Micro B: Problem Set 7.a

Repeated Games*

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Spring 2023

Question 1. Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Definition 8.2: Underspil. Et (egentligt) underspil (proper subgame) G af et udvidet-form spil Γ består af en enkelt singleton knude, $x \in X \setminus Z$ samt alle dennes efterkommere i Γ med den egenskab, at hvis $x' \in G$ og $x'' \in h(x')$, så er $x'' \in G$. (I.e. all information sets are either fully included or excluded from the subgame G.)
Underspillet G er selv et spiltræ, som arver sine informationsmængder og payoffs fra Γ.

Question 2. Let G be the following game

	C	D
A	27,-3	-0,0
B	6,6	-2,7

Consider the repeated game $G(T)$, where G is repeated T times and the outcomes of each round are observed by both players before the next round.

(a) If $T = 2$, is there a Subgame Perfect Nash Equilibria such that (B, C) is played during the first round?

(b) What if $T = 42$

Question 3. Consider the two times repeated game, $G(2)$, where the stage game, G, is

	X	Y	Z
A	6,6	0,8	0,0
B	7,1	2,2	1,1
C	0,0	1,1	4,5

G

(a) Find a subgame perfect Nash equilibrium such that the outcome of the first stage is (B, Y). Make sure to write down the full equilibrium.

(b) Find a subgame perfect Nash equilibrium such that the outcome of the first stage is (C, Z). Make sure to write down the full equilibrium.

(c) Can you find a subgame perfect Nash equilibrium such that the total payoffs that the players receive are 10 for player 1 and 11 for player 2? If yes, write down the full equilibrium.

*For prior contributions, thanks to Jeppe Dinsen.

Question 4. Consider the situation of two flatmates. They both prefer having a clean kitchen, but cleaning is a tedious task, so that it is individually rational not to clean regardless of what the other does. This results in the following game G:

	Clean	Do not clean
Clean	4,4	0,6
Do not clean	6,0	1,1

G

Now consider the situation where the two flatmates have to decide every day whether to clean or not, i.e. consider the infinitely repeated game $G(\infty, \delta)$.

(a) Define trigger strategies such that the outcome of all stages will be $(Clean, Clean)$.

(b) Find the lowest value of δ such that the trigger strategies from (a) constitute a SPNE in $G(\infty, \delta)$. *Recall:* you have to check for deviations both on and off the equilibrium path.

Question 5. Consider again the game from above

	C	NC
C	4,4	0,6
NC	6,0	1,1

(a) Define a tit-for-tat strategy such that the outcome of all stages will be (C, C).

(b) Check for which δ tit-for-tat is optimal on the equilibrium path against the following strategy: 'Always play 'Do not clean".

(c) Check for which δ tit-for-tat is optimal on the equilibrium path against the following strategy: 'Start by playing 'Do not clean', then play 'tit-for-tat' forever after that'.