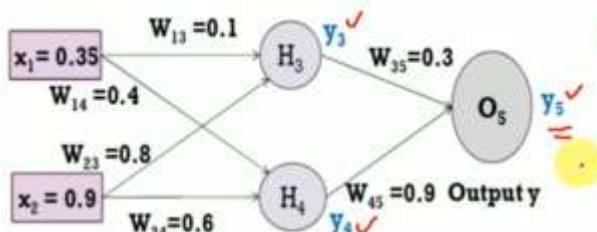
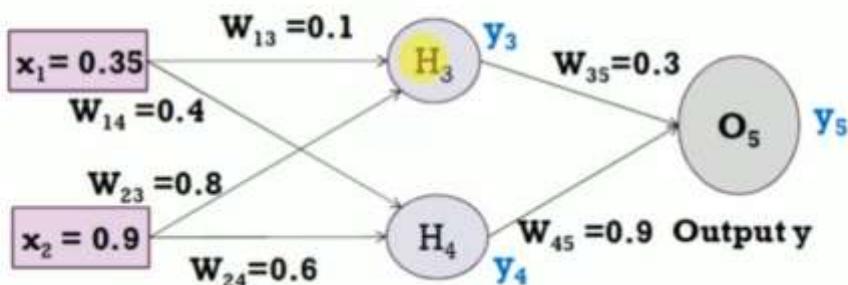


- Assume that the neurons have a sigmoid activation function, perform a forward pass and a backward pass on the network. Assume that the actual output of  $y$  is 0.5 and learning rate is 1. Perform another forward pass.



- Forward Pass: Compute output for  $y_3$ ,  $y_4$  and  $y_5$ .

$$a_j = \sum_l (w_{l,j} * x_l) \quad y_j = F(a_j) = \frac{1}{1 + e^{-a_j}}$$

$$\begin{aligned} a_1 &= (w_{13} * x_1) + (w_{23} * x_2) \\ &= (0.1 * 0.35) + (0.8 * 0.9) = 0.755 \\ y_3 &= f(a_1) = 1 / (1 + e^{-0.755}) = 0.68 \end{aligned}$$

$$\begin{aligned} a_2 &= (w_{14} * x_1) + (w_{24} * x_2) \\ &= (0.4 * 0.35) + (0.6 * 0.9) = 0.68 \\ y_4 &= f(a_2) = 1 / (1 + e^{-0.68}) = 0.6637 \end{aligned}$$

$$\begin{aligned} a_3 &= (w_{35} * y_3) + (w_{45} * y_4) \\ &= (0.3 * 0.68) + (0.9 * 0.6637) = 0.801 \\ y_5 &= f(a_3) = 1 / (1 + e^{-0.801}) = 0.69 \text{ (Network Output)} \end{aligned}$$

Error =  $y_{\text{target}} - y_5 = -0.19$

$0.5 - 0.69$

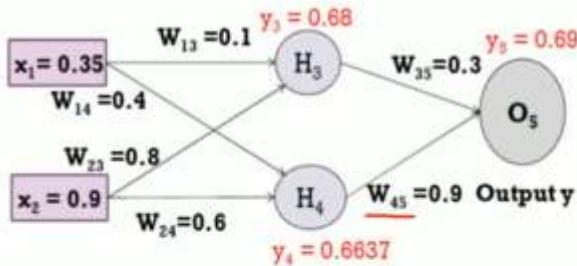
- Each weight changed by:

$$\Delta w_{ji} = \eta \delta_j o_i$$

$$\delta_j = o_j(1 - o_j)(t_j - o_j) \quad \text{if } j \text{ is an output unit}$$

$$\delta_j = o_j(1 - o_j) \sum_k \delta_k w_{kj} \quad \text{if } j \text{ is a hidden unit}$$

- where  $\eta$  is a constant called the learning rate
- $t_j$  is the correct teacher output for unit  $j$
- $\delta_j$  is the error measure for unit  $j$



Compute new weights

$$\Delta w_{ji} = \eta \delta_j o_i$$

$$\Delta w_{45} = \eta \delta_5 y_4 = 1 * -0.0406 * 0.6637 = -0.0269$$

$$w_{45} (\text{new}) = \Delta w_{45} + w_{45} (\text{old}) = -0.0269 + 0.9 = 0.8731$$

• Backward Pass: Compute  $\delta_3$ ,  $\delta_4$  and  $\delta_5$ .

For output unit:

$$\begin{aligned}\delta_5 &= y(1-y) (y_{\text{target}} - y) \\ &= 0.69 * (1 - 0.69) * (0.5 - 0.69) = -0.0406\end{aligned}$$

For hidden unit:

$$\begin{aligned}\delta_3 &= y_3(1-y_3) w_{35} * \delta_5 \\ &= 0.68 * (1 - 0.68) * (0.3 * -0.0406) = -0.00265\end{aligned}$$

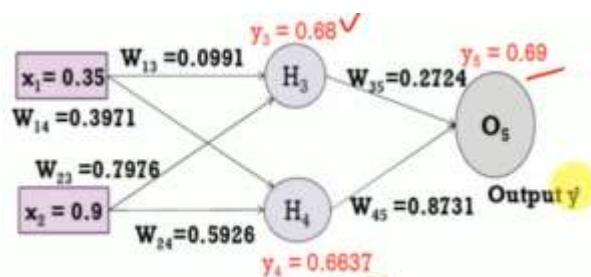
$$\Delta w_{14} = \eta \delta_4 x_1 = 1 * -0.0082 * 0.35 = -0.00287$$

$$w_{14} (\text{new}) = \Delta w_{14} + w_{14} (\text{old}) = -0.00287 + 0.4 = 0.3971$$

$$\begin{aligned}\delta_4 &= y_4(1-y_4) w_{45} * \delta_5 \\ &= 0.6637 * (1 - 0.6637) * (0.9 * -0.0406) = -0.0082\end{aligned}$$

- Similarly, update all other weights

i	j	w <sub>ij</sub>	$\delta_i$	x <sub>i</sub>	$\eta$	Updated w <sub>ij</sub>
1	3	0.1	-0.00265	0.35	1	0.0991
2	3	0.8	-0.00265	0.9	1	0.7976
1	4	0.4	-0.0082	0.35	1	0.3971
2	4	0.6	-0.0082	0.9	1	0.5926
3	5	0.3	-0.0406	0.68	1	0.2724
4	5	0.9	-0.0406	0.6637	1	0.8731



• Forward Pass: Compute output for y3, y4 and y5.

$$a_j = \sum_l (w_{l,j} * x_l) \quad yj = F(a_j) = \frac{1}{1 + e^{-a_j}}$$

$$\begin{aligned}a_1 &= (w_{13} * x_1) + (w_{23} * x_2) \\ &= (0.0991 * 0.35) + (0.7976 * 0.9) = 0.7525 \\ y_3 &= f(a_1) = 1 / (1 + e^{-0.7525}) = 0.6797\end{aligned}$$

$$\begin{aligned}a_2 &= (w_{14} * x_1) + (w_{24} * x_2) \\ &= (0.3971 * 0.35) + (0.5926 * 0.9) = 0.6723 \\ y_4 &= f(a_2) = 1 / (1 + e^{-0.6723}) = 0.6620\end{aligned}$$

$$\begin{aligned}a_3 &= (w_{35} * y_3) + (w_{45} * y_4) \\ &= (0.2724 * 0.6797) + (0.8731 * 0.6620) = 0.7631 \\ y_5 &= f(a_3) = 1 / (1 + e^{-0.7631}) = 0.6820 \text{ (Network Output)}$$

$$\text{Error} = y_{\text{target}} - y_5 = -0.182$$