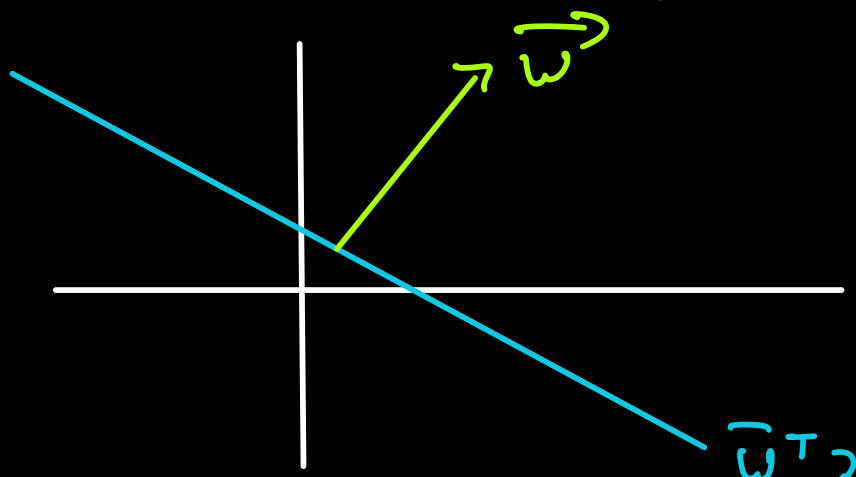
Loss Minimization in Classification
[Linear Algebra]

- → Distance b/w parallel lines
- → Circles
- → Unit vectors
- → Projections
- → Learning $\vec{w}$

Recap

→ Geometric meaning of $\vec{w}$



$\vec{w} \perp w^T x + w_0 = 0$

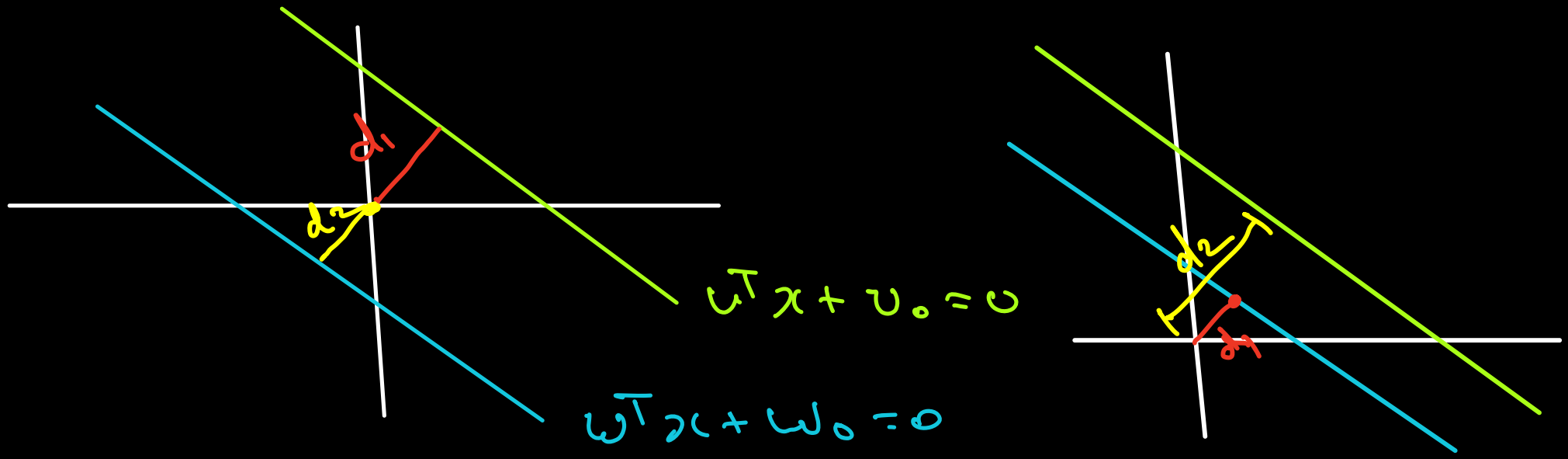
→ Distance of a pt from a h-plane

$$d = \frac{\vec{w}^T \vec{x}_0 + w_0}{\| \vec{w} \|}$$

→ take |d| for length

→ Half Space : if d is +ve ⇒ +ve half space
else −ve half space

Distance b/w parallel lines



Q: Lines are parallel, is $w = v$??

a) Yes → Desmos

b) No

c) $\frac{v_i}{v_j} = \frac{w_i}{w_j}$

distance = $|d_1| + |d_2|$ in b/s 1

$|d_2| - |d_1|$ in b/s 2

But distance already has a sign

Hence

distance b/w 2 lines $= |d_1 - d_2|$

Make $\vec{w} = \vec{v}$

$= \dfrac{|w_0 - v_0|}{\|w_0\|}$

in case 1

$v_0 = -ve$

$w_0 = +ve$

$\Rightarrow |w_0 - v_0|$

$= |w_0 - (-v_0)|$

$= |w_0 + v_0| / \|v\|$

$\Rightarrow |d_1| + |d_2|$

in case 2

$v_0 = +ve$

$w_0 = +ve$

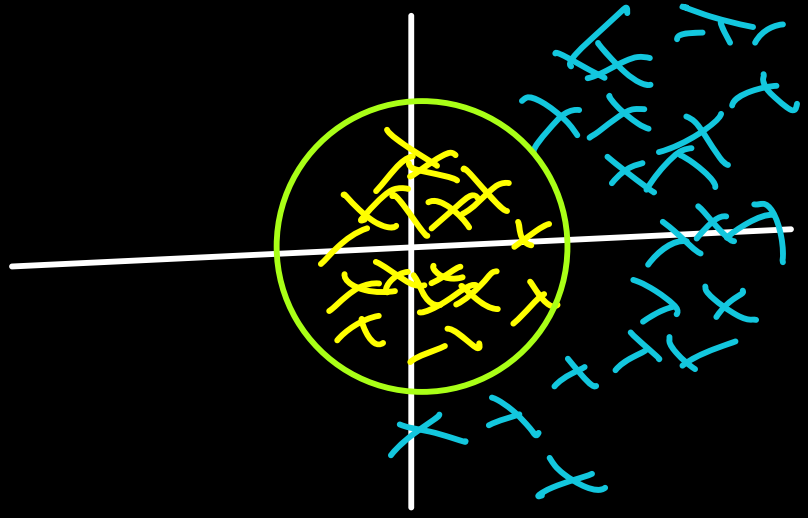
$|w_0 - v_0|$

$= ||v_0| - |v_0||/\|w\|$

$= |d_1 - d_2|$

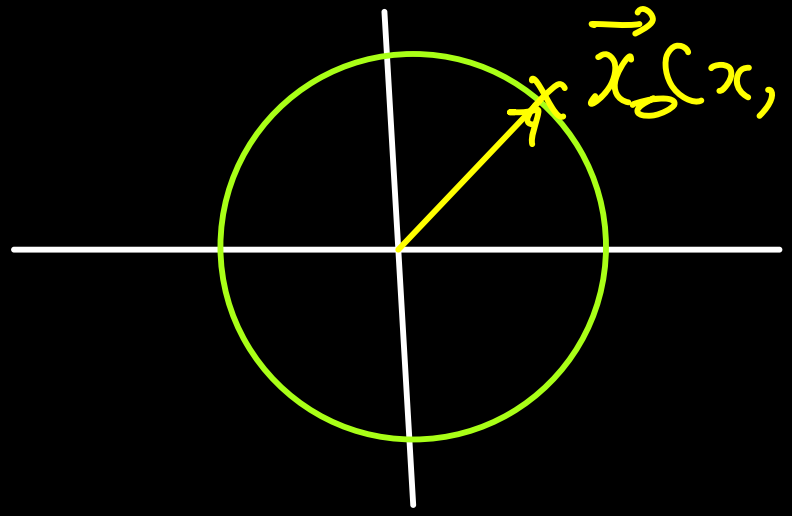
Circles

Sometimes you might need a circular decision boundary.



Q: What is the relatⁿ b/w x and y in a circle?

→ Distance from origin = radius



$$x^2 + y^2 = r^2$$

↳ $y^2 = r^2 - x^2$

$y = \pm\sqrt{r^2 - x^2}$ ↱ For each x, there are 2 y

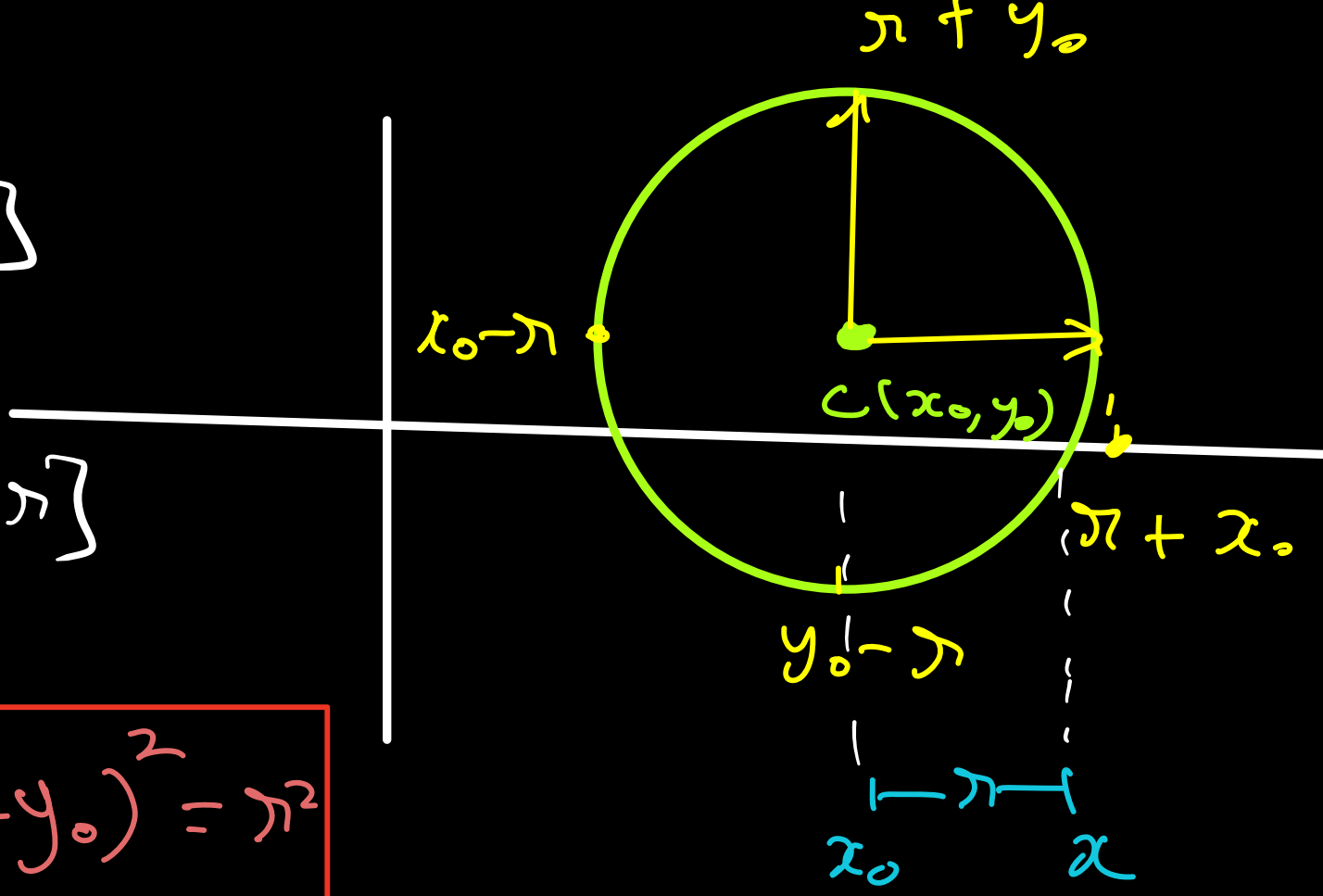
Q: What if the center is not the origin?

x coordinates →

$[x_0 - r,\ x_0 + r]$

y coordinates →

$[y_0 - r,\ y_0 + r]$



Eqn

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

[Extra]

For n-d : hyper-sphere

$$(x_1 - x_1^0)^2 + (x_2 - x_2^0)^2 \dots (x_n - x_n^0)^2 = r^2$$

For n-d hypersphere centered at origin

$$\| x_0 \| = r$$

Quiz: Is $(4, 4)$ in the circle $x^2 + y^2 = 25$

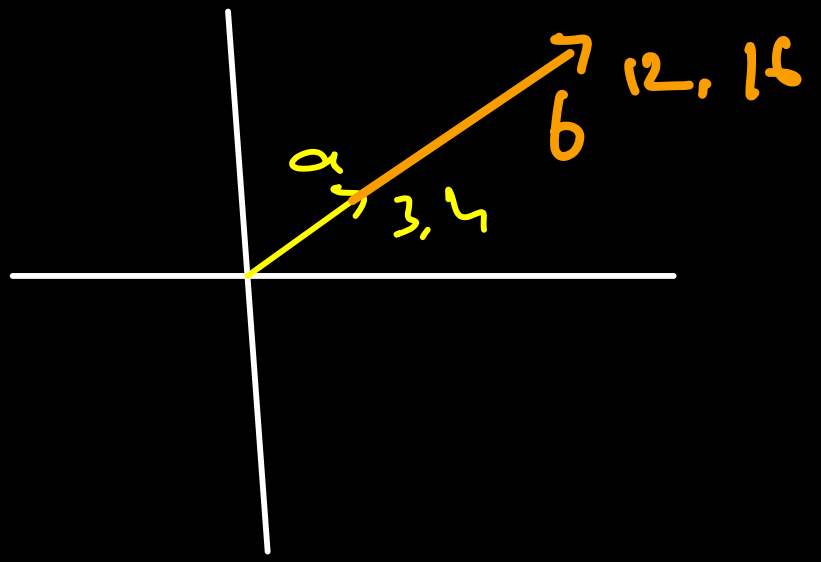
a) Inside b) outside c) on the circle

Unit Vectors

→ vectors whose magnitude is '1'

Consider 2 vectors,

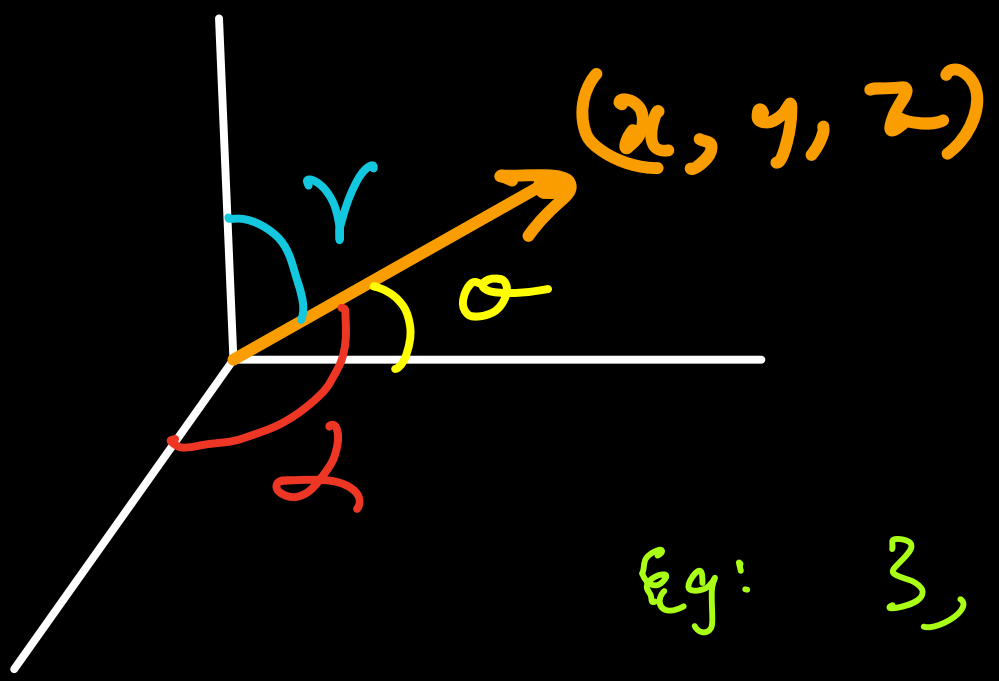
3, 4 and 12, 16



Do they have the same direction? Yes

Direction can be expressed as the angle w.r.t all axis OR

Unit Vector



But we know that angle depends only on ratios, not on absolute values,

Eg: 3, 4 & 12, 16

Angle w.r.t $\rightarrow \cos\theta = \frac{\text{base}}{\text{hypo}} = \frac{3}{\sqrt{3^2+4^2}}, \frac{12}{\sqrt{12^2+16^2}}$

$= \frac{3}{5}, \frac{12}{20} = \frac{3}{5}$

Hence we see that angle is the same

So we can create a new vector which can just represent this angle

Why? → I can multiply any scalar with the unit vector (mag = 1) to get any desired magnitude in the same direction.

Unit Vector

$$\hat{v} = \frac{\vec{v}}{\|v\|}$$

- They are the same for parallel vectors

Now we can write above 2 points as:

$\hat{v} = [3/5, \ 4/5]$

$\therefore a = 5 \times \hat{v} \rightarrow 5 \times 3/5 , \ 5 \times 4/5 = 3, 4$

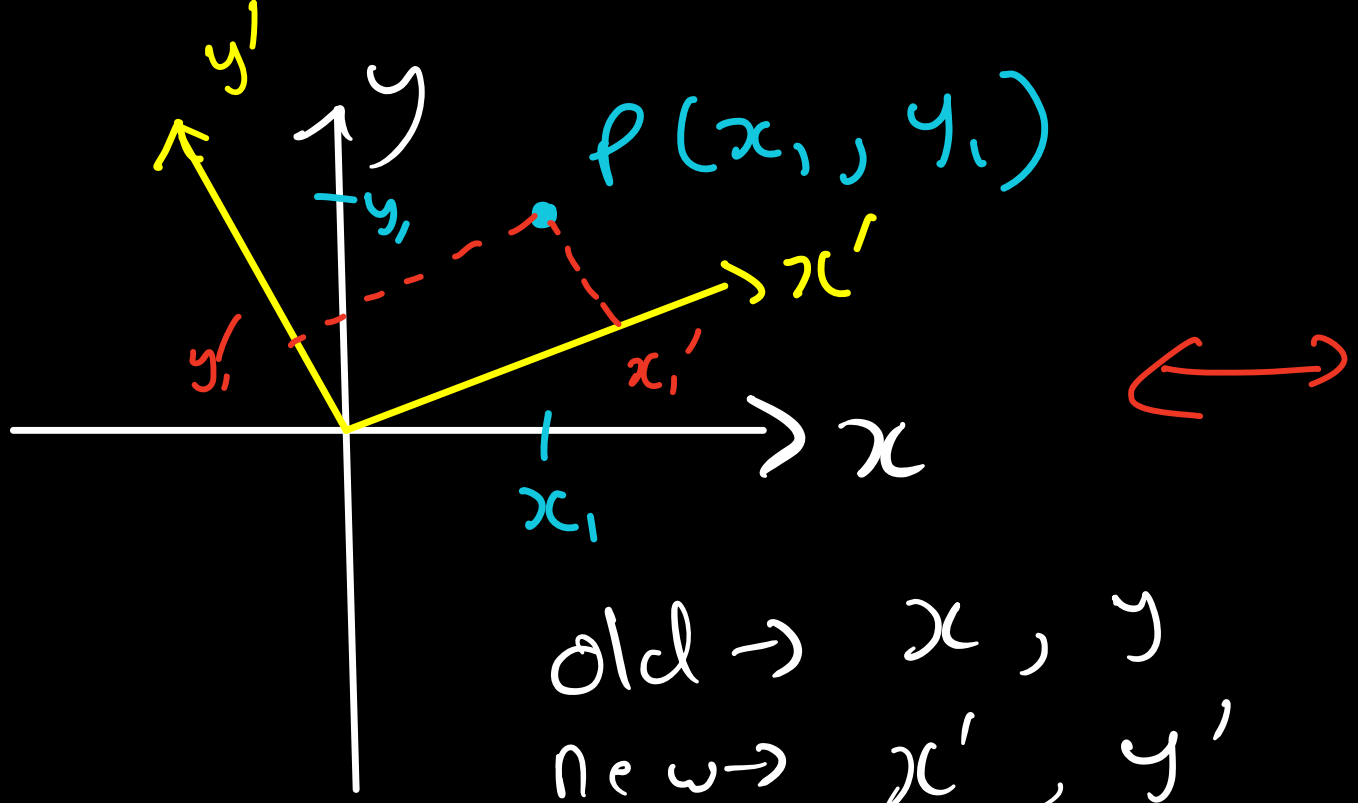
$\therefore b = 20 \times \hat{v} \rightarrow 20 \times 3/5 , \ 20 \times 4/5 = 12, 16$

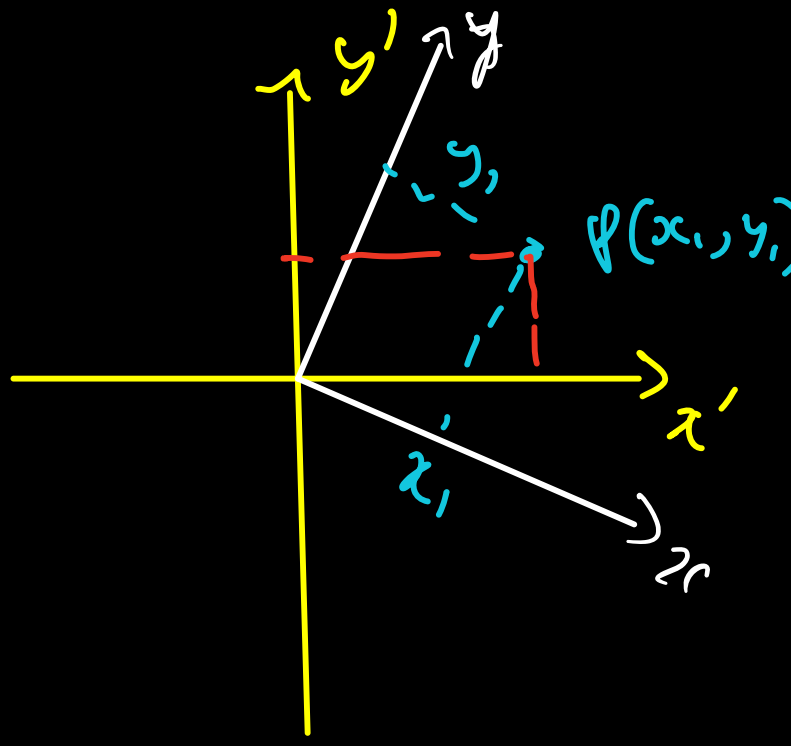
Quiz : calc unit vector of $(6, -8)$

a) $3/5, 4/5$ b) $3/5, -4/5$ c) $3, -4$ d) $3, 4$

Projections [Extra]

→ only see the intuition ; formulae not important





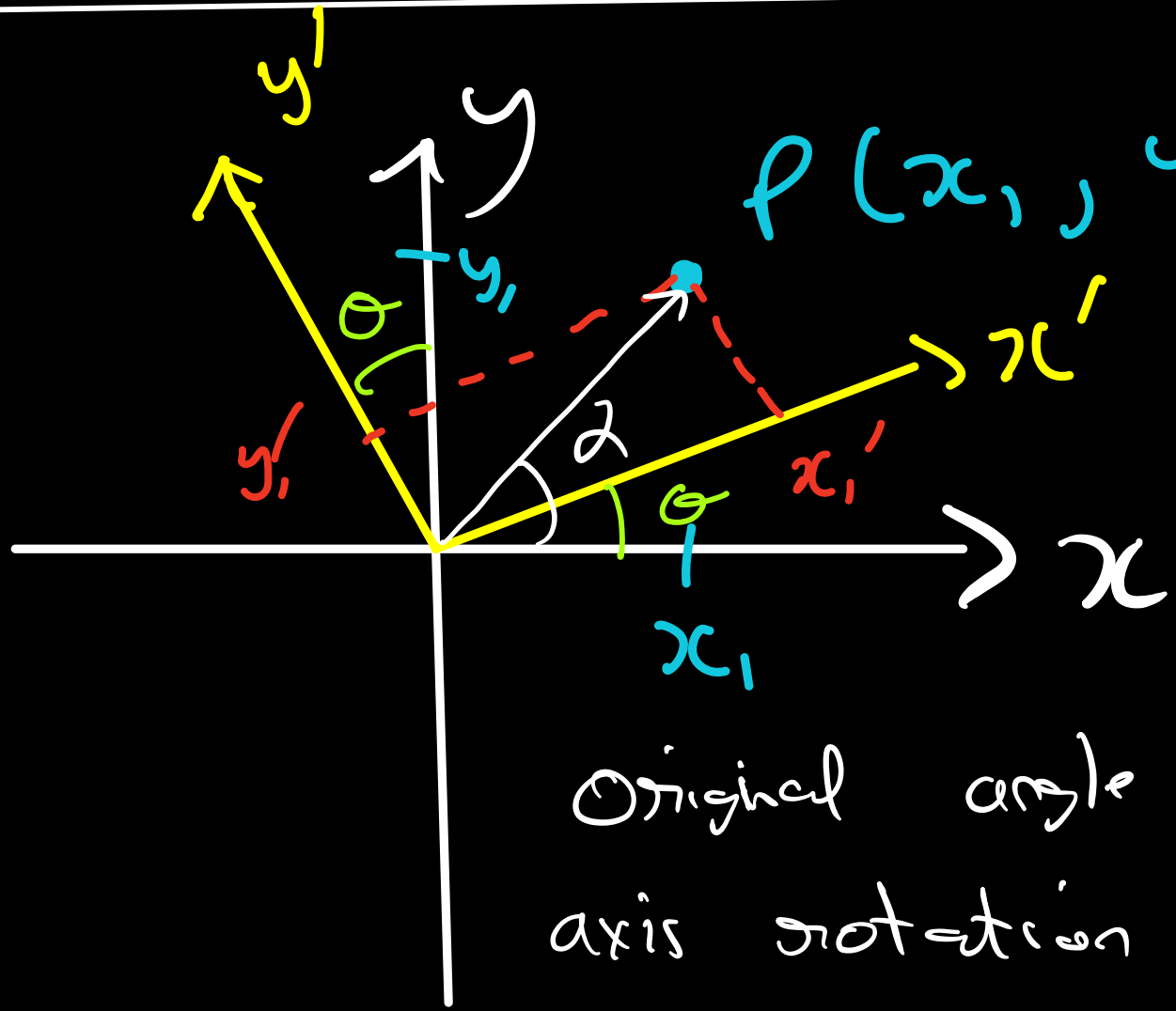
old → x, y

new → x', y'

Quiz: For point $P(x_1, y_1)$ which is true?

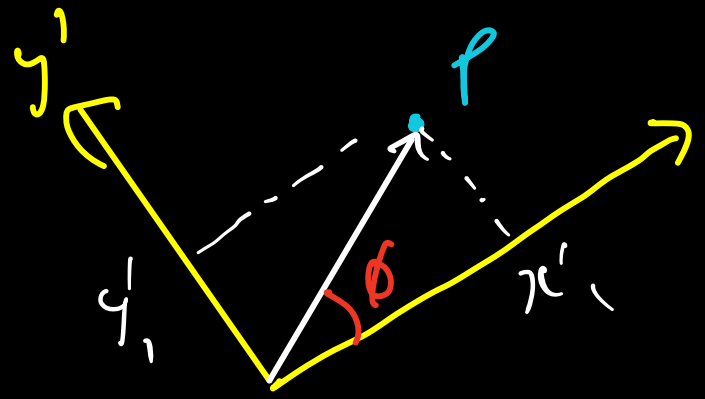
a) $x_1 < x_1'$ b) $x_1 > x_1'$ c) $y_1 < y_1'$ d) $y_1 = x_1'$

Calculating new coordinates



Original angle = $\alpha = \cos^{-1}\left(\frac{y_1}{x_1}\right)$

axis rotation angle = θ [Known]



$$\phi = \alpha - \theta$$

$$x'_1 = \|P\| \cdot \cos\phi$$

$$y'_1 = \|P\| \cdot \sin\phi$$

General form: (Not very important)

$$x'_1 = x\cos\theta + y\sin\alpha$$

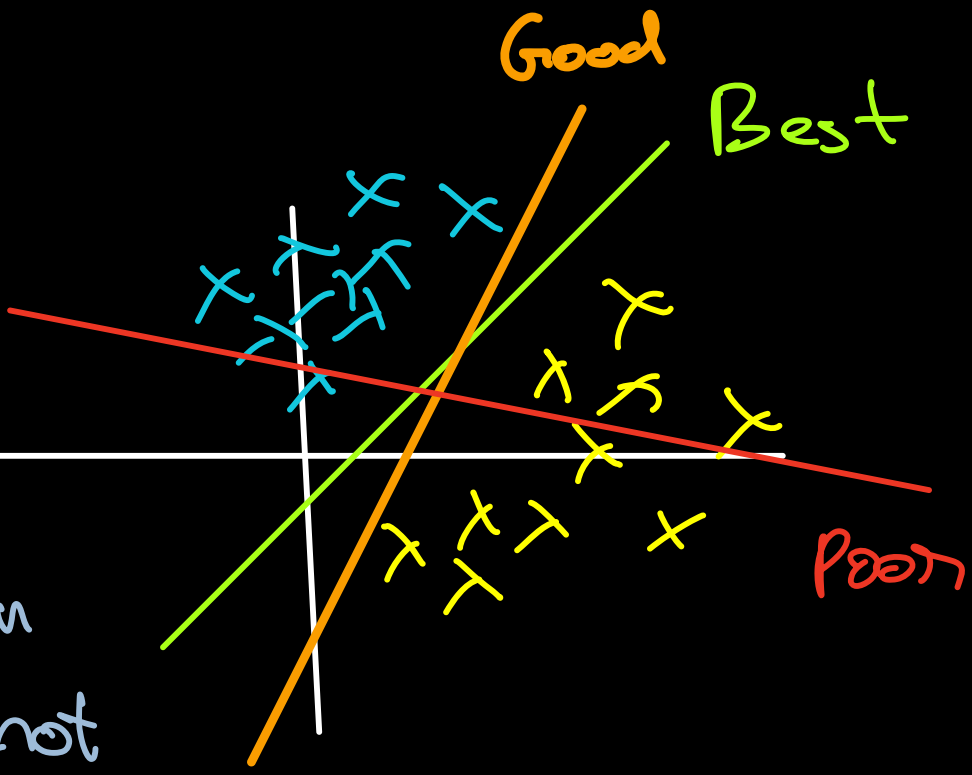
$$y'_1 = -x\sin\alpha + y\cos\theta$$

} Derivation out of scope (not needed)

Learning $\vec{w}$

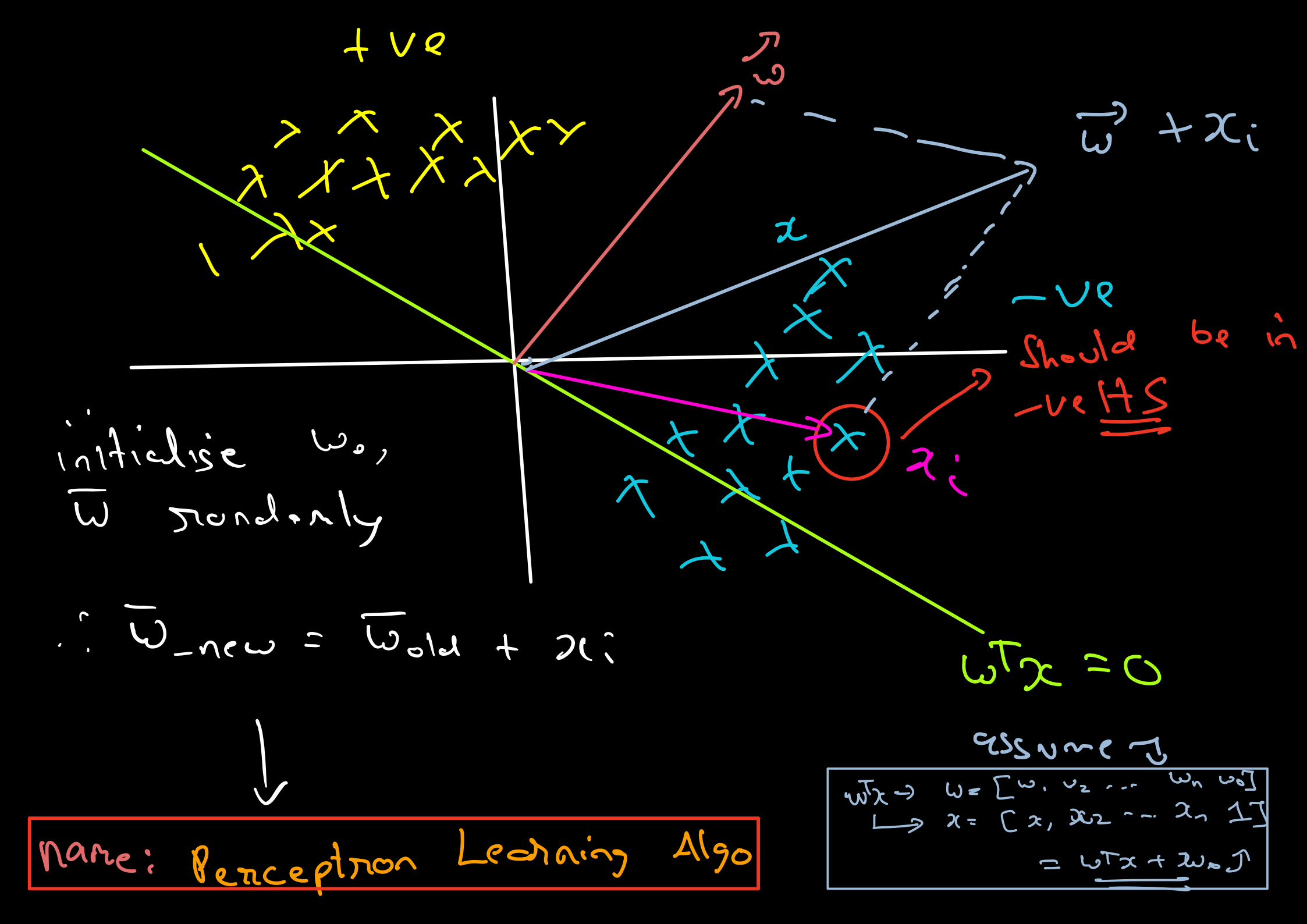
How to find the best line?

→ Today we will learn how to find good but not best



This algo is not really used much and is being discussed to build intuition.

	← coordinates →		← color
	age	income	credit score	Loan
$\vec{p_1}$	33	150K	800	Y
$\vec{p_2}$	47	100K	650	N
$\vec{p_3}$	25	30K	850	Y
	⋮	⋮	⋮	



∴ initialise w_0, $\bar{w}$ randomly

$\therefore \bar{w}_{new} = \bar{w}_{old} + x_i$

↓

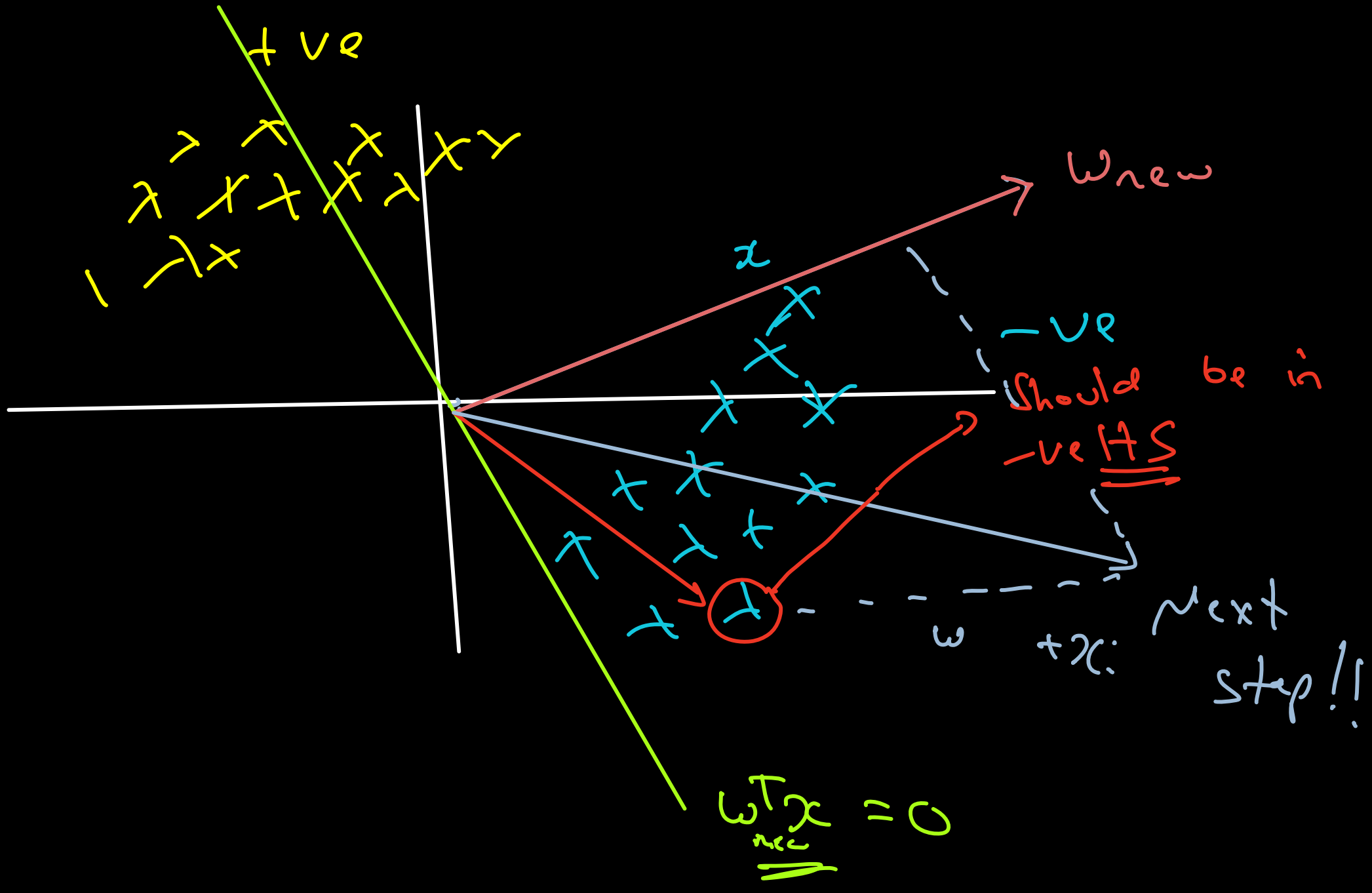
Name: Perceptron Learning Algo

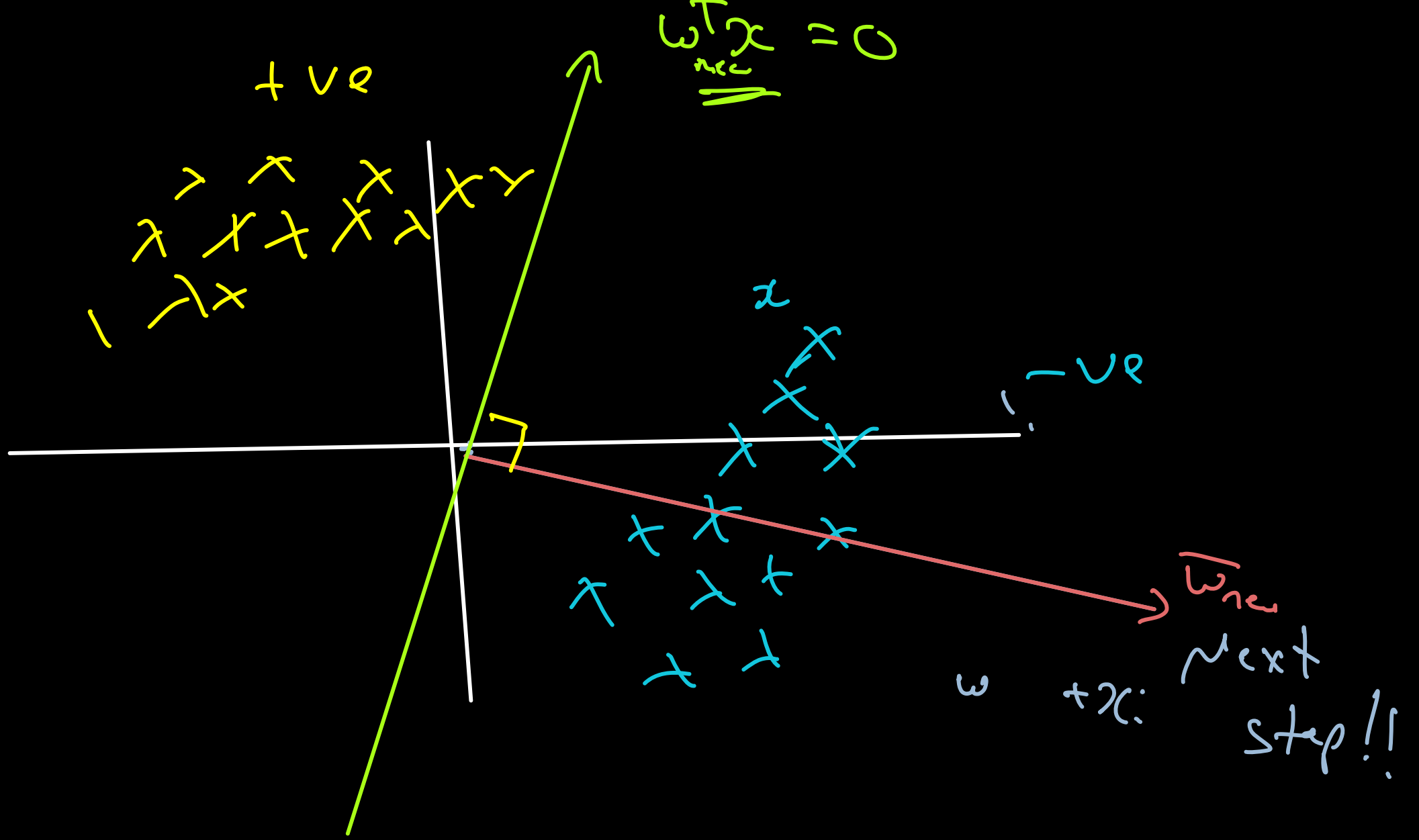
assume →

$w^T x \rightarrow w = [w_1, w_2 \dots w_n \; w_0]$

$\hookrightarrow x = [x_1, x_2 \dots x_n \; 1]$

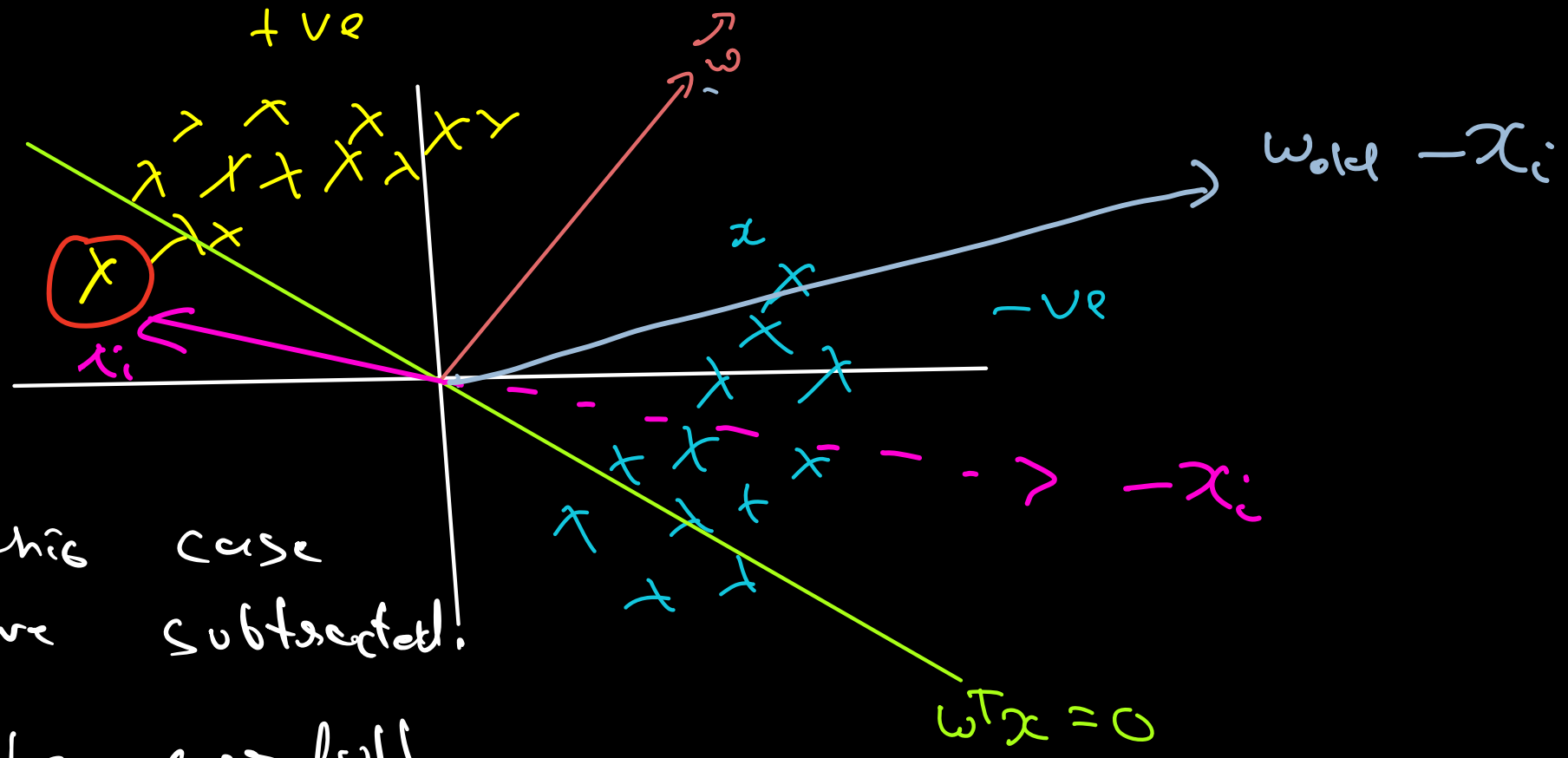
$= w^T x + w_0$


+ ve
w_{new}
x
- ve
Should be in
-ve LHS
$w^T x = 0$
$w + x$:
Next step!!



→ Stop no new mis classified points

I'm not going into anti clockwise, and subtraction etc. Choose which side you want to turn and use math acc.



In this case we have subtracted!

So be careful where you want to turn!