Problem Solving

[Linear Algebra]

Cheat Sheet

- Vectors: $\vec{x}, \vec{y} \in R^d \rightarrow \vec{x} = [x_1, x_2 \dots x_d]^T$
- Dot product: $\langle \bar{x}, \bar{y} \rangle = \vec{x} \cdot \vec{y} = x^T y = \sum_i x_i y_i$
- Norm: [Length], [Magn], [Mod] $= \sqrt{x \cdot x} = \sqrt{\sum x_i^2}$
- Projection of $\vec{x}$ on $\vec{y}$ $= \dfrac{\vec{x} \cdot \vec{y}}{\|y\|}$
- Angle: $\cos \theta = \dfrac{\bar{x} \cdot \bar{y}}{\|x\| \ \|y\|}$
- Hyperplane: $w^T x + w_0 = 0$
- Half space: $\text{Sign}(w^T x_0 + w_0) = \begin{cases} +ve \rightarrow H^+ \\ 0 \rightarrow \text{on} \\ -ve \rightarrow H^- \end{cases}$
- Distance b/w x_0 & π_d $= \dfrac{|w^T x_0 + w_0|}{\|w\|}$

Questions

Length of the projection

Let x = [2, 1, -3] and y = [5, 8, 6] be two vectors. What is the length of the projection of x on y?

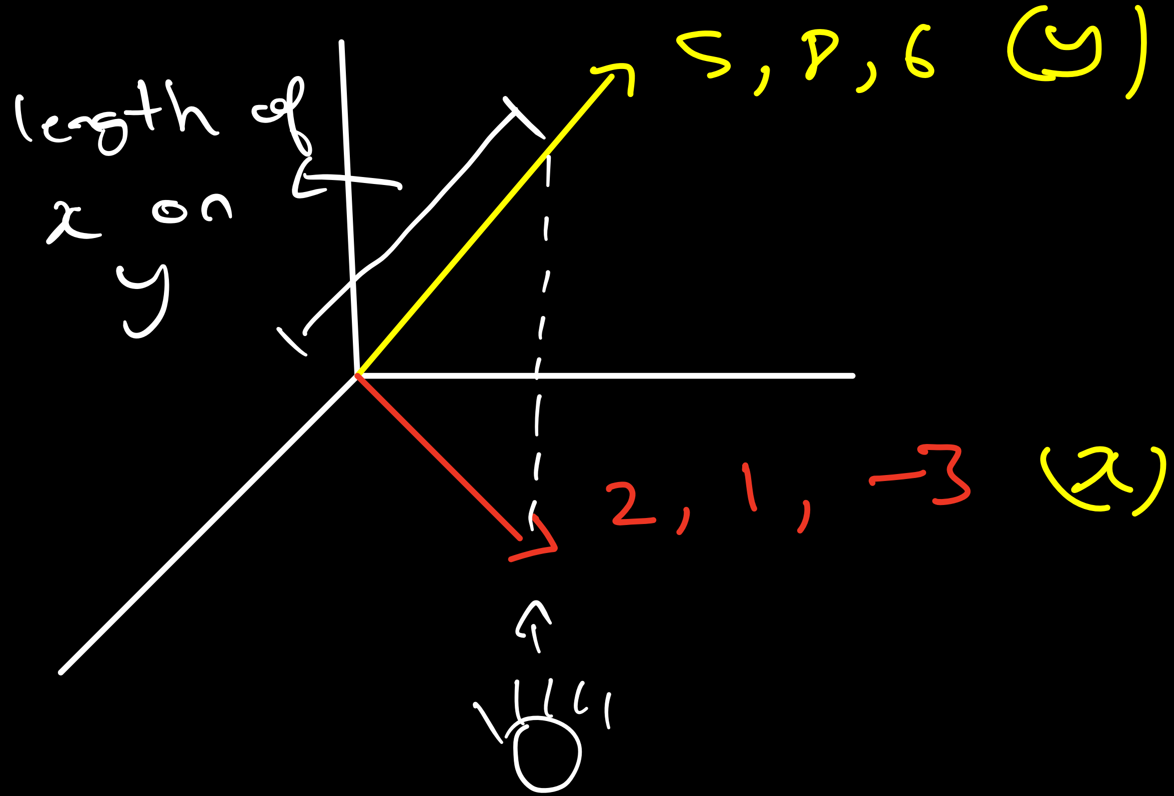
- 0
- -1
- 3
- 7

$$= \frac{\vec{x} \cdot \vec{y}}{\|y\|}$$

$$= \frac{\sum_i x_i y_i}{\sqrt{\sum y_i^2}}$$

$$= \frac{2.5 + 1.8 + -3.6}{\sqrt{5^2 + 8^2 + 6^2}}$$

$$= 0$$



$$\boxed{x \perp y}$$

Separating plane

If the label of **point**(1, 2) is -1. Given that we have a separating plane 4x + 3y - 12 = 0. What is the correct update equation for w ?
We will use the perceptron update rule for updating the coefficients of classifier, which says that:
1. If a label 1 datapoint is misclassified in the given , we will update w using w = w+x, and
2. If a label -1 datapoint is misclassified, we will update w using w = w - x, where w is the coefficient vector and x is the variables vector.

Note: A datapoint should be labelled 1 if wTx + w0 > 0 and -1 if wTx + wb< 0, where w is the vector consisting of coefficients of variables and wb is the constant

w = w + x

w = w - x

w = w - 1/2x

No update required

$x_0 = [1, 2]$, $L_0 = -1$

$D.B = l: \quad 4x + 3y - 12 = 0$

① Which Half space?

$\rightarrow 4(1) + 3(2) - 12$

$\rightarrow 4 + 6 - 12$

$= \underline{\underline{-2}} \rightarrow$ -ve $\underline{\underline{HS}}$

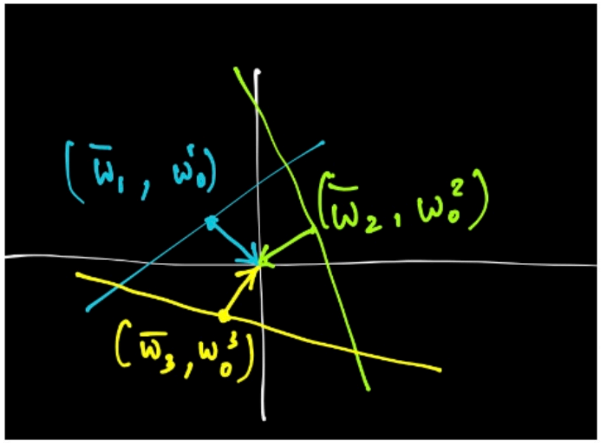
$\text{sign}(w^T x_- + w_0) == \text{sign}(H)$

$\rightarrow$ "Do nothing"

Three Biases

Three lines are shown as given in the image below.

$\bar{w_1}$, $\bar{w_2}$ and $\bar{w_3}$ are the weight vectors and w_0^1, w_0^2 and w_0^3 are the three biases respectively.



Based on the above information, which of the following statements are true?

Note : w0i represents the bias of the i[th] line

- **If w01 < 0, w02 > 0 and w03 > 0, then the origin lies in the positive half space for all lines**
- **If w01 < 0, w02 > 0 and w03 > 0, then the origin lies in the positive half space for all lines**
- **If w01 > 0, w02 < 0 and w03 > 0, then the origin lies in the positive half space for all lines**
- **If w01 > 0, w02 > 0 and w03 > 0, then the origin lies in the positive half space for all lines**

Tip : Just use Desmos

$\rightarrow$ origin $= 0, 0, 0 \ldots$

$$\text{sign}(\vec{w}^T \vec{0} + w_0) = \text{sign}(w_0)$$

$\therefore$ all w_0 should be +ve

we want +ve HS for all lines

Updated Equation

Let x_i be a datapoint with respect to ground truth label y_i. Which of the following is the correct updated equation for w?

The ground truth label of a point is +1 if the point lies in the positive halfspace and -1 if it lies in the negative halfspaces of the plane.

The coefficients of an equation are updated using the following rules:

1. If a label 1 datapoint is misclassified in prediction, we will update w using w = w+x, and

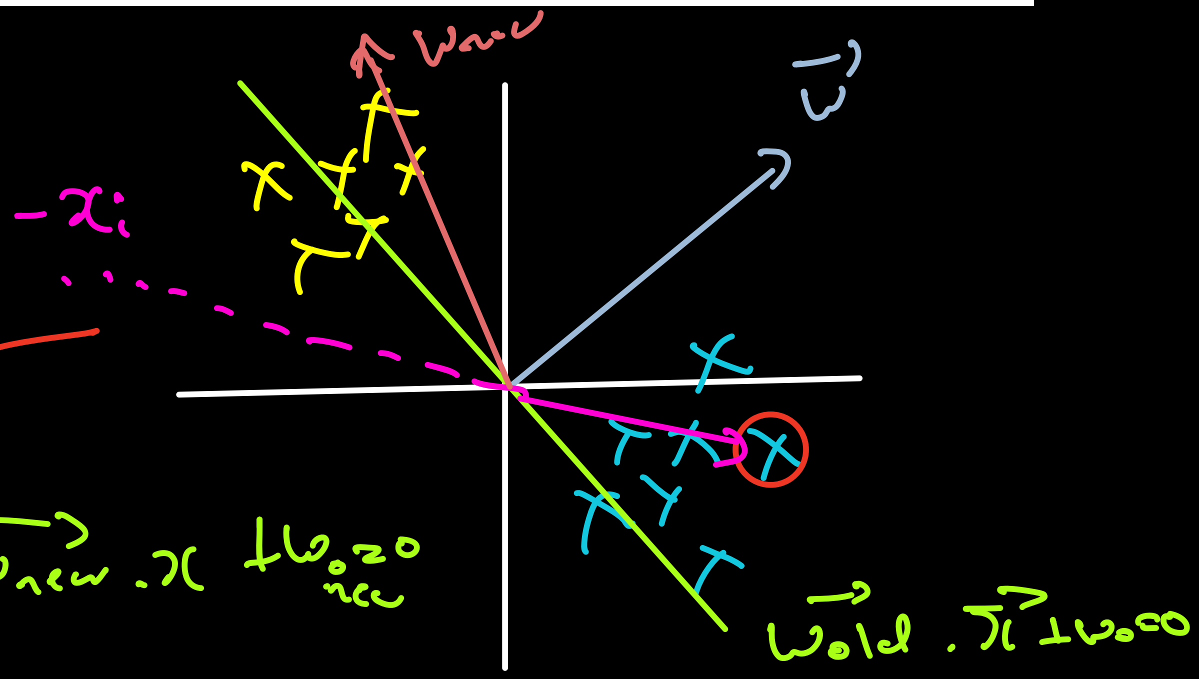
2. If a label -1 datapoint is misclassified in prediction, we will update w using w = w - x, where w is the coefficient vector of the plane and x is the variables vector.

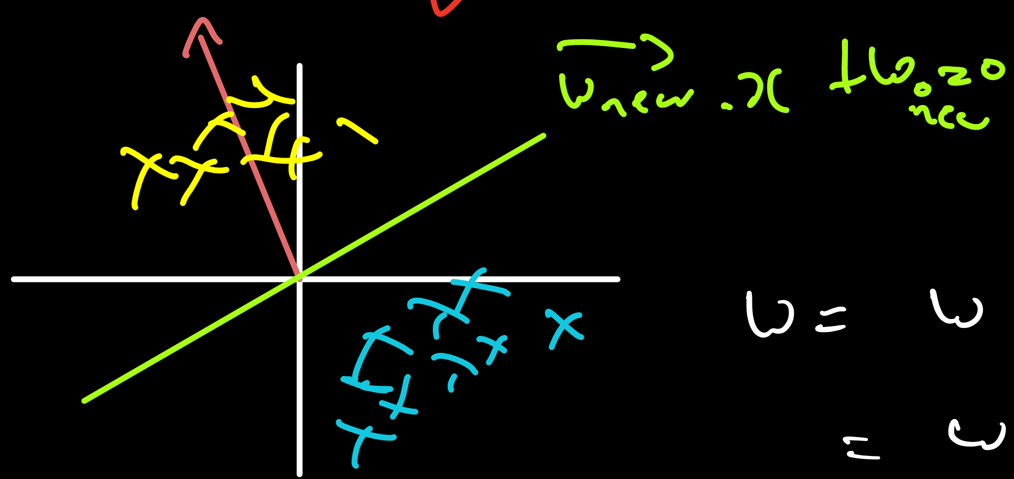
~~**if sign(w.xi - w0) != sign(yi): w = w + xi.sign(yi)**~~

if sign(w.xi + w0) != sign(yi): w = w + xi.sign(yi)

if sign(w.xi + w0) != sign(yi): w = w - xi.sign(yi)

~~**if sign(w.xi - w0) != sign(yi): w = w - xi.sign(yi)**~~





$$w = w + x_i(\text{sign}) \checkmark$$

$$= w + x_i(-ve)$$

$$= w - \underline{x_i}$$

Unit vector angle

Let a and b be two unit vectors. If the vectors **c = a + 2b** and **d = 5a − 4b** are perpendicular to each other, then the angle (in radians) between a and b is :

- π/4
- π/2
- 2π/3
- π/3

$c = a + 2b$

$d = 5a - 4b$ $\quad c \perp d \rightarrow \vec{c} \cdot \vec{d} = 0$

$$\underbrace{(\vec{p} + \vec{q}) \cdot (\vec{r} + \vec{s}) = \vec{p} \cdot \vec{r} + \vec{q} \cdot \vec{r} + \vec{p} \cdot \vec{s} + \vec{q} \cdot \vec{s}}_{\text{property}}$$

$\therefore (a + 2b) \cdot (5a - 4b) = 0$

$\therefore 5\, a \cdot a + 10\, a \cdot b - 4\, a \cdot b - 8\, b \cdot b = 0$

$\therefore 5\, \|a\|^2 - 8\, \|b\|^2 + 6\, \vec{a} \cdot \vec{b} = 0$ $\quad (\|a\| = 1,\ \|b\| = 1)$

$\therefore \vec{a} \cdot \vec{b} = 3/6 = 1/2$

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{\|a\| \, \|b\|}\right) = \cos^{-1}\left(1/2\right) = \boxed{\pi/3}$$

Straight line or not

You are given a list of (X, Y) coordinates, check whether the coordinates lie on a straight line or not. If they lie on a straight line return the slope and intercept else return -1.

Input Format

```
coordinates (list of tuples)
```

Output Format

```
if straight line exists :
(M, B)
tuple of M and B, consisting of float values rounded upto one decimal place

else:
-1
```

Example 1 Input

```
[(1.0, 5.0), (-3.0, -3.0), (2.5, 8.0)]
```

Example 1 Output

```
(2.0, 3.0)
```

Example 2 Input

```
[(5.0, 5.0), (-2.0, -3.0), (0.0, 0.0), (14.0, 6.7), (-3.0, -6.3)]
```

Example 2 Output

```
-1
```

$p1 = 1, 5$ $\qquad$ $P_3 = 2.5, 8$

$p2 = -3, -3$

$$\text{slope} : \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$m(1 \to 2) \to -3 - 5 / -8 - 1 = {}^{-8}/_{-4} = 2$

$m(2 \to 3) \to 8 - (-3) / 2.5 - (-3) = {}^{11}/_{5.5} = 2$

```
slopes = []

for i in range(1, len(p)):
    a = p[i]
    b = p[i-1]
    slopes.append((b[1] - a[1]) / (b[0] - a[0]))
if len(set(slopes)) == 1
    return slopes[0]
else
    return -1
```

Decision function

Suppose you want to implement a decision function to classify the data points into two classes => {0, 1}.

If the data point lies towards the normal then we can classify it as class 1, otherwise class 0.

Which option correctly implements the given condition?

Note: **x** represents the data point, **w** and w0 represent the weights of the plane.

a.

```
import numpy as np
def classifyDataPoint(w, w0, x):
  if np.dot(w, x) + w0 < 0:
    return 1
  else:
    return 0
```

b.

```
import numpy as np
def classifyDataPoint(w, w0, x):
  if np.dot(w, w0) + x > 0:
    return 1
  else:
    return 0
```

c.

```
import numpy as np
def classifyDataPoint(w, w0, x):
  if np.dot(w, x) + w0 > 0:
    return 1
  else:
    return 0
```

d.

```
import numpy as np
def classifyDataPoint(w, w0, x):
  if np.dot(w, w0) + x < 0:
    return 1
  else:
    return 0
```

Towards Normal

↳ if $w^T x_o + w_o > 0$

↳ return 1

Away from normal

↳ $w^T x_o + w_o < 0$

↳ return 0

Accuracy

Accuracy is one of the metrics used for classification problems. It is defined as :
Accuracy = (number of correctly classified points)/(total number of points).

Given an equation of the separating plane as $3x_1 + 4x_2 + 1 = 0$,determine the accuracy of this plane on the given below dataset.

x1	x2	label
1	4	positive
-2	-1	negative
3	-1	negative

Our model is designed in such a way that any point lying towards the normal will be classified as positive otherwise negative.

- 0.5
- 1
- 0
- 0.67

$$\text{Accuracy} = \frac{\# \text{ correct}}{\# \text{ total}}$$

$l: \; 3x_1 + 4x_2 + 1 = 0$

		label	
1) $1, 4 \rightarrow 3(1) + 4(4) + 1 > \underline{\underline{0}}$	+ve	positive	✓
2) $-2, -1 \rightarrow 3(-2) + 4(-1) + 1 < \underline{\underline{0}}$	-ve	negative	✓
3) $3, -1 \rightarrow 3(3) + 4(-1) + 1 > \underline{\underline{0}}$	+ve	negative	✗

$\rightarrow \text{Acc} = 2/3 = \underline{\underline{0.67}}$