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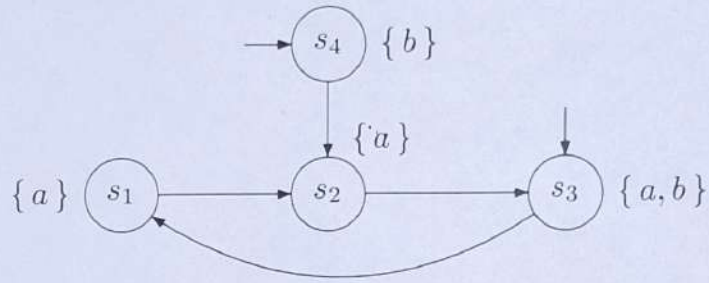
Indian Institute of Technology Kanpur
CS637 Embedded and Cyber-Physical Systems
Homework Assignment 3
Deadline: November 8, 2024

Total: **40 marks**

Problem 1. (10 points)
Consider the following state machine over the set of atomic propositions $\{a, b\}$:



Decide for each of the following LTL specifications whether the model satisfies it. For the positive outcome, provide a proof. For the negative outcome, provide a counterexample trace.
Note that the symbols $\bigcirc$, $\Box$, $\Diamond$, and U represent the "next", "always", "eventually", and "until" temporal operators respectively.

(a) $\bigcirc\bigcirc\bigcirc a$
(b) $\Box b$
(c) $\Box\Diamond a$
(d) $\Box(b \, \mathtt{U} \, a)$
(e) $\Diamond(a \, \mathtt{U} \, b)$

Solutions:

* (a) $\bigcirc\bigcirc\bigcirc a$: This represents that after three "next" the proposition 'a' should hold.

- since s_1, s_2, s_3 are in a cycle and at every state a is satisfied any transition from initial state s_1, s_2 or s_3 will end up in a state which satisfy a.
- possible sequences:

$s_1 \to s_2 \to s_3 \to s_1$: s_1 $\{a\}$
$s_2 \to s_3 \to s_1 \to s_2$: s_2 $\{a\}$
$s_3 \to s_2 \to s_2 \to s_3$: s_3 $\{a, b\}$
$s_4 \to s_1 \to s_2 \to s_3$: s_3 $\{a, b\}$

$\{a\}$ is held in all seq.
$\Rightarrow$ Model satifies $\bigcirc\bigcirc\bigcirc a$.

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* (b) $\Box b$: This formula requires that b always hold in every state along every possible path.

since, In state s_1 and s_2 only a is true (not b)

counter example path: $s_4 \to s_1 \to s_2$
(b doesnt always hold)

$\Rightarrow$ Model doesn't satisfy $\Box b$.

* (c) $\Box \Diamond a$: a should eventually hold at some point on every path and this should be true for every position in any infinite path.

All paths return to s_3 which contains a and path starting from s_4 terminates at s_1, s_2 or s_3 in which a holds.

$\Rightarrow$ Model satisfies $\Box \Diamond a$.

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Indian Institute of Technology Kanpur
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Homework Assignment 3
Deadline: November 8, 2024

* (d) $\Box(b \cup a)$ ⇒ along every path, b should hold until a holds.

In state s_1, b is false but a is true; so for path $s_4 \to s_1$

⇒ Model doesn't satisfy $\Box(b \cup a)$.

* (e)

$\Diamond(a \cup b)$: At some point on every path a will hold until b eventually holds.

In : $s_4 \to s_1 \to s_2 \to s_3$, b eventually holds after a.

- for any path starting from s_3 as s_1, s_2, s_3 are in cycle and s_3 has both a and b this trivially satisfies $a \cup b$ because b is already there.

⇒ Model satisfies $\Diamond(a \cup b)$.

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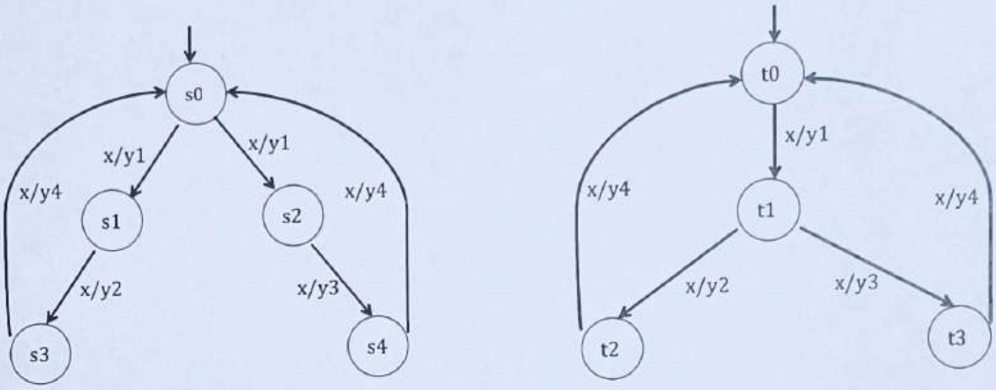
Problem 2. (10 points)
Consider the two state machines in Figure 1 and answer the following questions:



(a) M_1 (b) M_2

Figure 1

(a) Does the state machine M_1 simulate the state machine M_2? If yes, provide the simulation relation. If no, provide a transition of M_2 that M_1 cannot match.

(b) Does the state machine M_2 simulate the state machine M_1? If yes, provide the simulation relation. If no, provide a transition of M_1 that M_2 cannot match.

(c) Are the two state machines bisimilar? If yes, provide the bisimulation relation. If no, provide one reason.

Solution:

- Establishing the type equivalence between state machines M_1 and M_2.

Let P_i denote the set of input ports of machine i. O_i denotes the set of output ports of the machine i

V_p', V_q' denote the type of input and output ports respectively where $p \in P_2$ $q \in O_2$

Name: HIMESH BARAIK
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→ Required condition for type equivalence are

1) $P_2 = P_1$ II) $Q_1 = Q_2$ III) $V_p = V_{p'}$; $\forall p \in P_2$

IV) $V_{q'} = V_q$; $\forall q \in Q_1$

Given, $P_1 = P_2 = \{x\}$; x is pure

$Q_1 = Q_2 = \{y_1, y_2, y_3, y_4\}$; y_i is pure $\forall i \in \{1, 2, 3, 4\}$ $V_x = V_x' =$ pure signal

let $q \in Q_i$; $V_q' = V_q =$ pure signal.

- All the four necessary condition are satisfied

M1 and M2 are type equivalent.

States 1 = $\{s_0, s_1, s_2, s_3, s_4\}$

States 2 = $\{t_0, t_1, t_2, t_3\}$

* (a) NO, M1 doesn't simulate M2.

consider the following sequence of a simulation.

M2 : $t_0 \xrightarrow{x/y_1} t_1 \xrightarrow{x/y_3} t_3$

M1 : $s_0 \xrightarrow{x/y_1} s_1 \xrightarrow{x/} $ cant match

only o/p y_1 is possible

Name: HIMESH BARAIK
Roll No: 210447 Dept.: EE

* (b) Yes, M_2 can simulate M_1
consider the simulation relation S_{12}

$S_{12} = \{(s_0, t_0), (s_1, t_1), (s_2, t_1), (s_3, t_2), (s_4, t_3)\}$

clearly $S_{12} \subseteq$ states$_1$ × states$_2$

proof:

I) Initial condition = (initial condⁿ 1, initial condⁿ 2) = $(s_0, t_0) \in S_{12}$

II) To show that if $(s_i, t_j) \in S_{12}$ x is present and $(s_k', y_1) \in$ possible updates$_1$ (s_i, a) then $\exists$ $(t_m', y_m) \in$ possible updates (t_j, a) such that

a) $(s_k', t_m') \in S_{12}$ and

b) $y_1 = y_m$

consider current state $= (s_2, t_1) \in S_{12}$, let x be present possible updates $(s_2, x) = (s_4, y_3)$ (only element)

consider $(t_3, y_3) \in$ possible updates (t_1, x)

Now the next continued state $(s_4, t_3) \in S_{12}$

Name: HIMESH BARAIK

Roll No: 210447 (e.g. 170001) Dept.: EE (e.g. CSE)

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and the update on both machine is y_3.

∴ All conditions are satisfied, we can perform the above steps in all possible state, to show that the proof holds.

⇒ <u>M_2 simulates M_1</u>.

* (c) We have shown above that while M_2 simulates M_1, the inverse is not true. M_1 doesnt simulate M_2

∴ The state machine M_1 and M_2 are not dissimilar.

- The necessary (but not sufficient) condn for two machines, M_1 and M_2 to be dissimilar is that both M_1 and M_2 must simulate each other.

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Roll No: 210447 (e.g. 170001) Dept.: EE (e.g. CSE)

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Indian Institute of Technology Kanpur
CS637 Embedded and Cyber-Physical Systems
Homework Assignment 3
Deadline: November 8, 2024

Problem 3. (10 points)
Consider the processes P_1 and P_2 with the shared variables b_1, b_2, and x. Variables b_1 and b_2 are Boolean variables, while variable x can take either the value 1 or 2. Initially, each process P_i is in the non-critical section (i.e., P_i is in location $noncrit_i$). The scheduling strategy for giving the processes access to the critical section is realized using x as follows. If both processes want to enter the critical section (i.e., P_i is in location $wait_i$), the value of variable x decides which of the two processes may enter its critical section: if $x = i$, then P_i may enter its critical section $crit_i$ (for $i = 1, 2$). On entering location $wait_1$, process P_1 performs $x := 2$, thus giving privilege to process P_2 to enter the critical section. The value of x thus indicates which process has its turn to enter the critical section. Symmetrically, P_2 sets x to 1 when starting to wait. The variables b_i provide information about the current location of P_i. More precisely, b_i is set when P_i starts to wait, and is reset when the process exits the critical section. In pseudocode, P_1 performs as follows (the code for process P_2 is similar):

P_1 **loop** forever
$\vdots$ (*noncritical actions*)
$b_1 := true; x := 2$
wait until $(x = 1 \lor \neg b_2)$ (*request*)
do critical section **od**
$b_1 := false$ (*release*)
$\vdots$ (*noncritical actions*)
end loop

(a) Draw the state machines for P_1 and P_2.
(b) Show the state machine that is obtained by asynchronous composition of P_1 and P_2.
(c) How many total states are there in the composed state machine? How many of them are reachable?
(d) Provide an LTL formula that captures the requirement that the process P_1 and P_2 will not enter the critical section simultaneously. Using the composed state machine, determine whether the two systems satisfy the formula (property).

* solution: *(a)

- Given the process P_1 and P_2 with stored variable b_1, b_2 and x.
- Based on pseudocode.
  State machine is given by ⇒

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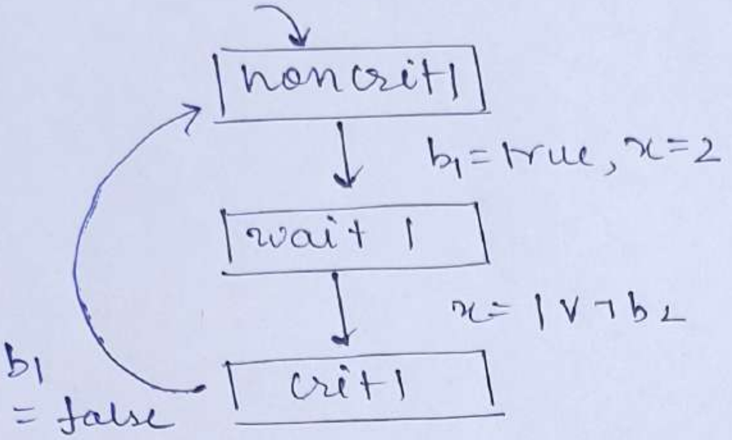
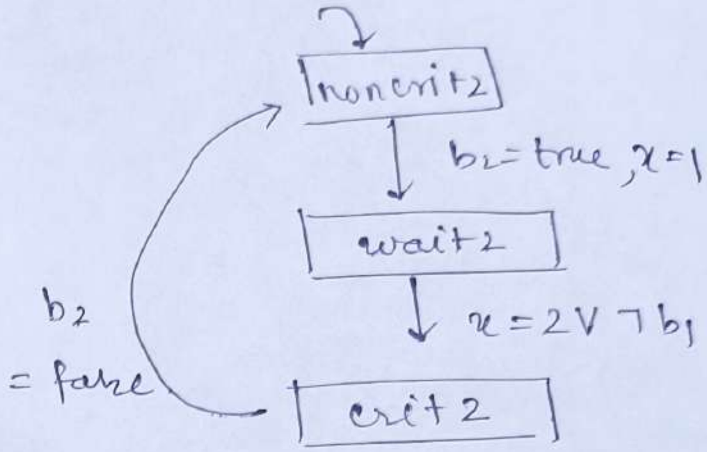
Roll No: 210447 (e.g. 170001) Dept.: EE (e.g. CSE)

Indian Institute of Technology Kanpur
CS637 Embedded and Cyber-Physical Systems
Homework Assignment 3
Deadline: November 8, 2024

state machine P1:



state machine for P2:



* (b) let us say, NC_i = noncrit i, C_i = criti, W_i = waiti for asynchronous composition, combined states of P1 and P2 are

1) $\langle NC_1, NC_2 \rangle$ Both are non critical

2) $\langle W_1, NC_2 \rangle$ 3) $\langle NC_1, W_2 \rangle$ 4) $\langle C_1, NC_2 \rangle$

5) $\langle NC_1, C_2 \rangle$ 6) $\langle W_1, W_2 \rangle$ 7) $\langle C_1, W_2 \rangle$

8) $\langle W_1, C_2 \rangle$ 9) $\langle C_1, C_2 \rangle$

(diagram in next page)

Name: HIMESH BARAIK

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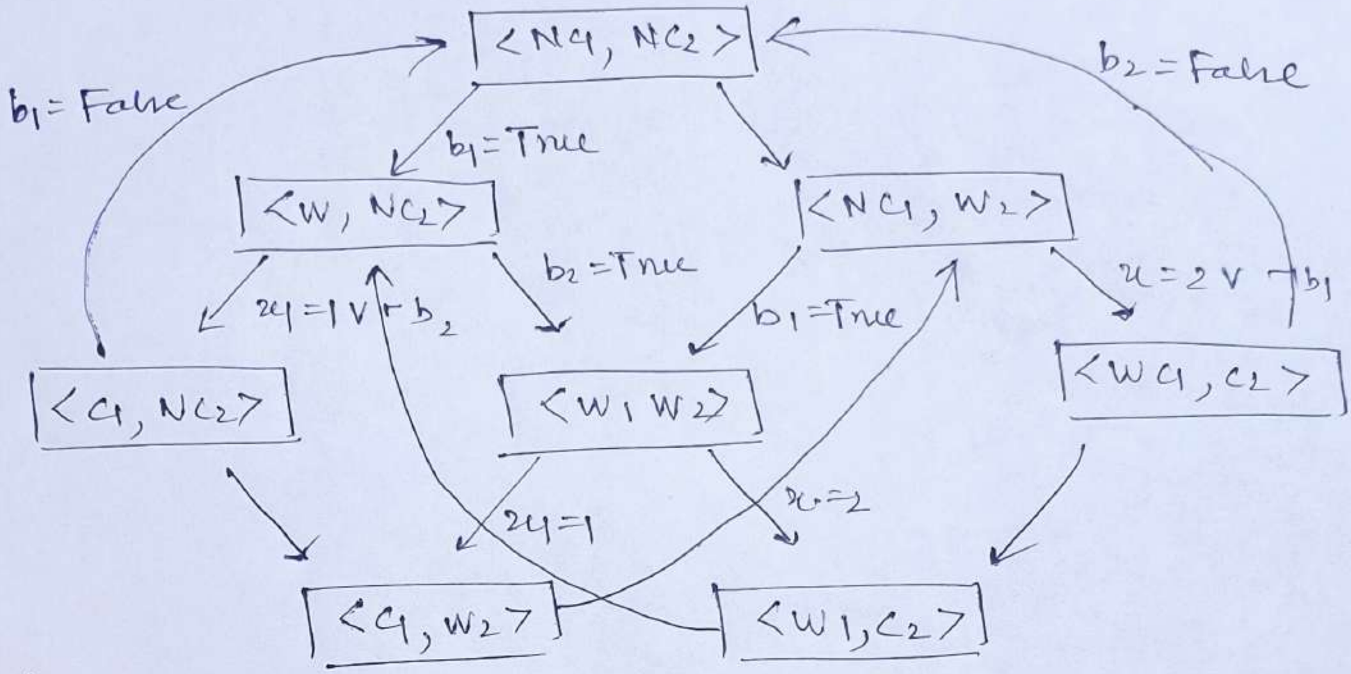
Indian Institute of Technology Kanpur
CS637 Embedded and Cyber-Physical Systems
Homework Assignment 3
Deadline: November 8, 2024



*(c)

- There are total of 9 states in the state diagram out of which 8 states are reachable and the state $\langle C_1, C_2 \rangle$ ie when both P_1 and P_2 are critical is unreachable

* (d) The LTL formula ensuring P_1 and P_2 are never both in critical section is

$$G\ \neg\ (P_1\_incritical \wedge P_2\_incritical)$$

- The formula means it is globally true that P_1 and P_2 are not in critical section simultenously.

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Roll No: 210447 (e.g. 170001) Dept.: EE (e.g. CSE)

Indian Institute of Technology Kanpur
CS637 Embedded and Cyber-Physical Systems
Homework Assignment 3
Deadline: November 8, 2024

→ verification of mutual exclusion,
from the state machine diagram,
we can observe that $\langle c_1, c_2 \rangle$ is
unreachable therefore, mutual exclusion
holds for system.

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Indian Institute of Technology Kanpur
CS637 Embedded and Cyber-Physical Systems
Homework Assignment 3
Deadline: November 8, 2024

Problem 4. (10 points)
Consider the following program:

```
int count (int a, int b)
{
    int count;
    for (count = 0; count < 2; count++)
    {
        if (a > b)
            b = a + 1;
        else
            b = a - 1;
    }
    return b;
}
```

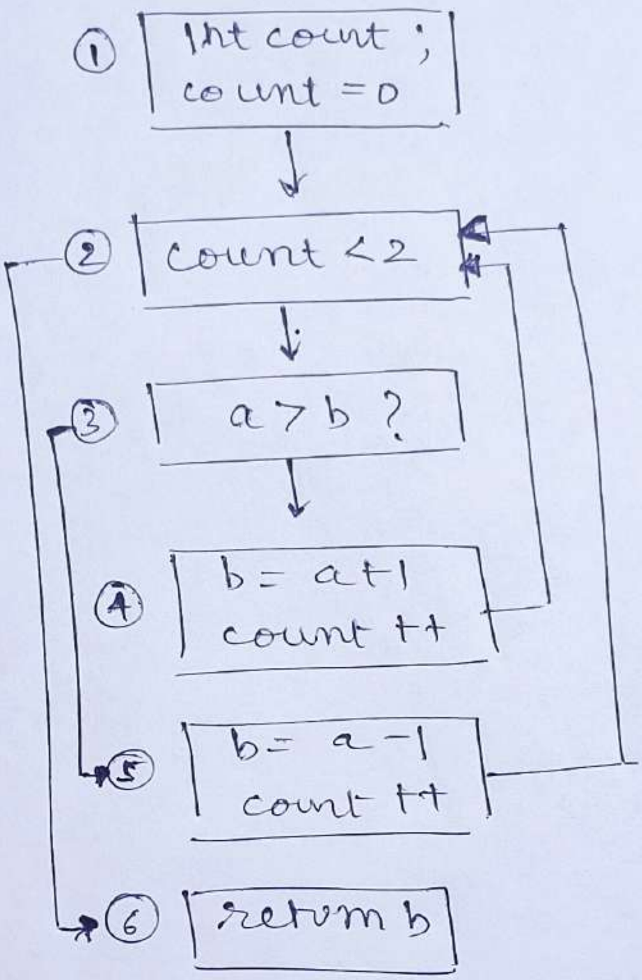
(a) Draw a control flow graph for the program.
(b) How many paths are there in the program? How many paths are feasible?
(c) Assume the following:

- An assignment statement (for example, `count = 0`) requires 2 unit time for execution.
- A statement involving an arithmetic operation followed by an assignment (for example, `count ++`, `b = a + 1`) requires 6 unit time for execution.
- A comparison statement (for example, `count < 2`) requires 4 unit time for execution.

Compute a tight bound on the worst-case execution time for the program.

Name: HIMESH BARAIK
Roll No: 210447 (e.g. 170001) Dept.: EE (e.g. CSE)

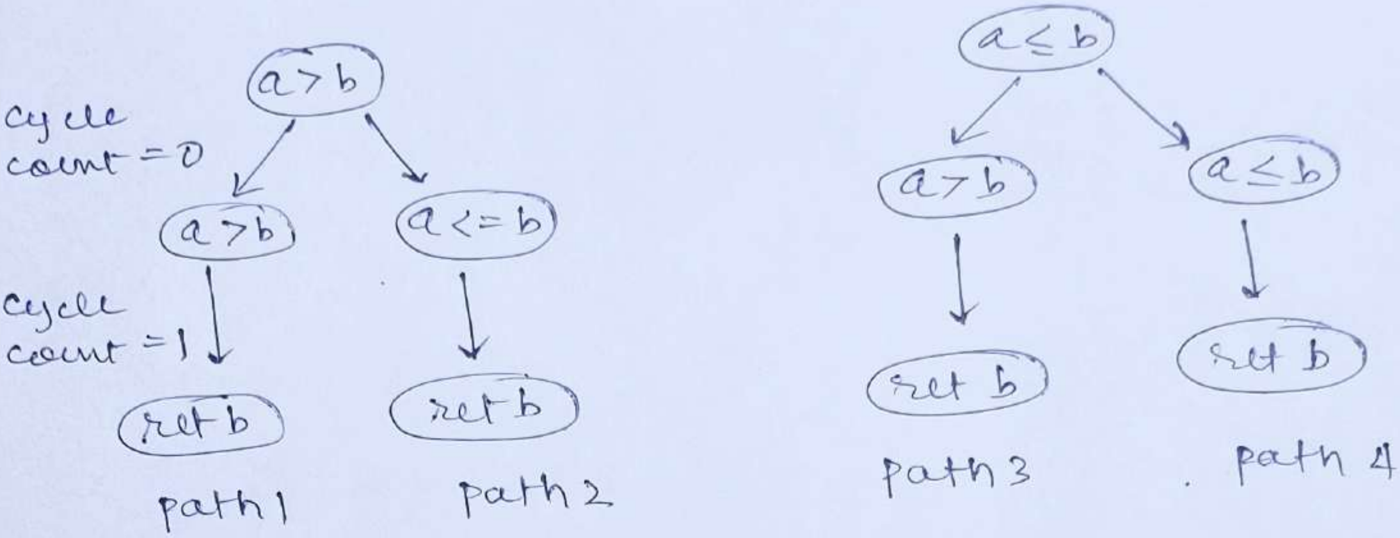
* (a) - control flow graph for the program.



* (b) since, the for loop runs for two cycles when count = 0 and count = 1.
In each cycle it has two possible paths.
If a > b is true and if it is false.
When count = 2, loop terminates and passes to return

Name: HIMESH BARAIK
Roll No: 210447 Dept.: EE

- therefore, we have total $2 \times 2 = 4$ paths as shown:



If $a > b$, then after one execution of for loop $b = a+1$ ie $b > a$ Hence in next cycle $a > b$ is not satisfied and path of $a <= b$ is followed ⇒
path 1 is not feasible
Path 2 is feasible

If initially $a \le b$, then after one execution, $b = a - 1$, hence only $a > b$ can be followed in next iteration
⇒ path 3 is feasible, Path 4 is not

⇒ Path 2 & Path 3 are FEASIBLE

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* (C) Given time units required for
execution assignment statement = 2 unit
statement and count++ = 6 unit
comparison = 4 units

$$WCET = \max \sum_{i=1}^{n} w_i x_i$$

w_i = time of exe.
x_i = no. of times the block exe.

calculation x_i:

$x_1 = 1 \quad x_6 = 1 \quad x_2 = x_3 + 1 \quad x_4 + x_5 = x_3$

each time when x_3 is executed either x_4 or x_5 is executed.
hence $x_4 + x_5 = x_3$

execution time w_i

$w_1 = 2$ unit (assignment)
$w_2 = 4$ unit (comparison)
$w_3 = 4$ unit (comparison)
$w_4 = 6$ unit
$w_5 = 6$ unit
$w_6 = 0$

statement and count++
$2 + 4 = 6$ units.

$\Rightarrow \sum w_i x_i = 34$ units

WCET = 34 units.

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Dept.: EE (e.g. CSE)