

Experiment: administer m_1 to covid patients

$$S = \left\{ \underset{25}{n_{1-7}}, \underset{20}{n_{8-14}}, \underset{50}{n_{15+}}, \underset{5}{d} \right\}$$

E_1 : recovered $P(E_1) = \frac{25+20+50}{25+20+50+5} = \frac{95}{100}$

E_2 : recovered in 14 days $P(E_2) = \frac{25+20}{25+20+50+5} = \frac{45}{100}$

E_3 : recovered in 20 days $\frac{45}{100} \leq P(E_3) \leq \frac{95}{100}$

not an
event
→

Dice:

$$S = \{1, 2, 3, 4, 5, 6\}$$

E_1 : odd

$$\{1, 3, 5\}$$

$$P(E_1) = \frac{3}{6}$$

E_2 : perfect sq

$$\{1, 4\}$$

$$P(E_2) = \frac{2}{6}$$

Coin Toss

$S = \{H, T\}$ (Fair coin)

E : getting a head

$$P(E) = \frac{1}{2}$$

Coin toss twice:

$$S = \{ HH, HT, TH, TT \}$$

$E1$: getting at least one head

$$E1 = \{ HH, HT, TH \}$$

$$P(E1) = \frac{3}{4}$$

Dice two Times

(D_1, D_2)

$$S = \left\{ \begin{array}{l} (1,1), (1,2), (1,3) \dots (1,6), \\ (2,1), (2,2), (2,3) \dots (2,6) \\ \vdots \\ (6,1), (6,2), \dots (6,6) \end{array} \right\}$$

36 outcomes

$D_1 = 2 \quad D_2 = 6$

$E : D_1 \text{ is } 3 \quad E = \{(3,1), (3,2), \dots, (3,6)\} \quad P(E) = \frac{6}{36}$

$F : D_1 + D_2 = 7 \quad F = \{(1,6), (2,5), \dots, (6,1)\} \quad P(F) = \frac{6}{36}$

$G : D_1^2 + D_2^2 = 5^2 \quad G = \{(3,4), (4,3)\} \quad P(G) = \frac{2}{36}$

$E \cap F = \{(3,4)\}$

$P(E \cap F) = \frac{1}{36}$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/36}{6/36} = \frac{1}{6}$$

Joint Prob

U : All covid patients $\rightarrow 1000$

Intersection

m_1 : patient is given medicine $\rightarrow 100$

s : patient survived $\rightarrow 700$

$$P[m_1] = \frac{100}{1000}$$

$$P[s] = \frac{700}{1000}$$

$$P[m_1 \text{ and } s] = \frac{95}{1000}$$

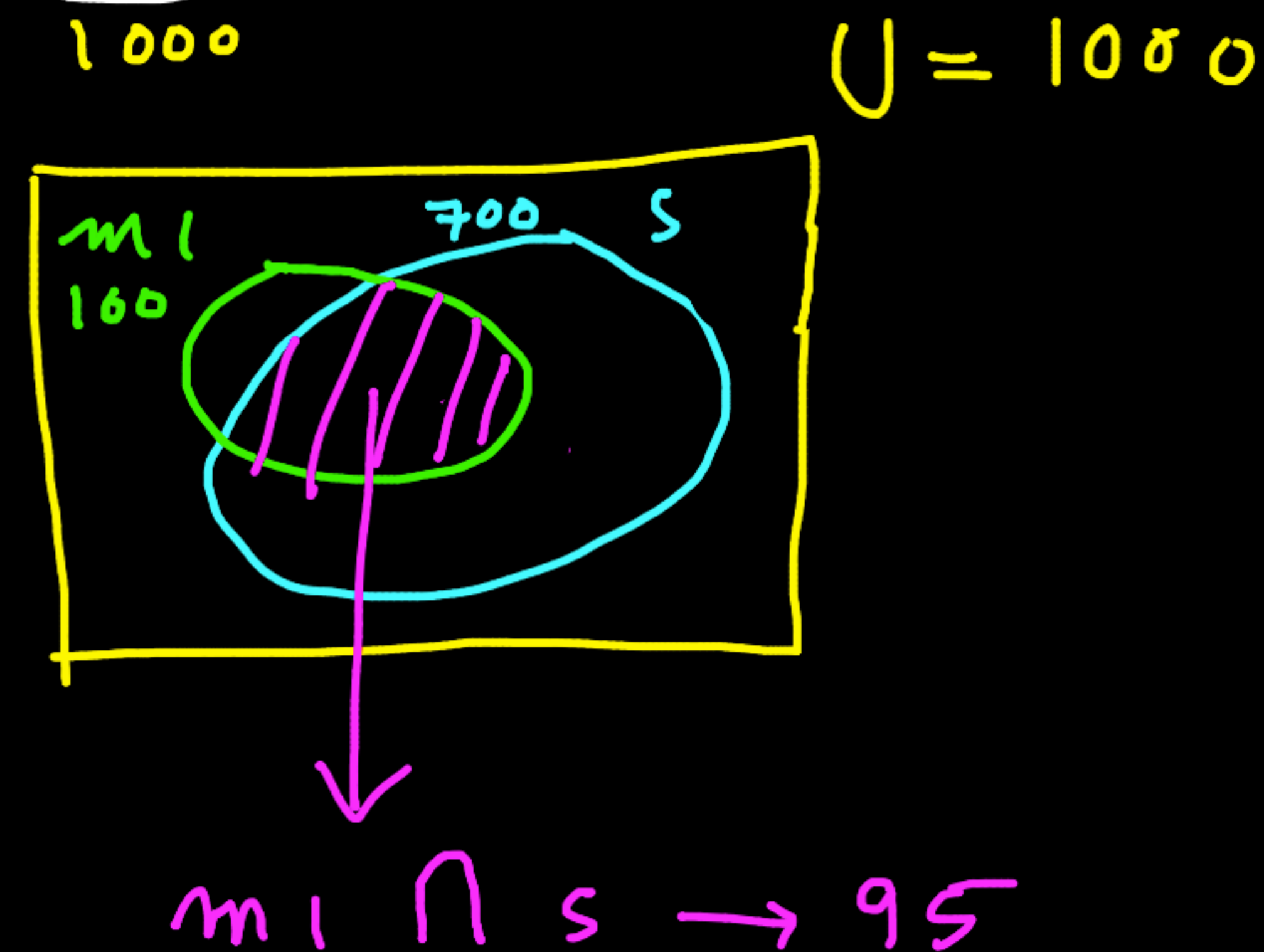
Conditional Probability

Prob of surviving given m_1

$$P[s | m_1] = \frac{95}{100} = \frac{P[s \cap m_1]}{P[m_1]}$$

Prob of m_1 given that patient survived

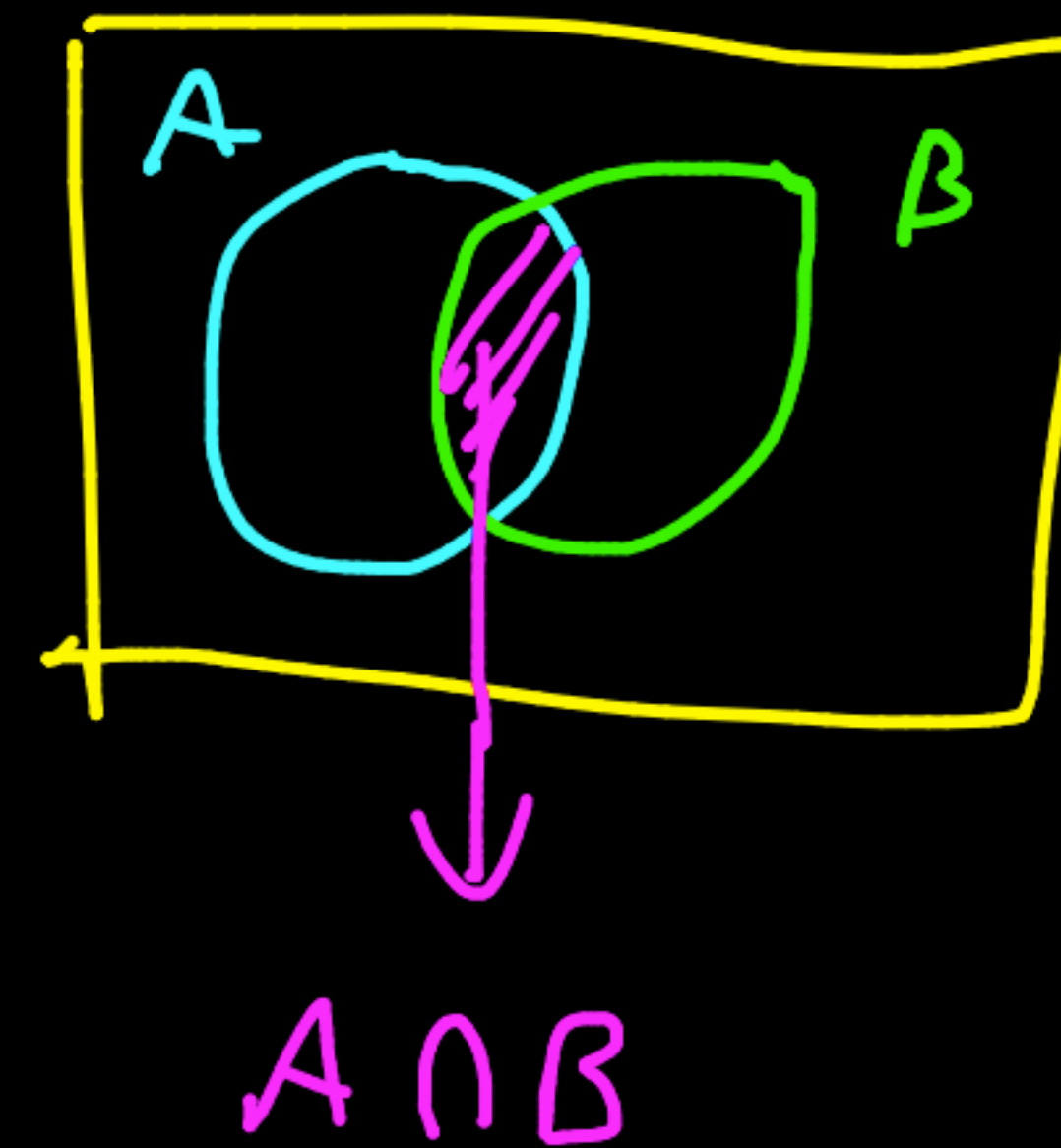
$$P[m_1 | s] = \frac{95}{700} = \frac{P[s \cap m_1]}{P[s]}$$



Definition: [Conditional Prob]

$$P[A|B] = \frac{P(A \cap B)}{P(B)}$$

$$P[B|A] = \frac{P[A \cap B]}{P(A)}$$



Covid example new medicine m2

Independence

U: all covid patients — 1000

m2: patients given m2 — 100

S: patients who survived — 700

$$P[m_1] = \frac{100}{1000} = 0.1$$

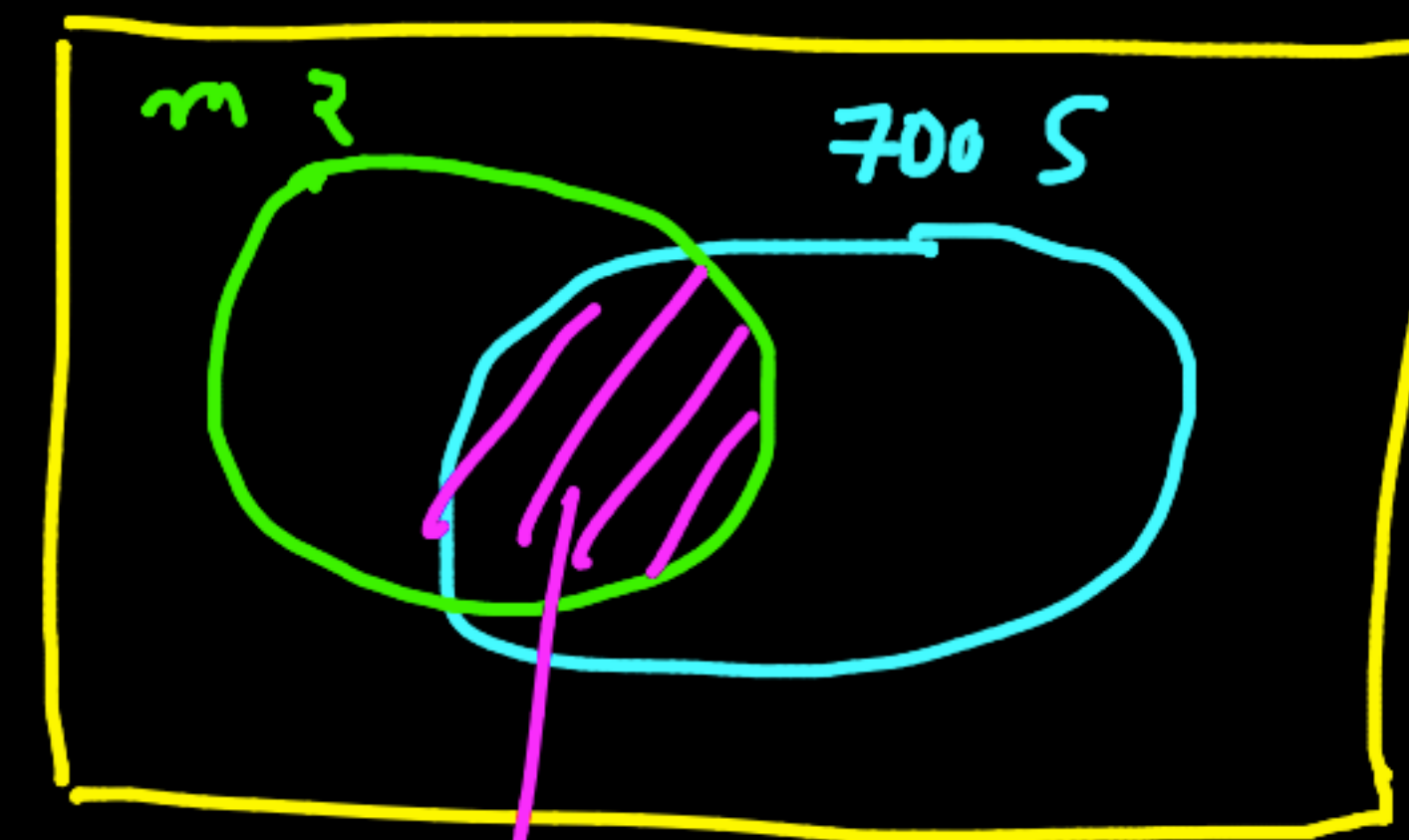
$$P[S] = \frac{700}{1000} = 0.7$$

Prob of surviving given m2

$$P[S|m_2] = \frac{70}{100} = 0.7$$

Prob of m2 given survived

$$P[m_2|S] = \frac{70}{700} = 0.1$$



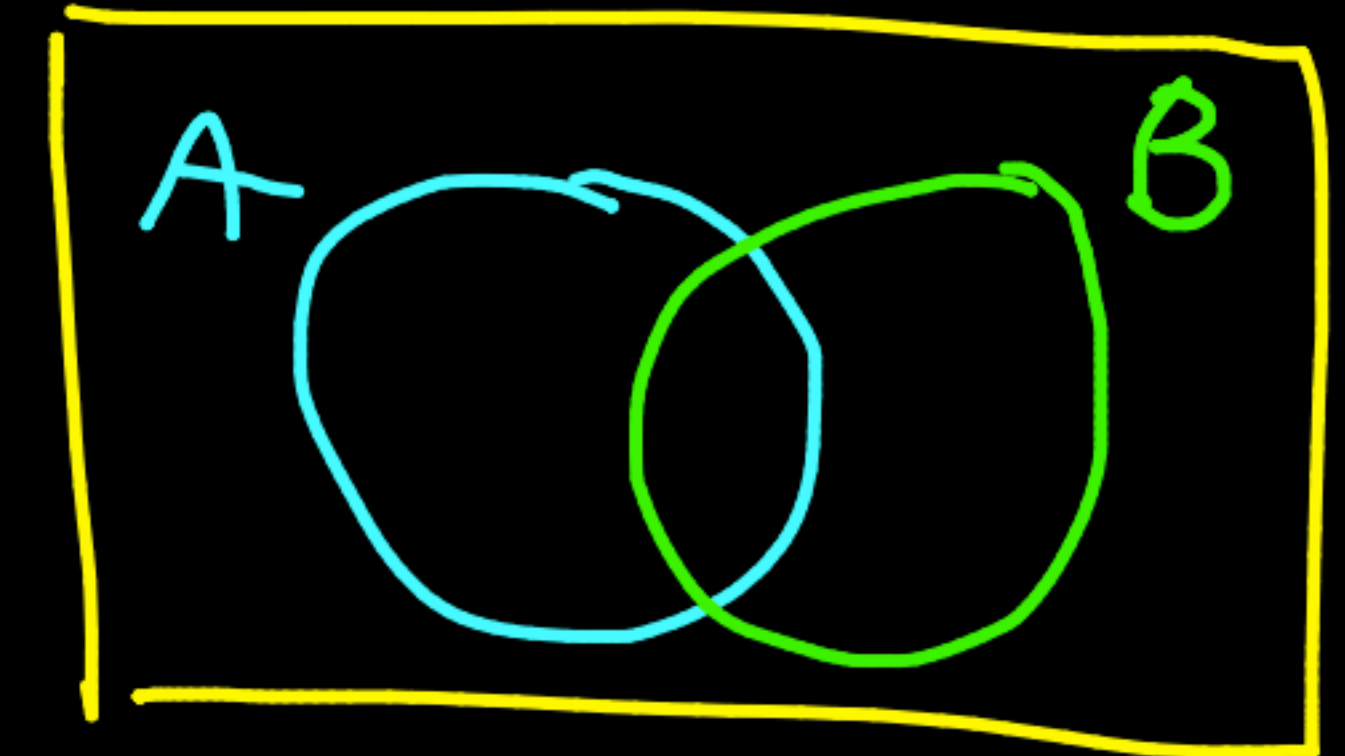
$$m_2 \cap S \rightarrow 70$$

Definition : [Independence]

We say that A and B are independent

if

$$P[A|B] = P[A]$$



Coin + Dice

$$S = \left\{ \begin{array}{l} (H, 1), (H, 2), \dots, (H, 6) \\ (T, 1), (T, 2), \dots, (T, 6) \end{array} \right\} \quad 12 \text{ outcomes}$$

E : coin is heads

$$E = \{ (H, 1), (H, 2), \dots, (H, 6) \} \quad P(E) = \frac{6}{12} = \frac{1}{2}$$

F : dice is 3

$$F = \{ (H, 3), (T, 3) \} \quad P(F) = \frac{2}{12} = \frac{1}{6}$$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/12}{2/12} = \frac{1}{2} = P(E)$$

$$\begin{aligned} E \cap F &= \{ (H, 3) \} \\ P(E \cap F) &= \frac{1}{12} \end{aligned}$$

$$P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{1/12}{1/2} = \frac{1}{6} = P(F)$$