Micro B: Problem Set 8

Bargaining

Anders Munk-Nielsen

Spring 2023

Question 1 (Finite horizon bargaining)**.** Consider a sequential Rubinstein bargaining game with T periods. The periods take turn at proposing an offer, $s_t \in [0; 1]$. If the proposal is accepted, the payoffs are $(s_t, 1 - s_t)$, and if rejected, the game proceeds to the next stage. If the final offer is rejected, the game ends with payoffs $(0, 0)$. Both players discount future payoffs with common discount factor $\delta \in [0; 1]$.

(a) Illustrate the game graphically (in a tree).

(b) Is there a first-mover advantage? Does your answer depend on the value of T?

Question 2 (Outside option)**.** Consider the bargaining game from question 1 with the single exception that if the final offer is rejected, the payoffs are $(0, x)$: that is, player 1 still gets zero, but player 2 gets x.

(a) Focusing on $T = 1$, describe what happens if $x < 0$ or if $x > 1$. Then, in the following, assume $0 < x < 1$.

(b) Suppose $T = 1$. Find the Nash equilibrium of the game.

(c) Do the same for $T = 2$.

(d) And for $T = 3$.

Question 3 (Asymmetric Patience)**.** Consider the Rubinstein infinite-horizon bargaining game and assume that player 1 has discount factor δ_1 and player 2 has discount factor δ_2, where $\delta_1, \delta_2 \in [0; 1)$. The backwards induction outcome of the game is that player 1 offers $(x^*, 1 - x^*)$, and player 2 offers $(1 - y^*, y^*)$, where x^* and y^* are the unique solutions to the equations.

$$\delta_1 x^* = 1 - y^* \wedge \delta_2 y^* = 1 - x^*. \tag{1}$$

(a) Explain the intuition behind (1).

(b) Show that the unique solution to (1) is

$$x^* = \frac{1 - \delta_1}{1 - \delta_1 \delta_2}, \quad y^* = \frac{1 - \delta_2}{1 - \delta_1 \delta_2}.$$

(c) Show that the equilibrium *outcome* of the game where player 1 gives the first offer is

$$\left(\frac{1 - \delta_2}{1 - \delta_1 \delta_2}, \frac{\delta_2 (1 - \delta_1)}{1 - \delta_1 \delta_2} \right) \tag{2}$$

Hint: Check out https://youtu.be/z0NHnrdjeeA.

Question 4 (Asymmetric Patience in Python)**.** Code up Equation (2) in Python.

(a) Suppose two countries borrow at the interest rates $r_1 = 5\%$ and $r_2 = 10\%$ and are bargaining in infinite horizon with alternating offers over the division of some surplus between the countries. What is the equilibrium?
Hint: The discount factor is $\delta = \frac{1}{1+r}$.

(b) The yields on 10 year government bonds are (as of May 2022)

- China: 2.82%,
- Vietnam: 3.37%,
- Namibia: 12.11%
- Germany: 0.94%,
- Denmark: 1.32%,
- Poland: 7.05%.

Compute the equilibrium bargaining outcome between each of the pairs of countries.

(c) Explain in your own words how patience help "the rich get richer."

Question 5 (Realistic Values)**.** You are at the Roskilde Festival and you have spotted a great restaurant that has no app. The restaurant is currently selling $Q = 100$ meals per hour at $p = 100$ DKK. With the app live, they will instead sell $Q' = 200$ meals.

(a) Set up the game as a bargaining game and draw the extensive form graph.

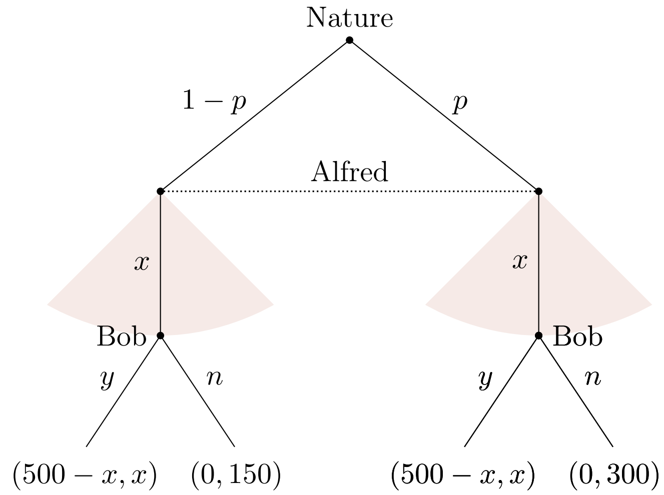
(b) What "take-it-or-leave-it" offer should you make to the restaurant?

(c) If the restaurant gets to make a counter-offer after which the game ends, what is the subgame perfect equilibrium?

(d) Suppose both parties get to make sequential counteroffers (starting with you) but each period there is a 1% probability that the game ends. What will be the unique subgame perfect equilibrium?

Question 6 (Asymmetric Information)**.** Suppose Alfred is considering offering Bob a wage contract but does not know Bob's current wage (i.e. outside option). With probability $p \in [0; 1]$, Bob is a strong type two currently earns 300 kr/hour, and with probability $1-p$, Bob only earns 150 kr. The value of Bob's work to Alfred is 500 kr. regardless of the type of Bob. Alfred must make a wage offer to Bob without knowing Bob's current wage. Bob will accept anything that is higher than his current wage. The game is depicted in Figure 1 below.

Figure 1: Wage Bargaining with Asymmetric Information



(a) Explain what Alfred will offer when $p \to 1$ or $p \to 0$.

(b) Argue that Alfred will only offer either $x = 150$ or $x = 300$ to Bob.

(c) Show that Alfred will offer $x = 150$ if $p < 42.9\%$ and $x = 300$ if $p > 42.9\%$.
Hint: Alfred maximizes expected profits and earns $500 - x$ from any Bob type that accepts. So 350 "per Bob" if the low wage offer is given and 200 if the high offer is given.

(d) Explain intuitively why a *market breakdown* occurs.
Hint: The market "breaks down" means that there are efficient transactions that do not occur.

Hint: See this video from Game Theory 101, which covers the exact setup, albeit with different values.