

$[3, -1, 2, 6]$
List / Array

Subarray \rightarrow contiguous segment
of an array.

Subsequences | Subset

$\begin{matrix} 0 & 0 \\ 0 & 1 \\ \hline 1 & 1 \end{matrix}$	Gray <u>code</u>	$\begin{matrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{matrix}$
$\begin{matrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{matrix}$		

You can delete 0 or more elements from the array and the sequence of elements that you get is called a subsequence.

ORDER

$[2, -3, 5, 6, 7, 10]$

MATTERS



Should be

Same as the

original array.

= $[-3, 7, 10]$ \rightarrow a subsequence

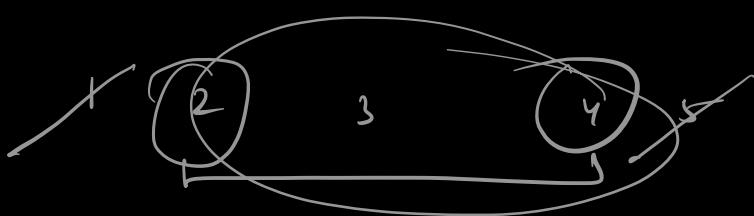
Subsets the order doesn't matter !

No!
[-3, 7, 1] is a subsequence
of [4, 7, 1, -3, 4]

" is a subset
Yes of _____

DNA sequence ATGC
String matching segment
LCC

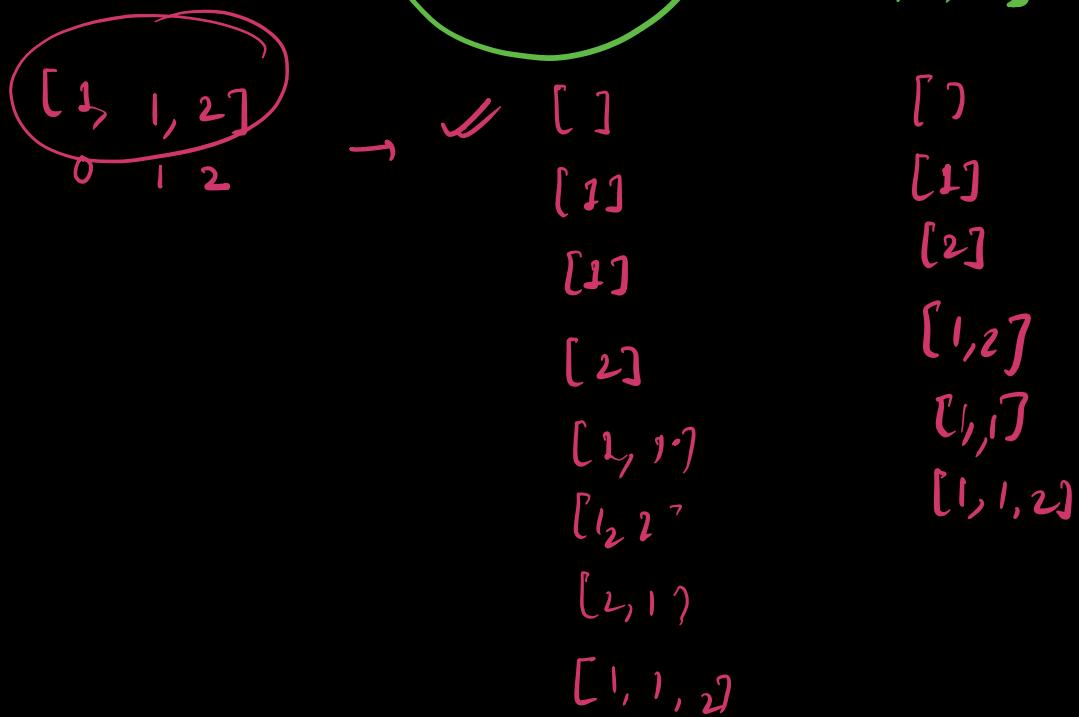
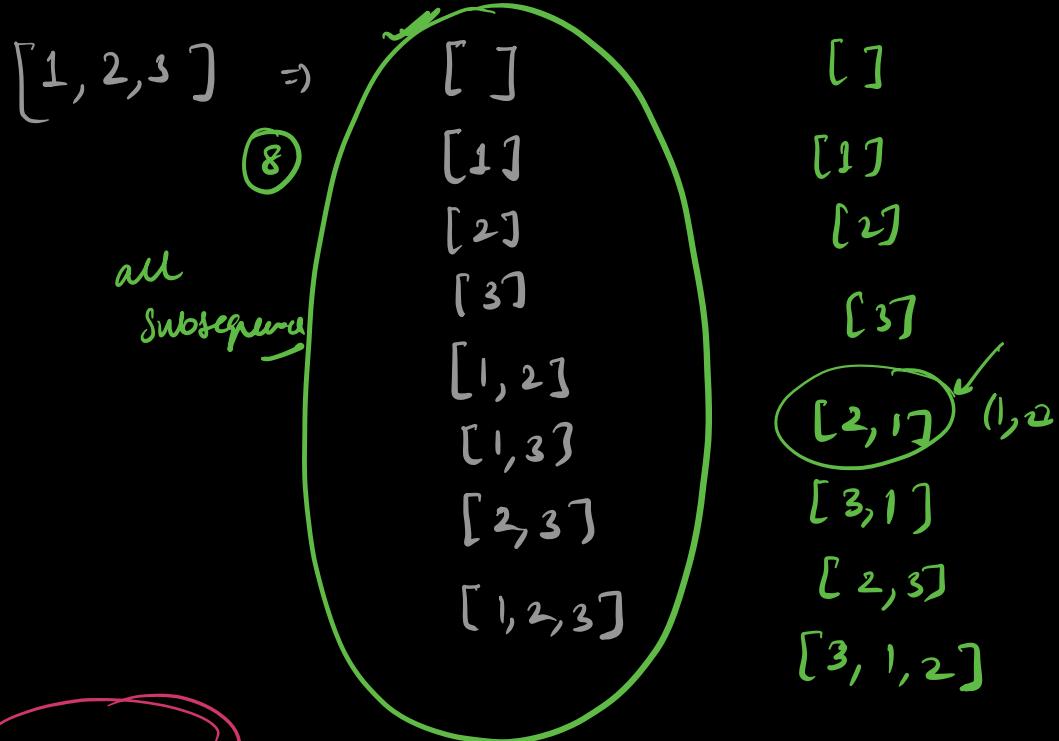
Subsequence \rightarrow Deleting 0 or
more element



- ① All subsequences are subsets
- ② All subsets are subsequences

Sort the array
and generate subsequence

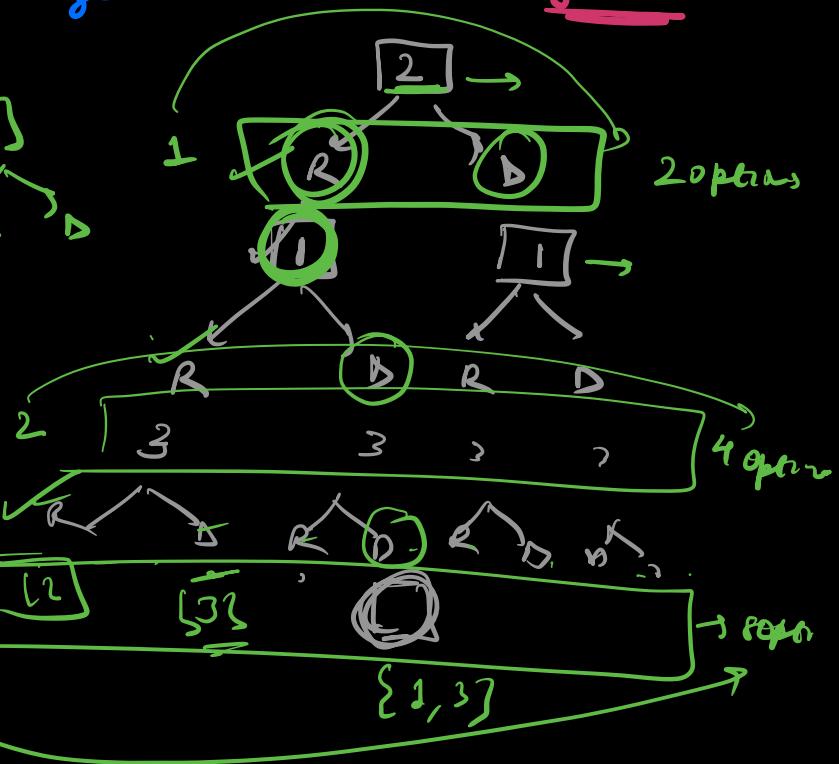
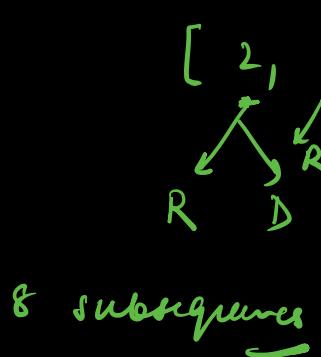
→ subsets of original array



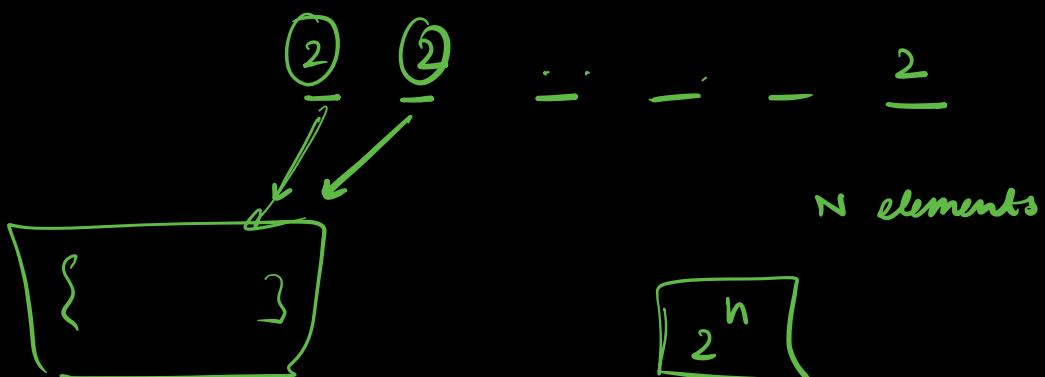
Q1

How many subsequences of an array
of size N?

Logic 1



Logic 2



Logic 3

Q 2

Given N elements. Check if there exists a subsequence which has sum of its elements as K .

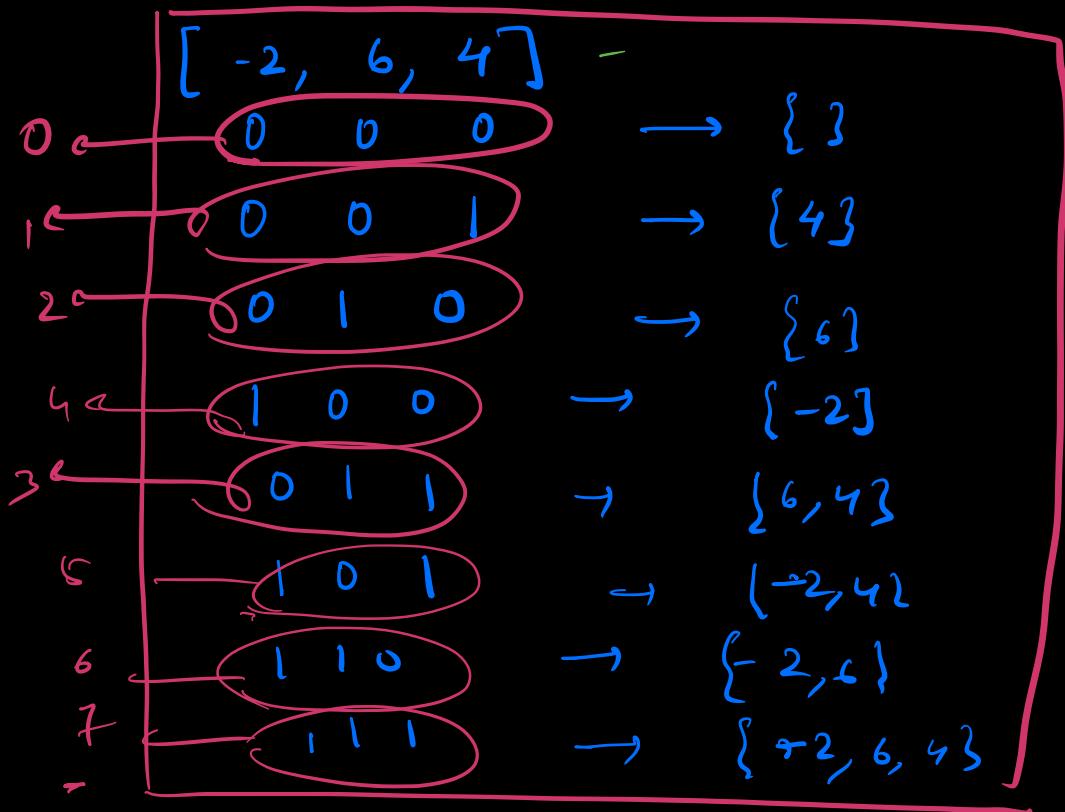
$$arr = [2, 1, 3, -1, 4]$$

$$\underline{K=0} \Rightarrow [\underline{1}, \underline{-1}] \Rightarrow \text{True}$$

$$arr = [\underline{3}, \underline{-5}, 0, \underline{6}, \underline{2}, -3, \underline{5}]$$
$$\underline{K=10} \quad \underline{\text{True}}$$

B.F Generate all possible subsequences
↳ Check the sum for all subsequence
 $= K$

$$TC \boxed{2^N * N}$$



$$0 \rightarrow 2^n - 1$$

$\downarrow^{2^{\text{nd}}}$

\downarrow^*

\downarrow^{0**}

\downarrow

\downarrow

\downarrow

Represents a possible
subsequence!

$n=3$

$i = [0, \dots, 7]$

Tracks
the
sum of
a
subsequence

check if
a no.
is
a part of

```

for i in range(2**n):
    sum = 0
    for j in range(N):
        if i & (1<<j) != 0:
            sum += arr[j]
    if sum == k:
        return True
  
```

subsequence

0, 1, 2 j

$\phi \& (1 << 0)$
 $\phi \& (1 << 1) X$
 $\phi \& (1 << 2)$ X

i =

0	sum = 0	{3}
1	sum = 0	{4}
2		
3		
4		
5	101	
6		
7		

2 + 1 = 1

0 0 1

$5 \& (1 << 0)$
 $5 \& (1 << 2)$

$[-2, 6, 4]$
sum = 2

2.

6 + 7 + (-3) 10

$k < 0$ X

check (arr, k, i) \leftarrow [i]

\Rightarrow It return True if there exists a subsequence with sum $= k$ in the otherwise return False

arr
↓
Starting at index i

a₁, a₂, a₃

~~a₄~~ (= k - a₄)

\downarrow + arr[i] \Leftrightarrow contribution to subseq sum

0

check (arr, k - arr[i], i+1)

check (arr, k, i+1)

def check (arr, k, i) :

if i == len(arr) \Rightarrow

if k == 0 :
return True

return False

if check (arr, k - arr[i], i+1) or

check (arr, k, i+1) :

return True

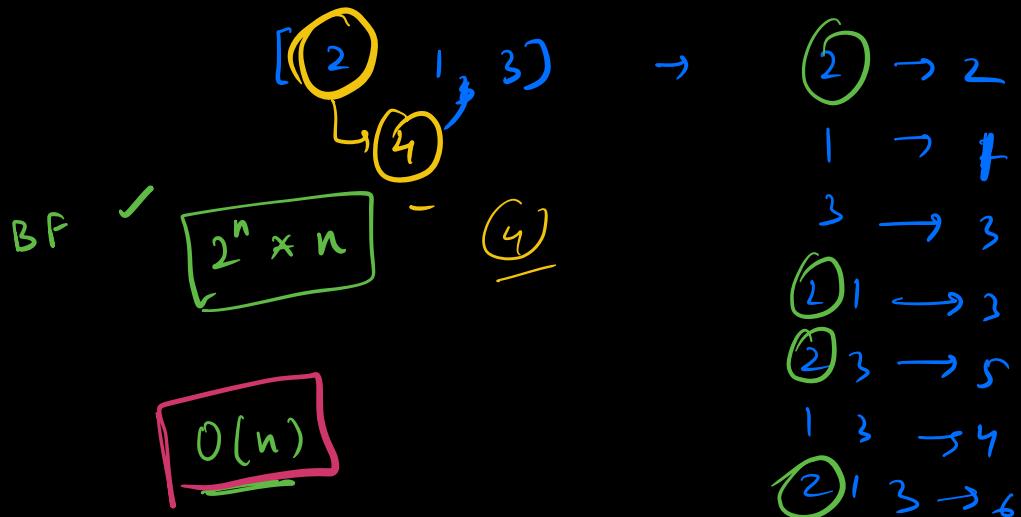
return False

$O(2^n)$

Base !

Break - 5 min.

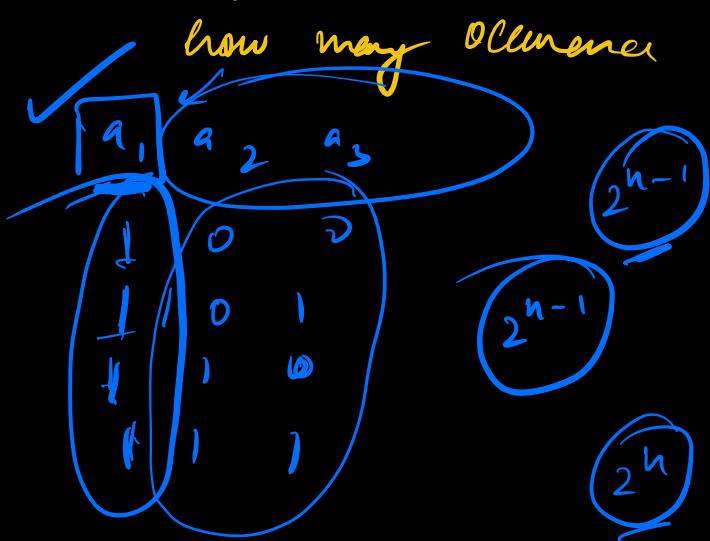
Q Find sum of all the subsequences



Contribution
Technique

↓

Final
sum will
have



2^{n-1}

a_1 + no. of times it occurs in all subseq

$$\frac{2^n}{2} = \left(2^{n-1}\right)$$

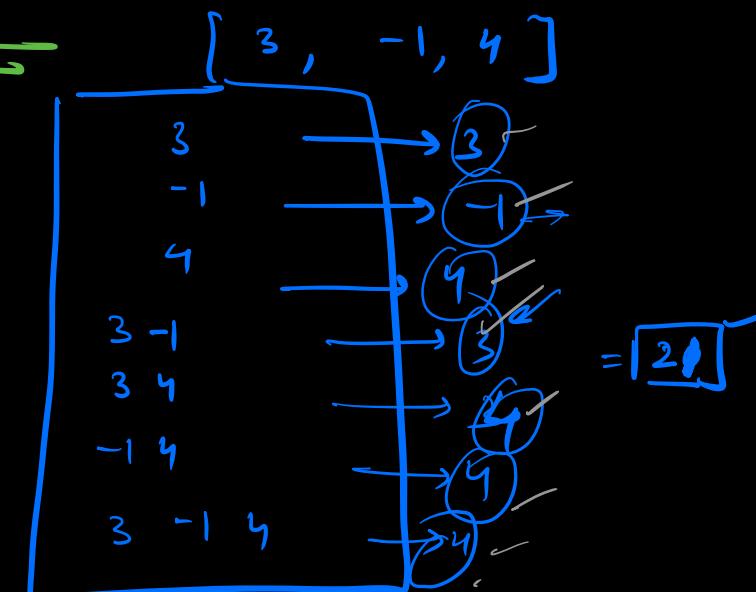
\checkmark
 ans = 0
 for i in range (N) :
 $\underline{\text{ans}} = \underline{\text{ans}} + (\text{arr}[i] * 2^{n-1})$
 return ans

\mathbb{Q} Given N array elements. Find the sum of max of every subsequence ! min ?

$$2^n * n$$

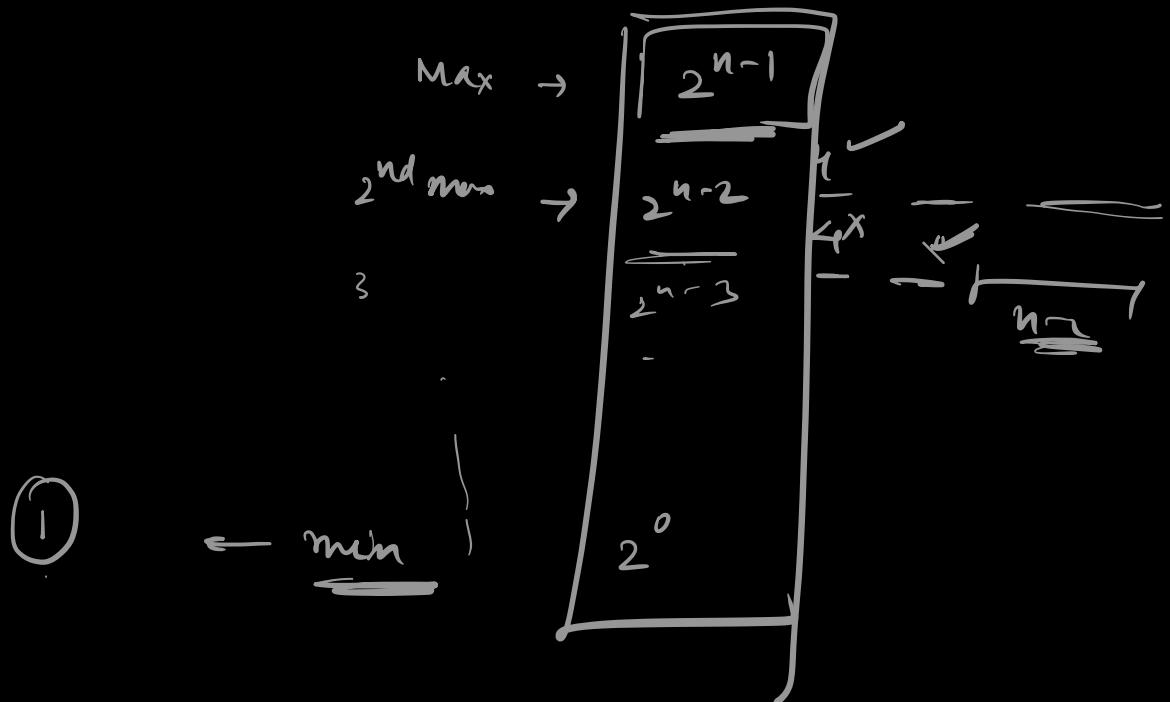
bf ✓

$$\cancel{*} \quad \cancel{3} \quad \cancel{\frac{n-2}{2^{n-2}}}$$



① $e.setk()$
 ② for i in range(n)
 $\quad \text{sum} += 2^i * \underline{\underline{\text{arr}[i]}}$

$T.C = \underline{\underline{n \log n}}$ ① $\underline{\underline{\text{arr}[0] \rightarrow \text{min}}}$



Q Find sum of $(\max - \min)$ of every subsequence.

$$\boxed{\sum (\max_i - \min_i)}$$

\downarrow

$$\sum \max_i - \sum \min_i$$

$\overbrace{15}^{\text{Ans}}$

Given array: $[-1, 3, 4]$

	Max	Min
$\{3\}$	3	0
$\{-1\}$	-1	-1
$\{3, 3\}$	3	3
$\{4\}$	4	4
$\{-1, 3\}$	3	-1
$\{-1, 4\}$	4	-1
$\{3, 4\}$	4	3
$\{-1, 3, 4\}$	4	-1

⊕ OOPS → Class
 ↳ Read ↳ Pillars of OOPS
Object