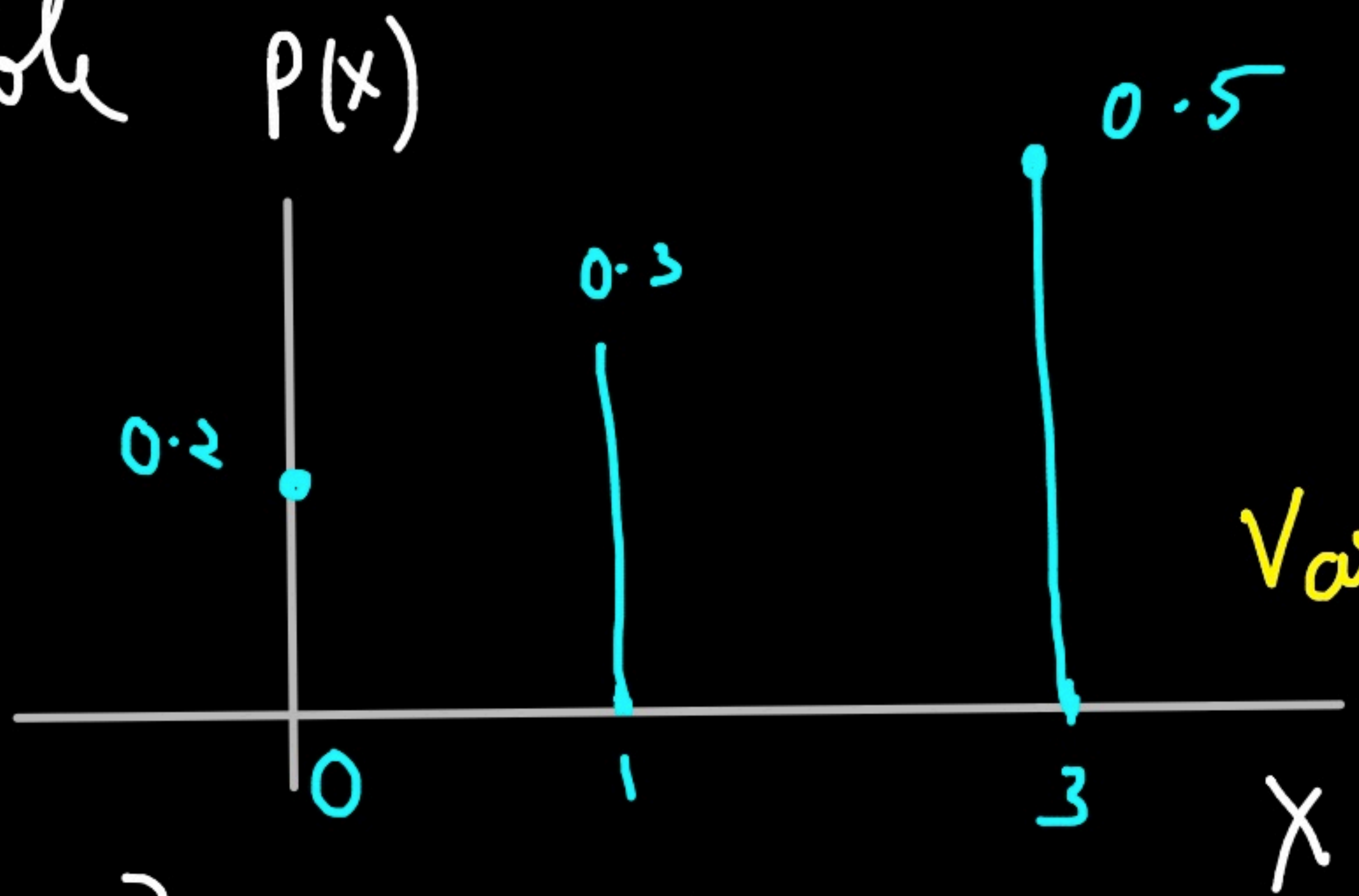
X : Random variable



$$\bar{X} = \frac{X_1 + X_2 \cdots + X_n}{n}$$

1) $n = 2$

2) $n = 3$

$$E\bar{X} = 1.8$$

$$Var[\bar{X}] = \frac{1}{n} Var[X]$$

$$E[X] = \sum k P[X=k] = (0)(0.2) + 1(0.3) + 3(0.5) = 1.8$$

$$E[X^2] = \sum k^2 P[X=k] = 0^2(0.2) + 1^2(0.3) + 3^2(0.5) = 4.8$$

$$Var[X] = EX^2 - (E[X])^2 = 4.8 - 1.8^2 = 1.56$$

10 fair dice are rolled. Approximate the prob. that the sum of the values is between 30 and 40

$$Y = X_1 + X_2 + \cdots + X_{10}$$

$EX_i = 3.5$ $\qquad EY = 35$

$Var X_i = 35/12$ $\qquad Var Y = \frac{350}{12}$

$$P[30 \le Y \le 40]$$

$$P\left[\underbrace{\frac{30-35}{\sqrt{\frac{350}{12}}}}_{y_1} \le \frac{Y-35}{\sqrt{\frac{350}{12}}} \le \underbrace{\frac{40-35}{\sqrt{\frac{350}{12}}}}_{y_2}\right]$$

norm.cdf(y_2) − norm.cdf(y_1)

A dice is rolled, an we keep adding the values. We roll till we get 450. Approximate the prob that this will require more than 140 rolls

The sum of the first 140 rolls < 450

$$Y = X_1 + X_2 + \cdots + X_{140}$$

$$E X_i = 3.5 \qquad \operatorname{Var} X_i = \frac{35}{12}$$

$$E Y = (140)(3.5) \qquad \operatorname{Var} Y = (140)\frac{35}{12}$$

$$P[Y \le 450] = P\left[\frac{Y - (140)3.5}{\sqrt{(140)\frac{35}{12}}} < \underbrace{\frac{450 - (140)(3.5)}{\sqrt{(140)\frac{35}{12}}}}_{y_1}\right]$$

$$y_1 = -1.98$$

$$= \text{norm.cdf}(y_1) = 0.023$$

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Battery → lifetime mean lifetime = 5 weeks.
std. deviation = 1.5 weeks.

Approx. the prob. of needing 13 or more batteries in one year.

$X_i \to$ lifetime of the i^{th} battery

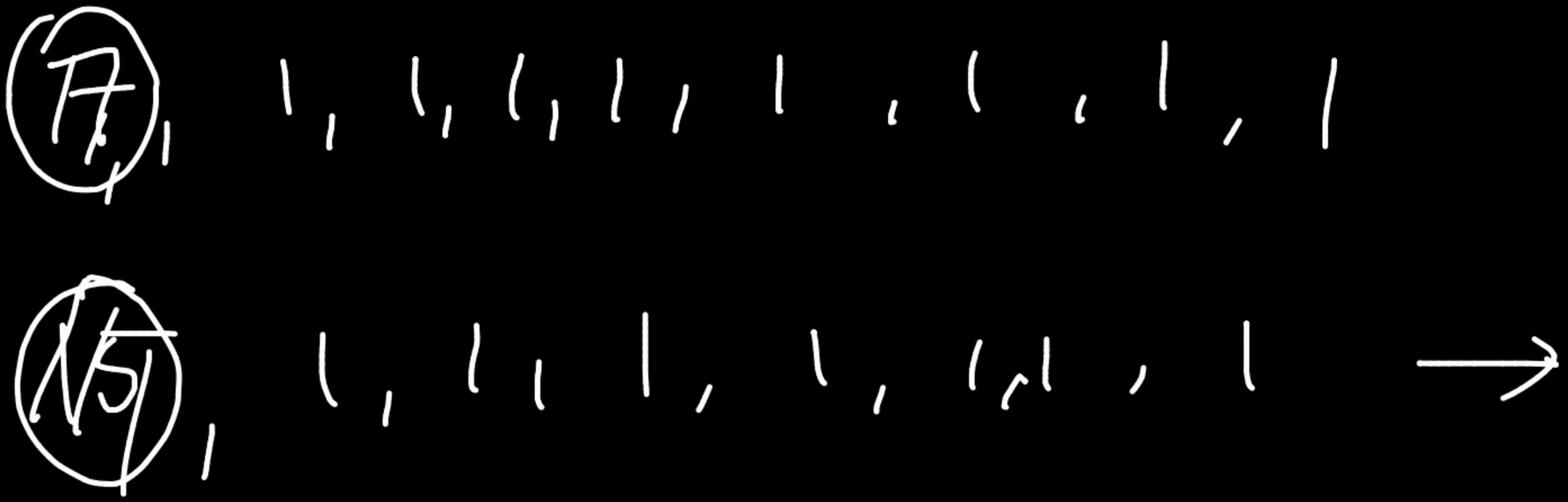
$$Y = \underline{X_1} + \underline{X_2} + \cdots + X_{12}$$

$EX_i = 5 \to EY = 60$

$\mathrm{Var}\,X_i = 1.5^2 \to \mathrm{Var}\,Y = 12(1.5)^2$

$$P[Y < 52] = P\left[\frac{Y-60}{\sqrt{12(1.5)^2}} < \underbrace{\frac{52-60}{\sqrt{12(1.5)^2}}}_{y_1}\right]$$

norm.cdf (y_1)



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Electrical part → lifetime mean is 100 hours and std deviation is 20 hours. If 16 parts are used, find the prob. that sample mean is less than 104

$$P[\bar{X} < 104]$$

$$= P\left[\frac{\bar{X} - E[\bar{X}]}{\sqrt{Var[\bar{X}]}} < \frac{104 - 100}{5}\right]$$

$$= P\left[Z < \frac{4}{5}\right]$$

$$= \text{norm.cdf}\left(\frac{4}{5}\right) = 0.788$$

$$E[\bar{X}] = 100$$

$$Var[\bar{X}] = \frac{1}{16} 20^2 = \frac{400}{16}$$

$$98 < \bar{X} < 104$$

$$cdf\left(\frac{4}{5}\right) - cdf\left(\frac{-8}{20}\right)$$

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Students mark : mean 77, std dev 15

→ Batch 1 has 25 students

→ Batch 2 has 64 students

1) Approx the prob. average score of Batch 1 is between 72 & 82

$\bar{X}_1 \rightarrow$ average of batch 1

$\bar{X}_2 \rightarrow$ average of batch 2

$P[\bar{X}_1 > \bar{X}_2]$

$\boxed{\bar{X}_1 - \bar{X}_2 > 0}$

$\bar{X}_1 - \bar{X}_2$

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$$Y = \bar{X}_1 - \bar{X}_2$$

sum of Gaussian

sum of Gaussian

same variance

$$P[Y > 0]$$

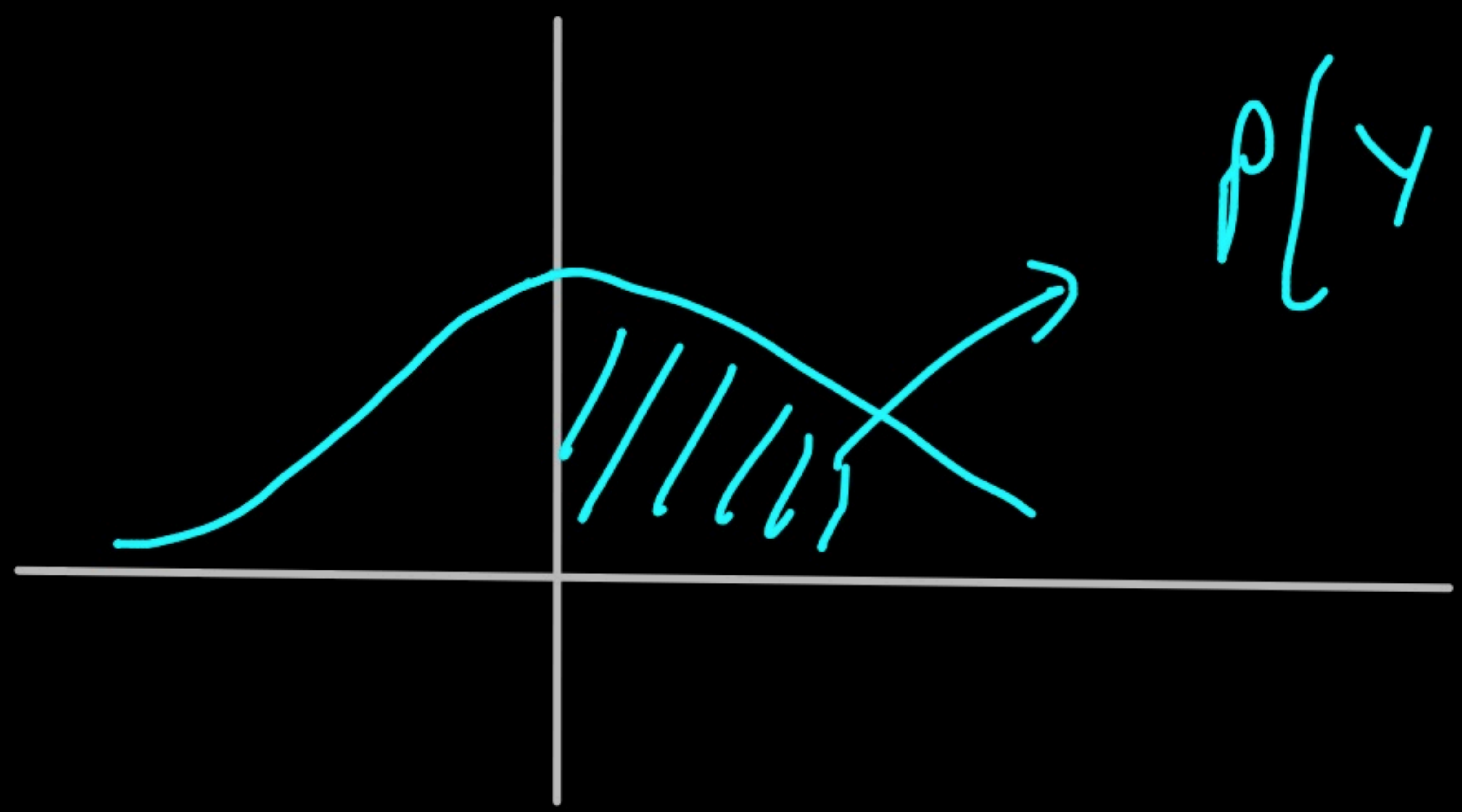
$Y \rightarrow$ is also a Gaussian

$$E[Y] = E[\bar{X}_1] - E[\bar{X}_2]$$

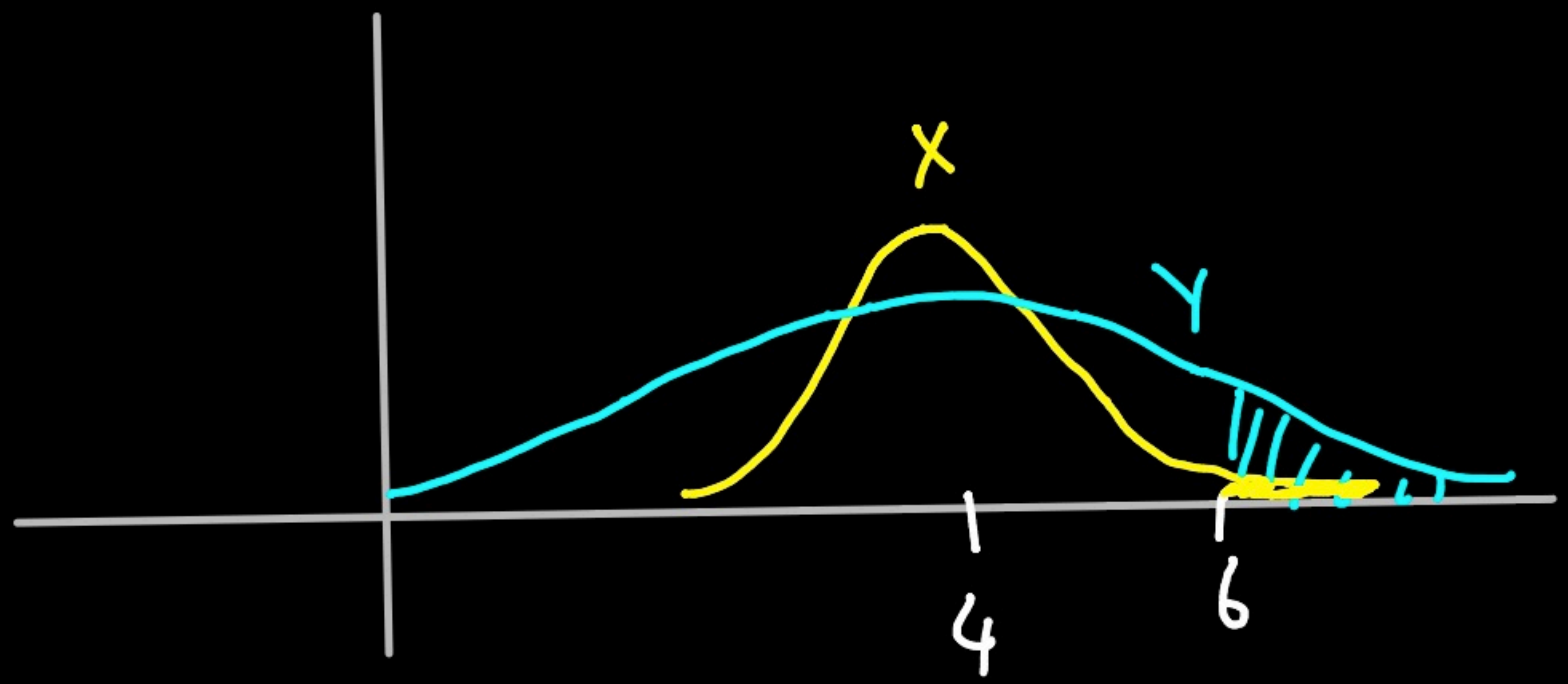
$$= 77 - 77$$

$$= 0$$

Y is a Gaussian with mean 0



$$P[Y > 0] = \frac{1}{2}$$



$E[X] = 4$

$E[Y] = 4$

$P[X > 6]$

$P[Y > 6]$