(13). L-4.

Dynamic Programming approach

Travelling Sales Person Problem: (TSP)

In Travelling sales person problem (TSP). our objective is to find a tour of minimum cost.

Let $G(V, E)$ be a directed graph with edge cost c_{ij}

$$c_{ii} = 0 \quad \text{for } i = 1, \cdots n$$
$$c_{ij} > 0 \quad \text{for all } (i, j) \in E$$

$$c_{ij} = \infty \quad \text{for all } (i, j) \notin E$$

→ A tour of G is a directed cycle that includes every Vertex in V.

→ the cost of a tour is the sum of the cost of edges on the tour.

→ A tour is a simple path that start and end at same Vertex.

Let $g(i, S)$ be the length of shortest path starting at vertex i, going through all Vertices in S and terminating at vertex 1.

→ the function $g(1, V-\{1\})$ is the length of an optimal sales Person tour and is given by

$$g(1, V-\{1\}) = \min_{2 \le k \le n} \left\{ c_{1k} + g(k, V-\{1, k\}) \right\} \quad -(1)$$

generalizing it and written as

$$g(i, S) = \min_{j \in S} \left\{ c_{ij} + g(j, g(j, S-\{j\} \right\} \quad -(11)$$

Example:- In the graph Given below, find the minimum cost of Travelling start from Vertex 1 and it cover All the Vertices of the Graph.

Scanned by CamScanner

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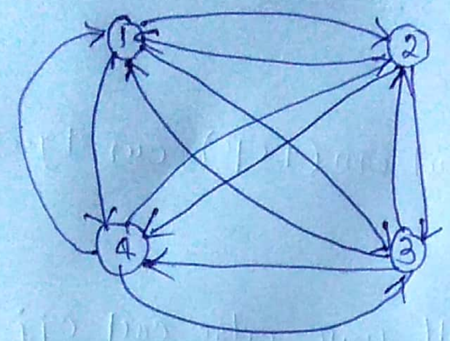

Fig: Directed graph

Adjacency cost matrix

$$A = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 10 & 15 & 20 \\ 2 & 5 & 0 & 9 & 10 \\ 3 & 6 & 13 & 0 & 12 \\ 4 & 8 & 8 & 9 & 0 \end{array}$$

we find a tour which start from Vertex 1 and cover all the Vertices of Graph G (Given).

$$g(1, \{2,3,4\}) = \min_{2 \le k \le n} \left(c_{ik} + g(k\ g(k\ (S - \{k\})) \right)$$

or

$$g(i, S) = \min_{j \in S} \left(c_{ij} + g(j,\ g(j,\ (S - \{j\}))) \right)$$

$\Rightarrow$

$$g(1, \{2,3,4\}) = \min_{j \in S} \left\{ c_{12} + g(2, \{3,4\}),\ c_{13} + g(3, \{2,4\}),\ c_{14} + g(4, \{2,3\}) \right\} \quad -①$$

Here starting Vertex 1 and $S = V - \{1\} = \{2,3,4\}$

For computing minimum cost of $g(1, \{2,3,4\})$ Here is a need to find $g(2, \{3,4\})$, $g(3, \{2,4\})$, $g(4, \{2,3\})$ and the value of c_{12}, c_{13} & c_{14} are known from cost Adjacency matrix A.

$\Rightarrow$ we compute

$$g(2, \{3,4\}) = \min_{j \in S} \left(c_{23} + g(3, \{4\}),\ c_{24} + g(4, \{3\}) \right)$$

Here starting vertex is 2 and $S = \{3,4\}$

then we compute

$$g(3, \{4\}) = \min\{ c_{34} + g(4, \phi) \} = 12 + 8 = 20$$

and $g(4, \phi) = c_{41} = 8$ = starting from Vertex 4 and going through no other node $\Rightarrow$ we have to reach Vertex 1.

$\Rightarrow g(3,\{4\}) = \min\{c_{34} + g(4,\phi)\}$

and we know that

$g(4,\phi) = c_{41} = 8$ (From given cost Adjacency matrix)

$\Rightarrow g(3,\{4\}) = c_{34} + g(4,\phi) = c_{34} + c_{41} = 12+8 = 20$

then we compute

$g(4,\{3\}) = \min\{c_{43} + g(3,\phi)\}$

$= c_{43} + c_{31} = 9+6 = 15$

(since $g(3,\phi) = c_{31} = 6$.

$\Rightarrow g(2,\{3,4\}) = \min\{c_{23} + g(3,\{4\}),\ c_{24} + g(4,\{3\})\}$

$= \min\{9+20,\ 10+15\} = \min\{29, 25\}$

$= 25$

Now we compute $g(3,\{2,4\}) = \min\{c_{32} + g(2,\{4\}),\ c_{34} + g(4,\{2\})\}$

$g(2,\{4\}) = \min\{c_{24} + g(4,\phi)\}$

$= c_{24} + c_{41} = 10+8 = 18$

$g(4,\{2\}) = \min\{c_{42} + g(2,\phi)\}$

$= c_{42} + c_{21} = 8+5 = 13$

$\Rightarrow g(3,\{2,4\}) = \min\{c_{32} + g(2,\{4\}),\ c_{34} + g(4,\{2\})\}$

$= \min\{13+18,\ 12+13\} = \min\{31, 25\}$

$= 25$

Now we compute

$$g(4,\{2,3\}) = \min\{c_{42} + g(2,\{3\}),\ c_{43} + g(3,\{2\})\}$$

Here $g(2,\{3\}) = \min\{c_{23} + g(3,\phi)\}$

$$= c_{23} + c_{31} = 9 + 6 = 15$$

$$g(3,\{2\}) = \min\{c_{32} + g(2,\phi)\}$$

$$= c_{32} + c_{21} = 13 + 5 = 18$$

$$\Rightarrow g(4,\{2,3\}) = \min\{c_{42} + g(2,\{3\}),\ c_{43} + g(3,\{2\})\}$$

$$= \min\{8 + 15,\ 9 + 18\}$$

$$= \min\{23,\ 27\} = 23$$

after knowing the value of all component of $g(1,\{2,3,4\})$, we compute

$$g(1,\{2,3,4\}) = \min\begin{cases} c_{12} + g(2,\{3,4\}), \\ c_{13} + g(3,\{2,4\}), \\ c_{14} + g(4,\{2,3\}) \end{cases}$$

$$= \min\{10 + 25,\ 15 + 25,\ 20 + 23\}$$

$$= \min\{35,\ 40,\ 43\}$$

$$= 35$$

⇒ the minimum cost for finding the tour starting from Vertex A is 35 and the tour is as follows:

$$1 \xrightarrow{10} 2 \xrightarrow{10} 4 \xrightarrow{9} 3 \xrightarrow{6} 1$$

⇒ total cost of tour $= 10 + 10 + 9 + 6 = 35$