

## Propositional Language

A. Let's consider a propositional language where

- p means "Paola is happy",
- q means "Paola paints a picture", and
- r means "Renzo is happy".

- Formalize the following sentences:
  1. "if Paola is happy and paints a picture then Renzo isn't happy"
  2. "if Paola is happy, then she paints a picture"
  3. "Paola is happy only if she paints a picture"

B. Let's consider a propositional language where

- p means "x is a prime number",
- q means "x is odd".

- Formalize the following sentences:
  1. "x being prime is a sufficient condition for x being odd"

C. Let A = "Aldo is Italian" and B = "Bob is English".

Formalize the following sentences:

1. "Aldo isn't Italian"
2. "Aldo is Italian while Bob is English"
3. "If Aldo is Italian then Bob is not English"
4. "Aldo is Italian or if Aldo isn't Italian then Bob is English"
5. "Either Aldo is Italian and Bob is English, or neither Aldo is Italian nor Bob is English"

D. Let's consider a propositional language where

- A = "Angelo comes to the party"
- B = "Bruno comes to the party",
- C = "Carlo comes to the party",
- D = "Davide comes to the party".

Formalize the following sentences:

1. "If Davide comes to the party then Bruno and Carlo come too"
2. "Carlo comes to the party only if Angelo and Bruno do not come"

3. "Davide comes to the party if and only if Carlo comes and Angelo doesn't come"
4. "If Davide comes to the party, then, if Carlo doesn't come then Angelo comes"
5. "Carlo comes to the party provided that Davide doesn't come, but, if Davide comes, then Bruno doesn't come"
6. "A necessary condition for Angelo coming to the party, is that, if Bruno and Carlo aren't coming, Davide comes"
7. "Angelo, Bruno and Carlo come to the party if and only if Davide doesn't come, but, if neither Angelo nor Bruno come, then Davide comes only if Carlo comes"

E. Socrate says:

"If I'm guilty, I must be punished;  
I'm guilty. Thus I must be punished."  
Is the argument logically correct?  
(Hint: Use Types of Inference Rules)

## Answer

A.

- "if Paola is happy and paints a picture then Renzo isn't happy"
  - $p \wedge q \rightarrow \neg r$
- "if Paola is happy, then she paints a picture"
  - $p \rightarrow q$
- "Paola is happy only if she paints a picture"
  - $\neg(p \wedge \neg q)$

B.

Solution. 1.  $p \rightarrow q$

C.

1.  $\neg A$
2.  $A \wedge B$
3.  $A \rightarrow \neg B$
4.  $A \vee (\neg A \rightarrow B)$  logically equivalent to  $A \vee B$
5.  $(A \wedge B) \vee (\neg A \wedge \neg B)$  logically equivalent to  $A \leftrightarrow B$

D.

1.  $D \rightarrow B \wedge C$
2.  $C \rightarrow \neg A \wedge \neg B$
3.  $D \leftrightarrow (C \wedge \neg A)$
4.  $D \rightarrow (\neg C \rightarrow A)$
5.  $(\neg D \rightarrow C) \wedge (D \rightarrow \neg B)$
6.  $A \rightarrow (\neg B \wedge \neg C \rightarrow D)$
7.  $(A \wedge B \wedge C \leftrightarrow \neg D) \wedge (\neg A \wedge \neg B \rightarrow (D \leftrightarrow C))$

E.

The argument is logically correct: if p means “I’m guilty” and q means “I must be punished”,

then:

$(p \rightarrow q) \wedge p \models q$  (modus ponens)