Micro B: Problem Set 3

Solution to "Undercutting with Shoppers and Loyals"

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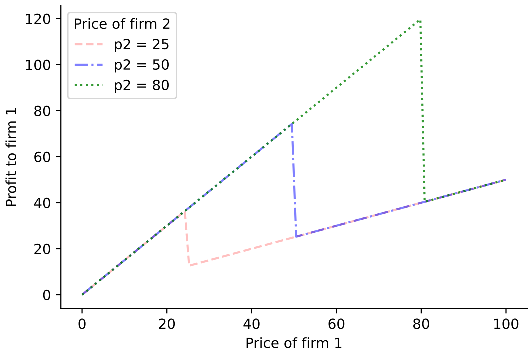
Question 1 (*Undercutting with Loyals and Shoppers*[1]).

(a) Figure 1 presents the profit function of one firm for three different values of the competitor's price. Explain the intuition for the shape and identify the best response to each case.

Figure 1: Profit to firm 1 for 3 values of competitor price



Answer: Since $p_i \mapsto s_i(p_i, p_j)$ is piecewise constant and $p_i \mapsto \pi_i(p_i, p_j)$ is linear in s_i, the profit function is piecewise linear. The slope changes when p_i moves past p_j, jumping down discontinuously, and proceeding at a lower slope. Exactly at the crossover point, the profit function takes the value precisely in between the two other lines (in the plot, it is shown as a vertical line). When p_j is higher, the crossover point occurs later, naturally.

(b) Argue intuitively (or prove mathematically) that there cannot exist a pure strategy Nash equilibrium.
Answer: Suppose the pair (p_i^*, p_j^*) comprises some Nash equilibrium. For simplicity, let us assume that the equilibrium is symmetric, $p_i^* = p_j^* \equiv p^*$ (the proof for asymmetric equilibria is more tedious). Initial profits are $\pi_i^* = pi_i(p^*, p^*) = M\frac{1}{2}p^*$.
The deviation: Consider the alternative $p_i' = p_j - \epsilon$, for some tiny $\epsilon > 0$. The deviation profit is

$$\pi_i' = pi_i(p_i', p^*) = M\frac{3}{4}(p^* - \epsilon).$$

[1]This example builds on an important and seminal paper by Varian (1980): "A Model of Sales", in *The American Economic Review*.

All that remains is to show that we can pick ϵ so small that $\pi_i' > \pi_i^*$. This occurs when,

$$M\frac{3}{4}(p^* - \epsilon) > M\frac{1}{2}p^*$$
$$\Leftrightarrow \frac{1}{4}p^* > \frac{3}{4}\epsilon$$
$$\Leftrightarrow \epsilon < \frac{1}{3}p^*.$$

This is possible so long as $p^* > 0$, so we still have to find a deviation when $p^* = 0$. In that case, however, the proposed equilibrium profits are zero, $pi^* = 0$, so any deviation to $p_i' > 0$ yields

$$\pi_i' = M\frac{1}{4}p_i' > 0.$$

Thus, there always exists a deviation to any proposed pure strategy equilibrium that would leave the firm better off: Either it undercuts or it hikes the price to exploit the loyal customers. Thus, no pure strategy equilibrium can exist. ■

Note that the same undercutting argument works in the pure Bertrand case. The big difference is that there now exists a deviation strategy when the opponent sets a price of zero.

To simplify the game, suppose $M = 2$ and let us discretize the action space and suppose that firms can only choose three prices:

$$S_i = \{38, 54, 80\}.$$

(c) Argue that this game can be represented by the following bimatrix

	$p_2 = 80$	$p_2 = 54$	$p_2 = 38$
$p_1 = 80$	$80, 80$	$40, 81$	$40, 57$
$p_1 = 54$	$81, 40$	$54, 54$	$27, 57$
$p_1 = 38$	$57, 40$	$57, 27$	$38, 38$

Answer: On the diagonal, $p_1 = p_2$, so $q_1 = q_2 = 1$. Then profits are $\pi_i(p, p) = p$ for all $p \in \{38, 54, 80\}$. On the off-diagonal, $p_1 \neq p_2$. The high price firm thus obtains $q = 2\frac{3}{4}$, while the low price firm gets $q = 2\frac{1}{4}$. For example, $\pi_1(38, 80) = 38 \cdot 2 \cdot \frac{3}{4} = 54$.

(d) Show that there is no Nash equilibria in pure strategies.
Answer: We mark the best responses for both players and note that none coincide.

	$p_2 = 80$	$p_2 = 54$	$p_2 = 38$
$p_1 = 80$	$80, 80$	$40, 81^*$	$40^*, 57$
$p_1 = 54$	$81^*, 40$	$54, 54$	$27, 57^*$
$p_1 = 38$	$57, 40^*$	$57^*, 27$	$38, 38$

(e) Confirm that the following strategy profile is a Nash equilibrium: each firm plays price 80 with probability 0.232, price 54 with probability 0.361, and price 38 with probability 0.407.
Answer: Denote the strategy vector by $\sigma = (0.232, 0.361, 0.407)'$, and let U_i be the 3×3 payoff matrix for player i. Then $EU_1 = U_1\sigma$ is the 3×1 vector of expected utilities for player 1, and $EU_2 = \sigma' U_2$ is the 1×3 vector of expected utilities for player 2. When we compute these, we get $EU_1 = (49.3, 49.3, 49.3) = EU_2'$. Since the expected utilities are identical for all the pure strategies, both players are indifferent and thus happy to choose (any) mixed strategy in response to σ. Thus, (σ, σ) is a Mixed Strategy Nash Equilibrium.