y$

$g: Y \to Z$ $g(y) = z$.

$g \circ f: X \to Z$. $g \circ f(x) = z$.



$z = g(y) = g(f(x) = g \circ f(x)$.

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- Composition of function is not commutative.
  i.e. $\boxed{fog \neq gof}$
- Composition of function is Associative.
  i.e. $\boxed{(hog)of = ho(gof)}$

Proof Let, $f(x) = y$ Let, $f : x \to y$
$g(y) = z$. $g : y \to z$
$h(z) = w$ $h : z \to w$.

LHS $(hog)of$
$\Rightarrow (hog)of(x)$.
$\Rightarrow (hog)f(x)$.
$\Rightarrow (hog)y$
$\Rightarrow h(g(y))$
$\Rightarrow h(z)$
$\Rightarrow w$.

RHS $ho(gof)$
$\Rightarrow ho(gof)(x)$.
$\Rightarrow ho(g(f(x)))$
$\Rightarrow ho(g(y))$.
$\Rightarrow h(z)$.
$\Rightarrow w$.

$\boxed{LHS = RHS.}$ (Hence Proved). composition of function is Associative.

Q. $f(x) = 3x+2$. on the set of real no's. find gof & fog.
$g(x) = 4x+8$

$fog(x) = f(g(x))$
$= 3(g(x))+2$.
$= 3(4x+8)+2$.
$= 12x+24+2$.
$\boxed{fog(x) = 12x+26.}$

$gof(x) = g(f(x))$
$= 4(f(x))+8$.
$= 4(3x+2)+8$
$= 12x+8+8$
$\boxed{gof(x) = 12x+16.}$

Q Let h, g & f be functions from N to N. such that $f(x) = x+1$, $g(x) = 2x$ & $h(x) = \begin{cases} 0, & \text{if } x \text{ is even} \\ 1, & \text{if } x \text{ is odd} \end{cases}$.
find fof, gof, fog, goh, hog, (fog)oh, go(fog)oh.

→ $fof(x) = f(f(x))$
$= f(x+1)$
$= x+1+1$
$fof(x) = x+2$

$gof(x) = g(f(x))$
$= 2f(x)$
$= 2(x+1) = 2x+2$.

$fog(x) = f(g(x))$
$= g(x)+1 = 2x+1$

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$$goh = g(h(x))$$
$$= 2(h(x)).$$

$\Rightarrow$ if x is even.

$$\begin{cases} goh = 0. \\ \Rightarrow \text{ if } x \text{ is odd.} \\ goh = 2 \times 1. = 2. \end{cases}$$

$$hog(x) = h(g(x))$$
$$= h(2x)$$
$$hog(x) = 0. \qquad \{ x \text{ is even} \}.$$

$$(fog)oh.(x) = \begin{cases} (fog)(h(x)) & \text{if } x \text{ is even} \\ (fog)(h(x)) & \text{if } x \text{ is odd} \end{cases}$$

$$= \begin{cases} (fog)\,0 & \text{if } x \text{ is even} \\ (fog)\,1 & \text{if } x \text{ is odd.} \end{cases}$$

$$= \begin{cases} f\circ(g(0)) & \text{if } x \text{ is even} \\ f\circ(g(1)) & \text{if } x \text{ is odd.} \end{cases}$$

$$= \begin{cases} f(0) & \text{if } x \text{ is zero.} \\ f(2) & \text{if } x \text{ is } 2. \end{cases}$$

$$= \begin{cases} 1 \\ 3 \end{cases}$$

$$go(fog)oh = \begin{cases} go(fog)(h(x)) & \text{if } x \text{ is even} \\ go(fog)(h(x)) & \text{if } x \text{ is odd.} \end{cases}$$

$$= \begin{cases} go\,fo(g(0)) & \text{if } x \text{ is even} \\ go\,fo(g(1)) & \text{if } x \text{ is odd.} \end{cases}$$

$$= \begin{cases} go\,fo(0) & \text{if } x \text{ is even} \\ go\,f(2). & \text{if } x \text{ is odd.} \end{cases}$$

$$= \begin{cases} g(1) & \text{if } x \text{ is even} \\ g(3) & \text{if } x \text{ is odd.} \end{cases}$$

$$= \begin{cases} 2 \\ 6. \end{cases}$$

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* **Inverse of a function** :- Let f be a <u>bijective function</u> from X to Y the inverse function of f is the function that assigns an element $y \in Y$ a unique element $x \in X$ such that $f(x) = y$.

Inverse of f denoted by f^{-1}.

i.e. [If $f(x) = y$ then $f^{-1}(y) = x$.]

e.g $f(x) = \frac{7-2x}{3}$.

$\Rightarrow y = \frac{7-2x}{3} \Rightarrow 3y = 7-2x$.

$\Rightarrow 2x = 7-3y$

$\Rightarrow x = \frac{7-3y}{2} \Rightarrow \boxed{f^{-1}(y) = \frac{7-3y}{2}}$

* **Theorem** :- If $f: X \to Y$ & $g: Y \to X$ then g is called inverse of f i.e. $\boxed{g = f^{-1}}$ iff [$fog = I_Y$ & $gof = I_X$]

<u>Proof</u> :- Functions f & g should be bijective functions.

$f: X \to Y$ $\forall y \in Y$ there exist $\exists x \in X$ such that $f(x) = y$

$g: Y \to X$ $\forall x \in X$ there exist $\exists y \in Y$ such that $g(y) = x$

$gof : X \to X$ & $fog : Y \to Y$

Now, $gof(x) = g(f(x))$ & $fog(y) = f(g(y))$

$= g(y)$ $= f(x)$.

$gof(x) = x$. $fog(y) = y$.

$\Rightarrow \boxed{gof(x) = I_X}$ $\Rightarrow \boxed{fog(y) = I_Y}$.

Q Let $f: R \to R$ & $g: R \to R$ where R is the set of real no.s. Find fog & gof where $f(x) = x^2 - 2$ & $g(x) = x+4$. State whether these functions are injective, surjective & bijective.

→ $fog(x) = f(g(x))$

$= (x+4)^2 - 2$.

$= x^2 + 16 + 8x - 2$.

$\boxed{fog(x) = x^2 + 8x + 14}$.

$gof(x) = g(f(x))$

$= (x^2 - 2) + 4$

$\boxed{gof(x) = x^2 + 2}$.

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Let, $fog(x_1) = fog(x_2)$

$\Rightarrow x_1^2 + 8x_1 + \cancel{14} = x_2^2 + 8x_2 + \cancel{14}$

$\Rightarrow x_1(x_1+8) = x_2(x_2+8)$

Let, $x_1 = 2$, $x_2 = 8 - 10$

$\Rightarrow 2(2+8) = -10(-10+8)$

$\Rightarrow 20 = +20$

$\Rightarrow \boxed{x_1 \neq x_2}$

It is not one to one function.

<u>for surjective</u>

$y = x^2 + 8x + 14.$

$y = x^2 + 8x + 14 + 2 - 2$

$\Rightarrow y = (x+4)^2 - 2.$

$\Rightarrow (x+4)^2 = y+2.$ <u>or</u> $x+4 = \sqrt{y+2}$

$x = \sqrt{y+2} - 4$

$\Rightarrow (x+4)^2 > 0.$

for $y < -2$

$\Rightarrow y+2 < 0$

So, it is not onto function

It is also not bijective function.

<u>for injective</u>

Let, $gof(x_1) = gof(x_2)$

$\Rightarrow x_1^2 + \cancel{2} = x_2^2 + \cancel{2}$

$\Rightarrow x_1^2 = x_2^2.$

$\Rightarrow x_1 = \pm x_2.$ $\Rightarrow \boxed{x_1 \neq x_2.}$

It is not one to one function

<u>for surjective :-</u>

Let, $gof = x^2 + 2$

$\Rightarrow y = x^2 + 2$

$\Rightarrow x^2 = y - 2.$

if $y < 2$ therefore there is no such x.

So, it is not onto.

So, It is also not bijective.

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Q $f: Z^+ \to Z^+$, $f(x) = x^4$.
$f: Z \to Z$, $f(x) = x + x^2$.
$f: R \to R$, $f(x) = \sin x$.
$f: R \to R$, $f(x) = |x|$

Whether these functions are injective or not.

1) Let, $f(x_1) = f(x_2)$

$x_1^4 = x_2^4$

$x_1^4 = \pm x_2$

but, $f: Z^+ \to Z^+$

$\Rightarrow \boxed{x_1 = x_2}$ for Z^+ set it is one to one.

2) Let, $f(x_1) = f(x_2)$

$\Rightarrow x_1 + x_1^2 = x_2 + x_2^2$.

$\Rightarrow x_1(1 + x_1) = x_2(1 + x_2)$

Let, $x_1 = 1$ & $x_2 = -2$.

$\Rightarrow 1(1+1) = -2(1-2)$

$\Rightarrow 2 = 2$

$\Rightarrow \boxed{x_1 \neq x_2}$. So, it is not one to one.

3) Let, $f(x_1) = f(x_2)$

$\Rightarrow \sin x_1 = \sin x_2$.

Let, $x_1 = 0$, $x_2 = \pi$.

$\Rightarrow \sin 0 = \sin \pi$

$\Rightarrow 0 = 0$.

$\Rightarrow \boxed{x_1 \neq x_2}$. So, it is not one to one.

4) Let, $f(x_1) = f(x_2)$

$\Rightarrow |x_1| = |x_2|$.

$\Rightarrow \boxed{x_1 = \pm x_2}$. So, it is not one to one.

Q State whether following functions are onto or not.

1) $f: R \to R$. $f(x) = e^x$.
2) $f: R \to R$ $f(x) = \sin x$.
3) $f: Z \to Z$ $f(x) = x^2 + 14x - 51$.

1) $f(x) = e^x$.

$\Rightarrow y = e^x$.

$\Rightarrow \boxed{x = \log y}$

$\therefore$ log is not defined for -ve real no.'s. So, for -ve value of y x does not exist. So, it is not onto.

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2) $y = \sin x$

$\boxed{x = \sin^{-1} y}$

For $-1 \le y \le 1$ sin does not have any value.

so, it is not onto.

3) $y = x^2 + 14x - 51$.

$y = x^2 + 14x - 51 + 49 - 49$.

$y = (x+7)^2 - 100$.

$(x+7)^2 = y + 100$.

$\Rightarrow (x+7)^2 > 0$.

for $y < -100$.

$\Rightarrow y + 100 < 0$.

so, it is not onto.

* Theorem :- If $f : X \to Y$ & $g : Y \to Z$ are invertible function then $gof : X \to Z$ is also invertible function and

$\boxed{(gof)^{-1} = f^{-1} o g^{-1}}$.

<u>Proof</u> Since f and g are invertible therefore they are bijective functions. so, $gof : X \to Z$ is bijective. Hence, gof is also invertible.

$f : X \to Y$ $\forall y \in Y$ there exist $\exists x \in X$ such that $f(x) = y \Rightarrow f^{-1}(y) =$

$g : Y \to Z$ $\forall z \in Z$ there exist $\exists y \in Y$ such that $g(y) = z \Rightarrow g^{-1}(z) =$

Since, $gof : X \to Z$ is bijective and invertible.

$\Rightarrow gof(x) = z$.

$\Rightarrow \boxed{(gof)^{-1}(z) = x}$

(OR)

$gof(x) = g(f(x))$

$= g(y)$

$gof(x) = z$

$\Rightarrow \boxed{(gof)^{-1}(z) = x}$. — ①

$f^{-1} o g^{-1}(z) = f^{-1}(g^{-1}(z))$

$= f^{-1}(y)$

$\Rightarrow \boxed{f^{-1} o g^{-1}(z) = x}$ — ② from eqn ① & ②.

$\therefore \boxed{(gof)^{-1}(z) = (f^{-1} o g^{-1})(z)}$

Q show that the function $f(x) = x^5$ and $g(x) = x^{1/5}$ are inverse of each other over the set of real numbers.

$\rightarrow fog(x) = f(g(x))$

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$$fog(x) = (x^{\frac{1}{5}})^5$$

$$\Rightarrow \boxed{fog(x) = x.} \Rightarrow \boxed{fog = I_x}$$

$$gof(x) = g(f(x))$$
$$= (x^5)^{1/5}$$

$$\Rightarrow \boxed{gof(x) = x.} \Rightarrow \boxed{gof = I_x}$$

Since, $fog = gof = I_x$

Therefore, they are inverse of each other.

Q Let, $X = \{1, 2, 3\}$ a function $f : X \to X$ is defined as $f = \{(1,2), (2,1), (3,3)\}$. Find f^2, f^3, f^{-1}.

→ $f^{-1} = \{(2,1), (1,2), (3,3)\}$.

$f(1) = 2$

$f(2) = 1$

$f(3) = 3$

$f^2 = f \circ f \Rightarrow f \circ f(1) = f(f(1)) = f(2) = 1.$

$f \circ f(2) = f(f(2)) = f(1) = 2.$

$f \circ f(3) = f(f(3)) = f(3) = 3.$

$f^2 = f \circ f = \{(1,1), (2,2), (3,3)\}.$

$f^3 = f^2 \circ f \Rightarrow f^2 \circ f(1) = f^2(f(1)) = f^2(2) = 2.$

$\Rightarrow f^2 \circ f(2) = f^2(f(2)) = f^2(1) = 1.$

$\Rightarrow f^2 \circ f(3) = f^2(f(3)) = f^2(3) = 3.$

$f^3 = f^2 \circ f = \{(1,2), (2,1), (3,3)\}.$

* **Permutation Function :-** Let, A be a finite set the set of all bijective functions from A to A is called the set of permutation functions from A to A. if set A contain n elements then there are n! different permutations on A.

The matrix form to represent a permutation function on a finite set is to list all the elements of the set accross the top row and the images of each element exactly below it.

Consider $A = \{x_1, x_2, x_3, \ldots x_n\}$.

Let, 'p' be a permutation on set A. then

$$P = \begin{pmatrix} x_1 & x_2 & x_3 & \ldots & x_n \\ P(x_1) & P(x_2) & P(x_3) & \ldots & P(x_n) \end{pmatrix}$$

where, $P(x_1), P(x_2), P(x_3) \ldots P(x_n) \in A$. ($P(x_1), P(x_2)$ & $\ldots P(x_n)$ are images.

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$P = 2! = 2.$

$P_1 = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$, $P_2 = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$,

$P = \{ P_1 , P_2 \}.$

- Inverse of Permutation :-

$A = \{1, 2, 3, 4\}.$ (OR) $P(1) = 2$, $P(2) = 3$, $P(3) = 1$, $P(4) = 4$

$P^{-1}(2) = 1$, $P^{-1}(3) = 2$, $P^{-1}(1) = 3$,

$P^{-1}(4) = 4$.

$P = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$

$P^{-1} = \begin{pmatrix} 2 & 3 & 1 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$

$P^{-1} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}.$

- Composition or Product of Permutation :-

Let P_1 & P_2 be two permutations then composition of $P(1)$ & $P(2)$ is denoted by $P_1 \circ P_2$ is another permutation defined by

$\boxed{P_1 \circ P_2 (x) = P_1 (P_2 (x))}$ where, $x \in X$.

e.g $A = \{ a, b, c \}.$

$P_1 = \begin{pmatrix} a & b & c \\ c & b & a \end{pmatrix}$, $P_2 = \begin{pmatrix} a & b & c \\ b & c & a \end{pmatrix}$

$P_1 \circ P_2 = \begin{pmatrix} a & b & c \\ b & c & a \\ b & a & c \end{pmatrix}$

$P_1 \circ P_2 = \begin{pmatrix} a & b & c \\ b & a & c \end{pmatrix}$

$P_1 \circ P_2 (a) = P_1 (P_2 (a)) = P_1 (b) = b.$

$P_1 \circ P_2 (b) = P_1 (P_2 (b)) = P_1 (c) = a.$

$P_1 \circ P_2 (c) = P_1 (P_2 (c)) = P_1 (a) = c.$

$P_2 \circ P_1 = \begin{pmatrix} a & b & c \\ c & b & a \\ a & c & b \end{pmatrix}$

$P_2 \circ P_1 = \begin{pmatrix} a & b & c \\ a & c & b \end{pmatrix}.$

Q $A = \{ 5, 6, 7 \}.$ $P_1 = \begin{pmatrix} 5 & 6 & 7 \\ 6 & 5 & 7 \end{pmatrix}$, $P_2 = \begin{pmatrix} 5 & 6 & 7 \\ 7 & 5 & 6 \end{pmatrix}$

$P_1 \circ P_2 = \begin{pmatrix} 5 & 6 & 7 \\ 7 & 5 & 6 \\ 7 & 6 & 5 \end{pmatrix}$

find P^{-1} ; P_2^{-1} , $P_1 \circ P_2$.

$P_1^{-1} \circ P_2$, $P_2^{-1} \circ P_1^{-1}$.

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$$P_1 \circ P_2 = \begin{pmatrix} 5 & 6 & 7 \\ 7 & 6 & 5 \end{pmatrix}$$

$P_1(5) = 6$, $P_1(6) = 5$, $P_1(7) = 7$

$P_1^{-1}(6) = 5$, $P_1^{-1}(5) = 6$, $P_1^{-1}(7) = 7$.

$$P_1^{-1} = \begin{pmatrix} 5 & 6 & 7 \\ 6 & 5 & 7 \end{pmatrix}$$

$$P_2^{-1} = \begin{pmatrix} 5 & 6 & 7 \\ 6 & 7 & 5 \end{pmatrix}$$

$$P_1^{-1} \circ P_2 = \begin{pmatrix} 5 & 6 & 7 \\ 7 & 6 & 5 \end{pmatrix}$$

$$P_2^{-1} \circ P_1^{-1} = \begin{pmatrix} 5 & 6 & 7 \\ 6 & 5 & 7 \\ 7 & 6 & 5 \end{pmatrix}$$

$$P_2^{-1} \circ P_1^{-1} = \begin{pmatrix} 5 & 6 & 7 \\ 7 & 6 & 5 \end{pmatrix}$$

* Mathematical Induction :-

Mathematical Induction is a technique of proving a proposition over the positive Integers.

A formal statement of principle of mathematical Induction can be stated as follows.

Let, $P(n)$ be a statement over the +ve Integers then

- Inductive Base : show that $P(1)$ is true for $n \geq 1$.
  $P(n_0)$ is true for $n \geq n_0$.
- Inductive Hypothesis : Let $P(k)$ is true for $n = k$. (arbitrary k)
- Inductive step : verify $P(k+1)$ is true for $n = k+1$ using Inductive Hypothesis.

Q Show that sum of square of first n +ve Integers is $\frac{n(n+1)(2n+1)}{6}$.

→ Let, $P(n) : 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

1) for $n = 1$, $P(1)$, LHS $= 1^2 = 1$

Inductive Base :-

$$RHS = \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{1(2)(3)}{6} = \frac{6}{6} = 1$$

$\therefore$ RHS $= 1$.

LHS = RHS.

$P(1)$ is verified.

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Let $P(k) = 1^2 + 2^2 + 3^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$ is true.

3) Inductive step :-

Now, ~~R(k+1)~~ For $n = k+1$

$$\text{LHS} = 1^2 + 2^2 + 3^2 + \cdots + (k+1)^2.$$

$$= 1^2 + 2^2 + 3^2 + \cdots + k^2 + (k+1)^2.$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \quad \text{[By using step 2]}$$

$$= (k+1)\left[\frac{k(2k+1) + 6(k+1)}{6}\right]$$

$$= \frac{(k+1)}{6}\left[2k^2 + k + 6k + 6\right]$$

$$= \frac{(k+1)}{6}\left[2k^2 + 7k + 6\right]$$

$$= \frac{(k+1)}{6}\left[2k^2 + 4k + 3k + 6\right]$$

$$= \frac{(k+1)}{6}\left[2k(k+2) + 3(k+2)\right]$$

$$= \frac{(k+1)(2k+3)(k+2)}{6}.$$

$\therefore P(k+1)$ is true. Proved.

Q Prove by mathematical Induction $1 \cdot \lfloor 1 + 2 \cdot \lfloor 2 + \cdots n \cdot (n!) = (n+1)! -$

→ Let, $P(n)$: $1 \cdot (1!) + 2 \cdot (2!) + \cdots + n(n!) = (n+1)! - 1$.

1.) Inductive Base :-

For, $n = 1$, $\text{LHS} = 1 \cdot (1!) = 1 \cdot 1 = 1$

$\text{RHS} = (n+1)! - 1 = 2! - 1 = 2 - 1 = 1.$

LHS = RHS.

Hence, $P(1)$ is true.

2) Inductive Hypothesis :-

Let, $P(k) = 1(1!) + 2(2!) + \cdots + k(k!) = (k+1)! - 1$ is true.

3) Inductive step :-

$P(k+1)$: $1(1!) + 2(2!) + \cdots + (k+1) \cdot (k+1)! = (k+2)! - 1.$

Now, For $n = k+1$.

$$\text{LHS} = 1(1!) + 2(2!) + \cdots + (k+1)(k+1)!$$

$$= 1 \cdot (1!) + 2(2!) + \cdots + k(k!) + (k+1)(k+1)!$$

$$= (k+1)! - 1 + (k+1)(k+1)!$$

$$= (k+1)!\left[1 + k + 1\right] - 1.$$

$$= (k+1)!\,(k+2) - 1.$$

$$= (k+2)! - 1.$$

$$= \text{RHS}.$$

$\therefore P(k+1)$ is true. Hence $(P(n))$ is true for all natural no's.

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Q $3 + 33 + 333 + \cdots + \underbrace{333\cdots3}_{n \text{ times}} = \frac{10^{n+1} - 9n - 10}{27}$

→ Let, $P(n)$: $3 + 33 + 333 + \cdots + 333\cdots3 = \frac{10^{n+1} - 9n - 10}{27}$

1) Inductive Base :-

for, $n = 1$. LHS $= 3$.

RHS $= \frac{10^2 - 9 - 10}{27} = \frac{100 - 19}{27} = \frac{\cancel{81}\,33}{\cancel{27}} = 33$.

LHS = RHS

∴ $P(1)$ is true.

2) Inductive Hypothesis :-

Let, $P(k)$: $3 + 33 + 333 + \cdots + \underbrace{333\cdots3}_{k \text{ times}} = \frac{10^{k+1} - 9k - 10}{27}$.

3) Inductive step :- for $n = k+1$.

$P(k+1)$: $3 + 33 + \cdots + \underbrace{333\cdots3}_{(k+1) \text{ times}} = \frac{10^{k+2} - 9(k+1) - 10}{27}$.

LHS $= 3 + 33 + 333 + \cdots + \underbrace{333\cdots3}_{k \text{ times}} + \underbrace{333\cdots3}_{(k+1) \text{ times}}$.

$= \frac{10^{k+1} - 9k - 10}{27} + \frac{(333\cdots3)3}{3}$

$= \frac{10^{k+1} - 9k - 10}{27} + \frac{999\cdots9}{3}$

$= \frac{10^{k+1} - 9k - 10}{27} + \frac{10^{k+1} - 1}{3}$

$\left\{ \begin{array}{l} 9 = 10^1 - 1 \\ 99 = 10^2 - 1 \\ 999 = 10^3 - 1 \end{array} \right\}$

$= \frac{10^{k+1} - 9k - 10 + 9 \cdot 10^{k+1} - 9}{27}$

$= \frac{10^{k+1}(1+9) - 9(k+1) - 10}{27}$.

$= \frac{10^{k+2} - 9(k+1) - 10}{27}$. = RHS

∴ $P(k+1)$ is true. (Proved)

Q Prove that $11^{n+2} + 12^{2n+1}$ is divisible by 133.

→ 1) Inductive Base :-

for $n = 1$, $P(1)$: $11^3 + 12^3 = 3059 = 133 \times 23$.

3059 is divisible by 133.

∴ $P(1)$ is divisible by 133.

2) Inductive Hypothesis :-

Let $P(k)$: $11^{k+2} + 12^{2k+1}$ is divisible by 133.

3) Inductive step :-

$P(k+1)$:- $11^{k+3} + 12^{2k+3}$ is divisible by 133.

$= 11 \cdot 11^{k+2} + 12^2 \cdot 12^{k+2}$.

$= 11 \cdot 11^{k+2} + 144 \cdot 12^{k+2}$.

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$= 11 \cdot 11^{k+2} + 133 \cdot 12^{2k+1} + 11 \cdot 12^{2k+1}$.

$= 11(11^{k+2} + 12^{2k+1}) + 133 \cdot 12^{2k+1}$.

$= 11 \cdot 133\, m + 133 \cdot 12^{2k+1}$. $\{\because m \in Z^+\}$.

$= (11m + 12^{2k+1})\, 133$.

$= 133 \cdot s$ $\{\because s \in Z^+\}$.

So, $P(k+1)$ is divisible by 133.

Q1 Prove that $8^m - 3^m$ is divisible by 5.
Q-2 Prove that $n^5 - n$ is divisible by 5.
Q-3 Prove that $n^3 + 2n$ is divisible by 3.

1) $P(m)$: $8^m - 3^m$ is divisible by 5.

1) Inductive Base :

For $m = 1$, $P(1) = 8^1 - 3^1 = 5$

$\therefore P(1)$ is divisible by 5.

2) Inductive Hypothesis :-

Let, $P(k)$: $8^k - 3^k$ is divisible by 5.

3) Inductive step :-

$P(k+1)$: $8^{k+1} - 3^{k+1}$ is divisible by 5.

$= 8 \cdot 8^k - 3 \cdot 3^{k+1}$.

$= (5+3)\, 8^k - 3 \cdot 3^k$.

$= 5 \cdot 8^k + 3 \cdot 8^k - 3 \cdot 3^k$.

$= 5 \cdot 8^k + 3(8^k - 3^k)$.

$= 5 \cdot 8^k + 3 \cdot 5 \cdot s$ $\{\because s \in Z^+\}$.

$= 5(8^k + 3s)$.

$= 5 \cdot m$. $\{\because m \in Z^+\}$.

$\therefore P(k+1)$ is true i.e. it is divisible by 5.

(2). $P(n)$: $n^5 - n$ is divisible by 5.

1) Inductive Base :-

For $n = 1$, $P(1) = (1)^5 - 1 = 0$.

$\therefore P(1)$ is divisible by 5.

2) Inductive Hypothesis :-

$P(k)$: $k^5 - k$ is divisible by 5.

3) Inductive Step :-

$P(k+1)$: $(k+1)^5 - (k+1)$ is divisible by 5.

$= ({}^5C_0 + {}^5C_1 \cdot k + {}^5C_2 \cdot k^2 + {}^5C_3 k^3 + {}^5C_4 k^4 + {}^5C_5 k^5) - (k+$

$= (\cancel{1} + 5k + 10k^2 + 10k^3 + 5k^4 + k^5) - k - \cancel{1}$.

$= (k^5 - k) + (5k^4 + 10k^3 + 10k^2 + 5k)$.

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$= 5m + 5(k^4 + 2k^3 + 2k^2 + k)$.

$= 5m + 5p$.

$= 5(m+p)$ $\{\because m \& p \in z^+\}$.

$= 5s$ $\{\because s \in z^+\}$

$\therefore$ $P(k+1)$ is divisible by 5.

2) $P(n)$: $n^3 + 2n$ is divisible by 3.

1) Inductive Base:-

For, $n=1$, $\{P(1)$: $1^3 + 2(1) = 3$

$\therefore$ $P(1)$ is divisible by 3.

2) Inductive Hypothesis:-

Let, $P(k)$: $k^3 + 2k$ is divisible by 3.

3) Inductive Step:-

$P(k+1)$: $(k+1)^3 + 2(k+1)$ is divisible by 3.

$= (k+1)(k+1)^2 + 2(k+1)$

$= k^3 + 1 + 3k^2 + 3 + 2k + 2$

$= (k^3 + 2k) + 3k^2 + 6$.

$= 3 \cdot m + 3(k^2 + 2)$

$= 3 \cdot m + 3 \cdot s$.

$= 3(m+s)$

$= 3p$.

$\therefore$ $P(k+1)$ is divisible by 3.

Q Using MI show that $2^n < n!$ for $n \geq 4$.

→ $P(n)$: $2^n < n!$.

1) Inductive Base:-

For, $n=4$, Q. $P(4)$: $2^4 < 4!$.

$16 < 24$.

$\therefore$ $P(4)$ is true:

2). Inductive Hypothesis:-

Let, $P(k)$: $2^k < k!$ is true for $k \geq 4$.

3) Inductive Step:-

$P(k+1)$: $2^{k+1} < (k+1)!$ is true for $(k+1) \geq 4$.

Using Inductive Hypothesis.

$2^k < k!$

$\Rightarrow 2^k \cdot 2 < 2(k!)$

$\Rightarrow 2^{k+1} < 2(k!)$ — ①

Again, $k \geq 4$

$\Rightarrow (k+1) \geq 5$

$\Rightarrow (k+1) > 2$.

$\Rightarrow (k+1)(k!) > 2(k!)$.

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$(k+1)! > 2(k+1) \quad \text{--- (2)}$

From eqn (1) & (2).

$\Rightarrow 2^{k+1} < (k+1)!$

$\therefore$ $P(k+1)$ is true.

Q Prove that. $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n}$ for $n \geq 2$.

→ $P(n)$: $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n}$.

1) Inductive Base:-

For. $n=2$, $P(2) = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} = 1.7017$.

$\sqrt{2} = 1.414$.

$\Rightarrow 1.7017 > 1.414$.

$\therefore$ $P(2)$ is true.

2) Inductive Hypothesis:-

$P(k)$: $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{k}} > \sqrt{k}$. is true for $k \geq 2$

3) Inductive step:-

$P(k+1)$: $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{k+1}} > \sqrt{k+1}$ is true for $k+1 \geq$

Using Inductive Hypothesis-

$\Rightarrow \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{k}} > \sqrt{k}$

$\Rightarrow \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \sqrt{k} + \frac{1}{\sqrt{k+1}}$

$\Rightarrow \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \frac{\sqrt{k}\cdot\sqrt{k+1} + 1}{\sqrt{k+1}}$

$\Rightarrow \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \frac{\sqrt{k}\cdot\sqrt{k+1}}{\sqrt{k+1}}$ $\quad$ [$\because \sqrt{k+1} >$

$\Rightarrow \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \frac{k+1}{\sqrt{k+1}}$

$\Rightarrow \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1}$

$\therefore$ $P(k+1)$ is true.

Q If a set has n elements then prove that no. of possible relations on set A is greater than no. of possible functions on set A. $[2^{n^2} > n^n]$.

→ $P(n)$: $2^{n^2} > n^n$.

1) Inductive Base:-

For, $n=1$, $2^{1^2} > 1^1 \Rightarrow 2 > 1$.

$\therefore$ $P(1)$ is true.

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2) Inductive Hypothesis :-

$P(k) : 2k^2 > k^k$.

3) Inductive step :-

$P(k+1) : 2(k+1)^2 > (k+1)^{k+1}$

* Methods of Proofs :-

$p \rightarrow q$ implication

- Direct Proof : In this method we will assume that hypothesis p is true in implication ($p \rightarrow q$). We can prove that q is true by using some rules and theorems. This will show $p \rightarrow q$ is true.

Q Show by using method of direct proof if n is even then n^2 is even.

→ Let p : n is even.
q : n^2 is even

$p \rightarrow q$: If n is even then n^2 is even.

Now, Let p is true ie. n is even.

$\Rightarrow n = 2k$, where $k \in \mathbb{Z}$.

$\Rightarrow n^2 = (2k)^2$

$\Rightarrow n^2 = 4k^2$

$\Rightarrow n^2 = 2 \cdot 2k^2$.

$\Rightarrow n^2 = 2 \cdot m$. (where, $m = 2k^2 \in \mathbb{Z}$)

So, n^2 is even, ie. q is true.

Therefore, $p \rightarrow q$ is true.

Q Using method of direct proof show that sum of 2 odd integers is even.

→ Let, p : x and y are odd integers.
q : $(x+y)$ is even.

$p \rightarrow q$: If x & y are two odd integers than $(x+y)$ is even.

Now, Let, p is true. ie. x & y are odd integers.

$x = 2n_1 + 1$.

$y = 2n_2 + 1$. , $n_1, n_2 \in \mathbb{Z}$.

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$= 2(m_1 + n_2 + 1)$

$= 2(k+1)$, $k \in \mathbb{Z}$, where, $k = (m_1 + n_2) \in \mathbb{Z}$.

$\Rightarrow$, $x+y$ is even. ie. q is true.

Therefore, $p \to q$ is true.

- Indirect Proof :-

Proof By contrapositive :- In this method we can prove $p \to q$ is true by showing $\sim q \to \sim p$ is true.

Q Prove that if n is integer and $3n+2$ is even then n is even.

$\to$ Let, p: $3n+2$ is even, $n \in \mathbb{Z}$.

q: n is even

$p \to q$: If $3n+2$ is even where $n \in \mathbb{Z}$ then n is even.

$\sim p$: $3n+2$ is odd, $n \in \mathbb{Z}$.

$\sim q$: n is odd.

$\sim q \to \sim p$: If n is odd then $(3n+2)$ is odd where $n \in \mathbb{Z}$.

Now, Let, $\sim q$ is true ie. n is odd.

$n = 2k+1$, $k \in \mathbb{Z}$

So, $3n+2 = 3(2k+1)+2$

$= 6k+5$

$= 6k+4+1$.

$= 2(3k+2)+1$

$= 2m+1$, where $m = 3k+2 \in \mathbb{Z}$.

$\Rightarrow$ $3n+2$ is odd ie. $\sim p$ is true.

Hence, $\sim q \to \sim p$ is true.

Therefore, $p \to q$ is true.

Proof By contradiction :- In this method we assume that statement is false ie. if p is statement then assume that $\sim p$ is true. then by using some rules & theorems we will show that given statement p is true as well as false ie. we will reach at a contradiction. Therefore, p must be true.

Imp Q Proof that $\sqrt{3}$ is irrational.

Let, $\sqrt{3}$ is rational.

$\Rightarrow$ $\sqrt{3} = \frac{p}{q}$, where, p & q are prime no's and $q \neq 0$. —— (1)

By squaring both sides-

$\Rightarrow$ $3 = \frac{p^2}{q^2}$

$\Rightarrow$ $p^2 = 3q^2$ —— (2)

$\Rightarrow$ 3 is a factor of 'p^2'.

$\Rightarrow$ 3 is a factor of 'p'.

Let, $p = 3n$, $n \in \mathbb{Z}$.

Put, $p = 3n$ in eqn (2).

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$\Rightarrow (3n)^2 = 3q^2$

$\Rightarrow 9n^2 = 3q^2$.

$\Rightarrow q^2 = 3n^2$.

$\Rightarrow$ 3 is a factor of q^2

$\Rightarrow$ 3 is a factor of q.

$\therefore$ 3 is a common factor of p & q both which means p and q are not prime which leads to a contradiction. i.e. p and q are not prime no. So, $\sqrt{3}$ is not a rational number.

i.e. $\sqrt{3}$ is Irrational.

Q Prove that $5-3\sqrt{2}$ is irrational.

→ Let, $5-3\sqrt{2}$ is rational. —(1)

$\Rightarrow 5-3\sqrt{2} = \frac{p}{q}$, where, p & q are prime & $q \neq 0$.

$\Rightarrow 3\sqrt{2} = 5 - \frac{p}{q}$

$\Rightarrow \sqrt{2} = \frac{5q-p}{3q}$

$\Rightarrow \sqrt{2} = \frac{r}{s}$, where, $r = 5q-p$ & $3q \neq 0$.

on squaring both sides-

$\Rightarrow 2 = \frac{r^2}{s^2}$

$\Rightarrow 2s^2 = r^2$. —(2)

$\Rightarrow r^2 = 2s^2$

$\Rightarrow$ 2 is factor of r^2.

$\Rightarrow$ 2 is factor of r.

Let, $r = 2n$, $n \in Z$.

Put in eqn (2) -

$\Rightarrow 2(2n)^2 = 2s^2$.

$\Rightarrow 4n^2 = 2s^2$.

$\Rightarrow 2n^2 = s^2$.

$\Rightarrow s^2 = 2n^2$.

$\Rightarrow$ 2 is factor of s^2

$\Rightarrow$ 2 is factor of s.

$\therefore$ 2 is a common factor of r & s both which means r & s are not prime which leads to a contradiction. i.e. r & s are not prime no. So, $\sqrt{2}$ is not a rational no.

i.e. $5-3\sqrt{2}$ is irrational.

-11 By counter Example :- In this method we will give an example for which given statement is false. This ~~sto~~ example is called counter example and negation of that statement is true.

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an counter example.

→ Let f & g be two functions such that $f(x) = x^2$

$g(x) = 2x+1$.

$f \circ g = f(g(x)) = f(2x+1)$.

$= (2x+1)^2 = 4x^2 + 4x + 1$.

$g \circ f = g(f(x)) = g(x^2)$

$= 2x^2+1$.

So, composition of function is not commutative.

Q Find a counter example to a statement that every +ve integ can be written as sum of square of 3 integers.

→ $1 = 1^2 + 0^2 + 0^2$

$2 = 1^2 + 1^2 + 0^2$

$3 = 1^2 + 1^2 + 1^2$

$4 = 2^2 + 0^2 + 0^2$

$5 = 2^2 + 0^2 + 1^2$.

$6 = 2^2 + 1^2 + 1^2$

$7 = \times$.

⇒ 7 can not be written as sum of square of 3 Integers so, 7 is the counter example to that statement. and statement is a

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