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Subject:
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problem set 8

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<<problem set 8.pdf>>

Micro B: Problem Set 8

Bargaining

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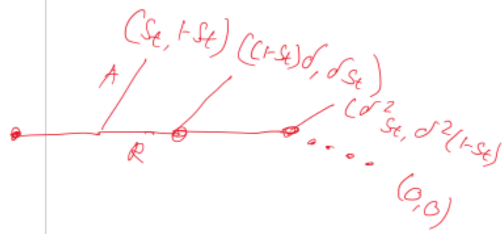
Spring 2023



Question 1 (Finite horizon bargaining). Consider a sequential Rubinstein bargaining game with T periods. The periods take turn at proposing an offer, $s_t \in [0;1]$. If the proposal is accepted, the payoffs are $(s_t, 1-s_t)$, and if rejected, the game proceeds to the next stage. If the final offer is rejected, the game ends with payoffs $(0,0)$. Both players discount future payoffs with common discount factor $\delta \in [0;1]$.

(a) Illustrate the game graphically (in a tree).

(b) Is there a first-mover advantage? Does your answer depend on the value of T?
No, instead last mover advantage.

Question 2 (Outside option). Consider the bargaining game from question 1 with the single exception that if the final offer is rejected, the payoffs are $(0, x)$: that is, player 1 still gets zero, but player 2 gets x.

(a) Focusing on $T = 1$, describe what happens if $x < 0$ or if $x > 1$. Then, in the following, assume $0 < x < 1$.
$x < 0$: P2 accepts anything $x > 1$: P2 rejects anything

(b) Suppose $T = 1$. Find the Nash equilibrium of the game.
$(1-x, x)$ SP2 indifferent on x nu.

(c) Do the same for $T = 2$.
Spiller 2 har magten og tilbyder 0 til SP1. SP1 indifferent.

(d) And for $T = 3$.
Tilbyder $((1-x)\delta^2, x\delta^2)$

Question 3 (Asymmetric Patience). Consider the Rubinstein infinite-horizon bargaining game and assume that player 1 has discount factor δ_1 and player 2 has discount factor δ_2, where $\delta_1, \delta_2 \in [0;1)$. The backwards induction outcome of the game is that player 1 offers $(x^*, 1-x^*)$, and player 2 offers $(1-y^*, y^*)$, where x^* and y^* are the unique solutions to the equations.

$$\delta_1 x^* = 1 - y^* \wedge \delta_2 y^* = 1 - x^*. \tag{1}$$

(a) Explain the intuition behind (1).
Vil ikke afvise hvis de får det samme som i næste periode diskonteret.

(b) Show that the unique solution to (1) is

$$x^* = \frac{1-\delta_1}{1-\delta_1\delta_2}, \quad y^* = \frac{1-\delta_2}{1-\delta_1\delta_2}.$$

b) $x = \frac{1-y}{\delta_1}$

$\delta_2 y = 1 - \frac{1-y}{\delta_1}$

$y^* = \frac{1-\delta_1}{1-\delta_2\delta_1}$

$x^* = \frac{1-\delta_1}{1-\delta_1\delta_2}$

(c) Show that the equilibrium *outcome* of the game where player 1 gives the first offer is

$$(x^*, 1-x^*) = \left(\frac{1-\delta_2}{1-\delta_1\delta_2}, \frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}\right) \tag{2}$$

Hint: Check out https://youtu.be/z0NHnrdjeeA.

Question 4 (Asymmetric Patience in Python). Code up Equation (2) in Python.

(a) Suppose two countries borrow at the interest rates $r_1 = 5\%$ and $r_2 = 10\%$ and are bargaining in infinite horizon with alternating offers over the division of some surplus between the countries. What is the equilibrium?
Hint: The discount factor is $\delta = \frac{1}{1+r}$.

$$(x^*, 1-x^*) = \left(\frac{1-\left(\frac{1}{1+0{,}1}\right)}{1-\left(\frac{1}{1{,}1}\right)\cdot\left(\frac{1}{1{,}05}\right)}, \frac{\frac{1}{1{,}1}\cdot\left(1-\frac{1}{1{,}05}\right)}{1-\frac{1}{1{,}1}\cdot\frac{1}{1{,}05}}\right)$$

(b) The yields on 10 year government bonds are (as of May 2022)

- China: 2.82%,
- Vietnam: 3.37%,
- Namibia: 12.11%
- Germany: 0.94%,
- Denmark: 1.32%,
- Poland: 7.05%.

Se interaction in Python.

Compute the equilibrium bargaining outcome between each of the pairs of countries.

(c) Explain in your own words how patience help "the rich get richer."
Tålmodige lande (lav rente) er bedre stillet i forhandlingen, og de vil derfor få en større del ...

Question 5 (Realistic Values). You are at the Roskilde Festival and you have spotted a great restaurant that has no app. The restaurant is currently selling $Q = 100$ meals per hour at $p = 100$ DKK. With the app live, they will instead sell $Q' = 200$ meals

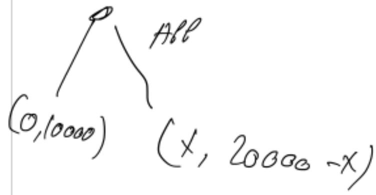
(a) Set up the game as a bargaining game and draw the extensive form graph.

(b) What "take-it-or-leave-it" offer should you make to the restaurant?

Marginally lower than gain 100 · 100.

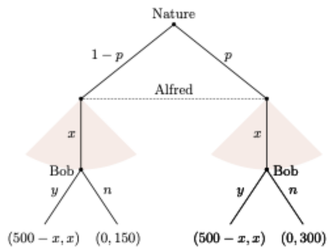
(c) If the restaurant gets to make a counter-offer after which the game ends, what is the subgame perfect equilibrium? Betale 0, da SR er indifferent.

(d) Suppose both parties get to make sequential counteroffers (starting with you) but each period there is a 1% probability that the game ends. What will be the unique subgame perfect equilibrium? 1% Sandsynlighed for (0, 10000). Tænker man vil tilbyde alt til 0.99 · 10000.



Question 6 (Asymmetric Information). Suppose Alfred is considering offering Bob a wage contract but does not know Bob's current wage (i.e. outside option). With probability $p \in [0;1]$, Bob is a strong type two currently earns 300 kr/hour, and with probability $1-p$, Bob only earns 150 kr. The value of Bob's work to Alfred is 500 kr. regardless of the type of Bob. Alfred must make a wage offer to Bob without knowing Bob's current wage. Bob will accept anything that is higher than his current wage. The game is depicted in Figure 1 below.

Figure 1: Wage Bargaining with Asymmetric Information



(a) Explain what Alfred will offer when $p \to 1$ or $p \to 0$.

$p=1 \Rightarrow x=300$

$p=0 \Rightarrow x=150$

(b) Argue that Alfred will only offer either $x = 150$ or $x = 300$ to Bob.

over 300 giver aldrig mening. Under 150 heller aldrig.

(c) Show that Alfred will offer $x = 150$ if $p < 42.9\%$ and $x = 300$ if $p > 42.9\%$.

Hint: Alfred maximizes expected profits and earns $500 - x$ from any Bob type that accepts. So 350 "per Bob" if the low wage offer is given and 200 if the high offer is given.

(d) Explain intuitively why a *market breakdown* occurs.

Hint: The market "breaks down" means that there are efficient transactions that do not occur.

Markedet bryder sammen hvis Bob ikke ansættes til en løn på 500 eller under. Hvis alfred f.eks. tilbyder 149.

Hint: See this video from Game Theory 101, which covers the exact setup, albeit with different values.

C) E(Alfred) = (1-p) * (500-150): Hvis bob er svag, hvis stærk => 0

E(Alfred) = 200: Hvis bob er stærk, vil altså acceptere

Finder punkter hvor han er indifferent.

(1-p) * (500-150) =200 -> p = 3/7