Backpropogation

30 March 2022 13:18

Backprop → Algo → train nn   How

training

<u>Pre requisite</u>

→ Gradient Descent (✓) →

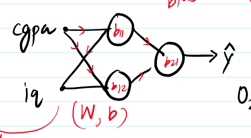
→ Forward Propogation (✓) →

cgpa	iq	lpa
8	80	8
7	70	7

reg

predict

weights, bias → correct value



6+3 → 9 trainable

$\sigma(z)$

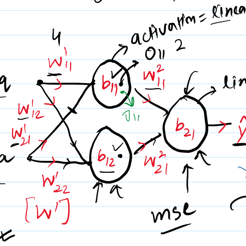
$O_{21} = W^2_{11} O_{11} + W^2_{21} O_{12} + b_{21}$

L min → $\hat{y} \to O_{21}$

Backprop

4 students

iq	cgpa	lpa	error
80	8	3	225
60	9	5	
70	5	8	
120	7	11	



(W, b) Random val

$\boxed{W \to 1,\ b \to 0}$

Steps →

0) Init w, b ✓

1) You select a point (row) ✓
   ↳ student

2) Predict (lpa) → forward prop [Dot product]

3) Choose a loss function (mse)

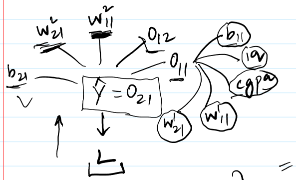
4) Weights and bias update ✓
   ↳ Gradient Descent

→ $\boxed{18 \text{ lpa}}$

$L = (y - \hat{y})^2$

$(3-18)^2 = 225$ error

Hierarchy ↓ Backprop



$$\boxed{W_{new} = W_{old} - \eta \frac{\partial L}{\partial W_{old}}} \checkmark$$

=1 (0.1)

$$\boxed{b_{new} = b_{old} - \eta \frac{\partial L}{\partial b_{old}}} \checkmark$$

$$\boxed{W^2_{11\,new} = W^2_{11\,old} - \eta \boxed{\frac{\partial L}{\partial W^2_{11}}}}$$ 9 derivatives ✓

$$\boxed{W^2_{21\,new} = W^2_{21\,old} - \eta \frac{\partial L}{\partial W^2_{21}}} \checkmark$$

$$\boxed{b_{21\,new} = b_{21\,old} - \eta \frac{\partial L}{\partial b_{21}}} \checkmark$$

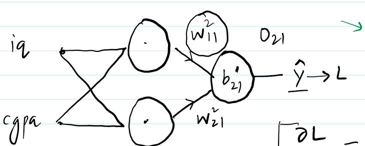
derivative of loss wrt weight

$\boxed{\frac{\partial L}{\partial W^2_{11}}}$ ← partial derivative

derivative $\frac{\partial L}{\partial W^2_{11}}$

$\frac{\partial L}{\partial W^2_{11}}, \frac{\partial L}{\partial W^2_{21}}, \frac{\partial L}{\partial b_{21}}$ | $\frac{\partial L}{\partial W^1_{11}}, \frac{\partial L}{\partial W^1_{21}}, \frac{\partial L}{\partial b_{11}}$ | $\frac{\partial L}{\partial W^1_{12}}, \frac{\partial L}{\partial W^1_{22}}, \frac{\partial L}{\partial b_{12}}$

$\hat{y}$, W_{11}, L (min)

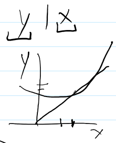


$$\frac{\partial L}{\partial W^2_{11}} = \boxed{\frac{\partial L}{\partial \hat{y}}} \times \boxed{\frac{\partial \hat{y}}{\partial W^2_{11}}}$$ → chain rule of differen

$\left\{\frac{dy}{dx}\right\}$

$$\frac{\partial L}{\partial \hat{y}} = \frac{\partial}{\partial \hat{y}} (y - \hat{y})^2 = \boxed{-2(y - \hat{y})}$$

y | x



$\hat{y}$ — W^2_{21} $\frac{\partial L}{\partial W^2_{21}}$ → L

$$\frac{\partial \hat{y}}{\partial W^2_{11}} = \frac{\partial}{\partial W^2_{11}} \left[O_{11} W^2_{11} + O_{12} W^2_{21} + b_{21} \right] = O_{11}$$

$$\boxed{\frac{\partial L}{\partial W^2_{11}} = -2(y - \hat{y}) O_{11}}$$ 1

$$\frac{\partial L}{\partial W^2_{21}} = \boxed{\frac{\partial L}{\partial \hat{y}}} \times \frac{\partial \hat{y}}{\partial W^2_{21}}$$

$$\frac{\partial \hat{y}}{\partial W^2_{21}} = \frac{\partial}{\partial W^2_{21}} \left[O_{11} W^2_{11} + O_{12} W^2_{21} + b_{21} \right] = O_{12}$$

$$\boxed{\frac{\partial L}{\partial W^2_{21}} = -2(y - \hat{y}) O_{12}}$$ 2

$\hat{y} \to b$, L

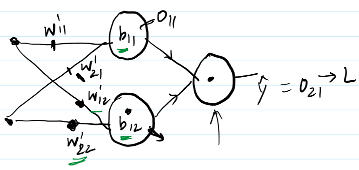
$$\frac{\partial \hat{y}}{\partial b_{21}} = 1$$

$$\boxed{\frac{\partial L}{\partial w^2_{21}} = -2(y-\hat{y})O_{12}}\quad 2$$

$\hat{y} \rightarrow b$, $L \uparrow$

$$\frac{\partial \hat{y}}{\partial b_{21}} = 1$$

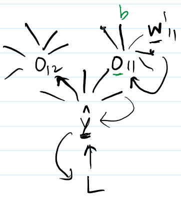
$$\boxed{\frac{\partial L}{\partial b_{21}} = -2(y-\hat{y})}\quad 3 \qquad \frac{\partial L}{\partial b_{21}} = \boxed{\frac{\partial L}{\partial \hat{y}}} \times \frac{\partial \hat{y}}{\partial b_{21}}$$



$$\frac{\partial L}{\partial w^1_{11}} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial O_{11}} \frac{\partial O_{11}}{\partial w^1_{11}} \rightarrow \boxed{-2(y-\hat{y})\, w^2_{11}\, x_{i1}}\quad 4$$

$$\frac{\partial L}{\partial w^1_{21}} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial O_{11}} \frac{\partial O_{11}}{\partial w^1_{21}} \rightarrow \boxed{-2(y-\hat{y})\, w^2_{11}\, x_{i2}}\quad 5$$

$$\frac{\partial L}{\partial b_{11}} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial O_{11}} \frac{\partial O_{11}}{\partial b_{11}} \rightarrow \boxed{-2(y-\hat{y})\, w^2_{11}}\quad 6$$

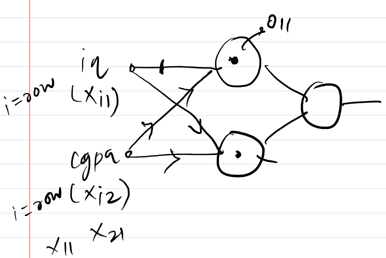


$$\frac{\partial L}{\partial w^1_{12}} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial O_{12}} \frac{\partial O_{12}}{\partial w^1_{12}} \rightarrow \boxed{-2(y-\hat{y})\, w^2_{21}\, x_{i1}}\quad 7$$

w^2_{11}

$$\frac{\partial L}{\partial w^1_{22}} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial O_{12}} \frac{\partial O_{12}}{\partial w^1_{22}} \rightarrow \boxed{-2(y-\hat{y})\, w^2_{21}\, x_{i2}}\quad 8 \qquad \frac{\partial \hat{y}}{\partial O_{12}} = w^2_{21}$$

$$\frac{\partial L}{\partial b_{12}} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial O_{12}} \frac{\partial O_{12}}{\partial b_{12}} \rightarrow \boxed{-2(y-\hat{y})\, w^2_{21}}\quad 9$$



$$\frac{\partial O_{11}}{\partial w^1_{11}} = \frac{\partial}{\partial w^1_{11}}[\underbrace{iq\, w^1_{11}} + cgpa\, w^1_{21} + b_{11}] = (iq) \qquad \uparrow x_{i1}$$

$$\frac{\partial O_{11}}{\partial w^1_{21}} = x_{i2}$$

$$\frac{\partial O_{11}}{\partial b_{11}} = 1$$

$$\frac{\partial O_{12}}{\partial w^1_{12}} = \frac{\partial}{\partial w^1_{12}}[\underbrace{iq\, w^1_{12}} + cgpa\, w^1_{22} + b_{12}] = iq\ (x_{i1})$$

$$\frac{\partial O_{12}}{\partial w^1_{22}} = x_{i2}$$

$$\frac{\partial O_{12}}{\partial b_{12}} = 1$$

multiple chance

epochs

Steps (once again)

0) weights/bias → init → random w=1, b=0

loop - 100/100, converge

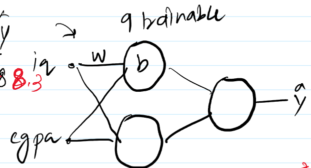
1) for i in range(4):

1a) 1 student → forward prop → lpa) →

1b) Loss calculate (mse) → $(y-\hat{y})^2$ → Backprop

1c) Adjust all weights and bias

cgpa	iq	lpa	$\hat{y}$
8	80	8	8.3
6	60	6	
7	70	7	
9	90		



L ↓

12 min

The How

$$\boxed{w_{new} = w_{old} - \eta \frac{\partial L}{\partial w_{old}}}$$

GD → 9 times

Back propagation Algorithm

epochs=5

for i in range(epochs):

for j in range(X.shape[0]):

$$\frac{\partial L}{\partial w^2_{11}} = -2(y-\hat{y})O_{11}$$

$$\frac{\partial L}{\partial w^2_{21}} = -2(y-\hat{y})O_{12}$$

Ready

```
for i in range(epochs):
    for j in range(x.shape[0]):
        → Select 1 row (random)
        → Predict ( using Forward prop)
        → Calculate loss (using Loss function → mse)
        → Update weights and bias using GD
              W_n = W_o - η ∂L/∂W
    → Calculate avg loss for the epoch
      L_avg
```

$$\frac{\partial L}{\partial w^2_{21}} = -2(y-\hat{y})O_{12}$$

$$\frac{\partial L}{\partial b_{21}} = -2(y-\hat{y})$$

$$\frac{\partial L}{\partial w^1_{11}} = -2(y-\hat{y})w^2_{11}x_{i1}$$

$$\frac{\partial L}{\partial w^1_{21}} = -2(y-\hat{y})w^2_{11}x_{i2}$$

$$\frac{\partial L}{\partial b_{11}} = -2(y-\hat{y})w^2_{11}$$

$$\frac{\partial L}{\partial w^1_{12}} = -2(y-\hat{y})w^2_{21}x_{i1}$$

$$\frac{\partial L}{\partial w^1_{22}} = -2(y-\hat{y})w^2_{21}x_{i2}$$

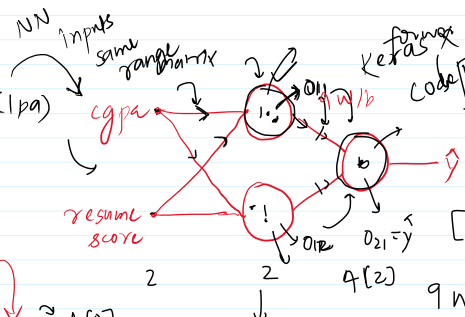
$$\frac{\partial L}{\partial b_{12}} = -2(y-\hat{y})w^2_{21}$$

Regression Problem

0-10 cgpa (x_{i1})	resume score (x_{i2})	package (lpa)
8	8	4
7	9	5
6	10	6
5	12	7

4 students

80-120



code → A[0] [8 8]

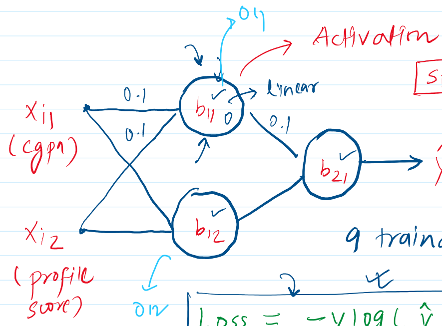
A[1] w = 0.1 b = 0

[O_{11} O_{12}]

$\hat{y} = A[2] = O_{21}$

Classification Example

cgpa	profile score	placement
8	8	1
7	9	1
6	10	0
5	5	0



Activation function

$\boxed{\text{sigmoid}}$

$Z = 0.1 \times 8 + 0.1 \times 8 + 0$

$\sigma(z) = O_{11}$

9 trainable parameters

$$\boxed{Loss = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})}$$

$(y-\hat{y})^2$

binary cross entropy

Backpropagation Algorithm

epochs = 5

```
for i in range(epochs):   (rows)
    for j in range(x.shape[0]):
        → Select 1 row (random) → 1 by 1
        → Predict ( using Forward prop) → predict
        → Calculate loss (using Loss function → ~~mse~~ bce)   { code convert }
        → Update weights and bias using GD
```

Cross entropy

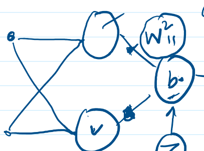
Backpropagation Algorithm

epochs = 5

```
for i in range(epochs): (rows)
    for j in range(X.shape[0]):
        → Select 1 row (random) → 1 by 1
        → Predict (using Forward prop) → predict
        → Calculate loss (using Loss function → bce)   → {code convert}
        → Update weights and bias using GD
              W_n = W_o - η (∂L/∂W)        9 w, b update
    → Calculate avg loss for the epoch
      L_avg
```

$z = W_{11}^2 O_{11} + W_{21}^2 O_{12} + b_{21}$

$\hat{y} = \sigma(z)$



der wrt L

$\hat{y} \to L$

$L = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$

$$\boxed{\frac{\partial L}{\partial W_{11}^2}} = \boxed{\frac{\partial L}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial z}} \times \frac{\partial z}{\partial W_{11}^2} = \boxed{-(y-\hat{y})\,O_{11}} \quad (3)$$

$-(y-\hat{y})$, O_{11}

$$\frac{\partial L}{\partial W_{21}^2} = \frac{\partial L}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial z} \times \frac{\partial z}{\partial W_{21}^2} = \boxed{-(y-\hat{y})\,O_{12}}$$

O_{12}

$z_{final} = W_{11}^2 O_{11} + W_{21}^2 O_{12} + b_{21}$

$$\frac{\partial L}{\partial b_{21}} = \frac{\partial L}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial z} \times \frac{\partial z}{\partial b_{21}} = \boxed{-(y-\hat{y})}$$

1

z_{final}

$\hat{y} \to W_{11}^2 \times W_{21}^2$

$L \to$

$$\boxed{\frac{\partial L}{\partial \hat{y}}} = \frac{\partial}{\partial \hat{y}}\left[-y\log(\hat{y}) - (1-y)\log(1-\hat{y})\right]$$

$$= \frac{-y}{\hat{y}} + \frac{(1-y)}{(1-\hat{y})} = \frac{-y(1-\hat{y}) + \hat{y}(1-y)}{\hat{y}(1-\hat{y})} = \frac{-y + \cancel{y\hat{y}} + \hat{y} - \cancel{y\hat{y}}}{\hat{y}(1-\hat{y})}$$

$$= \boxed{\frac{-(y-\hat{y})}{\hat{y}(1-\hat{y})}} \qquad \frac{\partial}{\partial z}\sigma(z) = \sigma(z)[1-\sigma(z)]$$

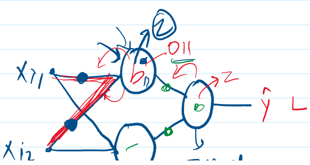
$\hat{y} = \sigma(z)$ $\qquad \sigma(z) = \hat{y}$

$$\boxed{\frac{\partial \hat{y}}{\partial z}} = \frac{\partial}{\partial z}(\sigma(z)) = \sigma(z)[1-\sigma(z)] = \hat{y}(1-\hat{y})$$

$$\frac{\partial L}{\partial \hat{y}} = \frac{-(y-\hat{y})}{\hat{y}(1-\hat{y})} \qquad \frac{\partial \hat{y}}{\partial z} = \hat{y}(1-\hat{y}) \Rightarrow \frac{\partial L}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial z} = \frac{-(y-\hat{y})}{\cancel{\hat{y}(1-\hat{y})}} \times \cancel{\hat{y}(1-\hat{y})}$$

$$= -(y-\hat{y})$$

$$\frac{\partial z_p}{\partial W_{11}^1} = W_{11}^1 x_{i1} + W_{21}^1 x_{i2} + b_{11}$$

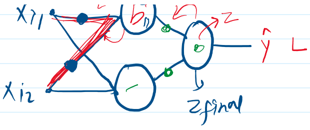


$$\frac{\partial L}{\partial W_{11}^1}, \quad \frac{\partial L}{\partial W_{21}^1}, \quad \frac{\partial L}{\partial b_{11}}$$

$\to -(y-\hat{y})$

$z_f = W_{11}^2 O_{11} + W_{21}^2 O_{12} + b_{21}$ $\quad O_{11}(1-O_{11})$

$O_{11} = \sigma(z_{prev})$



$\frac{\partial L}{\partial w_{11}^1}, \frac{\partial L}{\partial w_{21}^1}, \frac{\partial L}{\partial b_{11}}$

$z_f = w_{11}^2 O_{11} + w_{21}^2 O_{12} + b_{21}$ $\quad O_{11}(1-O_{11})$

$O_{11} = \sigma(z_{prev})$

$= \sigma(z_{prev})[1-\sigma(z_{prev})]$

$-(y-\hat{y})$

$L \rightarrow \hat{y} \rightarrow z_f \rightarrow O_{11} \rightarrow z_{prev} \rightarrow w_{11}^1$

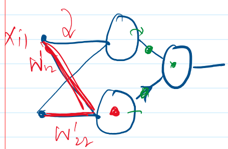
$$\frac{\partial L}{\partial w_{11}^1} = \boxed{\frac{\partial L}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial z_f}} \times \frac{\partial z_f}{\partial O_{11}} \times \frac{\partial O_{11}}{\partial z_{prev}} \times \frac{\partial z_{prev}}{\partial w_{11}^1}$$

$\frac{\partial z_f}{\partial O_{11}} \rightarrow w_{11}^2$, $\quad \frac{\partial O_{11}}{\partial z_{prev}} \rightarrow O_{11}(1-O_{11})$, $\quad \frac{\partial z_{prev}}{\partial w_{11}^1} \rightarrow x_{i1}$

$$\boxed{\frac{\partial L}{\partial w_{11}^1} = -(y-\hat{y}) w_{11}^2 \, O_{11}(1-O_{11}) x_{i1}}$$ ③

$$\boxed{\frac{\partial L}{\partial w_{21}^1} = -(y-\hat{y}) w_{11}^2 \, O_{11}(1-O_{11}) x_{i2}}$$

$$\boxed{\frac{\partial L}{\partial b_{11}} = -(y-\hat{y}) w_{11}^2 \, O_{11}(1-O_{11})}$$



$\frac{\partial L}{\partial w_{12}^1}, \frac{\partial L}{\partial w_{22}^1}, \frac{\partial L}{\partial b_{12}}$

$\frac{\partial z_f}{\partial O_{12}} = w_{11}^2 O_{11} + w_{21}^2 O_{12} + b_{21}$

$L \rightarrow \hat{y} \rightarrow z_f \rightarrow O_{12} \rightarrow z_p \rightarrow w_{12}^1$

$$\frac{\partial L}{\partial w_{12}^1} = \boxed{\frac{\partial L}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial z_f}} \times \frac{\partial z_f}{\partial O_{12}} \times \frac{\partial O_{12}}{\partial z_p} \times \frac{\partial z_p}{\partial w_{12}^1}$$

$\frac{\partial z_f}{\partial O_{12}} \rightarrow w_{21}^2$, $\quad \frac{\partial O_{12}}{\partial z_p} \rightarrow O_{12}(1-O_{12})$, $\quad \frac{\partial z_p}{\partial w_{12}^1} \rightarrow x_{i1}$

$$\boxed{\frac{\partial L}{\partial w_{12}^1} = -(y-\hat{y}) w_{21}^2 \, O_{12}(1-O_{12}) x_{i1}}$$ ③

$$\boxed{\frac{\partial L}{\partial w_{22}^1} = -(y-\hat{y}) w_{21}^2 \, O_{12}(1-O_{12}) x_{i2}}$$

$$\boxed{\frac{\partial L}{\partial b_{12}} = -(y-\hat{y}) w_{21}^2 \, O_{12}(1-O_2)}$$

Backpropagation → The why?

The intuition behind the algorithm ✓

Backpropagation Algorithm

epochs = 5

\# epochs × rows

for i in range(epochs):

    for j in range(x.shape[0]):

        → select 1 row (random)

        → Predict (using Forward prop) $\hat{y}$ y

        → Calculate loss (using loss function → mse)   bce

        → Update weights and bias using GD

            ↳ $W_n = W_0 - \boxed{\eta \frac{\partial L}{\partial W}}$ rule

            magic

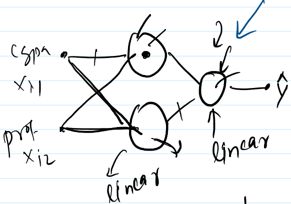
    → Calculate avg loss for the epoch
    L_{avg}

data → target

$L = (\cancel{y} - \hat{y})^2$

→ Loss function is a function of all trainable parameters

cgpa | profile | lpa



mse $= (y - \hat{y})^2$

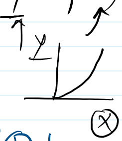
variable

$L = (y - \hat{y})^2$ → mathematical function

constant

$L(?)$ $L(\hat{y}) =$

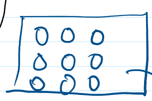
$y = f(x)$ 1 thing



$L(W, b)$ ↓

9 things

W, b



L ↓

$\hat{y} = W_{11}^2 O_{11} + W_{21}^2 O_{12} + b_{21}$

$\hat{y} = W_{11}^2 [W_{11}^1 x_{i1} + W_{21}^1 x_{i2} + b_{11}] + W_{21}^2 [W_{12}^1 x_{i1} + W_{22}^1 x_{i2} + b_{12}] + b_{12}$

→ Concept of Gradient → Gradient Descent

    ↳ fancy word for derivative

$z = f(x, y) = \boxed{x^2 + y^2}$

$y = f(x) = x^2 + x$

$\frac{dy}{dx} = \frac{d}{dx}(f(x)) = \frac{d}{dx}(x^2 + x) = 2x + 1$

derivative

$y \rightarrow x$ → derivative $\frac{d}{dx}$

$\frac{\partial z}{\partial x} = 2x$ $\frac{\partial z}{\partial y} = 2y$

gradient $\boxed{\frac{\partial}{\partial x}}$

gradient complex $L(W_{11}^1, W_{12}^1 \ldots \ldots W_{21}^2, b_{21}, b_{11}, b_{12})$

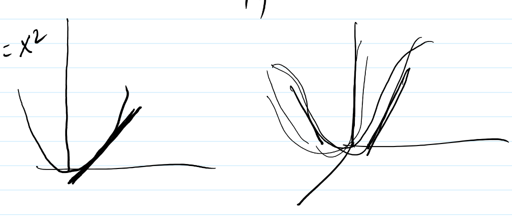
$\frac{\partial L}{\partial W}$, $\frac{\partial L}{\partial b}$

9 parameters

↳ 9D function → 9 diff slopes wrt each dim

3D → $z = x^2 + y^2$ $z = f(x, y)$

$y = f(x) = x^2$

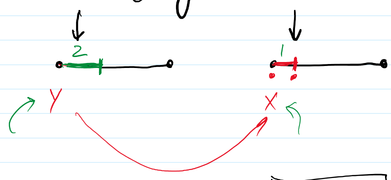


→ Concept of Derivative → Derivative at a point

↑ intuition

$\frac{dy}{dx}$ → rate of change ↑

y func x → x



$\frac{dy}{dx} = 2$ (+ve) ↗ mag ↘ sign

$\frac{dy}{dx} = -2$ (−ve)

x →

y →

derivative

$\boxed{x=5}$ → deriv

$y = x^2 + 2x$

$\frac{dy}{dx} = (2x+1)_{x=5}$

$\left(\frac{dy}{dx}\right)_{x=5} = 11$ → slope

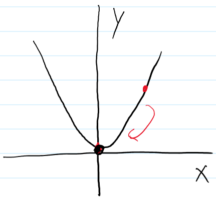
$\boxed{\frac{\partial L}{\partial w_{11}^1}}$

w_{11}^1 — 1 unit

L → ? (mag) (sign)

$W_{new} = W_{old} - \eta \frac{\partial L}{\partial w}$

→ the concept of minima



$\boxed{y = x^2}$

$\frac{dy}{dx} = 2x = 0$

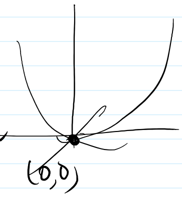
$\boxed{x=0}$

$z = x^2 + y^2$

$(x, y) \to z$ min

$\frac{\partial z}{\partial x} = 0$ x →

$\frac{\partial z}{\partial y} = 0$ y →



(x, y) (0,0)

$2x = 0 \Rightarrow x = 0$

$2y = 0 \Rightarrow y = 0$

L (9 param)

(w, b) min

L ↓

$\frac{\partial L}{\partial w_{11}^1} \quad \cdots \quad \frac{\partial L}{\partial b_{12}} = 0$

9 dim → minima

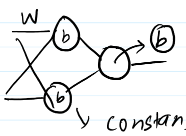
→ Backprop Intuition

$\eta = 1$

$\boxed{W_{new} = W_{old} - \eta \frac{\partial L}{\partial w}}$ ← 9 step

$\boxed{W_{new} = W_{old} - \frac{\partial L}{\partial w}}$

1 each parameters



constans

L = L(w, b) → 9 → 8 constant

$L(b_{21})$

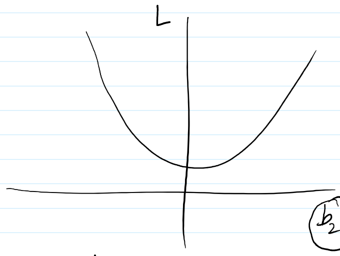
$L = b_{21}^2$

$L = (y - \hat{y})^2$

$L(b_{21})$ $\hat{y} = (b_{21})$

$b_{21} = \boxed{b_{21}} - \boxed{\frac{\partial L}{\partial b_{21}}}$ → +ve −ve

$b_{21} = 5$ w



b_{21} L min

$\boxed{\frac{\partial L}{\partial b_{21}}}$

smart

L derivative wrt b_{21}

↑ ? ↑ 1 unit → L ↓

b_{21} ↓

b_{21} ↓

L ↓

$\frac{\partial L}{\partial b_{21}} = +ve$ → b_{21} ↑ L ↑

b_{21} ↑

$\frac{\partial L}{\partial b_{21}} = -ve$

b_{21} ↑ L ↓

b_{21} ↑

$b_{21} = b_{21} - \frac{\partial L}{\partial b_{21}}$

game ... −ve of the gradient

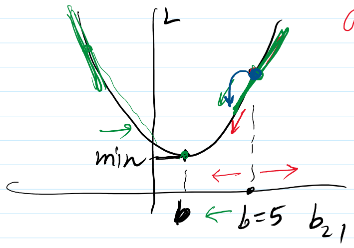
game

derivative (b_{21})

$-ve$ of the gradient

$$b_{21} = b_{21} - \boxed{\frac{\partial L}{\partial b_{21}}}$$

algo



$\frac{\partial L}{\partial b}$ = slope

+ve

$b = -5$

$\frac{\partial L}{\partial b} = -ve$

$-ve$

slope (gradient)

$\downarrow -ve$

$\boxed{\text{slope}}$

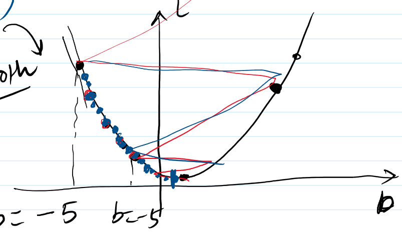
$$W_n = W_0 - \boxed{\frac{\partial L}{\partial W}} \qquad \boxed{\eta} \qquad \boxed{W_n = W_0 - \eta \frac{\partial L}{\partial W}}$$

→ Effect of Learning Rate (η)

smooth

$$W_n = W_0 - \eta \boxed{\frac{\partial L}{\partial W}}$$

0.1/0.01



train

$\boxed{\eta = 1}$

η = parameter

$\downarrow$

0.01

$\eta = 0.00001$

$$\frac{\partial L}{\partial b} = -50$$

$$b = -5 - (-50) = 45$$

$b = 45$

$$\frac{\partial L}{\partial b} = 50$$

$$b = 45 - 50$$

$b = -5 \quad \frac{\partial L}{\partial b} = -50 \quad \eta = 0.01$

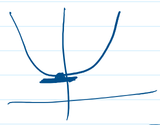
$$b = -5 + (0.01 \times 50) = 0.5$$

$$= -4.5$$

→ What is convergence?

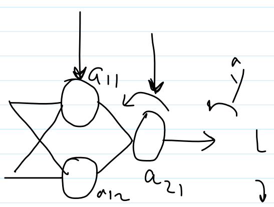
$$\boxed{W_n = W_0} \qquad W_n = W_0 - \eta \frac{\partial L}{\partial W} \approx 0$$

0



$\boxed{\text{epoch} \to 100}$

while → con → for 1000



$i = 1, 0$

$-(y - \hat{y})$

activation = ②

$\hat{y}(1 - \hat{y})\, O_{11}$

$-y \log(a_{21}) - (1-y) \log(1 - a_{21})$

$-(y - \hat{y})\, O_{11}$

$$-\frac{y}{\hat{y}} + \frac{(1-y)}{(1-\hat{y})}$$

$$\frac{\partial L}{\partial w_{11}^2} = \boxed{\frac{\partial L}{\partial a_{21}}} \times \frac{\partial a_{21}}{\partial w_{11}^2}$$

$\frac{\partial L}{} =$

$-\frac{y}{\hat{y}} + \frac{(1-y)}{(1-\hat{y})}$

$\frac{\partial L}{\partial w^2_{21}} =$

$\frac{\partial L}{\partial b_{21}} =$

curr_activa = $a[1]$

$$\left\{ \begin{array}{l} -\boxed{(y-\hat{y}) \quad a_{21}(1-a_{21})\, a_{11}} \\ -(y-\hat{y})\, a_{21}(1-a_{21})\, a_{12} \\ -(y-\hat{y})\, a_{21}(1-a_{21}) \end{array} \right\}$$

delta = $(y-\hat{y}) \quad a^{(2)}(1-a^{(2)})$

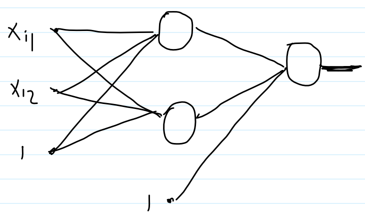
1×1

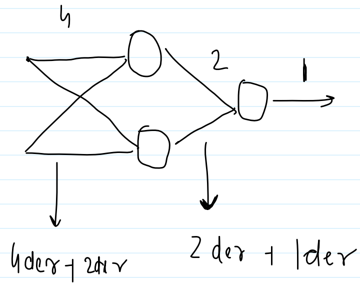
$\boxed{W[1]}$

$\boxed{1\times 2}$

$\cancel{2\times 1}$ $\boxed{1\times 2}$

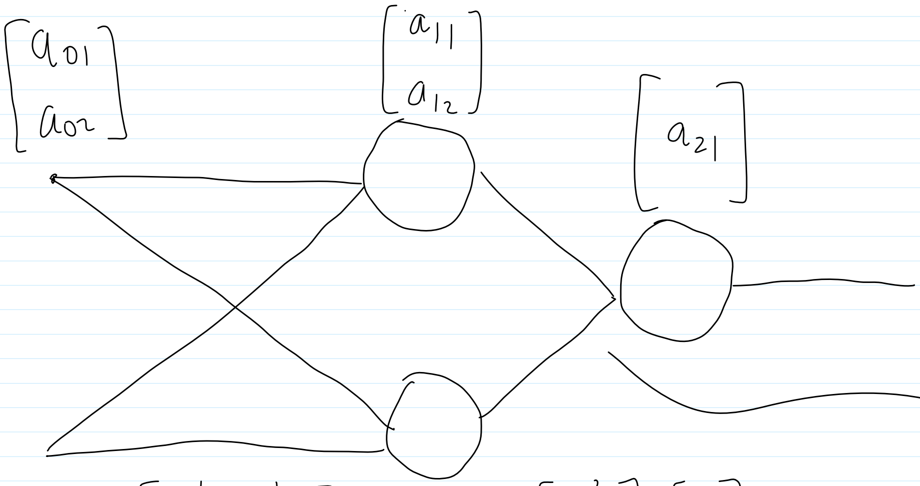
$\boxed{-(y-\hat{y})\, a_{21}(1-a_{21})} \times \begin{matrix} a[1] \\ \begin{bmatrix} a_{11} \\ a_{12} \end{bmatrix} \end{matrix}$





$1 \times 1 \quad 1\times 2$

$1\times 2 \quad \times \quad 2\times 2$



$L \qquad L \to a_2 \to a_1 \to$

$\frac{\partial L}{\partial a_2} \times \frac{\partial a_2}{\partial a_1} \; \frac{\partial a_1}{\partial w}$

$$\frac{\partial L}{\partial w} = \boxed{\frac{\partial L}{\partial a_2}} \times \boxed{\frac{\partial a_2}{\partial w}}$$

$\begin{bmatrix} W^1_{11} & W^1_{12} \\ W^1_{21} & W^1_{22} \end{bmatrix} \quad \begin{bmatrix} b_{11} \\ b_{12} \end{bmatrix} \quad \begin{bmatrix} W^2_{11} \\ W^2_{12} \end{bmatrix} \quad \begin{bmatrix} b_{21} \end{bmatrix}$

$\begin{bmatrix} \partial L/\partial w^1_{11} & \frac{\partial L}{\partial w^1_{12}} \\ \frac{\partial L}{\partial w^1_{21}} & \frac{\partial L}{\partial w^1_{22}} \end{bmatrix} \qquad \begin{bmatrix} \frac{\partial L}{\partial w^2_{11}} \\ \frac{\partial L}{\partial w^2_{12}} \end{bmatrix}$