Micro B: Problem Set 6

Dynamic Games of Perfect Information*

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Question 1. Find SPNE in the following four games

Figure 1: Game A

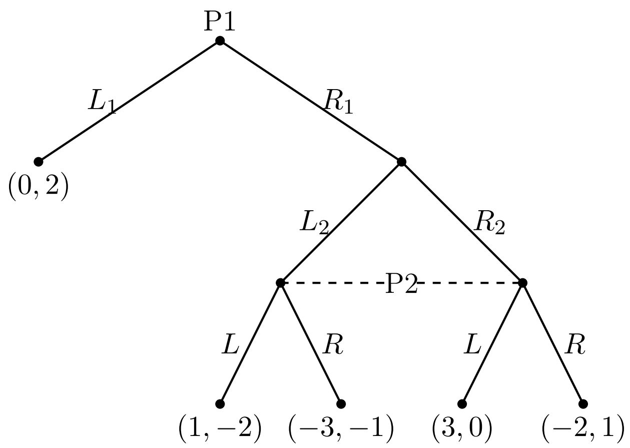
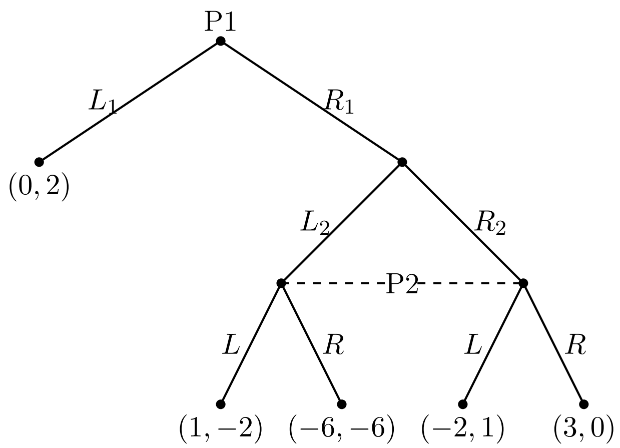


Figure 2: Game B



*For prior contributions, thanks to Jeppe Dinsen.

Figure 3: Game C

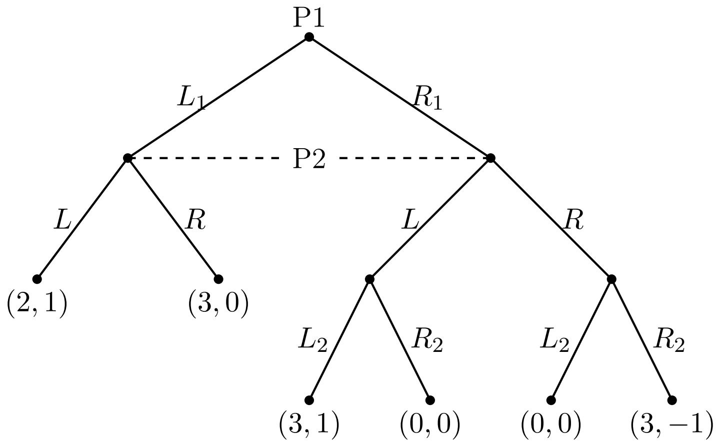
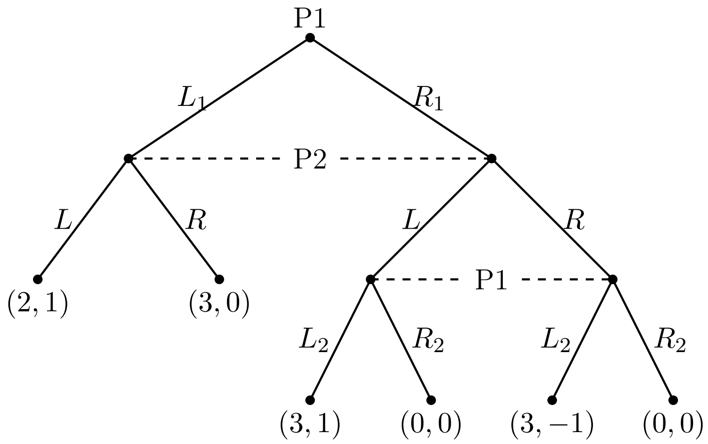


Figure 4: Game D



Question 2 (Mutually mutated seabass destruction game continued)**.** Go back to question 6 in Problem Set 5. Write up the game tree for the situation in part (c), where the choice to acquire the mutated seabass weapon is not observed. Find the SPNE. What has changed?

Question 3 (A two stage game with simultaneous moves)**.** On July 12, 2001, the presidents of Toyota and PSA Group, Fujio Cho and Jean-Martin Folz, decided to jointly develop a small city car. This project was called B-Zero. The outcome of this project were Toyota Aygo, Peugeot 107, and Citroen C1 which are essentially differently named versions of the same car. So the firms entered into a collusive agreement at the Research & Development (R&D) stage, but remained rivals in the final product market.
It is not surprising that the firms were not allowed to collude in the product market, as this would increase their monopoly power, which is costly for the consumers. But why was collusion allowed in the R&D market?
Below you are asked to show that if there are sufficient spillovers in R&D, collusion in R&D may be beneficial both for the firms and for the consumers.
Consider an industry consisting of two firms. They face the inverse demand function given by

$$P(Q) = 2 - Q \tag{1}$$

where $Q = q_1 + q_2$ is the total quantity produced. Before production, each firm can engage in research activities that lower the cost of production for the entire industry. More precisely, the marginal cost of each firm is identical, $c_1 = c_2 = c$, and it is a function of the total amount of research undertaken by the two firms, given by $(x_1 + x_2)$:

$$c = 1 - x_1 - x_2$$

Thus, each firm benefits from the research undertaken by the other firm. The R&D cost of x_i units of research to firm i is given by

$$x_i^2$$

The timing of the game is as follows: In the first stage the firms choose the levels of research x_1 and x_2. In the second stage, after observing x_1 and x_2, the firms simultaneously and independently (i.e., as in a standard Cournot game) decide on the amounts of output (q_1 and q_2).

Consider using CAS software (e.g. Maple, Mathematica, or SymPy) to simplify calculations in the following.

(a) Given the levels of research x_1, x_2, find the resulting levels of output ($q_1(x_1, x_2)$ and $q_2(x_1, x_2)$) in the second stage.

(b) Assume that the stage one decisions are made simultaneously and independently. That is, each firm i chooses x_i in order to maximize its own profit (foreseeing the outcome of stage two). Using your results from (a), find the levels of research and output in the subgame perfect Nash equilibrium: $x_1^*, x_2^*, q_1(x_1^*, x_2^*)$ and $q_2(x_1^*, x_2^*)$.

(c) Assume now that the firms collude in the first stage. That is, they choose x_1 and x_2 to maximize their joint profit while taking into account that q_1 and q_2 will be chosen simultaneously and independently in stage two. Find the resulting levels of research and output: $x_1^{**}, x_2^{**}, q_1(x_1^{**}, x_2^{**})$ and $q_2(x_1^{**}, x_2^{**})$.

(d) Based on your findings in (b) and (c), compare the outcomes in terms of consumer welfare [hint: it is enough to look at total output] and firms' profit [hint: no calculations are necessary]. Comment on the source of the difference.

Question 4. Consider the following 2×2 game where payoffs are monetary:

	L	R
T	$3, 3$	$0, 4$
B	$4, 0$	$1, 1$

Some game

Before this game is played, Player 1 can choose whether, after the game is played, players should keep their own payoffs or split the aggregate payoff evenly between them. Player 2 observes this choice.

a) Write down the game tree of this two-stage game: be careful to represent then simultaneous-move game in the second stage using information sets.

b) Find the Nash Equilibria.

c) Now suppose that Player 2 cannot observe Player 1's choice in the first stage. Draw the game tree (again using information sets) and find the Nash Equilibria.