Micro B: Problem Set 3

Mixed Strategies*

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Question 1 (*IESDS and Nash Equilibrium*). Show that for each of the following two games, the only Nash equilibrium is in pure strategies. Describe the intuition for this result. What do these two games have in common?

A

	L	R
U	$5,5$	$1,6$
D	$6,1$	$2,2$

B

	L	C	R
U	$1,0$	$1,2$	$0,1$
D	$0,3$	$0,1$	$2,0$

Question 2 (*Recap: Nash Equilibrium in Pure Strategies*). Solve for all pure strategy Nash equilibria in the following game. Which equilibrium do you find most reasonable?

	a	b	b
A	$2,2$	$0,0$	$-1,2$
B	$0,0$	$0,0$	$0,0$
C	$2,-1$	$0,0$	$1,1$

Question 3 (*Mixed Strategy Nash Equilibrium (MSNE) in a Bimatrix Game*). For each of the four games below, do the following two things:

(a) Plot the best response functions for both players together in the same graph.

(b) Find all Nash Equilibria (pure and mixed).

A

	L	R
T	$0,0$	$0,0$
D	$0,0$	$1,1$

B

	L	R
T	$1,3$	$1,0$
D	$1,1$	$5,5$

C

	L	R
T	$3,2$	$1,2$
D	$0,1$	$1,2$

D

	t_1	t_2
s_1	$2,1$	$3,0$
s_2	$1,2$	$4,3$
s_3	$0,1$	$0,3$

Question 4 (*MSNE in Python*). Repeat the above problem using Python using two approaches:

1. For 2×2 games, write a function to plot the best response curves and find the MSNE. **Hints**

*For prior contributions, thanks to Jeppe Dinsen.

- Make a grid over $[0;1]$, `pp`. Each possible mixed strategy of an opponent can be summarized by a single probability, $p_j = \Pr(a_j = 0)$ (the probability that the opponent plays her first action.)
- For each opponent strategy, $(p_j, 1-p_j)$, compute the expected utility for both actions to player i.
- Find the maximum of the two and return its number. However, think carefully about what to do when the two give the same expected utility...

2. Using the Python package `nashpy`. Use the provided scaffolding code.

Question 5 (*Undercutting with Loyals and Shoppers*[1]). Consider price competition between two firms, $N = \{1, 2\}$, when some consumers are informed about prices and others are not. Firms have zero marginal cost and their actions consist of setting prices, $p_i \in S_i = [0; \infty)$, which they set simultaneously. Payoffs are thus:

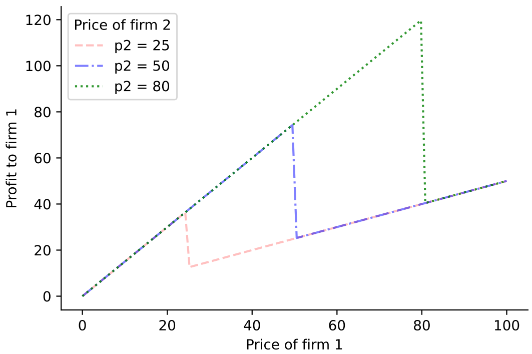
$$\pi_i(p_i, p_j) = p_i q_i(p_i, p_j),$$

where $q_i(\cdot)$ is the market demand, which we describe now: The market consists of two consumer types: loyals and shoppers. There are equally many of both types. Loyals always buy from their preferred brand regardless of the price, while shoppers always choose the cheapest product. That is, firms always sell to 25% of consumers (their share of the loyal segment), but win an additional 50% of consumers if they are cheapest. If they tie, they share the shoppers equally. Suppose that the mass of consumers is M (e.g. in million people). Mathematically, demand is $q_i(p_i, p_j) = M s_i(p_i, p_j)$, where s_i is the market share of firm i,[2] given by

$$s_i(p_i, p_j) = \begin{cases} \frac{1}{4} & \text{if } p_i > p_j \\ \frac{3}{4} & \text{if } p_i < p_j \\ \frac{1}{2} & \text{if } p_i = p_j. \end{cases}$$

(a) Figure 1 presents the profit function of one firm for three different values of the competitor's price. Explain the intuition for the shape and identify the best response to each case.

Figure 1: Profit to firm 1 for 3 values of competitor price



(b) Argue intuitively (or prove mathematically) that there cannot exist a symmetric pure strategy Nash equilibrium. Discuss what makes this setting different from the classical

[1] This example builds on an important and seminal paper by Varian (1980): "A Model of Sales", in *The American Economic Review.*

[2] The market share of a firm is the fraction of the total mass of consumers it sells to, $s_i \equiv q_i / \sum_{j \in N} q_j$.

Bertrand case (where the pure strategy equilibrium is that both firms set a price of zero). *Hint: Start from a proposed symmetric equilibrium, (p^*, p^*). What could be a deviation if $p^* > 0$? And is there a deviation if $p^* = 0$?*

To simplify the game, suppose $M = 2$ and let us discretize the action space and suppose that firms can only choose three prices:

$$S_i = \{38, 54, 80\}.$$

(c) Argue that this game can be represented by the following bimatrix

	$p_2 = 80$	$p_2 = 54$	$p_2 = 38$
$p_1 = 80$	$80, 80$	$40, 81$	$40, 57$
$p_1 = 54$	$81, 40$	$54, 54$	$27, 57$
$p_1 = 38$	$57, 40$	$57, 27$	$38, 38$

(d) Show that there is no Nash equilibria in pure strategies

(e) Confirm that the following strategy profile is a Nash equilibrium: each firm plays price 80 with probability 0.232, price 54 with probability 0.361, and price 38 with probability 0.407.

Question 6 (*Penalty Kicks*)**.** Consider the penalty kick game below, where the row player is the kicker and the column player is the goalie.

	L	R
L	$0, 0$	$1, -1$
R	$\frac{1}{2}, -\frac{1}{2}$	$0, 0$

Penalty Kick Game

- Describe the intuition.
- Before computing, write down your expectations regarding the equilibrium.
- Find the MSNE and discuss the intuition.

Question 7 (*MSNE in a 2×3 bimatrix game*)**.** Find all (pure and mixed) Nash equilibria in the following game:

	L	C	R
T	$4, 1$	$2, 3$	$0, 4$
D	$2, 3$	$1, 2$	$5, 0$

A 3×2 game