Microeconomics B - Problem Set 2

Nash equilibria in pure strategies and oligopoly

1) Consider the following games and questions

a) Find the Nash equilibria (In pure strategies) in the following game.

	L	R
T	9, 9	0, 0
B	0, 0	8, 8

We plot in for the best responses (marked in the bimatrix with an $\underline{\text{underline}}$)

- If player 1 plays T, then player 2's best response is to play L.
- If player 1 plays B, then player 2's best response is to play R
- If player 2 plays L, then player 1's best response is to play T
- If player 2 plays R, then player 1's best response is to play B

$$
\begin{array}{c|c|c|}
 & L & R \\ \hline
T & 9,9 & 0,0 \\ \hline
B & 0,0 & 8,8 \\ \hline
\end{array}
\rightarrow
\begin{array}{c|c|c|}
 & L & R \\ \hline
T & 9,\underline{9} & 0,0 \\ \hline
B & 0,0 & 8,8 \\ \hline
\end{array}
\rightarrow
\begin{array}{c|c|c|}
 & L & R \\ \hline
T & 9,\underline{9} & 0,0 \\ \hline
B & 0,0 & 8,\underline{8} \\ \hline
\end{array}
\rightarrow
\begin{array}{c|c|c|}
 & L & R \\ \hline
T & \underline{9},\underline{9} & 0,0 \\ \hline
B & 0,0 & 8,\underline{8} \\ \hline
\end{array}
\rightarrow
\begin{array}{c|c|c|}
 & L & R \\ \hline
T & \underline{9},\underline{9} & 0,0 \\ \hline
B & 0,0 & \underline{8},\underline{8} \\ \hline
\end{array}
$$

such that there is two nash equilibria in pure strategies

$$\text{PSNE: } \{(T, L), (B, R)\}$$

b) Find all the Nash equilibria (In pure strategies) in the following game

	L	R
T	9, 9	0, 7
B	7, 0	8, 8

We plot in for the best responses in the bi-matrix

- If player 1 plays T, then player 2's best response is to play L.
- If player 1 plays B, then player 2's best response is to play R
- If player 2 plays L, then player 1's best response is to play T
- If player 2 plays R, then player 1's best response is to play B

$$
\begin{array}{c|c|c|}
 & L & R \\ \hline
T & 9,9 & 0,7 \\ \hline
B & 7,0 & 8,8 \\ \hline
\end{array}
\rightarrow
\begin{array}{c|c|c|}
 & L & R \\ \hline
T & 9,\underline{9} & 0,7 \\ \hline
B & 7,0 & 8,8 \\ \hline
\end{array}
\rightarrow
\begin{array}{c|c|c|}
 & L & R \\ \hline
T & 9,\underline{9} & 0,7 \\ \hline
B & 7,0 & 8,\underline{8} \\ \hline
\end{array}
\rightarrow
\begin{array}{c|c|c|}
 & L & R \\ \hline
T & \underline{9},\underline{9} & 0,7 \\ \hline
B & 7,0 & 8,\underline{8} \\ \hline
\end{array}
\rightarrow
\begin{array}{c|c|c|}
 & L & R \\ \hline
T & \underline{9},\underline{9} & 0,7 \\ \hline
B & 7,0 & \underline{8},\underline{8} \\ \hline
\end{array}
$$

such that there is two nash equilibria in pure strategies

$$\text{PSNE: } \{(T, L), (B, R)\}$$

c) In each of the two games above, which equilibrium do you find most reasonable as a prediction? Why? What is the difference between the two games?

Lets compare the two games

A	L	R
T	$\underline{9},\underline{9}$	$0,0$
B	$0,0$	$\underline{8},\underline{8}$

B	L	R
T	$\underline{9},\underline{9}$	$0,7$
B	$7,0$	$\underline{8},\underline{8}$

Game A:
In game A, we see a Nash equilibrium in (T, L), that pareto dominates the other Nash equilibrium (B, R), because both players payoff in the first Nash equilibria is higher for both players.

- Therefor the Nash equilibria (T, L) seems most likely

Game B:
In the B game, we see that the nash equilibrium (T, L) pareto dominates the other nash equilibrium (B, R), because payoff to both players is larger in the fist NE.

- Therefor the Nash equilibria (T, L) seems most likely

but in game B, the other Nash equilibria is now a more safe bet, because if the other player chooses to deviate you will get 7 compared to 0 in game A.

Difference between games:
The difference in the two games, is that in game B one of the Nash equilibria will be a more safe bet than the other, because if the other player deviates you will not lose all but only some payoff.

2) Find all pure strategy Nash equilibria in the following game

	a	b	c
A	$7,7$	$3,0$	$1,6$
B	$2,8$	$5,4$	$9,3$
C	$3,0$	$5,4$	$2,1$

To find the nash equilibria in pure strategies, lets plot in for the best responses in the bimatrix.

- If player 1 plays A then player 2's best response is to play a
- If player 1 plays B then player 2's best response is to play a
- If player 1 plays C then player 2's best response is to play b

- If player 2 plays a then player 2's best response is to play A
- If player 2 plays b then player 2's best response is to play either B or C
- If player 2 plays c then player 2's best response is to play B

	a	b	c
A	$7,7$	$3,0$	$1,6$
B	$2,8$	$5,4$	$9,3$
C	$3,0$	$5,4$	$2,1$

→

	a	b	c
A	$7,\underline{7}$	$3,0$	$1,6$
B	$2,8$	$5,4$	$9,3$
C	$3,0$	$5,4$	$2,1$

→

	a	b	c
A	$7,\underline{7}$	$3,0$	$1,6$
B	$2,\underline{8}$	$5,4$	$9,3$
C	$3,0$	$5,4$	$2,1$

→

	a	b	c
A	$7,\underline{7}$	$3,0$	$1,6$
B	$2,\underline{8}$	$5,4$	$9,3$
C	$3,0$	$5,\underline{4}$	$2,1$

→

	a	b	c
A	$\underline{7},\underline{7}$	$3,0$	$1,6$
B	$2,\underline{8}$	$5,4$	$9,3$
C	$3,0$	$5,\underline{4}$	$2,1$

→

	a	b	c
A	$\underline{7},\underline{7}$	$3,0$	$1,6$
B	$2,\underline{8}$	$\underline{5},4$	$9,3$
C	$3,0$	$\underline{5},\underline{4}$	$2,1$

→

	a	b	c
A	$\underline{7},\underline{7}$	$3,0$	$1,6$
B	$2,\underline{8}$	$\underline{5},4$	$\underline{9},3$
C	$3,0$	$\underline{5},\underline{4}$	$2,1$

such that there are two nash equilibria in pure strategies in the game

$$\text{PSNE: } \{(A,a),(C,b)\}$$

3) See Python solution

4) See Python solution

5) There are two bakeries in the same village. Every morning, they simultaneously decide how many breads to produce. Denote the quantities they produce by q_1 and q_2. The price for which they can sell the bread is a function of the overall quantity, such that $p = 11 - (q_1 + q_2)$. The cost of producing one bread is 2.

a) Compute the quantities in the Cournot equilibrium, i.e., the Nash Equilibrium of the game where the firms simultaneously choose quantities.

Instead of using an inverse demand function $p = 11 - Q$, lets instead change it to $p = a - Q$ and lets assume that the marginal costs are just c.

How do we solve a cournot game?

i) Start by finding best response function

ii) Find the firms supply, by maximizing with the other firms best response

iii) Set supply and demand equal to find price

Firm i will maximize his profit function

$$\begin{aligned} \max_{q_i} \pi_i &= p \cdot q_i - c \cdot q_i \\ &= (a - (q_i + q_j)) \cdot q_i - c \cdot q_i \end{aligned} \tag{1}$$

taking the first order condition

$$\begin{aligned} \frac{\partial \pi_i}{\partial q_i} &= -1 \cdot q_i + (a - q_i - q_j) \cdot 1 - c = 0 \\ &\Leftrightarrow -2q_i + a - q_j - c = 0 \\ &\Leftrightarrow 2q_i = a - q_j - c \\ &\Leftrightarrow q_i = \frac{a - q_j - c}{2} \equiv R_{q_i}(q_j) \end{aligned} \tag{2}$$

because of symmetry, between the two firms the best response function for firm j is identical

$$\begin{aligned}\frac{\partial \pi_j}{\partial q_j} &= -1 \cdot q_i + (a - q_i - q_j) \cdot 1 - c = 0 \\ &\Leftrightarrow -2q_j + a - q_i - c = 0 \\ &\Leftrightarrow 2q_j = a - q_i - c \\ &\Leftrightarrow q_j = \frac{a - q_i - c}{2} \equiv R_{q_j}(q_i)\end{aligned} \tag{3}$$

now we need to find what is optimal compared to what the other firms best response. Inserting firm j's best response into the first order condition for q_i, and isolate for q_i

$$\begin{aligned}q_i = \frac{a - q_j - c}{2} &\Rightarrow q_i = \frac{a - R_{q_j}(q_i) - c}{2} \\ &\Rightarrow q_i = \frac{a - \frac{a - q_i - c}{2} - c}{2} \\ &\Leftrightarrow 2q_i = a - \frac{a - q_i - c}{2} - c \\ &\Leftrightarrow 4q_i = 2a - (a - q_i - c) - 2c \\ &\Leftrightarrow 4q_i = a + q_i - c \\ &\Leftrightarrow 3q_i = a - c \\ &\Leftrightarrow q_i^* = \frac{a - c}{3}\end{aligned} \tag{4}$$

which is the optimal level of q_i. Now firm j is exactly identical, so we will have

$$q_j^* = \frac{a - c}{3} \tag{5}$$

The method used here is the way to solve the problem, even if the two firms were not identical and symmetric, but because the two firms are identical, can use a trick to solve the question faster.

We have assumed two firms, who are exactly identical. This implies that in optimum the two firms must produce the same amount, therefor we can set $q_i = q_i$ in one of the best response function, to find the optimal quantity.

$$\begin{aligned}R_{q_i}(q_j) = q_i &= \frac{a - \overbrace{q_i}^{q_j} - c}{2} \\ &\Leftrightarrow 2q_i = a - q_i - c \\ &\Leftrightarrow 3q_i = a - c \\ &\Leftrightarrow q_i = \frac{a - c}{3}\end{aligned}$$

now we can insert the two supplies and find the price in equilibrium

$$\begin{aligned} p(q_i^*, q_j^*) &= a - \left(q_i^* + q_j^*\right) \\ &= a - \left(\frac{a-c}{3} + \frac{a-c}{3}\right) \\ &= a - \frac{a-c}{3} - \frac{a-c}{3} \\ &= \frac{3a}{3} - \frac{a-c}{3} - \frac{a-c}{3} \\ &= \frac{3a - a + c - a + c}{3} \\ &= \frac{a+2c}{3} \end{aligned}$$

and we can find the profit for firm i

$$\begin{aligned} \pi_i^* &= p(q_i^*, q_j^*) \cdot q_i^* - c \cdot q_i^* \\ &= \frac{a+2c}{3} \cdot \frac{a-c}{3} - c \cdot \frac{a-c}{3} \\ &= \frac{a^2 - ac + 2ac - 2c^2}{9} - \frac{ac + c^2}{3} \\ &= \frac{a^2 + ac - 2c^2}{9} - \frac{3ac + 3c^2}{9} \\ &= \frac{a^2 + ac - 2c^2 - 3ac + 3c^2}{9} \\ &= \frac{a^2 - 2ac + c^2}{9} \\ &= \frac{(a-c)^2}{9} \end{aligned}$$

Lets now insert the information, about a and c from the problem to find exact values

$$\begin{aligned} q_i^* &= \frac{a-c}{3} = \frac{11-2}{3} = \frac{9}{3} = 3 \\ q_j^* &= \frac{a-c}{3} = \frac{11-2}{3} = \frac{9}{3} = 3 \\ p(q_i^*, q_j^*) &= \frac{a+2c}{3} = \frac{11+2\cdot 2}{3} = \frac{11+4}{3} = \frac{15}{3} = 5 \\ \pi_i^* &= \frac{(a-c)^2}{9} = \frac{(11-2)^2}{9} = \frac{9^2}{9} = 9 \end{aligned}$$

b) Draw a diagram of the best response functions, and check whether they really intersect in the Nash Equilibrium.

Now we need to draw the two best response functions. We need to draw them in a (q_i, q_j) diagram, which implies that we need to have best response function in in terms of q_i as the explanaroty variable.

We can directly draw the best response function for firm j, but for firm i, we need to isolate for q_j first

$$\begin{aligned} R_{q_i}\left(q_j\right) = q_i &= \frac{a - q_j - c}{2} \\ \Leftrightarrow 2q_i &= a - q_j - c \\ \Leftrightarrow q_j &= a - c - 2q_i \end{aligned} \tag{6}$$

and

$$R_{q_j}(q_i) = q_j = \frac{a - q_i - c}{2} \tag{7}$$

such that if we illustrate the best responses as.

We have two function which we need to draw

$$R_{q_i}\left(q_j\right) = q_j\left(q_i\right) = a - c - 2q_i$$
$$R_{q_j}\left(q_i\right) = q_j\left(q_i\right) = \frac{a - q_i - c}{2}$$

Best Response firm i

The best response function with q_j as a function of q_i for firm i is

$$R_{q_i}\left(q_j\right) = q_j\left(q_i\right) = a - c - 2q_i$$

the intersect on the 2nd axis is

$$\begin{aligned} R_{q_i}\left(q_j\right) = q_j\left(0\right) &= a - c - 2 \cdot 0 \\ \Leftrightarrow &= a - c \end{aligned}$$

and the intercept with the first axis

$$\begin{aligned} 0 = a - c - 2q_i &\Leftrightarrow 2q_i = a - c \\ &\Leftrightarrow q_i = \frac{a-c}{2} \end{aligned}$$

such that we can draw it

Best Response firm j

The best response function with q_j as a function of q_i for firm j is

$$R_{q_j}\left(q_i\right) = q_j\left(q_i\right) = \frac{a - q_i - c}{2}$$

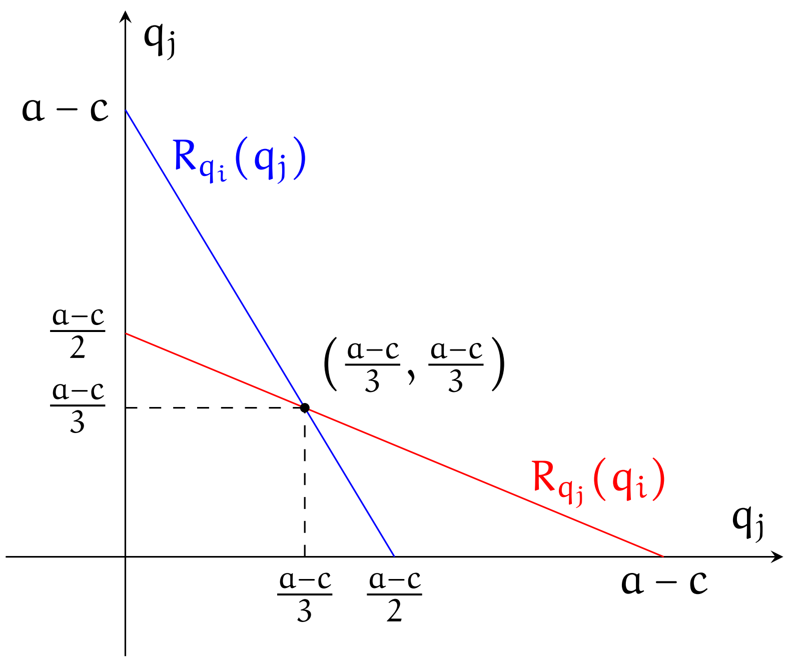
The intercept with the 2nd axis is

$$\begin{aligned} R_{q_j}\left(0\right) = q_j\left(0\right) &= \frac{a - 0 - c}{2} \\ &= \frac{a-c}{2} \end{aligned}$$

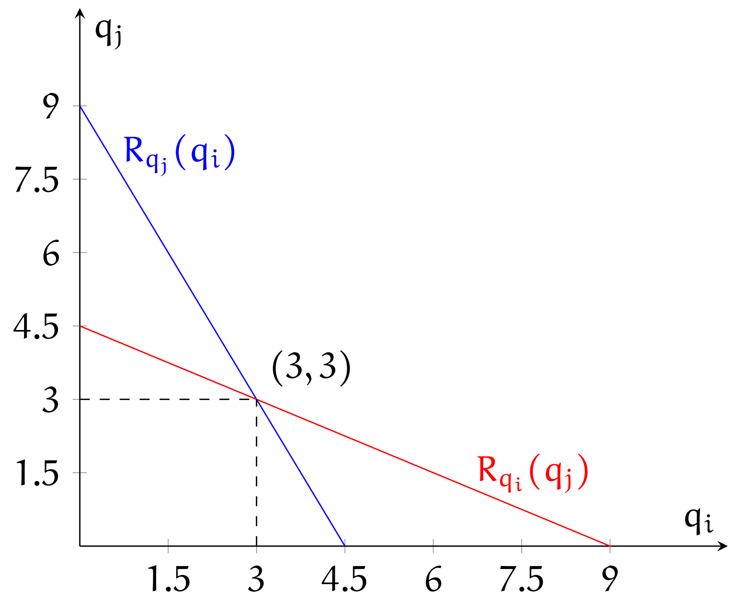
and the intercept with the first axis

$$\begin{aligned} 0 = \frac{a - q_i - c}{2} &\Leftrightarrow 0 = a - q_i - c \\ &\Leftrightarrow q_i = a - c \end{aligned}$$

such that we can draw it



And we can insert our information of $a = 11$ and $c = 2$ and draw the figure again



6) Find the nash equilibria of the following game

	a	b
A	$100, 100$	$1, 99$
B	$99, 1$	$0, 0$

Now add 2 to each player's payoff from the action B (for each action of the other player):

	a	b
A	$100, 100$	$1, 101$
B	$101, 1$	$2, 2$

Find the Nash equilibrium of this game and comment. What has changed?

Lets start by looking at game 1.

We see that for player 1, the strategy A strictly dominates strategy B, and we can use eliminate strategy B.

	a	b
A	$100, 100$	$1, 99$

now we can remove strategy b for player 2

	a
A	$100, 100$

and we have a unique solution that survives IESDS, which is therefor also the unique nash equilibria in both pure and mixed strategies

$$NE = (A, a)$$

Lets look at game 2

	a	b
A	$100, 100$	$1, 101$
B	$101, 1$	$2, 2$

In the second game, we don't have any strictly dominating strategies, so lets plug in for the best responses instead

- If player 1 plays A then player 2's optimal choice is to play b
- If player 1 plays B then player 2's optimal choice is to play b
- If player 2 plays a then player 1's optimal choice is to play B
- If player 2 plays a then player 1's optimal choice is to play B

	a	b
A	$100, 100$	$1, 101$
B	$101, 1$	$2, 2$

$\rightarrow$

	a	b
A	$100, 100$	$1, \underline{101}$
B	$101, 1$	$2, 2$

$\rightarrow$

	a	b
A	$100, 100$	$1, \underline{101}$
B	$101, 1$	$2, \underline{2}$

$\rightarrow$

	a	b
A	$100, 100$	$1, \underline{101}$
B	$\underline{101}, 1$	$2, \underline{2}$

$\rightarrow$

	a	b
A	$100, 100$	$1, \underline{101}$
B	$\underline{101}, 1$	$\underline{2}, \underline{2}$

such that we have one nash equilibria in

$$NE = (B, b)$$

What has changed?

A small change in the payoffs, has changed the results from an efficient results $(NE = (A, A))$ to an inefficient equilibrium $(NE = (B, B))$.