# Discrete Mathematics (CSA103)

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**Mathematical Logic:** Propositional and Predicate Logic, Propositional Equivalences, Normal Forms, Predicates and Quantifiers, Nested Quantifiers, Rules of Inference.

**Sets and Relations:** Set Operations, Representation and Properties of Relations, Equivalence Relations, Partially Ordering. Counting,

**Mathematical Induction and Discrete Probability:** Basics of Counting, Pigeonhole Principle, Permutations and Combinations, Inclusion- Exclusion Principle, Mathematical Induction, Probability, Bayes Theorem.

**Group Theory:** Groups, Subgroups, Semi Groups, Product and Quotients of Algebraic Structures, Isomorphism, Homomorphism, Automorphism, Rings, Integral Domains, Fields, Applications of Group Theory.

**Graph Theory:** Simple Graph, Multigraph, Weighted Graph, Paths and Circuits, Shortest Paths in Weighted Graphs, Eulerian Paths and Circuits, Hamiltonian Paths and Circuits, Planner graph, Graph Coloring, Bipartite Graphs, Trees and Rooted Trees, Prefix Codes, Tree Traversals, Spanning Trees and Cut-Sets.

**Boolean Algebra:** Boolean Functions and its Representation, Simplifications of Boolean Functions.

**Optimization:** Linear Programming - Mathematical Model, Graphical Solution, Simplex and Dual Simplex Method, Sensitive Analysis; Integer Programming, Transportation and Assignment Models,

**PERT-CPM:** Diagram Representation, Critical Path Calculations, Resource Levelling, Cost Consideration in Project Scheduling.

**Books Recommended:**

- J.P. Trembley and R.P.Manohar, Discrete Mathematical Structures with Applications to Computer Science, McGraw Hill.
- Dornhoff and Hohn, Applied Modern Algebra, McMillan.
- N. Deo, Graph Theory with Applications to Engineering and Computer Science, PHI.
- C.L. Liu, Elements of Discrete Mathematics, McGraw-Hill.
- Kenneth H Rosen, Discrete Mathematics, Tata McGraw Hill.
- K.L.P. Mishra, N. Chandrasekaran, Theory of Computer Science: Automata, Languages and Computation, PHI.

**Boolean Algebra:**

A boolean algebra consists of a nonempty set B containing two distinct elements 0 and 1, two binary operators + and., and a unary operator ′ satisfying the following conditions for all x,y, and z in $(B, +, ., ', 0, 1)$:

1. Commutative laws
$$x + y = y + x; \qquad x.y = y.x$$
2. Associative laws
$$x + (y + z) = (x + y) + z; \qquad x.(y.z) = (x.y).z$$
3. Distributive laws
$$x.(y + z) = (x.y) + (x.z); \qquad x + (y.z) = (x + y).(x + z)$$
4. Identity laws
$$x + 0 = x; \qquad x.1 = x$$
5. Complement laws
$$x + x' = 1; \qquad x.x' = 0$$

**Properties:** Let x and y be arbitrary elements in a boolean algebra $(B, +, ., ', 0, 1)$.

1. Idempotent laws

$$x + x = x; \qquad x.x = x$$

2. Boundedness laws/Domination laws

$$x + 1 = 1; \qquad x.0 = 0$$

3. Involution laws

$$(x')' = x; \qquad 0' = 1; \qquad 1' = 0$$

4. Absorption laws

$$x + xy = x; \qquad x(x + y) = x$$

5. De Morgan's laws

$$(x + y)' = x'y'; \qquad (xy)' = x' + y'$$

**Set Theoretic Operations using Boolean Operations:**

1. Union $\cup$ as binary operation $+$ or $\vee$
2. Intersection $\cap$ as binary operation $\cdot$ and $\wedge$
3. Compliment $-$ as unary operation $\prime$

**Boolean Function:** A boolean variable assumes the value 0 or 1. A boolean function is a function $f : B^n \to B$, where $B^n = BxBx...xB$, the cartesian product of B with itself to n factors.
Boolean functions can also be defined by boolean expressions made up of boolean variables and boolean operators. For instance, if x, y, and z are boolean variables, then $xy$, $(xy)'$, $x + yz$, and $x + (xy)'$ are boolean expressions.

**Equality of Boolean Expressions:** Two boolean expressions are equal, if they yield the same value for all $(x1, x2, ..., Xn) \in B^n$.

**Example:** $(x + y)' = x'y'$.

**Dual of a Boolean Expressions:** It is obtained by interchanging Boolean sums and Boolean products and interchanging $0s$ and $1s$.

**Example:** Dual of $x(y + 0)$ is $x + (y.1)$

**Simplifications of Boolean Functions:**

1. Laws of boolean algebra

$$xy + \bar{x}y = y$$

2. Karnaugh map (K-map): This method is used for simplifying **sum of product** expansions.

$$x\bar{y} + \bar{x}y + \bar{x}\bar{y} = \bar{x} + \bar{y}$$

**Representation of Boolean Functions/Normal Form:**

1. Disjunctive normal form (DNF) or sum-of-products (SOP)

$$F(x, y, z) = (x + y)\bar{z} = xy\bar{z} + x\bar{y}\bar{z} + \bar{x}y\bar{z}$$

2. Conjunctive normal form (CNF) or products-of-sums (POS)

$$F(x, y, z) = x + y + z = x + y + z$$

$$F(x, y, z) = (x+y)z = (x+y+z)(x+y+\bar{z})(x+\bar{y}+z)(\bar{x}+y+z)(\bar{x}+y+\bar{z})$$

Questions/Query ?