TIME SERIES PROJECT

FORECASTING THE TOTAL DAILY COVID-19 VACCINATIONS ACROSS THE WORLD

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INTRODUCTION:

Time series analysis is a specific way of analyzing a sequence of data points collected over an interval of time. In time series analysis, analysts record data points at consistent intervals over a set period of time rather than just recording the data points intermittently or randomly. There are different types of models available for the analysis of time series data. We apply different types of models depending on the nature of the time series data set.

ABOUT THE DATASET:

The data is collected from OWID (Our World in Data) GitHub repository, which is updated on daily bases and we have collected it from Kaggle.

Link: https://www.kaggle.com/rsrishav/covid-vaccination-dataset

It contains the records of vaccination doses received by people from all the countries.

- location: name of the country (or region within a country).
- iso_code: ISO 3166-1 alpha-3 – three-letter country codes.
- **date**: date of the observation.
- total_vaccinations: total number of doses administered. This is counted as a single dose, and may not equal the total number of people vaccinated, depending on the specific dose regime (e.g. people receive multiple doses). If a person receives one dose of the vaccine, this metric goes up by 1. If they receive a second dose, it goes up by 1 again.
- total_vaccinations_per_hundred: total_vaccinations per 100 people in the total population of the country.
- daily_vaccinations_raw: daily change in the total number of doses administered. It is only calculated for consecutive days. This is a raw measure provided for data checks and transparency, but we strongly recommend that any analysis on daily vaccination rates be conducted using daily_vaccinations instead.
- **daily_vaccinations**: new doses administered per day (7-day smoothed). For countries that don't report data on a daily basis, we assume that doses changed equally on a daily basis over any periods in which no data was reported. This produces a complete series of daily figures, which is then averaged over a rolling 7-day window.
- daily_vaccinations_per_million: daily_vaccinations per 1,000,000 people in the total population of the country.
- people_vaccinated: total number of people who received at least one vaccine dose. If a person receives the first dose of a 2-dose vaccine, this metric goes up by 1. If they receive the second dose, the metric stays the same.

- people_vaccinated_per_hundred: people_vaccinated per 100 people in the total population of the country.
- people_fully_vaccinated: total number of people who received all doses prescribed by the vaccination protocol. If a person receives the first dose of a 2-dose vaccine, this metric stays the same. If they receive the second dose, the metric goes up by 1.
- people_fully_vaccinated_per_hundred: people_fully_vaccinated per 100 people in the total population of the country.

We have dataset of the COVID-19 vaccinated people on daily basis from February 22, 2021 to July 9, 2021. Thus, we have the dataset of dimension of 32411 rows and 12 columns.

OBJECTIVE:

The main objective of this project is to 'fit an appropriate time series model and to forecast the total number of COVID-19 vaccinated people across the world for the upcoming days.

Thus, we require a series of columns 'daily_vaccinations' and 'date' for our analysis.

IN THIS PROJECT WE INTEND TO:

- Cleaning out the data to obtain the series for our analysis.
- Perform Box-Cox method to stabilize the variance of the series.
- Test the stationarity of the series and make it stationary.
- Find out which model fits the data and estimate it's parameters.
- Forecast the total daily covid-19 vaccinations across the world.

METHODOLOGY:

We use the Box-Jenkins methodology for the time series modelling and forecasting.

ARIMA:

An ARIMA model is a class of statistical model for analyzing and forecasting time series data. ARIMA is an acronym that stands for **A**uto**R**egressive **I**ntegrated **M**oving **A**verage. It is a generalization of the simpler AutoRegressive Moving Average and adds the notion of integration.

- **AR**: *Autoregression*. A model that uses the dependent relationship between an observation and some number of lagged observations.
- **I**: *Integrated*. The use of differencing of raw observations (i.e. subtracting an observation from an observation at the previous time step) in order to make the time series stationary.

- **MA**: *Moving Average*. A model that uses the dependency between an observation and residual errors from a moving average model applied to lagged observations.

A standard notation is used of *ARIMA(p,d,q)* where the parameters are substituted with integer values to quickly indicate the specific ARIMA model being used.
The parameters of the ARIMA model are defined as follows:

- ***p***: The number of lag observations included in the model, also called the lag order.
- ***d***: The number of times that the raw observations are differenced, also called the degree of differencing.
- ***q***: The size of the moving average window, also called the order of moving average.

ARIMA(p,d,q) model is given by the equation below:

$$Y_t = c + \emptyset_1 Y_{t-1} + \ldots + \emptyset_p Y_{t-p} + \theta_1 e_{t-1} + \ldots + \theta_q e_{t-q} + e_t$$

Where:

- Y_t = the variable that will be explained in time t;
- c = constant or intercept;
- $\emptyset$ = coefficient of each parameter p;
- θ = coefficient of each parameter q;
- e_t = Residuals or errors in time t.

BOX-JENKINS METHODOLOGY:

The approach starts with the assumption that the process that generated the time series can be approximated using an ARMA model if it is stationary or an ARIMA model if it is non-stationary. The 2016 5th edition of the textbook (Part Two, page 177) refers to the process as a stochastic model building and that it is an iterative approach that consists of the following 3 steps:

1. **Identification**
2. **Estimation**
3. **Diagnostic Checking**

It is an iterative process, so that as new information is gained during diagnostics, we can circle back to step 1 and incorporate that into new model classes.

1. IDENTIFICATION:

The identification step is further broken down into:

- Assess whether the time series is stationary, and if not, how many differences are required to make it stationary.
- Identify the parameters of an ARMA model for the data.

1.1 DIFFERENCING:

- **Unit Root Tests**. Use unit root statistical tests on the time series to determine whether or not it is stationary. Repeat after each round of differencing.
- **Avoid over differencing**. Differencing the time series more than is required can result in the addition of extra serial correlation and additional complexity.

1.2 CONFIGURING AR AND MA:

Two diagnostic plots can be used to help choose the *p* and *q* parameters of the ARMA or ARIMA. They are:

- **Autocorrelation Function (ACF)**. The plot summarizes the correlation of an observation with lag values. The x-axis shows the lag and the y-axis shows the correlation coefficient between -1 and 1 for negative and positive correlation.

$$\rho_k = E[(\mathrm{Y}_t - \mu)(\mathrm{Y}_{t+k} - \mu)]/\sqrt{(E[(\mathrm{Y}_t - \mu)^2]E[(\mathrm{Y}_{t+k} - \mu)^2])}$$

- **Partial Autocorrelation Function (PACF)**. The plot summarizes the correlations for an observation with lag values that is not accounted for by prior lagged observations.

Both plots are drawn as bar charts showing the 95% and 99% confidence intervals as horizontal lines. Bars that cross these confidence intervals are therefore more significant and worth noting.

- The model is AR if the ACF trails off after a lag and has a hard cut-off in the PACF after a lag. This lag is taken as the value for *p*.
- The model is MA if the PACF trails off after a lag and has a hard cut-off in the ACF after the lag. This lag value is taken as the value for *q*.
- The model is a mix of AR and MA if both the ACF and PACF trail off.

2. ESTIMATION:

We use the data to estimate the parameters coefficients of the model.

3. DIAGNOSTIC CHECKING:

The idea of diagnostic checking is to look for evidence that the model is not a good fit for the data e.g. residual errors diagnostics.

DATA CLEANING:

- First, we check the dimension of the dataset and we found that it has 32411 rows and 12 columns.
- We check for NaN values in the dataset.

```
location                                0
iso_code                                0
date                                    0
total_vaccinations                  13317
people_vaccinated                   14163
people_fully_vaccinated             17161
daily_vaccinations_raw              16266
daily_vaccinations                    249
total_vaccinations_per_hundred      13317
people_vaccinated_per_hundred       14163
people_fully_vaccinated_per_hundred 17161
daily_vaccinations_per_million        249
```

- Now we replace all the NaN values by 0.
- We check for unique values in ‘date’ column.

```
2021-06-05    221
2021-06-02    221
2021-06-10    221
2021-06-03    221
2021-06-04    221
          ...
2020-12-07      6
2020-12-05      6
2020-12-04      6
2020-12-03      4
2020-12-02      4
Name: date, Length: 221
```

Which shows how many countries are having vaccination on a particular date.

- We drop the ‘location’ and ‘iso_code’ columns from dataset which is not neces sary for us. So, dimension of the dataset reduces to 32411 rows and 10 colum ns.
- We make a list of numerical-value columns in dataset having length 9.

```
['total_vaccinations',
 'people_vaccinated',
 'people_fully_vaccinated',
 'daily_vaccinations_raw',
 'daily_vaccinations',
 'total_vaccinations_per_hundred',
 'people_vaccinated_per_hundred',
 'people_fully_vaccinated_per_hundred',
 'daily_vaccinations_per_million']
```

- Sort the dataframe by ‘date’ in ascending order.

- Group dataframe by ‘date’ and summing them on same date. After this, dimension of the dataframe reduces to 221 rows × 10 columns.
- Now we make our dataframe continuous by adding a random number using uniform (0,1).
- Take ‘daily_vaccinations’ and ‘date’ columns only in new dataframe for further analysis.

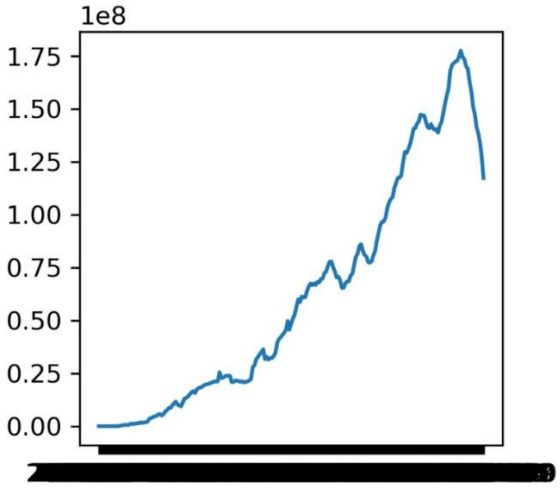
	daily_vaccinations	Date
0	4.985070e-01	2020-12-02
1	5.121923e-01	2020-12-03
2	3.733408e-01	2020-12-04
3	5.216472e-01	2020-12-05
4	5.475862e-01	2020-12-06
...	...	...
216	1.413055e+08	2021-07-06
217	1.385741e+08	2021-07-07
218	1.338450e+08	2021-07-08
219	1.271848e+08	2021-07-09
220	1.172732e+08	2021-07-10

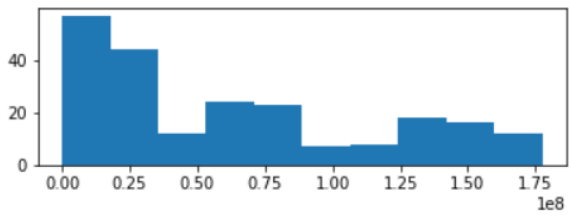
221 rows × 2 columns

	daily_vaccinations
count	2.210000e+02
mean	6.232140e+07
std	5.455110e+07
min	1.650159e-01
25%	1.667417e+07
50%	4.970357e+07
75%	1.031357e+08
max	1.774500e+08

LINE PLOT AND HISTOGRAM OF ‘daily_vaccinations’:

Now, as we get our time series of ‘daily_vaccinnations’ across the world on daily basis, we plot the line diagram to see the nature of the series. From the line plot, we can see that variance is not stable, hence we need to apply the ‘Box-Cox’ transformation on the series to stablize the variance.





BOX-COX TRANSFORMATION FOR VARIANCE STABILIZATION OF THE SERIES:

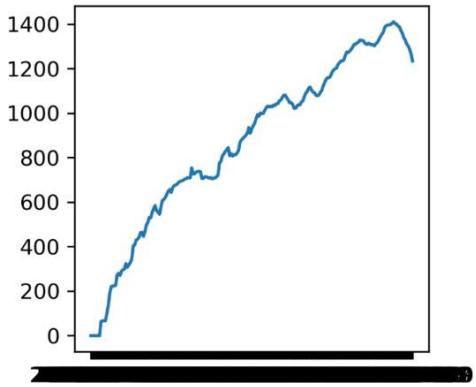
A Box cox transformation is defined as a way to transform non-normal dependent variables in our data to a normal shape through which we can run a lot more tests than we could have. Box-cox Transformation only cares about computing the value of λ which varies from – 5 to 5. A value of λ is said to be best if it is able to approximate the non-normal curve to a normal curve. This function requires input to be positive.

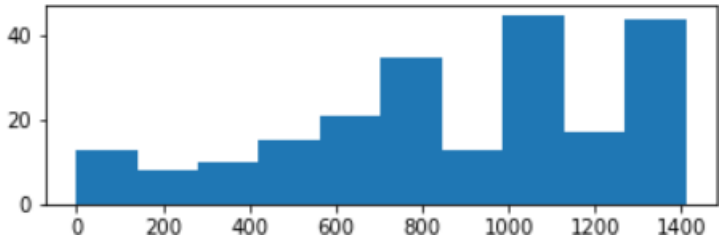
The transformation equation is as follows:

$$y(\lambda) = \begin{cases} \left(y^{\lambda} - 1\right)/\lambda \text{ if } \lambda \neq 0 \\ \log y \quad \text{if } \lambda = 0 \end{cases}$$

After applying the Box-Cox transformation, we get λ = 0.322258 and the series after the transformation updates as the following:

```
0         -0.623579
1         -0.601844
2         -0.844168
3         -0.587057
4         -0.547400
             ...
216     1309.856208
217     1301.623385
218     1287.105166
219     1266.057016
220     1233.303050
Name: daily_vaccinations, Length: 221, dtype: float64
```





Above plots represents the line plot and histogram of the updated series after applying the transformation.

DIVIDING THE SERIES INTO TRAINING AND TEST SETS:

We divide our series into two parts - training and test set. Training and test sets are disjoint portions of the series which is used to fit the model while the test set is used

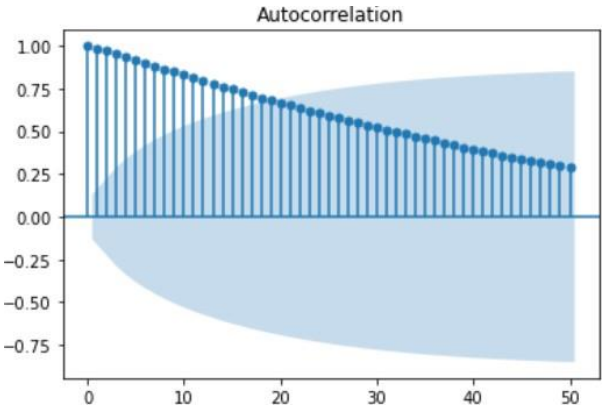
to see how accurate our fitted model is. Here, train set contains first 154 observations (approx. 70% of total) and test set contains remaining last 67 observations (approx. 30% of total) of the series.

ACF PLOT OF THE SERIES WITH NO DIFFERENCING UPTO LAG 50:

ACF (auto-correlation function) is a function which gives us values of auto-correlation of any series with its lagged values. We plot these values along with the confidence band. In simple terms, it describes how well the present value of the series is related with its past values.

In time series analysis, the partial autocorrelation function (PACF) gives the partial correlation of a stationary time series with its own lagged values, regressed the values of the time series at all shorter lags.

We plot the ACF plot of our series and see that it goes to zero in linear fashion, suggesting that series is non-stationary. Now, we check for the stationarity of the series by different tests.



TESTING THE STATIONARITY OF THE SERIES

Time series is said to be stationary if they do not have trend or seasonal effects. Summary statistics calculated on the time series are consistent over time, like the mean or the variance of the observations. When the time series is not stationary then it is called non-stationary time series.

When a time series is stationary, it can be easier to model. Statistical modelling methods assume or require the time series to be stationary to be effective. We use ADF and KPSS test for this purpose.

- **ADF test**:

The Augmented Dickey-Fuller test is a type of statistical test called a unit root test.

Null Hypothesis (H0): series has a unit root, that is series is not stationary.

Alternate Hypothesis (H1): series does not have a unit root, that is series is stationary.

We apply the ADF test on the series with no differencing and see that the p-value is 0.029776 which is less than 0.05 level of significance, hence we reject the null hypothesis and conclude that the series is stationary.

```
ADF Statistic: -3.058553
p-value: 0.029776
Critical Values:
        1%: -3.461
        5%: -2.875
        10%: -2.574
```

- **KPSS test**:

The KPSS test, short for, Kwiatkowski-Phillips-Schmidt-Shin (KPSS), is a type of Unit root test that tests for the stationarity of a given series around a deterministic trend.

Null Hypothesis (H0): series does not have a unit root, that is series is stationary.

Alternate Hypothesis (H1): series has a unit root, that is series is not stationary.

We apply the KPSS test on the series with no differencing and see that the p-value is 0.01 which is less than 0.05 level of significance, hence we reject the null hypothesis and conclude that the series is non-stationary.

```
(1.4238626879574532, 0.01, 15, {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%': 0.739})
```

As we can see that, ADF and KPSS tests give different results, conclude that the series is non-stationary. For making the series stationary, we need differencing of it.

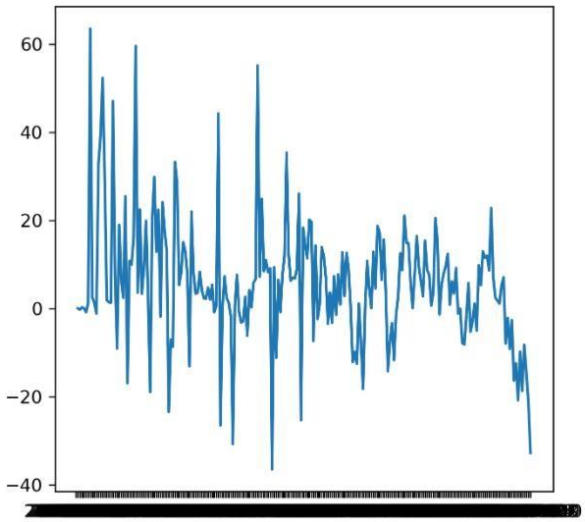
FIRST ORDER DIFFERENCING OF THE SERIES:

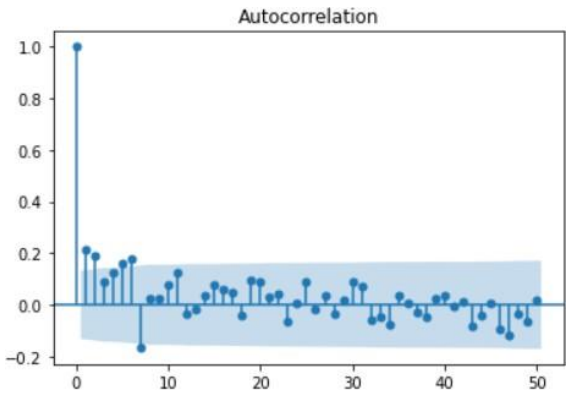
In the below table, the column ‘daily_vaccinations_with_diff_1’ shows the series with order of differencing 1.

	daily_vaccinations	Date	daily_vaccinations_with_diff_1
0	-0.623579	2020-12-02	NaN
1	-0.601844	2020-12-03	0.021735
2	-0.844168	2020-12-04	-0.242324
3	-0.587057	2020-12-05	0.257111
4	-0.547400	2020-12-06	0.039657
...	...	...	...
216	1309.856208	2021-07-06	-18.686686
217	1301.623385	2021-07-07	-8.232824
218	1287.105166	2021-07-08	-14.518219
219	1266.057016	2021-07-09	-21.048151
220	1233.303050	2021-07-10	-32.753966

221 rows × 3 columns

LINE PLOT OF THE TIME SERIES WITH ORDER OF DIFFERENCING 1 AND IT’S ACF PLOT:





By looking at the ACF plot, we can see that it cuts off the region and goes to zero quickly which is kind of sign of stationarity of the series. Now, we again apply the ADF and KPSS tests to check the stationarity of the series with order of differencing 1.

TESTING THE STATIONARITY OF THE SERIES WITH ORDER OF DIFFERENCING 1:

- **ADF test:**

Null Hypothesis (H0): series has a unit root, that is the series is not stationary.

Alternate Hypothesis (H1): series does not have a unit root, that is the series is stationary.

We apply the ADF test on the series with first order differencing and see that the p-value is 0.225578 which is greater than 0.05 level of significance, hence we fail to reject the null hypothesis and conclude that the series is non-stationary.

```
ADF Statistic: -2.148205
p-value: 0.225578
Critical Values:
        1%: -3.462
        5%: -2.875
        10%: -2.574
```

- **KPSS test:**

Null Hypothesis (H0): series does not have a unit root, that is the series is stationary.

Alternate Hypothesis (H1): series has a unit root, that is the series is non-stationary.

We apply the KPSS test on the series with first order differencing and see that the p-value is 0.01948 which is less than 0.05 level of significance, hence we reject the null hypothesis and conclude that the series is non-stationary.

```
(0.6346958167554975, 0.019482198476772954, 15, {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%': 0.739})
```

As both the tests results in non-stationarity of the series with order of differencing 1, we need to difference it again and check for the stationarity.

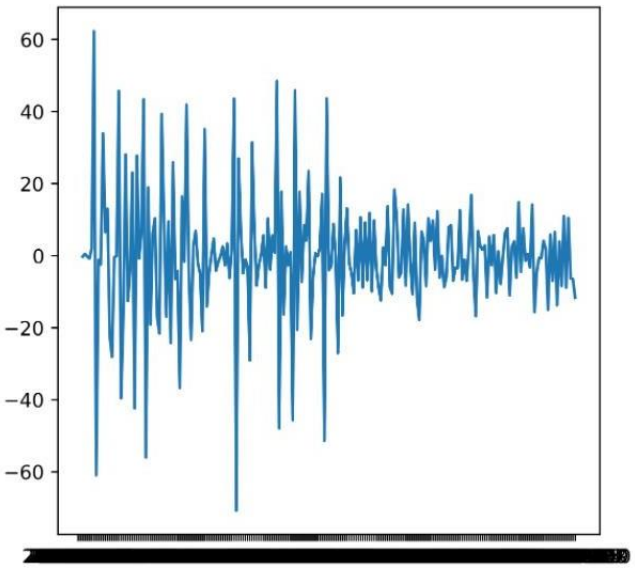
SECOND ORDER DIFFERENCING OF THE SERIES:

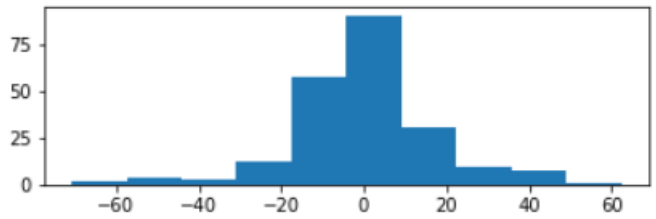
As series is not stationary with order of differencing 1, We need to perform 2nd order differencing on the series which is given below in the column 'daily_vaccinations_with_diff_2'.

	daily_vaccinations	Date	daily_vaccinations_with_diff_1	daily_vaccinations_with_diff_2
0	-0.623579	2020-12-02	NaN	NaN
1	-0.601844	2020-12-03	0.021735	NaN
2	-0.844168	2020-12-04	-0.242324	-0.264059
3	-0.587057	2020-12-05	0.257111	0.499435
4	-0.547400	2020-12-06	0.039657	-0.217454
...	...	...	...	...
216	1309.856208	2021-07-06	-18.686686	-8.863791
217	1301.623385	2021-07-07	-8.232824	10.453863
218	1287.105166	2021-07-08	-14.518219	-6.285395
219	1266.057016	2021-07-09	-21.048151	-6.529932
220	1233.303050	2021-07-10	-32.753966	-11.705815

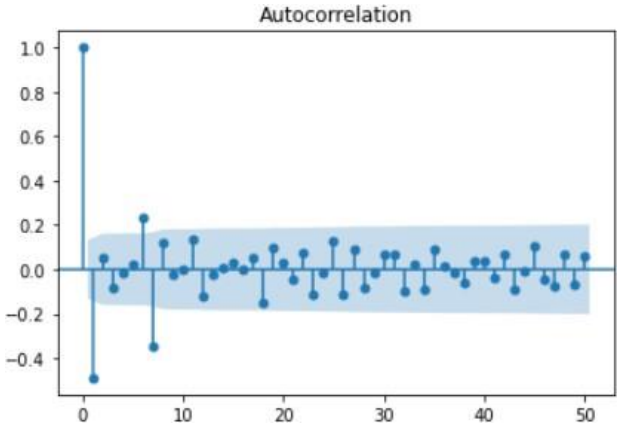
221 rows × 4 columns

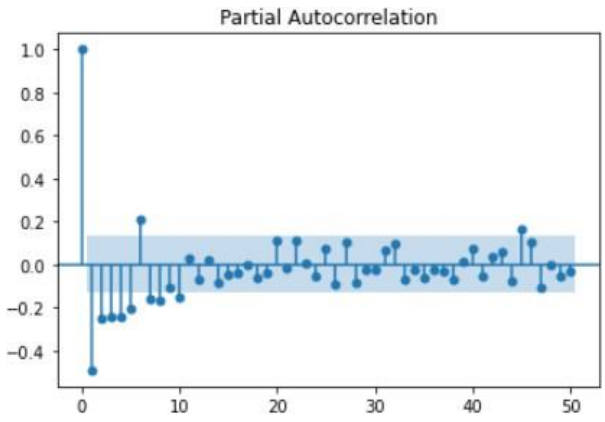
LINE PLOT AND HISTOGRAM OF THE SERIES WITH 2ND ORDER OF DIFFERENCING:





ACF AND PACF PLOTS OF THE SERIES WITH 2ND ORDER OF DIFFERENCING:





TESTING THE STATIONARITY OF THE SERIES WITH 2ND ORDER DIFFERENCING:

- **ADF test**:

Null Hypothesis (H0): series has a unit root, that is the series is not stationary.

Alternate Hypothesis (H1): series does not have a unit root, that is the series is stationary.

We apply the ADF test on the series with 2nd order of differencing and see that the p-value is 0.00 which is less than 0.05 level of significance, hence we reject the null hypothesis and conclude that the series is now stationary.

```
ADF Statistic: -7.989885
p-value: 0.000000
Critical Values:
        1%: -3.462
        5%: -2.875
        10%: -2.574
```

- **KPSS test**:

Null Hypothesis (H0): series does not have a unit root, that is the series is stationary.

Alternate Hypothesis (H1): series have a unit root, that is the series is non-stationary.

We apply the KPSS test on the series with 2nd order differencing and see that the p-value is 0.1 which is greater than 0.05 level of significance, hence we fail to reject the null hypothesis and conclude that the series is stationary.

```
(0.12533126967433905, 0.1, 15, {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%': 0.739})
```

With the second order of differencing our series has become stationary. We can proceed for the modelling stage.

ESTIMATING AR AND MA PARAMETERS:

Further, as we got our series is stationary with 2nd order of differencing, we find out the appropriate order of (p, q). We'll fit the ARIMA model of order (p, d=2, q) and calculate the value of Akaike Information Criteria and consider the corresponding value of p and q for which AIC is minimum.

d=2

p\q	1	2	3	4
1	1269.365	1271.681	1272.102	1274.099
2	1270.982	1272.103	1265.547	1267.540
3	1272.543	1274.097	1267.541	1269.504
4	1274.510	1276.085	1269.404	1271.223

Value of minimum AIC: 1265.547

Values of estimated p and q: 2, 3

Thus, we obtain ARIMA (2,2,3)

ARIMA (2,2,3) MODEL RESULTS:

ARIMA Model Results

Dep. Variable:	D2.daily_vaccinations	No. Observations:	152
Model:	ARIMA(2, 2, 3)	Log Likelihood	-625.774
Method:	css-mle	S.D. of innovations	14.404
Date:	Fri, 13 Aug 2021	AIC	1265.547
Time:	23:44:53	BIC	1286.715
Sample:	2	HQIC	1274.146

	coef	std err	z	P>|z|	[0.025	0.975]
const	-0.0725	0.030	-2.440	0.015	-0.131	-0.014
ar.L1.D2.daily_vaccinations	-0.2383	0.061	-3.886	0.000	-0.358	-0.118
ar.L2.D2.daily_vaccinations	-0.8098	0.056	-14.489	0.000	-0.919	-0.700
ma.L1.D2.daily_vaccinations	-0.6876	0.028	-24.226	0.000	-0.743	-0.632
ma.L2.D2.daily_vaccinations	0.6874	0.033	20.544	0.000	0.622	0.753
ma.L3.D2.daily_vaccinations	-0.9999	0.037	-26.963	0.000	-1.073	-0.927

Roots

	Real	Imaginary	Modulus	Frequency
AR.1	-0.1471	-1.1015j	1.1113	-0.2711
AR.2	-0.1471	+1.1015j	1.1113	0.2711
MA.1	1.0000	-0.0000j	1.0000	-0.0000
MA.2	-0.1562	-0.9878j	1.0001	-0.2750
MA.3	-0.1562	+0.9878j	1.0001	0.2750

The obtained **ARIMA (2,2,3)** model is given by:

$$Y_t=(-0.0725)-(0.2383)Y_{t-1}-(0.8098)Y_{t-2}+\varepsilon_t-(0.6876)\varepsilon_{t-1}+(0.6874)\varepsilon_{t-2}-(0.9999)\varepsilon_{t-3}$$

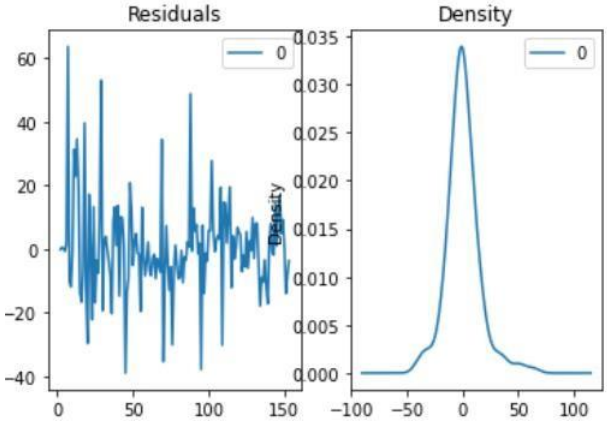
CORRELATION MATRIX OF THE PARAMETERS ESTIMATES:

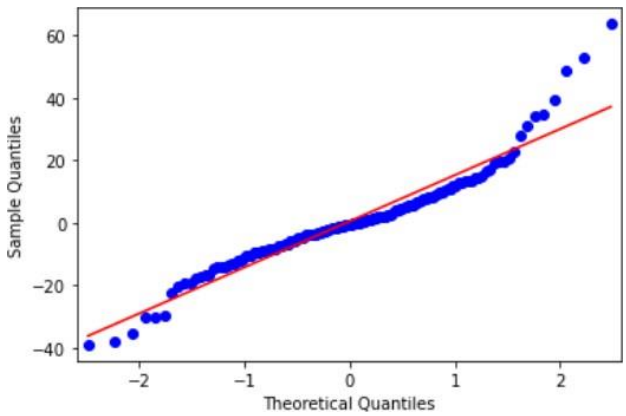
```
[[ 1.00000000e+00 -4.58600679e-03  2.65377603e-02  6.50590754e-03  -5.60483681e-03  7.61952248e-05]
 [-4.58600679e-03  1.00000000e+00  6.59723438e-02 -3.99086634e-01   3.37797371e-01  6.48927910e-04]
 [ 2.65377603e-02  6.59723438e-02  1.00000000e+00  1.24565244e-01  -1.08142475e-01  2.23521059e-03]
 [ 6.50590754e-03 -3.99086634e-01  1.24565244e-01  1.00000000e+00  -1.38282521e-01  2.32334624e-01]
 [-5.60483681e-03  3.37797371e-01 -1.08142475e-01 -1.38282521e-01   1.00000000e+00 -5.65097784e-01]
 [ 7.61952248e-05  6.48927910e-04  2.23521059e-03  2.32334624e-01  -5.65097784e-01  1.00000000e+00]]
```

We can see from correlation matrix that all the parameters are uncorrelated.

RESIDUALS DIAGNOSTICS:

Below are the line plot and density plot and Q-Q plot of the residuals. All represents that the residuals are nearly normally distributed around mean zero.





ANDERSON-DARLING TEST FOR NORMALITY OF THE RESIDUAL DISTRIBUTION:

The Anderson-Darling test (Stephens, 1974) is used to test if a sample of data came f rom a population with a specific distribution. It is a modification of the Kolmogorov-Sm irnov (K-S) test and gives more weight to the tails than does the K-S test. The K-S te st is distribution free in the sense that the critical values do not depend on the specifi c distribution being tested (note that this is true only for a fully specified distribution, i. e. the parameters are known). The Anderson-Darling test makes use of the specific d istribution in calculating critical values. The Anderson-Darling test is an alternative to t he chi-square and Kolmogorov-Smirnov goodness-of-fit tests.

The Anderson-Darling test is defined as:

H_0: The data follow a specified distribution.
H_a: The data do not follow the specified distribution

Test statistics, A is given by,

$$A^2 = -n - S,$$

where

$$S = \sum_{i=1}^{n} \frac{2i-1}{n} \left[\ln(F(Y_i)) + \ln(1 - F(Y_{n+1-i}))\right].$$

Where**,**
F is the cumulative distribution function of the specified distribution.
$\mathbf{Y_i}$ are the *ordered* statistics.

We reject the null hypothesis if the p-value is less than 0.05 level of significance whic h allows us to claim that our data is statistically different from a specified distribution. On the other hand, if p-value is higher than 0.05, we can state that our data is not sta tistically different from a specified distribution.

Critical values in a statistical test are a range of pre-defined significance boundaries a t which the H0 can be failed to be rejected if the calculated statistic is less than the cr itical value.

Rather than just a single p-value, the test returns a critical value for a range of differe nt commonly used significance levels.

Statistic: 2.810

Significance level:critical values
15.000: 0.562, data does not look normal (reject H0)
10.000: 0.640, data does not look normal (reject H0)
5.000: 0.768, data does not look normal (reject H0)
2.500: 0.895, data does not look normal (reject H0)
1.000: 1.065, data does not look normal (reject H0)

We apply the Anderson-Darling test and find that at each significance level, p values are lesser than critical value and we conclude that the residuals do not follow a norm al distribution.

SHAPIRO-WILK TEST FOR NORMALITY TESTING OF RESIDUAL DISTRIBUTION:

The Shapiro–Wilk test tests the null hypothesis that a sample x_1, ..., x_n come from a normally distributed population.

Statistics:

$$W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\sum_{i=1}^{n}(x_i - \overline{x})^2},$$

Where,
$x_{(i)}$ is the i^{th} order statistic.

The coefficients a_i are given by:[1]

$$(a_1, \ldots, a_n) = \frac{m^{\mathsf{T}} V^{-1}}{C},$$

where C is a vector norm:[2]

$$C = \|V^{-1} m\| = (m^{\mathsf{T}} V^{-1} V^{-1} m)^{1/2}$$

and the vector m,

$$m = (m_1, \ldots, m_n)^{\mathsf{T}}$$

Which is made of the expected values of the order statistics of independent and ident ically distributed random variables sampled from the standard normal distribution and V is the covariance matrix of those normal order statistics.

Null Hypothesis (H0): residuals are normally distributed.
Alternate Hypothesis (H1): residuals are not normally distributed.

If the *p* value is less than 0.05, then the null hypothesis is rejected means that there i s evidence that the residuals are not normally distributed. On the other hand, if the *p* v alue is greater than the 0.05, then the null hypothesis cannot be rejected means ther e is evidence that the residuals are normally distributed.

Statistics=0.931, p=0.000

We see that p value is less than 0.05 level of significance and hence we reject the nu ll hypothesis and conclude that residuals are not normally distributed.

MANN-WHITNEY U TEST (non-parametric test):

Null hypothesis H_0: the distributions of two populations are equal.

Alternative hypothesis H_1: the distributions are not equal.

Let X1, X2,…,Xn be an i.i.d. random sample from population X, and Y1, Y2,…Yn be an i.i.d. random sample from population Y, and both samples independent of each other. The corresponding Mann-Whitney U statistic is defined as:

$$U = \sum_{i=1}^{n} \sum_{j=1}^{m} S(X_i, Y_j),$$

with

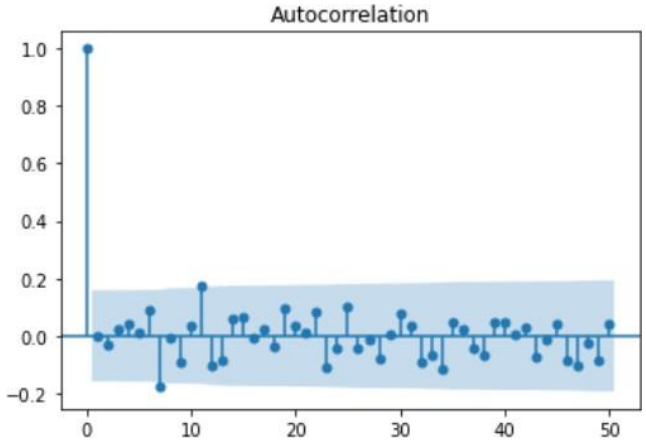
$$S(X,Y) = \begin{cases} 1, & \text{if } Y < X, \\ \frac{1}{2}, & \text{if } Y = X, \\ 0, & \text{if } Y > X. \end{cases}$$

Here, we have taken a random sample from N(0,1) to compare the distribution of residuals.

Statistics=4824.000, p=0.268

After applying the test, we can see that p value is 0.268 which is greater than 0.05 le vel of significance hence we fail to reject the null hypothesis and conclude that both samples have the same distribution that means residuals are normally distributed.

ACF PLOT OF RESIDUALS:



LJUNG BOX TEST TO CHECK WHETHER RESIDUALS ARE CORRELATED AT DIFFERENT LAGS:

Ljung Box test is a test for testing the *absence* of serial autocorrelation, up to a specified lag *k*.

Null hypothesis H_0: residuals are uncorrelated.

Alternate hypothesis H_a: residuals are correlated.

Test statistics:

$$Q(m) = n(n+2) \sum_{j=1}^{m} \frac{r_j^2}{n-j},$$

Where,

r_j = the accumulated sample autocorrelations,

m = the time lag.

If p value is less than level of significance 0.05, then we reject the null hypothesis and conclude that residuals are correlated else we fail to reject the null hypothesis and conclude that residuals are uncorrelated.

	lb_stat	lb_pvalue
1	0.000264	0.987031
2	0.180994	0.913477
3	0.266900	0.966129
4	0.512444	0.972278
5	0.527548	0.991080
10	8.372105	0.592540
12	15.301489	0.225361
14	17.076177	0.252141
15	17.871433	0.269485
17	17.990221	0.389458

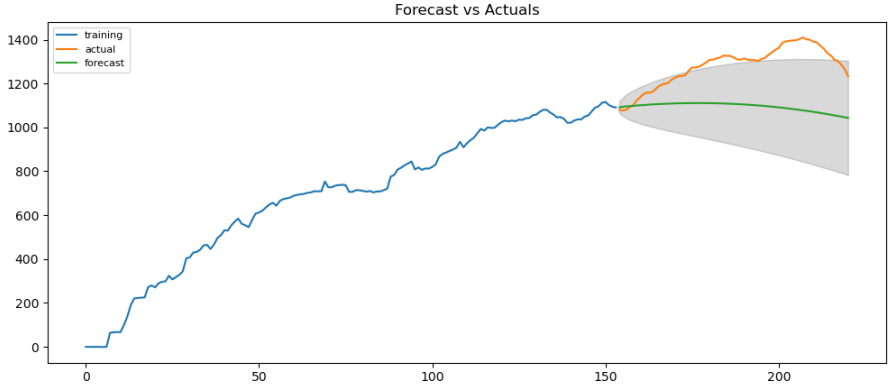
From the above, we see that p values are greater than 0.05 level of significance at each lag. This suggests that residuals are uncorrelated.

FORECASTING:

Now we are ready for forecasting stage. We forecasted the observations and show these with it's actual values.

```
>Predicted=1091.764, Actual=1079.103
>Predicted=1093.917, Actual=1077.794
>Predicted=1095.920, Actual=1080.561
>Predicted=1096.837, Actual=1093.100
>Predicted=1097.987, Actual=1101.820
>Predicted=1099.811, Actual=1122.861
>Predicted=1101.138, Actual=1137.756
>Predicted=1101.888, Actual=1152.561
>Predicted=1103.030, Actual=1158.684
>Predicted=1104.397, Actual=1158.790
>Predicted=1105.245, Actual=1166.756
>Predicted=1105.886, Actual=1183.164
>Predicted=1106.848, Actual=1192.663
>Predicted=1107.752, Actual=1198.854
>Predicted=1108.262, Actual=1201.617
>Predicted=1108.764, Actual=1216.995
>Predicted=1109.438, Actual=1225.743
>Predicted=1109.930, Actual=1233.399
>Predicted=1110.176, Actual=1234.054
>Predicted=1110.481, Actual=1237.764
```

```
>Predicted=1110.822, Actual=1258.280
>Predicted=1110.958, Actual=1273.680
>Predicted=1110.965, Actual=1272.322
>Predicted=1111.021, Actual=1277.730
>Predicted=1111.020, Actual=1285.712
>Predicted=1110.846, Actual=1295.329
>Predicted=1110.609, Actual=1307.748
>Predicted=1110.380, Actual=1308.602
>Predicted=1110.051, Actual=1314.726
>Predicted=1109.591, Actual=1318.176
>Predicted=1109.095, Actual=1327.349
>Predicted=1108.564, Actual=1326.187
>Predicted=1107.923, Actual=1326.183
>Predicted=1107.187, Actual=1318.347
>Predicted=1106.415, Actual=1310.194
>Predicted=1105.580, Actual=1308.360
>Predicted=1104.640, Actual=1314.151
>Predicted=1103.628, Actual=1308.886
>Predicted=1102.569, Actual=1306.060
>Predicted=1101.432, Actual=1307.177
>Predicted=1100.203, Actual=1302.148
>Predicted=1098.910, Actual=1311.936
>Predicted=1097.559, Actual=1317.256
>Predicted=1096.124, Actual=1330.214
>Predicted=1094.608, Actual=1341.731
>Predicted=1093.031, Actual=1353.685
>Predicted=1091.385, Actual=1362.308
>Predicted=1089.657, Actual=1385.093
>Predicted=1087.855, Actual=1392.144
>Predicted=1085.989, Actual=1394.616
>Predicted=1084.049, Actual=1396.415
>Predicted=1082.031, Actual=1397.545
>Predicted=1079.942, Actual=1402.792
>Predicted=1077.785, Actual=1409.851
>Predicted=1075.553, Actual=1401.888
>Predicted=1073.246, Actual=1399.736
>Predicted=1070.869, Actual=1390.572
>Predicted=1068.421, Actual=1387.936
>Predicted=1065.897, Actual=1371.595
>Predicted=1063.301, Actual=1359.171
>Predicted=1060.634, Actual=1338.366
>Predicted=1057.895, Actual=1328.543
>Predicted=1055.081, Actual=1309.856
>Predicted=1052.196, Actual=1301.623
>Predicted=1049.239, Actual=1287.105
>Predicted=1046.209, Actual=1266.057
>Predicted=1043.105, Actual=1233.303
```



PYTHON CODES THAT IS USED FOR THIS PROJECT:

```
#importing the libraries
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import random

#reading dataset into data frame as data1
data1 = pd.read_csv("C:/Users/Lenovo/Desktop/time series project summer/vaccinations.csv")
data1.head(5)

#checking dimension of
data1data1.shape

#CHECKING FOR NULL VALUES in data1
data1.isnull().sum()

#Replacing NA by 0
data1 = data1.fillna(0)
data1

#CHECKING FOR NULL VALUES IN EACH COLUMNS
data1.isnull().sum()

#CHECKING COUNT OF UNIQUE VALUES IN COLUMNS
data1['date'].value_counts()

#DROPPING THE locations and ISO code COLUMNs FROM data1
data1.drop(['location','iso_code'], axis=1, inplace=True)
data1.head()
data1.shape
#finding list of numerical value columns into list num_col
num_col = [i for i in data1.columns if data1.dtypes[i]!='object']
num_col

#sorting dataframe by Date in ascending order
data1 = data1.sort_values(by = 'date')
data1
```

```
#grouping dataframe by Date and summing them date wise data1 as data2

data2 = data1.groupby(['date'], as_index = False).sum()

data2

#copying the date column

col_date = data2['date'].copy()

len(num_col)

np.random.seed(10)

#now we make our dataframe data2 continuous by adding random number using unniform(0,1) and name it as added_df

added_df = data2.iloc[:,1:].add(np.random.rand(221, 9))

added_df

#adding col_date column in added_df dataframe at right most column

added_df['Date'] = col_date

added_df

#making list of column names of added_df dataframe as cols

cols=list(added_df.columns)

#taking total_vaccinations and date columns in new dataframe df_date_tot to fit the model in this dataframe to forecast
total_vaccinations

df_date_tot = added_df[[cols[4]] + [cols[-1]]]

df_date_tot

#plotting the df_date_tot dataframe date vs total_vaccinations all over the wworld

plt.figure(figsize=(3,3),dpi=300)

plt.plot(df_date_tot['Date'],df_date_tot['daily_vaccinations'])

plt.show()

plt.subplot(212)

plt.hist(df_date_tot['daily_vaccinations'])

plt.show()

# splitting dataframe by row index into two parts

part_1 = df_date_tot.iloc[:111,:]

part_2 = df_date_tot.iloc[111:,:]

df_date_tot.describe()

part_1.shape

part_2.shape

part_1.var()

part_2.var()

df_daily_vaccinations = df_date_tot.copy()

df_daily_vaccinations.shape

#box cox method for variance stablization

from scipy.stats import boxcox

df_daily_vaccinations['daily_vaccinations'], lam = boxcox(df_daily_vaccinations['daily_vaccinations'])
```

```
print('Lambda: %f' % lam)
# line plot
plt.figure(figsize=(3,3),dpi=300)
plt.plot(df_daily_vaccinations['Date'],df_daily_vaccinations['daily_vaccinations'])
plt.show()
# histogram
plt.subplot(212)
plt.hist(df_daily_vaccinations['daily_vaccinations'])
plt.show()
df_daily_vaccinations['daily_vaccinations']
train = df_daily_vaccinations['daily_vaccinations'][:154]
test = df_daily_vaccinations['daily_vaccinations'][154:]
plt.figure(figsize=(3,3),dpi=300)
plt.plot(df_daily_vaccinations['Date'],df_daily_vaccinations['daily_vaccinations'])
plt.show()
#testing the stationarity by DF test which results it as stationary time series
#by looking at the graph and adf test it is clearly  non_stationary.
from statsmodels.tsa.stattools import adfuller
result = adfuller(df_daily_vaccinations['daily_vaccinations'])
print('ADF Statistic: %f' % result[0])
print('p-value: %f' % result[1])
print('Critical Values:')
for key, value in result[4].items():
                                                                                print('\t%s: %.3f' %
(key, value))
# KPSS test
from statsmodels.tsa.stattools import kpss
result = kpss(df_daily_vaccinations['daily_vaccinations'])
print(result)
#shows series is non stationary
#ACF plot upto lag 50
from statsmodels.graphics.tsaplots import plot_acf
plot_acf(df_daily_vaccinations['daily_vaccinations'], lags = 50)
plt.show()
#first order differencing of raw data
df_daily_vaccinations['daily_vaccinations_with_diff_1'] = df_daily_vaccinations['daily_vaccinations'] -
df_daily_vaccinations['daily_vaccinations'].shift(1)
df_daily_vaccinations
#ploting the time series after first order differning
plt.figure(figsize=(5,5),dpi=300)
```

```
plt.plot(df_daily_vaccinations['Date'],df_daily_vaccinations['daily_vaccinations_with_diff_1'])
plt.show()
#acf plot of dataframe of diff 1
from statsmodels.graphics.tsaplots import plot_acf
plot_acf(df_daily_vaccinations['daily_vaccinations_with_diff_1'][1:], lags = 50)
plt.show()
#pacf plot of dataframe of diff 1
from statsmodels.graphics.tsaplots import plot_pacf
plot_pacf(df_daily_vaccinations['daily_vaccinations_with_diff_1'][1:], lags = 50)
plt.show()
#DF test on data with diff 1
from statsmodels.tsa.stattools import adfuller
result = adfuller(df_daily_vaccinations['daily_vaccinations_with_diff_1'][1:])
print('ADF Statistic: %f' % result[0])
print('p-value: %f' % result[1])
print('Critical Values:')
for key, value in result[4].items():
                                                                                print('\t%s: %.3f' %
(key, value))
#shows non stationary
# KPSS test
from statsmodels.tsa.stattools import kpss
result = kpss(df_daily_vaccinations['daily_vaccinations_with_diff_1'][1:])
print(result)
#shows series is non stationary
#second order differencing of raw data
df_daily_vaccinations['daily_vaccinations_with_diff_2'] = df_daily_vaccinations['daily_vaccinations_with_diff_1'] -
df_daily_vaccinations['daily_vaccinations_with_diff_1'].shift(1)
df_daily_vaccinations
#ploting the time series after second order differning
plt.figure(figsize=(5,5),dpi=500)
plt.plot(df_daily_vaccinations['Date'],df_daily_vaccinations['daily_vaccinations_with_diff_2'])
plt.show()
# histogram
plt.subplot(212)
plt.hist(df_daily_vaccinations['daily_vaccinations_with_diff_2'])
plt.show()
#acf plot of dataframe of diff 2
from statsmodels.graphics.tsaplots import plot_acf
plot_acf(df_daily_vaccinations['daily_vaccinations_with_diff_2'][2:], lags = 50)
```

```
plt.show()
#pacf plot of dataframe of diff 2
from statsmodels.graphics.tsaplots import plot_pacf
plot_pacf(df_daily_vaccinations['daily_vaccinations_with_diff_2'][2:], lags = 50)
plt.show()
#df test for data with diff 2
from statsmodels.tsa.stattools import adfuller
result = adfuller(df_daily_vaccinations['daily_vaccinations_with_diff_2'][2:])
print('ADF Statistic: %f' % result[0])
print('p-value: %f' % result[1])
print('Critical Values:')
for key, value in result[4].items():
                                                                                print('\t%s: %.3f' %
(key, value))
#shows series is stationary
# KPSS test
from statsmodels.tsa.stattools import kpss
result = kpss(df_daily_vaccinations['daily_vaccinations_with_diff_2'][2:])
print(result)
#shows series is stationary
df_date_tot
#fitting arima models
from statsmodels.tsa.arima_model import ARIMA
from statsmodels.tsa.arima_model import ARMAResults
import re
import statsmodels.api as sm
from statsmodels.graphics.gofplots import qqplot
import pylab
import scipy.stats as stats
from scipy.stats import anderson
from scipy.stats import shapiro
from scipy.stats import normaltest
#ARIMA Model
def arima(p,d,q):
    model = ARIMA(train, order=(p,d,q))
    model_fit = model.fit(disp=0)
    print(model_fit.summary())
    print(ARMAResults.cov_params(model_fit))
    diag_elt = np.sqrt(ARMAResults.cov_params(model_fit).diagonal())
```

```
D = np.diag(diag_elt)
corr_mat = np.linalg.inv(D).dot(ARMAResults.cov_params(model_fit)).dot(np.linalg.inv(D))
print(corr_mat)



# Plot residual errors
residuals = pd.DataFrame(model_fit.resid)
fig, ax = plt.subplots(1,2)
residuals.plot(title="Residuals", ax=ax[0])
residuals.plot(kind='kde', title='Density', ax=ax[1])
plt.show()
plt.savefig('C:/Users/Lenovo/Desktop/resdi_plots.png',dpi=100)

# q-q plot of residuals
plt.subplot(1,1,1)
qqplot(model_fit.resid, line='r', ax=plt.gca())
plt.show()
plt.savefig('C:/Users/Lenovo/Desktop/qq_plots.png',dpi=500)

#anderson darling test of normality of residual distribution
result = anderson(model_fit.resid)
print('Statistic: %.3f' % result.statistic)
p = 0
for i in range(len(result.critical_values)):
    sl, cv = result.significance_level[i], result.critical_values[i]
    if result.statistic < result.critical_values[i]:
        print('%.3f: %.3f, data looks normal (fail to reject H0)' % (sl, cv))
    else:
        print('%.3f: %.3f, data does not look normal (reject H0)' % (sl, cv))

# Shapiro-Wilk Test of normality of residual distribution
stat, p = shapiro(model_fit.resid)
print('Statistics=%.3f, p=%.3f' % (stat, p))
# interpret
alpha = 0.05
if p > alpha:
    print('Sample looks Gaussian (fail to reject H0)')
else:
    print('Sample does not look Gaussian (reject H0)')
```

```
# Mann-Whitney U test
from numpy.random import randn
from scipy.stats import mannwhitneyu
data1 = randn(67)
stat, p = mannwhitneyu(data1, model_fit.resid)
print('Statistics=%.3f, p=%.3f' % (stat, p))
# interpret
alpha = 0.05
if p > alpha:
    print('Same distribution (fail to reject H0)')
else:
    print('Different distribution (reject H0)')



#stats.probplot(model_fit.resid, dist="norm", plot=plt)



# acf plot of residuals
plot_acf(residuals, lags = 50)
plt.show()
plt.savefig('C:/Users/Lenovo/Desktop/acf_resid.png',dpi=100)
# Forecast
fc, se, conf = model_fit.forecast(67, alpha=0.05)  # 95% conf


for i in range(len(test)):
    print('>Predicted=%.3f, Actual=%.3f' % (fc.tolist()[i], test.reset_index(drop=True).tolist()[i]))


#ljung box test
print(sm.stats.acorr_ljungbox(residuals, lags=[1,2,3,4,5,10,12,14,15,17], return_df=True)) #if statistics p value is less than
critical value than reject H0
#means residuals are correlated


# Make as pandas series
fc_series = pd.Series(fc, index=test.index)
lower_series = pd.Series(conf[:, 0], index=test.index)
upper_series = pd.Series(conf[:, 1], index=test.index)


# Plot
```

```
    plt.figure(figsize=(12,5), dpi=100)
    plt.plot(train, label='training')
    plt.plot(test, label='actual')
    plt.plot(fc_series, label='forecast')
    plt.fill_between(lower_series.index, lower_series, upper_series,
                     color='k', alpha=.15)
    plt.title('Forecast vs Actuals')
    plt.legend(loc='upper left', fontsize=8)
    plt.savefig('C:/Users/Lenovo/Desktop/forecast_f.png',dpi=100)
    plt.show()

arima(1,1,1) #bad
arima(2,1,1) #bad
arima(1,1,2) #bad
arima(2,1,2) #second best
arima(2,2,4)
arima(3,1,3) #third best
arima(1,2,1) #less AIC than 2,1,2 but not all variables are signifi
arima(2,2,1) #bad
arima(1,2,2)  #bad
arima(2,2,2)  #bad
arima(2,2,3) #best
arima(0,1,0)
import pmdarima as pm
model = pm.auto_arima(train, start_p=1, start_q=1,
                      test='adf',       # use adftest to find optimal 'd'
                      max_p=4, max_q=4, # maximum p and q
                      m=1,              # frequency of series
                      d=None,           # let model determine 'd'
                      seasonal=False, # No Seasonality
                      start_P=0,
                      D=0,
                      trace=True,
                      error_action='ignore',
                      suppress_warnings=True,
                      stepwise=True)

print(model.summary())
model.plot_diagnostics(figsize=(7,5))
```

plt.show()

arima(4,2,4)

arima(1,2,1)

arima(1,2,0)

arima(0,2,2)

arima(1,2,2)

arima(2,2,2)

arima(2,2,1)

arima(2,2,0)

arima(0,2,3)

arima(1,2,3)

arima(2,2,3)

arima(3,2,3)

arima(3,2,2)

arima(3,2,1)

arima(1,2,4)

REFERENCES:

- Machine Learning Mastery
- Wikipedia
- Time Series Analysis: Forecasting and Control Fifth Edition by Box and Jenkins
- Kaggle
- Stackexchange