Micro B: Problem Set 4

Dynamic Games of Perfect Information*

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Spring 2024

Day 1/2

Question 1 (MSNE)**.** Find all equilibria (pure and mixed) in the following games. Plot the best response functions. *Hint: you can solve the problem analytically or using Python as you please.*

	L	R
T	3,3	0,0
B	0,0	4,4

A

	L	R
T	1,1	0,0
B	1,0	2,1

B

Question 2 (Entry deterrence)**.** Consider the following dynamic game: firm 1 owns a shop in town A. Firm 2 decides whether to enter the market in town A. If firm 2 enters, firm 1 chooses whether to fight or accommodate the entrant. If firm 2 does not enter, firm 1 receives a profit of 2 and firm 2 gets 0. If firm 2 enters and firm 1 accommodates, they share the market and each of them receives a profit of 1. If firm 2 enters and firm 1 decides to fight, firm 2 suffers a loss of 1 (so that the payoff is -1), but fighting is costly for firm 1, lowering its payoff to 0.

(a) Draw the game tree

(b) Solve the game by backwards induction

Question 3 (Generalized Battle of the Sexes)**.** Consider the following game, where $N > 1$:

	$C1$	$C2$
$C1$	$N, 1$	$0, 0$
$C2$	$0, 0$	$1, N$

Battle of the sexes ($N > 1$)

(a) How can you interpret the parameter N?

(b) Solve for the mixed strategy Nash equilibrium. When N becomes very large, what happens to the probability of successful coordination?

Question 4 (Palacios-Heurta (2003): "Professionals Play Minimax" *Review of Economic Studies*)**.** With data from 1417 FIFA games, Ignacio Palacios-Heurta estimates the following payoffs in soccer penalty kicks:

*For prior contributions, thanks to Jeppe Dinsen.

	L	R
L	0.58,0.42	0.95, 0.05
R	0.93,0.07	0.70, 0.30

Penalty Kicks: 1 = kicker, 2 = goalie

Furthermore, the observed frequencies of play are:

$$\underbrace{(\sigma_{1L}, \sigma_{1R}) = (0.40, 0.60)}_{\text{kicker}}, \quad \underbrace{(\sigma_{2L}, \sigma_{2R}) = (0.42, 0.58)}_{\text{goalie}}.$$

(a) Show that the MSNE of the game is

$$MSNE = \{(\underbrace{(0.38, 0.62)}_{\text{kicker}}, \underbrace{(0.42, 0.58)}_{\text{goalie}})\}.$$

(b) What would be your advice to players?

Question 5 (MSNE)**.** *North-Atlantic, 1943:* An allied convoy, counting 100 ships, is heading east and it can choose between a northern route where icebergs are known to be numerous or a more southern route. The northern route is dangerous - because of the icebergs - and it is estimated that 6 ships will get lost due to icebergs. Below the surface, the wolf-pack lures. If the u-boats catch the convoy on the southern route, it is a field day, and 40 ships from the convoy are estimated to get lost. If the u-boats catch the convoy on the northern route, they do not have as much time hunting down the convoy - due to petrol shortages - and they are only expected to be able to sink 20 ships from the convoy. The wolf-pack does not have time to check both locations, north and south. Each headquarter (allied or nazi) has to decide whether to go north or south. Unfortunately, there is no radar etc, so one cannot observe the move of the enemy before taking a decision. Each headquarter has a simple payoff function. For the allied headquarter it equals the number of ships making it across the Atlantic. For the nazi headquarter payoff equals the number of ships lost by the allies.

(a) Write down this strategic situation in a bi-matrix

(b) Find the Nash Equilibrium (equilibria?)

(c) In equilibrium, what is the expected number of ships that make it across the Atlantic?

Day 2/2

Question 6 (Stackelberg)**.** Two neighbors, $N = \{1, 2\}$ are building a common playground for their children. The time spent on the project by neighbor i is x_i. Each neighbor chooses from the action set $S_i = [0; \infty)$. The resulting quality of the playground is

$$q(x_1, x_2) = x_1 + x_2 - x_1 x_2$$

Spending time on the project is costly. More precisely, the cost function of the neighbors are:

$$C_i(x_i) = x_i^2, \quad i = 1, 2.$$

The payoff of neighbor i, U_i is equal to the quality of the playground minus his cost.

(a) Suppose the neighbors decide how much time to spend on the project simultaneously and independently. Derive the best response functions. Find the Nash equilibrium of this game.

(b) Suppose now that the game is played in two stages. First, neighbor 1 decides how much time to spend on the project. Neighbor 2 observes this and then chooses how much time to put in himself. Solve the game by backwards induction.

(c) Compare the games from (a) and (b) with respect to the payoff that each neighbor obtains. Is it best to move first or last? Give an intuitive explanation.

Question 7 (Dynamic game of imperfect information)**.** Consider the following 2x2 bimatrix game where payoffs are monetary

	L	R
T	$3, 3$	$0, 4$
B	$4, 0$	$1, 1$

Before this game is played, Player 1 can choose whether, after the game is played, players should keep their own payoffs or split the aggregate payoff evenly between them.

(a) Draw the game tree of this two-stage game (assuming that Players 1's choice of whether to split payoffs is revealed to Player 2 before the second stage).

(b) Solve by backwards induction.

If Time Permits

Question 8 (MSNE in a bimatrix game)**.** Thomas and Alice want to meet on a Friday night. There are two bars in their home town: "The Focal Point" and "The Other Place". They have to decide independently where they go. If they meet in the same bar, they both get utility of 1. If they end up in different bars, they get utility of 0.

(a) Find all equilibria (pure and mixed). Which equilibrium do you consider the most realistic? Where would you go if you were one of them?

(b) Now assume that Thomas wants to meet Alice, but Alice does not want to meet Thomas. Thomas gets a payoff of 1 if he meets Alice, and -1 otherwise. Alice gets a payoff of -1 for meeting Thomas, and 1 otherwise. Find all equilibria (pure and mixed).

(c) Now assume again that Thomas and Alice both want to meet (so that payoffs are as in part (a)), but now there are N bars in town, where N can be very large. Show that there are $2^N - 1$ equilibria (pure and mixed). Say that the bars have names: "The First Bar in Town", "The Second Bar in Town", and so on. Which equilibrium is the most realistic?

Question 9 (Free-rider problem)**.** As in Problem Set 2, there are $n \geq 2$ people observing someone trying to steal a parked bike. Each of the witnesses would like the thief to be stopped, but prefers not to do it him/herself (because it is unpleasant and perhaps even dangerous). More precisely, if the thief is stopped by someone else, each of the witnesses gets a utility of $v > 0$. Every person who stops the thief gets a utility of $v - c > 0$, where c is the cost of interaction with the thief. Finally, if nobody stops the thief and the bike gets stolen, every witness gets a utility of 0. The witnesses decide whether or not to stop the thief simultaneously and independently.

1. Solve for a symmetric mixed strategy equilibrium of this game, where each witness stops the thief with probability $p \in [0, 1]$.

2. Discuss what happens to p as the number of witness becomes very large. What happens then to the probability that the thief will get stopped? What is the intuition for this result?