Micro B: Problem Set 11

Revenue Equivalence

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Throughout this problem set, we will be considering an auction setting where n symmetric bidders draw their valuations, v_i, as

$$v_i \sim \text{IID}\Gamma(1, 2).$$

In Python, you can draw from this distribution using `np.random.gamma(1,2,(n,R))`.

For this problem set, you may find it helpful to review these videos: https://youtu.be/qwWk_Bqtue8 and https://youtu.be/eYTGQCGpmXI.

Question 1 (Simulating SPSB Auctions)**.** We start by simulating a dataset of winning auction payments, which we will later pretend we forgot the parameters for and try to reverse engineer.

(a) Show a histogram of the underlying distribution as well as the $n-1$'th and n'th order statistics.

(b) Suppose $n = 5$ bidders compete in a Second Price Sealed Bid auction. Simulate $R = 10,000$ auctions and find the winning bid and payment for each. Save these $R \times 2$ numbers in a csv file, `out/SPSB.csv`.

Hint: Remember to use a seed, e.g. `np.random.seed(1337)`*.*

Question 2 (Computing the Distance Between Two Distributions)**.** This question asks you to compute the distance between two distributions: the standard Gamma and a $\chi^2(\mu)$ distribution. And then to minimize the distance.

(a) Simulate $N = 13,337$ draws from a standard Gamma distribution, which will be our "data".
Hint: Use `np.random.standard_gamma(1,2,13337)`*.*

(b) Write a function, `predict_distribution(mu)`, which returns $R = 20,000$ draws of a $\chi^2(\mu)$ distribution, using `np.random.chisquare(mu,20000)`.

(c) Evaluate the empirical CDF of the two distributions over a grid, `xx = np.linspace(0,10,G)` for $G = 50$ grid points, and plot the two together.
Hint: Use `ECDF` *from* `statsmodels.distributions.empirical_distribution`*. You call it on a vector of observations and then it returns a lambda function.*

(d) Write a function, $Q(\mu)$, which returns the L_2 distance between the empirical CDF of the two distributions on the grid `xx`:

$$Q(\mu) = \sum_{x \in \mathtt{xx}} \left[\hat{F}^{sim}(x) - \hat{F}^{data}(x) \right]^2.$$

(e) Estimate μ by minimizing the distance:

$$\hat{\mu} = \arg\min_{\mu > 0} Q(\mu).$$

Question 3 (Learning the Bid Distribution)**.** Suppose now that you receive the dataset, `out/SPSB.csv`, but only know that the results come from a SPSB auction where the distribution comes from the $\Gamma(k, \theta)$ family, indexed by the parameters $k > 0$ ("shape") and $\theta > 0$ ("scale").

1. Estimate k and θ.

Hint: Use the same approach as in question 2.

Question 4 (Drawing from a truncated distribution)**.**

1. Plot the histogram of a truncated $\Gamma(k, \theta)$ with truncation points $\bar{v} = 1, 2, 3$.
   Hint: One way of drawing from a truncated distribution is to draw from the underlying distribution and then delete all observations that are greater than the truncation point.

2. Compute the expected value of the maximum when drawing $n = 4$ values from the truncated distribution with truncation points $\bar{v} = 1, 2, 3$. Write a function that works for any truncation point $\bar{v}$.

Question 5. Read in the dataset, `out/SPSB.csv`. Compute the Bayesian Nash Equilibrium, $b^*(v)$, on a grid $\mathcal{V} = \{1.1, 1.2, ..., 5.0\}$ using revenue equivalence. Use the k and θ estimated in question 3.
Hint: For each input value, $v \in \mathcal{V}$, compute the expected maximum from a truncated $\Gamma(\hat{k}, \hat{\theta})$ with truncation point equal to v, denoted $\Gamma^{trunc}(\hat{k}, \hat{\theta}, v)$ and with $n-1$ bidders. Use the algorithm outlined in 1.

Algorithm 1: FPSB from SPSB

Input: v, R, n
Output: x: the expected payment for an *winning* bidder with valuation v.

for $r = 1, ..., R$ **do**
  Draw $v_i^r \sim \Gamma^{\text{trunc}}(\hat{k}, \hat{\theta}, v)$ for $i = 1, ..., n-1$
  $x^r \leftarrow \max_i v_i^r$
$x \leftarrow \frac{1}{R} \sum_{r=1}^{R} x^r$