

# GRAMMAR

- \* Grammer consist of set of rule to define a language.
  - \* It can be define through 4 Tuples.

(\*) V stands for set of non terminal values or symbols. It can produce a new item or new non terminal value.

(\*) T stands for set of terminal symbols or input alphabet. It can generally denote through all the symbol and character except capital letter.

(\*) P stands for production rules. @  
P is the starting symbol.

(\*) S is the starting symbol.

## • TYPES OF GRAMMER

- TYPES OF GRAMMARS
  - ⇒ Based on string, the Grammar may be recursive or non-recursive.
  - ⇒ Based on derivation tree, the Grammar may be ambiguous and unambiguous.
  - ⇒ Based on chomsky classification,  
chomsky classify the Grammar in 4 different types.

### (i) Type O

## (ii) Type 1

(iii) Type 2

(iv) Type 3

Grammer Type	Grammer Accepted	Language Accepted	Automation
Type-0	unrestricted	Recursively Enumerable Language	Turing Machine (TM)
Type-1	Context Sensitive	Context Sensitive	Bounded Automaton (LBA)
Type-2	Context free	Context free	Push down Automaton (PDA)
TYPE-3	Regular	Regular	finite state machine

• Grammer type 0 is unrestricted and it generates all languages.

• Grammer type 1 is context sensitive and it generates all recursive enumerable languages.

• Grammer type 2 is context free and it generates all regular languages.

• Grammer type 3 is regular and it generates all finite languages.

- **TYPE-3 GRAMMARI :-**
- (\*) Type 3 is a regular grammar and it supports regular language.
- (\*) Here the grammar must have a single non-terminal in the left side and consist a single terminal or followed by single or more non-terminal symbol in the production.

(\*) The General form of production will be:

$$A \rightarrow a$$

$$A \rightarrow aBv \rightarrow \text{Trace} \in T^*$$

(\*) If  $a$  is not allowed to produce in right hand side of any production rule,

Example -

$$\left. \begin{array}{l} x \rightarrow \epsilon \\ x \rightarrow aYb \\ Y \rightarrow b \end{array} \right\} \text{Type 3 P.D.O. to generate}$$

- **TYPE-2 GRAMMARI :-**
- (\*) Type-2 Grammer generates context-free language.
- (\*) The production rule must be in the form of:

$A \rightarrow aBvA \rightarrow \text{Trace}$  where  $A \in V$

Example -

$$\left. \begin{array}{l} x \rightarrow \epsilon \\ x \rightarrow aYb \\ Y \rightarrow Ba \end{array} \right\} \text{D.P.O. to generate}$$

## • TYPE-1 GRAMMER :-

- \* Type 1 is called context-sensitive Grammar and the language is called context-sensitive language.
- \* It is recognized by linear bound automata.
- \* The production rule must be in the form of  $\alpha A\beta \rightarrow \alpha\gamma\beta$ , where  $\alpha, \beta, \gamma \in (V \cup T)^*$ .
- \*  $\gamma$  should not be empty.

(1)  $A \rightarrow \epsilon$  is allowed if  $A$  does not appear right side of any production rule.

$$ABA \rightarrow ABCAABA$$

## • TYPE-0 GRAMMER :-

- \* Type-0 is most restricted type of grammar.
- \* Production rule of Type-0

$$\alpha \rightarrow \beta$$

where  $\alpha, \beta \in (V \cup T)^*$

$\alpha \neq \epsilon$

## • REGULAR GRAMMER :-

⇒ The regular Grammar is type-3 Grammar again classified into two categories.

(1) Left linear grammar.

(2) Right linear grammar.

(1) LEFT Linear Grammar - If the right hand side of any production contains a non-terminal in left most position then the grammar is said to be left linear Grammar.

\* Hence the derivation tree will expand towards left direction.

$$A \rightarrow Ba$$



(2) Right Linear Grammar - If the right hand side of any production contains a non-terminal in right most position then the grammar is said to be right linear Grammar.

\* Hence the derivation tree will expand towards right direction.

$$A \rightarrow QB$$



$$A \xrightarrow{\epsilon} \{B\}$$

$$A \xrightarrow{\epsilon} \{A\}$$

$$A \xrightarrow{\epsilon} \{A, B\}$$

$$A \xrightarrow{\epsilon} \{B\}$$

## Conversion from Grammar to FA :-

### RULES

The tuple of the Grammar are  $\langle V T P S \rangle$ .  
 Whereas as the tuple of finite automata are  $\langle Q, \Sigma, S, q_f, f \rangle$  by considering the tuples, we can find  $\langle Q, \Sigma, S, q_f, f \rangle$  by considering the tuples, we can find similarity between grammar and finite Automata.

$$\begin{aligned} \text{Grammar} &: \langle V T P S \rangle \\ \text{FA} &: \langle Q, \Sigma, S, q_f, f \rangle \end{aligned}$$

To convert Grammar to finite Automata followings are the steps :-

### Step 1 :-

S, which is the starting symbol in the grammar, will be considered as starting state in the finite automata.

Step 2 :- for each production rule in the form of  $S \rightarrow w q_f \rightarrow w q_f$ , will be converted to transition of  $S(q_i, w) \rightarrow q_f$ . for the production rule in the form of  $q_i \rightarrow w$ , that can be represented as  $S(q_i, w) \rightarrow q_f$ , where  $q_f$  is the final state.

(\*) Convert the following Grammar to finite Automata

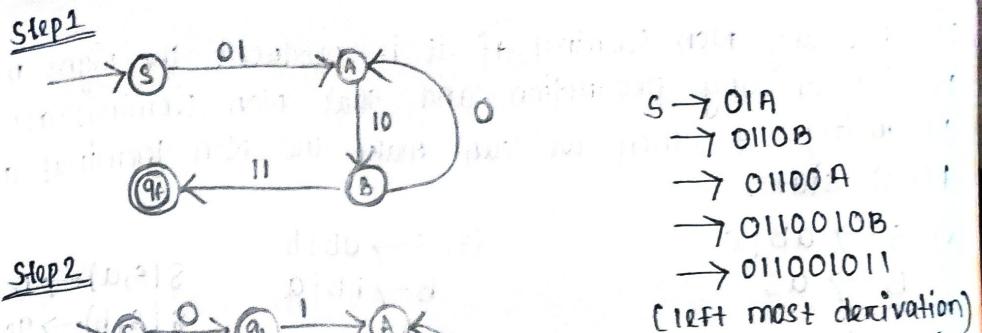
$$S \rightarrow 01A, A \rightarrow 10B, B \rightarrow 0A / 11$$

$$\delta(S, 01) \rightarrow A$$

$$\delta(A, 10) \rightarrow B$$

$$\delta(B, 0) \rightarrow A$$

$$\delta(B, 11) \rightarrow q_f$$



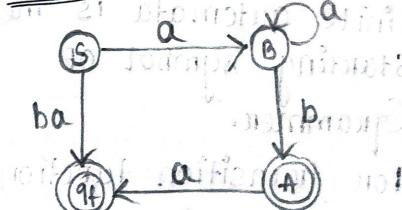
$$\begin{aligned} S &\rightarrow 01A \\ &\rightarrow 0110B \\ &\rightarrow 01100A \\ &\rightarrow 0110010B \\ &\rightarrow 011001011 \end{aligned}$$

(left most derivation)

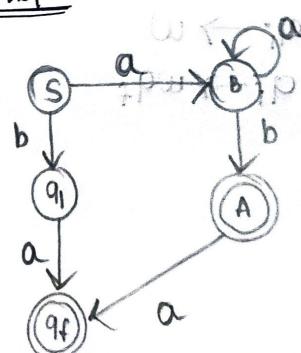
∴ This string also accepted by finite automata.

$$\begin{aligned} (*) \quad S &\rightarrow ABC | BA \\ B &\rightarrow BA | AB \\ A &\rightarrow a | \epsilon \end{aligned}$$

### Step 1



### Step 2



$$\begin{aligned} \delta(S, a) &\rightarrow B \\ \delta(S, ba) &\rightarrow q_f \\ \delta(B, b) &\rightarrow A \\ \delta(B, ba) &\rightarrow B \\ \delta(A, a) &\rightarrow q_f \\ \delta(A, \epsilon) &\rightarrow A \end{aligned}$$

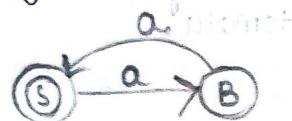
final state

NOTE :-

- for any non terminal if it is produced the right most position of any production - and that non terminal also producing  $\epsilon$  then we can make the non terminal as final state.

$$\begin{array}{l} \text{: } S \rightarrow aB|\epsilon \\ \quad B \rightarrow aS \end{array}$$

$$\begin{array}{l} S(s,a) \rightarrow B \\ S(B,a) \rightarrow S \\ S(s,\epsilon) \rightarrow q_f \end{array}$$



$$(*) S \rightarrow aB|b$$

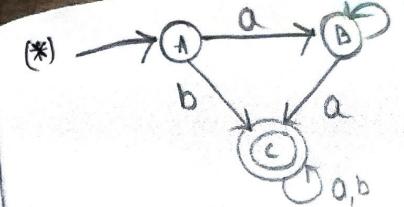
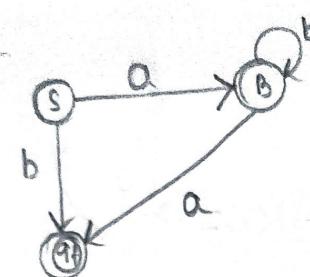
$$B \rightarrow bB|a$$

$$\delta(s,a) \rightarrow B$$

$$\delta(s,b) \rightarrow q_f$$

$$\delta(B,b) \rightarrow B$$

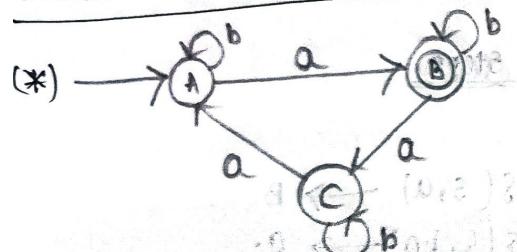
$$\delta(B,a) \rightarrow q_f$$



$$\begin{array}{l} A \rightarrow aB|bC|b \\ B \rightarrow bB|aC|a \\ C \rightarrow aC|bC|\epsilon \end{array}$$

$$\text{or } aC|bC|a|b$$

### FINITE AUTOMATA TO GRAMMER :-



$$\begin{array}{l} \delta(A,b) \rightarrow A \\ \delta(A,a) \rightarrow B \\ \delta(B,b) \rightarrow B \\ \delta(B,a) \rightarrow C \end{array}$$

$$\begin{array}{l} A \rightarrow BA|ab|a^2 \\ B \rightarrow bB|ac|\epsilon \\ C \rightarrow aA|bc \end{array}$$

(\*) Starting state of finite automata is the starting symbol of Grammer.

for transition function

$$\delta(q_i, w) = \text{final state}$$

$$q_i \rightarrow w$$

$$q_i \rightarrow wq_f$$

## UNIT PRODUCTION :-

(\*) A production is said to be ~~one~~ a unit production, IF right hand side (RHS) OF Rule contain only one non terminal.

$A \rightarrow B$ , where both A & B are non terminal.

$$\begin{array}{l} A \rightarrow B \\ B \rightarrow a \end{array}$$

(Q)  $S \rightarrow AB | d | A | B | \epsilon$

$$A \rightarrow a$$

$$B \rightarrow bA | d b$$

Remove a :-

$$S \rightarrow AB | d | B | \epsilon$$

$$\begin{array}{l} \text{Remove } B \\ S \rightarrow AB | d | A | B | d b \end{array}$$

final production -

$$\begin{array}{l} S \rightarrow AB | \epsilon | a | BA | d | b \\ B \rightarrow dA | d b \\ A \rightarrow a \end{array}$$

chomsky Normal form.

A grammar is said to be CNF, if all the products are either of  $A \rightarrow a$  or  $A \rightarrow BC$  where  $A, B, C \in V$  and  $a \in T$ .

\*  $S \rightarrow \epsilon$  is allowed if  $S$  does not appear right side of any production.

$$A \rightarrow \frac{a}{B_1} A_2 A_3 A_4 \mid \frac{A_4 A_3 A_2}{B_2}$$

$$A_2 \rightarrow az \mid A_4 A_3 A_2 z$$

$$z \rightarrow \epsilon \mid A_3 A_2 z$$

$$A_1 \rightarrow A_2 A_3 \mid A_4 A_3$$

$$A_4 \rightarrow b$$

$$A_3 \rightarrow d$$

$$\begin{aligned} A_2 &\rightarrow az \mid b A_4 A_3 z \\ z &\rightarrow e \mid d A_2 z \\ A_1 &\rightarrow az A_3 \mid b A_4 A_3 z A_3 \\ A_4 &\rightarrow b \\ A_3 &\rightarrow d \end{aligned}$$

(Q) Remove left recursion from the grammar

$$A \rightarrow ab \mid ABD \mid cda$$

$$B \rightarrow b$$

$$D \rightarrow d$$

$$A \rightarrow \underline{ab} \mid \underline{AbD} \mid \underline{cdA}$$

~~Left Recursion~~

$$A \rightarrow ABZ \mid CDAZ$$

$$Z \rightarrow \epsilon \mid bDZ$$

$$B \rightarrow b$$

$$D \rightarrow d$$

### CYK ALGORITHM : COCKE YOUNGER KASAMI

→ This algorithm is used to check a string to be accepted by grammar or not.

→ To check this thing the grammar must be in CNF.

$$(Q) S \rightarrow AB$$

$$A \rightarrow BB \mid a$$

$$B \rightarrow AB \mid b$$

Given string "abbba"

Find out the number of alphabet.

Step 1

(1) find out the number of alphabet in a string.

$$n=4$$

(2) Design a table with 4 row and 4 column.

Step 1 may be skip as the grammar is already in CNF.

	4	3	B2	B1
1	{S, B}	{A}	{S, B}	{A}
2	{S, B}	{A}	{B}	
3	{A}	{B}		
4	{B}			

"abb" (start, end)

✓(1,1)  $\rightarrow A$  ( $\because A \rightarrow a$ )

✓(2,2)  $\rightarrow B$  ( $\because B \rightarrow b$ )

✓(3,3)  $\rightarrow B$  ( $\because B \rightarrow b$ )

✓(4,4)  $\rightarrow B$  ( $\because B \rightarrow b$ )

✓(1,2)  $\rightarrow (1,1)$  (2,2)  $\rightarrow AB = S, B$  ( $\because S \rightarrow AB, B \rightarrow AB$ )

✓(2,3)  $\rightarrow (2,2)$  (3,3)  $\rightarrow BB = A$  ( $\because AB \rightarrow BB$ )

✓(3,4)  $\rightarrow (3,3)$  (4,4)  $\rightarrow BB = A$  ( $\because A \rightarrow BB$ )

✓(1,3)  $\rightarrow (1,1)$  (2,3)  $\rightarrow AA = \emptyset$  OR, (1,2) (3,3)  $\rightarrow \{S, B\} \{B\} = SB, BB = \emptyset, A$

✓(2,4)  $\rightarrow (2,2)$  (3,4)  $\Rightarrow \{B\} \{A\} = BA$

OR, (2,3) (4,4)  $\Rightarrow \{A\} \{S, B\} = AS, AB = \emptyset, B$  ( $\because S \rightarrow AB, B \rightarrow AB$ )

✓(1,4)  $\rightarrow (1,1)$  (2,4)  $\rightarrow \{A\} \{S, B\} = AS, AB = \emptyset, B$

OR, (1,2) (3,4)  $\rightarrow \{S, B\} \{A\} = \emptyset$

OR, (1,3) (4,4)  $\rightarrow \{A\} \{B\} = AB = S, B$

$$\begin{array}{l} S \rightarrow A_1 \\ A_1 \rightarrow A_2 A_3 \mid A_4 A_4 \\ A_2 \rightarrow a \mid A_1 A_4 \\ A_3 \rightarrow b \\ A_4 \rightarrow c \\ B \rightarrow A_4 \end{array}$$

$$A_1 \rightarrow A_2 A_3 \mid A_4 A_4$$

$$A_2 \rightarrow a \mid A_1 A_4$$

$$A_4 \rightarrow b$$

$$A_3 \rightarrow d$$

→ Consider, for the production  $A_i \rightarrow A_j$ ,  $i$  should always less than  $j$ .

( $A_i$  always considered as first non-terminal in RHS)

$$A_1 \rightarrow A_2 A_3 \mid A_4 A_4$$

$$A_2 \rightarrow a \mid A_1 A_3 \mid A_4 A_4$$

$$A_4 \rightarrow b$$

$$A_3 \rightarrow d$$

Now,  $A_2 \rightarrow A_1 A_3 A_4$  is a case of left recursion, so

this has to removed from grammar.

(\*) The recursion is a process where a non-terminal produce it self in right hand side of production.

→ There are 3 case of Recursion.

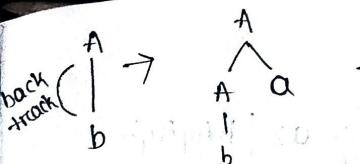
(1) Normal :  $A \rightarrow \alpha A \beta$

(2) Left recursion :  $A \rightarrow A \alpha$

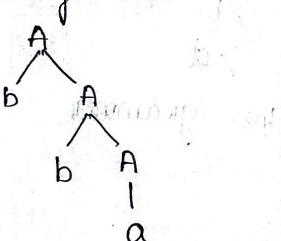
(3) Right recursion :  $A \rightarrow \alpha A$

→ All the three case, difficulty level case is left recursion.

WHY? Suppose the production rule is  $A \rightarrow A \alpha \mid B \beta$  and generate the string "baa".

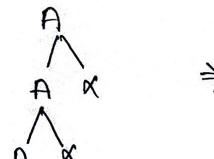


(\*)  $A \rightarrow b A \mid a$ , generate the string "bba"



But in right recursion, we will not face such kind of problem as we can visualize the first symbol and as per requirement we can choose the production.

$$(Q) A \rightarrow A\alpha \mid B$$



$$\begin{array}{l} A \rightarrow B \alpha^* \\ \alpha^* \rightarrow \epsilon \mid \alpha \alpha^* \end{array}$$

$$\begin{array}{l} A \rightarrow B Z \\ Z \rightarrow \epsilon \mid \alpha Z \end{array}$$

$$\begin{array}{l} (Q) A \rightarrow A\alpha \mid A\beta \mid B_1 \mid B_2 \\ B_1 \rightarrow B_1 Z \mid B_2 Z \\ Z \rightarrow \epsilon \mid \alpha Z \end{array}$$

$$\begin{array}{l} A \rightarrow Y \alpha^* \mid Y B^* \\ \alpha^* \rightarrow \epsilon \mid \alpha \alpha^* \\ B^* \rightarrow \epsilon \mid B B^* \end{array}$$

$$\begin{array}{l} A \rightarrow Y \beta \mid Y Y \\ \beta \rightarrow \epsilon \mid \alpha X \\ Y \rightarrow \epsilon \mid BY \end{array}$$

{ TWO  
left  
Recursion

Conversion of a CFG to CNF (Step 1 of 4 steps)

(1) Remove useless production and unreachable code from the Grammar.

(2) Remove Null production.

(3) Remove unit production

(4) Convert all the production into CNF format.

(Q1)  $S \rightarrow aAD$

$A \rightarrow aB | bAB$

$B \rightarrow bA | Aa | Aa | Aa$

$D \rightarrow d$

$D \leftarrow A$

$d / b / Ad \leftarrow A$

$b / a / B / a / Aa / Aa / Aa \leftarrow A$

$a / D \leftarrow A$

$b / a / B / a / Aa / Aa / Aa \leftarrow A$

$a / D \leftarrow A$

(1) There is no unit production and null production in grammar. Hence this step may be skipped.

(2) After checking the format of CNF, convert all the terminal into non-terminal from rest of production.

$S \rightarrow xAD$

$B \rightarrow b$

$D \rightarrow d$

$X \rightarrow a$

$Y \rightarrow b$

$S \rightarrow PD$

$P \rightarrow XA$

$A \rightarrow XB | QB$

$Q \rightarrow YA$

$B \rightarrow b$

$D \rightarrow d$

$X \rightarrow a$

$Y \rightarrow b$

(Q2)  $S \rightarrow \sim S | [(S)S] | P | d$

$S \rightarrow P$

$S \rightarrow d$

$S \rightarrow XS$

$X \rightarrow \text{any first of the terminals}$

$P \rightarrow [$

$Q \rightarrow C$

$R \rightarrow ]$

$T \rightarrow ]$

$S \rightarrow \overline{PQ} \overline{SR} \overline{ST}$

$S \rightarrow UVW$

$U \rightarrow PQ$

$V \rightarrow SR$

$W \rightarrow ST$

$Z \rightarrow UV$

Finally we can conclude that if the starting symbol is available on top most corner of table then, the string is accepted.

So here the string is accepted by Grammar.

$$\begin{aligned} Q) S &\rightarrow AB/BC \\ A &\rightarrow BA/A \\ B &\rightarrow CC/b \\ C &\rightarrow AB/A \end{aligned}$$

(\*) Here number of alphabet is 5.  
So we should draw a matrix of 5 row x 5 column

1	$\{S, CA\}$	$\{B\}$	$\{B\}$	$\{S, C\}$	$\{A\}$
2	$\{B\}$	$\{S, C\}$	$\{A, S\}$	$\{B\}$	
3	$\{B\}$	$\{S, C\}$	$\{A, S\}$		
4	$\{S, A\}$	$\{B\}$			
5	$\{A\}$				

Given string is

"ababa".

So we have to check whether this string is accepted or not.

Step 1:  $a \rightarrow 1$  (1, 2)  $b \rightarrow 2$  (2, 3)  $a \rightarrow 3$  (3, 4)  $b \rightarrow 4$  (4, 5)  $a \rightarrow 5$  (5, 1)

+ one combination

$$\checkmark (1, 1) = a = A \{ \text{as } A \rightarrow a \} \leftarrow (A, C)$$

$$\checkmark (2, 2) = b = B \{ \text{as } B \rightarrow b \}$$

$$\checkmark (3, 3) = a = A, C \{ \text{as } A \rightarrow a, C \rightarrow c \}$$

$$\checkmark (4, 4) = a = B$$

$$\checkmark (5, 5) = a = A, C$$

two combination

$$\checkmark (1, 2) = (1, 1)(2, 2) = \{S, C\}(S, C) = \{A, C\}(B) = AB, BC = \{S, C\}, \emptyset$$

$$\checkmark (2, 3) = (2, 2)(3, 3) = \{B\}(S, C) = \{A, S\}(B) = AS, BC = \{S, C\}, \emptyset$$

$$\checkmark (3, 4) = (3, 3)(4, 4) = \{A, C\}(B) = AB, CB = \{S, C\}, \emptyset$$

$$\checkmark (4, 5) = (4, 4)(5, 5) = \{B\}(A, C) = BA, BC = A, S$$

three combination

$$\checkmark (1, 3) = (1, 1)(2, 3) = \{A, C\}\{S, C\} = AA, AC, CA, CS = \emptyset, \emptyset, \emptyset, \emptyset$$

$$\text{OR, } (1, 2)(3, 3) = \{S, C\}\{S, C\} = SS, SC, CS, CC = \emptyset, \emptyset, \emptyset, \underline{B}$$

$$\checkmark (2, 4) = (2, 2)(3, 4) = \{B\}\{S, C\} = BS, BC = \emptyset, \underline{S}$$

$$\text{OR, } (2, 3)(4, 4) = \{A, S\}\{B\} = \del{AB, SB} = \{S, C\}, \emptyset$$

$$\checkmark (3, 5) = (3, 3)(4, 5) = \{A, C\}\{S, A\} = AS, AA, SC, CA = \emptyset, \emptyset, \emptyset, \emptyset$$

$$\text{OR, } (3, 4)(5, 5) = \{S, C\}\{A, C\} = SA, CA, SA, CC = \emptyset, \emptyset, \emptyset, \underline{B}$$

four Combination

$$\checkmark (1, 4) = (1, 1)(2, 4) = \{A, C\}\{S, C\} = \emptyset$$

$$\text{OR, } (1, 2)(3, 4) = \{S, C\}\{S, C\} = SS, SC, CS, CC = \underline{B}$$

$$\text{OR, } (1, 3)(4, 4) = \{B\}\{B\} = \emptyset$$

$$\checkmark (2, 5) = (2, 2)(3, 5) = \{B\}\{B\} = BB = \emptyset$$

$$\text{OR, } (2, 3)(4, 5) = \{A, S\}\{S, A\} = AS, AA, SS, SA = \emptyset$$

$$\text{OR, } (2, 4)(5, 5) = \{S, C\}\{A, C\} = SA, SC, CA, CC = \underline{B}$$

five combination

$$\checkmark (1, 5) = (1, 1)(2, 5) = (A, C)(B) = AB, CB = \{S, C\}, \emptyset$$

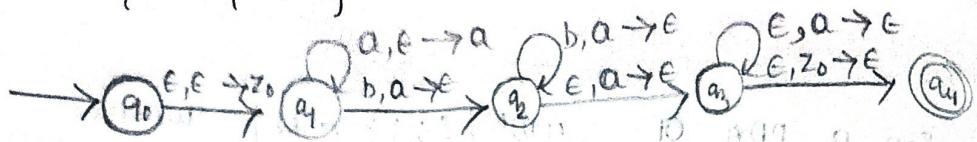
$$\text{OR, } (1, 2)(3, 5) = (S, C)(B) = SB, CB = \emptyset$$

$$\text{OR, } (1, 3)(4, 5) = \{B\}\{S, A\} = BS, BA = \emptyset, A$$

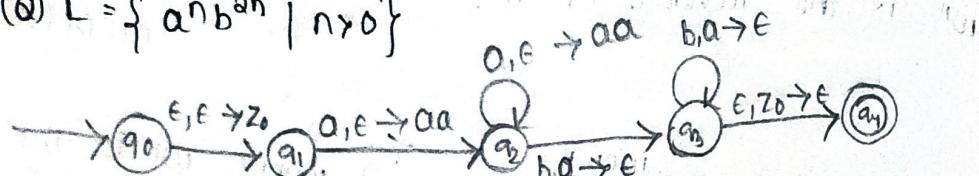
$$\text{OR, } (1, 4)(5, 5) = \{B\}\{A, C\} = BA, BC = \{S, A\}$$

$$\text{OR, } (1, 5)(2, 5) = \{S, C\}\{A, C\} = SA, SC = \emptyset$$

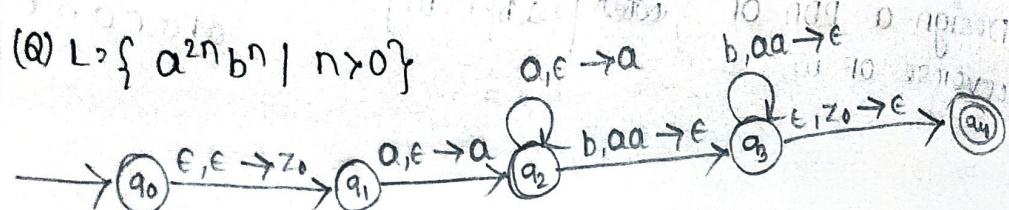
(Q)  $L = \{a^m b^n \mid m > n\}$



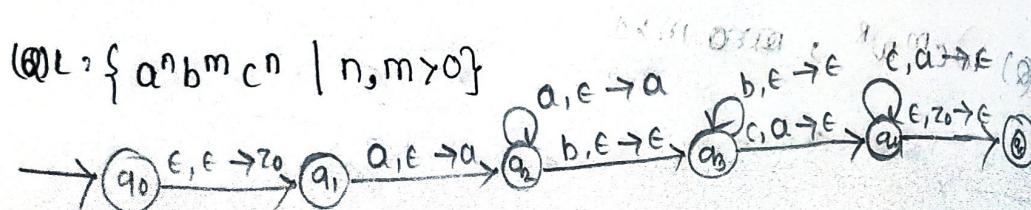
(Q)  $L = \{a^n b^n \mid n > 0\}$



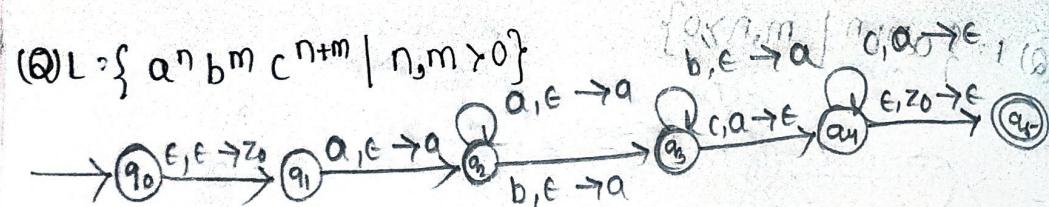
(Q)  $L = \{a^{2n} b^n \mid n > 0\}$



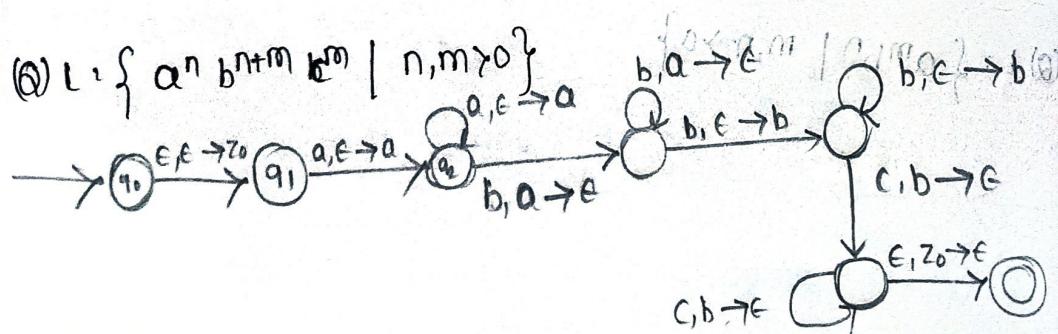
(Q)  $L = \{a^n b^m c^n \mid n, m > 0\}$



(Q)  $L = \{a^n b^m c^{n+m} \mid n, m > 0\}$



(Q)  $L = \{a^n b^{n+m} c^m \mid n, m > 0\}$



### Instantaneous Description ID -

- \* Instantaneous Description, ID, describe the current instance of a PDA.
- \* It can be described through three tuple.

$(q_i, w, s)$

$q \rightarrow$  current state,  $w \rightarrow$  Remaining string input  
 $s \rightarrow$  stack symbol

### (\*) TYPE OF PDA :-

PDA can be broadly categorized into D-PDA and N-PDA.

#### (a) NPDA :-

Non Deterministic Push Down Automata

- \* In this PDA, there is no unique transition for single input symbol.
- \* Generally in designing of PDA, NPDA is used.

#### (b) DPDA

Deterministic Push Down Automata

- \* Similar to DFA, DPDA has also unique transition for every input symbol.

\* If  $\delta(q, a, x)$  is defined as unique  
where as  $a \in \Sigma, x \in \Gamma$

- \* If  $\delta(q, a, x) \neq \emptyset$  then  $\delta(q, a, x) = \emptyset$
- \*  $\delta(q, a, x) = \emptyset$  then  $\delta(q, a, x) \neq \emptyset$

## How to Represent Transition function :-

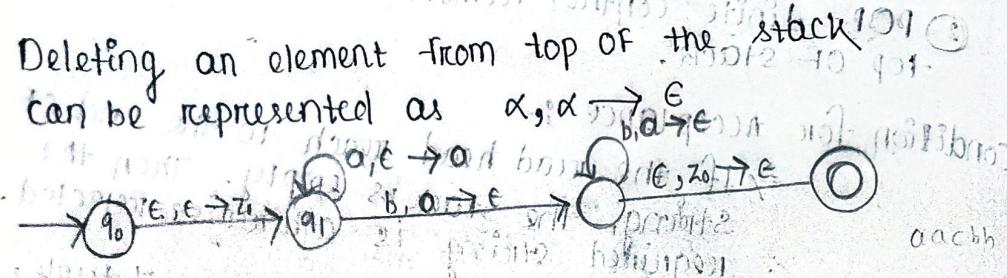
Transition function for push down automata is represented through following notation i.e instantaneous description.

\* Here a triplet format is used i.e.  $(q, w, z)$  where  $q$  is the state,  $w$  is the input and  $z$  is the stack contains.

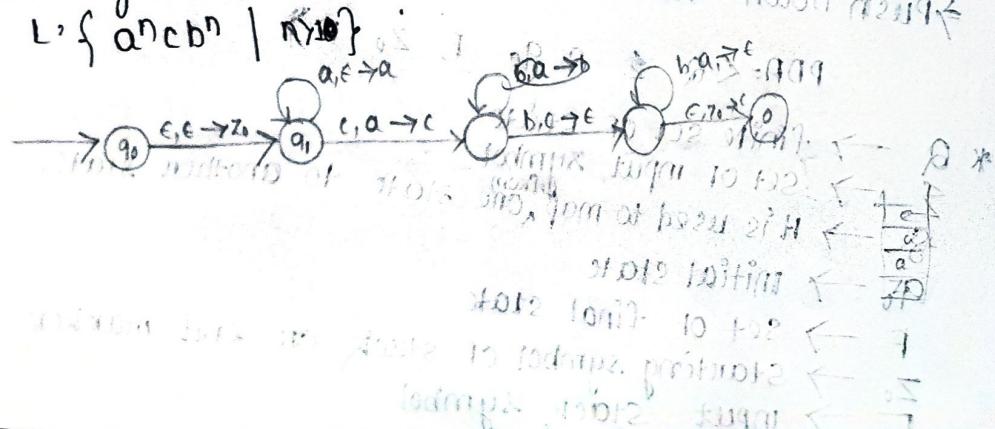
Here,  $a = \text{input symbol}$   
 $b = \text{popped value from stack}$   
 $c = \text{pushed symbol into top of stack.}$

(Q) Design a PDA, that accept the language  $L = \{a^n b^n | n \geq 0\}$

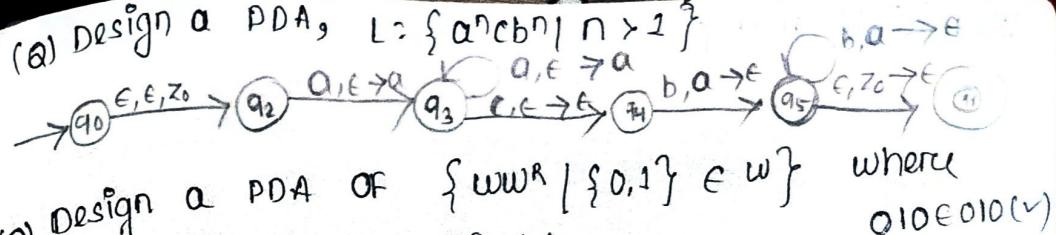
NOTE :-  
 Pushing an element into the stack can be represented as  $\alpha, \epsilon \rightarrow \alpha \alpha$ .



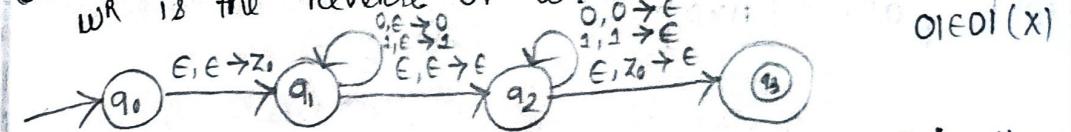
(Q) Design a PDA for  $L = \{a^n c^n | n \geq 0\}$



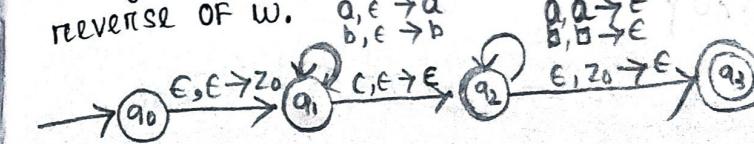
(Q) Design a PDA,  $L = \{a^n c^n | n >= 1\}$



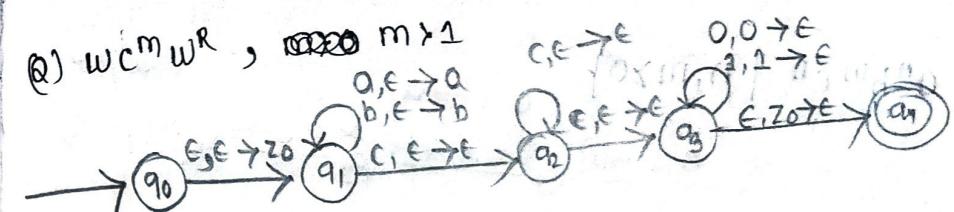
(Q) Design a PDA of  $\{wwr | \{0,1\}^* \in w\}$  where  $wr$  is the reverse of  $w$ .



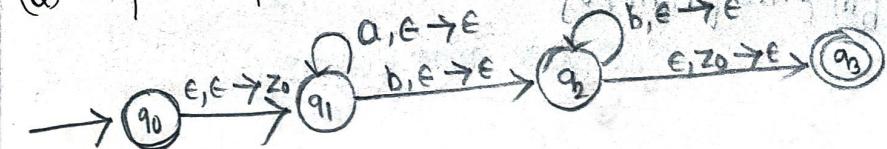
(Q) Design a PDA of  $\{wcr | \{0,1\}^* \in w\}$  where  $wr$  is the reverse of  $w$ .



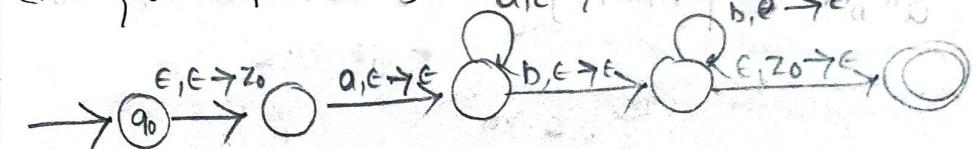
(Q)  $w^m w^r$ ,  $m > 1$



(Q)  $L = \{amb^n | m, n > 0\}$

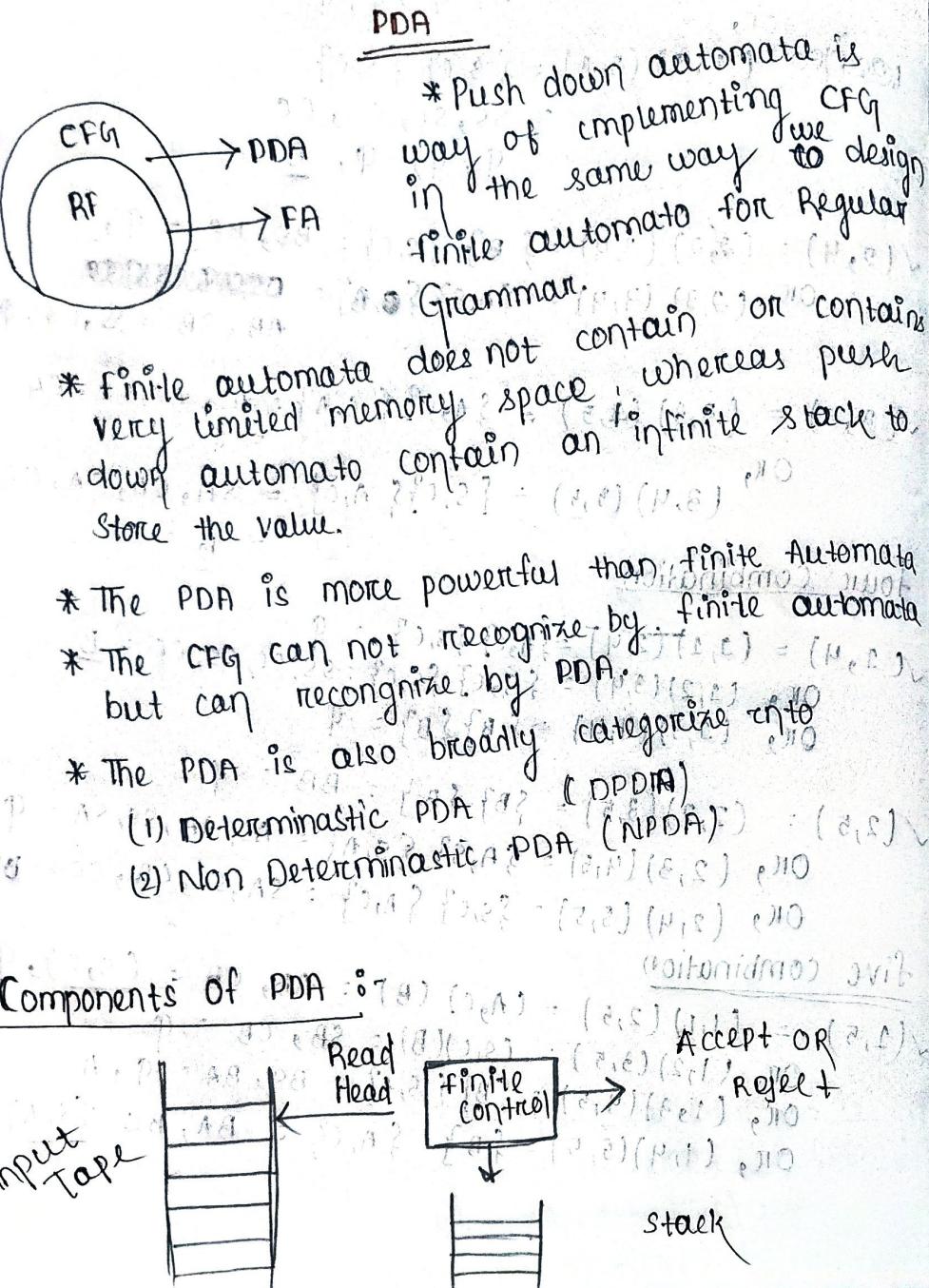


(Q)  $L = \{amb^n | m, n > 0\}$



## PUSHDOWN AUTOMATA

### UNIT-03



1) Input tape -  
 \* It is a infinite tape contain the input string.  
 \* Every field of the input tape, can contain atmost one symbol.

2) Read head -  
 \* It read the symbol of input tape.

3) Finite control -  
 It read the content from input tape. Then it may push or pop to stack and give it final decision.

4) Stack -  
 Stack is a infinite data structure that can store the ~~information~~ stack input. Here two operation can be perform.

- ① PUSH - finite control store an element from top of stack.
- ② POP - finite control remove an element from top of stack.

Condition for acceptance :-  
 ⇒ If the read head reach to the end of string, the stack is empty. Then the required string is otherwise rejected.

⇒ Push Down Automata defined through 7 tuple.

$$PDA = \langle Q, \Sigma, \Gamma, \delta, q_0, F, Z \rangle$$

- \*  $Q \rightarrow$  finite set of state
- $\Sigma \rightarrow$  set of input symbol
- $\delta \rightarrow$  It is used to map one state to another state.
- $q_0 \rightarrow$  initial state
- $F \rightarrow$  set of final state
- $Z \rightarrow$  starting symbol of stack or end marker
- $\Gamma \rightarrow$  input stack symbol

$$S \rightarrow S(q_i, a) \rightarrow (q_j, x, D)$$

$q_0$  → initial state,  $\epsilon Q$

$F \subseteq Q$  → set of final states,  $F \subseteq Q$

$B$  → special kind of symbol is used to represent the blank cell on the input tape.

$\Gamma$  → set of tape symbols.

$$S(q_i, a) \rightarrow (q_j, x, D)$$

where  $q_i, q_j \in Q$

$a \in \Sigma, x \in \Gamma$

$D = \{R, L\}$ , // D symbolizes movement of Read, write, Head

(Q) Design Turing machine that can accept the string '011'.

# | 0 | 1 | 1 | #

$$S(q_0, 0) \rightarrow (q_1, x, R)$$

$$S(q_1, 1) \rightarrow (q_2, y, R)$$

$$S(q_2, 1) \rightarrow (q_3, y, R)$$

$$S(q_3, \#) \rightarrow \text{Accept}$$

	Input	Output	Transition
$q_0$	$(q_1, x, R)$		blank cell to x
$q_1$	$(q_2, y, R)$		current position to y
$q_2$	$(q_3, y, R)$		current position to y
$q_3$	-	Accept	

(Q) Design a turing machine that accept the string ending over 0 and 1 and end with 1.

(6)  $S \rightarrow aAB \mid bA \mid ab$

$A \rightarrow aA \mid b$

$B \rightarrow bB \mid a$

String "aaabbba"

Derivation:  $S \Rightarrow aAB \Rightarrow aaAB \Rightarrow aaabbA \Rightarrow aaabbba$

$a^2 \mid a^3 \mid a^2 0$

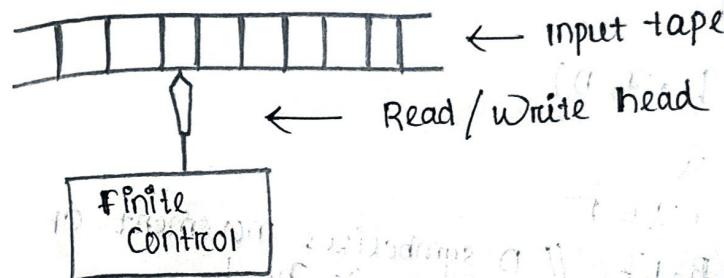
$0 \mid 1 \mid \#$

$\# \mid . \mid 0$

"0001000" = 0

## TURING MACHINE

- \* Turing machine can recognize type-0 Grammer.
- \* Alan Turing in year 1936 has invented Turing machine. He was an English mathematician.
- \* Turing machine is a generalized form of PDA.



\* There input tape is an infinite tape, that can contain or accommodate infinite input symbol or output symbol. It is containing one special symbol for denoting blank space. (#) (\$) -- so it combine the features of input tape and stack in one unit.

(\* ) Read/Write head points the cell in the input tape where the finite control want to perform operation.

(\* ) Read/Write head can move in the both the direction

(\* ) finite control control all the operation through read/write head as per requirement.

Defining Turing Machine :-

\* Turing machine can also define through 7 tuples.

$\langle Q \sqsubseteq S \sqsubseteq F \mid B, \delta \rangle$

\*  $Q$  = finite set of states

$\sqsubseteq$  = Set of input symbol

## Conversion

### CONVERSION FROM CFG TO PDA

Context free Language

Defined by or

Represented by CFG

Accepted or Recognized

by PDA

→ To convert the CFG to PDA two rules are apply as general form of CFG  $A \rightarrow \alpha$  where  $A$  is single non-terminal value and  $\alpha$  is any combination of terminal and non-terminal value.

RULE:1

(1) for Non-terminal

$$\delta(q, \epsilon, A) \rightarrow (q, \alpha)$$

RULE:2

(2) for terminal

$$\delta(q, a, a) \rightarrow (q, \epsilon)$$

(Q) convert this CFG to PDA.

$$S \rightarrow OS1 | O0 | 11$$

$$S \rightarrow OS1 \quad S(q, \epsilon, S) \rightarrow (q, OS1) \rightarrow ①$$

$$S \rightarrow O0 \quad S(q, \epsilon, S) \rightarrow (q, O0) \rightarrow ②$$

$$S \rightarrow 11 \quad S(q, \epsilon, S) \rightarrow (q, 11) \rightarrow ③$$

for terminals  $p$   $p \in \{0, 1, S\}$

$$\delta(q, 0, 0) \rightarrow (q, \epsilon) \rightarrow ④$$

$$\delta(q, 1, 1) \rightarrow (q, \epsilon) \rightarrow ⑤$$

(Q) check for the string 0111 for acceptance or Rejection.

$$\delta(q, 0111, S20) \xrightarrow{①} \delta(q, 0111, OS120) \xrightarrow{②}$$

$$\vdash \delta(q, 111, S120) \xrightarrow{③}$$

$$\vdash \delta(q, 111, 11120) \xrightarrow{④}$$

$$\vdash \delta(q, 11, 1120) \xrightarrow{⑤}$$

$$\vdash \delta(q, 1, 120) \xrightarrow{⑥}$$

$$\vdash \delta(q, \epsilon, 20) \text{ (ACCEPT)}$$

$$(Q) S \rightarrow OS1 | O0 | 1B$$

$$A \rightarrow 1A | 0$$

$$B \rightarrow 0B | 1$$

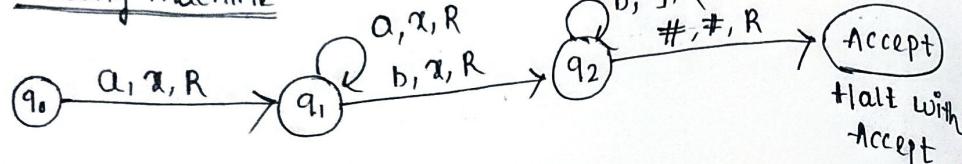
$$w = "0001000"$$

(Q) Design a turing machine for  $L = \{a^n b^m \mid n, m \geq 0\}$

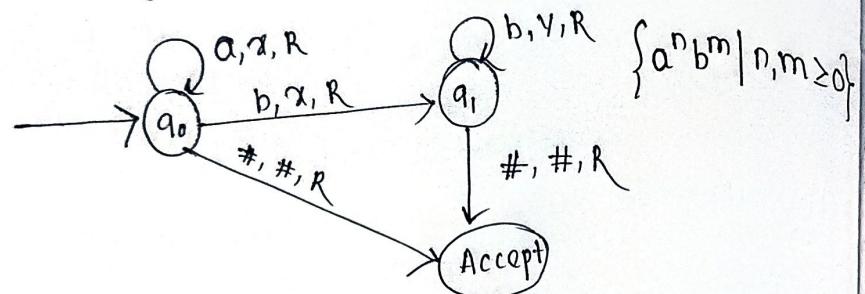
DFA



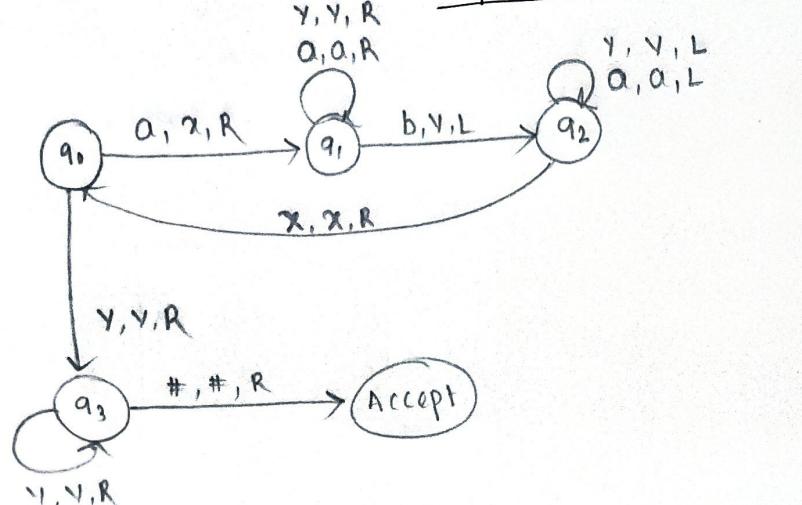
Turing machine



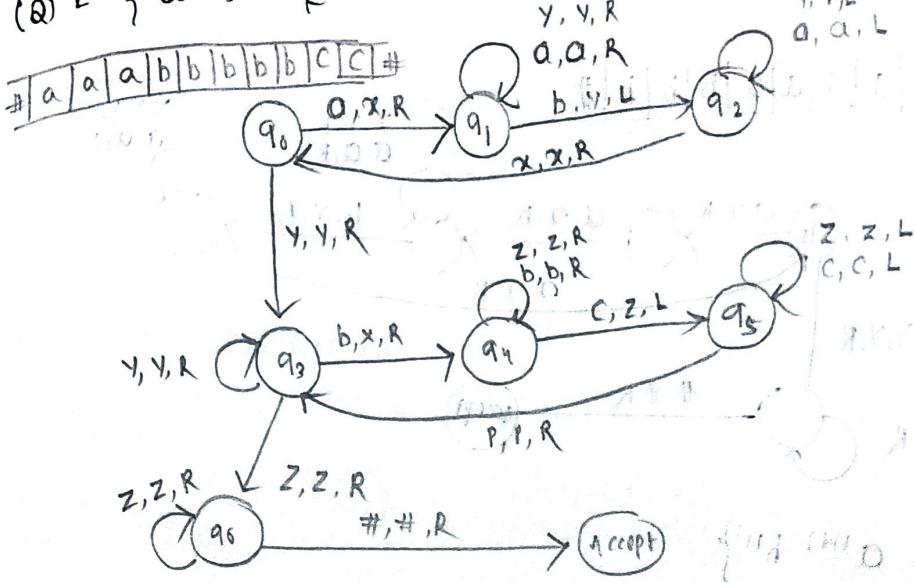
\*\* +halt with Reject - dead state



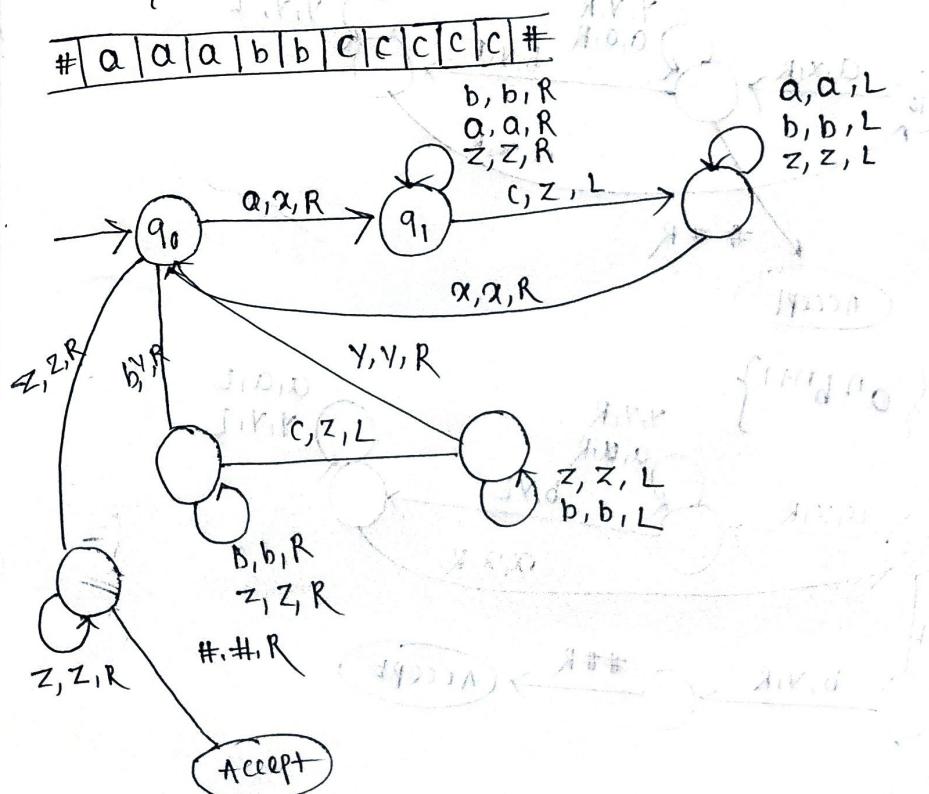
(Q)  $L : \{a^n b^n \mid n > 0\}$



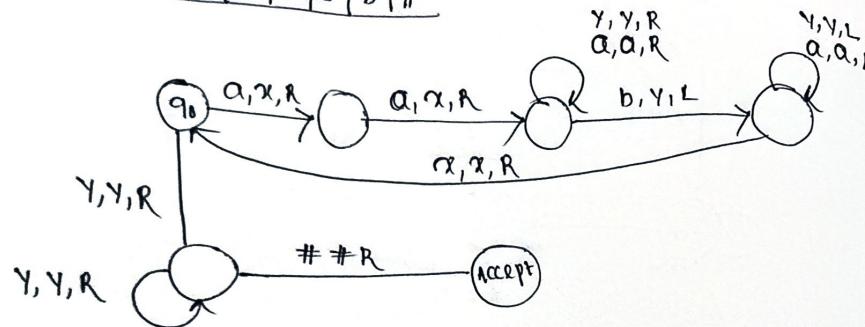
(Q)  $L = \{a^n b^{n+m} c^m \mid n, m \geq 0\}$



(Q)  $L = \{a^n b^m c^{m+n}\}$

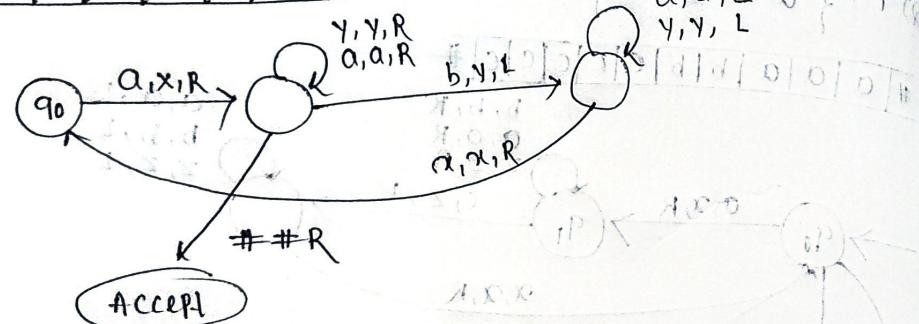


# a a a a a a b b | b #

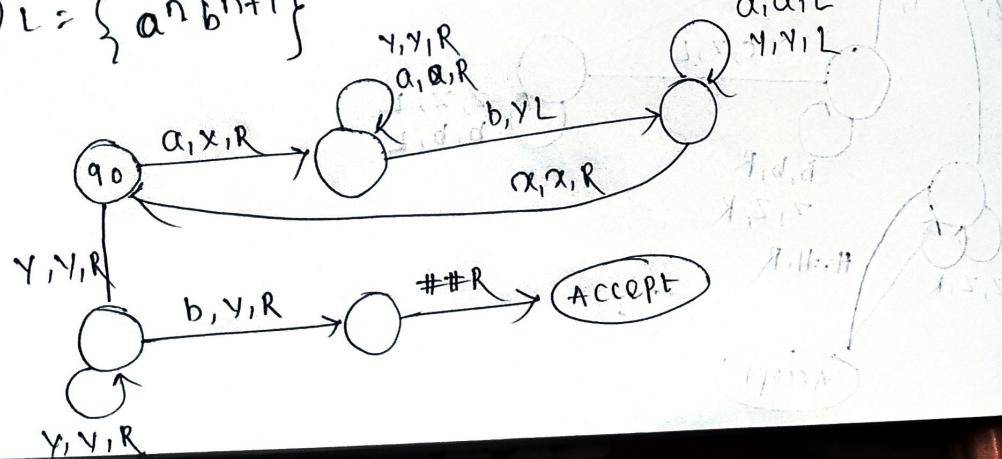


(Q)  $L : \{ a^{n+1} b^n \}$

# a a a a b b | b #



(Q)  $L = \{ a^n b^{n+1} \}$



Turing Machine can also used for Evaluation.  
Evaluation

- 1) Find 1's complement
- 2) find 2's complement
- 3) Addition of two number

(1) Find 1's Complement

# 0 1 0 0 1 0 #

