Micro B: Problem Set 10

Auctions*

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Question 1 (FPSB)**.** Consider a First Price Sealed Bid (FPSB) auction between two bidders. The bidders draw their valuations independently as

$$v_i \sim \text{IID}\mathcal{U}(1,3) \quad i = 1,2.$$

(a) Show that there exists a symmetric Bayesian Nash Equilibrium in Linear strategies, i.e. where strategies come from the family

$$b_i(v_i; c, d) = cv_i + d \quad i = 1,2, \text{ and } c, d \in \mathbb{R},$$

where $c = \frac{1}{2}$ and $d = \frac{1}{2}$.
Note: in the lectures, it was proven that that $b(v) = \frac{1}{2}v$ is the BNE when valuations are $\mathcal{U}(0,1)$.

(b) Calculate the expected revenue to the seller.

Question 2 (Python: Ex ante vs. interim utility)**.** Consider again the 2-bidder auction game from 1, where valuations are drawn as

$$v_i \sim \text{IID}\mathcal{U}(1,3) \quad i = 1,2.$$

Simulate $R = 10,000$ auctions and assume that bidder 2 uses the BNE strategy

$$b_2(v) = \frac{1}{2}v + \frac{1}{2},$$

and where bidder 1 tries different strategies. The goal is to show that $b_1(v) = \frac{1}{2}v + \frac{1}{2}$ is the best response, i.e. that it is a BNE. Your results should look like 1.

(a) Compute the *ex ante* utility of bidder 1 from committing to the strategy

$$b_1(v) = cv + \frac{1}{2}, \qquad \text{for } c \in \left\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\right\}.$$

Hint: Simulate v_{ir} for $i = 1,2$ and R auctions. Compute b_{ir} from the strategies of each, and compute the expected utility of bidder 1 as $\hat{\mathbb{E}}(u_1) = \frac{1}{R}\sum_{r=1}^{R} u^{[r]}$, where

$$u^{[r]} = \begin{cases} v_{1r} - b_{1r} & \text{if } b_{1r} \geq b_{2r} \\ 0 & \textit{otherwise} \end{cases} \tag{1}$$

*For prior contributions, thanks to Jeppe Dinsen.

(b) Compute the interim utility of bidder 1 having drawn some v_1 from any bid $b \in [1; v_1]$, and do this for $v_1 \in \{1.1, 2, 3\}$.
Hint: As before, simulate R auctions and compute $b_{2r} = b_2(v_{2r})$ and $\hat{\mathbb{E}}(u_1) = \frac{1}{R}\sum_{r=1}^{R} u^{[r]}$, where this time

$$u^{[r]}(b) = \begin{cases} v_{1r} - b & \text{if } b \geq b_{2r} \\ 0 & \text{otherwise} \end{cases}$$

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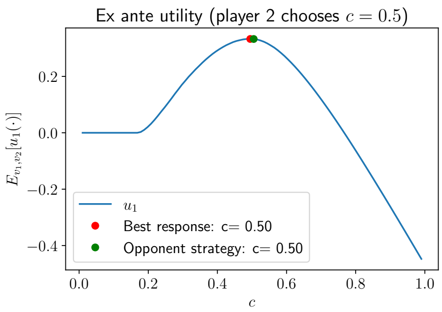
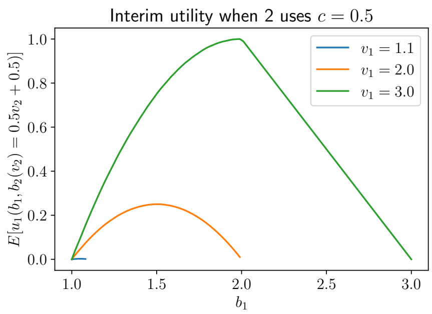
Figure 1: Solution to 2

(a) Ex ante utility



(b) Interim utility



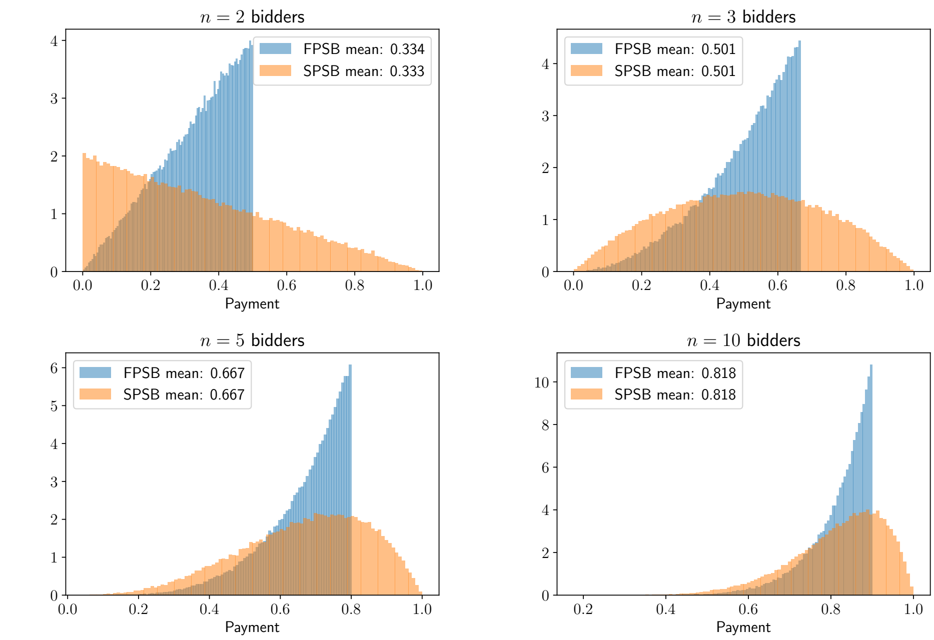
Question 3 (Python: FPSB vs. SPSB)**.** Simulate $R = 10,000$ auctions between n bidders under both a First Price Sealed Bid (FPSB) and a Second Price Sealed Bid (SPSB). Do this for $n \in \{2, 3, 5, 10\}$.
Hint: For the FPSB, use the BNE from (2). For the SPSB, use the (weakly) dominant strategy, $b(v) = v$.

(a) Compute the average revenue to the seller (over the R simulations),

(b) Plot a histogram comparing the distribution of the payment to the seller for the two formats.

Figure 2 shows the solutions.

Figure 2: Distribution of payments



Question 4 (Analytical Formula, General n)**.** Consider a First Price Sealed Bid (FPSB) auction between n bidders where valuations are

$$v_i \sim \text{IID}\,\mathcal{U}(0,1) \quad i = 1, \dots, n.$$

Show graphically that a symmetric BNE is

$$b_i^*(v) = \frac{n-1}{n}v, \quad i = 1, ..., n. \tag{2}$$

Note that the formula only applies to $v_i \sim \mathcal{U}(0,1)$.
Hint: Use the same approach as in question 1. Also,

$$\Pr(i \text{ wins with } b) = \prod_{j \neq i} \Pr(b_j^*(v_j) > b),$$

where $b_j^*(v_j) = \frac{n-1}{n}v_j$.