Microeconomics B Problem Set 1

Normal form, bi-matrix, dominated strategies and iterated dominance solvability, and some Nash Equilibrium

1. Solve this game by iterative elimination of strictly dominated strategies

	t_1	t_2	t_3
s_1	$5, 0$	$3, 3$	$1, 1$
s_2	$3, 4$	$2, 2$	$3, 1$
s_3	$2, 2$	$1, 1$	$0, 5$

To solve this by IESDS (iterateive elimination of strictly dominated strategies) lets, start by rewriting the bimatrix, with a little bit more information.

	t_1	t_2	t_3
s_1	$5^{s_1}, 0^{t_1}$	$3^{s_1}, 3^{t_1}$	$1^{s_1}, 1^{t_1}$
s_2	$3^{s_2}, 4^{t_2}$	$2^{s_2}, 2^{t_2}$	$3^{s_2}, 1^{t_2}$
s_3	$2^{s_3}, 2^{t_3}$	$1^{s_3}, 0^{t_3}$	$0^{s_3}, 5^{t_3}$

just to specify where each of the numbers comes from.

Lets start by looking at player 1, if any of his strategies strictly dominates another strategy.

Player 1's strategy s_1 strictly dominates s_2 if he will recieve a higher payoff no matter player 2's choice. That is if all of the below is true at the same time

Strategy s^2		Strategy s^1	
3^{s_2}	$<$	5^{s_1}	True
2^{s_2}	$<$	3^{s_1}	True
3^{s_2}	$<$	1^{s_1}	Not True

is is clear that 1 is not larger than 3. Therefor strategy s_1 does not strictly dominate strategy s_2.

It is also clear that 5>3, and therefor we a strictly dominates strategy the other way around i.e. we don't have that s_2 strictly dominates s_1.

Lets see if strategy s_1 strictly dominates strategy s_3, which it does if all of the below is true

Strategy s^3		Strategy s^1	
2^{s_3}	$<$	5^{s_1}	True
1^{s_3}	$<$	3^{s_1}	True
0^{s_3}	$<$	1^{s_1}	True

all of these inequalities is true, therefor player 1's strategy s_1 strictly dominated strategy s_3.

We have that for player 1 strategy s_1 strictly dominates strategy s_3, and because of this player will will never pålay s_3, because he could insted play s_1 and gain a higher payoff no matter what player 2 does. Therefor we can eliminate strategy s_3 from the game.

The game therefor reduces to

	t_1	t_2	t_3
s_1	$5^{s_1}, 0^{t_1}$	$3^{s_1}, 3^{t_1}$	$1^{s_1}, 1^{t_1}$
s_2	$3^{s_2}, 4^{t_2}$	$2^{s_2}, 2^{t_2}$	$3^{s_2}, 1^{t_2}$

now player 1 does not have anymore strictly dominates strategies at the moment, so we change our focus to player 2.

Player 2's strategy t_2 strictly dominates strategy strategy t_3 if

Strategy t^3		Strategy t^2	
1^{t_3}	$<$	3^{t_2}	True
1^{t_3}	$<$	2^{t_2}	True

which is true, and therefor player 2 will never play t_3 and we can eliminate this strategy from the game.

	t_1	t_2
s_1	$5^{s_1}, 0^{t_1}$	$3^{s_1}, 3^{t_2}$
s_2	$3^{s_2}, 4^{t_2}$	$2^{s_2}, 2^{t_2}$

in the remaining game, player 1 has a strictly dominated strategy in s_1 beceause

Strategy s^2		Strategy s^1	
3^{s_2}	$<$	5^{s_1}	True
2^{s_2}	$<$	3^{s_1}	True

and therefor player 1 will never play strategy s_2, so we remove it from the game

	t_1	t_2
s_1	$5^{s_1}, 0^{t_1}$	$3^{s_1}, 3^{t_2}$

Now it is optimal for player 2 to choose t_2, because it will give the highest payoff, because $3^{t_2} > 0^{t_1}$

	t_2
s_1	$3^{s_1}, 3^{t_2}$

and we find a unique solution so survive IESDS is

$$\text{IESDS Solution} = (s_1, t_2)$$

2. (Old version) The Travellers' Dilemma: "An airline loses two suitcases belonging to two different travelers. Both suitcases look identical and contain identical items. An airline manager tasked to settle the claims of both travelers explains that the

airline is liable for a maximum of \$100 per suitcase, and in order to determine an honest appraised value of the antiques the manager separates both travelers so they can't confer, and asks them to write down the amount of their value no less than \$0 and no larger than \$100. He also tells them that if both write down the same number, he will treat that number as the true dollar value of both suitcases and reimburse both travelers that amount. However, if one writes down a smaller number than the other, this smaller number will be taken as the true dollar value, and both travelers will receive that amount along with the following: \$1 extra will be paid to the traveler who wrote down the lower value and a \$1 fine imposed on the person who wrote down the higher amount."

a) Write down the normal form of this game: players, strategy sets, payoffs

To specify the normal form of the game, we need to specify the 3 things we commented in question 1.[1]

Players	There are two players denoted traveler 1 and traveler 2.
Strategy Sets	Illustrated in discrete time, the strategies is to write a number between 0 and 100 $s_i = \{0; 0,1; 0,2; 0,3; ..., 99,8; 99,9; 100\}$ for $i = 1, 2$
Payoff's	We can illustrate the payoff's in a bracket function

$$U_i(s_i, s_j) = \begin{cases} s_i & \text{when } s_i = s_j \\ s_i + 1 & \text{when } s_i < s_j \\ s_i - 1 & \text{when } s_i > s_j \end{cases}$$

also with the assumption that $U_i(s_i, s_j) \geq 0$.

Illustrating this in a bimatrix, could be something like

Player j

Player j		$s_j = 100$	$s_j = 99.9$	$s_j = 99.8$	$\cdots$	$s_j = 0.2$	$s_j = 0.1$	$s_j = 0$
	$s_i = 100$	$100, 100$	$98.9, 100.9$	$98.8, 100.8$	$\cdots$	$0, 1.2$	\$0,1.1	$0, 1$
	$s_i = 99.9$	$100.9, 98.9$	$99.9, 99.9$	$98.8, 100.8$	$\cdots$	$0, 1.2$	$0, 1.1$	$0, 1$
	$s_i = 99.8$	$100.8, 98.8$	$100.8, 99.8$	$99.8, 99.8$	$\cdots$	$0, 1.2$	$0, 1.1$	$0, 1$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
	$s_i = 0.2$	$1.2, 0$	$1.2, 0$	$1.2, 0$	$\cdots$	$0.2, 0.2$	$0, 0.2$	$0, 1$
	$s_i = 0.1$	$1.1, 0$	$1.1, 0$	$1.1, 0$	$\cdots$	$1.1, 0$	$0.1, 0.1$	$0, 1$
	$s_i = 0$	$1, 0$	$1, 0$	$1, 0$	$\cdots$	$1, 0$	$1, 0$	$0, 0$

b) Can you solve this game by IESDS?

To solve a game by IESDS (Iterated elimination of strictly dominated strategies), we need to have a strictly dominating strategy.

In this game there is no strategy for either player, that will give a strictly higher payoff than any of the other strategies. Therefor there is no strictly dominating strategy, and it is not possible to solve the game by IESDS.

[1] We have assumed that the two players can write decimal numbers, which is quite important because it removes a nash equilibria where both player's writes the same numberm, and only leaves one NE.

Lets compare some strategies x, y and z. Let player j play $z < \min\{x, y\}$. Then player i will get the same payoff from playing either x or y. Therefor there cannot be any strictly dominating strategies.

This is also clear by looking at the bimatrix above. There is no strategy which strictly dominates. Therefor we do not have a solution to IESDS.

c) What number do you think each traveler will write down? Why? An informal discussion of the reasoning will suffice.

Each player has complete information, and knows that the other player has it ass well. Therefor each player will write his best response, to what the player expect the other player will do i.e. we will need to find the nash equilibria (in pure strategies).

Lets plug in the best responses for player j

- Lets assume player i plays $s_i = 100$, then player j's best response is to write $s_j = 99.9$.
- Lets assume player i plays $s_i = 99.9$, then player j's best response is to write $s_j = 99.8$.
- Continuing this
- Lets assume player i plays $s_i = 0.2$, then player j's best response is to write $s_j = 0.1$.
- Lets assume player i plays $s_i = 0.1$, then player j's best response is to write $s_j = 0$.
- Lets assume player i plays $s_i = 0$, then player j's best response is to write $s_j = 0$.

and lets plug in the best responses for player i

- Lets assume player j plays $s_j = 100$, then player i's best response is to write $s_i = 99.9$.
- Lets assume player j plays $s_j = 99.9$, then player i's best response is to write $s_i = 99.8$.
- Continuing this
- Lets assume player j plays $s_j = 0.2$, then player i's best response is to write $s_i = 0.1$.
- Lets assume player j plays $s_j = 0.1$, then player i's best response is to write $s_i = 0$.
- Lets assume player j plays $s_j = 0$, then player i's best response is to write $s_i = 0$.

so it is optimal for both player's to write 0, which is also a nash equilibrium in pure strategies.

$$\text{PSNE: } \left\{(s_i, s_j) = (0, 0)\right\}$$

3. Solve these games by iterative elimination of strictly dominated strategies

	t_1	t_2	t_3
s_1	$5, 0$	$2, 3$	$1, 1$
s_2	$2, 4$	$2, 2$	$3, 1$
s_3	$2, 2$	$1, 1$	$0, 5$

	t_1	t_2	t_3
s_1	$5, 0$	$2, 3$	$1, 1$
s_2	$2, 4$	$2, 2$	$3, 1$
s_3	$2, 2$	$1, 1$	$1, 5$

Lets start with the first game.

For player 1 it is clear that s_1 strictly dominates strategy s_1, because no matter what strategy player 2, chooses player 1 gets a higher payoff from s_1. That is $5 > 2$ and $2 > 1$ and $1 > 0$. Therefor player 1 will never play s_3, and we remove it from the game

	t_1	t_2	t_3
s_1	$5, 0$	$2, 3$	$1, 1$
s_2	$2, 4$	$2, 2$	$3, 1$

Now for player 2, we see that she will always get a higher payoff from playing t_2 instead of playing t_3 no matter what strategy player 1 chooses. Therefor player 2 will never play t_3 and we can remove it from the game

	t_1	t_2
s_1	$5, 0$	$2, 3$
s_2	$2, 4$	$2, 2$

there is no more strategies, that strictly dominates, and we need to look at different solution concepts to solve the remaining of the game.

Lets look at the second game:

In the second game, there is no strategies, that strictly dominates the other strategies. Note it is required to have strict domination.

4) See code solution

5. We can also write games with more than two players. Consider the game below where player 1 chooses the bi-matrix (A or B), player 2 chooses the row (C or D), and player 3 chooses the column (E or F). In each cell, the first number gives the payoff of Player 1, the second number the payoff of Player 2, and the third number the payoff of Player 3.

	E	F
C	$0, 2, 2$	$2, 1, 1$
D	$0, 1, 1$	$3, 0, 0$

A

	E	F
C	$1, 0, 1$	$3, 1, 2$
D	$1, 1, 0$	$5, 2, 1$

B

Find the pure strategy profile that survives iterated elimination of strictly dominated strategies.

Lets start by looking if player 1, has any strictly dominating strategies. To do this we need to compare

	E	F
C	0, 2, 2	2, 1, 1
D	0, 1, 1	3, 0, 0

A

	E	F
C	1, 0, 1	3, 1, 2
D	1, 1, 0	5, 2, 1

B

it is clear that all the colored numbers, in the matrix to the right, is strictly larger than the same number in the right bimatrix.

Strategy A		Strategy B
0	<	1
2	<	3
0	<	1
3	<	5

And therefor strategy B strictly dominates strategy A for player 1, and player 1 will never play B. We remove the bimatrix A.

	E	F
C	1, 0, 1	3, 1, 2
D	1, 1, 0	5, 2, 1

B

Now player 2 gets a strictly higher payoff from playing D compared to playing C, therefor we can remove C from the game.

	E	F
D	1, 1, 0	5, 2, 1

B

Now player 3, gets the highest payoff from playing F, and therefor we can remove E from the game

	F
D	5, 2, 1

B

and the unique strategy that survives IESDS is

$$\{B, D, F\}$$

6) Michael and Jonas are playing a game instead of working. The game has the following rules: Both secretly pick a (natural) number between 1 and 5. Then they reveal the numbers to each other. If both have picked the same number, nobody gets anything. If Jonas' number is higher than Micheal's number, Michael has to pay Jonas 1 DKK. If Micheal's number is higher than Jonas', Jonas has to pay 10 DKK to Michael.

a) Does this game seem fair to you?

There is no definition of a fair game. Maybe check whether it is symmetric.

b) Write the game in bimatrix form.

To do this lets start by stating the normal form representation

Players There are two players denoted Jonas and Michael.

Strategy Sets The strategies for the two players are $s_i = \{1, 2, 3, 4, 5\}$ for $i = J, M$.

Payoff's We can illustrate the payoff's in two bracket functions

$$U_J(s_J, s_M) = \begin{cases} 0 & \text{when } s_J = s_M \\ 1 & \text{when } s_J > s_M \\ -10 & \text{when } s_J < s_M \end{cases} \qquad U_M(s_J, s_M) = \begin{cases} 0 & \text{when } s_M = s_J \\ 10 & \text{when } s_M > s_J \\ -1 & \text{when } s_M < s_J \end{cases}$$

Lets illustrate these in a bi matrix

Player 2 (Michael)

Player 1 (Jonas)	"1"	"2"	"3"	"4"	"5"
"1"	$0, 0$	$-10, 10$	$-10, 10$	$-10, 10$	$-10, 10$
"2"	$1, -1$	$0, 0$	$-10, 10$	$-10, 10$	$-10, 10$
"3"	$1, -1$	$1, -1$	$0, 0$	$-10, 10$	$-10, 10$
"4"	$1, -1$	$1, -1$	$1, -1$	$0, 0$	$-10, 10$
"5"	$1, -1$	$1, -1$	$1, -1$	$1, -1$	$0, 0$

c) Are there any strictly dominated strategies? Solve the game by iterated elimination of strictly dominated strategies.

We see that player 1 (Jonas) gets a higher payoff from playing "5" compared to playing "1", no matter what player 2 does i.e. strategy "5" strictly dominates strategy "1", and we can therefor eliminate "1" for player 1.

Player 2 (Michael)

Player 1 (Jonas)	"1"	"2"	"3"	"4"	"5"
"2"	$1, -1$	$0, 0$	$-10, 10$	$-10, 10$	$-10, 10$
"3"	$1, -1$	$1, -1$	$0, 0$	$-10, 10$	$-10, 10$
"4"	$1, -1$	$1, -1$	$1, -1$	$0, 0$	$-10, 10$
"5"	$1, -1$	$1, -1$	$1, -1$	$1, -1$	$0, 0$

Now it is clear that for player 2, it is always optimal to play "5" compared to playing "1", not matter what strategy player 1 chooses. Therefor strategy "5" strictly dominates strategy "1" and player 2 will never play "1"

Player 2 (Michael)

Player 1 (Jonas)	"2"	"3"	"4"	"5"
"2"	$0, 0$	$-10, 10$	$-10, 10$	$-10, 10$
"3"	$1, -1$	$0, 0$	$-10, 10$	$-10, 10$
"4"	$1, -1$	$1, -1$	$0, 0$	$-10, 10$
"5"	$1, -1$	$1, -1$	$1, -1$	$0, 0$

For player 1, strategy "5" now strictly dominates strategy "2"

Player 2 (Michael)

Player 1 (Jonas)	"2"	"3"	"4"	"5"
"3"	$1, -1$	$0, 0$	$-10, 10$	$-10, 10$
"4"	$1, -1$	$1, -1$	$0, 0$	$-10, 10$
"5"	$1, -1$	$1, -1$	$1, -1$	$0, 0$

For player 2 strategy "5" strictly dominates strategy "2", so she will never play strategy 2

Player 2 (Michael)

Player 1 (Jonas)	"3"	"4"	"5"
"3"	$0, 0$	$-10, 10$	$-10, 10$
"4"	$1, -1$	$0, 0$	$-10, 10$
"5"	$1, -1$	$1, -1$	$0, 0$

for player 1 strategy "5" now strictly dominates strategy "3" so we remove that from the game

Player 2 (Michael)

Player 1 (Jonas)	"3"	"4"	"5"
"4"	$1, -1$	$0, 0$	$-10, 10$
"5"	$1, -1$	$1, -1$	$0, 0$

now for player 2, strategy "5" strictly dominates strategy "3", so he will never player strategy "3"

Player 2 (Michael)

Player 1 (Jonas)	"4"	"5"
"4"	$0, 0$	$-10, 10$
"5"	$1, -1$	$0, 0$

for player 1, strategy "5" strictly dominates strategy "4"

Player 2 (Michael)

Player 1 (Jonas)	"4"	"5"
"5"	$1, -1$	$0, 0$

and lastly player 2 now gets the highest payoff from choosing "5" compared to "4".

Player 2 (Michael)

Player 1 (Jonas)	"5"
"5"	$0, 0$

therefor the strategies that survives IESDS is

$$\text{IESDS Outcome: } \{\text{"5"}, \text{"5"}\}$$

Note: Notice that we started by removing a strategy for player 1, we could instead have started with removing a strategy for player 2, but because we use strictly dominating strategies this would

yield the same results.
If we instead looking at weakly dominating strategies, than which strategy we start with removing will be relevant. We will return to this fact in a later problem set.

d) What is the outcome of the game if both Michael and Jonas are rational, know the other is rational, know that the other knows that they are rational ect.?

If we assume that the two player are rational, it would lead to the IESDS outcome, because a rational player will never play a strictly dominated strategy, because no matter what the other player do, they will recieve a higher payoff from playing the dominating strategy than the dominated strategy.

Therefor the outcome if they were rational is the IESDS outcome

$$\text{IESDS Outcome: } \{\text{"5"}, \text{"5"}\}$$

A Problem 2 with whole numbers

Lets take a look at question 3), with the assumption that the numbers the two travelers write, needs to be whole numbers between 0 and 100.

The normal form representation of the game is then

Players There are two players denoted traveler 1 and traveler 2.

Strategy Sets Illustrated in discrete time, the strategies is to write a number between 0 and 100 $s_i = \{0, 1, 2, 3, ..., 97, 98, 99, 100\}$ for $i = 1, 2$

Payoff's We can illustrate the payoff's in a bracket function

$$U_i(s_i, s_j) = \begin{cases} s_i & \text{when } s_i = s_j \\ s_i + 1 & \text{when } s_i < s_j \\ s_i - 1 & \text{when } s_i > s_j \end{cases}$$

also with the assumption that $U_i(s_i, s_j) \geq 0$.

Illustrating this in a bimatrix, could be something like

Player j

Player j		$s_j = 100$	$s_j = 99$	$s_j = 98$	$\cdots$	$s_j = 2$	$s_j = 1$	$s_j = 0$
	$s_i = 100$	$100, 100$	$98, 100$	$97, 99$	$\cdots$	$1, 3$	$0, 2$	$0, 1$
	$s_i = 99$	$100, 98$	$99, 99$	$97, 99$	$\cdots$	$1, 3$	$0, 2$	$0, 1$
	$s_i = 98$	$99, 97$	$99, 97$	$98, 98$	$\cdots$	$1, 3$	$0, 2$	$0, 1$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
	$s_i = 2$	$3, 1$	$3, 1$	$3, 1$	$\cdots$	$2, 2$	$0, 2$	$0, 1$
	$s_i = 1$	$2, 0$	$2, 0$	$2, 0$	$\cdots$	$2, 0$	$1, 1$	$0, 1$
	$s_i = 0$	$1, 0$	$1, 0$	$1, 0$	$\cdots$	$1, 0$	$1, 0$	$0, 0$

there is still no strategy which strictly dominates another strategy, so there is no solution to IESDS.

Plugging in for the best responses to find the nash equilibria, is a little different than without the assumption of pure numbers.

- Lets assume player i plays $s_i = 100$, then player j's best response is to write $s_j = 100$ or $s_j = 99$.
- Lets assume player i plays $s_i = 99$, then player j's best response is to write $s_j = 99$ or $s_j = 98$.
- Continuing this
- Lets assume player i plays $s_i = 2$, then player j's best response is to write $s_j = 2$ or $s_j = 1$.
- Lets assume player i plays $s_i = 1$, then player j's best response is to write $s_j = 1$ or $s_j = 0$.
- Lets assume player i plays $s_i = 0$, then player j's best response is to write $s_j = 0$.

and lets plug in the best responses for player i

- Lets assume player j plays $s_j = 100$, then player i's best response is to write $s_i = 100$ or $s_i = 99$.
- Lets assume player j plays $s_j = 99$, then player i's best response is to write $s_i = 99$ or $s_i = 98$.
- Continuing this
- Lets assume player j plays $s_j = 2$, then player i's best response is to write $s_i = 2$ or $s_i = 1$.
- Lets assume player j plays $s_j = 1$, then player i's best response is to write $s_i = 1$ or $s_i = 0$.
- Lets assume player j plays $s_j = 0$, then player i's best response is to write $s_i = 0$.

therefor we will have nash equilibria in all the diagonals

$$\text{PSNE: } \left\{(s_i, s_j) \mid (100, 100), (99, 99), (98, 98), ..., (2, 2), (1, 1), (0, 0)\right\}$$