B Tech-III (CO) 6th semester

Course: Computer Graphics (CS-3) (CO306)

Solution: Tutorial – 5

Based On: 2D Transformation

1. Prove that 2D rotation and scaling are commutative if
   1. $S_x = S_y$
   2. $\Theta = n\pi$.

Solution : The matrix notation for scaling along S_x and S_y is as given below

$$S = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \text{ and}$$

The matrix notation for rotation is as given below.

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$S \cdot R = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} S_x\cos\theta & S_x\sin\theta \\ -S_y\sin\theta & S_y\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} S_x\cos\theta & S_x\sin\theta \\ -S_x\sin\theta & S_x\cos\theta \end{bmatrix} \qquad \because S_x = S_y \quad \ldots\text{I}$$

or

$$= \begin{bmatrix} -S_x & 0 \\ 0 & -S_y \end{bmatrix} \qquad \because \theta = n\pi \text{ where n is integer } \ldots\text{II}$$

$$R \cdot S = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

$$= \begin{bmatrix} S_x\cos\theta & S_y\sin\theta \\ -S_x\sin\theta & S_y\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} S_x\cos\theta & S_x\sin\theta \\ -S_x\sin\theta & S_x\cos\theta \end{bmatrix} \qquad \because S_x = S_y \quad \ldots\text{III}$$

or

$$= \begin{bmatrix} -S_x & 0 \\ 0 & -S_y \end{bmatrix} \qquad \because \theta = n\pi \text{ where n is integer } \ldots\text{IV}$$

From equations I and III, and equations II and IV it is proved that 2D rotation and scaling commute if $S_x = S_y$ or $\theta = n\pi$ for integral n and that otherwise they do not.

2. Consider the square A(1,0), B(0,0), C(0,1) and D(1,1). Rotate the square ABCD by 45° clockwise about A(1,0).

Ans . HINT:-

1) **First, translate the square by Tx= -1 and Ty=0.**
2) **Then rotate the square by 45°.**
3) **Again translate the square by Tx=1 and Ty=0.**

3. The reflection along the line y=x is equivalent to the reflection along the X-axis followed by counter clockwise rotation by Θ degrees. Find the value of Θ.

Hint: 1. Find T for line y=x, reflection.
2. Find T'= [A][B],where [A]= reflection along the X-axis and [B]= counter clockwise rotation by Θ degrees.
3. Find Θ using equation T=T'.

4. Prove that two scaling transformations are commutative i.e. S1.S2=S2.S1

Ans. Hint take $S_1 = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $S_2 = \begin{pmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Then solve $\mathbf{S_1.S_2}$ and then $\mathbf{S_2.S_1}$ and prove .

5. a) Find the matrix that represents rotation of an object by 45°about the origin.
b) What are the new coordinates of the point P(2 , -4) after the rotation?

Solution: Refer Tutorial for Hint.

6. A triangle is defined by
$\begin{bmatrix} 2 & 2 \\ 4 & 2 \\ 4 & 4 \end{bmatrix}$
Find the transformed coordinates after the following transformation
   1. 90° rotation about origin.
   2. Reflection about line y = -x.
   3.

Solution: Refer Tutorial for Hint.

7. Magnify the triangle with vertices A(0,0), B(1,1) and C(5,2) to twice its size while keeping C(5,2) fixed.

Ans . HINT:-

1) **First, translate the triangle by Tx= -5 and Ty=-2**
2) **Then Magnify the triangle by twice its size** (Use Scaling factors, S_x=S_y=2)
3) **Again translate the triangle by Tx= 5 and Ty= 2.**