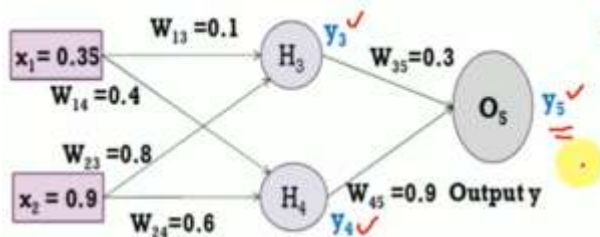
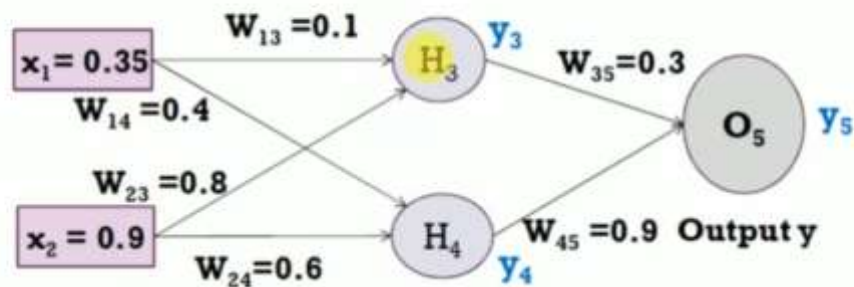


- Assume that the neurons have a sigmoid activation function, perform a forward pass and a backward pass on the network. Assume that the actual output of y is 0.5 and learning rate is 1. Perform another forward pass.



$$\text{Error} = y_{\text{target}} - y_5 = -0.19$$

$0.5 - 0.69$

- Forward Pass: Compute output for y_3 , y_4 and y_5 .

$$a_j = \sum_i (w_{ij} * x_i) \quad y_j = F(a_j) = \frac{1}{1 + e^{-a_j}}$$

$$a_1 = (w_{13} * x_1) + (w_{23} * x_2) = (0.1 * 0.35) + (0.8 * 0.9) = 0.755$$

$$y_3 = f(a_1) = 1 / (1 + e^{-0.755}) = 0.68$$

$$a_2 = (w_{14} * x_1) + (w_{24} * x_2) = (0.4 * 0.35) + (0.6 * 0.9) = 0.68$$

$$y_4 = f(a_2) = 1 / (1 + e^{-0.68}) = 0.6637$$

$$a_3 = (w_{35} * y_3) + (w_{45} * y_4) = (0.3 * 0.68) + (0.9 * 0.6637) = 0.801$$

$$y_5 = f(a_3) = 1 / (1 + e^{-0.801}) = 0.69 \text{ (Network Output)}$$

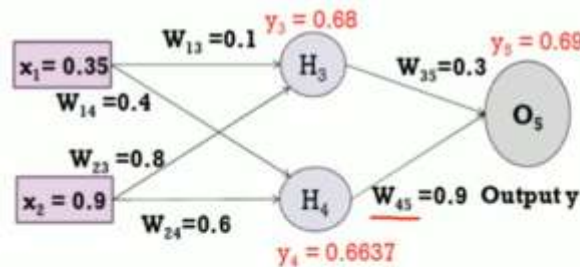
- Each weight changed by:

$$\Delta w_{ji} = \eta \delta_j o_i$$

$$\delta_j = o_j(1 - o_j)(t_j - o_j) \quad \text{if } j \text{ is an output unit}$$

$$\delta_j = o_j(1 - o_j) \sum_k \delta_k w_{kj} \quad \text{if } j \text{ is a hidden unit}$$

- where η is a constant called the learning rate
- t_j is the correct teacher output for unit j
- δ_j is the error measure for unit j



• Backward Pass: Compute δ_3 , δ_4 and δ_5 .

For output unit:

$$\delta_5 = y(1-y) (y_{\text{target}} - y) = 0.69*(1-0.69)*(0.5-0.69) = -0.0406$$

For hidden unit:

$$\delta_3 = y_3(1-y_3) w_{35} * \delta_5 = 0.68*(1 - 0.68)*(0.3 * -0.0406) = -0.00265$$

Compute new weights

$$\Delta w_{ji} = \eta \delta_j o_i$$

$$\Delta w_{45} = \eta \delta_5 y_4 = 1 * -0.0406 * 0.6637 = -0.0269$$

$$w_{45}(\text{new}) = \Delta w_{45} + w_{45}(\text{old}) = -0.0269 + (0.9) = 0.8731$$

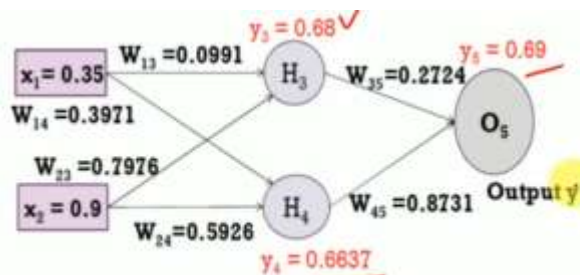
$$\delta_4 = y_4(1-y_4)w_{45} * \delta_5 = 0.6637*(1 - 0.6637)*(0.9 * -0.0406) = -0.0082$$

$$\Delta w_{14} = \eta \delta_4 x_1 = 1 * -0.0082 * 0.35 = -0.00287$$

$$w_{14}(\text{new}) = \Delta w_{14} + w_{14}(\text{old}) = -0.00287 + 0.4 = 0.3971$$

• Similarly, update all other weights

i	j	w_{ij}	δ_i	x_i	η	Updated w_{ij}
1	3	0.1	-0.00265	0.35	1	0.0991
2	3	0.8	-0.00265	0.9	1	0.7976
1	4	0.4	-0.0082	0.35	1	0.3971
2	4	0.6	-0.0082	0.9	1	0.5926
3	5	0.3	-0.0406	0.68	1	0.2724
4	5	0.9	-0.0406	0.6637	1	0.8731



• Forward Pass: Compute output for y_3 , y_4 and y_5 .

$$a_j = \sum_i (w_{ij} * x_i) \quad y_j = F(a_j) = \frac{1}{1 + e^{-a_j}}$$

$$a_1 = (w_{13} * x_1) + (w_{23} * x_2) = (0.0991 * 0.35) + (0.7976 * 0.9) = 0.7525$$

$$y_3 = f(a_1) = 1 / (1 + e^{-0.7525}) = 0.6797$$

$$a_2 = (w_{14} * x_1) + (w_{24} * x_2) = (0.3971 * 0.35) + (0.5926 * 0.9) = 0.6723$$

$$y_4 = f(a_2) = 1 / (1 + e^{-0.6723}) = 0.6620$$

$$a_3 = (w_{35} * y_3) + (w_{45} * y_4) = (0.2724 * 0.6797) + (0.8731 * 0.6620) = 0.7631$$

$$y_5 = f(a_3) = 1 / (1 + e^{-0.7631}) = 0.6820 \text{ (Network Output)}$$

$$\text{Error} = y_{\text{target}} - y_5 = -0.182$$