Recap:

Sample space, Events

$S = \{1, 2, 3, 4, 5, 6\}$ → $E = \{3, 4, 6\} \subseteq S$

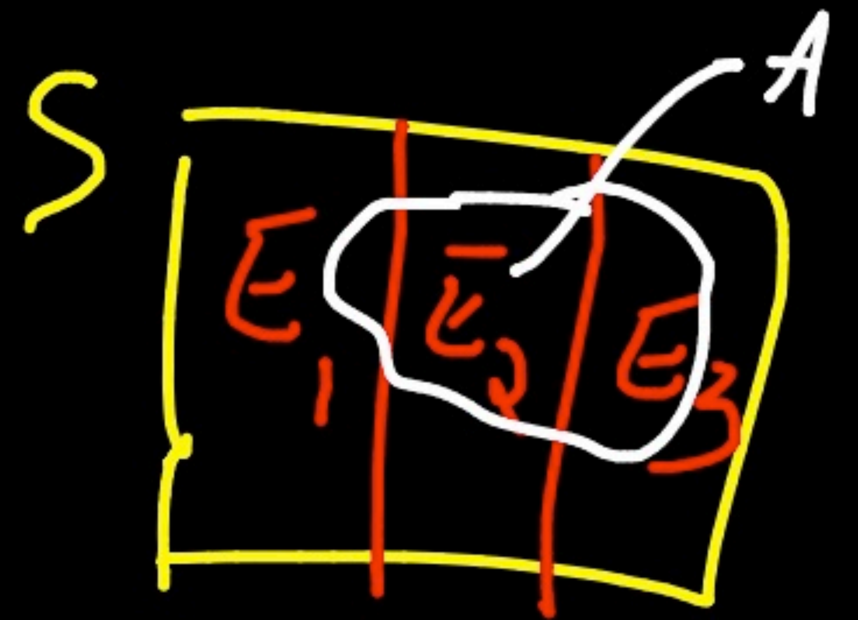
Intersection, Union

$A \cap B \rightarrow$ in both / and

$A \cup B$ "or"

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad \text{— } 1$$

Mutually Exclusive Exhaustive



$E_1 \cap E_2 = \{\ \}$

$E_2 \cap E_3 = \{\ \}$

$E_1 \cap E_3 = \{\ \}$

$E_1 \cup E_2 \cup E_3 = S$ → 2

Conditional probability $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ → always true

Independence: A and B are independent if $P(A \mid B) = P(A)$ → 3

Bayes Theorem: $P(E_1 \mid A) = \dfrac{P(A \mid E_1)\, P(E_1)}{P(A)}$ → 4

E_1, E_2, E_3 are MEE → $P(A) = P(A \mid E_1) P(E_1) + P(A \mid E_2) P(E_2) + P(A \mid E_3) P(E_3)$ → 5

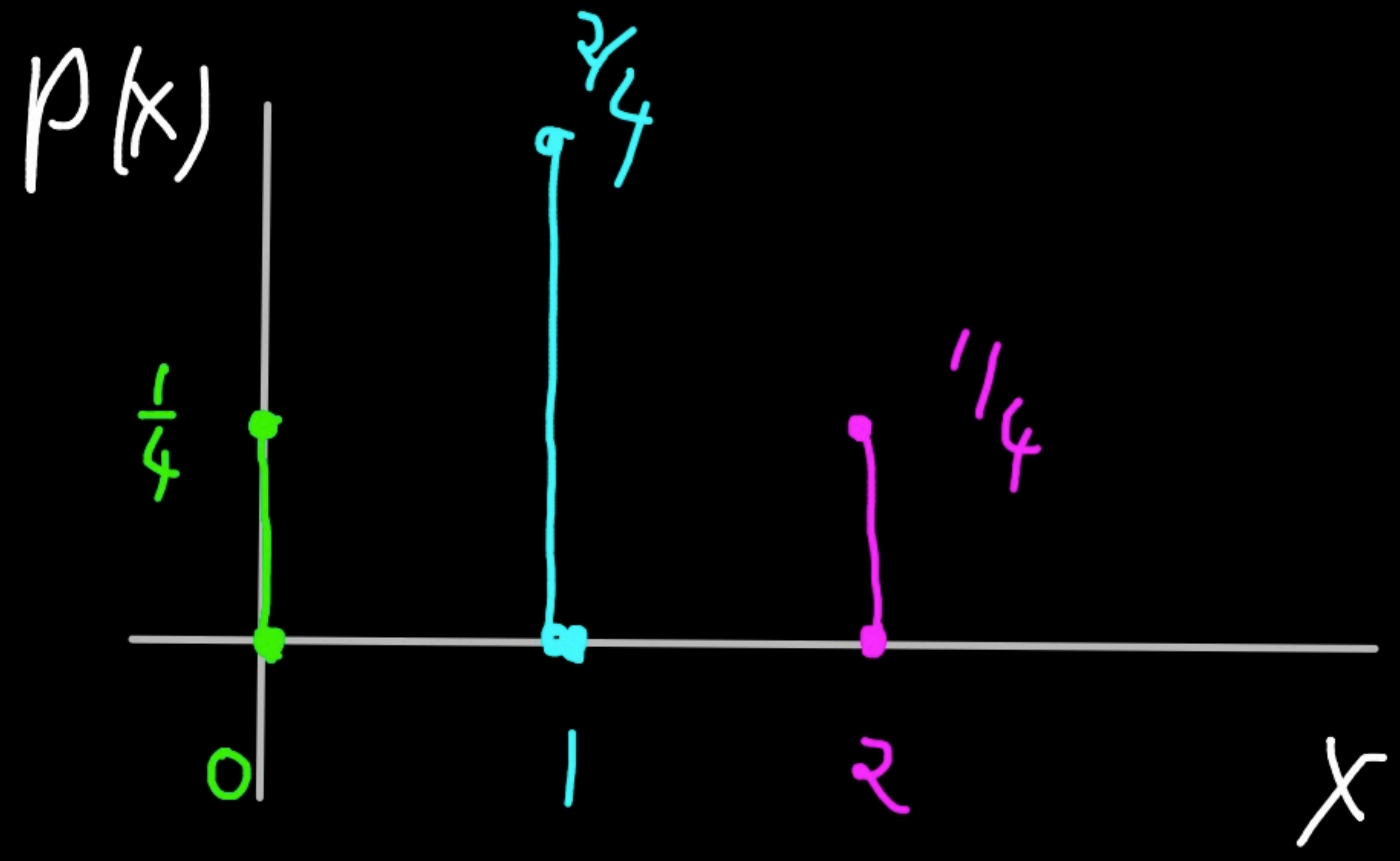
Coin toss twice

R.V. $S = \{HH, HT, TH, TT\}$

X: no. of heads in 2 tosses

$$X \to \begin{cases} 0 & \{TT\} \to \frac{1}{4} \\ 1 & \{HT, TH\} \to \frac{3}{4} \\ 2 & \{HH\} \to \frac{1}{4} \end{cases}$$

probability mass function



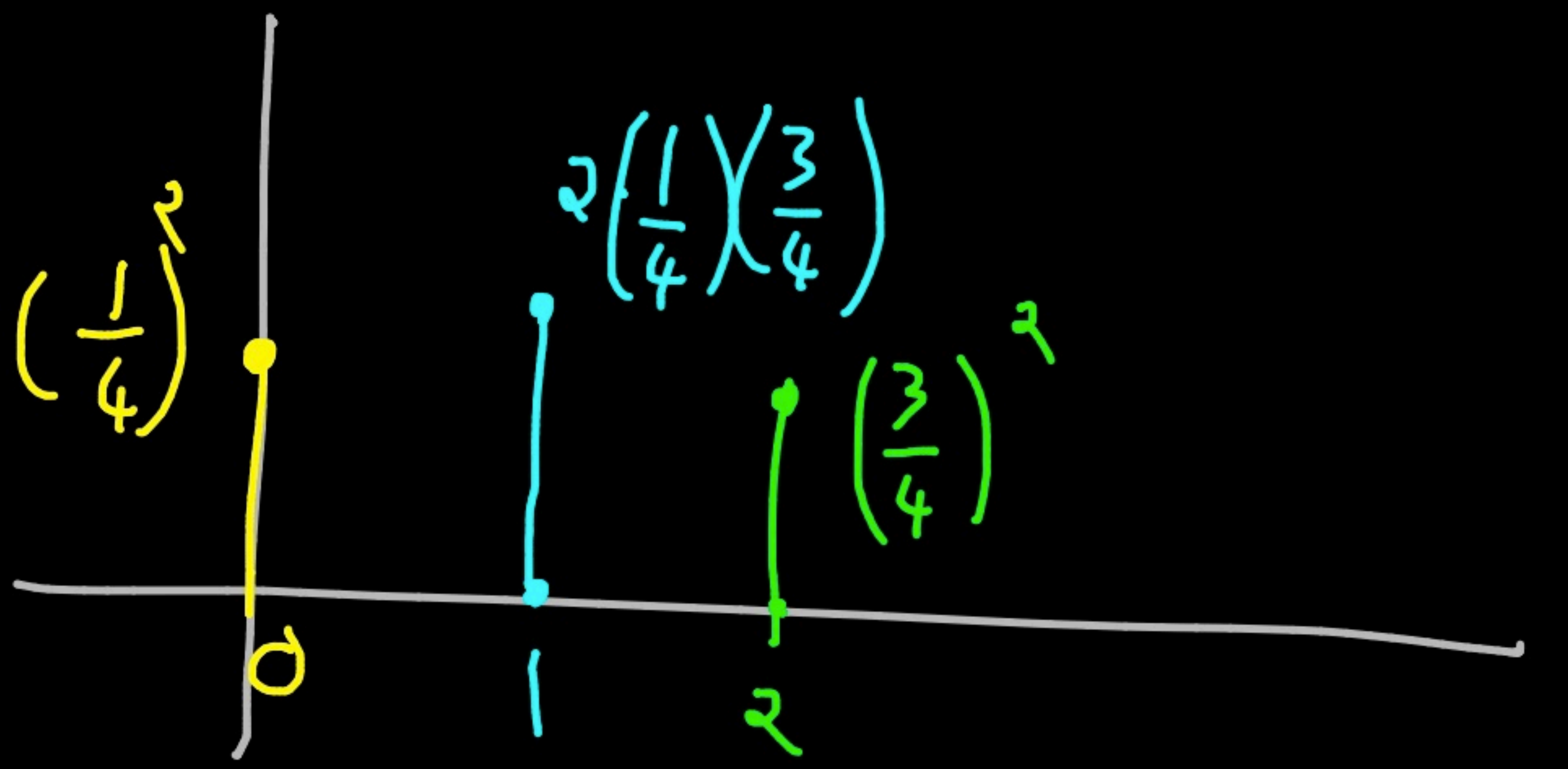
mapping S (strings "HH" etc) $\longrightarrow$ numbers

$S \longrightarrow R$

$R \to$ all real number

$$EX = (0)\frac{1}{4} + 1\left(\frac{2}{4}\right) + 2\left(\frac{1}{4}\right) = 1$$

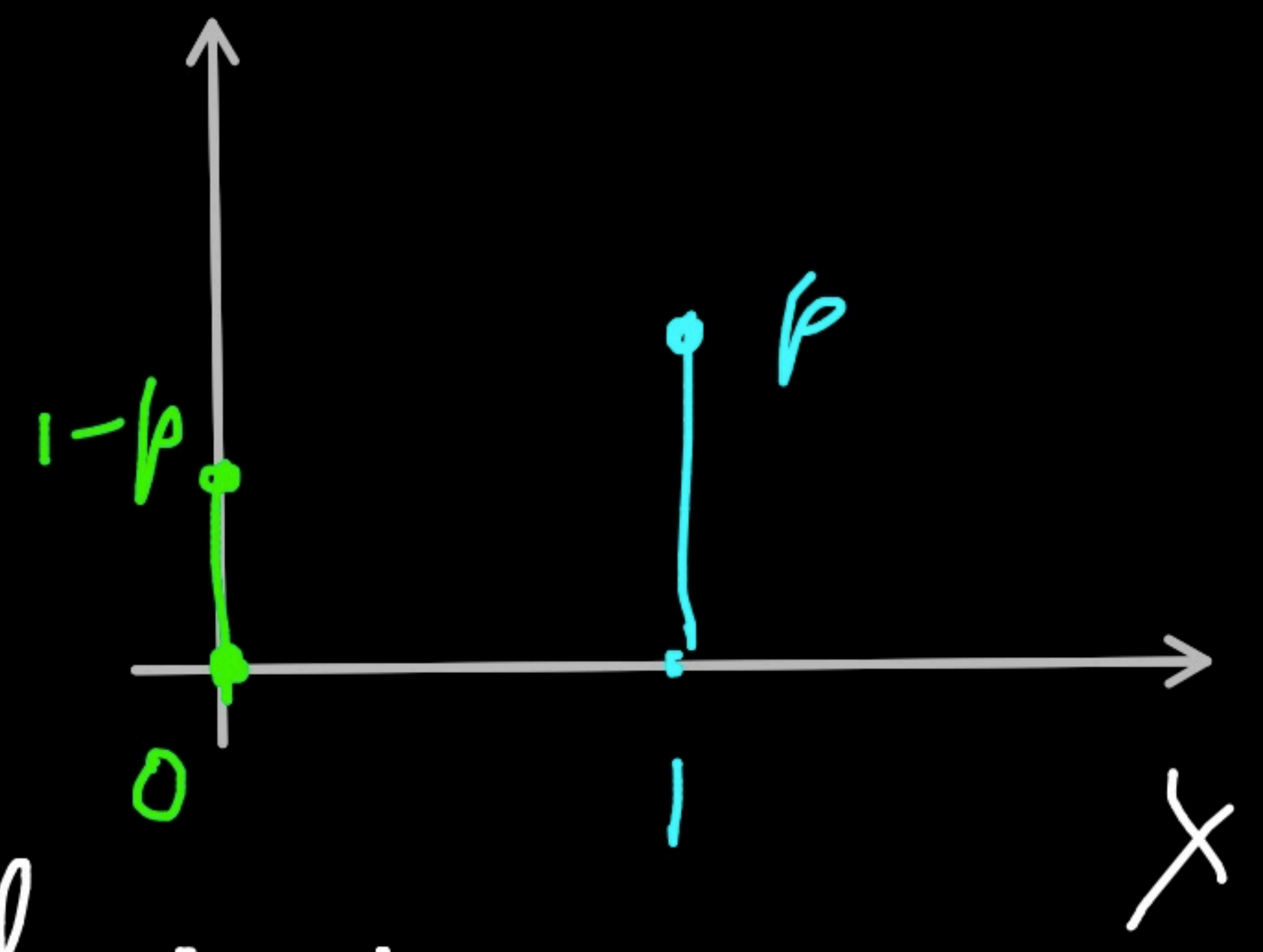
$$EX = 0 \cdot P[X=0] + 1\, P[X=1] + 2\, P[X=2]$$



$$EX = 0\left(\frac{1}{4}\right)^2 + 1 \cdot 2 \cdot \left(\frac{1}{4}\right)\left(\frac{3}{4}\right) + 2\left(\frac{3}{4}\right)^2 = \frac{3}{2}$$

Bernoulli Random variable : "p" → parameter $p \in [0, 1]$

$$X \begin{cases} \to 0 , & 1-p \\ \to 1 , & p \end{cases}$$



Coin toss with prob of heads being p.

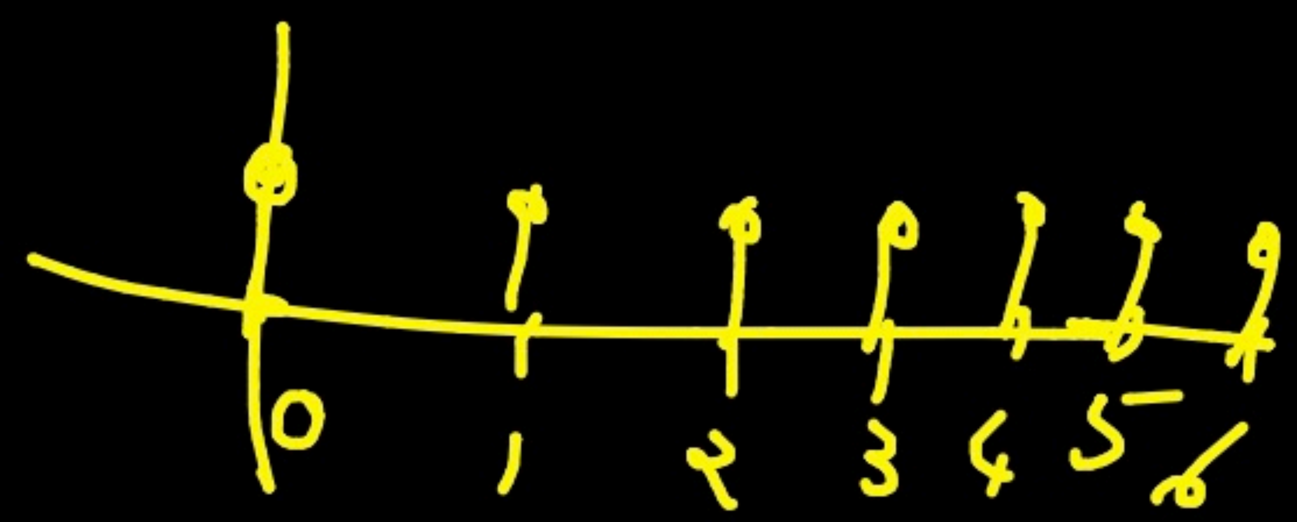
$S = \{H, T\}$

$H \longrightarrow X = 1$

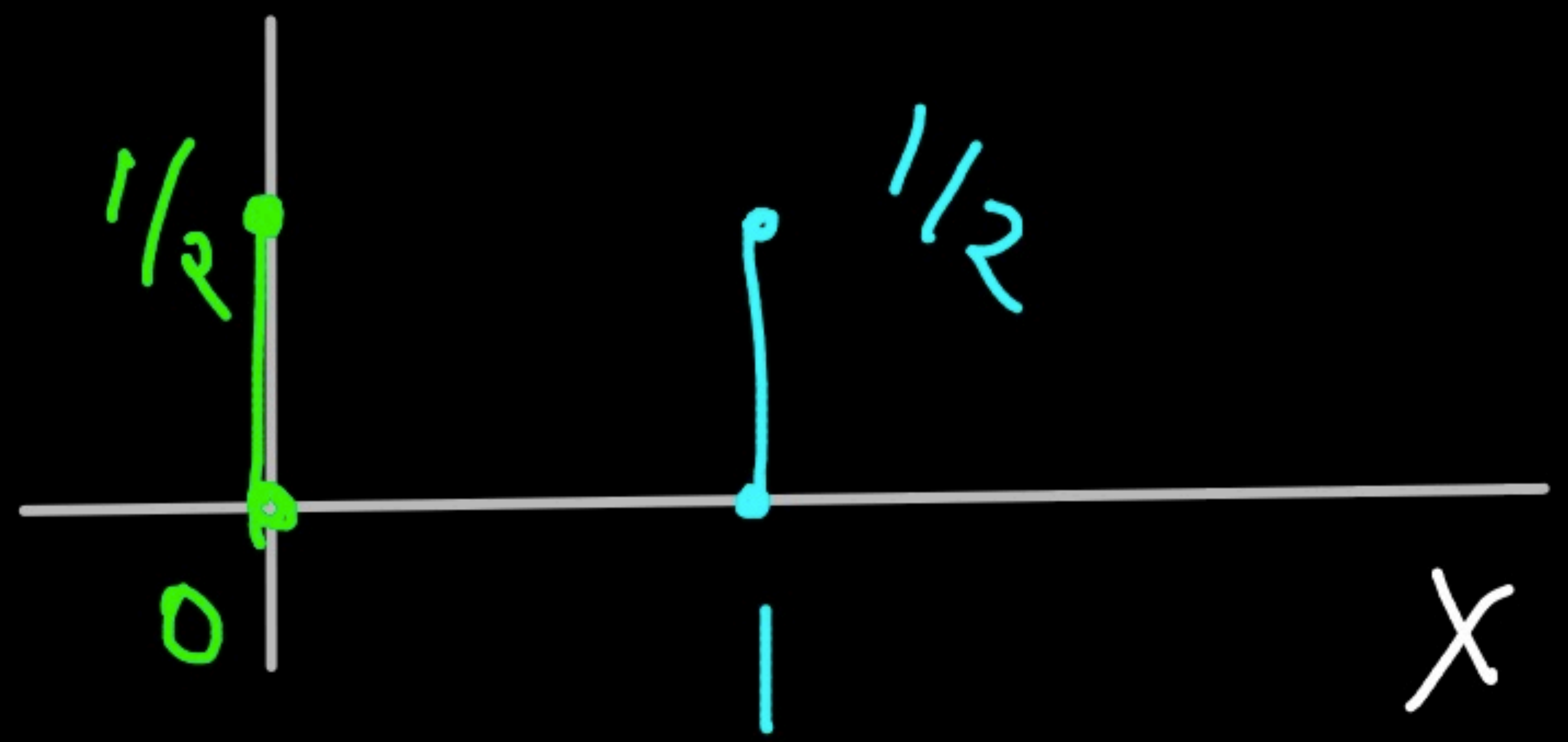
$T \longrightarrow X = 0$

Dice:

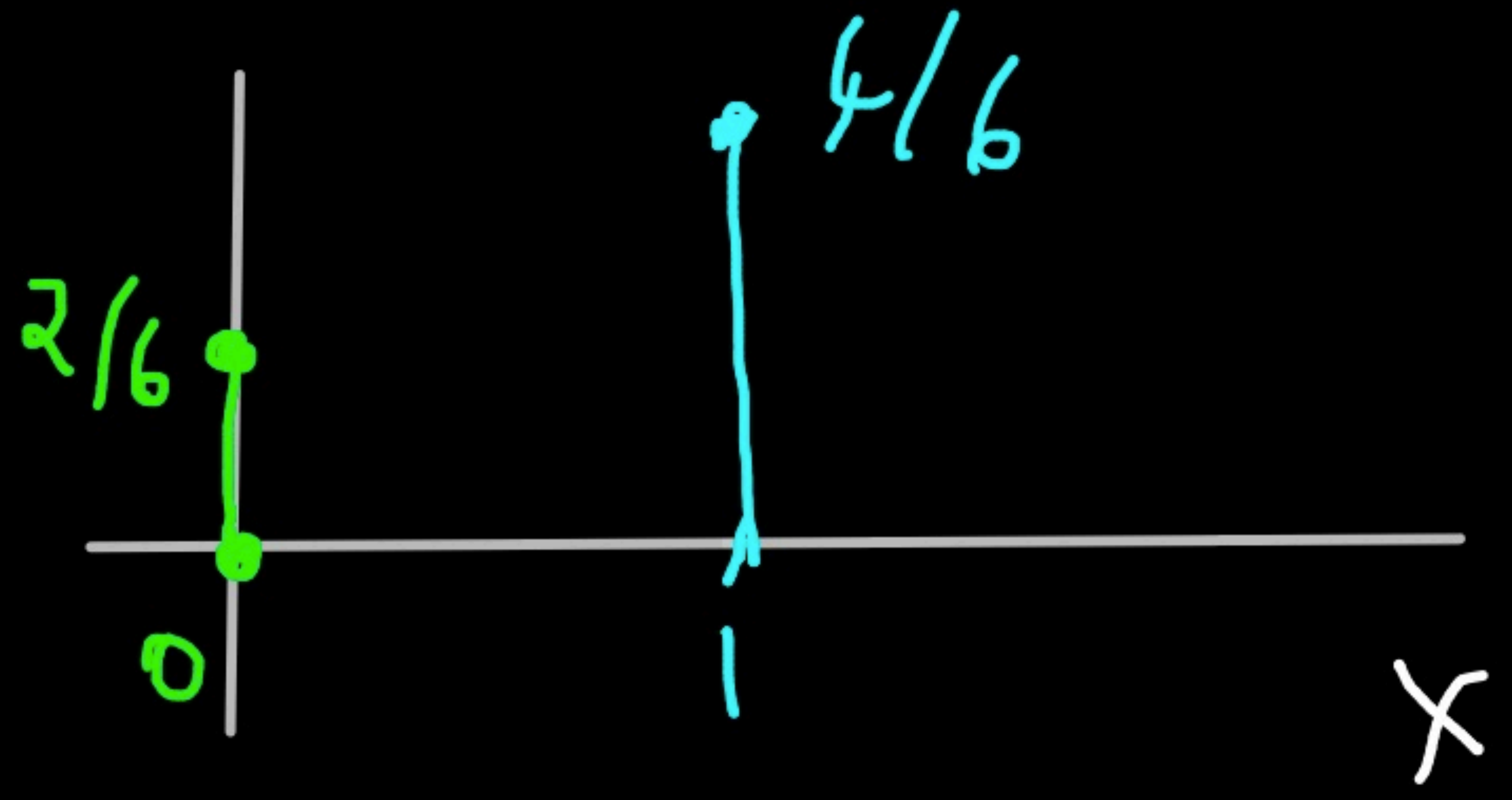
$S = \{1, 2, 3, 4, 5, 6\} \longrightarrow Z$



$X = \begin{cases} 0, & \text{odd} \\ 1, & \text{even} \end{cases}$



$Y = \begin{cases} 0, & \{1, 2\} \rightarrow 2/6 \\ 1, & \{3, 4, 5, 6\} \rightarrow 4/6 \end{cases}$



X or Y or Z

$[X = i] \qquad [X = j]$

Binomial Random variable : $n, p \rightarrow$ parameters

We toss a coin "n". Prob. of heads is "p"

Let X denote number of heads. Possible values of X : $\{0, 1, 2, 3\}$

$\boxed{n=3}$ $S = \{\underbrace{HHH}_{3}, \underbrace{HHT}_{2}, \underbrace{HTH}_{2}, \underbrace{HTT}_{1}, \underbrace{THH}_{2}, \underbrace{THT}_{1}, \underbrace{TTH}_{1}, \underbrace{TTT}_{0}\}$

$X = 0 \rightarrow 1$ times

$X = 1 \rightarrow 3$ times

$X = 2 \rightarrow 3$ times

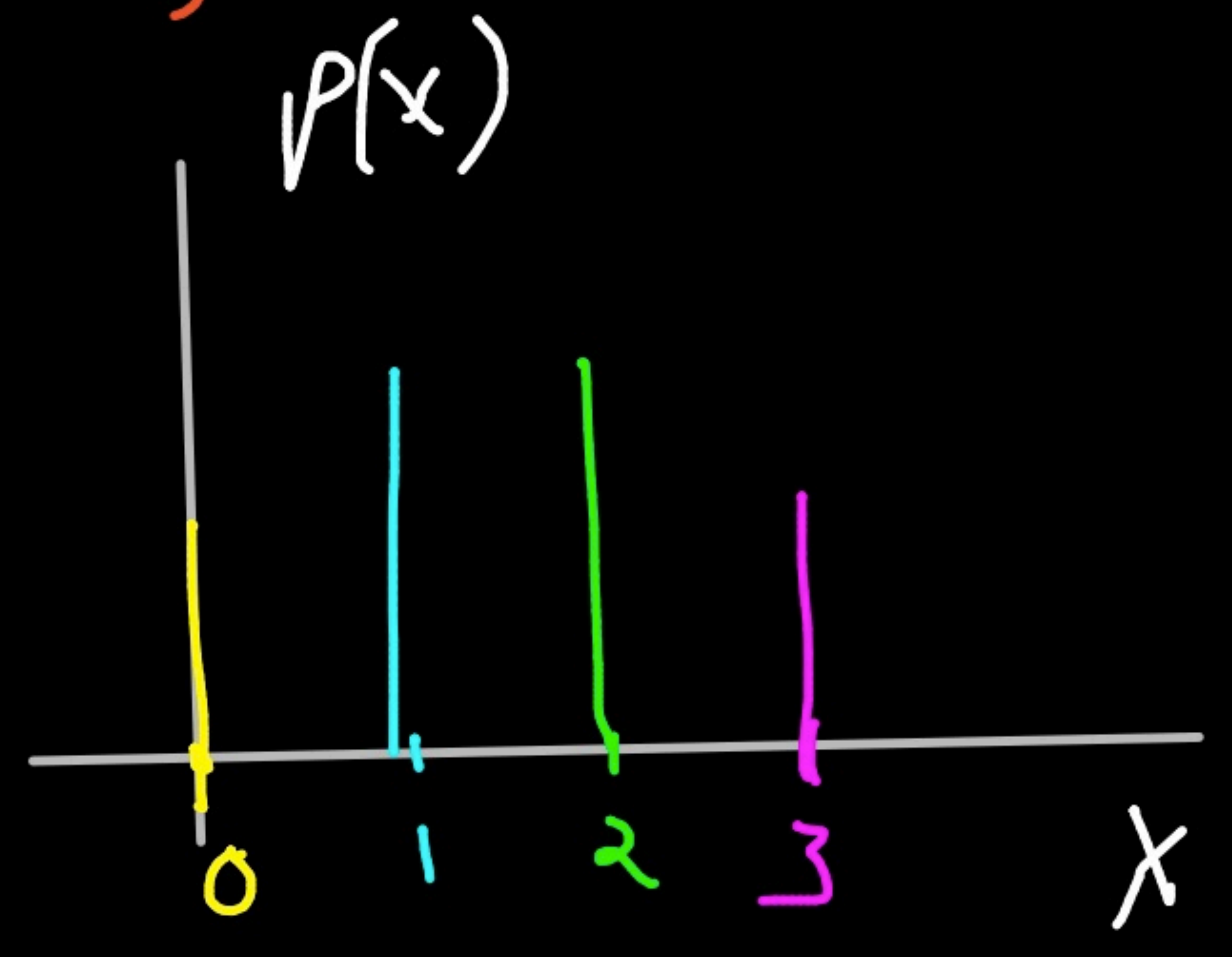
$X = 3 \rightarrow 1$ times

$P[X=0] = (1-p)^3$

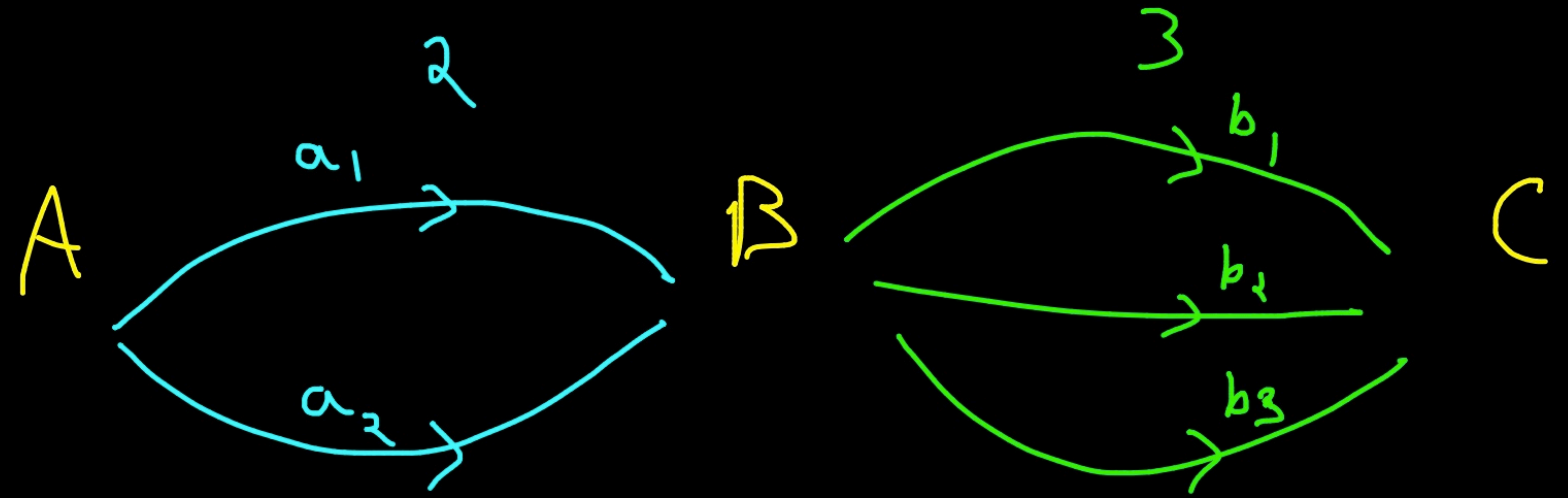
$P[X=1] = 3p(1-p)^2$

$P[X=2] = 3p^2(1-p)$

$P[X=3] = p^3$



Counting



How many ways to go from A to C

$a_1 b_1$
$a_1 b_2$
$a_1 b_3$
$a_2 b_1$
$a_2 b_2$
$a_2 b_3$

$$2 \times 3 = 6$$

$$n_1 \times n_2$$

$\boxed{10}\ \boxed{9}\ \boxed{8}\ \boxed{7} \longrightarrow$ 4 boxes

10 balls $\longrightarrow$ 4 boxes , one in each box

$$\left.(10 \cdot 9 \cdot 8 \cdot 7) \cdot \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{10!}{6!}\right\} \rightarrow {}^{10}P_{4}$$

" permutation "

4 letters : A, B, C, D.

1) A B C D

2) A B D C

3) A D B C

⋮

$4! = 4 \times 3 \times 2 \times 1$

$\lfloor 4$

4 balls

$4 \times 3 \times 2 \times 1 \longrightarrow$ "factorial"

4	3	2	1

4 boxes

"principle of counting"

google / Youtube

We toss a coin 10 times.

"Khan academy"

permutation & combination

$S = \{ \underset{①}{HHH\cdot\ H}, \underset{②}{HHH\cdots T}, \underset{③}{HH\cdots TH}, \ldots$

2	2	2	2	2	2	2	2	2	2

$\longrightarrow 2\times2\times2\cdots\times2 = 2^{10}$

$= 1024$

Outcomes

How many of these 2^{10} outcomes have 3 heads?

Choose 3 "locations" to place heads

$^{10}C_3 \qquad \dfrac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1}$

$\longrightarrow 3, 7, 8 \rightarrow H$ everything else tails

$\longrightarrow 1, 2, 5$

$\longrightarrow 6, 7, 9$

$5, 2, 1 \longrightarrow$ permutations

3×2

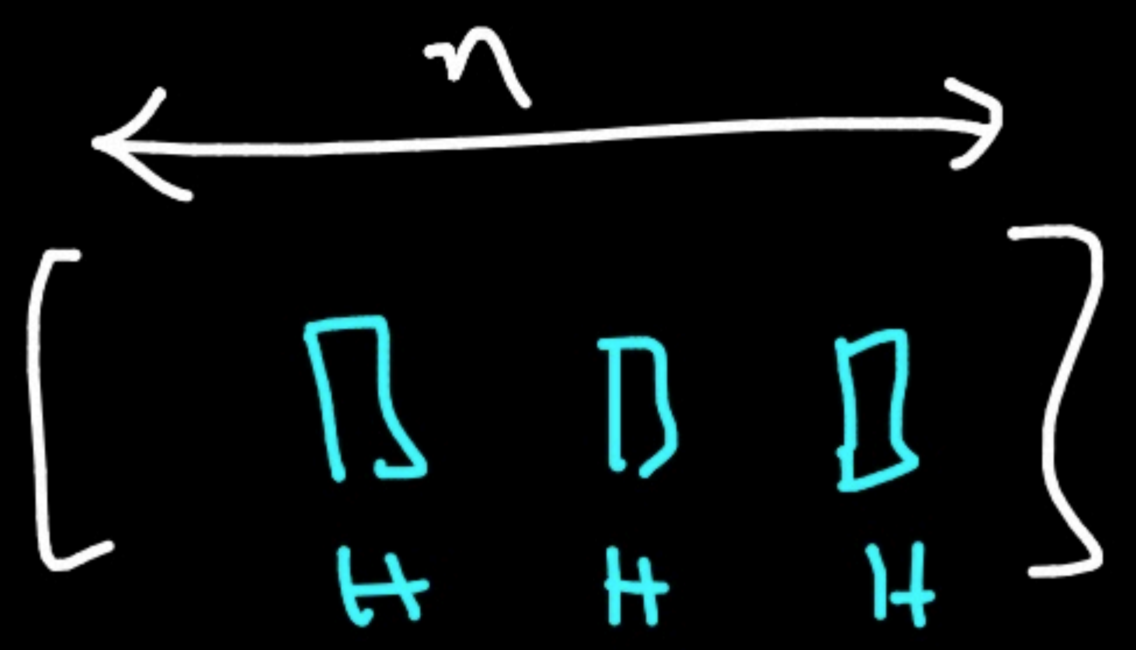
"Combination"

Binomial: n, p

X : no. heads in n tosses, $p \rightarrow$ prob. of heads

$$P[X = k] = \underbrace{{}^{n}C_{k}}_{\text{total no. of outcomes with } k \text{ heads}} \, p^{k}(1-p)^{n-k}$$

Separate Remedial Session



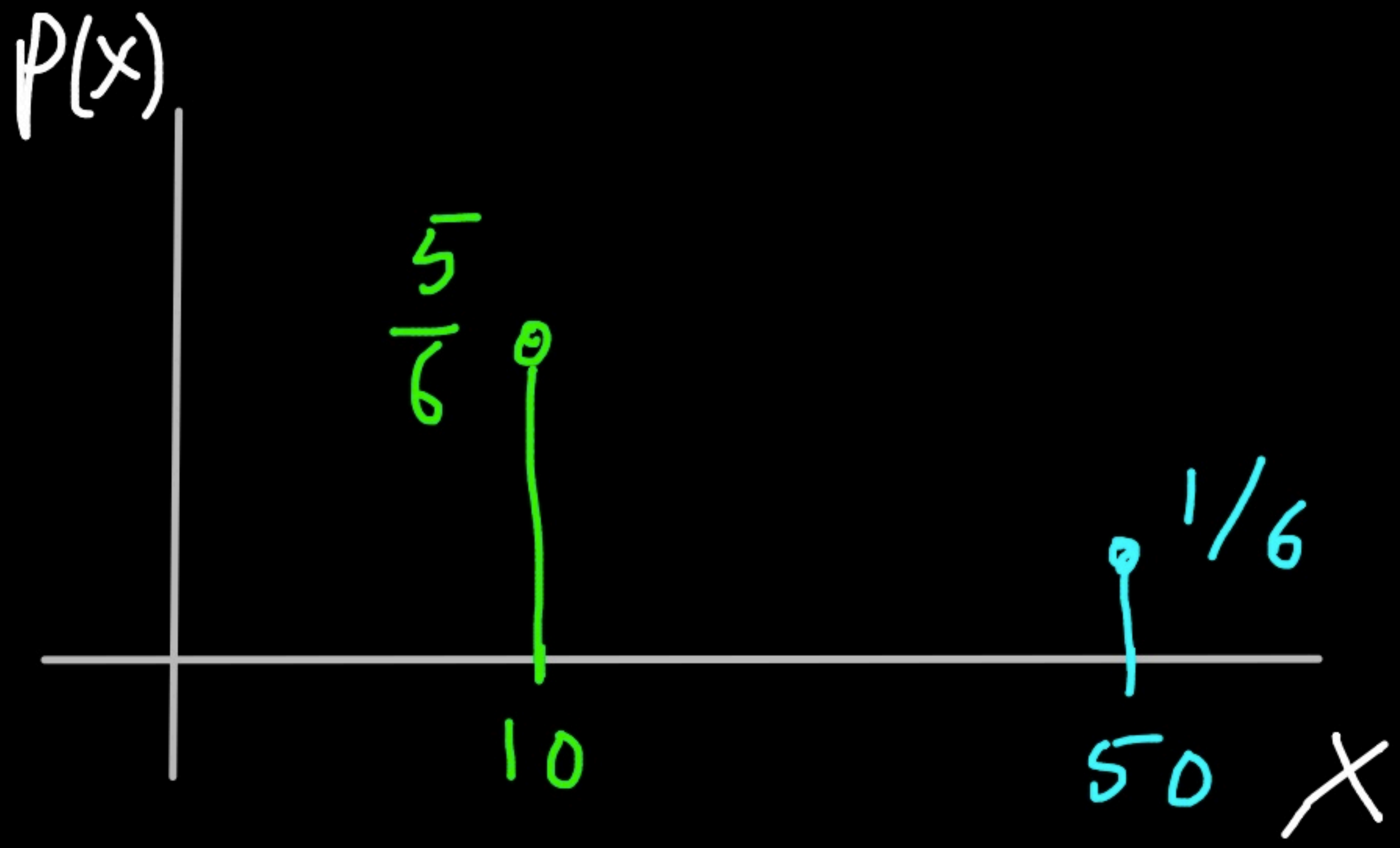
total no. of outcomes with k heads

$$\left.\begin{array}{l} k \rightarrow \text{heads} \\ n-k \rightarrow \text{tails} \end{array}\right\} \rightarrow p^{k}(1-p)^{n-k}$$

Dice

$S = \{1, 2, 3, 4, 5, 6\}$

$$X = \begin{cases} 10, & \{1,2,3,4,5\} \\ 50, & \{6\} \end{cases}$$



suppose we get 'x' rupees in the above manner. We toss 1000 times. How much do you "expect" to make "per toss"

$k_1 + k_2 = 1000$

$$\frac{k_1 \cdot 50 + k_2 \cdot 10}{k_1 + k_2} \rightarrow 50 \cdot \frac{1}{6} + 10 \cdot \frac{5}{6}$$

$$50\, P[X=50] + 10\, P[X=10]$$

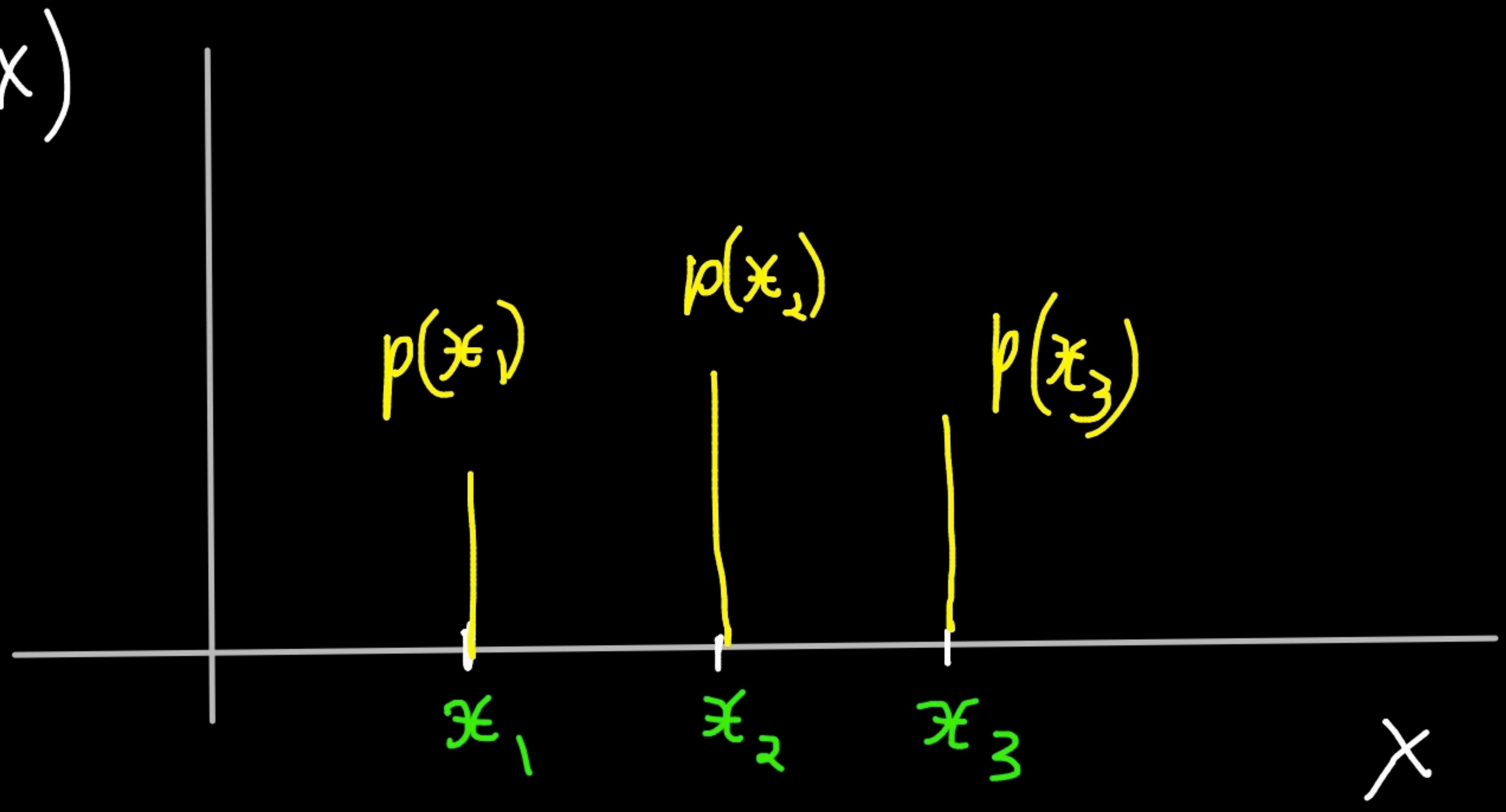
$$\frac{320 \times 50 + 680 \times 10}{1000}$$

K_1 320 → $\{6\}$

K_2 680 → $\{1,2,3,4,5\}$

$$\frac{k_1}{k_1 + k_2} \rightarrow \frac{1}{6} \qquad \frac{k_2}{k_1 + k_2} \rightarrow \frac{5}{6}$$

Expectation



$$\boxed{10 : 45}$$

$$E[X] = x_1\, p(x_1) + x_2\, p(x_2) + x_3\, p(x_3)$$

$$1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \cdots + 6 \cdot \frac{1}{6}$$

Sehwag Vs Dravid → "average" 50

49.34 — 52.31 → nearly same

inconsistent — consistent

$|70 - 52.31|$ → actual score

$|40 - 52.31|$

12/- is lost

You have to bet on their average.

The difference $|\text{actual score} - \text{average}|$

lose this much

$319 - 49.31 \rightarrow$

$|0 - 49.31| \rightarrow$ lose 49.31/-

Sehwag → $\overset{x_1}{105}, \overset{x_2}{13}, \overset{x_3}{20}, 66, \ldots 319, 309$ "sample mean"

$$\frac{105+13+20+66}{4}$$

Dravid → 70, 40, 50, 70, 110,

data tells us that Var(Sehwag) > Var(dravid)

$X_1 - \bar{X}$ "variation"

$$\bar{X} = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

"Sample Variance" $\dfrac{(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + \cdots + (X_n - \bar{X})^2}{n}$

Median

$\{30, 45, 27, 32, 12, \underline{\underline{350}}\}$

sample mean is low

sample mean is high

$\{12, 27, \underbrace{30, 32}, 45, 350\}$

average of them

$\underline{\underline{31}}$

"central tendency"

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Sehwag $\{5, 10, 300, 20, 270, 10, 0,\}$

$\{0, 5, 10, \underset{\downarrow \text{median}}{\underline{\underline{10}}}, 20, \underbrace{270, 300}_{\downarrow}\}$

median: sort & take center

atypical / rare

"Outlier"

"and" "or"

$X = 1$ $\quad \{\underset{A}{HTT},\ \underset{B}{THT},\ \underset{C}{TTH}\}$ $\quad$ A "or" B "or" C

$$P[A \cup B \cup C] = P[A] + P[B] + P[C]$$

$$= p(1-p)^2 + p(1-p)^2 + p(1-p)^2$$

$$= 3\underset{1H}{p}\,\underset{2T}{(1-p)^2}$$

$A \cap B = \{\}$

$X = 2$ $\quad \{HHT,\ HTH,\ THH\}$

$$p^2(1-p) + p^2(1-p) + p^2(1-p)$$

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$$P[A \cap B] = P[A] \cdot P[B] \quad \text{if independent}$$

$$P[A \cap \underline{B \cap C}] = P[A] \cdot P[B \cap C]$$

$$= P[A] \cdot P[B]\, P[C]$$

$$P[TTT] = P[T]\, P[T]\, P[T]$$

$$= (1-p)(1-p)(1-p)$$

$$P[HTT] = P(H)\, P(T)\, P(T)$$

$$= p(1-p)(1-p)$$