## Total: 50 marks

**Problem 1.** (10 points) Provide the state-space representation of the dynamics of a DC Motor. Assume that there is no additional load on the motor. Next, Design a Simulink model to capture the dynamics and simulate the model for an input PWM voltage signal with magnitude 1V, frequency 1 kHz and duty cycle 0.1. Assume that the kinetic friction of the motor is negligible. Take the values of the other parameters from Example 7.13 in [LS15]. Provide the plot showing how the angular velocity of the motor varies with time.

[LS15] Edward A. Lee and Saniit A. Seshia. Introduction to Embedded Systems. A Cyber-Physical Systems

[LS15] Edward A. Lee and Sanjit A. Seshia, Introduction to Embedded Systems, A Cyber-Physical Systems Approach, Second Edition, http://LeeSeshia.org, ISBN 978-1-312-42740-2, 2015.

#### Parameters of motor

The motor parameters are defined as follows:

$$I = 3.88 \times 10^{-7} \text{ kg} \cdot \text{m}^2$$
  
 $k_b = 2.75 \times 10^{-4} \text{ volts/RPM}$   
 $k_T = 5.9 \times 10^{-3} \text{ newton} \cdot \text{meters/amp}$   
 $R = 1.71 \text{ ohms}$   
 $L = 1.1 \times 10^{-4} \text{ henrys}$ 

## Mathematical Modelling:

Initially, at  $t = 0^+$ ,

$$V(t) = Ri(t) + L\frac{di(t)}{dt} \iff V(s) = (R + sL)i(s)$$

Due to this current i(t), torque is generated in the motor shaft,

$$\tau(t) = k_T i(t) \iff \tau(s) = k_T i(s)$$

This leads to the torque equation (with constant load torque  $\tau_L$ , which equals 0 in this case):

$$I\frac{d\omega(t)}{dt} = \tau(t) - \tau_L \quad \Longleftrightarrow \quad sI\omega(s) = \tau(s) - \tau_L \tag{1}$$

Now, due to the rotation of the motor, there is an armature reaction leading to back e.m.f generated,

$$E_b(t) = k_b \omega(t) \iff E_b(s) = k_b \omega(s)$$

This leads to a change in the original equation:

$$V(t) - E_b(t) = Ri(t) + L\frac{di(t)}{dt} \iff V(s) - k_b\omega(s) = Ri(s) + sLi(s)$$
(2)

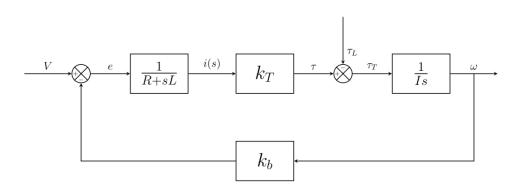


Figure 1: Block Diagram for Voltage Controlled DC Motor

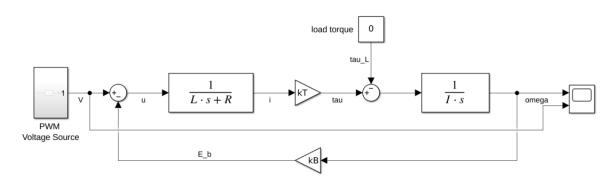


Figure 2: Transfer Function

For obtaining the state-space equations, consider equations (1) and (2), we get:

$$\begin{bmatrix} \dot{\omega} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} 0 & \frac{k_T}{I} \\ -\frac{k_B}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{I} \\ \frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} V \\ \tau_L \end{bmatrix}$$
 (3)

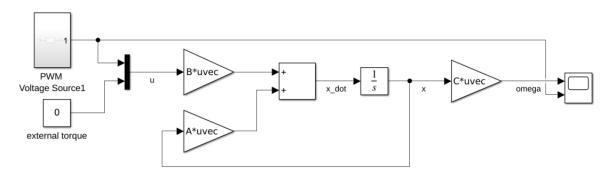


Figure 3: Steady Space

Resultant response of both representation for 1V DC Input with 0.1 duty cycle at 1kHz:

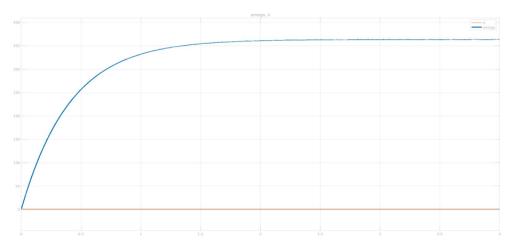


Figure 4: Response to 0.1 V

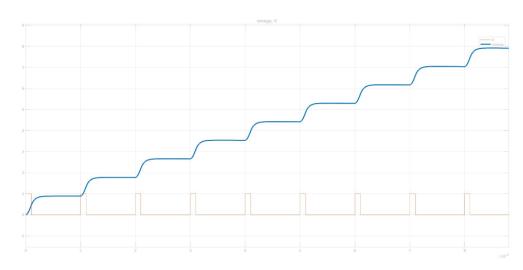


Figure 5: Initial response (blue), Input (red)

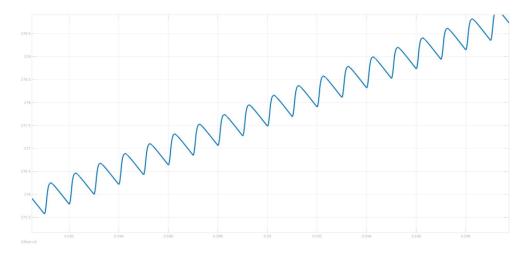


Figure 6: PWM rise and fall visible in response curve

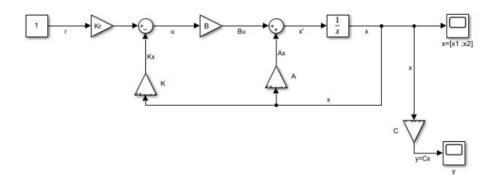
**Problem 2.** (10 points) Consider the vehicle steering control problem in Example 6.4 in [AM09]. Assume that  $k_1 = 1$ ,  $k_2 = 1.6$ , and  $k_r = 1$ . Model the control system in Simulink using double precision floating point arithmetic. Now replace the model of the controller with the ones that use 16 bit and 8-bit fixed-point arithmetic. In each case, determine the fixed-point data types precisely. Plot the difference between the first state for the floating-point controller and that for the fixed-point controllers. Generate code for both the floating point controller and the fixed-point controllers using different optimization options. Describe the steps you have followed for the code generation.

[AM09] K. J. Astrom and R. M. Murray. Feedback Systems: An Introduction for Scientists and Engineers. Princeton University Press, 2009.

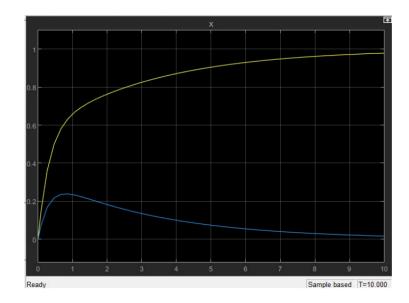
http://www.cds.caltech.edu/~murray/books/AM05/pdf/am08-complete\_22Feb09.pdf.

#### **Solution:**

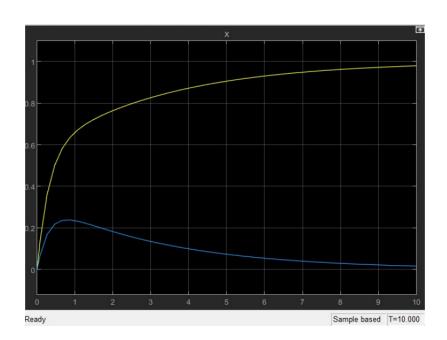
#### Simulink Model:



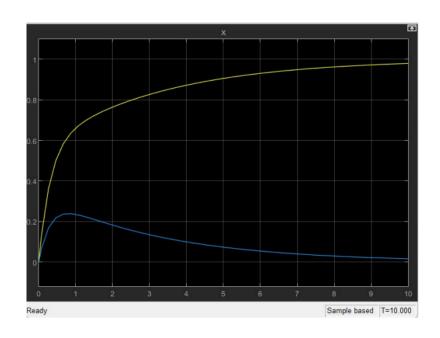
#### Double Precision Point Arithmetic:



### 8 Bit Fixed Point Arithmetic:



## 16 Bit Fixed Point Arithmetic:



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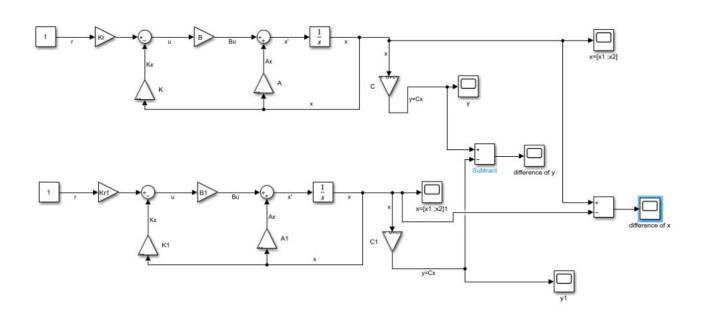
Indian Institute of Technology Kanpur CS637 Embedded and Cyber-Physical Systems
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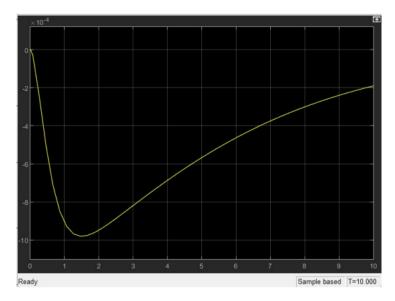
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>> A=sfi(A,8); >> gamma=sfi(gamma,8); >> B=sfi(B,8); >> K=sfi(K,8) K = 1.0000 1.5938 DataTypeMode: Fixed-point: binary point scaling Signedness: Signed WordLength: 8 FractionLength: 6 >> Kr=sfi(Kr,8); >> C=sfi(Kr,8); >> C=sfi(C,8); >> C=[1 0]; >> C=sfi(C,8); >> A=sfi(A, 16); >> gamma=sfi(gamma,8); >> gamma=sfi(gamma, 16); >> B=sfi(B,16); >> K=sfi(K, 16); >> C=sfi(C,16); >> Kr=sfi(Kr, 16);

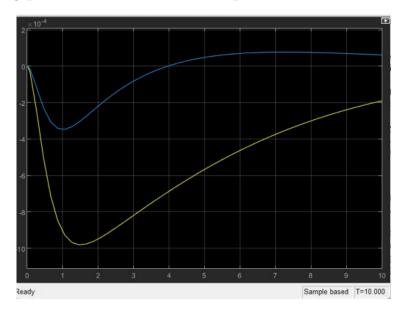


## Differnce of Y in floating point controller and fixed point Controller



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# Differnce of X in floating point controller and fixed point Controller



**Problem 3.** (10 points) Work out Problem 1 in the Exercises of Chapter 9 in [LS15]. [LS15] Edward A. Lee and Sanjit A. Seshia, Introduction to Embedded Systems, A Cyber-Physical Systems Approach, Second Edition, http://LeeSeshia.org, ISBN 978-1-312-42740-2, 2015.

#### **Solution:**

```
int data[N];
int compute_variance() {
    int sum1 = 0, sum2 = 0, result;
    int i;

    for(i=0; i < N; i++) {
        sum1 += data[i];
    }
    sum1 /= N;

    for(i=0; i < N; i++) {
        sum2 += data[i] * data[i];
    }
    sum2 /= N;

    result = (sum2 - sum1*sum1);

    return result;
}</pre>
```

## Cache Memory Analysis

#### Main Memory:

Main memory is divided into blocks of 4 words (16 bytes), with a total of 4094 blocks, making up 64 kB.

#### Cache:

The cache is divided into 128 blocks (each block has 4 words = 16 bytes), giving a total cache size of 2 kB. The cache is direct-mapped, meaning each memory block i is mapped to cache block i mod 128.

#### Part 1

Let N = 16.

### Loop 1:

- 1. Step 1 (data[0]): Cache miss occurs as data[0] is not in cache. Memory block 0x00 to 0x1F is loaded into cache set 0.
- 2. Step 2 (data[1]): Cache hit; data[1] is already in cache set 0.
- 3. Step 3 (data[2]): Cache miss occurs. Memory block 0x20 to 0x3F is loaded into cache set 1.
- 4. Step 4 (data[3]): Cache hit; data[3] is already in cache set 1.

This pattern continues, resulting in:

• Every even-indexed access results in a cache miss, while every odd-indexed access results in a cache hit.

#### **Total Cache Misses:**

8 out of 16 accesses (50% miss rate).

#### Loop 2:

Since the data array is already in cache, all accesses will be cache hits.

#### **Total Cache Misses:**

0 out of 16 accesses (0\% miss rate).

#### Summary for N=16:

Loop 1: 8 misses

Loop 2: 0 misses

Total: 8 out of 32 accesses (25\% miss rate).

#### <u>Part 2:</u>

Let N = 32.

#### Loop 1:

The first 16 elements (data[0] to data[15]) are loaded into cache similarly as in Part 1, with 8 cache misses. When accessing data[16], a cache miss occurs as it maps to cache set 0, which now contains data[0] and data[1]. A cache miss occurs, loading block 0x100 to 0x11F, evicting data[0] and data[1].

This pattern continues, resulting in: A cache miss for every even-indexed access from data[16] onwards.

## **Total Cache Misses:**

16 out of 32 accesses (50% miss rate).

### Loop 2:

After Loop 1, accessing data[0] results in a cache miss (loading back block 0x00 to 0x1F). Every two iterations result in a cache miss as blocks are reloaded into cache.

#### **Total Cache Misses:**

16 out of 64 accesses (25\% miss rate).

### Summary for N = 32:

Loop 1: 16 misses Loop 2: 16 misses

Total: 32 out of 96 accesses (33% miss rate).

### Part 3:

Let N = 16 with Cache Configuration (m, S, E, B) = (32, 8, 2, 4).

### Cache Configuration:

- 8 sets (S = 8)
- 2 lines per set (E=2)
- Each block in the cache stores only one word (B=4).

## Loop 1:

Iteration 0 (data[0]): Cache miss occurs, loading block 0x00 to 0x0F into cache set 0, line 0.

Iteration 1 (data[1]): Cache miss occurs, loading block 0x10 to 0x1F into cache set 1, line 0.

This continues until data[8], which maps back to set 0. Since there are 2 lines per set, data[8] is stored in line 1 of cache set 0 without evicting data[0]. This prevents immediate evictions.

### **Total Cache Misses:**

16 cache misses out of 16 accesses in Loop 1.

#### Loop 2:

All data is already loaded in the cache from Loop 1, resulting in all accesses being cache hits.

#### **Total Cache Misses:**

0 cache misses out of 16 accesses.

#### Summary for N = 16 (2-way Set-Associative Cache):

Loop 1: 16 misses

Loop 2: 0 misses

Total: 16 out of 48 accesses (33\% miss rate).

## Conclusion

Part 1 (N = 16):

8 misses in Loop 1, 0 in Loop 2. Total: 8 out of 32 accesses (25\% miss rate).

Part 2 (N = 32):

32 misses out of 96 accesses (33% miss rate).

Part 3 (N = 16, 2-way associative cache):

16 misses in Loop 1, 0 in Loop 2. Total: 16 out of 48 accesses (33\% miss rate).

**Problem 4.** (10 points) Work out Problem 3 in the Exercises of Chapter 12 in [LS15].

[LS15] Edward A. Lee and Sanjit A. Seshia, Introduction to Embedded Systems, A Cyber-Physical Systems Approach, Second Edition, http://LeeSeshia.org, ISBN 978-1-312-42740-2, 2015.

#### **Solution:**

This problem compares RM and EDF schedules.

Consider two tasks with periods p1 = 2 and p2 = 3 and execution times e1 = e2 = 1. Assume that the deadline for each execution is the end of the period.

#### 1. RM Schedule and Processor Utilization

### Task Characteristics

Task 1 (T1): 
$$p_1 = 2$$
,  $e_1 = 1$   
Task 2 (T2):  $p_2 = 3$ ,  $e_2 = 1$ 

#### RM Schedule

Using the Rate Monotonic (RM) scheduling, we assign priorities based on the periods of the tasks: the task with the shorter period gets a higher priority. Thus:

• T1 has a higher priority than T2.

We can construct the schedule as follows:

- Time 0: T1 executes from 0 to 1 (remaining time 1)
- Time 1: T1 completes, T2 executes from 1 to 2 (remaining time 0)
- Time 2: T1 starts executing again from 2 to 3 (remaining time 1)
- Time 3: T1 completes; T2 executes from 3 to 4 (remaining time 0)
- Time 4: T1 starts executing again from 4 to 5 (remaining time 1)
- Time 5: T1 completes; T2 executes from 5 to 6 (remaining time 0)
- Time 6: T1 starts executing again from 6 to 7 (remaining time 1)
- Time 7: T1 completes; T2 executes from 7 to 8 (remaining time 0)

This pattern continues, and the schedule can be repeated every 6 time units since

$$LCM(2,3) = 6.$$

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#### **Processor Utilization**

The utilization U for RM scheduling can be calculated as follows:

$$U = \frac{e_1}{p_1} + \frac{e_2}{p_2}$$

$$= \frac{1}{2} + \frac{1}{3}$$

$$= \frac{3}{6} + \frac{2}{6}$$

$$= \frac{5}{6} \approx 0.8333.$$

## Liu and Layland Utilization Bound

The utilization bound for n tasks is given by:

$$U(n) = n \cdot \left(2^{1/n} - 1\right).$$

For n=2:

$$U(2) = 2 \cdot (2^{1/2} - 1)$$

$$= 2 \cdot (1.414 - 1)$$

$$= 2 \cdot 0.414$$

$$= 0.828.$$

Thus, we have:

$$U = 0.8333, \quad U(2) \approx 0.828.$$

Since U > U(2), this implies the tasks are schedulable under RM but close to the utilization bound.

## 2. Feasibility with Increased Execution Times

### Increase in $e_1$ or $e_2$

If we increase either  $e_1$  or  $e_2$ :

• If  $e_1$  increases:

$$U = \frac{e_1}{p_1} + \frac{1}{2} + \frac{1}{3} > 1$$

implies

$$U = \frac{2}{e_1} + \frac{1}{3} > 1$$
 (infeasible)

• If  $e_2$  increases:

$$U = \frac{1}{2} + \frac{e_2}{3} + \frac{1}{3} > 1$$

implies

$$U = \frac{1}{2} + \frac{3}{e_2} > 1$$
 (infeasible)

Thus, any increase in execution time makes the RM schedule infeasible.

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## Reducing $p_1$

If we hold  $e_1 = e_2 = 1$  and  $p_2 = 3$  constant, we can try reducing  $p_1$ :

• If 
$$p_1 = 1$$
:

$$U = \frac{1}{1} + \frac{1}{3} = 1 + 0.333 > 1$$
 (infeasible)

• If 
$$p_1 = 1.5$$
:

$$U = \frac{1}{1.5} + \frac{1}{3} = 0.666 + 0.333 = 1$$
 (feasible)

Thus, we can reduce  $p_1$  to a maximum of 1.5.

## Reducing $p_2$

If we hold  $e_1 = e_2 = 1$  and  $p_1 = 2$  constant, we can try reducing  $p_2$ :

• If 
$$p_2 = 2.5$$
:

$$U = \frac{1}{2} + \frac{1}{2.5} = 0.5 + 0.4 = 0.9$$
 (feasible)

• If 
$$p_2 = 2$$
:

$$U = \frac{1}{2} + \frac{1}{2} = 1$$
 (infeasible)

Thus, we can reduce  $p_2$  to a maximum of 2.5.

#### 3. EDF Schedule with Increased Execution Time of Task 2

**New Task Characteristics** 

• Task 1 
$$(T_1)$$
:

$$p_1 = 2, \quad e_1 = 1$$

• Task 2 
$$(T_2)$$
:

$$p_2 = 3, \quad e_2 = 1.5$$

## **EDF Schedule**

Using the Earliest Deadline First (EDF) scheduling, we schedule the tasks based on their deadlines:

- Time 0:  $T_1$  executes from 0 to 1 (remaining time 1)
- Time 1:  $T_2$  executes from 1 to 2.5 (remaining time 0.5)
- Time 2.5:  $T_1$  executes from 2.5 to 3 (remaining time 0)
- Time 3:  $T_2$  executes from 3 to 4 (remaining time 0)
- Time 4:  $T_2$  executes from 4 to 5 (remaining time 0)

We can see that:

- At Time 2:  $T_1$  is scheduled first due to its earlier period.
- At Time 3:  $T_2$  has remaining time to complete but its deadline is missed, as it was only supposed to run until time 3, which is infeasible.

### Feasibility

Since  $T_2$  cannot complete within its deadline, the EDF schedule is not feasible.

## **Processor Utilization**

The utilization U for EDF can be calculated as follows:

$$U = \frac{e_1}{p_1} + \frac{e_2}{p_2} = \frac{1}{2} + \frac{1.5}{3} = 0.5 + 0.5 = 1$$

Thus, the processor utilization is U = 1, which means it is fully utilized but cannot meet the deadlines due to the higher execution time of  $T_2$ .

**Problem 5.** (10 points) Work out Problem 6 in the Exercises of Chapter 12 in [LS15].

[LS15] Edward A. Lee and Sanjit A. Seshia, Introduction to Embedded Systems, A Cyber-Physical Systems Approach, Second Edition, http://LeeSeshia.org, ISBN 978-1-312-42740-2, 2015.

#### **Solution:**

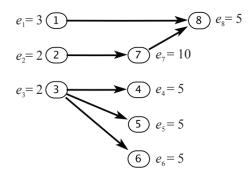


Figure 12.16: Precedence Graph for Exercise 6.

#### **Precedence Constraints**

- $T_1$  must finish before  $T_8$
- $T_2$  must finish before  $T_7$
- $T_7$  must finish before  $T_8$
- $T_3$  must finish before  $T_4$ ,  $T_5$ , and  $T_6$

#### Task Execution Times

- Task 1  $(T_1)$ :  $e_1 = 3$
- Task 2  $(T_2)$ :  $e_2 = 2$
- Task 3  $(T_3)$ :  $e_3 = 2$
- Task 4  $(T_4)$ :  $e_4 = 5$
- Task 5  $(T_5)$ :  $e_5 = 5$
- Task 6  $(T_6)$ :  $e_6 = 5$
- Task 7  $(T_7)$ :  $e_7 = 10$
- Task 8  $(T_8)$ :  $e_8 = 5$

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## (a) Scheduling on Two Processors

### Schedule:

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• Processor 1 (P1):

$$\boxed{\text{T1 (3)}} \rightarrow \boxed{\text{T3 (2)}} \rightarrow \boxed{\text{T4 (5)}}$$

• Processor 2 (P2):

$$\boxed{\text{T2 (2)}} \rightarrow \boxed{\text{T7 (10)}} \rightarrow \boxed{\text{T8 (5)}}$$

### **Execution Order:**

- $T_1(3)$  on  $P_1$ : 0 to 3
- $T_2(2)$  on  $P_2$ : 0 to 2
- $T_3(2)$  on  $P_1$ : 3 to 5
- $T_4(5)$  on  $P_1$ : 5 to 10
- $T_7(10)$  on  $P_2$ : 2 to 12
- $T_8(5)$  on  $P_2$ : 12 to 17

### Makespan:

The final completion time is 17 time units.

## (b) Scheduling on Three Processors

## Schedule:

• Processor 1 (P1):

$$\boxed{\text{T1 (3)}} \rightarrow \boxed{\text{T3 (2)}}$$

• Processor 2 (P2):

$$\boxed{\text{T2 (2)}} \rightarrow \boxed{\text{T4 (5)}}$$

• Processor 3 (P3):

$$\boxed{\text{T5 (5)}} \rightarrow \boxed{\text{T6 (5)}}$$

### **Execution Order:**

- $T_1(3)$  on  $P_1$ : 0 to 3
- $T_2(2)$  on  $P_2$ : 0 to 2
- $T_3(2)$  on  $P_1$ : 3 to 5
- $T_4(5)$  on  $P_2$ : 2 to 7
- $T_5(5)$  on  $P_3$ : 5 to 10
- $T_6(5)$  on  $P_3$ : 10 to 15
- $T_7(10)$  on  $P_2$ : 7 to 17
- $T_8(5)$  on  $P_2$ : 17 to 22

### Makespan:

The final completion time is 22 time units.

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## (c) Scheduling on Two Processors with Reduced Execution Times

### **Execution Times:**

- Task 1  $(T_1)$ :  $e_1 = 2$
- Task 2  $(T_2)$ :  $e_2 = 1$
- Task 3  $(T_3)$ :  $e_3 = 1$
- Task 4  $(T_4)$ :  $e_4 = 4$
- Task 5  $(T_5)$ :  $e_5 = 4$
- Task 6  $(T_6)$ :  $e_6 = 4$
- Task 7  $(T_7)$ :  $e_7 = 9$
- Task 8  $(T_8)$ :  $e_8 = 4$

### Schedule:

• Processor 1 (P1):

$$\boxed{\text{T1 (2)}} \rightarrow \boxed{\text{T3 (1)}} \rightarrow \boxed{\text{T4 (4)}}$$

• Processor 2 (P2):

$$\boxed{\text{T2 (1)}} \rightarrow \boxed{\text{T7 (9)}} \rightarrow \boxed{\text{T8 (4)}}$$

## **Execution Order:**

- $T_1(2)$  on  $P_1$ : 0 to 2
- $T_2(1)$  on  $P_2$ : 0 to 1
- $T_3(1)$  on  $P_1$ : 2 to 3
- $T_4(4)$  on  $P_1$ : 3 to 7
- $T_7(9)$  on  $P_2$ : 1 to 10
- $T_8(4)$  on  $P_2$ : 10 to 14

### Makespan:

The final completion time is 14 time units.

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## **Summary of Makespans**

• Part (a): 17 time units (two processors)

• Part (b): 22 time units (three processors)

• Part (c): 14 time units (two processors with reduced execution times)

## Conclusions

- The makespan is greater when using three processors (22) compared to using two processors (17).
- Reducing the execution times of tasks results in a smaller makespan (14) compared to the original two-processor scenario (17).