Micro B: Problem Set 2

Nash equilibria in pure strategies and oligopoly*

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Spring 2024

Everywhere in this problem set, the term Nash Equilibrium refers to the equilibrium in pure strategies.

Question 1 (*Nash equilibria*)**.** Consider the following two games

Game A

	L	R
T	9,9	0,0
B	0,0	7,7

Game B

	L	R
T	9,9	0,x
B	x,0	7,7

(a) Find all the Nash equilibria (in pure strategies) in the two games for $x \in \{6, 8, 10\}$

(b) In both games above, which equilibrium do you find most reasonable as a prediction? Why? What is the difference between the two games?

Question 2. Find all Nash equilibria in the following game

	a	b	c
A	$7,7$	$3,0$	$1,6$
B	$2,8$	$5,4$	$9,3$
C	$3,0$	$5,4$	$2,1$

Question 3 (*Brute force Nash equilibrium solver*)**.** Code up a *Brute Force* Nash equilibrium solver. Proceed as follows:

(1) First code up a function that finds all best response actions to a given competitor strategy:

$$BR_i(s_{-i}) = \arg\max_{s_i \in S_i} u_i(s_i, s_{-i}).$$

Note that $BR_i(s_{-i})$ is a set with $BR_i(s_{-i}) \subset S_i$. There is at least one element in $BR_i(s_{-i})$ and at most $|S_i|$ (the number of strategies for player i).
Hint:

(1) Using that function, find the two sets, $BR_1(s_2)$ and $BR_2(s_1)$ for all s_1 and s_2 in S. Then, the Nash equilibria are found as the "intersection" points:

$$\text{NE} = \{(s_1, s_2) \in S | s_1 = BR_1(s_2) \wedge s_2 = BR_2(s_2)\}$$

(1) Use your solver on the 2-player 3×3 game from Question 2.

*For prior contributions, thanks to Jeppe Dinsen.

Question 4 (*(Cournot in Python)*)**.** Consider the following Cournot game from Tadelis (2013, p. 65): two firms, $N = \{1, 2\}$, set prices $q_i \in S_i = [0; 90]$. Their payoff is given by the profit

$$\pi_i(q_i, q_j) = q_i p(q_i + q_j) - 10q_i,$$

where the market demand curve is $p(Q) = 100 - Q$.

(a) Solve for the Nash equilibria by brute force using our algorithm from Question 3.

(b) Solve the game by IESDS on a grid. Use e.g. `np.linspace(0,90,100)`.

(c) Solve the game by using the analytic best response functions:

  (i) First take the first-order conditions, which give

$$q_i^* = \frac{90 - q_j}{2}.$$

  (ii) Solve the two equations in two unknowns:

$$q_1^* = \frac{90 - q_2^*}{2}, \quad (1)$$
$$q_2^* = \frac{90 - q_1^*}{2}. \quad (2)$$

  This can be done either by hand or using `scipy.optimize`'s `fsolve`, which we will later have to use in settings where we cannot solve the best response equations in close form.

(d) Plot the solutions together with the best response functions *(this is done for you)* and wonder at the beauty.

Question 5 (Cournot)**.** There are two bakeries in the same village. Every morning, they simultaneously decide how many breads to produce. Denote the quantities they produce by q_1 and q_2. The price for which they can sell the bread is a function of the overall quantity, such that $p = a - (q_1 + q_2)$. The cost of producing one bread is c, and profits are given by

$$\pi_i = q_i(p - c), \quad i = 1, 2.$$

(a) Compute the quantities in the Cournot equilibrium, i.e., the Nash Equilibrium of the game where the firms simultaneously choose quantities.

(b) Draw a diagram of the best response functions, and check whether they really intersect in the Nash Equilibrium.

1 If time permits

Question 6 (*A tipping point*)**.**

(a) Find the Nash equilibria of the following game

	a	b
A	100,100	1,99
B	99,1	0,0

(b) Now add 2 to each player's payoff from the action B (for each action of the other player). Find the Nash equilibrium of this game and comment on what changed.[1]

	a	b
A	100,100	1,101
B	101,1	0,0

Question 7 (Weak Domination)**.** This exercise illustrates a simple point not covered in class: Eliminating weakly dominated strategies can result in a Nash equilibrium being removed.

Definition 1 (Weakly Dominant Strategy)**.** Let $s_i, s_i' \in S_i$ be two strategies for player i. We say that s_i *weakly dominates* s_i' if

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) \quad \forall s_{-i} \in S_{-i}.$$

That is, regardless of what opponents do, s_i' cannot make player i better off than playing s_i.

(a) Is the iterated elimination of *weakly* dominated pure strategies in finite games independent of the order of elimination? For example, you can use the game below to make your argument

	L	R
T	0,1	0,0
B	0,0	1,0

Game A

(b) Find the Nash Equilibria of the above game. Show that it is possible to eliminate Nash Equilibria by iterated elimination of weakly dominated pure strategies.

[1] Pretend this is an exam question: Discuss what constitutes a too brief answer, and what is too much. When you are asked to comment like this, there is typically some key insight you should realize.