Fuzzy Logic

Fuzzy Logic

- A computational paradigm that is based on how humans think.
- Fuzzy Logic looks at the world in imprecise terms, in much the same way that our brain takes in information (e.g. temperature is hot, speed is slow), then responds with precise actions.
- Fuzzy Logic is an approach to computing based on "degrees of truth" rather than the usual "true or false" logic.
- The human brain can reason with uncertainties, vagueness, and judgments. Computers can only manipulate precise valuations. Fuzzy logic is an attempt to combine the two techniques.

Fuzzy Logic

- Fuzzy logic is in fact, a precise problem-solving methodology.
- Fuzzy logic differs from classical logic in that statements are no longer black or white, true or false, on or off.
- In traditional logic an object takes on a value of either zero or one.
- In fuzzy logic, a statement can assume any real value between 0 and 1, representing the degree to which an element belongs to a given set.
- Works with imprecise statements such as:

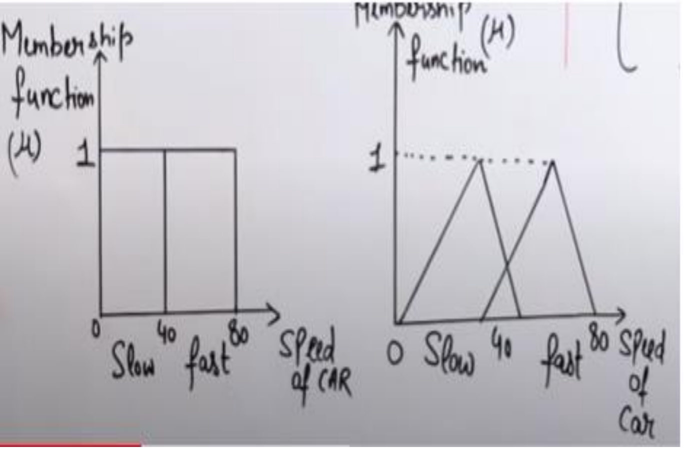
  In a process control situation, "*If* the temperature is moderate and the pressure is high, *then* turn the knob slightly right"

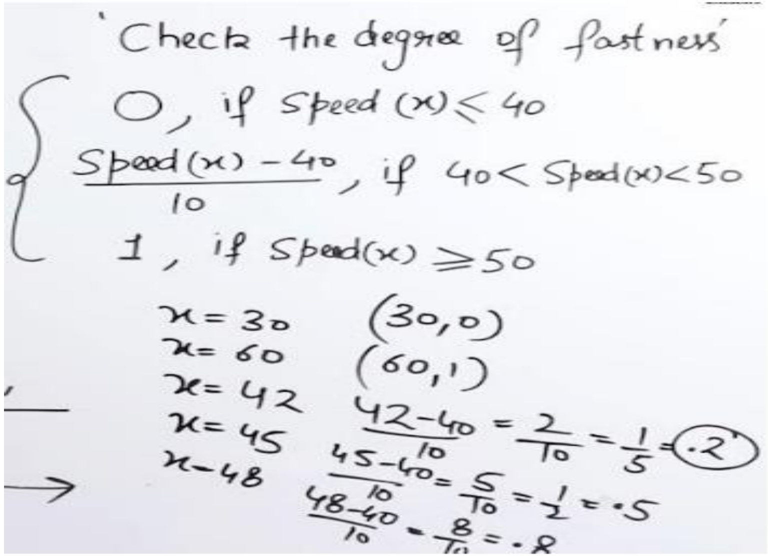
Fuzzy Logic

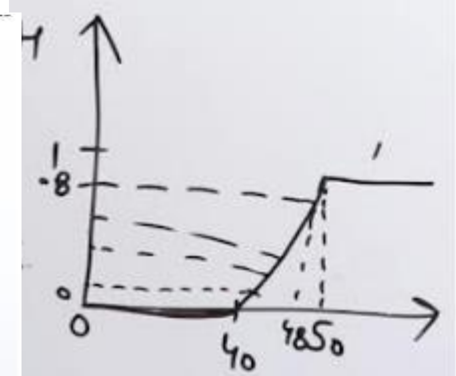
- Represent uncertainty
- Represent with degree
- Represent the belongingness of a member of a crisp set to fuzzy set.
- Example: U = {1,2,3,4,5} and S= {1,2} . S ⊂ U?

  {(1,1),(2,1),(3,0),(4,0),(5,0)}

  x μ







Fuzzy Logic Applications

- For **washing machines**, Fuzzy Logic control is almost becoming a standard feature: fuzzy controllers to load-weight, fabric-mix, and dirt sensors and automatically set the wash cycle for the best use of power, water, and detergent.
- **NASA** has studied fuzzy control for **automated space docking**: simulations show that a fuzzy control system can greatly reduce fuel consumption
- **Canon** developed an **auto-focusing camera** that uses a charge-coupled device (CCD) to measure the clarity of the image in six regions of its field of view and use the information provided to determine if the image is in focus. It also tracks the rate of change of lens movement during focusing, and controls its speed to prevent overshoot.

Fuzzy Logic Applications

- Nissan – fuzzy automatic transmission, fuzzy anti-skid braking system
- CSK, Hitachi – Hand-writing Recognition
- Ricoh, Hitachi – Voice recognition
- Automotive system for speed control, traffic control.

Fuzzy Logic-Set Theory

- The **Classical/Crisp set** is defined in such a way that the universe of discourse is spitted into two groups **members** and **non-members**. Hence, In case classical sets, **no partial membership exists**.
- In a Crisp set, membership or non-membership of element ‘x’ in set A is described by a characteristic function

$$\mu_A(x), \text{ where } \mu_A(x) = 1 \text{ if } x \in A \text{ and } \mu_A(x) = 0 \text{ if } x \notin A.$$

- **Fuzzy set** is a set having **degrees of membership** between 1 and 0. Fuzzy sets are represented with tilde character(~).
- A fuzzy set A on a universe of discourse U is characterized by a membership function $\mu_{\tilde{A}}(y)$ that takes values in the interval [0, 1].

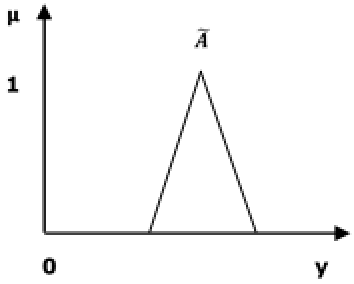
Fuzzy Logic-Set Theory

- **Classical set** contains elements that satisfy **precise properties** of membership while **fuzzy set** contains elements that satisfy **imprecise properties** of membership.
- Fuzzy sets allows partial membership which means that it contain elements that have varying degrees of membership in the set.

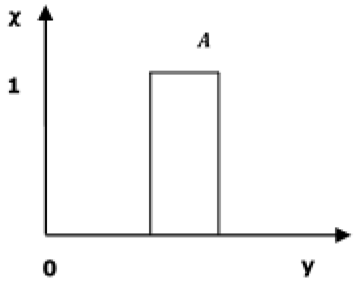
  Examples: {hot, warm, cool,cold}, { not cold, barely cold, bit cold, not quit cold, cold}
- A **fuzzy set** is a mapping of a **set** of real numbers (x_i) onto membership values (u_i) that (generally) lie in the range [0, 1]. In this **fuzzy** package a **fuzzy set** is **represented** by a **set** of pairs u_i/x_i, where u_i is the membership value for the real number x_i. We can **represent** the **set** of values as $\{ u_1/x_1 \ u_2/x_2 \ ... \ u_n/x_n \}$.

Fuzzy Logic-Set Theory

- From this, we can understand the difference between classical set and fuzzy set. Classical set contains elements that satisfy precise properties of membership while fuzzy set contains elements that satisfy imprecise properties of membership.



Membership Function of Fuzzy set $\tilde{A}$



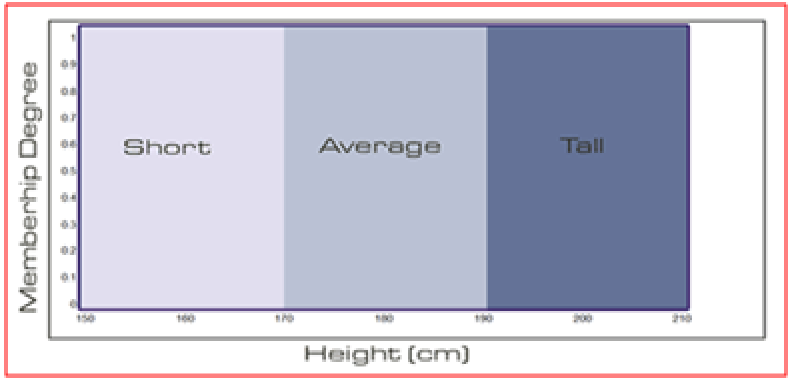
Membership Function of classical set A

Fuzzy Logic-Set Theory

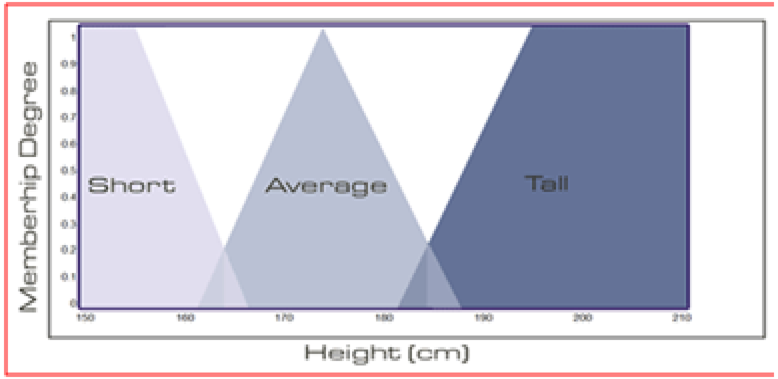
Example: Is a man whose height is 5’ 11-1/2” average or tall?

- A fuzzy system might say that he is partly medium and partly tall.

Boolean representation



Fuzzy representation



- In fuzzy terms, the height of the man would be classified within a range of [0, 1] as average to a degree of 0.6, and tall to a degree of 0.4.

Fuzzy Logic-Set Theory

- In other words, FL recognizes not only clear-cut, black-and-white alternatives, but also the infinite gradations in between.
- Fuzzy reasoning eliminates the vagueness by assigning specific numbers to those gradations. These numeric values are then used to derive exact solutions to problems.

Fuzzy Logic-Set Theory

- A fuzzy set $\widetilde{A}$ in the universe of information U can be defined as a set of ordered pairs and it can be represented mathematically as −

$$\widetilde{A} = \{(y, \mu_{\widetilde{A}}(y)) \,|y \in U\}$$

Here $\mu_{\widetilde{A}}(y)$ = degree of membership of y , assumes values in the range from 0 to 1, i.e., $\mu_{\widetilde{A}}(y) \in [0, 1]$

OR

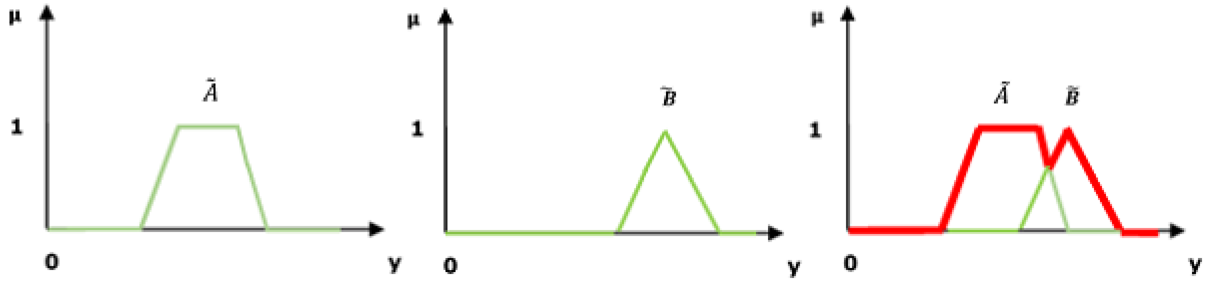
$$\widetilde{A} = \left\{ \frac{\mu_{\widetilde{A}}(y_1)}{y_1} + \frac{\mu_{\widetilde{A}}(y_2)}{y_2} + \frac{\mu_{\widetilde{A}}(y_3)}{y_3} + \ldots \right\}$$

$$= \left\{ \sum_{i=1}^{n} \frac{\mu_{\widetilde{A}}(y_i)}{y_i} \right\}$$

Operations of Fuzzy Set

- Union/Fuzzy OR

$$\mu_{A \cup B}(x) = \max\left[\mu_A(x), \mu_B(x)\right]$$

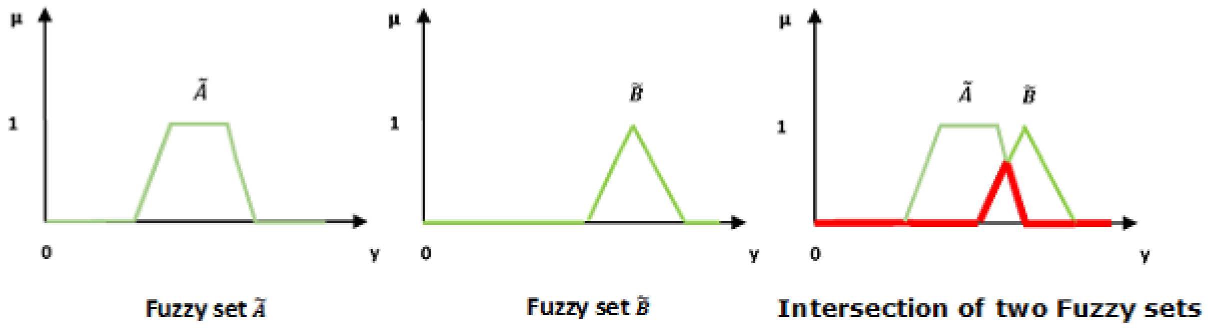


Fuzzy set $\tilde{A}$ Fuzzy set $\tilde{B}$ Union of two Fuzzy sets

Operations of Fuzzy Set

- Intersection/Fuzzy 'AND'

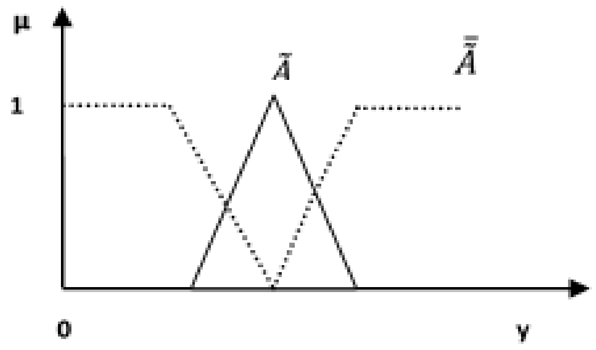
$$\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)]$$



Operations of Fuzzy Set

- Complement/Fuzzy ‘NOT’

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$



Properties of Fuzzy Set

- Commutative Property: Having two fuzzy sets $\widetilde{A}$ and $\widetilde{B}$, this property states –

$$\widetilde{A} \cup \widetilde{B} = \widetilde{B} \cup \widetilde{A}$$

$$\widetilde{A} \cap \widetilde{B} = \widetilde{B} \cap \widetilde{A}$$

- Distributive Property: Having three fuzzy sets $\widetilde{A}$, $\widetilde{B}$ and $\widetilde{C}$, this property states –

$$\widetilde{A} \cup \left(\widetilde{B} \cap \widetilde{C}\right) = \left(\widetilde{A} \cup \widetilde{B}\right) \cap \left(\widetilde{A} \cup \widetilde{C}\right)$$

$$\widetilde{A} \cap \left(\widetilde{B} \cup \widetilde{C}\right) = \left(\widetilde{A} \cap \widetilde{B}\right) \cup \left(\widetilde{A} \cap \widetilde{C}\right)$$

Properties of Fuzzy Set

- Idempotency Property: For any fuzzy set $\widetilde{A}$, this property states –

$$\widetilde{A} \cup \widetilde{A} = \widetilde{A}$$

$$\widetilde{A} \cap \widetilde{A} = \widetilde{A}$$

- Identity Property: For fuzzy set $\widetilde{A}$ and universal set U, this property states –

$$\widetilde{A} \cup \varphi = \widetilde{A}$$

$$\widetilde{A} \cap U = \widetilde{A}$$

$$\widetilde{A} \cap \varphi = \varphi$$

$$\widetilde{A} \cup U = U$$

Properties of Fuzzy Set

- Transitive Property: Having three fuzzy sets $\widetilde{A}$, $\widetilde{B}$ and $\widetilde{C}$, this property states –

$$If\ \widetilde{A} \subseteq \widetilde{B} \subseteq \widetilde{C},\ then\ \widetilde{A} \subseteq \widetilde{C}$$

- Involution Property: For any fuzzy set $\widetilde{A}$, this property states –

$$\overline{\overline{\widetilde{A}}} = \widetilde{A}$$

Properties of Fuzzy Set

- De Morgan’s Law:

This law plays a crucial role in proving tautologies and contradiction. This law states −

$$\overline{\widetilde{A} \cap \widetilde{B}} = \overline{\widetilde{A}} \cup \overline{\widetilde{B}}$$

$$\overline{\widetilde{A} \cup \widetilde{B}} = \overline{\widetilde{A}} \cap \overline{\widetilde{B}}$$

Fuzzy Logic Rule Base

- ➢ It contains the set of rules and the IF-THEN conditions provided by the experts to govern the decision making system, on the basis of linguistic information.
- ➢ Recent developments in fuzzy theory offer several effective methods for the design and tuning of fuzzy controllers. Most of these developments reduce the number of fuzzy rules.
- Examples

R1: <u>if</u> temperature is high <u>then</u> Climate is hot

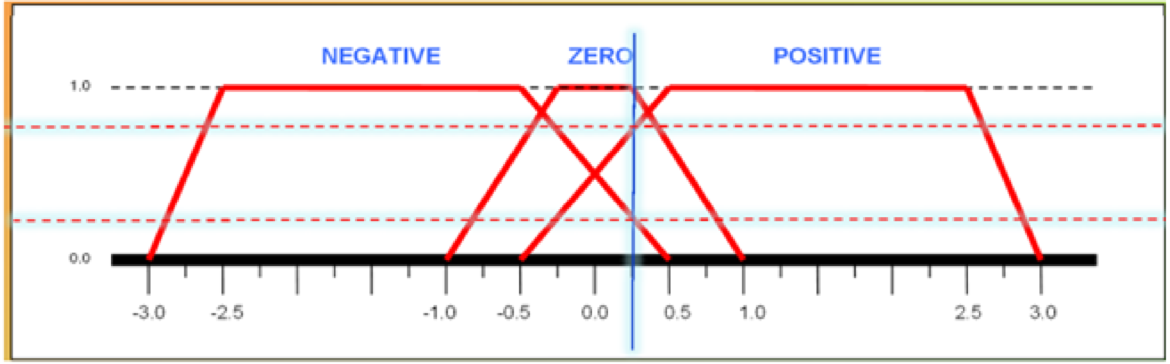
R2: <u>If</u> outlook is sunny and <u>if</u> humidity is high <u>then</u> Decision is <u>No</u>.

R3: <u>If</u> outlook is sunny and <u>if</u> humidity is low <u>then</u> Decision is <u>Yes</u>.

R4: <u>if</u> Tree_distance is SOMEWHAT close AND Tree_angle is small_positive <u>then</u> Turn slightly left

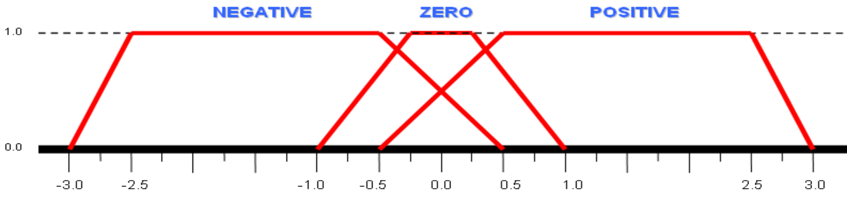
Fuzzification

- Input variables are assigned degrees of membership in various classes
- The purpose of fuzzification is to map the inputs from a set of sensors (or features of those sensors such as amplitude or spectrum) to values from 0 to 1 using a set of input membership functions.
- **Example**: Fuzzy Sets = { Negative, Zero, Positive }



- ➢ Assuming that we are using trapezoidal membership functions.
- ➢ Crisp Input: x = 0.25

Sample Calculations



$F_{zero}(0.25)$

$$F_{ZE}(0.25) = \max\left(\min\left(\frac{x-a}{b-a},1,\frac{d-x}{d-c}\right),0\right)$$

$$= \max\left(\min\left(\frac{0.25-(-1)}{-0.25-(-1)},1,\frac{1-0.25}{1-0.25}\right),0\right)$$

$$= \max\left(\min\left(1.67,1,1\right),0\right)$$

$$= 1$$

$F_{positive}(0.25)$

$$F_{P}(0.25) = \max\left(\min\left(\frac{0.25-(-0.5)}{0.5-(-0.5)},1,\frac{3-0.25}{3-0.25}\right),0\right)$$

$$= \max\left(\min\left(0.75,1,5.5\right),0\right)$$

$$= 0.75$$

$F_{negative}(0.25)$

$$F_{N}(0.25) = \max\left(\min\left(\frac{0.25-(-3)}{-2.5-(-3)},1,\frac{0.5-0.25}{0.5-(-0.5)}\right),0\right)$$

$$= \max\left(\min\left(6.5,1,0.25\right),0\right)$$

$$= 0.25$$

menu

Sample Calculations

$F_{zero}(-0.25)$

$$F_{ZE}(-0.25) = \max\left(\min\left(\frac{-0.25-(-1)}{-0.25-(-1)}, 1, \frac{1-(-0.25)}{1-0.25}\right), 0\right)$$
$$= \max\left(\min\left(1,1,1.67\right),0\right)$$
$$= 1$$

$F_{positive}(-0.25)$

$$F_P(-0.25) = \max\left(\min\left(\frac{-0.25-(-3)}{0.5-(-0.5)}, 1, \frac{3-(-0.25)}{3-2.5}\right), 0\right)$$
$$= \max\left(\min\left(0.25,1,6.5\right),0\right)$$
$$= 0.25$$

$F_{negative}(-0.25)$

$$F_N(-0.25) = \max\left(\min\left(\frac{-0.25-(-3)}{-2.5-(-3)}, 1, \frac{0.5-(-0.25)}{0.5-(-0.5)}\right), 0\right)$$
$$= \max\left(\min\left(5.5,1,0.75\right),0\right)$$
$$= 0.75$$

Defuzzification

- ➢ it is used to convert the fuzzy sets obtained by inference engine into a crisp value.
- ➢ There are several defuzzification methods available and the best suited one is used with a specific expert system to reduce the error.
- Different methods of Defuzzification are listed below:
- ➢ Maxima Methods
- ➢ Centroid Method
- ➢ Weighted Average Method
- ➢ Mean-Max Membership

Fuzzy Inference System

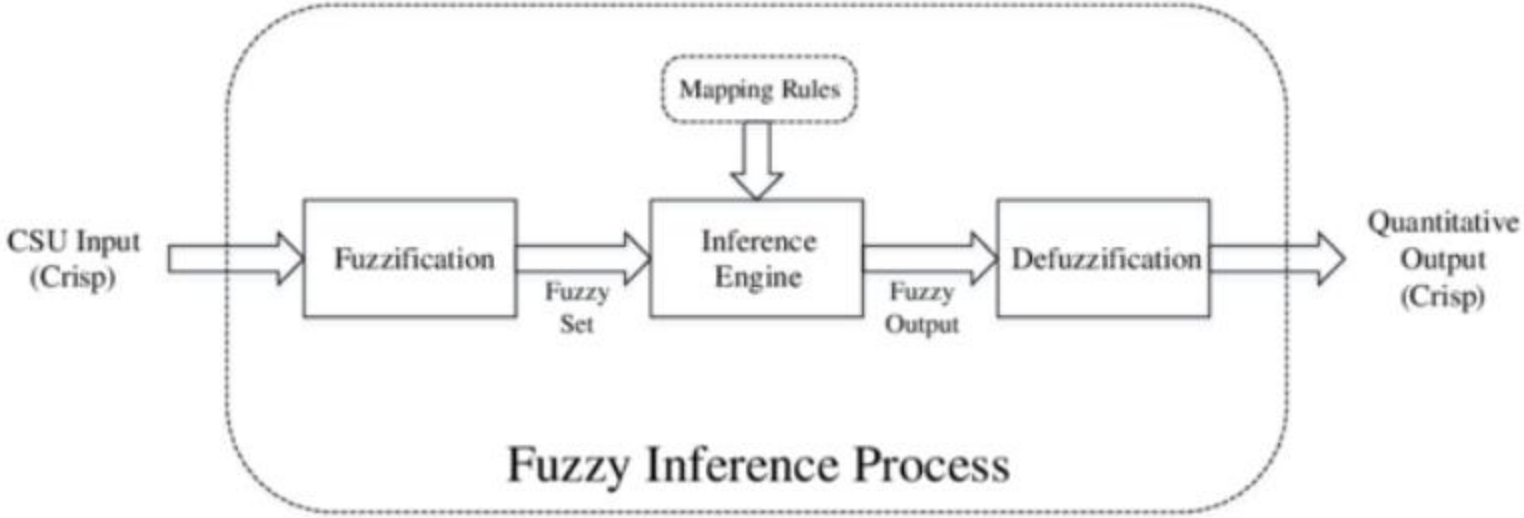
- A fuzzy inference system (FIS) is a system that uses fuzzy set theory to map inputs (*features* in the case of fuzzy classification) to outputs (*classes* in the case of fuzzy classification).
- This is a method to map an input to an output using fuzzy logic. Based on this mapping process, the system takes decisions and distinguishes patterns.
- **Characteristics** of Fuzzy Inference System

➢ The output from FIS is always a fuzzy set irrespective of its input which can be fuzzy or crisp.

➢ It is necessary to have fuzzy output when it is used as a controller.

➢ A defuzzification unit would be there with FIS to convert fuzzy variables into crisp variables.

Fuzzy Inference System Architecture



- The working of the FIS consists of the following steps −
- ➢ A **fuzzification** unit converts the crisp input into fuzzy input.
- ➢ A **knowledge base** - collection of rule base and database is formed upon the conversion of crisp input into fuzzy input.
- ➢ The **defuzzification** unit fuzzy input is finally converted into crisp output.

Fuzzy Inference System

- **Functional Blocks of FIS**
- ➢ **Rule Base** − It contains fuzzy IF-THEN rules.
- ➢ **Database** − It defines the membership functions of fuzzy sets used in fuzzy rules.
- ➢ **Decision-making Unit** − It performs operation on rules.
- ➢ **Fuzzification Interface Unit** − It converts the crisp quantities into fuzzy quantities.
- ➢ **Defuzzification Interface Unit** − It converts the fuzzy quantities into crisp quantities. Following is a block diagram of fuzzy interference system.

Fuzzy Inference System

- **Fuzzy Logic Rule Base**

➢ **fuzzy rule-based** systems are **rule-based** systems, where **fuzzy** sets and **fuzzy logic** are used as tools for representing different forms of knowledge about the problem at hand, as well as for modeling the interactions and relationships existing between its variables.

➢ Examples

R1: IF temperature is hot AND humidity is high. THEN fan speed is fast.

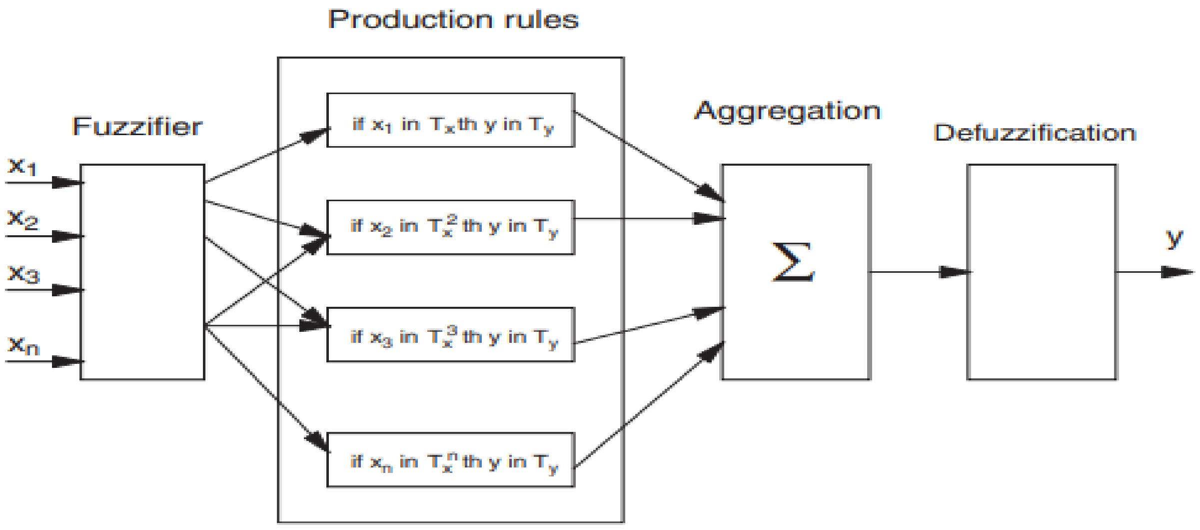
- The degree of truth assigned to temperature is hot and to humidity is high.

Fuzzy Inference System

- **Fuzzification**
- ➢ It is used to convert inputs i.e. crisp numbers into fuzzy sets. Crisp inputs are basically the exact inputs measured by sensors and passed into the control system for processing, such as temperature, pressure, etc.
- **Inference Engine**
- ➢ It determines the matching degree of the current fuzzy input with respect to each rule and decides which rules are to be fired according to the input field. Next, the fired rules are combined to form the control actions.
- **Defuzzification**
- ➢ It is the process of producing a quantifiable result in Crisp logic, given fuzzy sets and corresponding membership degrees. It is the process that maps a fuzzy set to a crisp set.

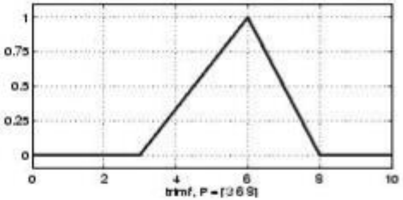
Fuzzy Inference System

- Schematic diagram of a fuzzy inference system

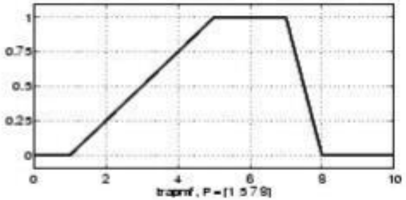


Fuzzy Membership Functions

- 4 common types of fuzzy membership functions:
  - triangular (3 parameters)   trapezoidal(4 para)

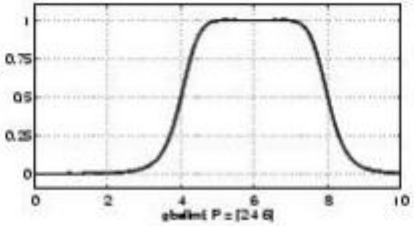


trimf



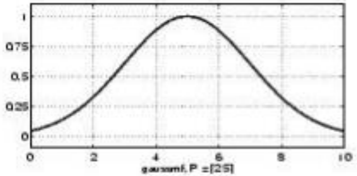
trapmf

  - generalised bell (3 parameters)

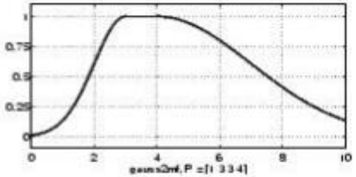


gbellmf

  - gaussian (2 parameters)



gaussmf



gauss2mf

Example

- **Design of a Fuzzy System**
- In this section, a simple example system will be constructed and executed to visualise the design and execution of a fuzzy inference system.
- The hypothetical system considered here **controls the speed of a fan has according to the environment's temperature and humidity**.
- Therefore, our system consists of two inputs, temperature and humidity and a single output, that is the fan speed.

Example..

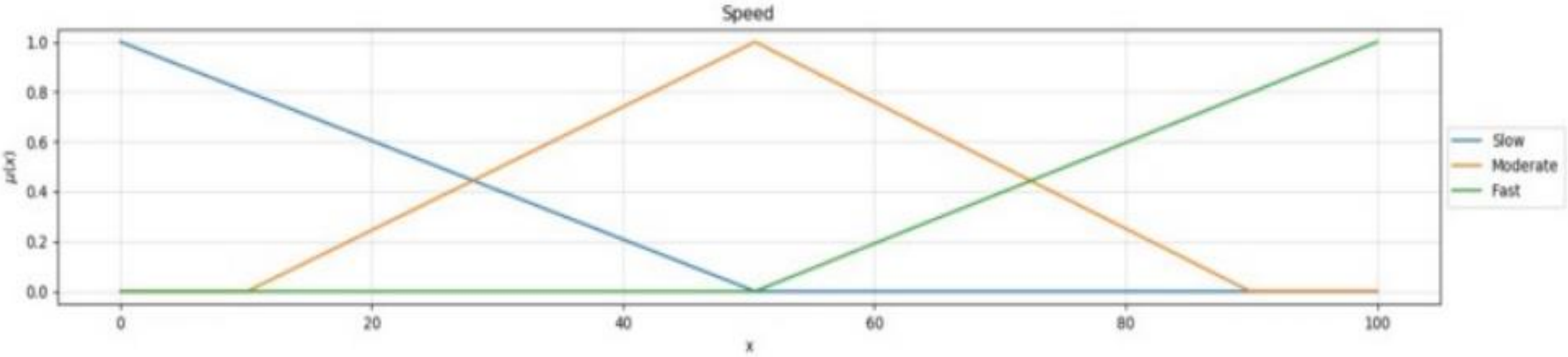
- The **first step** in the design of our system is to **define fuzzy sets** to describe the input and output variables.

➢ Temperature: Cold, Medium, Hot

➢ Humidity: Dry, Normal, Wet

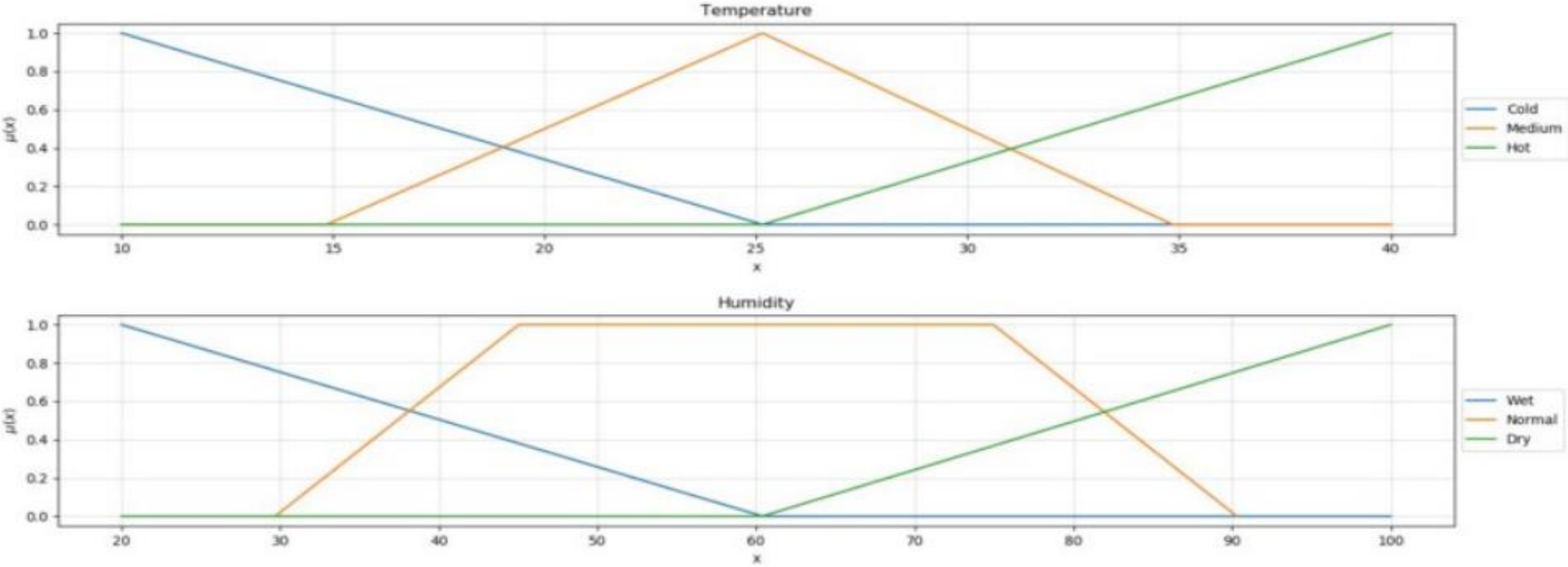
➢ Fan Speed: Slow, Moderate, Fast

- The diagram below shows a graphical representation of the input and output variables of our system and their respective sets.



Example..

- The diagram below shows a graphical representation of the input and output variables of our system and their respective sets.



Example..

- It can be noted that **triangular sets** were used to describe most of the sets of this system; however, ‘normal’ humidity is specified using a **trapezoidal set**. Fuzzy sets reflect the knowledge is the user designing the system, so they can take a wide variety of shapes.
- Note that the output was described using fuzzy sets as well; therefore the system that is being considered is a **Mamdani-type**, that links fuzzy sets related to the inputs of the system to fuzzy sets associated with the output of the system using fuzzy rules.

Example..

- A total of nine rules are used to describe the knowledge necessary to operate our fan:

```
If Temperature is Cold and Humidity is Dry Then Fan Speed is Slow
If Temperature is Medium and Humidity is Dry Then Fan Speed is Slow
If Temperature is Cold and Humidity is Dry Then Fan Speed is Slow


If Temperature is Hot and Humidity is Dry Then Fan Speed is Moderate
If Temperature is Medium and Humidity is Normal Then Fan Speed is
Moderate
If Temperature is Cold and Humidity is Wet Then Fan Speed is Moderate


If Temperature is Hot and Humidity is Normal Then Fan Speed is Fast
If Temperature is Hot and Humidity is Wet Then Fan Speed is Fast
If Temperature is Medium and Humidity is Wet Then Fan Speed is Fast
```

Example..

- These rules can be visualised if we use a combined fuzzy rule base, that is a grid where the input-fuzzy sets occupy the edges so that each cell in the grid defines a rule. The following diagram shows the **rule base** for this system.

Humidity		Temperature: Cold	Temperature: Medium	Temperature: Hot
Humidity	Dry	Moderate	Fast	Fast
Humidity	Normal	Slow	Moderate	Fast
Humidity	Wet	Slow	Slow	Moderate

Example..

- The following steps take place when an input combination is fed to the system, let us as an example say that we have a temperature of 18 degrees and humidity of 60%:
- The degree of membership for each set of the input variables is determined.
- **temperature of 18 degrees** is
  - 0.48 Cold
    0.29 Medium
    0.00 Hot
- **humidity of 60%** is
  - 0.0 Wet
    1.0 Normal
    0.0 Dry

Example..

- With this input combination, two rules are fired with a degree higher than zero, as can be seen in the updated fuzzy rule base below:

Humidity		Temperature		
		Cold (0.48)	Medium(0.29)	Hot(0.0)
Humidity	Dry(0.0)	Moderate	Fast	Fast
	Normal(1.0)	Slow	Moderate	Fast
	Wet(0.0)	Slow	Slow	Moderate

Example..

- And therefore our **fuzzy output** will consist of the **speed Slow**, that has activation of **0.48** and **Moderate** that has **0.29** activation.
- The output set is finally defuzzified using centre-of-gravity defuzzification and a crisp value of 36.814 is obtained to drive the fan.
- It is shown how control of systems can be achieved using linguistic terms to represent human knowledge.

Example 2

- Example of the two-input, one-output, three-rule tipping problem from The Basic Tipping Problem. The fuzzy inference system for this problem takes ***service* and *food quality*** as inputs and computes a **tip percentage** using the following rules.
- If the service is poor or the food is rancid, then tip is cheap.
- If the service is good, then tip is average.
- If the service is excellent or the food is delicious, then tip is generous.
- The first step is to take the inputs and determine the degree to which they belong to each of the appropriate fuzzy sets via membership functions (*fuzzification*).
- The output is a fuzzy degree of membership in the qualifying linguistic set (always the interval from 0 through 1).

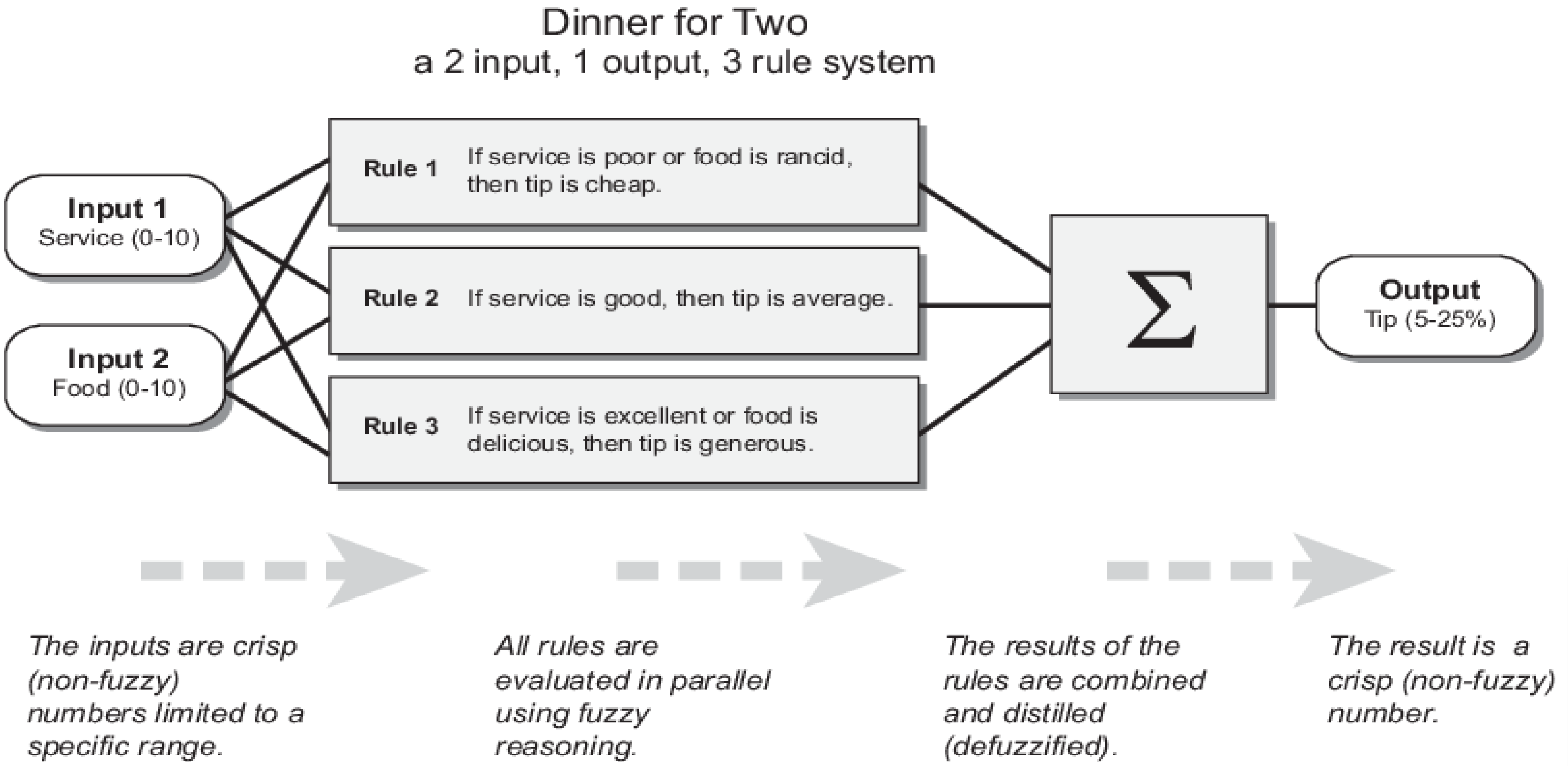
Example 2

- This example is built on three rules, and each of the rules depends on resolving the inputs into several different fuzzy linguistic sets:

{ service is **poor**, service is **good**, food is **rancid**, food is **delicious**, and so on…}

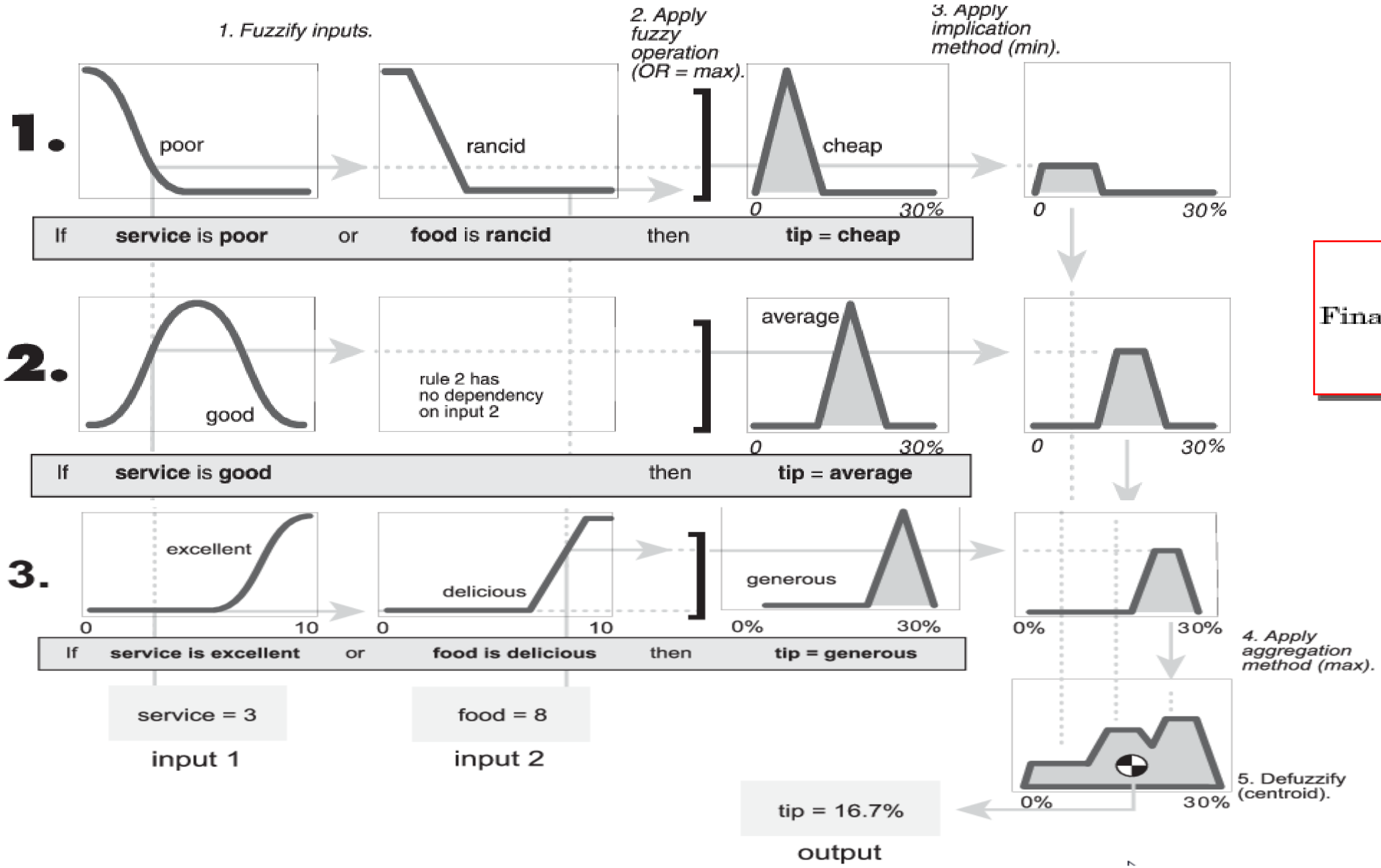
- Before the rules can be evaluated, the inputs must be fuzzified according to each of these linguistic sets.
- For example, to what extent is the food delicious? figure shows how well the **food** at the hypothetical restaurant (rated on a scale from **0 through 10**) qualifies as the linguistic variable delicious using a membership function.
- In this case, we rate the **food as an 8**, which, given the graphical definition of **delicious**, corresponds to $\boldsymbol{\mu = 0.7}$ for the delicious membership function.

Example 2



Dinner for Two
a 2 input, 1 output, 3 rule system
Input 1
Service (0-10)
Input 2
Food (0-10)
Rule 1
If service is poor or food is rancid, then tip is cheap.
Rule 2
If service is good, then tip is average.
Rule 3
If service is excellent or food is delicious, then tip is generous.
Σ
Output
Tip (5-25%)
The inputs are crisp (non-fuzzy) numbers limited to a specific range.
All rules are evaluated in parallel using fuzzy reasoning.
The results of the rules are combined and distilled (defuzzified).
The result is a crisp (non-fuzzy) number.

Example 2..



$$\text{Final Output} = \frac{\sum_{i=1}^{N} w_i z_i}{\sum_{i=1}^{N} w_i}$$

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Example 3

- When using a **washing machine**, the user manually **sets the washing time, based on the volume of clothes and the type and degree of dirtiness**. In order to transform the washing process into an automatic one, we can **use sensors to detect the volume of clothes and their dirtiness**. A certain washing time will be chosen, based on this data.
- Unfortunatelly, a precise mathematical relation between the inputs (volume of clothes, dirtiness) and the output (washing time) cannot be defined. Thus, the washing time is manually set by the user, based on previous experience and trials.

Example 3

- Building a washing machine with automatic washing time determination means building the following two subsystems:
  - *the* **sensor system** - collects data from the outer environment (the clothes) and send them to the controller
  - *the* **controller system** - sets the washing time, based on the information received from the sensor system.
- Given the fact that there is no mathematical relation between the inputs and output, a **"fuzzy logic controller"** will be used.

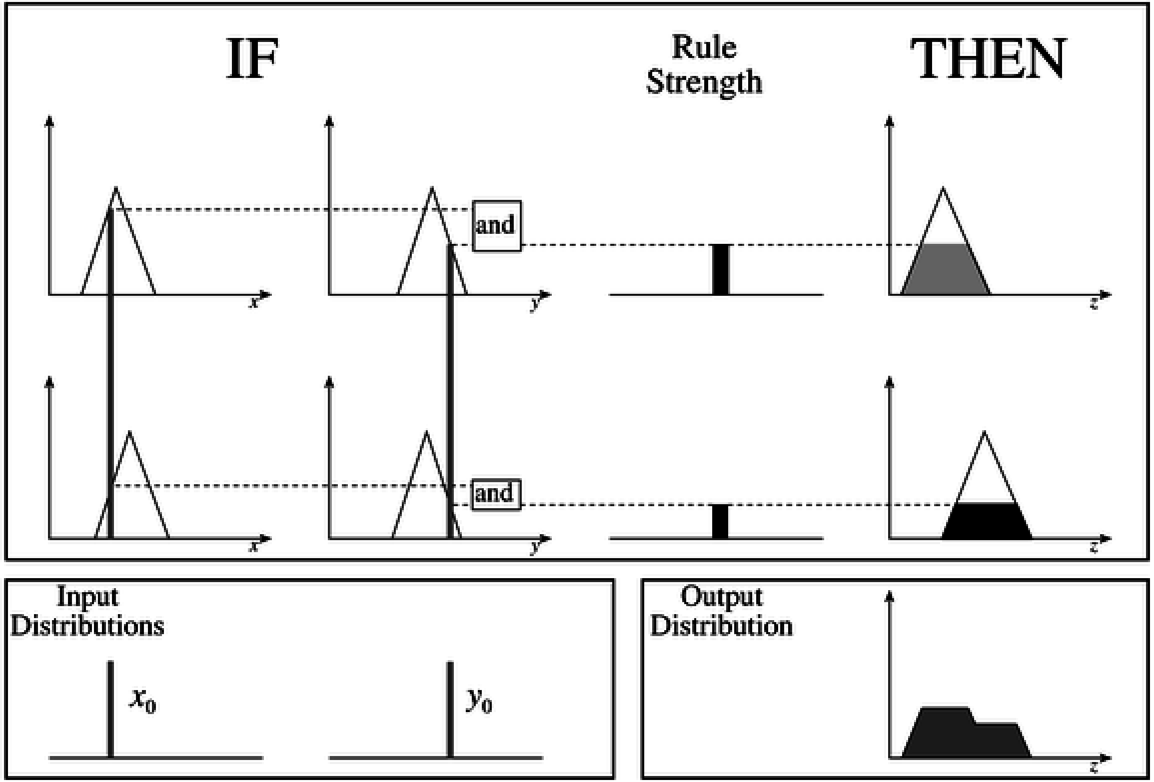
Fuzzy Inference System

- There are two main types of fuzzy inference systems
- **Mamdani Fuzzy Inference System**
  - ➢ The Mamdani fuzzy inference system was proposed by Ebhasim Mamdani.
  - ➢ Basically, it was anticipated to control a steam engine and boiler combination by synthesizing a set of fuzzy rules obtained from the experienced human operators.
- **Takagi-Sugeno Fuzzy Model (TS Method)**
  - ➢ This fuzzy inference system was proposed by Takagi, Sugeno, and Kang to develop a systematic approach for generating fuzzy rules from a given input-output dataset.

Mamdani Fuzzy Inference System

- Steps for Computing the Output
- **Step 1 −** determining a set of fuzzy rules
- **Step 2 −** fuzzifying the inputs using the input membership functions,
- **Step 3 −** combining the fuzzified inputs according to the fuzzy rules to establish a rule strength,
- **Step 4 −** finding the consequence of the rule by combining the rule strength and the output membership function,
- **Step 5 −** For getting output distribution combine all the consequents.
- **Step 6 −** defuzzifying the output distribution (this step is only if a crisp output (class) is needed).

Mamdani Fuzzy Inference System



- Fig.1: A two input, two rule Mamdani FIS with crisp inputs

Mamdani Fuzzy Inference System

1. **Creating fuzzy rules**

- Fuzzy rules are a collection of linguistic statements that describe how the FIS should make a decision regarding classifying an input or controlling an output.
- Fuzzy rules are always written in the following form:

if (input1 is membership function1) **and/or**
(input2 is membership function2) **and/or**
then (output is output membership function).

➢ **Example:**

R_1 : *IF size is small and weight is small THEN quality is bad,*
also
R_2 : *IF size is small and weight is large THEN quality is medium,*
also
R_3 : *IF size is large and weight is small THEN quality is medium,*
also
R_4 : *IF size is large and weight is large THEN quality is good*

Mamdani Fuzzy Inference System

2. **Fuzzification**

- The purpose of fuzzification is to map the inputs from a set of sensors (or features of those sensors such as amplitude or spectrum) to values from 0 to 1 using a set of input membership functions.
- In the example shown in Fig.1, their are two inputs, x_0 and y_0 shown at the lower left corner.
- For example, x_0 could be the EMG energy coming from the front of the forearm and y_0 could be the EMG energy coming from the back of the forearm. The membership functions could then represent "large" amounts of tension coming from a muscle or "small" amounts of tension. When choosing the input membership functions, the definition of what we mean by "large" and "small" may be different for each input.

Mamdani Fuzzy Inference System

3. **Fuzzy combinations (T-norms)**

- In making a fuzzy rule, we use the concept of "and", "or", and sometimes "not".
- Fuzzy combinations are also referred to as "T-norms".

4. **Consequence**

- The consequence of a fuzzy rule is computed using two steps:
  - Computing the rule strength by combining the fuzzified inputs using the fuzzy combination process. In example, the fuzzy "and" is used to combine the membership functions to compute the rule strength.
  - Clipping the output membership function at the rule strength.

Mamdani Fuzzy Inference System

5. **Combining Outputs into an Output Distribution**

- The outputs of all of the fuzzy rules must now be combined to obtain one fuzzy output distribution.
- The output membership functions on the right hand side of the figure are combined using the fuzzy "or" to obtain the output distribution shown on the lower right corner of the figure.

6. **Defuzzification of Output Distribution**

- In many instances, it is desired to come up with a single crisp output from a FIS.
- This crisp number is obtained in a process known as defuzzification.

Mamdani Fuzzy Inference System

- **Advantages**
  - It has widespread acceptance.
  - It is well suited to human input.
- **Summary**
  - Output membership function is present
  - Crisp result is obtained through defuzzification of rules’ consequent
  - MISO (Multiple Input Single Output) and MIMO (Multiple Input Multiple Output) systems
  - Expressive power and Interpretable rule consequents
  - Less flexibility in system design
- **Application**
  - Medical Diagnosis System

Takagi-Sugeno Fuzzy Inference System

- Sugeno FIS is similar to the Mamdani method in many respects.
- The first two parts of the fuzzy inference process, fuzzifying the inputs and applying the fuzzy operator, are exactly the same.
- The main difference between Mamdani and Sugeno is that the Sugeno output membership functions are either linear or constant.
- Format of the rule is given as −

IF x is A and y is B THEN Z = f(x,y)

Where, A and B are fuzzy sets in the antecedent

z = f(x,y) is a crisp function in the consequent.

- **Example**: A typical rule in a Sugeno fuzzy model has the form:

If Input 1 = x and Input 2 = y, then Output is z = ax + by + c

- For a zero-order Sugeno model, the output level z is a constant (a=b =0).

Takagi-Sugeno Fuzzy Inference System

- Takagi-Sugeno Fuzzy Model (TS Method) works in the following way :
- **Step 1: Fuzzifying the inputs** − Here, the inputs of the system are made fuzzy.
- **Step 2: Applying the fuzzy operator** − In this step, the fuzzy operators must be applied to get the output.
- <u>Advantages of the Sugeno Method</u>
- It is computationally efficient.
- It can be used to model any inference system in which the output membership functions are either linear or constant.
- It works well with linear techniques (e.g., PID (proportional–integral–derivative controller)).
- It works well with optimization and adaptive techniques.
- It is well suited to mathematical analysis.

Takagi-Sugeno Fuzzy Inference System

- **Summary**
  - No output membership function is present
  - No defuzzification: crisp result is obtained using weighted average of the rules' consequent
  - Only MISO systems
  - More flexibility in system design
- **Application**
  - To keep track of the change in aircraft performance with altitude

Fuzzy Inference System

MAMDANI FIS	SUGENO FIS
Output membership function is present	No output membership function is present
The output of surface is discontinuous	The output of surface is continuous
Distribution of output	Non distribution of output, only Mathematical combination of the output and the rules strength
Less flexibility in the system design	More flexibility in the system design
It has more accuracy in security evaluation block cipher algorithm	It has less accuracy in security evaluation block cipher algorithm
It is using in MISO (Multiple Input and Single Output) and MIMO (Multiple Input and Multiple Output) systems	It is using only in MISO (Multiple Input and Single Output) systems
Well suited to human input	Well suited to mathematically analysis

Example 4

- Rainfall events prediction using rule-based fuzzy inference system

Inputs:

  - Relative humidity
  - Total cloud cover
  - Wind direction
  - Temperature and
  - Surface pressure

Output

  - Rainfall events

S. Asklany et al. / Atmospheric Research 101 (2011) 228–236

Fig. 1. Mersa Matruh and Cairo cities on the Egypt map.

Example…

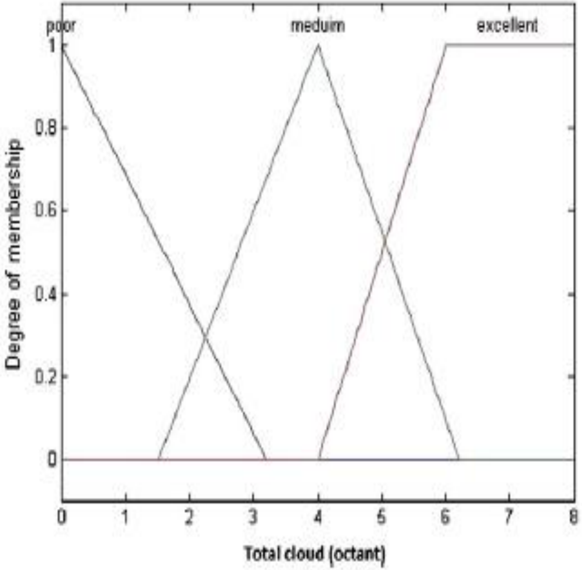


Fig. 3. Membership functions associated with the total cloud are referred to as "poor", "medium" and "excellent."

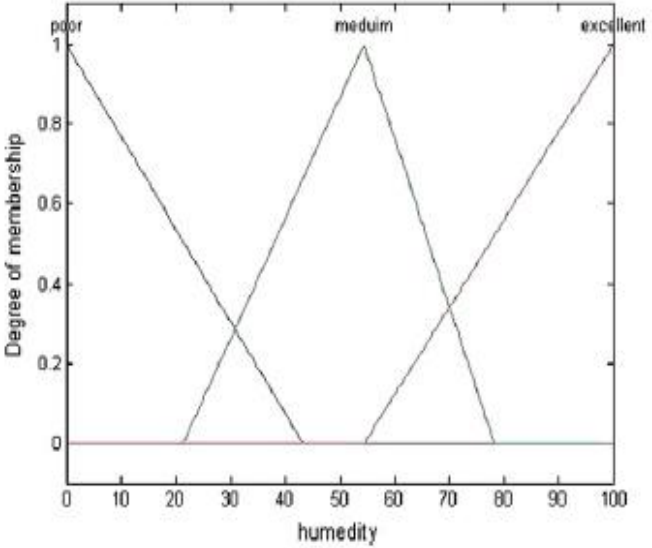
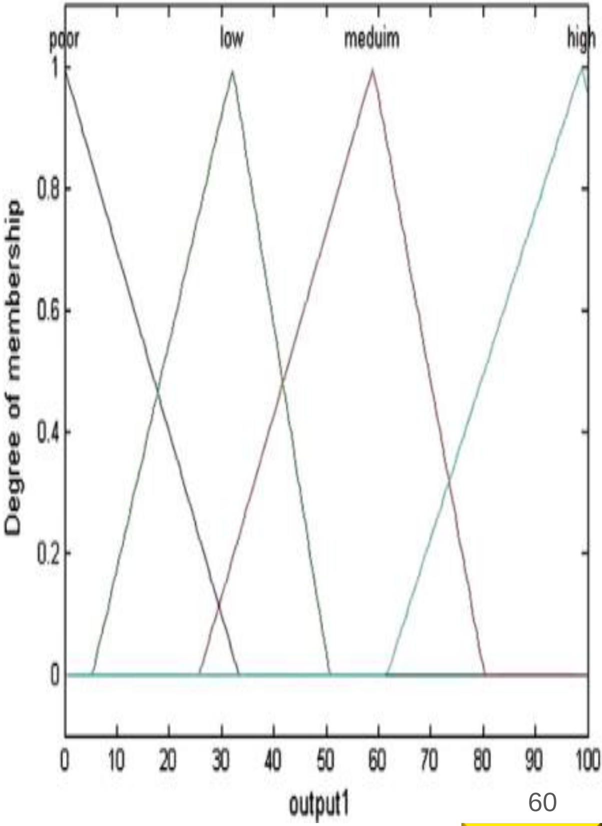


Fig. 2. Membership functions associated with the relative humidity are referred to as "poor", "medium" and "excellent."



b. IF humidity is poor AND IF total cloud is poor AND IF wind direction is poor AND IF pressure is high AND IF temperature is poor THEN rain percentage is low.

Example…

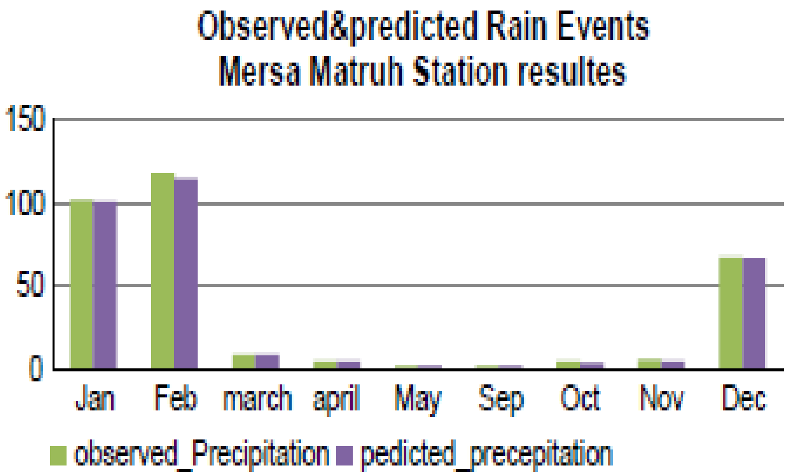


Fig. 8. FIS Output compared with observed rain events for Mersa Matruh station.

Table 1
Twenty years tested rainfall events for Cairo airport station (HECA).

Month	No. of rain events	No. of success forecasts
Jan.	388	301
Feb.	337	316
March	285	279
April	116	100
May	38	32
June	21	12
July	41	32
Aug.	47	45
Sep.	42	37
Oct	70	62
Nov	134	105
Dec	313	275

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Fuzzy Inference System

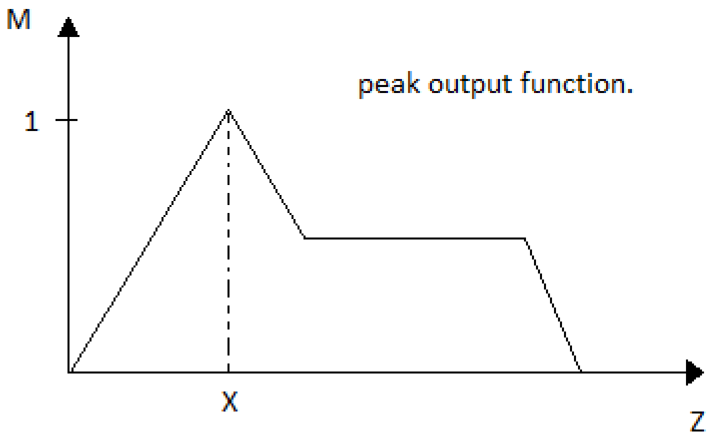
- **Applications**
  - ➢ It is used in the aerospace field for altitude control of spacecraft and satellite.
  - ➢ It has used in the automotive system for speed control, traffic control.
  - ➢ It is used for decision making support systems and personal evaluation in the large company business.
  - ➢ It has application in chemical industry for controlling the pH, drying, chemical distillation process.
  - ➢ Fuzzy logic are used in Natural language processing and various intensive applications in Artificial Intelligence.
  - ➢ Fuzzy logic are extensively used in modern control systems such as expert systems.

Defuzzification Methods

- Max membership principle
- Centroid method
- Weighted average method
- Mean max membership
- Smallest of Maximum
- Largest of Maximum
- Center of Maxima

Defuzzification Methods

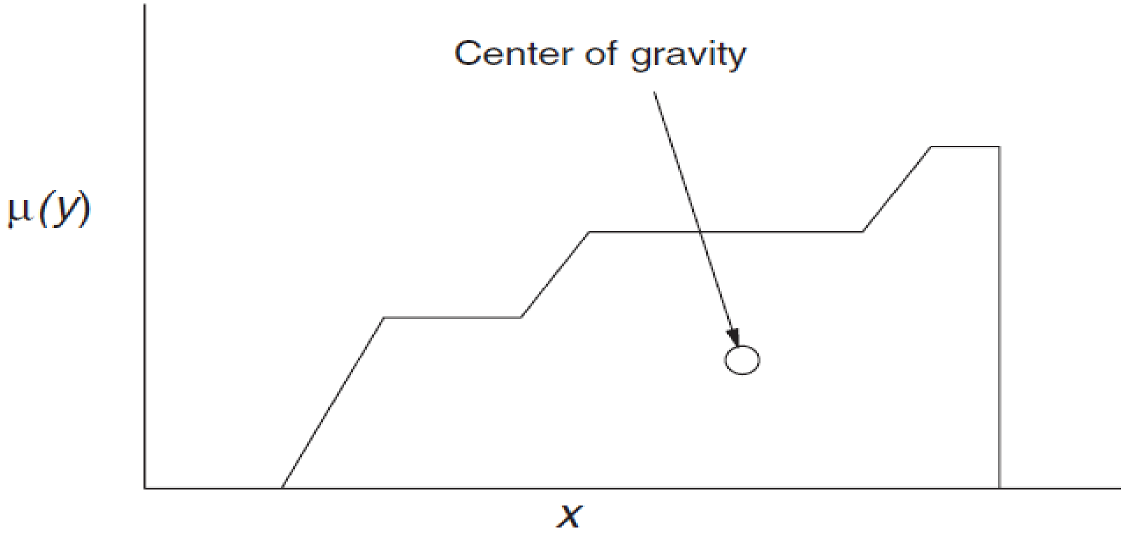
- **Max membership principle**: In this method, the defuzzifier examines the aggregated fuzzy set and chooses that output *y for which* $\mu_B(y)$ *is the maximum*



Defuzzification Methods

- **Centroid method:** centre of mall, centre of gravity or area.
- ➢ In this method, the defuzzifier determines the center of gravity (centroid) y_i' of B and uses that value as the output of the FLS.
- ➢ For a continuous aggregated fuzzy set, the centroid is given by

$$y' = \frac{\int_s y_i \mu_B(y)\,dy}{\int_s \mu_B(y)\,dy}$$



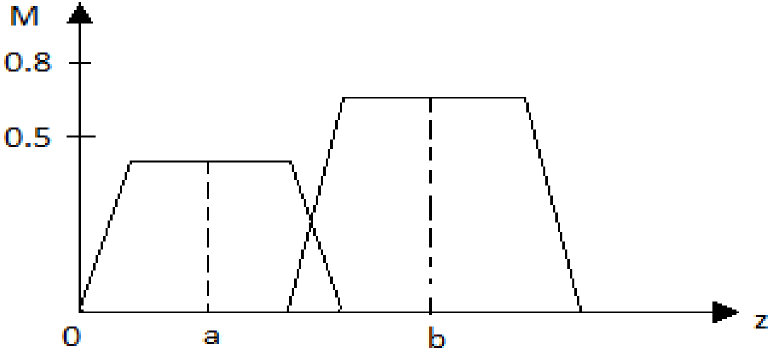
Defuzzification Methods

- **Weighted average method:** Valid for symmetrical output membership function.
- Each membership function is weighted by its max membership value.

$$X^* = \frac{\sum Mc(\bar{xi}).\bar{xp}}{\sum MC(\bar{xi})}$$

$\bar{X}i$ = maximum of with member function.

$\sum$ = algebraic sum.

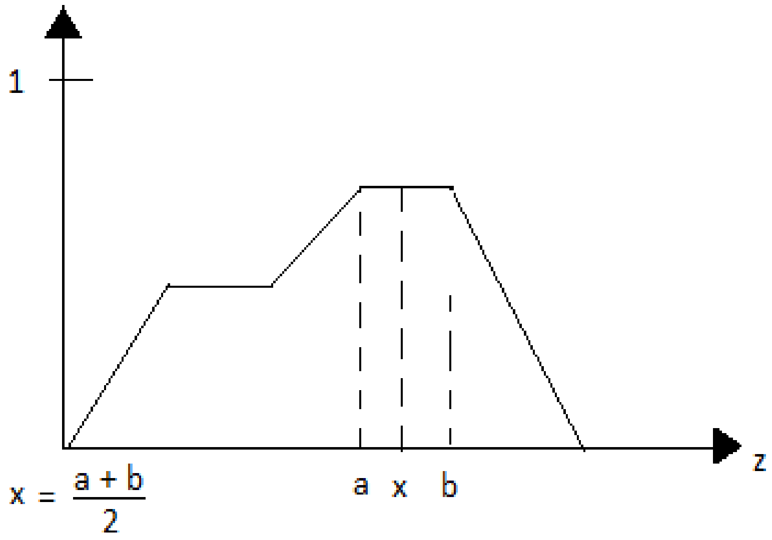


$$x^* = \frac{0.5a+0.8b}{0.5+0.8}$$

Defuzzification Methods

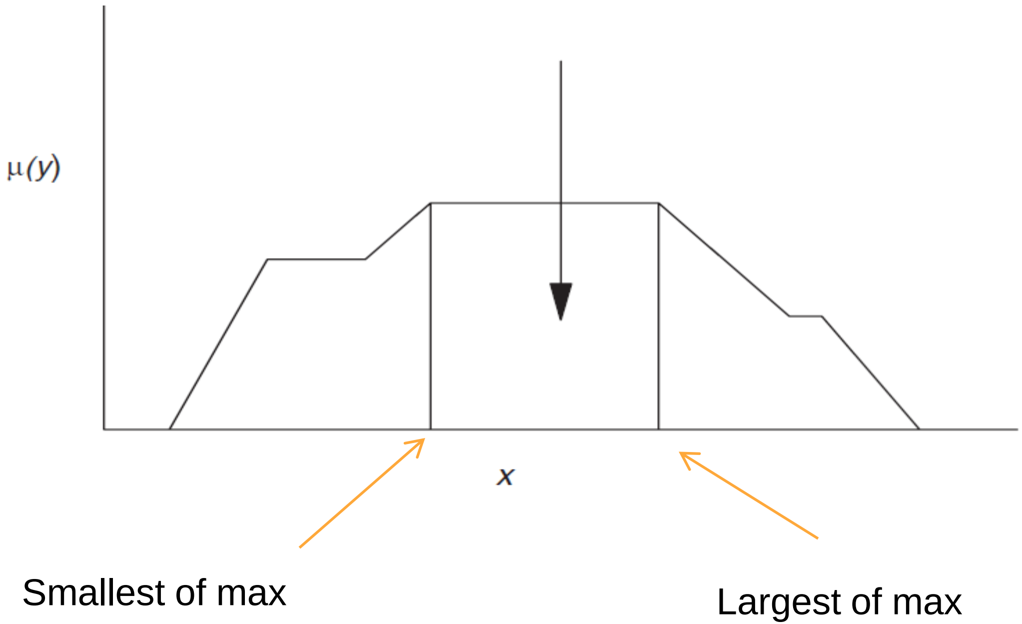
- **Mean max membership method:** This is known as middle of the maxima.

$$X^* = \frac{\sum_{i=1}^{n} \bar{x}p}{n}$$



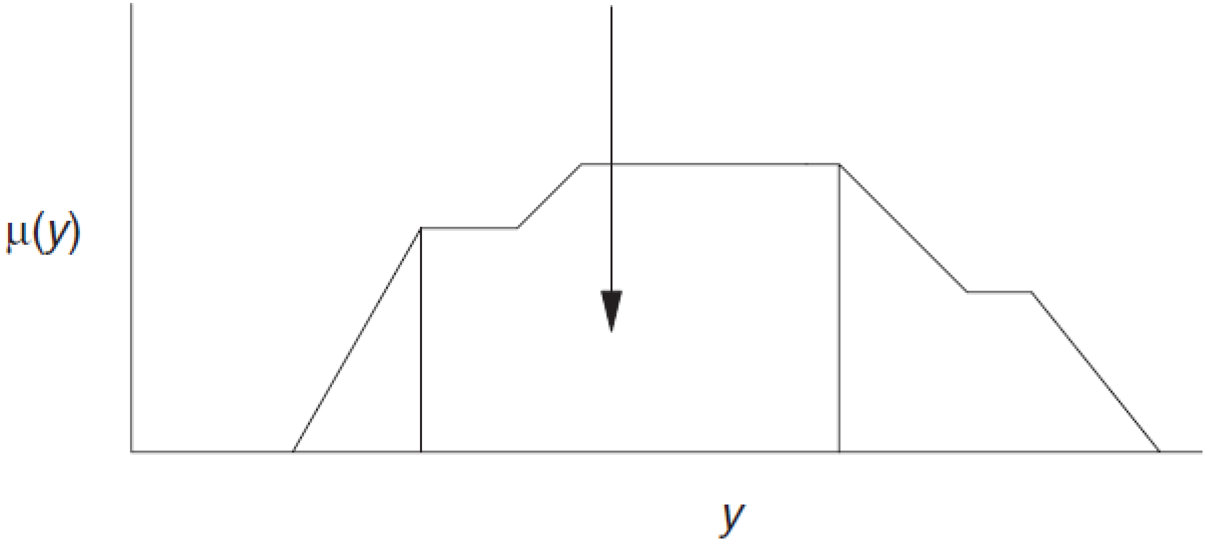
Defuzzification Methods

- **Smallest of Maximum and Largest of Maximum**



Defuzzification Methods

- **Center of maxima:** In a multimode fuzzy region, the center-of-maxima technique finds the highest plateau and then the next highest plateau. The midpoint between the centers of these plateaus is selected



Thank you.