Fuzzy Set:

There are two kinds of set

(i) Classical Set (Crisp Set)
(ii) Fuzzy Set

Classical Set:

If x be a universal set, then the set A can be represented for all elements $x \in X$ by its characteristics functions.

$$x_c(x) = \begin{cases} 1 \text{ if } x \in X \\ 0 \text{ otherwise} \end{cases}$$

In the classical set theory, $x_c(x)$ has only the values 0 (false) and 1 (true). Such set are called crisp sets.

Consider the set S as

$$S = \{x \in R : x \geq 6\}$$

The representation implies that if $x \geq 6$, then x is a member of S; otherwise x is not a member of S.

(Note: In crisp set either it will belong or not belong to the set)

Fuzzy Set: In the fuzzy set theory, the characteristic function defined on a set F generalized to a membership function that assigns to every $x \in X$, a value from the unit interval $[0,1]$ instead of the two elements set $\{0,1\}$.

The membership function μ_F of a fuzzy set F is a function

$$\mu_F : X \to [0,1]$$

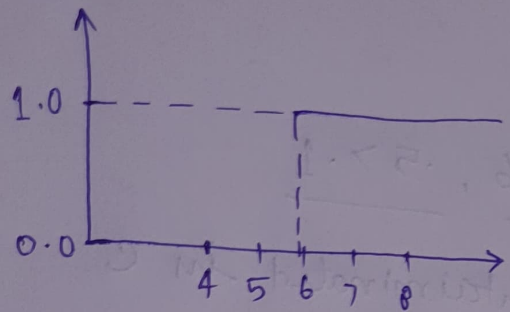
So every element x from X has a membership degree $\mu_F(x) \in [0,1]$.

F is completely determined by the set of tuples:

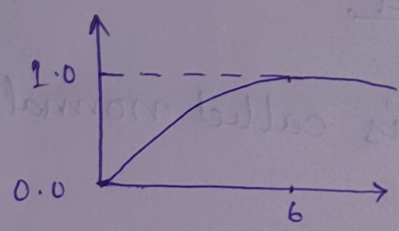
$$F = \{x, \mu_F(x) \mid x \in X\}$$

Membership Function: A fuzzy set is any set that allows its members to have different degree of membership, called membership function in interval $[0,1]$

Crisp versus Fuzzy Set:



Crisp Set



Fuzzy Set

Containment in Fuzzy Set:

Let X be a set and A, B are two fuzzy set of X with membership function μ_A and μ_B respectively. We say that the fuzzy set A is contained in the fuzzy set B if and only if

$$\mu_A(x) \leq \mu_B(x) \text{ for all } x \in X$$

Example: If $X = \{1, 2, 3\}$ and A, B, C are three fuzzy set given

$A = \left\{ \frac{.1}{1}\ \frac{.5}{2}\ \frac{.1}{3} \right\}$ $B = \left\{ \frac{.1}{1}, \frac{.4}{2}, \frac{.7}{3} \right\}$, $C = \left\{ \frac{.1}{1}\ \frac{.6}{2}\ \frac{.5}{3} \right\}$

then $A \subseteq C$

since $\underline{.1 = .1}$, $\underline{.5 < .6}$, $\underline{.5 > .1}$

Fuzzy set A is containminated in C

Normal and Abnormal fuzzy set:

A fuzzy set A of a set X is called normal fuzzy set if and only if

$$\max_{x \in X} \mu_A(x) = 1$$

($\mu_A(x) = 1$ for atleast one $x \in X$)

otherwise, fuzzy set is subnomal

Example: if $X = \{1, 2, 3, 4\}$ and $A = \left\{ \frac{.1}{1}, \frac{1}{2}, \frac{.2}{3}, \frac{.5}{4} \right\}$

there is a value present $\mu_A(x) = 1$, so we can say it is nomal fuzzy set

Height of Fuzzy Set:

The height of a fuzzy set A on the universal set X is the largest membership grade attained by an element

$$\text{Height}(A) = \max_{x \in X} \{\mu_A(x)\}$$

Core of a Fuzzy Set:

The core of a fuzzy set A on the universal set X is the crisp set that contains only those elements of X for which

$$\mu_A(x) = 1$$

$$\text{Core}(A) = \{x \in X : \mu_A(x) = 1\}$$

Convex:

A fuzzy set A on the universe X is said to be convex is said to be convex if for any three element $x, y, z \in A$ where $x < y < z$, we have

$$\mu_A(y) \geq \min\{\mu_A(x), \mu_A(z)\}$$

Support of A fuzzy set

The support of fuzzy set A of the set X is crisp set that contains all the elements of X that have non zero membership grade in A denoted by supp (A)

$$Supp(A) = \{x \in X : \mu_A(x) > 0\}$$

α- Cut or α-Level Set:

The α- level set is a crisp set that belong to the fuzzy set A at least to the degree α.

$$A_\alpha = \{x \in X : \mu_A(x) \geq \alpha\}$$

Value of α can be $0 \leq \alpha \leq 1$

Strong α- Level set:

It is defined as

$$A_\alpha^+ = \{x \in X : \mu_A(x) > \alpha\}$$

Example: Suppose $X = \{1, 2, 3, 4, 5, 6\}$ and consider A fuzzy set on X given by

$A = \{(1, 0.2), (2, .5), (3, .7), (4, 1), (5, 0.8), (6, .3)\}$

Then all the possible α - sets are

$A_{0.2} = \{1, 2, 3, 4, 5, 6\}$

$A_{0.3} = \{2, 3, 4, 5, 6\}$

$A_{0.5} = \{2, 3, 4, 5\}$

$A_{0.7} = \{3, 4, 5\}$

$A_{0.8} = \{4, 5\}$

$A_1 = \{4\}$

Basic Operations on Fuzzy Sets:

(i) Equal Set:

Two fuzzy sets A and B are equal if

$$\mu_A(x) = \mu_B(x) \text{ for all } x \in X$$

Example of two equal set

$A = \{(x_1, 0.3), (x_2, 0.5),\}$

$B = \{(x, 0.3), (x_2, 0.5)\}$

A and B are equal fuzzy set.

(ii) Complement:

The complement of a fuzzy set A is denoted by A^c or A', and it's defined by membership function as $\mu_A^c(x) = 1 - \mu_A(x)$ for all X.

If $A = \{(x_1, 0), (x_2, 0.3), (x_3, 0.5)\}$

$A^c = \{(x_1, 1-0), (x_2, 1-0.3), (x_3, 1-0.5)\}$

$A^c = \{(x_1, 1), (x_2, 0.7), (x_3, 0.5)\}$

(iii) Union:

The union of two fuzzy set A and B

$C = A \cup B$

where $\mu_C(x) = \max[\mu_A(x), \mu_B(x)]; \ x \in X$

$A = \{(4, 0.1), (6, 0.5), (8, 0.6)\}$

$B = \{(4, 0.2), (6, 1), (8, 0.4)\}$

$C = A \cup B$

$C = \{(4, 0.2), (6, 1), (8, 0.6)\}$

(iii) Intersection:

The intersection of two fuzzy set A and B is a fuzzy set C given by

$C = A \cap C$

where $\mu_C(x) = \min[\mu_A(x), \mu_B(x)]; \ x \in X$

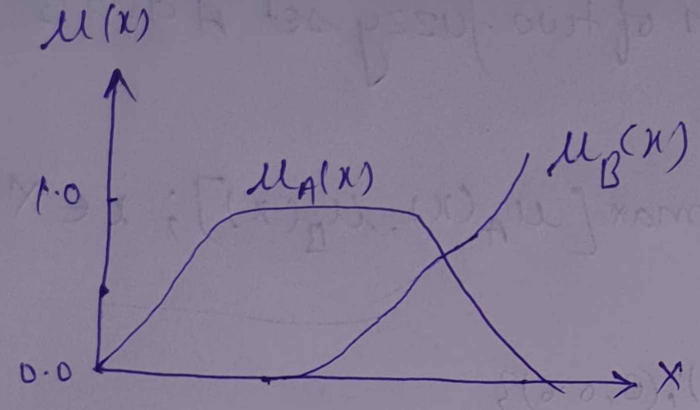
$A = \{(3, 0.1), (5, 0.7), (7, 0.7)\}$

$B = \{(3, 0.4), (5, 0.8), (7, 0.3)\}$

$C = A \cap C$

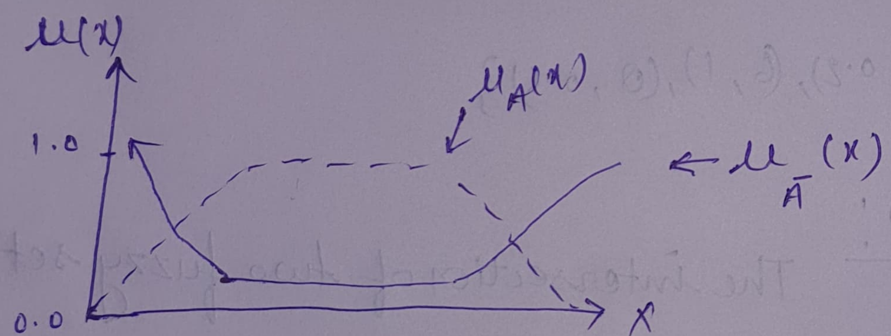
$\mu_C(x) = \{(3, 0.1), (5, 0.7), (7, 0.3)\}$

Graph Representation of Union, Intersection and Complement

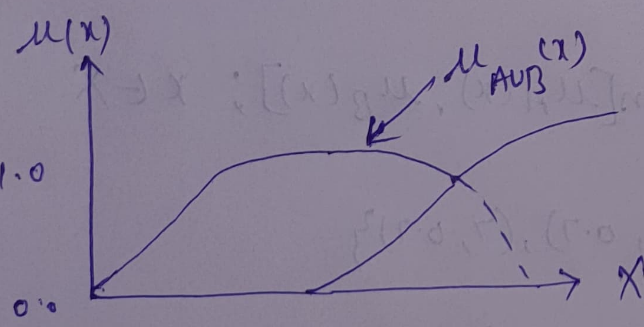


$\mu_A(x)$ and $\mu_B(x)$

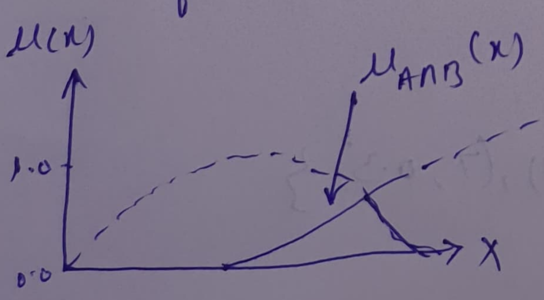
Complement: of $\mu_A(x)$



Union of μ_A and μ_B



Intersection of μ_A and μ_B



(iv) Difference of fuzzy set: The difference of two fuzzy sets A and B is defined by

$$A - B = A \cap B^c$$

Properties of fuzzy set operation: The properties of fuzzy set operations that are common to crisp set operations are

(1) Idempotent: $A \cup A = A$, $A \cap A = A$

(2) Commutative: $A \cup B = B \cup A$, $A \cap B = B \cap A$

(3) Associative: $\left.\begin{array}{l}(A \cup B) \cup C = A \cup (B \cup C) \\ (A \cap B) \cap C = A \cap (B \cap C)\end{array}\right\}$

(4) Distributive: $\left.\begin{array}{l}A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \\ A \cup (B \cap C) = (A \cup B) \cap (A \cup C)\end{array}\right\}$

(5) Absorption: $\left.\begin{array}{l}A \cup (A \cap B) = A \\ A \cap (A \cup B) = A\end{array}\right\}$

(6) Identity:

$$A \cup \phi = A, \; A \cap U = A$$

(7) Involution:

If A is a fuzzy set, then $(A^c)^c = A$

(8) De Morgan's Law:

If A and B are two fuzzy sets then

(i) $(A \cup B)^c = A^c \cap B^c$

(ii) $(A \cap B)^c = A^c \cup B^c$

Algebraic Sum: The algebraic sum of two fuzzy sets A and B is defined by the membership function as

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x)\,\mu_B(x) \text{ for all } x \in X$$

Algebraic Product: The algebraic product of two fuzzy set A and B is defined by two membership function as

$$\mu_{A \cdot B} = \mu_A(x) \cdot \mu_B(x) \text{ for All } x \in X$$

Fuzzy Function and Linguistic Variable :

Linguistic variables is quite useful in representing values in Fuzzy Set. In fuzzy systems, variable ranges over states - which are fuzzy numbers rather than real numbers.

Fuzzy numbers are often represented as linguistic variables such as HOT, COLD, WARM etc.

Each linguistic variable consist of the following elements :

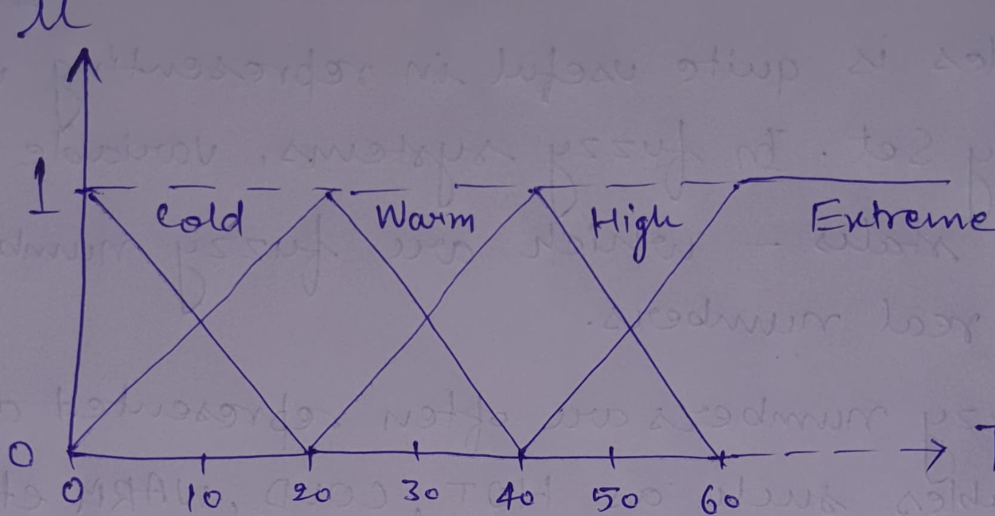
→ A name, which should be capture the meaning of the bare variable involved.

→ A bare variable with its range of values.

→ A set of linguistic terms that refer to values of the bare variable.

→ A semantic rule, which assigned to each linguistic term its meaning.

Example of linguistic variable



We can categorize linguistic variable as follow:

Quantification Terms: All, most, many, some

Usually Terms: Always, sometimes, seldoms,

Likelihood Terms: certain, likely, possible, certainely not,

Example of Linguistic Variable:

A = cold climate with $\mu_A(x)$ as the MF

B = Hot climate with $\mu_B(x)$ as the MF

Not Cold Climate: $A' = 1 - \mu_A(x)$

Not Hot Climate: $B' = 1 - \mu_B(x)$

Extreme Climate: $A \cup B = \max(\mu_A(x), \mu_B(x))$

Pleasant Climate: $A \cap B = \min(\mu_A(x), \mu_B(x))$

Fuzzification and Defuzzification:

Fuzzification is a process of tranfoming crisp set into fuzzy set or fuzzy set into fuzzier set.

Defuzzification: is a process of reducing a fuzzy set into crisp set or convert a crisp member into fuzzy member.

FIS (Fuzzy Inference System):

It is the key unit of fuzzy logic system having decision making as its primary work. It uses "IF _ _ THEN" rules along with connectors "OR" and "AND" for drawing essential decision rules.

Functional Block of FIS:

(1) Rule Base: It contains the fuzzy 'IF _ _ THEN' rules.

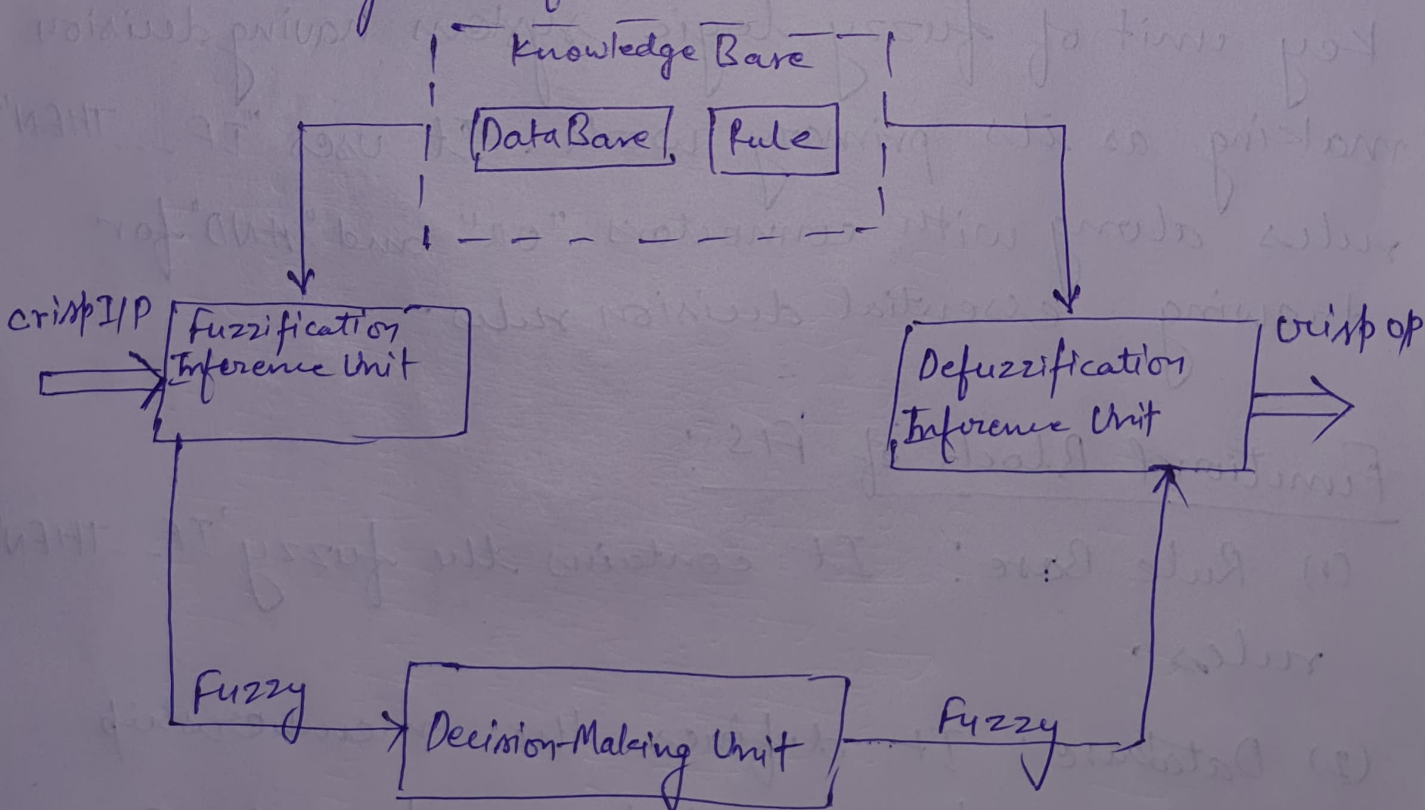
(2) Database: It defines the membership function of fuzzy sets used in fuzzy rules.

(3) Decision-Making Unit: It perfoms operations on rules.

(4) Fuzzification Inferenne Unit: It converts crisp quantities to fuzzy quantities.

(5) Defuzzification Inference Unit! It converts the fuzzy quantites into crisp quatities.

Block Diagram of FIS:



Methods of FIS:

There are two methods

(i) Mamdani Fuzzy Inference System

(ii) Takagi - Sugeno Fuzzy Model

(i) Mamdani Fuzzy Inference System:

Steps for computing the Output:

(1) Set of fuzzy rules needs to be determined

(2) In this step, by using input membership funtion, the input would be made fuzzy.

(3) Establish the rule strength by combining the fuzzified input

(4) In this step, determine the consequent of rules by combining the rule strength.

(5) For getting output distribution combine all the consequent

(6) Finally, A defuzzified output distribution is obtained.

Takagi - Sugeno Method (TS)

Format of this rule is given as:

IF x is A and Y is B THEN Z = f(x, y)

Here AB are fuzzy set in ~~ad~~ antencendent and z = f(x, y) is a crisp function of consequent.

Fuzzy Inference process in TS Method

TS Method work in this way of following

Step 1: Fuzzifying the inputs

Step 2: Applying the fuzzy operator

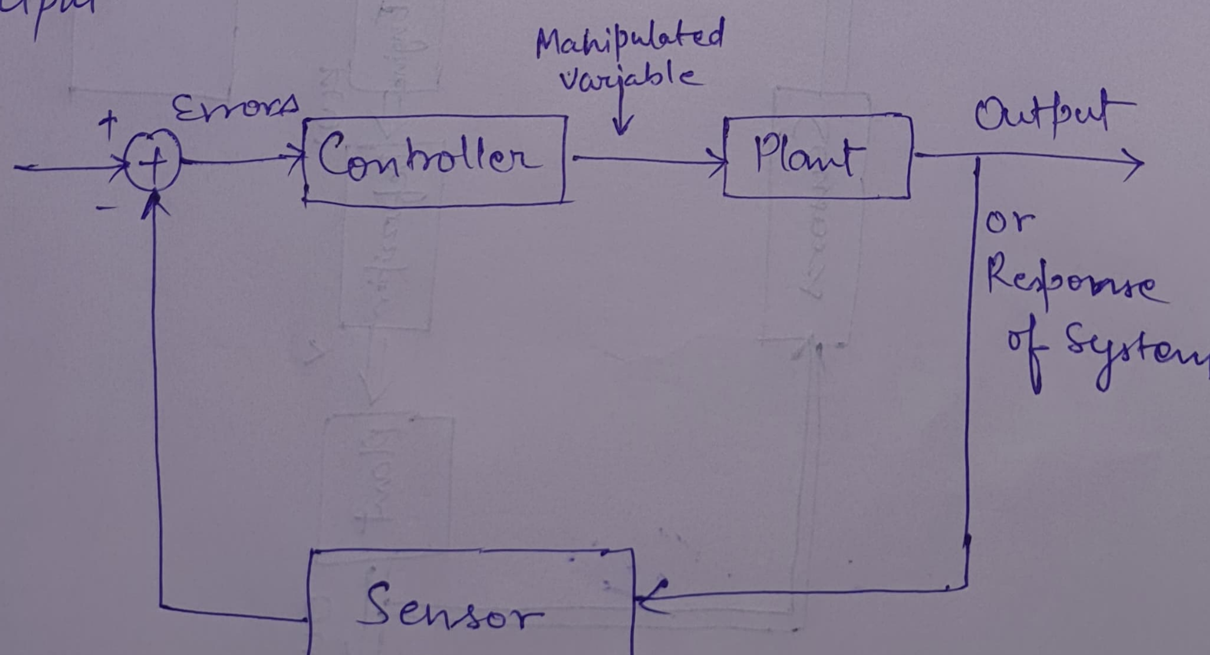
Fuzzy Logic Control System:

A control system is an arrangement of physical componentsdesigned to alter another physical system so that this system exhibits certain desired charalteristics

There exist two types of Control System

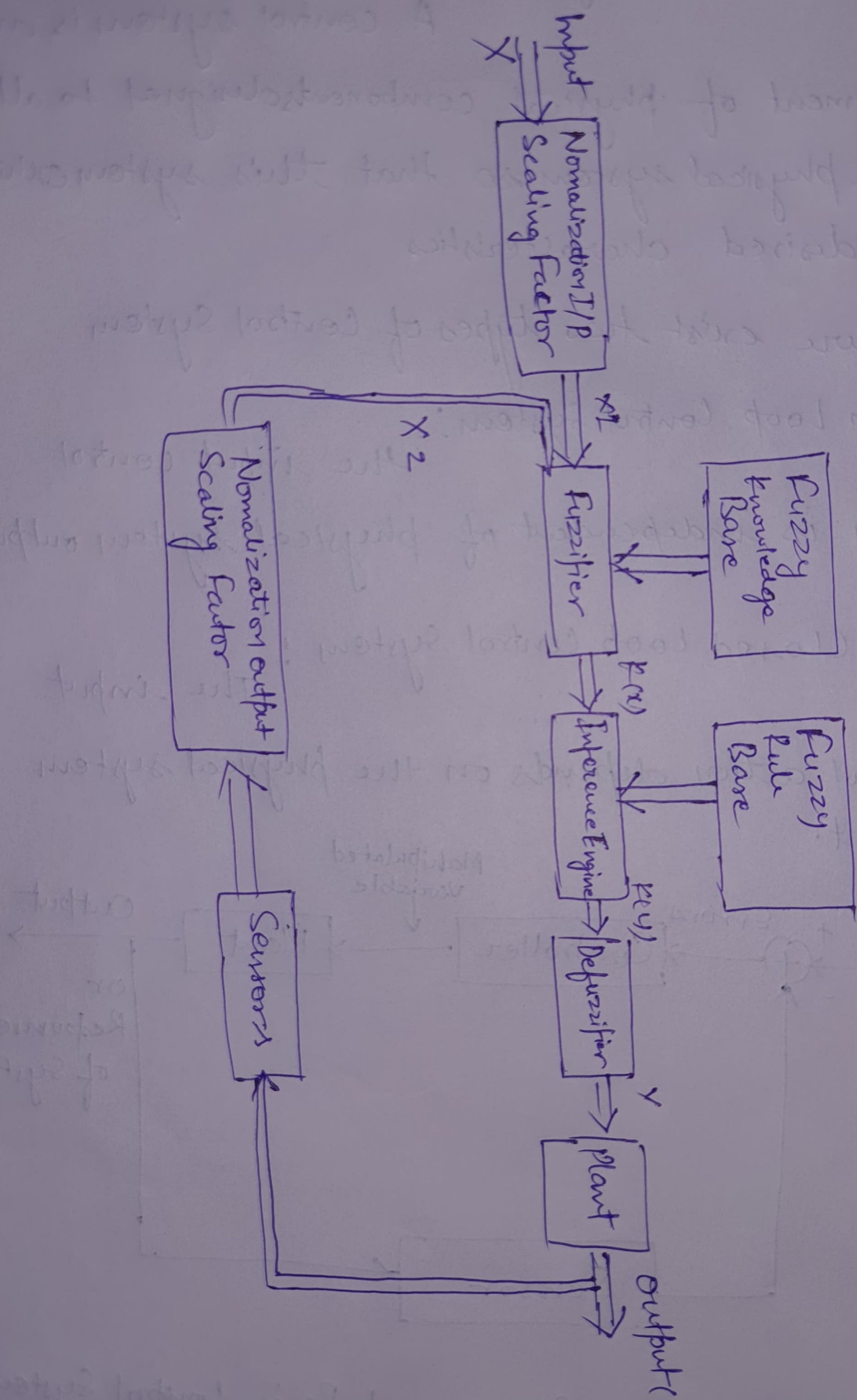
(i) Open Loop Control System:
the input control action is independent of physical system output.

(ii) Closed Loop Control System:
The input control action depends on the physical system output.



Block Diagram of Basic Control System

Basic Diagram of FLC System:



The principal component of FLC system is a Fuzzifier, a fuzzy rule base, a fuzzy knowledge base, an inference engine, and defuzzifier.

Various Steps are involved in Designing FLC:

(i) Locate the input, output and state variable

(ii) Split the complete universe into a number of fuzzy subsets, assigning Linguistic variable.

(iii) Obtain the membership function

(iv) Assign the fuzzy relationship between Input and output.

(v) choose appropriate scaling factors for inputs and Outputs.

(vi) Carry out the fuzzification process.

(vii) Identify the output with approximate logic.

(viii) Combine the fuzzy output obtained

(ix) Finally, apply diffuzzification for crisp output.

Applications of FLC Systems:

(i) Traffic Control

(ii) Steem Engine

(iii) Automatic Running Control

(iv) Adaptive Control

(v) Robotic Control

Block Diagram: Rule Based Fuzzy Systems