INVENTORY CONTROL

AND

MANAGEMENT

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Introduction

- **Inventories** are **materials and supplies** that a business or institution carries **either for sale** or **to provide inputs** or **supplies** to **the production process.**
- Often they are a **substantial part of total assets**. On the balance sheet, **inventories** usually represent **from 20% to 60%** of **total assets.**
- As **inventories** are **used**, their value is **converted into cash**, which **improves cash flow** and **return on investment**.
- There is a **cost for carrying inventories**, which **increases operating costs** and **decreases profits.**

Introduction (2)

- **Good inventory management** is **essential**.
- **Inventory management** is **responsible** for **planning and controlling inventory** from **the raw material stage** to **the customer.**
- Since **inventory** either **results from production** or **supports it**, the **two cannot be managed separately** and, therefore, **must be coordinated**.
- Inventory must be considered at **each of the planning levels** and is thus part of **production planning**, **master production scheduling**, and **material requirements planning.**
- **Production planning** is concerned with **overall inventory**, **master planning** with **end items**, and **material requirements planning** with **component parts** and **raw material.**

Aggregate Inventory Management

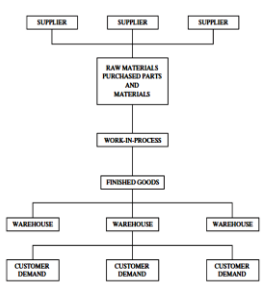
- **Aggregate inventory management** deals with **managing inventories** according to **their classification** (raw material, work-in-process, and finished goods) and **the function** they **perform rather than** at **the individual item level.**
- It is **financially oriented** and is concerned with **the costs and benefits** of **carrying the different** classifications of **inventories.**
- As such, **aggregate inventory management involves:**
  - ✓**Flow** and **kinds of inventory** needed.
  - ✓**Supply** and **demand patterns.**
  - ✓**Functions** that **inventories perform**.
  - ✓**Objectives** of inventory management.
  - ✓**Costs associated** with inventories.

Item Inventory Management

- **Inventory** is **not only** managed **at the aggregate level** but **also at the item level.**
- **Management** must **establish decision rules** about inventory items so the staff responsible for inventory control can do their job effectively.
- These rules include the following:
  - ✓ **Which individual inventory** items are **most important**.
  - ✓ **How** individual **items** are to be **controlled.**
  - ✓ **How much** to **order at one time**.
  - ✓ **When** to **place an order**.

Classification of Inventory and The Flow of Material

- Inventories are classified as per **flow of materials into, through**, and **out of a manufacturing organization**, as shown in Figure.
- **Raw materials**. These are **purchased items received** that **have not entered the production process**. They include **purchased materials**, **component parts**, and **subassemblies**.
- **Work-in-process (WIP)**. **Raw materials** that **have entered the manufacturing process** and **are being worked on** or **waiting to be worked on**.
- **Finished goods**. The **finished products** of the production process that are **ready to be sold** as completed items. They may be held **at a factory** or **central warehouse** or **at various points** in **the distribution system.**



Classification of Inventory and The Flow of Material

- **Distribution inventories**. **Finished goods** located in the **distribution system**.
- **Maintenance, repair, and operational supplies (MROs)**. Items used in production **that do not become part of the product.** These include **hand tools**, **spare parts**, **lubricants**, and **cleaning supplies.**
  - ✓ **Classification** of an item into a particular inventory **depends** on **the production environment.**
  - ✓ For instance, **sheet steel** or **tires** are **finished goods for the supplier** but are **raw materials and component parts** for **the car manufacturer.**

Build up of Inventories: Supply and Demand

- If **supply met demand** exactly, there would be **little need for inventory**. Goods could be made **at the same rate** as **demand**, and **no inventory would build up.**
- For this situation to exist, **demand must be predictable**, **stable**, and **relatively constant** over a long time period.
- If this is so, **manufacturing section** can produce **goods** on **a line-flow basis**, **matching production to demand**. Using this system, **raw materials are fed to production** as required, **work flow** from one workstation to another is **balanced** so **little work-in process inventory** is required, and **goods are delivered** to the customer at the rate the customer needs them.

Build up of Inventories: Supply and Demand (2)

- **Because the variety of products** they can make is so limited, **demand has to be large enough** to justify economically setting up the system. These systems are characteristic of **just-in-time manufacturing**.
- **Demand** for most products is **neither sufficient nor constant** enough **to warrant setting up** a line-flow system, and these products are usually made in **lots** or **batches.**
- **Workstations are organized by function**—for example, all machine tools in one area, all welding in another, and assembly in another. Work moves in lots from one workstation to another as required by the routing.
- ***By the nature of the system, inventory will build up in raw materials, work-in-process, and finished goods.***

Functions of Inventories

- In batch manufacturing, **the basic purpose** of **inventories** is to **decouple supply and demand**. Inventory serves as a buffer between:
  - ✓**Supply** and **demand**.
  - ✓**Customer demand** and **finished goods.**
  - ✓**Finished goods** and **component availability.**
  - ✓**Requirements for an operation** and the **output from the preceding operation.**
  - ✓**Parts and materials to begin production** and **the suppliers of materials.**

Classification of Inventories based on purpose

1) **Anticipation Inventory: Anticipation inventories are built up in anticipation of future demand.**

For example, they are created **ahead of a peak selling season**, **a promotion program**, **vacation shutdown**, or **possibly** the **threat of a strike.** They are built up to **help level production** and **to reduce the costs** of **changing production rates.**

2) **Fluctuation Inventory (Safety Stock): Fluctuation inventory is held to cover random unpredictable fluctuations in supply** and demand or lead time.

If demand or lead time is greater than forecast, a stock-out will occur. S**afety stock is carried to protect against this possibility. Its purpose is to** prevent disruptions in manufacturing or deliveries to customers. Safety stock is also called buffer stock or reserve stock.

Classification of Inventories based on purpose (2)

3) **Lot-Size Inventory:** Items **purchased** or **manufactured** in **quantities greater than needed immediately,** create **lot-size inventories.**

- Purpose is **to take advantage of quantity discounts;** to reduce **shipping**, **clerical**, and **setup costs;** and in cases where it is **impossible** to **make** or **purchase items** at **the same rate** that they will be **used or sold.**
- Lot-size inventory is sometimes called **cycle stock. It is the portion of inventory that depletes gradually as customers'** orders come in and is **replenished cyclically** when suppliers' orders are received.

Classification of Inventories based on purpose (3)

4) **Transportation Inventory**

- **Transportation inventories exist because of the time needed to move goods from one** location to another such as from a plant to a distribution center or a customer.
- They are sometimes called **pipeline or movement inventories. The average amount of** inventory in transit is:

$$I = \frac{tA}{365}$$

where ***I*** *is* ***the average annual inventory*** *in* ***transit, t*** *is* ***transit time*** *in* ***days,*** *and* ***A*** *is* **annual demand**.

- The **transit inventory** does not depend upon **the shipment size** but on the **transit time** and **the annual demand.**
- The only **way to reduce the inventory in transit**, and its cost, is **to reduce the transit time.**

Classification of Inventories based on purpose (4)

5) **Hedge Inventory**

- Some products such as **minerals and commodities**—for example, **grains** or **animal products**—are traded on a worldwide market.
- The **price for these products fluctuates** according to world supply and demand.
- If buyers expect prices to rise, they can purchase **hedge inventory when prices are low. Hedging is complex and beyond the scope** of our course.

6) **Maintenance, Repair and Operating Supplies (MROs)**

- **MROs are items** used to support **general operations** and **maintenance but** that **do not become** directly **part of a product.**
- They include **maintenance supplies**, **spare parts**, and **consumables** such as **cleaning compounds**, **lubricants**, **pencils**, and **erasers.**

Inventory Models

An inventory system can be modelled quantitatively based on demand patterns. They are

1. Deterministic inventory models (demand rate = constant)
   - Simple Economic Order Quantity (EOQ) model
   - Manufacturing/Production model
   - EOQ model with discounts
   - EOQ model with shortages
2. Probabilistic inventory models (demand rate = variable)
   - Perishable goods
   - Spare parts

Simple EOQ model

- The simple EOQ model is can be used if the demand is known with certainty.
- The demand and lead time are known.
- The item will be purchased from outside and that demand will continue well into the future.
- It is assumed that not only the demand is known with certainty, but that is the same from day to day and that stockouts, are not allowed.
- The model is described under the following situation:

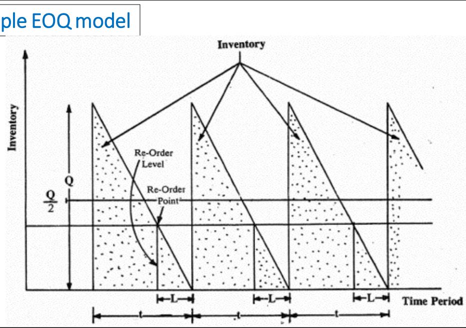
1. Planning period is one year.
2. Demand is deterministic and indicated by parameter D units per year.
3. Cost of purchase, or price of one unit is C.

Simple EOQ model

4. Cost of holding stock (also known as inventory carrying cost) is C_h per unit per year expressed either in terms of cost per unit per period or in terms of percentage charge of the purchase price.
5. Shortage cost (mostly it is back order cost) is C_s per unit per year,
6. Lead time is L, expressed in units of time.
7. Cycle period of replenishment is t.
8. Order size is Q

Instantaneous supply case when shortages are not allowed will be discussed here. That is, whatever is demanded is supplied immediately after the lead time. With these assumptions, a graph of inventory against time will be a regular saw-tooth pattern. Since shortages are not allowed, it implies that shortage cost is prohibitive or C_s, is very very large or infinite.

Simple EOQ model



Simple EOQ model

Total relevant cost equation for this case will be:

$$\underset{(TC)}{\text{Total annual relevant cost}} = \underset{(PC)}{\text{Annual Purchase Cost}} + \underset{\text{Cost (OC)}}{\text{Annual Ordering}} + \underset{(CC)}{\text{Annual Carrying Cost}}$$

$$= (\text{Price/Unit}) \begin{pmatrix}\text{Quantity} \\ \text{Purchased} \\ \text{per year}\end{pmatrix} + \begin{pmatrix}\text{Cost of} \\ \text{Ordering} \\ \text{per order}\end{pmatrix} \begin{pmatrix}\text{Number} \\ \text{of orders} \\ \text{placed/yr}\end{pmatrix} + \begin{pmatrix}\text{Cost of} \\ \text{Carrying} \\ \text{one unit}\end{pmatrix} \begin{pmatrix}\text{Average} \\ \text{no. of unit} \\ \text{carried}\end{pmatrix} \quad \ldots (1)$$

$$TC = CD + C_o \frac{D}{Q} + C_h \frac{Q}{2} \quad \ldots (2)$$

Simple EOQ model

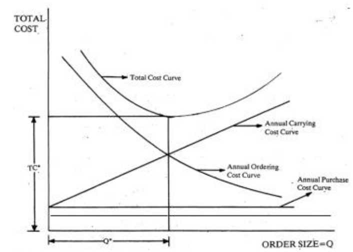
- The economic order quantity or optimal order size is that quantity which minimizes the total cost. Total cost is the sum of fixed cost and variable cost. Fixed cost (CD) is independent of order size while the variable cost is dependent on the order size Q.
- Since the fixed cost does not play any role in minimization or maximization process, only variable cost will be minimized here.
- For total cost to be minimum, the first order derivative of TC is zero, i.e.,

$$\frac{dTC}{dQ} = \frac{-C_oD}{Q^2} + \frac{C_h}{2} = 0 \quad \dots (3)$$

or $$\frac{C_oD}{Q} = \frac{C_hQ}{2} \quad \dots (4)$$

or Annual ordering cost = Annual carrying cost ... (5)

Simple EOQ model



Cost Trade-offs in inventory control

Simple EOQ model

- Further simplifying equation (4), we get Economic Order Quantity (EOQ).

$$= \sqrt{\frac{2C_oD}{C_h}} \quad \dots (6)$$

The cycle period t is given by $\frac{\text{Optimal Order Quantity}}{\text{Annual demand}}$

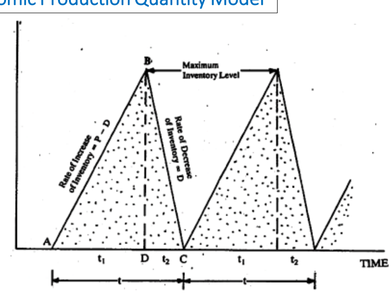
or $$t^* = \frac{Q^*}{D} = \sqrt{\frac{2C_o}{C_hD}} \quad \dots (7)$$

The total number of orders per year (=N) is the reciprocal of the cycle period i.e.

$$N = \frac{D}{Q^*} = \sqrt{\frac{C_hD}{2C_o}} \quad \dots (8)$$

Total annual cost = $TC = CD + \sqrt{2C_oC_hD}$ ... (9)

Economic Production Quantity Model



Economic Production Quantity Model

- An economic production quantity (EPQ) is associated with manufacturing environment, while EOQ is more common in retail situations. EPQ realistically shows that inventory is gradually built over a period of time because production and the consumption go side by side where production rate is higher than the consumption rate.
- All the assumptions of EOQ model holds good.
- Order size is taken as production size, the annual production rate is taken as P such that P>D, otherwise, if P is less than or equal to D, the item will be used as fast as it is produced.
- Cycle time t is the sum of the production time t_1, plus the depletion time, t_2 of maximum inventory level BD. Production starts at point A, and stops at point E as soon as the level of inventory becomes BE.

Economic Production Quantity Model

- Production time $t_1 = \frac{Q}{P}$ and cycle time, $t = \frac{Q}{D}$
- Maximum inventory level BE = $(P - D) * t_1 = (P - D) * \frac{Q}{P}$
- Minimum inventory level = 0

$$\text{Therefore, average inventory carried} = \frac{(P-D)\frac{Q}{P} + O}{2} = \frac{(P-D)Q}{2P}$$

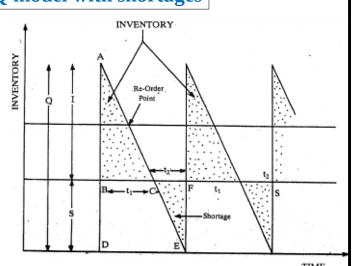
Total variable cost per year = TC = Annual Set up Cost + Annual Carrying Cost

$$= \frac{C_o D}{Q} + C_h \frac{(P-D)}{P} \frac{Q}{2}$$

$$\text{Thus, } Q^* = \sqrt{\frac{2C_o D}{\frac{C_h(P-D)}{P}}} \text{ and Total Variable Cost} = TC^* = \sqrt{2C_o C_h (P-D)\frac{D}{P}}$$

EOQ model with shortages

- In this case stock-outs are permitted which implies that shortage cost is finite or it is not large.
- All the assumptions of EOQ model hold good here also.



EOQ model with shortages

- Triangle ABC represents the inventory and triangle CEF, the shortage. I is the inventory level, and S, the shortage level.
- Order size Q=I+S. Cycle period t= $t_1 + t_2$,
- where t_1, is the portion of cycle period for inventory holding and t_2, is the time of stock-out.

Total variable cost = Annual (Ordering Cost + Holding Cost + Shortage Cost)

Total variable cost = Annual (Ordering Cost + Holding Cost + Shortage Cost)

$$= \frac{C_o D}{Q} + \frac{I^2 C_h}{2Q} + \frac{(Q-I)^2 C_s}{2Q}$$

EOQ model with shortages

From this we obtain

$$Q^* = EOQ = \sqrt{\left(\frac{2C_oD}{C_h}\right)\left(\frac{C_o+C_h}{C_s}\right)}$$

Inventory level $I^* = \sqrt{\left(\frac{2C_oD}{C_h}\right)\left(\frac{C_s}{C_o+C_h}\right)}$

Shortage level $= Q^* - I^*$

Cycle period $t^* = Q^*/D$

Number of orders/yr $= \frac{1}{t^*}$

Total variable cost $= TC^* = \sqrt{\frac{2C_oC_hC_sD}{(C_h+C_s)}}$

Probabilistic Models and Safety Stock

- All the inventory models we have discussed so far make the assumption that demand for a product is constant and certain. We now relax this assumption.
- The certain inventory models apply when product demand is not known but can be specified by means of a probability distribution. These types of models are called probabilistic models.
- An important concern of management is maintaining an adequate service level in the face of uncertain demand. The service level is the complement of the probability of a stockout.
- For instance, if the probability of a stockout is 0.05, then the service level is 95.
- Uncertain demand raises the possibility of a stockout. One method of reducing stockouts is to hold extra units in inventory.

Probabilistic Models and Safety Stock

- As we noted, such inventory is usually referred to as safety stock. It involves adding a number of units as a buffer to the reorder point. Hence,

Reorder point = ROP = d X L

Where,
d= Daily demand
L = Order lead time, or number of working days it takes to deliver an order

- The inclusion of safety stock (ss) changes the expression to:

ROP = d X L + ss

The amount of safety stock maintained depends on the cost of incurring a stockout and the cost of holding the extra inventory.

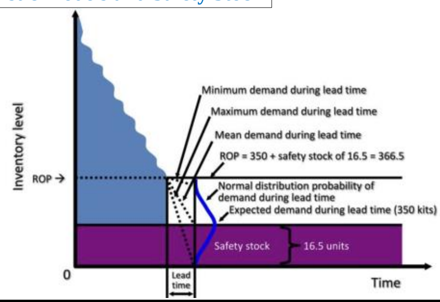
Probabilistic Models and Safety Stock

- Annual stockout cost is computed as follows:

Annual stockout costs = The sum of the units short for each demand level × The probability of that demand level × The stockout cost/unit × The number of orders per year

- When it is difficult or impossible to determine the cost of being out of stock, a manager may decide to follow a policy of keeping enough safety stock on hand to meet a prescribed customer service level, For instance, figure shows the use of safety stock when demand (for hospital resuscitation kits) is probabilistic. We see that safety stock in figure is 16.5 units and the reorder point is also increased by 16.5.
- The manager may want to define the service level as meeting 95% of the demand (or, conversely, having stockouts only 5% of the time).

Probabilistic Models and Safety Stock



Probabilistic Models and Safety Stock

- Assuming that demand during lead time (the reorder period) follows a normal curve, only the mean and standard deviation are needed to define the inventory requirements for any given service level. Sales data are usually adequate for computing the mean and standard deviation.
- In that cases, we use a normal curve with a known mean (μ) and standard deviation (σ) to determine the reorder point and safety stock necessary for a 95% service level.

We use the following formula:

ROP = Expected demand during lead time + Z.σ_{dLT}

Where, Z = Number of standard deviations,

σ_{dLT} = Standard deviations of demand during lead time

P System → Fixed Order Period System

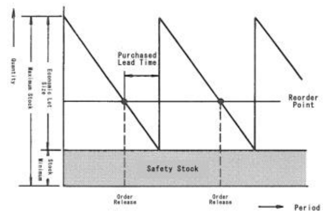
Q System → Fixed Order Quantity System

Q systems

- The fixed order quantity system is also known as the Q system.
- In this system, whenever the stock on hand reaches the reorder point, a fixed quantity of materials is ordered. The fixed quantity of material ordered each time is actually the economic order quantity.
- Whenever a new consignment arrives, the total stock is maintained within the maximum and the minimum limits.
- The fixed order quantity method is a method that facilitates for a predetermined amount of a given material to be ordered at a particular period of time.
- This method helps to limit reorder mistakes, conserve space for the storage of the finished goods, and block those unnecessary expenditures that would tie up funds that could be better utilized elsewhere.

Q systems

- The fixed order quantity may be bridged to an automatic reorder point where a particular quantity of a good is ordered when stock at hand reaches a level which is already determined.



Advantages of Q system

- Each material can be procured in the most economical quantity.
- Purchasing and inventory control people automatically gives their attention to those items which are required only when are needed.
- Positive control can easily be handled to maintain the inventory investment at the desired level only by calculating the predetermined maximum and minimum values.

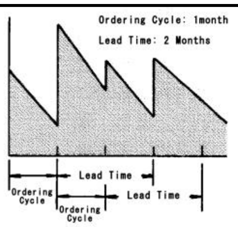
Disadvantages of Q system

- Sometimes, the orders are placed at the irregular time periods which may not be convenient to the producers or the suppliers of the materials.
- The items cannot be grouped and ordered at a time since the reorder points occur irregularly.
- If there is a case when the order placement time is very high, there would be two to three orders pending with the supplier each time and there is likelihood that he may supply all orders at a time.
- EOQ may give an order quantity which is much lower than the supplier minimum and there is always a probability that the order placement level for a material has been reached but not noticed in which case a stock out may occur.
- The system assumes stable usage and definite lead time. When these change significantly, a new order quantity and a new order point should be fixed, which is quite cumbersome.

P system

- In this system, the stock position of each material of a product is checked at regular intervals of time period.
- When the stock level of a given product is not sufficient to sustain the operation of production until the next scheduled tested, an order is placed destroying the supply.
- The frequency of reviews varies from organization to organization.
- It also varies among products within the same organization, depending upon the importance of the product, predetermined production schedules, market conditions and so forth.
- The order quantities vary for different materials.

P system



Order Quantity = Scheduled Demand of (Ordering Cycle + Lead Time) + Safety Stock - Inventory Quantity - Released Order

Advantages of P system

- The ordering and inventory costs are low. The ordering cost is considerably reduced though follow up work for each delivery may be necessary.
- The suppliers will also offer attractive discounts as sales are guaranteed.
- The system works well for those products which exhibit an irregular or seasonal usage and whose purchases must be planned in advance on the basis of sales estimates.

Disadvantages of P system

- The periodic testing system tends to peak the purchasing work around the review dates.
- The system demands the establishment of rather inflexible order quantities in the interest of administrative efficiency.
- It compels a periodic review of all items; this itself makes the system somewhat inefficient.

Distinction between P and Q system

Point of difference	Q system	P system
Initiation of order	Stock on hand reaches to reorder point	Based on fixed review period and not stock level
Period of order	Any time when stock level reaches to reorder point	Only after the predetermined period
Record keeping	Continuously each time a withdrawal or addition is made	Only at the review period
Order quantity	Constant the same quantity ordered each time	Quantity of order varies each time order is placed
Size of inventory	less than the P system	Larger than the Q system
Time to maintain	Higher due to perpetual record keeping	Less than due to only at the review period

Selective Inventory Control

Classification	Full form	Criterion employed
ABC Analysis	Always Better Control	Usage value
GOLF Analysis	Govt, Ordinary, Local, Foreign	Source procurement
SOS Analysis	Seasonal and Off seasonal	Seasonality
HML Analysis	High, Medium, Low	Unit price (does not take consumption into account)
FSND Analysis	Fast, Slow, Non-monetary material demands	Issues from store
SDE Analysis	Scarce, Difficult, Easy	Procurement difficulties
VED Analysis	Vital, Essential, Desirable	Criticality usage (Maintenance works)
XYZ Analysis	High value, Medium value, Low value	Storage cost

ABC Analysis

- Classifies items based on the annual usage value (AUV)
- Identify a small percentage of items which account for most of the total inventory value

Basic Principle

- 20/80 – Rule
- Pareto's Law – Vilfredo Pareto – Italian Economist "Few are vital' and 'many are trivial'
- AUV = Annual demand X Price

Pareto's law applied to inventories

- The relationship between the percentage of items and the percentage of AUV follows a pattern
- A – about 20 % of items account for about 80 % of the AUV
- B - about 30 % of items account for about 15 % of the AUV
- C - about 50 % of items account for about 5 % of the AUV

Steps in Making ABC Analysis

1. Determine the annual usage for each item
2. Calculate the AUV of each item
3. List the items according to their AUV (descending order)
4. Calculate the cumulative AUV and the cumulative percentage of items
5. Examine the annual usage distribution and group the items into A, B, C based on percentage of AUV

Using ABC approach, there are two general rules to follow:

- Have plenty of low-value items
- Use the money and control effort to reduce the inventory of high-value items

ABC Analysis

Different controls used with different classes

- A Items: High priority – Tight control including complete accurate records, regular and frequent review by management, frequent review of demand forecast and close follow-up and expediting to reduce lead time
- B Items: Medium priority – Normal Control
- C Items: Lowest priority – Simplest possible control. Perhaps use a two-bin system or periodic review system. Order larger quantities and carry sufficient safety stock

ABC Analysis: An Example

Problem:

A small firm inventories only ten items, but decide to setup an ABC inventory system with 20 % A items, 30 % B items, and 50 % C items. The company records provide the information shown below.

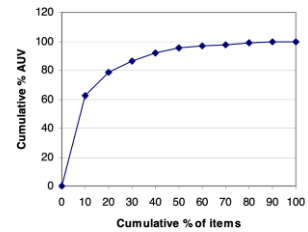
Part No.	1	2	3	4	5	6	7	8	9	10	
Unit Usage	1100	600	100	1300	100	10	100	1500	200	500	5510
Unit Cost	20	400	40	10	600	250	20	20	20	10	
AUV	22000	240000	4000	13000	60000	2500	2000	30000	4000	5000	382500

ABC Analysis: An Example

Solution:

Part No.	AUV in Descending order	Cumulative AUV	Cumulative % AUV	Cumulative % of items	Class
2	240000	240000	62.75	10	A
5	60000	300000	78.43	20	A
8	30000	330000	86.27	30	B
1	22000	352000	92.03	40	B
4	13000	365000	95.42	50	B
10	5000	370000	96.73	60	C
3	4000	374000	97.77	70	C
9	4000	378000	98.82	80	C
6	2500	380500	99.48	90	C
7	2000	382500	100	100	C

ABC Analysis



Graphical representation of ABC Analysis