Ops : Speed

17% → slow

81% → fine → feedback is not reflecting this

2% → fast

Resources:

MIT OCW 6 ↑ 29 ↓

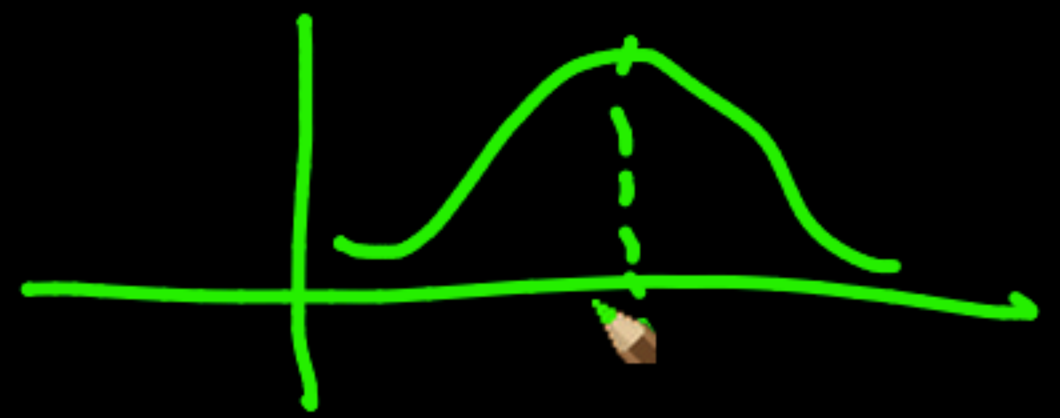
Recap:

Cond. prob | Bayes rule

Bernoulli / Binomial → Prob. mass fn.

Gaussian → mean, "scale", variance

↳ pdf

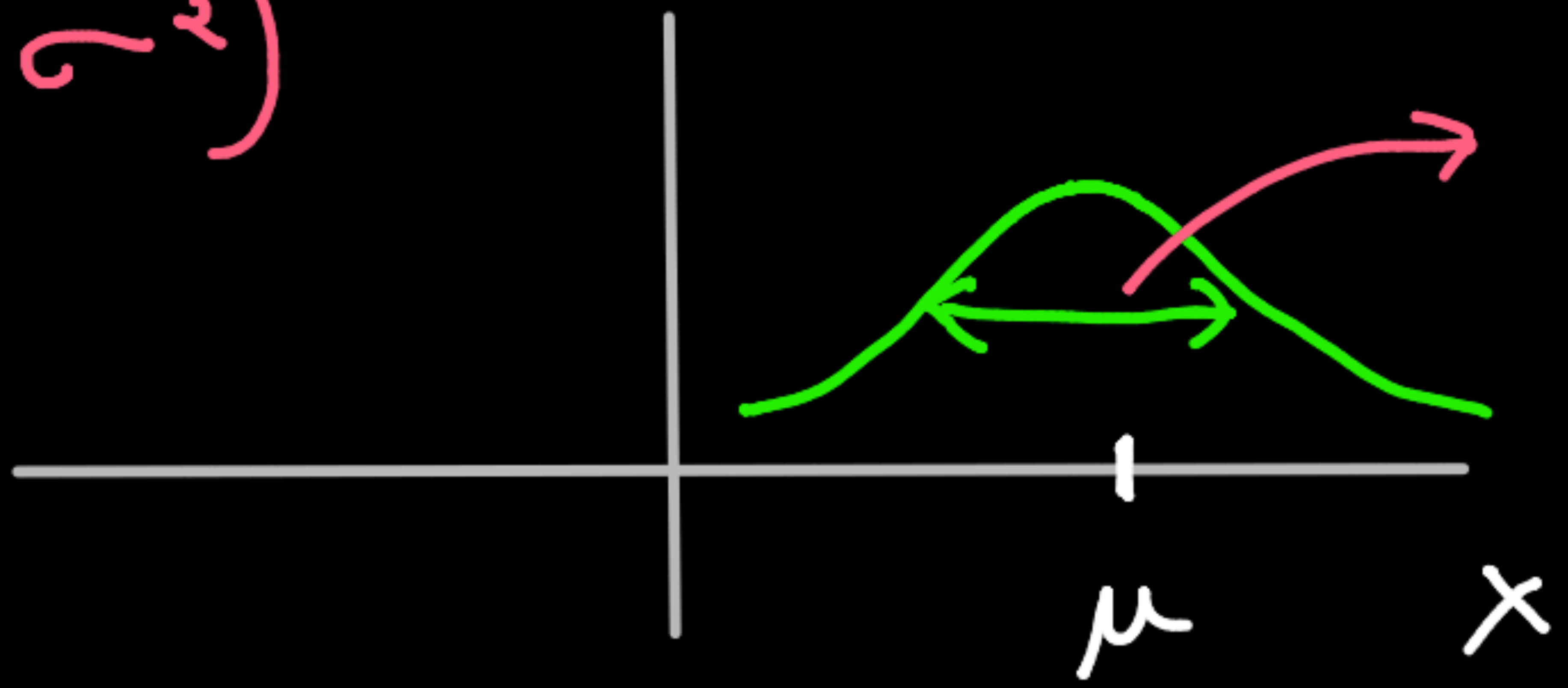


Gaussian (μ, σ^2)

μ: "mean"

σ: std. deviation

σ^2: variance



"scale"

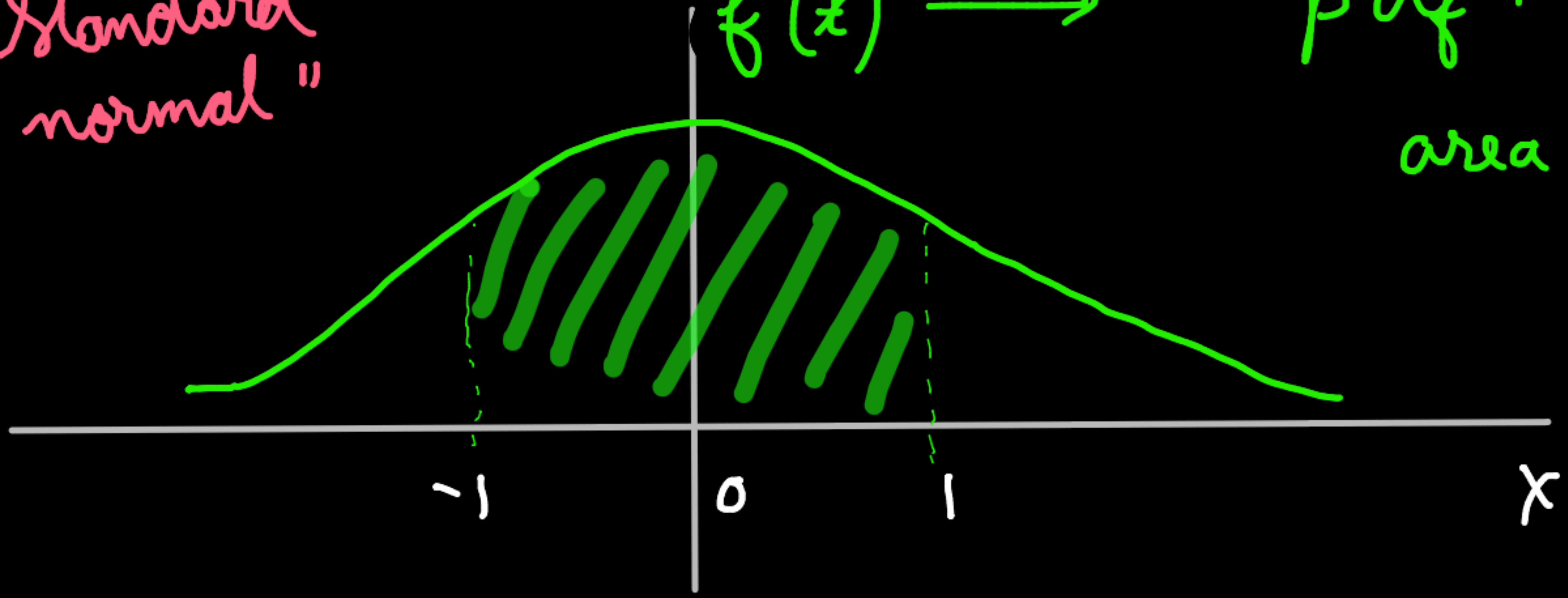
std. deviation / variance

$\left.\begin{array}{l}\mu = 0 \\ \sigma = 1\end{array}\right\} \Rightarrow$ "Standard normal"

$f(x) \longrightarrow$ "pdf"

$f(x) \geq 0$

area under f is probability



$$F(1) - F(-1) = \underline{\underline{0.682}}$$

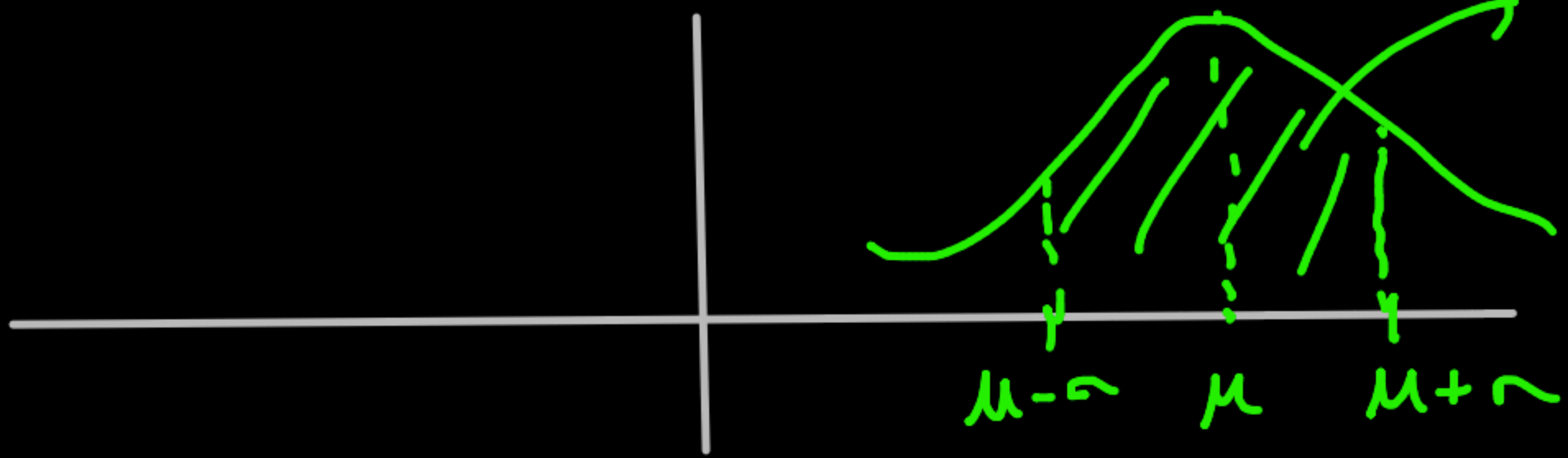
area b/w $(-\infty, 1]$ − area between $(-\infty, -1]$ = area b/w $(-1, 1]$

$$\mu \pm \sigma \longrightarrow 68.2 \simeq 68$$

$$\mu \pm 2\sigma \longrightarrow 95.4 \simeq 95$$

$$\mu \pm 3\sigma \longrightarrow 99.7 \simeq 99$$

68% of total area



$\mu = 5$

$\sigma = 0.3$

Eg : Height

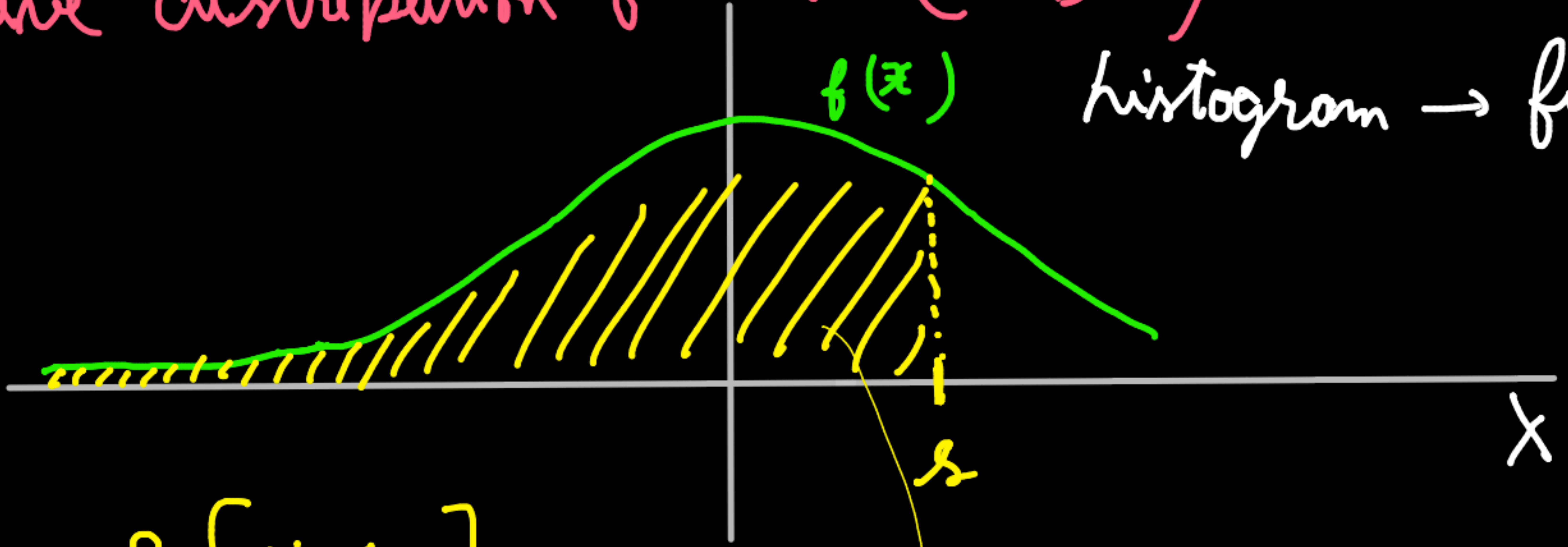
$\mu = 5.5$ np.mean

$\sigma = 0.2$ (exact no. may be different) np.var

$(5.5 - 0.2 \;,\; 5.5 + 0.2)$

68%

Cumulative distribution function (CDF)



histogram → finer → pdf

"area"

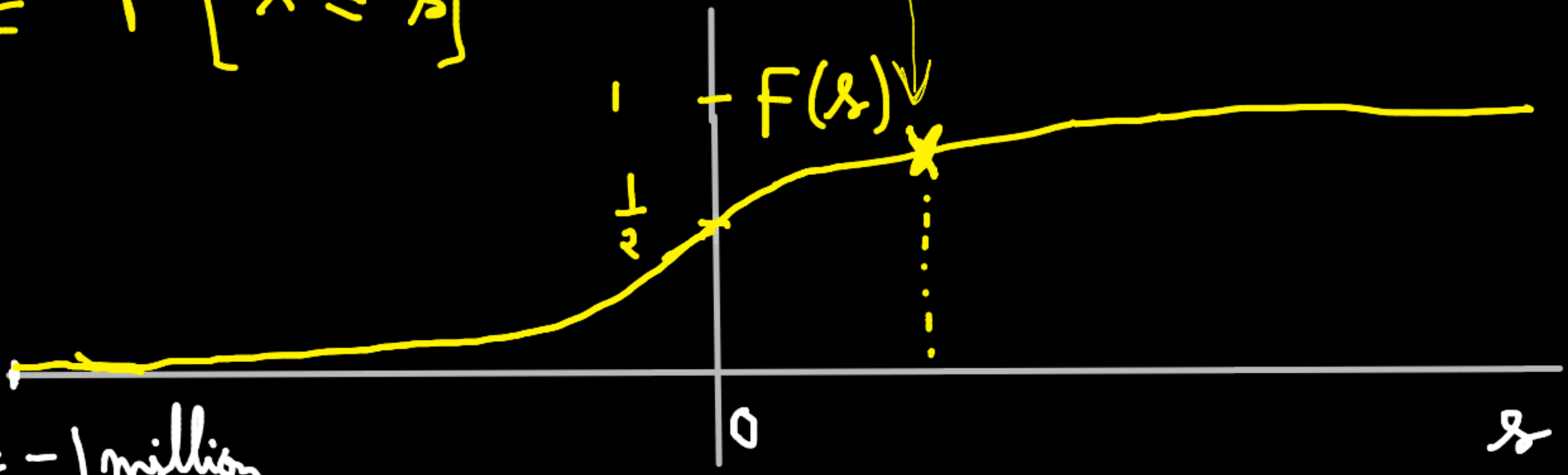
↓

probability

$$F(s) = P[X \le s]$$

↓

norm.cdf



$s = -1$ million

Y : Gaussian (μ, σ) (Height : $\mu = 5.5$, $\sigma = 0.2$)

$$X : \underbrace{\frac{Y - \mu}{\sigma}}_{\text{standardization}} \rightarrow \text{Gaussian } \mu = 0, \ \sigma = 1$$

$X \rightarrow$ Gaussian $\quad \mu = 3 \quad \sigma = 4 \quad \sigma^2 = 16$

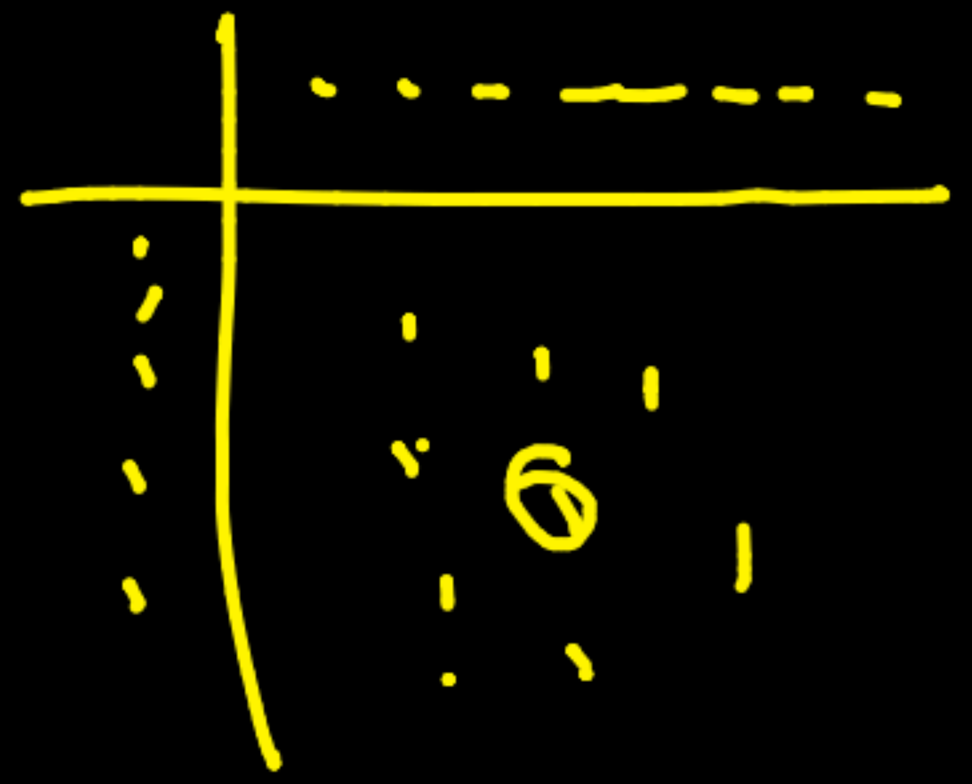
$$Z = \frac{X - 3}{4} \rightarrow \text{Gaussian } \mu = 0, \ \sigma = 1$$

$$P[X \leq 11] = P\left[\frac{X-3}{4} \leq \frac{11-3}{4}\right]$$

$$= P[Z \leq 2]$$

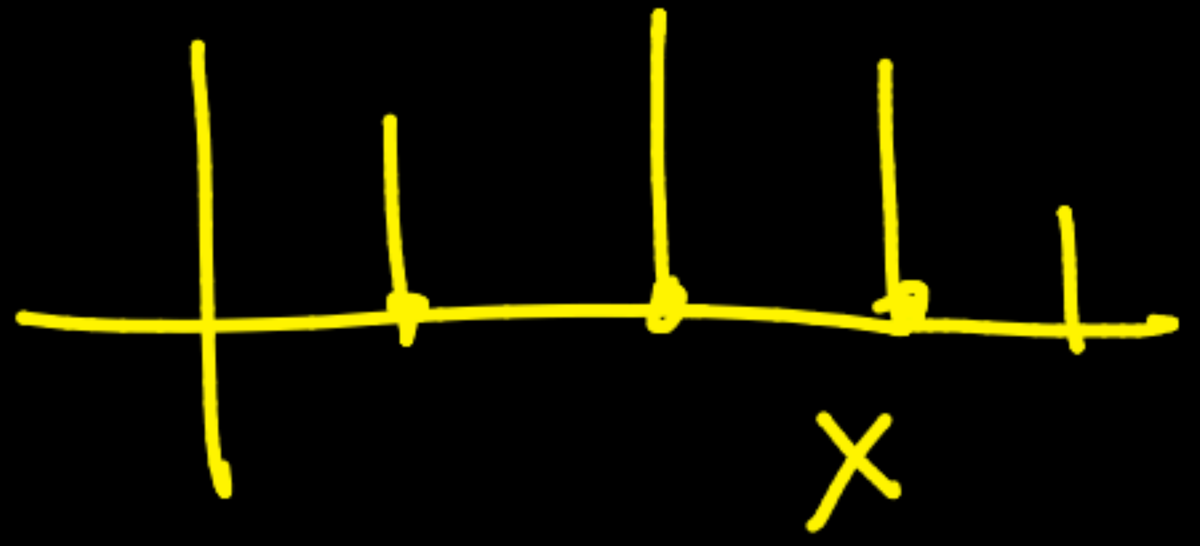
$$= \text{norm.cdf}(2)$$

(Pre-historic) table for stdGau



Mean , Variance

$X \rightarrow P(X)$



Expectation $\quad E[X] = \sum_{k} k \, P[X = k] \rightarrow$ weighted average

$$X \longrightarrow E[X] = \sum_{k} k \, P[X = k]$$

$$Y = aX + b$$

a, b are real number

$$E[Y] = \sum_{k} (ak + b)\, P[X = k]$$

$$= a \sum_{k} k\, P[X = k] + b \sum_{k} P[X = k]$$

$$E[Y] = a\,E[X] + b$$

$\{3, 1, 2\} \longrightarrow$ mean is 2

add 10

$b=10$

$\{13, 11, 12\} \longrightarrow$ mean is $2 + 10$

mult. 2

$a=2$

$\{6, 2, 4\} \longrightarrow$ mean is $2 \cdot 2$

Variance: $\mu = E[X]$

$$\mathrm{Var}[X] = E(X-\mu)^2 = E[X^2] - (E[X])^2$$

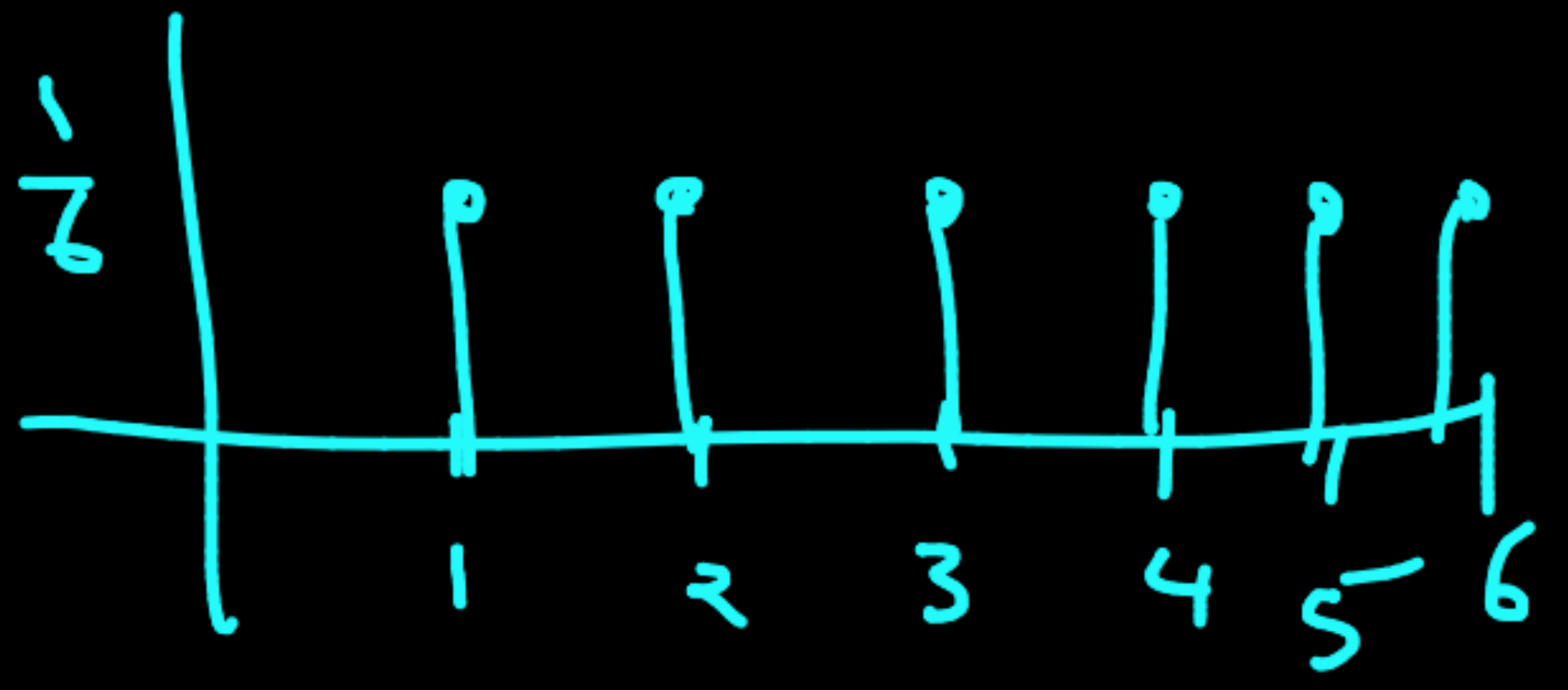
X : dice $\{1, 2, 3, 4, 5, 6\}$

$$E[X] = 3.5 \qquad (E[X])^2 = \left(\frac{7}{2}\right)^2$$

$$E[X^2] = \frac{91}{6}$$

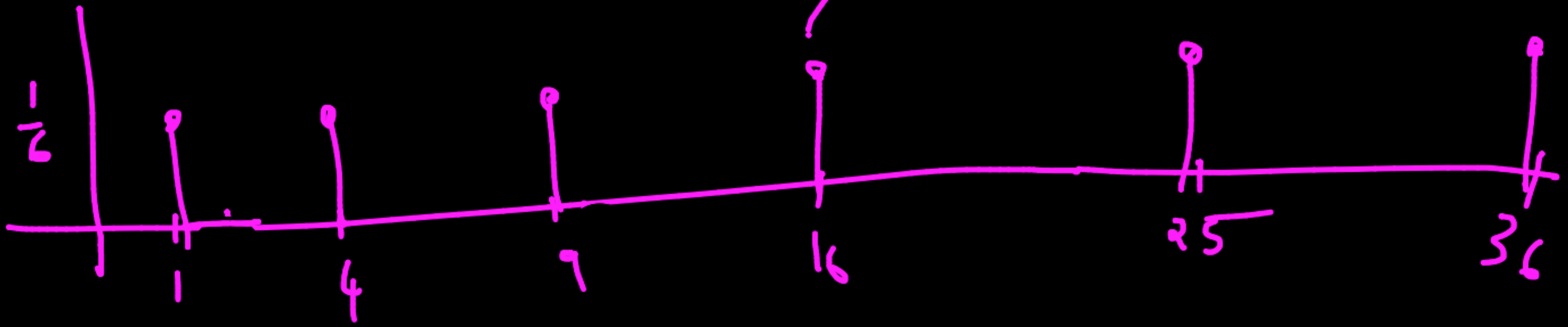
$$\mathrm{Var}[X] = E[X^2] - (E[X])^2 = \frac{91}{6} - \left(\frac{7}{2}\right)^2 = \frac{35}{12}$$

$X \in \{1,2,3,4,5,6\}$



$X^2 \in \{1,4,9,16,25,36\}$

$P[X^2=16] = P[X=4] = \frac{1}{6}$



$E[X^2] = 1\cdot\frac{1}{6} + 4\cdot\frac{1}{6} + 9\cdot\frac{1}{6} + 16\cdot\frac{1}{6} + 25\cdot\frac{1}{6} + 36\cdot\frac{1}{6}$

$= 91/6$

$$\left(1-\frac{7}{2}\right)^2\frac{1}{6}+\left(2-\frac{7}{2}\right)^2\frac{1}{6}+\cdots$$

$$X \rightarrow \mathrm{Var}[X] = E(X - E(X))^2 = E[X^2] - (E[X])^2$$

$$Y = aX + b \qquad E[Y] = a\mu + b \qquad \mathrm{Var}\, Y = E(Y - E(Y))^2$$

$$\mathrm{Var}(Y) \overset{1}{=} E[aX + b - a\mu - b]^2$$

$$\overset{2}{=} E[aX - a\mu]^2$$

$$\overset{3}{=} E[a^2 (X - \mu)^2]$$

$$\overset{4}{=} a^2 E(X - \mu)^2$$

$$\overset{5}{=} a^2 \mathrm{Var}\, X$$

Chrome File Edit View History Bookmarks Profiles Tab Window Help Mon 13 Jun 11:09 PM

Covariance: Does one data (X) influence another (Y)

$\mu_X = E[X]$

$\mu_Y = E[Y]$

More depth on Wednesday

$$Cov(X, Y) = E[(X-\mu_X)(Y-\mu_Y)]$$

$$Cov(X, X) = E[(X-\mu_X)(X-\mu_X)]$$

$$= E[(X-\mu_X)^2]$$

$$= Var X$$

21 / 22

Joint distribution (Coin + Dice)

$$\left\{ \begin{matrix} H1, & H2, & H3 & \cdots & H6 \\ T1, & T2, & T3 & \cdots & T6 \end{matrix} \right\}$$

12 outcomes

X : Coin $\rightarrow \{H, T\}$ $\{H = 1 \quad T = 0\}$

Y : Dice $\rightarrow \{1, 2, 3, 4, 5, 6\}$

H	T
H1, H2	T1, T2
H3, H4	T3, T4
H5, H6	T5, T6

$$P[X = 0, Y = 3] = \frac{1}{12}$$

$$\{Y = 3\} = \left(\{Y = 3\} \cap \{X = 0\}\right) \cup \left(\{Y = 3\} \cap \{X = 1\}\right)$$

$$= P[Y = 3 \cap X = 0] + P[Y = 3 \cap X = 1]$$

Chrome File Edit View History Bookmarks Profiles Tab Window Help   100%   Mon 13 Jun 11:20 PM

X, Y have a "joint" pmf $P[X=i, Y=j]$

(Y : 1 to 100

$$P[X=i] = (X=i \cap Y=1) \cup (X=i \cap Y=2) \cdots (\quad)$$

$$= \sum_{k} P[X=i \cap Y=k]$$

Marginal

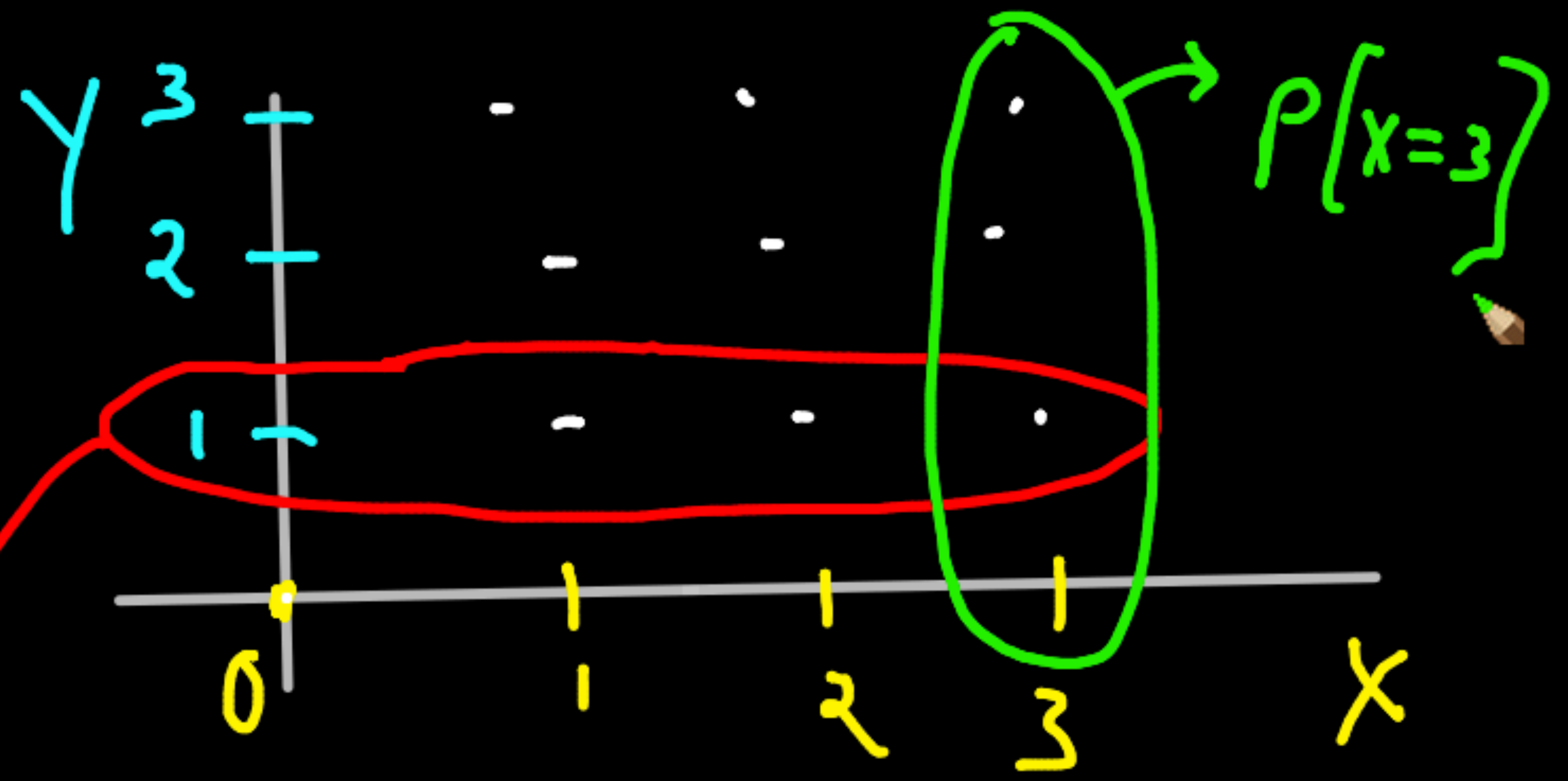
3 types of Batteries

3	4	5
New	working	defective

Pick 3 batteries randomly.

X : no. of new batteries

Y : no of working batteries



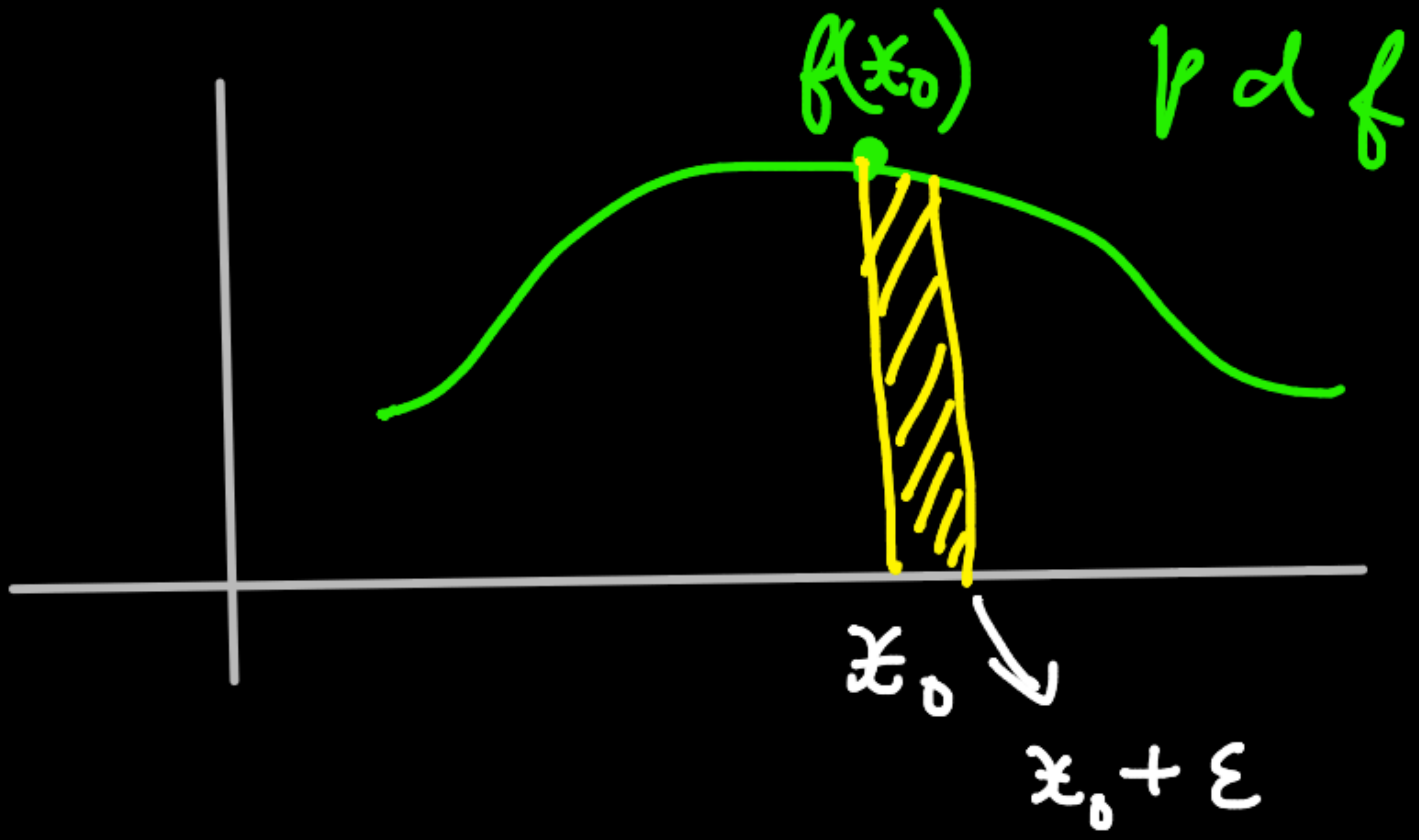
$$P[X=0, Y=0] = \frac{{}^5C_3}{{}^{12}C_3} =$$

$$P[X=0, Y=1] = {}^4C_1\, {}^5C_2 / {}^{12}C_3 =$$

$$P[X=1, Y=1] = {}^3C_1\, {}^4C_1\, {}^5C_1 / {}^{12}C_3 =$$

$$P[X=1, Y=2] = \frac{{}^3C_1\, {}^4C_2}{{}^{12}C_3} =$$

$$P[Y=1] = P[X=0, Y=1] + P[X=1, Y=1] + P[X=2, Y=1] + P[\text{[illegible]}]$$



$$P\left[x_0 \le X \le x_0 + \varepsilon\right] \simeq f(x_0) \cdot \varepsilon$$

$$\lim_{\varepsilon \to 0} \frac{P\left[x_0 \le X \le x_0 + \varepsilon\right]}{\varepsilon} = f(x_0)$$

$X : \{1, 2, 3, 4, 5, 6\}$ $\qquad 10X + 3$

$Y : \{13, 23, 33, 43, 53, 63\}$

Exercise : $E[Y]$

$Var[Y]$