CS6363.004 Design and Analysis of Computer Algorithms
Homework 3
Assigned: Wednesday 3/23/2022   **Due:** 11:59PM, Thursday, 3/31/2022

Please submit your solutions file ".pdf", ".docx" or "doc" format.

For algorithm design questions, provide your answer using pseudocode only; you should not provide a computer program implemented in a programming language. Otherwise, you will lose at least of half of the points.

1. Exercise 15.4-2 on page 396 from CLRS. **(30 pt)**

2. Consider the following variant of coin changing problem. Given a set of coin values $C = \{c_1, c_2, \ldots, c_m\}$, find the minimum number of coins to reprensent n cents with coins in C. For instance, if the coin value set is $C = \{1, 4, 9\}$ (i.e., there are one-cent, four-cent and nine-cent coins), then the best way to make $n = 16$ cents is to use 4 four-cents coins. Therefore, the optimal solution is 4.

   Design an $O(mn)$-time **dynamic programming** algorithm to solve this problem. Answer the following questions:

   - **Input: ?**
   - **Output: ?**
   - Explain what subproblems one must solve for this problem?
   - Find the recursive solution?
   - Give the O(mn)-time algorithm in pseudocode format?

   Explain your work in detail. Answers without explanation will not receive full credit. **(40 pt)**

3. Tribonacci numbers are a generalization of Fibonacci numbers and defined as follows:

$$T(n) = \begin{cases} 0 & \text{if } n = 0 \\ 0 & \text{if } n = 1 \\ 1 & \text{if } n = 2 \\ T(n-1) + T(n-2) + T(n-3) & \text{if } n \geq 3 \end{cases}$$

   To compute n-th Tribonacci number,

   - Design a recursive algorithm.
   - Design a bottom-up dynamic programming algorithm.
   - Design a top-down dynamic programming algorithm.

   Write the pseudocode for each algorithm clearly. **(30 pt)**