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Introduction

Fuzzy controllers appear in consumer products – such as washing machines, video cameras, cars – and in industry, for controlling cement kilns, underground trains, and robots. A *fuzzy controller* is an *automatic controller*, a self-acting or self-regulating mechanism that controls an object in accordance with a desired behaviour. The object can be, for instance, a robot set to follow a certain path. A fuzzy controller acts or regulates by means of rules in a more or less natural language, based on the distinguishing feature: fuzzy logic. The rules are invented by plant operators or design engineers, and fuzzy control is thus a branch of intelligent control.

1.1 What Is Fuzzy Control?

Traditionally, computers make rigid *yes* or *no* decisions, by means of decision rules based on two-valued logic: *true–false*, *yes–no*, or $1 - 0$. An example is an air conditioner with thermostat control that recognizes just two states: above the desired temperature or below the desired temperature. *Fuzzy logic,* on the other hand, allows a graduation from *true* to *false*. A fuzzy air conditioner may recognize 'warm' and 'cold' room temperatures. The rules behind this are less precise, for instance:

If the room temperature is warm and slightly increasing, then increase the cooling.

Many classes or *sets* have *fuzzy* rather than sharp boundaries, and this is the mathematical basis of fuzzy logic; the set of 'warm' temperature measurements is one example of a fuzzy set.

The core of a fuzzy controller is a collection of *verbal* or *linguistic* rules of the *if–then* form. Several variables may occur in each rule, both on the *if*-side and the *then*-side. Reflecting expert opinions, the rules can bring the reasoning used by computers closer to that of human beings.

In the example of the fuzzy air conditioner, the controller works on the basis of a temperature measurement. The room temperature is just a number, and more information

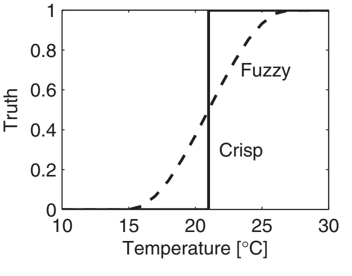


Figure 1.1: A warm room. The crisp air conditioner considers any temperature above 21 °C warm. The fuzzy air conditioner considers gradually warmer temperatures. (figwarm.m)

is necessary to decide whether the room is warm. Therefore the designer must incorporate a human's perception of warm room temperatures. A straightforward implementation is to evaluate beforehand all possible temperature measurements. For example, on a scale from 0 to 1, *warm* corresponds to 1 and *not warm* corresponds to 0:

Measurements (°C):	...	15	17	19	21	23	25	27	...
		|	|	|	|	|	|	|	
Grade:	...	0	0.1	0.3	0.5	0.7	0.9	1	...

The example uses *discrete* temperature measurements, whereas Figure 1.1 shows the same idea graphically in the form of a *continuous* mapping of temperature measurements to truth-values. The mapping is arbitrary, that is, based on preference and not mathematical reason.

1.2 Why Fuzzy Control?

If PID control (proportional-integral-derivative control) is inadequate – for example, in the case of higher-order plants, systems with a long deadtime, or systems with oscillatory modes (Åström and Hägglund 1995) – fuzzy control is an option. But first, let us consider why one would *not* use a fuzzy controller:

- The PID controller is well understood, easy to implement – both in its digital and analog forms – and it is widely used. By contrast, the fuzzy controller requires some knowledge of fuzzy logic. It also involves building arbitrary membership functions.
- The fuzzy controller is generally nonlinear. It does not have a simple equation like the PID, and it is more difficult to analyse mathematically; approximations are required, and it follows that stability is more difficult to guarantee.
- The fuzzy controller has more tuning parameters than the PID controller. Furthermore, it is difficult to trace the data flow during execution, which makes error correction more difficult.

On the other hand, fuzzy controllers are used in industry with success. There are several possible reasons:

- Since the control strategy consists of *if–then* rules, it is easy for a plant operator to read. The rules can be built from a vocabulary containing everyday words such as 'high', 'low', and 'increasing'. Plant operators can embed their experience directly.
- The fuzzy controller accommodates many inputs and many outputs. Variables can be combined in an *if–then* rule with the connectives *and* and *or*. Rules are executed in parallel, implying a recommended action from each. The recommendations may be in conflict, but the controller resolves conflicts.
  Fuzzy logic enables non-specialists to design control systems, and this may be the main reason for its success.

Artificial Intelligence - Fuzzy Logic Systems

Fuzzy Logic Systems (FLS) produce acceptable but definite output in response to incomplete, ambiguous, distorted, or inaccurate (fuzzy) input.

What is Fuzzy Logic?

Fuzzy Logic (FL) is a method of reasoning that resembles human reasoning. The approach of FL imitates the way of decision making in humans that involves all intermediate possibilities between digital values YES and NO.

The conventional logic block that a computer can understand takes precise input and produces a definite output as TRUE or FALSE, which is equivalent to human's YES or NO.

The inventor of fuzzy logic, Lotfi Zadeh, observed that unlike computers, the human decision making includes a range of possibilities between YES and NO, such as −

CERTAINLY YES
POSSIBLY YES
CANNOT SAY
POSSIBLY NO
CERTAINLY NO

The fuzzy logic works on the levels of possibilities of input to achieve the definite output.

Implementation

It can be implemented in systems with various sizes and capabilities ranging from small micro-controllers to large, networked, workstation-based control systems.

It can be implemented in hardware, software, or a combination of both.

Why Fuzzy Logic?

Fuzzy logic is useful for commercial and practical purposes.

It can control machines and consumer products.

It may not give accurate reasoning, but acceptable reasoning.

Fuzzy logic helps to deal with the uncertainty in engineering.

Fuzzy Logic Systems Architecture

It has four main parts as shown −

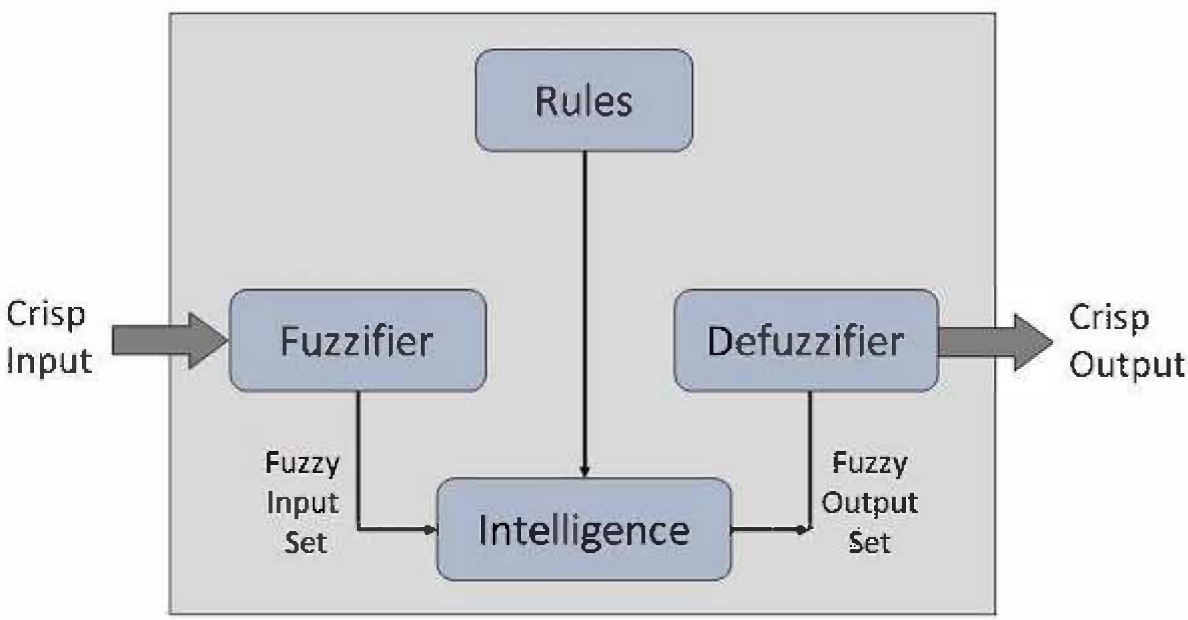
Fuzzification Module − It transforms the system inputs, which are crisp numbers, into fuzzy sets. It splits the input signal into five steps such as −

LP	x is Large Positive
MP	x is Medium Positive
S	x is Small
MN	x is Medium Negative
LN	x is Large Negative

Knowledge Base − It stores IF-THEN rules provided by experts.

Inference Engine − It simulates the human reasoning process by making fuzzy inference on the inputs and IF-THEN rules.

Defuzzification Module − It transforms the fuzzy set obtained by the inference engine into a crisp value.



The **membership functions work on** fuzzy sets of variables.

Membership Function

Membership functions allow you to quantify linguistic term and represent a fuzzy set graphically. A **membership function** for a fuzzy *set A* on the universe of discourse X is defined as $\mu_A: X \rightarrow [0,1]$.

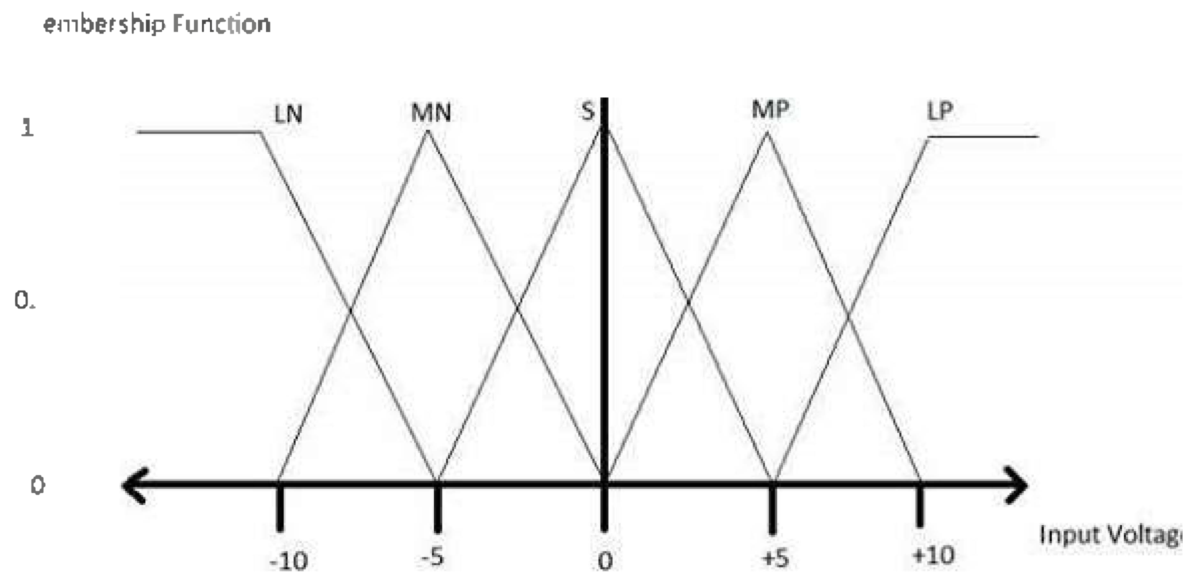
Here, each element of *X* is mapped to a value between 0 and 1. It is called **membership value** or **degree of membership**. It quantifies the degree of membership of the element in *X* to the fuzzy set *A*.

x axis represents the universe of discourse.

y axis represents the degrees of membership in the [0, 1] interval.

There can be multiple membership functions applicable to fuzzify a numerical value. Simple membership functions are used as use of complex functions does not add more precision in the output.

All membership functions for **LP, MP, S, MN,** and **LN** are shown as below −

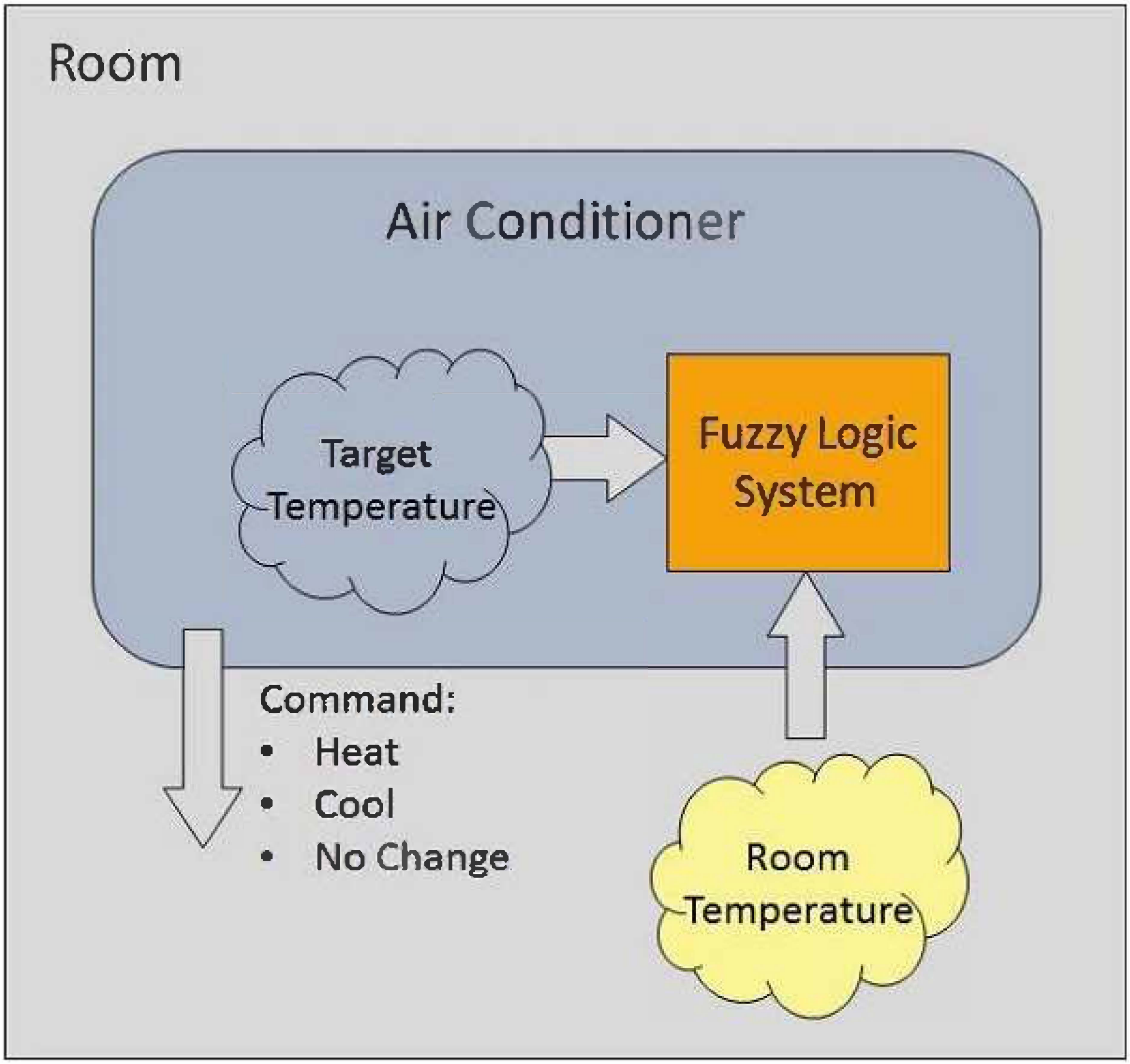


The triangular membership function shapes are most common among various other membership function shapes such as trapezoidal, singleton, and Gaussian.

Here, the input to 5-level fuzzifier varies from -10 volts to +10 volts. Hence the corresponding output also changes.

Example of a Fuzzy Logic System

Let us consider an air conditioning system with 5-level fuzzy logic system. This system adjusts the temperature of air conditioner by comparing the room temperature and the target temperature value.



Algorithm

Define linguistic variables and terms.

Construct membership functions for them.

Construct knowledge base of rules.

Convert crisp data into fuzzy data sets using membership functions. (fuzzification)

Evaluate rules in the rule base. (Inference Engine)

Combine results from each rule. (Inference Engine)

Convert output data into non-fuzzy values. (defuzzification)

Logic Development

Step 1: Define linguistic variables and terms

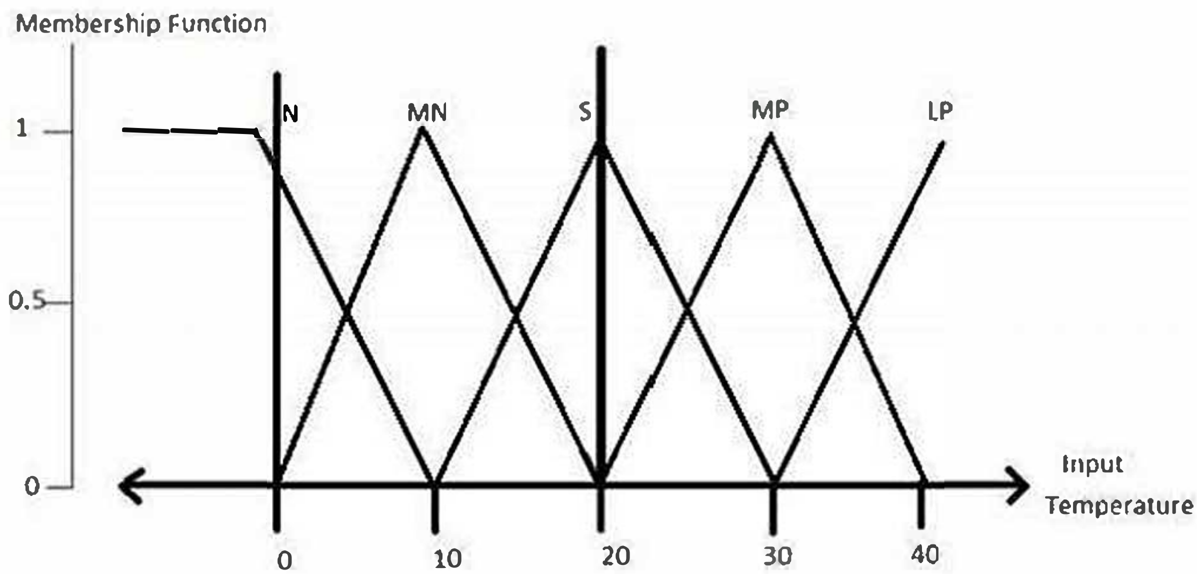
Linguistic variables are input and output variables in the form of simple words or sentences. For room temperature, cold, warm, hot, etc., are linguistic terms.

Temperature (t) = {very-cold, cold, warm, very-warm, hot}

Every member of this set is a linguistic term and it can cover some portion of overall temperature values.

Step 2: Construct membership functions for them

The membership functions of temperature variable are as shown −



Step3: Construct knowledge base rules

Create a matrix of room temperature values versus target temperature values that an air conditioning system is expected to provide.

RoomTemp. /Target	Very_Cold	Cold	Warm	Hot	Very_Hot
Very_Cold	No_Change	Heat	Heat	Heat	Heat
Cold	Cool	No_Change	Heat	Heat	Heat
Warm	Cool	Cool	No_Change	Heat	Heat
Hot	Cool	Cool	Cool	No_Change	Heat
Very_Hot	Cool	Cool	Cool	Cool	No_Change

Build a set of rules into the knowledge base in the form of IF-THEN-ELSE structures.

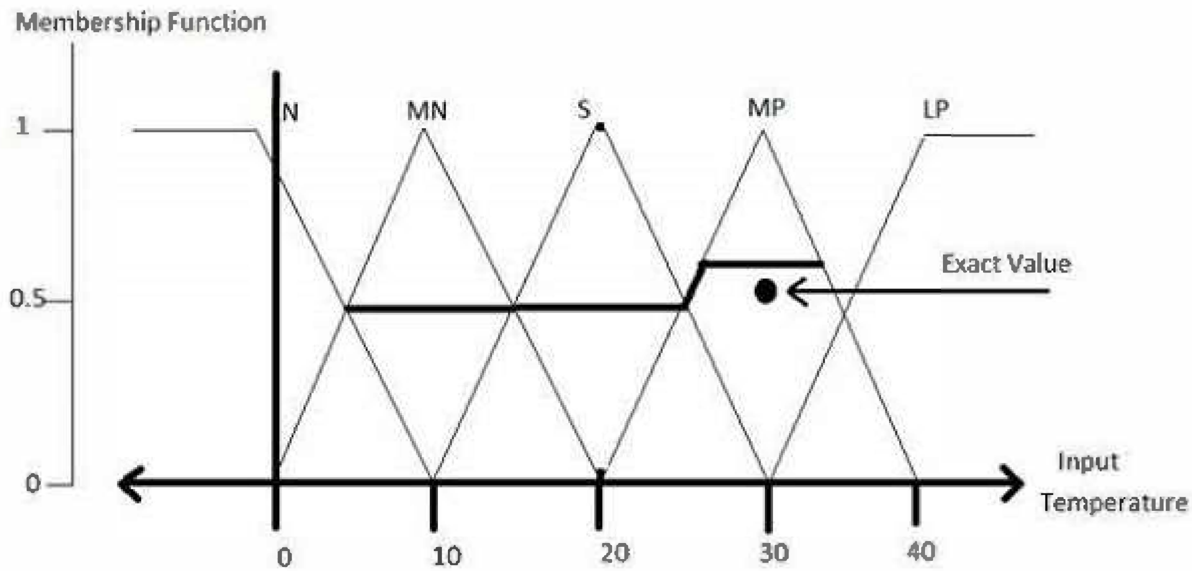
Sr. No.	Condition	Action
1	IF temperature=(Cold OR Very_Cold) AND target=Warm THEN	Heat
2	IF temperature=(Hot OR Very_Hot) AND target=Warm THEN	Cool
3	IF (temperature=Warm) AND (target=Warm) THEN	No_Change

Step 4: Obtain fuzzy value

Fuzzy set operations perform evaluation of rules. The operations used for OR and AND are Max and Min respectively. Combine all results of evaluation to form a final result. This result is a fuzzy value.

Step 5: Perform defuzzification

Defuzzification is then performed according to membership function for output variable.



Application Areas of Fuzzy Logic

The key application areas of fuzzy logic are as given −

Automotive Systems

Automatic Gearboxes

Four-Wheel Steering

Vehicle environment control

Consumer Electronic Goods

Hi-Fi Systems

Photocopiers

Still and Video Cameras

Television

Domestic Goods

Microwave Ovens

Refrigerators

Toasters

Vacuum Cleaners

Washing Machines

Environment Control

Air Conditioners/Dryers/Heaters

Humidifiers

Advantages of FLSs

Mathematical concepts within fuzzy reasoning are very simple.

You can modify a FLS by just adding or deleting rules due to flexibility of fuzzy logic.

Fuzzy logic Systems can take imprecise, distorted, noisy input information.

FLSs are easy to construct and understand.

Fuzzy logic is a solution to complex problems in all fields of life, including medicine, as it resembles human reasoning and decision making.

Disadvantages of FLSs

There is no systematic approach to fuzzy system designing.

They are understandable only when simple.

They are suitable for the problems which do not need high accuracy.

Defuzzification Methods

Fuzzy rule based systems evaluate linguistic if-then rules using fuzzification, inference and composition procedures. They produce fuzzy results which usually have to be converted into crisp output. To transform the fuzzy results in to crisp, defuzzification is performed.

Defuzzification is the process of converting a fuzzified output into a single crisp value with respect to a fuzzy set. The defuzzified value in FLC (Fuzzy Logic Controller) represents the action to be taken in controlling the process.

Different Defuzzification Methods

The following are the known methods of defuzzification.

- Center of Sums Method (COS)
- Center of gravity (COG) / Centroid of Area (COA) Method
- Center of Area / Bisector of Area Method (BOA)
- Weighted Average Method
- Maxima Methods
    - First of Maxima Method (FOM)
    - Last of Maxima Method (LOM)
    - Mean of Maxima Method (MOM)

Center of Sums (COS) Method

This is the most commonly used defuzzification technique. In this method, the overlapping area is counted twice.

The defuzzified value x^* is defined as :

$$x^* = \frac{\sum_{i=1}^{N} x_i \cdot \sum_{k=1}^{n} \mu_{A_k}(x_i)}{\sum_{i=1}^{N} \quad \sum_{k=1}^{n} \mu_{A_k}(x_i)} \quad ,$$

Here, n is the number of fuzzy sets, N is the number of fuzzy variables, $\mu_{A_k}(x_i)$ is the membership function for the k-th fuzzy set.

Example

The defuzzified value x^* is defined as :

$$x^* = \frac{\sum_{i=1}^{k} A_i \times \overline{x_i}}{\sum_{i=1}^{k} A_i} \quad ,$$

Here, A_i represents the firing area of i^{th} rules and k is the total number of rules fired and $\overline{x_i}$ represents the center of area.

The aggregated fuzzy set of two fuzzy sets C_1 and C_2 is shown in Figure 1. Let the area of this two fuzzy sets are A_1 and A_2.

A_1 = ½ * [(8-1) + (7-3)] * 0.5 = ½ * 11 * 0.5 = 55/20=2.75

A_2 = ½ * [(9-3) + (8-4)] * 0.3 = ½ * 10 * 0.3 = 3/2 =1.5

Now the center of area of the fuzzy set C_1 is let say $\overline{x_1}$ = (7+3)/2= 5 and

the center of area of the fuzzy set C_2 is $\overline{x_2}$= (8+4)/2=6.

Now the defuzzified value $x^* = \frac{(A_1.\overline{x_1} + A_2.\overline{x_2})}{A_1 + A_2} = \frac{(2.75 * 5 + 1.5 * 6)}{(2.75 + 1.5)}$ = 22.75/4.25 = 5.35

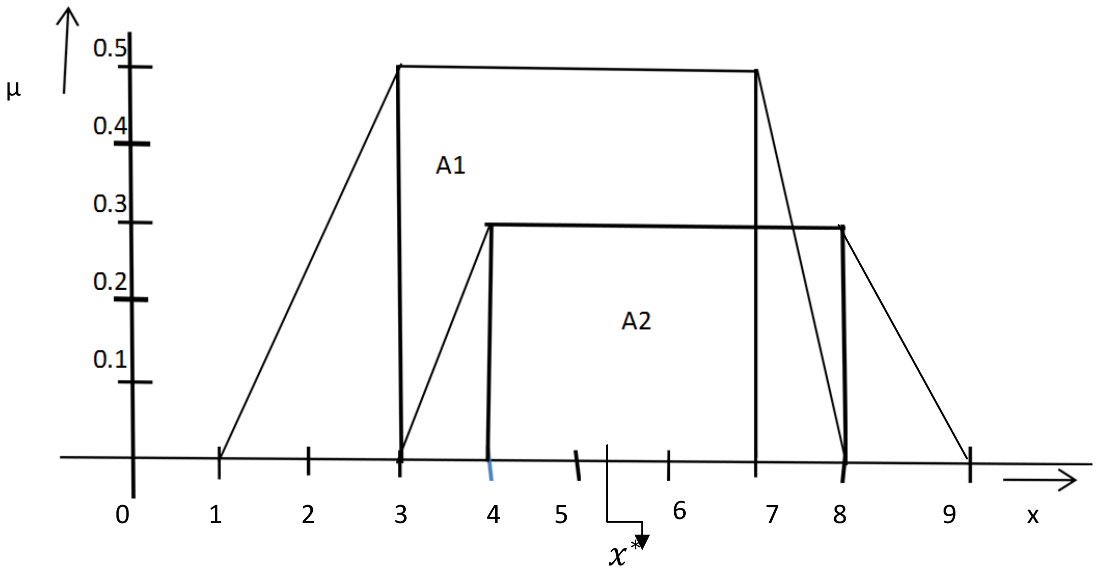


Figure 1 : Fuzzy sets $C_1$and C_2

Center of gravity (COG) / Centroid of Area (COA) Method

This method provides a crisp value based on the center of gravity of the fuzzy set. The total area of the membership function distribution used to represent the combined control action is divided into a number of sub-areas. The area and the center of gravity or centroid of each sub-area is calculated and then the summation of all these sub-areas is taken to find the defuzzified value for a discrete fuzzy set.

For discrete membership function, the defuzzified value denoted as x^*using COG is defined as:

$$x^* = \frac{\sum_{i=1}^{n} x_i \, .\mu(x_i)}{\sum_{i=1}^{n} \mu(x_i)}$$

, Here x_i indicates the sample element, $\mu(x_i)$ is the membership function, and n represents the number of elements in the sample.

For continuous membership function, x^* is defined as :

$$x^* = \frac{\int x \, \mu_A(\mathrm{x}) \, \mathrm{dx}}{\int \mu_A(\mathrm{x}) \, \mathrm{dx}}$$

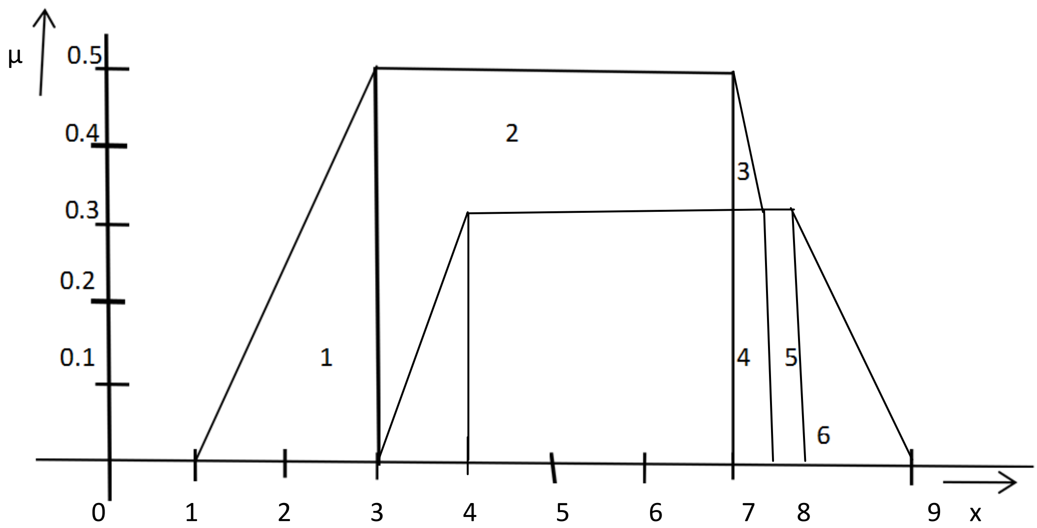


Figure 2 : Fuzzy sets C1 and C2

Example:

The defuzzified value x^*using COG is defined as:

$$x^* = \frac{\sum_{i=1}^{N} A_i \times \overline{x_i}}{\sum_{i=1}^{N} A_i}$$

, Here *N* indicates the number of sub-areas, A_i and $\overline{x_i}$ represents the area and centroid of area, respectively, of i^{th} sub-area.

In the aggregated fuzzy set as shown in figure 2. , the total area is divided into six sub-areas. For COG method, we have to calculate the area and centroid of area of each sub-area. These can be calculated as below.

The total area of the sub-area 1 is ½ * 2 * 0.5 = 0.5
The total area of the sub-area 2 is (7-3) * 0.5 = 4 * 0.5 = 2
The total area of the sub-area 3 is ½ * (7.5-7) * 0.2 = 0.5 * 0.5 *0.2 =0.05
The total area of the sub-area 4 is 0.5* 0.3 = 0.15
The total area of the sub-area 5 is 0.5* 0.3 = 0.15
The total area of the sub-area 6 is ½ *1* 0.3 = 0.15

Now the centroid or center of gravity of these sub-areas can be calculated as

Centroid of sub-area1 will be (1+3+3)/3 = 7/3 =2.333
Centroid of sub-area2 will be (7+3)/2 = 10/2 = 5 Centroid of sub-area3 will be (7+7+7.5)/3 = 21.5/3 =7.166 Centroid of sub-area4 will be (7+7.5)/2 =14.5/2=7.25 Centroid of sub-are5 will be (7.5+8)/2 =15.5/2 = 7.75 Centroid of sub-area6 will be (8+8+9)/3 = 25/3 = 8.333 Now we can calculate $A_i\,.\overline{x_\iota}$ and is shown in table 1.

Table 1

Sub-area number	Area(A_i)	Centroid of area($\overline{x_\iota}$)	$A_i\,.\overline{x_\iota}$
1	0.5	2.333	1.1665
2	02	5	10
3	.05	7.166	0.3583
4	.15	7.25	1.0875
5	.15	7.75	1.1625
6	.15	8.333	1.2499

The defuzzified value x^* will be $\frac{\sum_{i=1}^{N} A_i \times \overline{x_\iota}}{\sum_{i=1}^{N} A_i}$

$$= \frac{(1.1665+10+0.3583+1.0875+1.1625+1.2499)}{(0.5+2+.05+.15+.15+.15)}$$

= (15.0247)/3 =5.008

$$x^* = 5.008$$

Center of Area / Bisector of Area Method (BOA)

This method calculates the position under the curve where the areas on both sides are equal. The BOA generates the action that partitions the area into two regions with the same area.

$\int_{\alpha}^{x^*} \mu_A(x)\ dx = \int_{x^*}^{\beta} \mu_A(x)dx$, where α = min {x| $x \in X$} and β = max {x| $x \in X$}

Weighted Average Method

This method is valid for fuzzy sets with symmetrical output membership functions and produces results very close to the COA method. This method is less computationally intensive. Each membership function is weighted by its maximum membership value. The defuzzified value is defined as :

$$x^* = \frac{\sum \mu(x).x}{\sum \mu(x)}$$

Here $\sum$ denotes the algebraic summation and *x* is the element with maximum membership function.

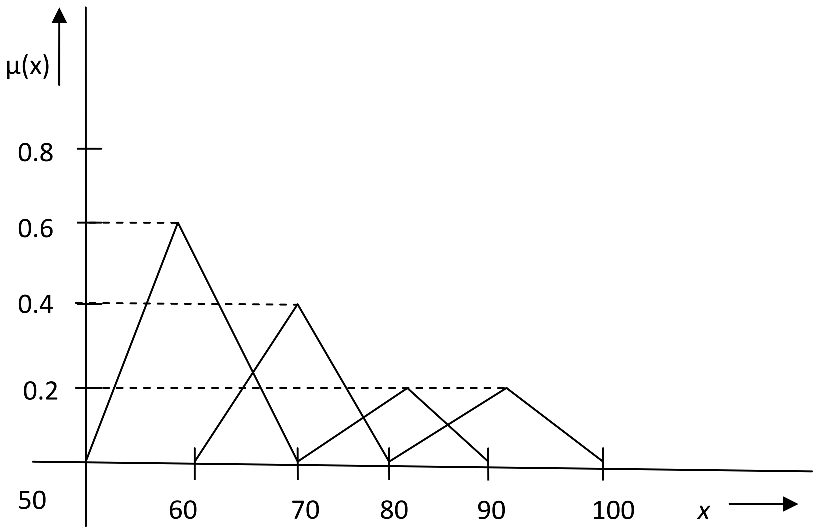


Figure 3: Fuzzy set A

Example:

Let A be a fuzzy set that tells about a student as shown in figure 3 and the elements with corresponding maximum membership values are also given.

A = {(P, 0.6), (F, 0.4),(G, 0.2),(VG, 0.2), (E, 0)}

Here, the linguistic variable P represents a Pass student, F stands for a Fair student, G represents a Good student, VG represents a Very Good student and E for an Excellent student.

Now the defuzzified value x^* for set A will be

$$x^* = \frac{(60*0.6+70*0.4+80*0.2+90*0.2+100*0)}{0.6+0.4+0.2+0.2+0}$$

= 98/1.4=70

The defuzzified value for the fuzzy set A with weighted average method represents a Fair student.

Maxima Methods

This method considers values with maximum membership. There are different maxima methods with different conflict resolution strategies for multiple maxima.

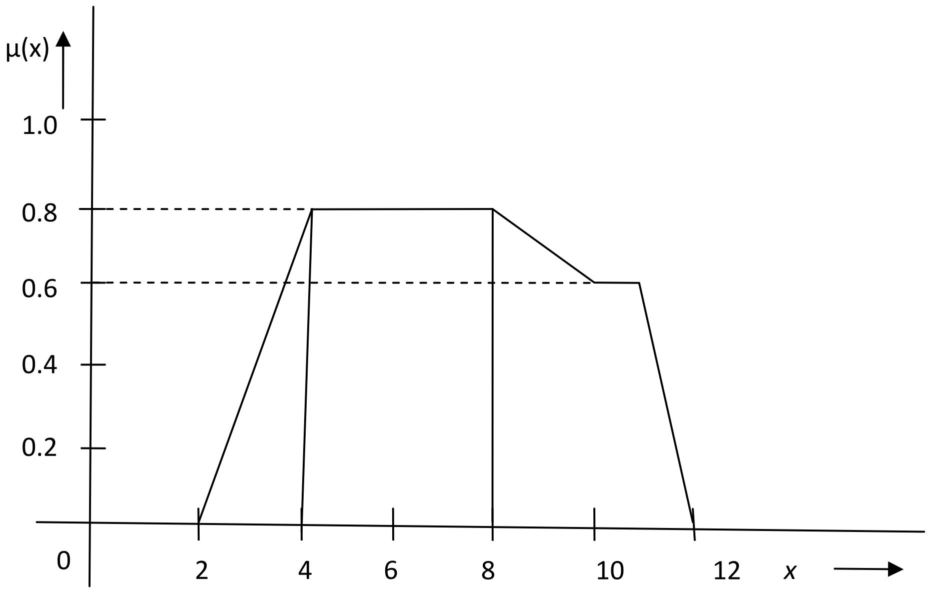
- o First of Maxima Method (FOM)
- o Last of Maxima Method (LOM)
- o Mean of Maxima Method (MOM)

▪ First of Maxima Method (FOM)

This method determines the smallest value of the domain with maximum membership value.

Example:

The defuzzified value x^* of the given fuzzy set will be x^*=4.



▪ Last of Maxima Method (LOM)

Determine the largest value of the domain with maximum membership value.

In the example given for FOM, the defuzzified value for LOM method will be x^*= 8

▪ Mean of Maxima Method (MOM)

In this method, the defuzzified value is taken as the element with the highest membership values. When there are more than one element having maximum membership values, the mean value of the maxima is taken.

Let A be a fuzzy set with membership function $\mu_A(x)$ defined over $x \in X$, where X is a universe of discourse. The defuzzified value is let say x^* of a fuzzy set and is defined as,

$$x^* = \frac{\sum_{x_i \in M} x_i}{|M|},$$

Here, *M* = {x_i| $\mu_A(x_i)$ is equal to the height of the fuzzy set A} and |*M*| is the cardinality of the set *M.*

Example

In the example as shown in Fig. , x = 4, 6, 8 have maximum membership values and hence |M| = 3

According to MOM method, $x^* = \frac{\sum_{x_i \in M} x_i}{|M|}$

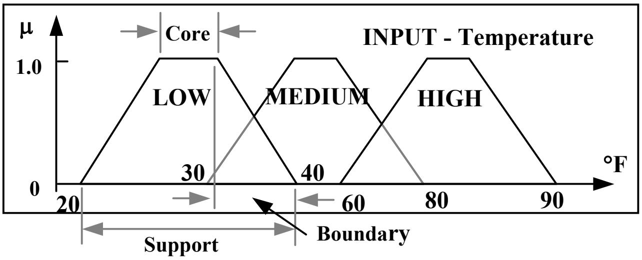
Now the defuzzified value x^* will be $x^* = \frac{4+6+8}{3} = \frac{18}{3} = 6.$

2.5.4 The Lookup Table

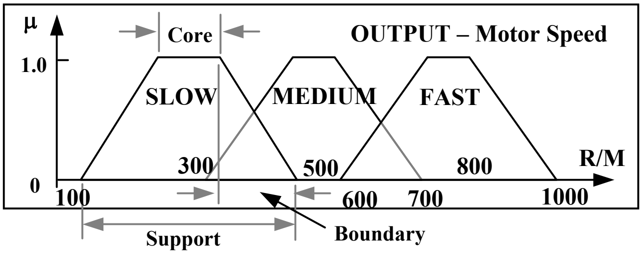
The terminal product of defuzzification is the lookup table. Defuzzification needs to be performed for each subset of a membership function, both inputs and outputs. For instance, in the air conditioner system, one needs to perform defuzzification for each subset of temperature input such as LOW, MEDIUM and HIGH based on the associated fuzzy rules. The defuzzification result for each subset needs to be stored in the associated location in the lookup table according to the current temperature and temperature change rate. In the following we use the air conditioner system as an example to illustrate the defuzzification process and the creation of the lookup table.

To make this illustration simple, we make two assumptions: I. assume that the membership function of the change rate of the temperature can be described as in Figure 2.9; II. only four rules are applied to this air conditioner system, which are

1) IF the temperature is LOW, and the change rate of the temperature is LOW, THEN the heater motor speed should be FAST
2) IF the temperature is MEDIUM, and the change rate of the temperature is MEDIUM, THEN the heater motor speed should be SLOW
3) IF the temperature is LOW, and the change rate of the temperature is MEDIUM, THEN the heater motor speed should be FAST
4) IF the temperature is MEDIUM, and the change rate of the temperature is LOW, THEN the heater motor speed should be MEDIUM



(a) Membership function of the input



(b) Membership function of the output

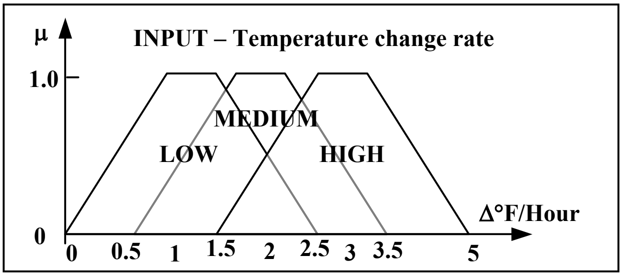


Figure 2.9. The membership function of the change rate of temperature

Based on the assumption made for the membership function and fuzzy rules, we can illustrate this defuzzification process using a graph. Four fuzzy rules can be interpreted as functional diagrams, as shown in Figure 2.10.

As an example, consider the current input temperature is 35 °F and the change rate of the temperature is 1 °F per hour.

From Figure 2.10, it can be found that the points of intersection between the temperature values of 35 °F and the graph in the first column (temperature input T) have the membership functions of 0.6, 0.8, 0.5 and 0.8. Likewise, the second column (temperature change rate ΔT) shows that a temperature change rate of 1 °F per hour has the membership functions of 1.0, 0.4, 0.4 and 1.0. The fuzzy output for the four rules is the intersection of the paired values obtained from the graph, or the AND result between the temperature input and the temperature change rate input. According to Equation (2.13), this operation result should be: min (0.6, 1.0), min (0.8, 0.4), min (0.5, 0.4) and min (0.8, 1.0), which produces to 0.6, 0.4, 0.4 and 0.8, respectively.

Now, for a pair of temperature and temperature change rate, four sets of fuzzy outputs exist. To determine the crisp value of action to be taken from these contributions, one can either choose the maximum value using the MOM method or use the Center of Gravity method (COG). In this example, the Weighted Average method is used and the action is given by the center of the summed area, which is contributed by the different fuzzy outputs. Furthermore, the COG method gives a more reliable lookup table compared with the MOM operation.

Thus, for a temperature of 35 deg. F and a change rate of temperature is 1 deg. F per hour, the fuzzy output element y for this input pair is

$$y = \frac{0.6 \times 800 + 0.4 \times 300 + 0.4 \times 800 + 0.8 \times 500}{0.6 + 0.4 + 0.4 + 0.8} = 600 \text{ R/M} \qquad (2.21)$$

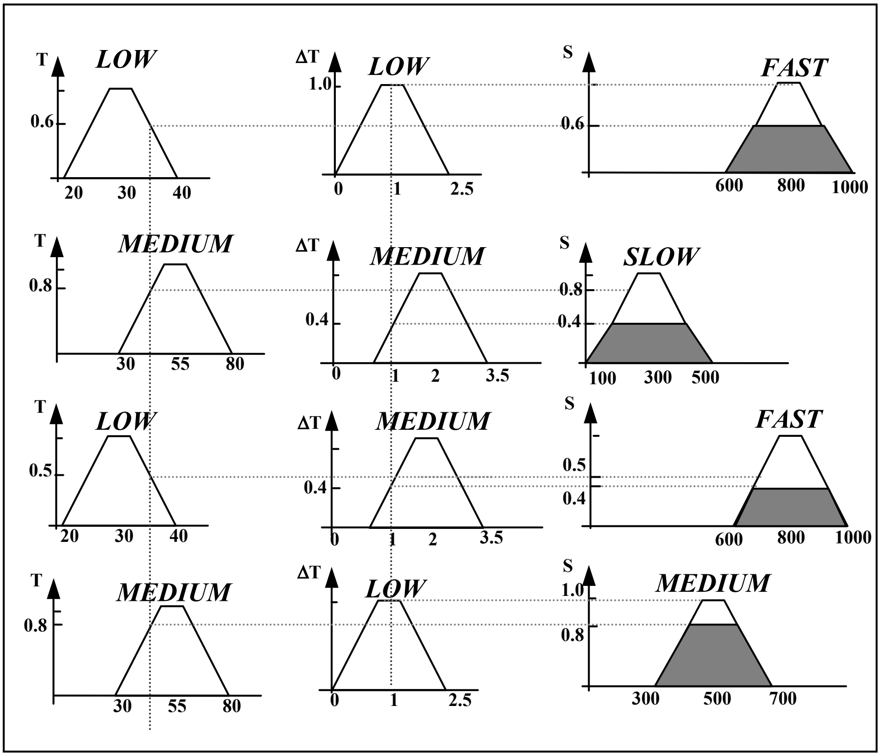


Figure 2.10. An illustration of fuzzy output calculation

The membership functions representing the control adjustment are weighted according to the input change and the different control contributions as shown in Figure 2.11.

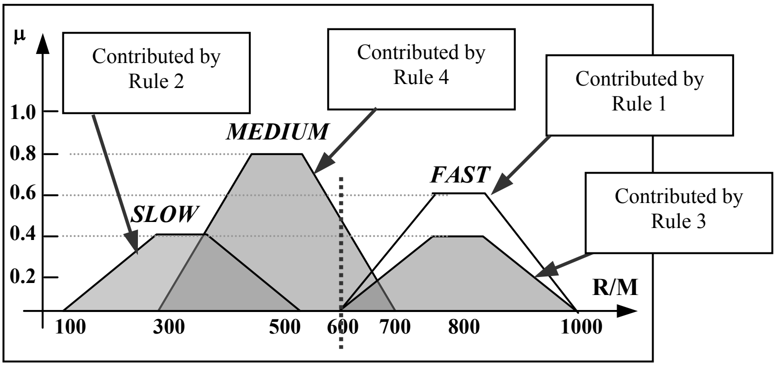


Figure 2.11. Determination of fuzzy output by Weighted Average Method

This defuzzified fuzzy output is a crisp or classical value, and should be entered into a certain location in a table called the lookup table. Since this fuzzy output element is associated with a temperature input pair with a temperature of 35 °F (belonging to LOW in the temperature membership function) and a change rate of temperature of 1 °F/h (also belonging to LOW in the membership function of change rate of temperature), this output value should be located in the cross point between the LOW temperature T (row) and the LOW change rate T (column) as shown in Table 2.2.

Table 2.2 shows an example of a lookup table.

Table 2.2. An example of a lookup table

$\dot{T}$ \ T	**LOW**	**MEDIUM**	**HIGH**
LOW	600	?	?
MEDIUM	?	?	?
HIGH	?	?	?

To fill this table, one needs to use the defuzzification technique to calculate all other fuzzy output values and locate them in the associated positions in the lookup table as we did in this example. Generally, the dimensions of the fuzzy rules should be identical with the dimensions of the lookup table as shown in this example. To obtain more accurate control accuracy or fine fuzzy output element values, one can divide the inputs, say the temperature input and change rate of the temperature, into multiple smaller subsets to get finer membership functions. For instance, in this example, one can define a VERY LOW subset that covers 20 ~ 30 °F for the temperature input, a LOW subset that contains a range of 30 ~ 40 °F, and so on. Performing the same process to the change rate of temperature and the heater motor output, one can get a much finer lookup table.

For higher control accuracy applications, an interpolation process can be added after the lookup table to obtain finer output.

When implementing a fuzzy logic technique to a real system, the lookup table can be stored in a computer's memory, and the fuzzy output can be obtained based on the current inputs. In practice, there are two ways to calculate the lookup table using fuzzy inference process in the real control applications: off-line and on-line methods.