Contents

Imperial College London

1. Drag and Cruise Performance (Range and Endurance)
2. Longitudinal Static Stability
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The early days. Students Seddon, Hopkins, Harris and Wood and Squadron Leader I.A. Robertson (Robbie)

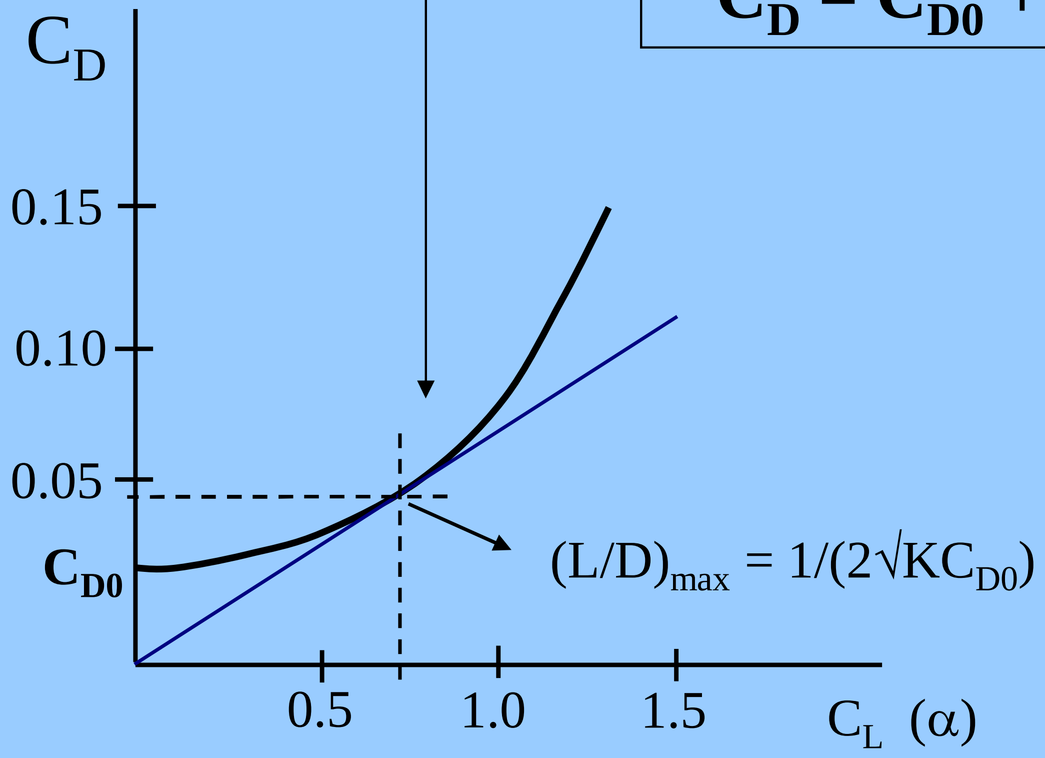
1. Drag & Cruise Performance

- Lift and Drag – Best lift to drag ratio and Minimum drag speed V_{MD}
- *Performance d*epends on aerodynamic characteristic & powerplant:
  - Endurance – time in the air
  - Range – fuel burn per nautical air mile

Aircraft Drag Equation

Quadratic approximation acceptable for flight test

$$C_D = C_{D0} + KC_L^{\,2}$$

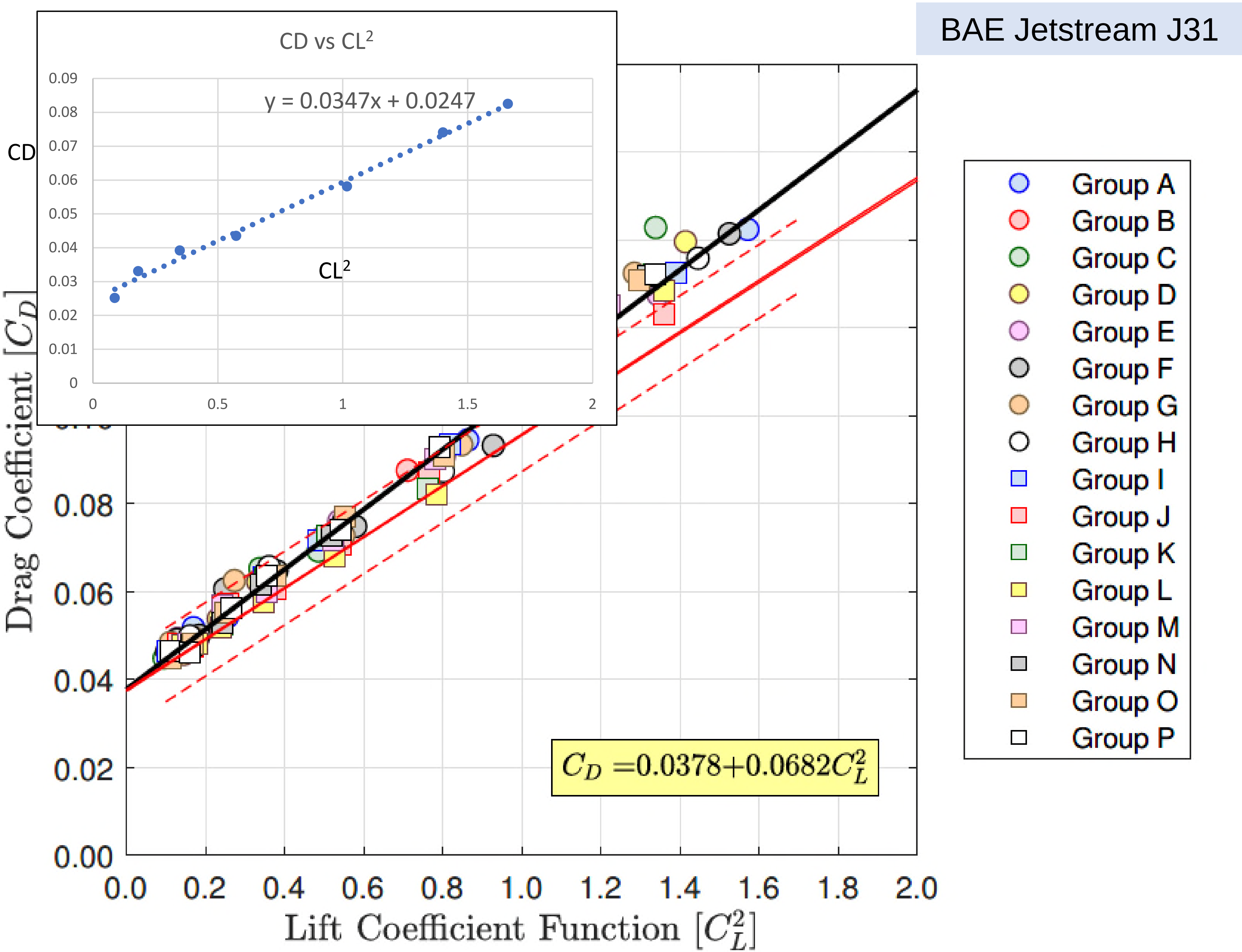


[Assume in region where C_L proportional α]

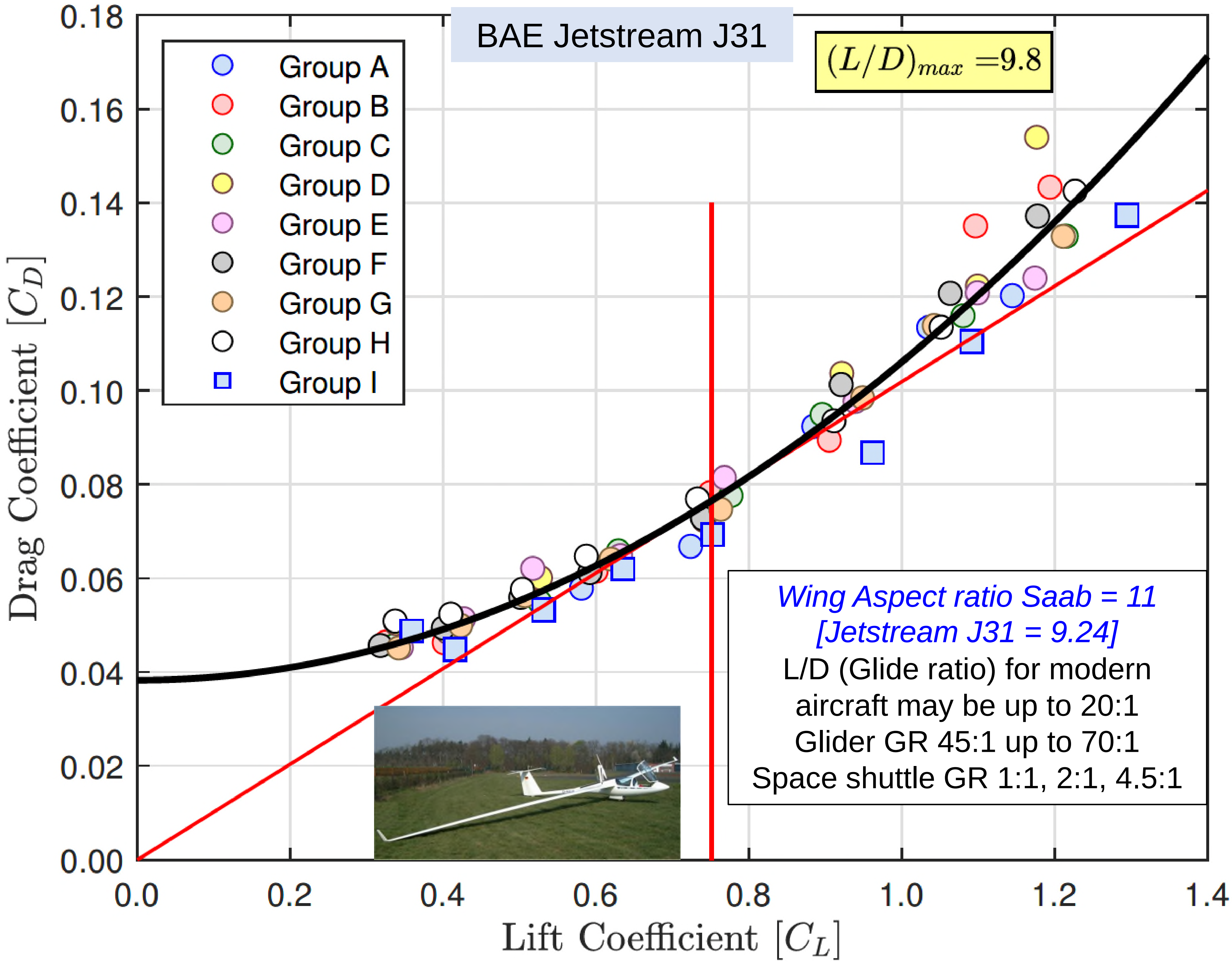
Approximation as K includes drag from:

- Wing
- Tail surfaces
- Fuselage
- Engine nacelles

these vary at different rates with Angle of Attack



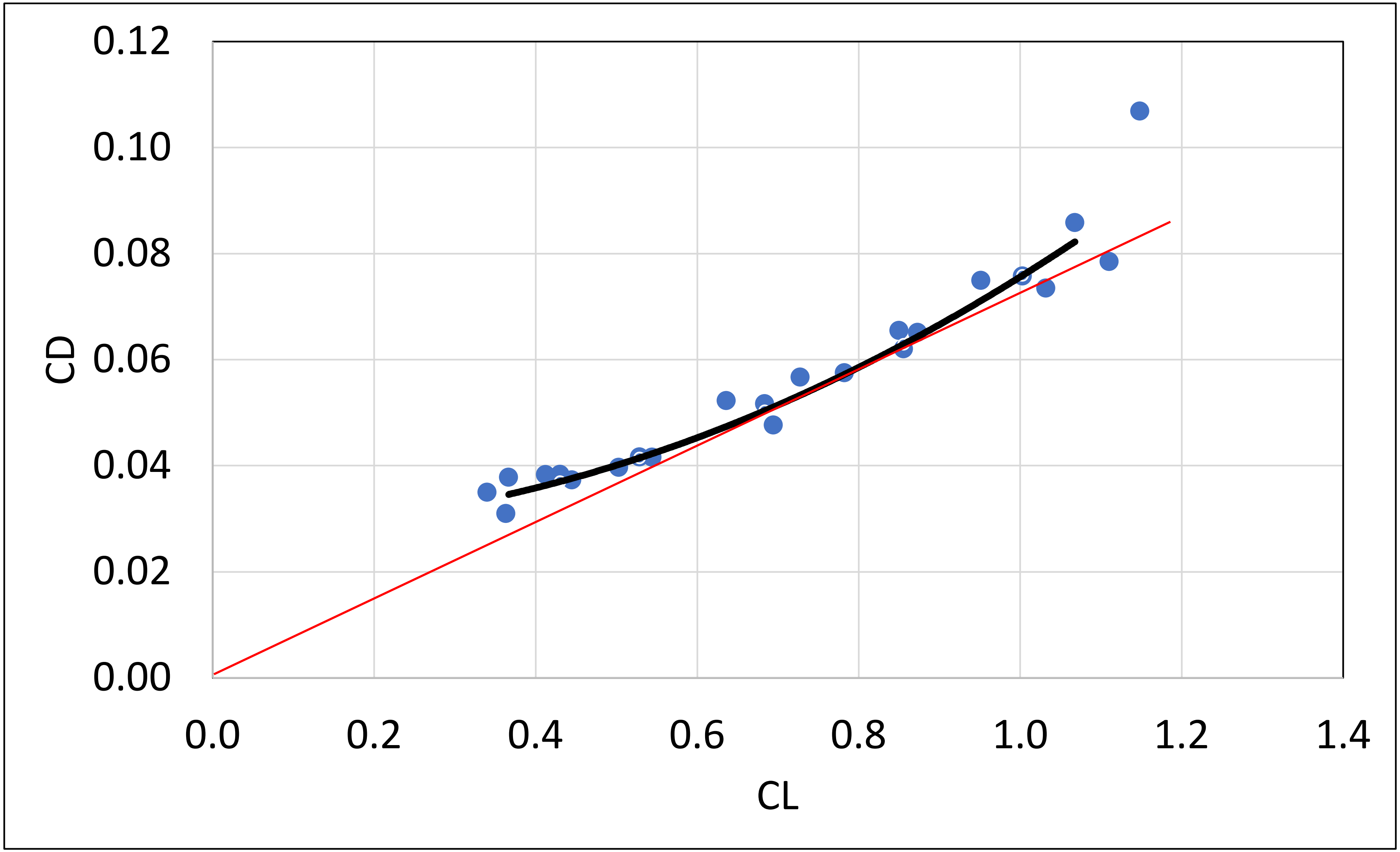
BAE Jetstream J31
CD vs CL2
y = 0.0347x + 0.0247
CD
CL2
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.5
1
1.5
2
Drag Coefficient [C_D]
0.08
0.06
0.04
0.02
0.00
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
Lift Coefficient Function [C_L^2]
$C_D = 0.0378 + 0.0682 C_L^2$
Group A
Group B
Group C
Group D
Group E
Group F
Group G
Group H
Group I
Group J
Group K
Group L
Group M
Group N
Group O
Group P



BAE Jetstream J31
$(L/D)_{max} = 9.8$
Group A
Group B
Group C
Group D
Group E
Group F
Group G
Group H
Group I
Wing Aspect ratio Saab = 11
[Jetstream J31 = 9.24]
L/D (Glide ratio) for modern
aircraft may be up to 20:1
Glider GR 45:1 up to 70:1
Space shuttle GR 1:1, 2:1, 4.5:1
Drag Coefficient $[C_D]$
Lift Coefficient $[C_L]$
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Estimating L/D ratio – CL vs CD

Typical Saab 340B data



From graph
Lift / Drag ratio 13.7

Aspect ratio, Fineness ratio and Glide ratio (L/D)

AR = aspect ratio; GD = glide ratio; FR = Fineness ratio

- Gimli Glider (Boeing 767) had a reported glide ratio of 12:1 in an emergency situation. A 767-200 has an aspect ratio of 8~, along with a glide ratio of (according to Wikipedia) 12:1.

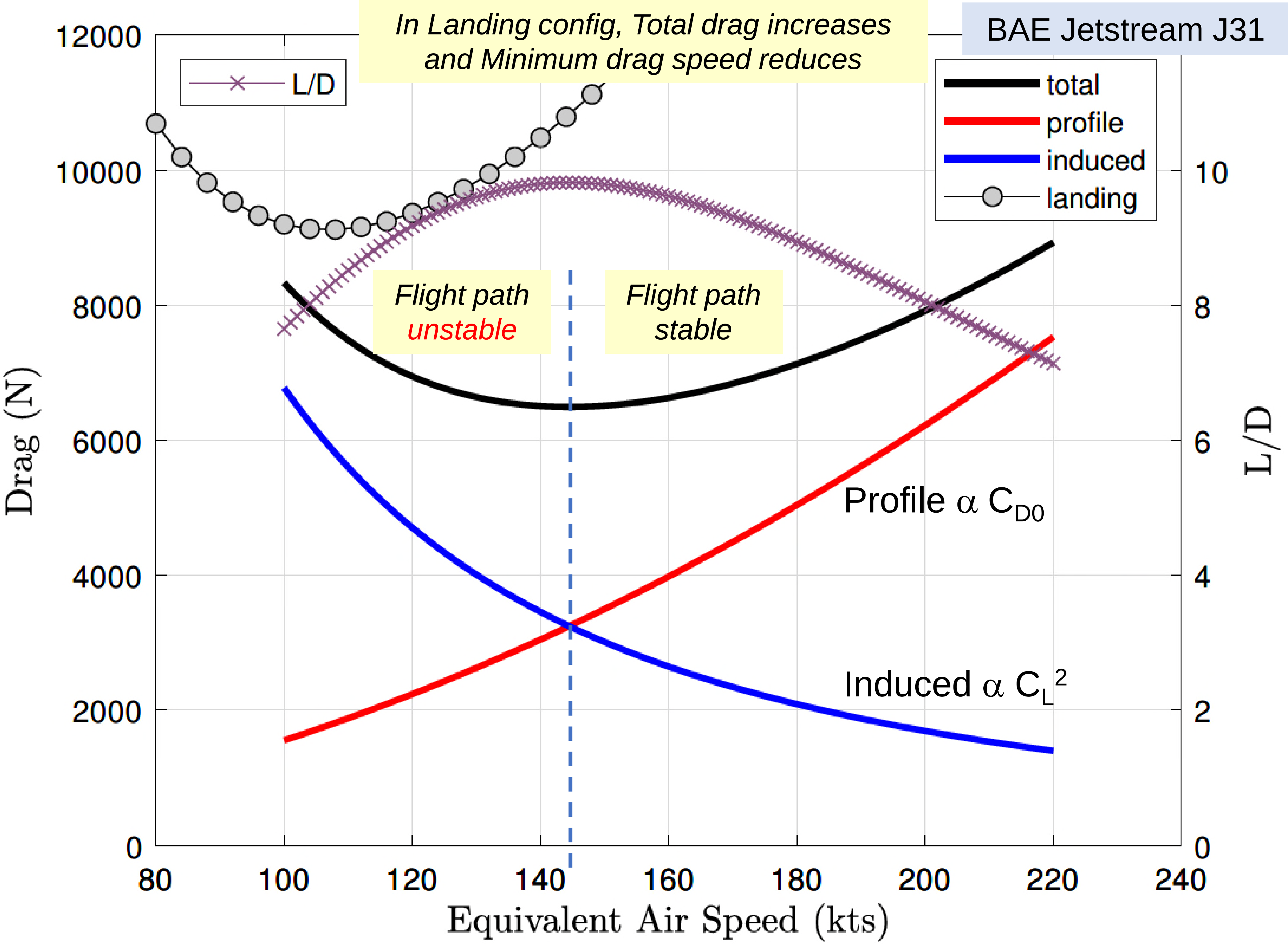
- An Airbus A340-300 has an aspect ratio of 10~, and a 737-NG with winglets has an aspect ratio also of 10~. Boeing 787 has an aspect ratio of 11; with GR of 21

Aircraft	Aspect ratio	Glide ratio
LS4	21.7	40
Ventus 1	23.7	44
Discus	21.3	42
Nimbus 3	35.9	58
ASH25	39.8	60
ETA	51	72

Fineness ratio (FR) Length / width

Jetstream J31 ~ 14 / 2 = 7 Saab 340B FR ~ 19 / 2.3 = 8.3

https://aviation.stackexchange.com/users/988/fboja



In Landing config, Total drag increases
and Minimum drag speed reduces
BAE Jetstream J31
L/D
total
profile
induced
landing
Flight path
unstable
Flight path
stable
Profile α C_{D0}
Induced α C_L^2
Drag (N)
L/D
Equivalent Air Speed (kts)
12000
10000
8000
6000
4000
2000
0
80
100
120
140
160
180
200
220
240
10
8
6
4
2
0

Cruise Performance

A pilot is interested in either:

1. how much longer will the fuel last (endurance)
2. the distance the current quantity of fuel will take the aircraft (range)

ENDURANCE: a function of fuel flow - kg/hr

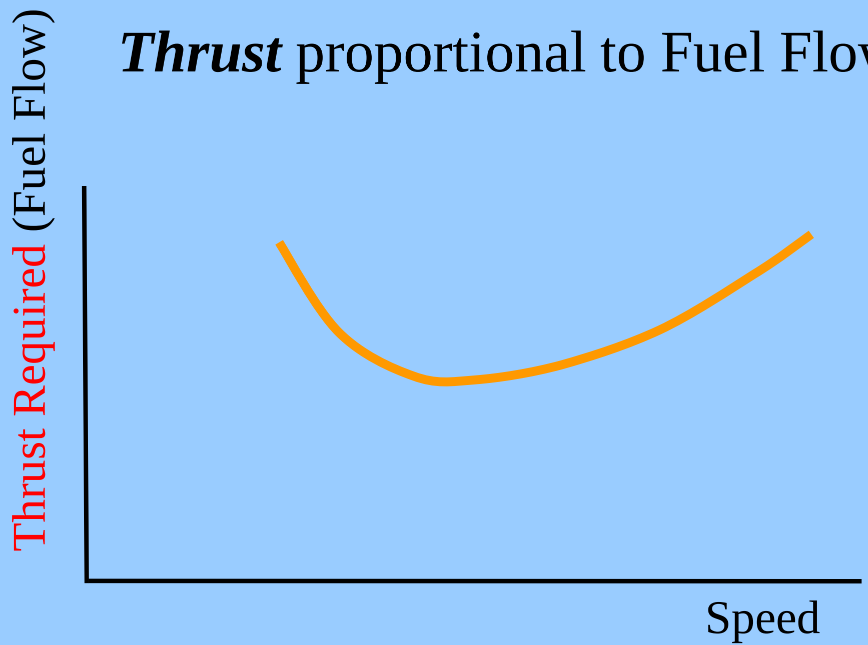
RANGE: a function of distance per unit quantity of fuel - nm/kg

$$\textbf{S}\text{pecific }\textbf{A}\text{ir }\textbf{R}\text{ange} = \frac{\text{True airspeed (knots = nm/hr)}}{\text{Fuel Flow (kg/hr)}}$$

Aircraft Powerplant – Fuel Flow Characteristics

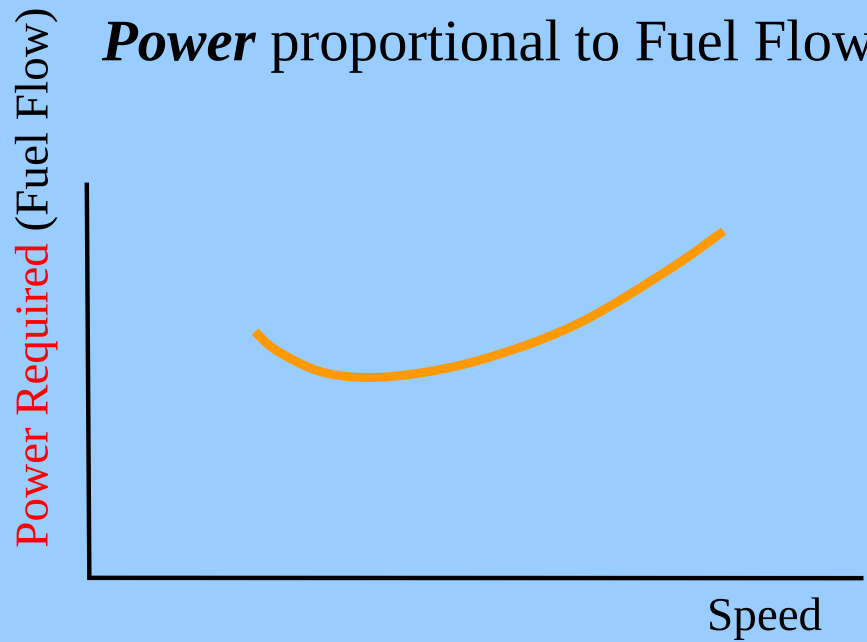
Jet engine produces thrust

Thrust proportional to Fuel Flow



Piston engine produces power

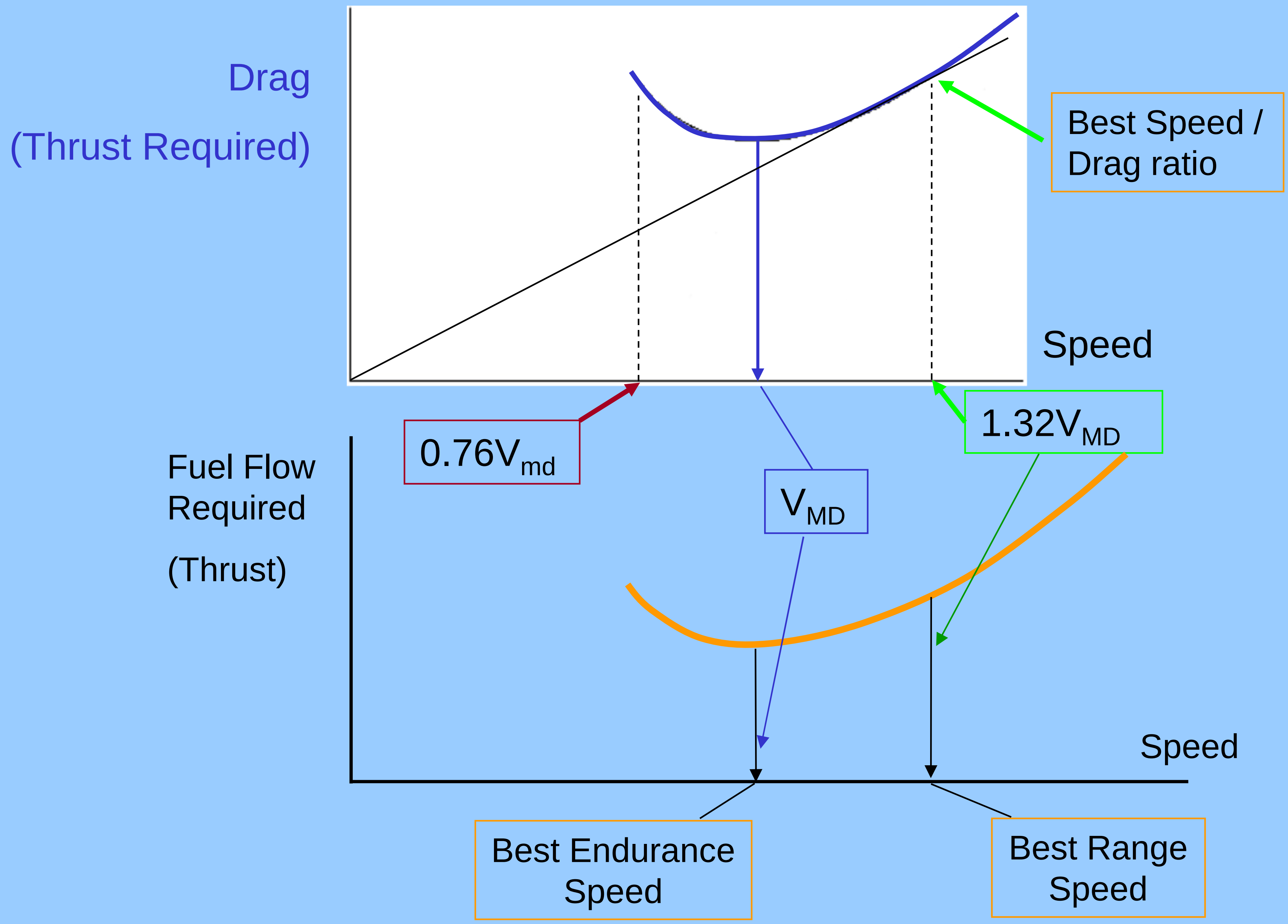
Power proportional to Fuel Flow



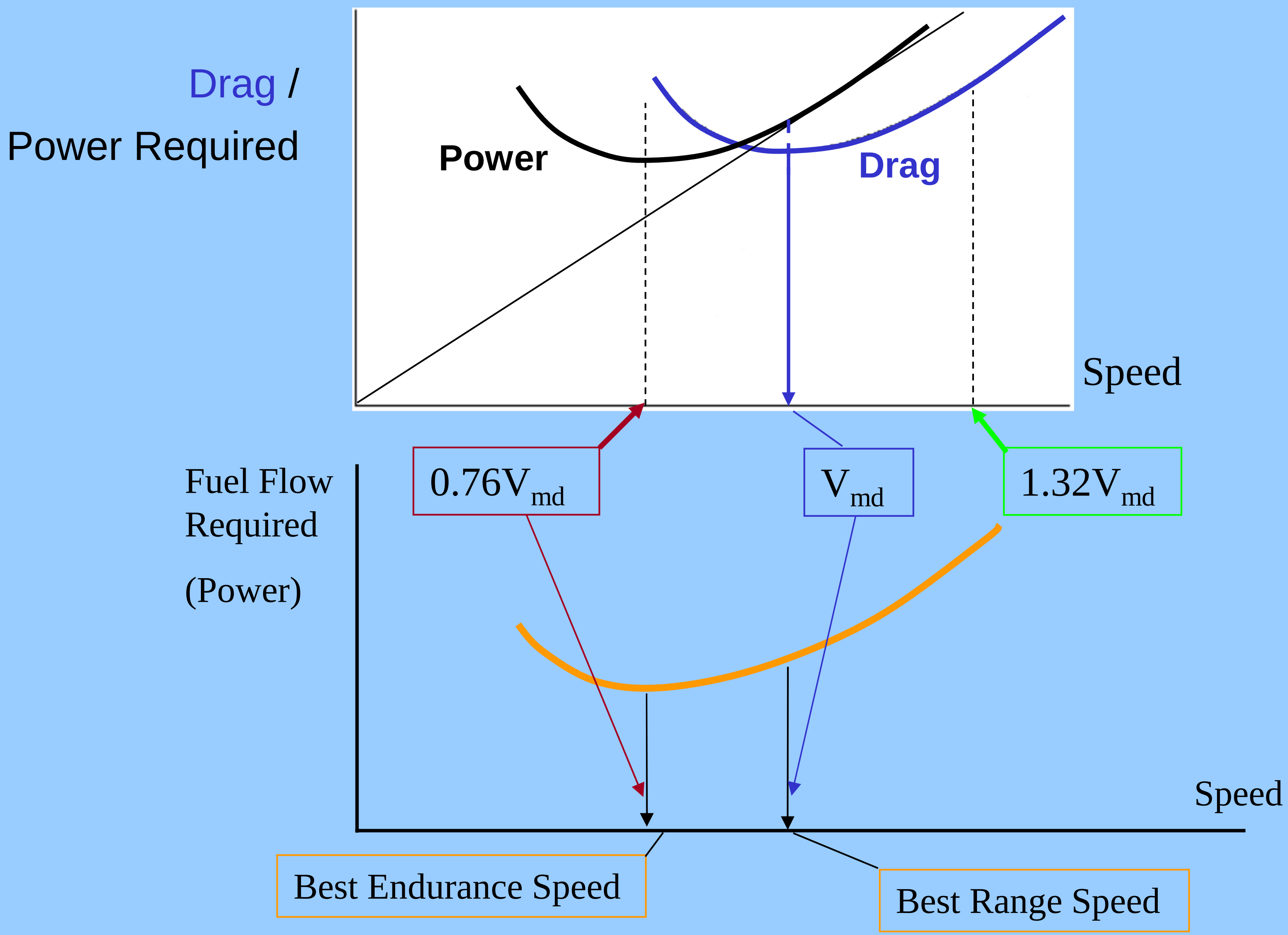
Tactical Short Landing US C-17 vs Euro A400M

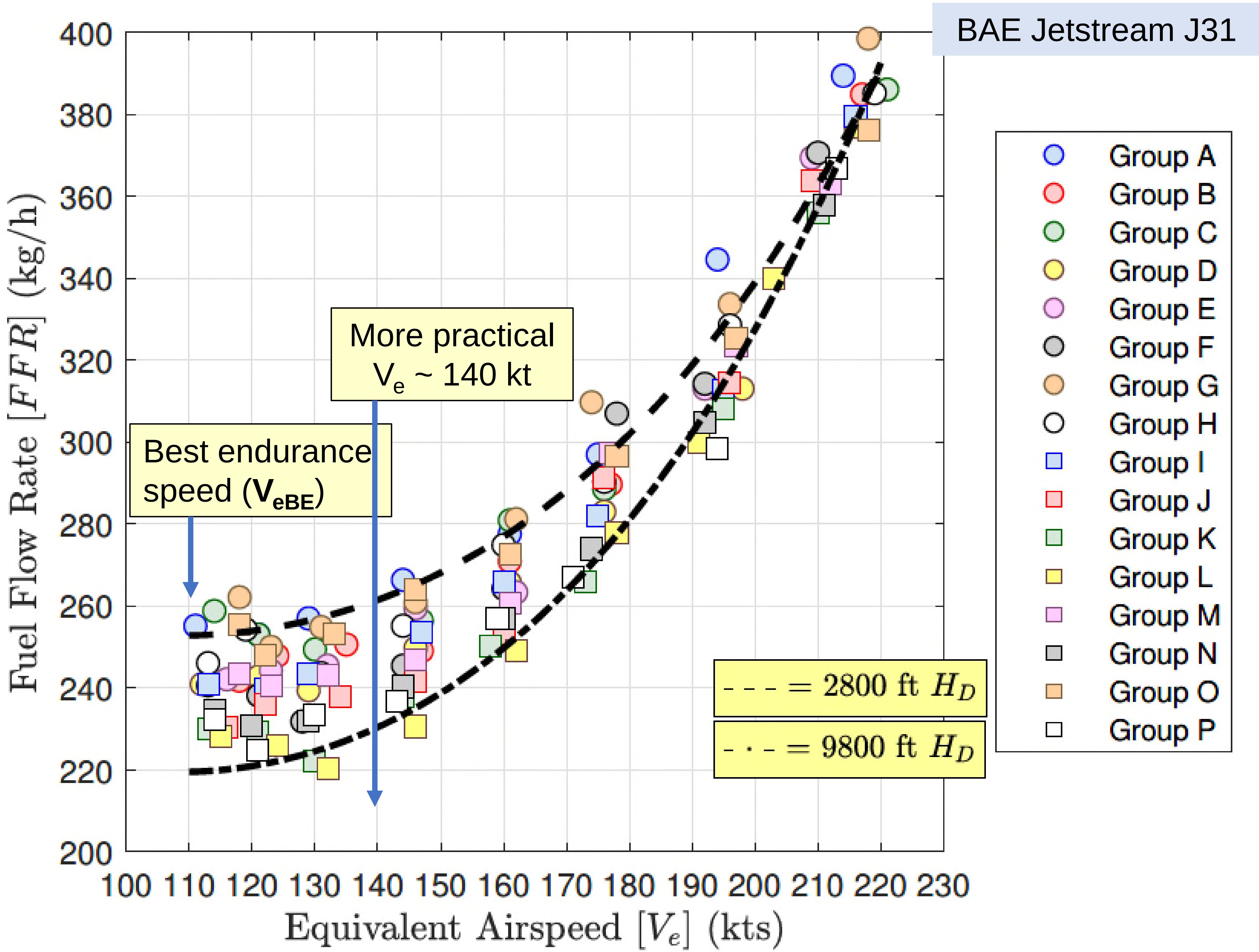
http://www.youtube.com/watch?v=HYwFlM3HMuQ

Jet Engine Characteristic

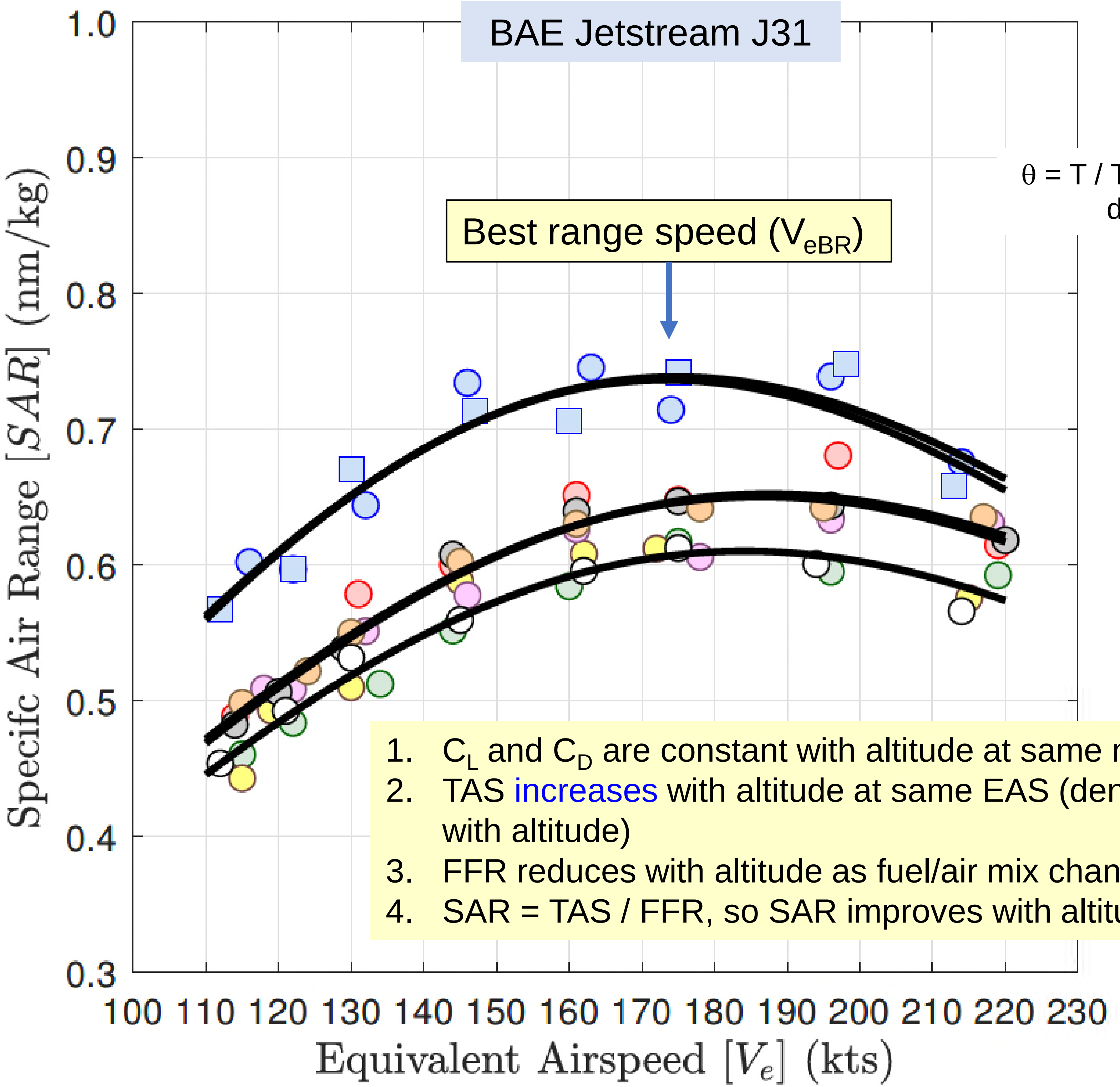


Piston Engine Characteristic





BAE Jetstream J31
Fuel Flow Rate [FFR] (kg/h)
400
380
360
340
320
300
280
260
240
220
200
100 110 120 130 140 150 160 170 180 190 200 210 220 230
Equivalent Airspeed [V_e] (kts)
More practical V_e ~ 140 kt
Best endurance speed ($\mathbf{V_{eBE}}$)
- - - = 2800 ft H_D
- · - = 9800 ft H_D
Group A
Group B
Group C
Group D
Group E
Group F
Group G
Group H
Group I
Group J
Group K
Group L
Group M
Group N
Group O
Group P

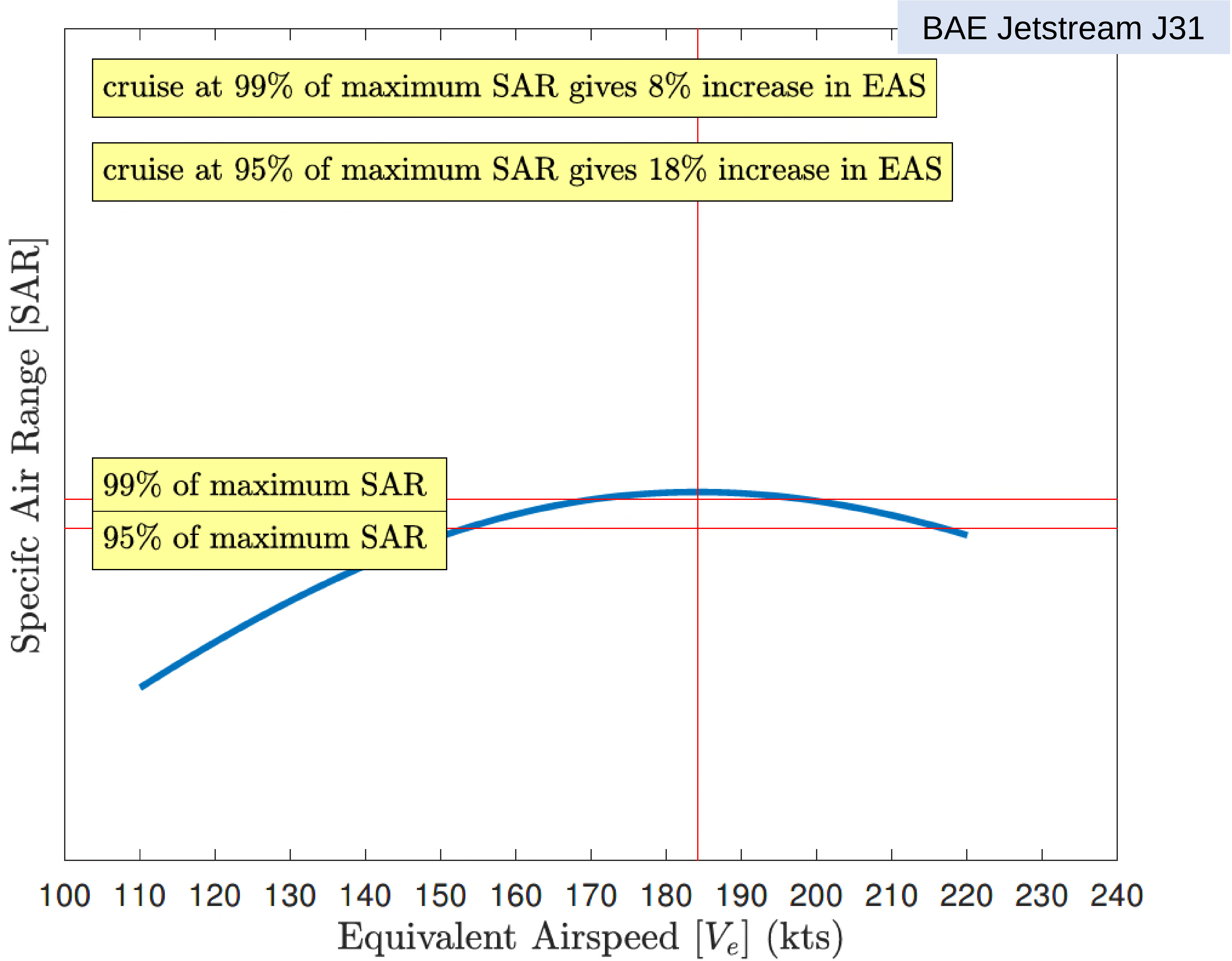


$$V \equiv V_t = V_e \sqrt{\frac{\theta}{\delta}}$$

$\theta = T / T_0$ and pressure ratio δ decreases with altitude

Using constant EAS, gives same C_L (lift) at all altitudes; so C_D (drag) is also same

1. C_L and C_D are constant with altitude at same mass and EAS
2. TAS increases with altitude at same EAS (density reduces with altitude)
3. FFR reduces with altitude as fuel/air mix changes
4. SAR = TAS / FFR, so SAR improves with altitude


BAE Jetstream J31
cruise at 99% of maximum SAR gives 8% increase in EAS
cruise at 95% of maximum SAR gives 18% increase in EAS
99% of maximum SAR
95% of maximum SAR
Specifc Air Range [SAR]
100 110 120 130 140 150 160 170 180 190 200 210 220 230 240
Equivalent Airspeed [V_e] (kts)

Saab 340B *Performance data summary*

From in-flight data

Measured best endurance speed ≈ Minimum power speed
Measured best range speed ≈ Best Speed /Drag ratio

0.76 V_{MD}

1.32 V_{MD}

V_{MP}	V_{MD}	$V_{(V/D)max}$	V_{BE}	Min FFR	V_{BR}	Max SAR	FFR @ V_{BR}
109	144	190	115	285	170	0.54	342
113 calc	148	196 calc	123 (data)	300	196	0.56	414

V_{MP} = Minimum power speed V_{MD} = Minimum drag speed	FFR = Fuel flow rate V_{BR} = Best range speed
$V_{(V/D)max}$ = Maximum speed to drag ratio V_{BE} = Best endurance speed	SAR = Specific air range H_D = Density altitude

Power-producing vs Thrust-producing engines

Piston-engined aircraft

- Power available approximately proportional to fuel flow
- V_{BE} is the minimum power speed ($0.76V_{MD}$)
- V_{BR} is the speed that maximises the speed to power ratio and that is minimum drag speed (V_{MD})

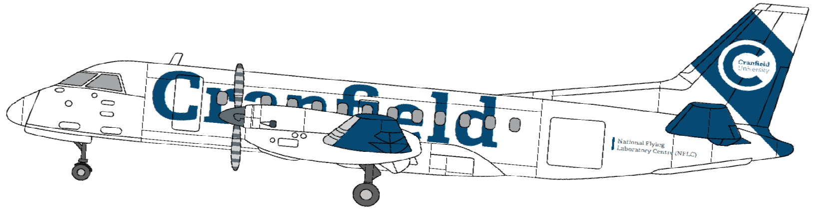
Jet-engined aircraft

- Thrust available approximately proportional to fuel flow
- V_{BE} is the minimum drag speed (V_{MD})
- V_{BR} is the speed that maximises the speed to drag ratio ($1.32V_{MD}$)

As the aircraft speed increases the overall thrust produced by the engines reduces. However the thrust from the "Jet pipe" reduces proportionately less than that of the propellers. Hence the <u>proportion of thrust</u> from the jet pipe <u>increases</u> as speed increases

C-17 vs A400M http://www.youtube.com/watch?v=HYwFlM3HMuQ

2 Longitudinal Static Stability



Definitions

- **Static stability** – tendency of an aircraft to revert back to equilibrium following disturbance from trim
- Dynamic stability – transient motion in the recovery to equilibrium after disturbance
- **Manoeuvre stability** – (Longitudinal) **manoeuvre** stability – relates to aircraft subject to a steady normal acceleration in response to a pitch control input
- ***Controls-fixed –*** elevator held at constant settings for trimmed condition
- ***Controls-free*** – elevator is allowed to float to position for trimmed position; so elevator tab is used.

[Cook, 2013]

Longitudinal Stability

Measurement of Longitudinal Static Stability

Object:

To assess the longitudinal static stability of the aircraft, to estimate the neutral points, controls fixed and controls free, and to suggest an aft limit for the CG range.

Measurement of Longitudinal Manoeuvre Stability

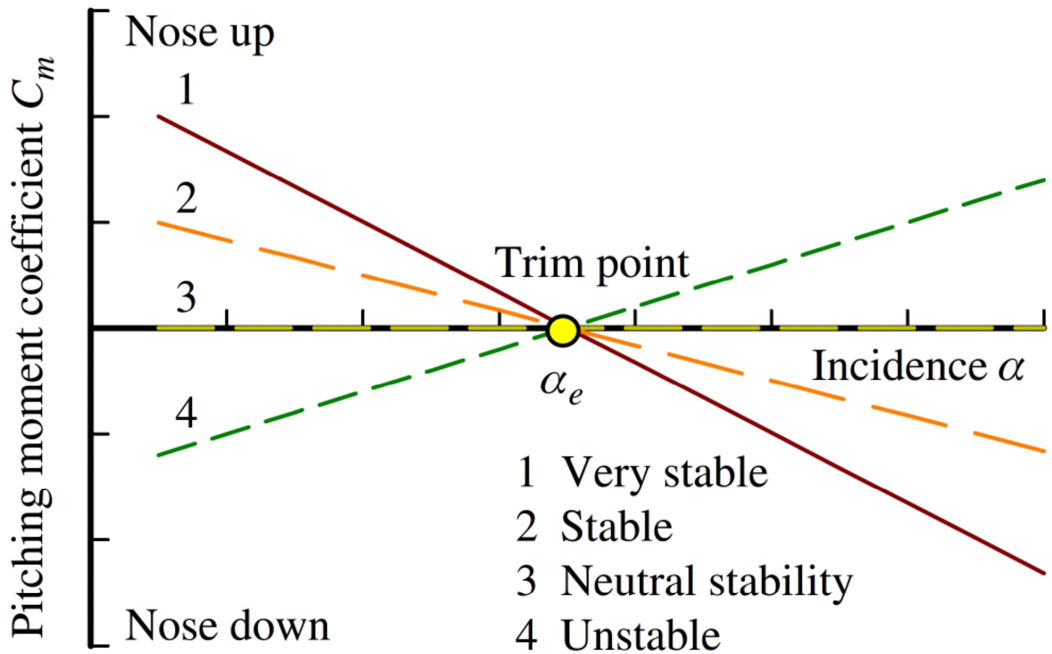
Object:

To determine the stick travel per g and stick force per g and to determine the manoeuvre points, controls fixed and controls free.

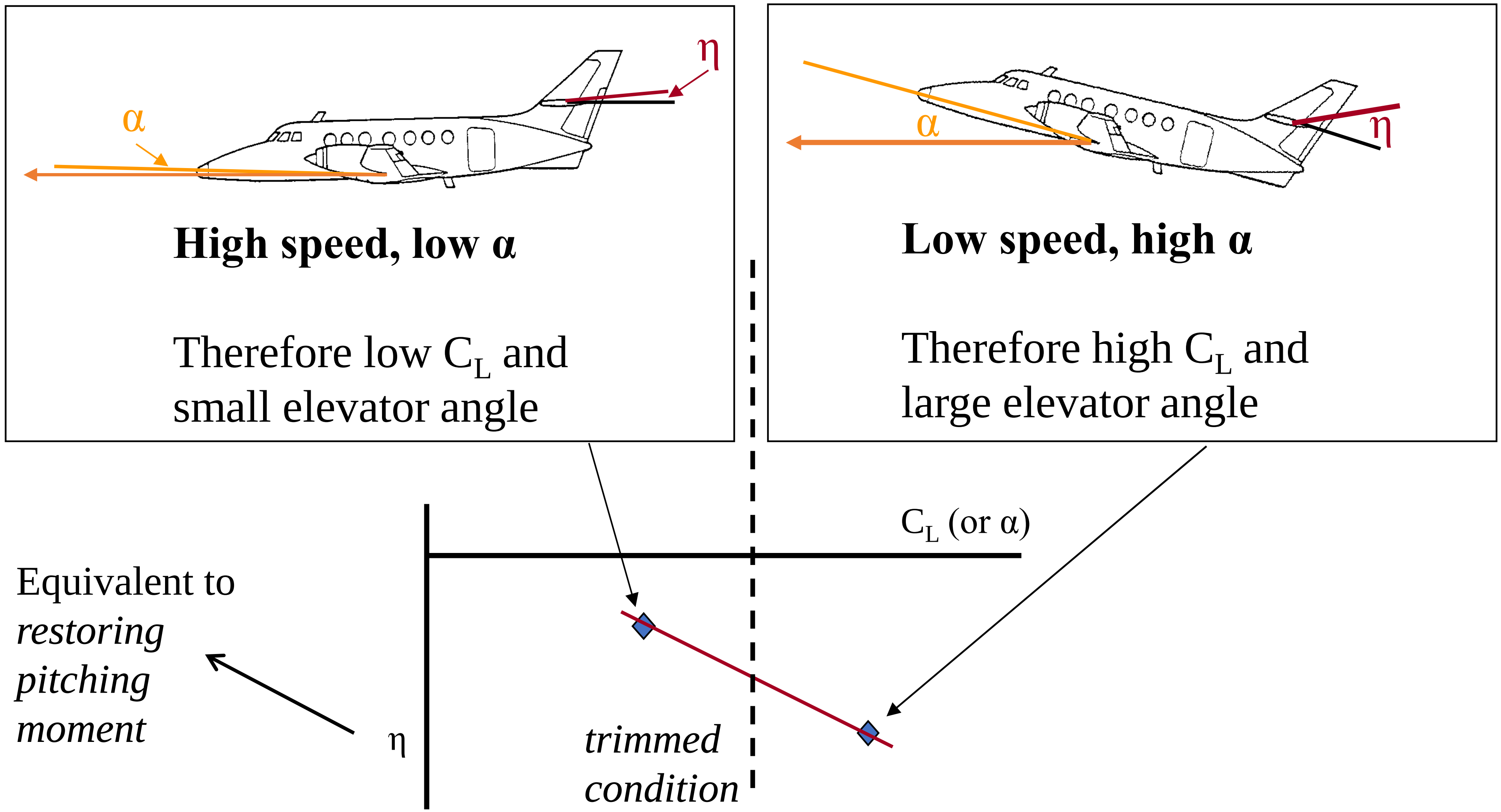
Introduction - definitions

- For a stable aircraft "… *a positive (nose-up) change of angle of attack should produce a negative (nose-down) change in pitching moment*" [Barnard & Philpott 2010]
- *Neutral point - a point about which the moment is independent of incidence, dM/dα ≡ 0 [Carley, 2020].*

" …acceptable margins of stability therefore determine the permitted range of CG position in a given aircraft"
[Cook, 2013]



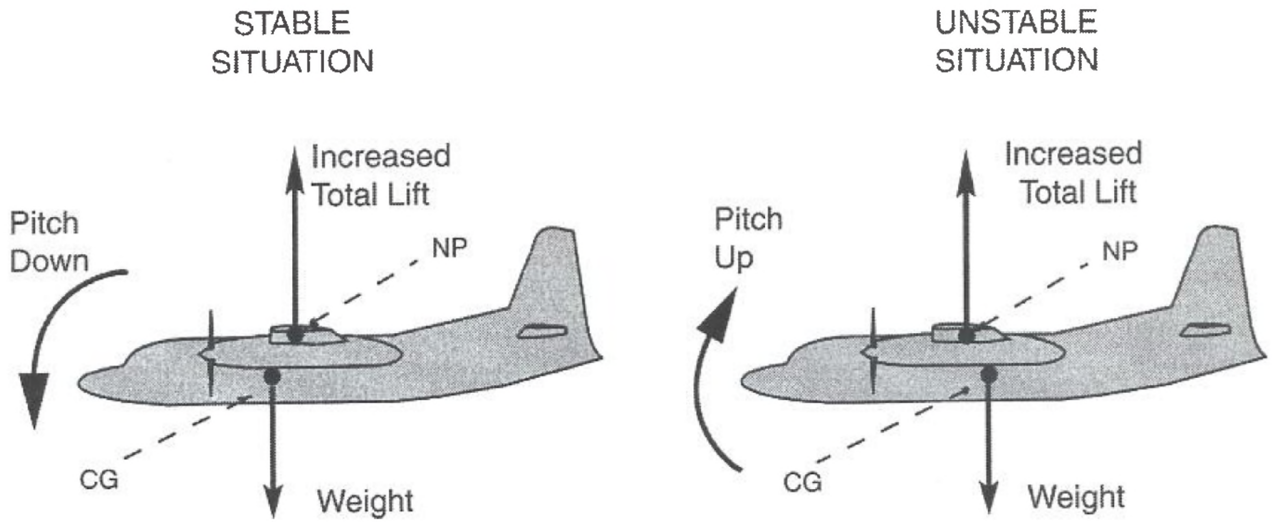
For free-flying aircraft, not possible to measure the pitching moment ΔM



Pitching moment ΔM is balanced by change in elevator angle η **about trimmed α**

Guernsey, 1999 - Freight aircraft

- Incorrect loading meant that CG was outside limits
- Flight conducted with marginal stability
- On approach, selection of Flap 40 led to pitch up and stall



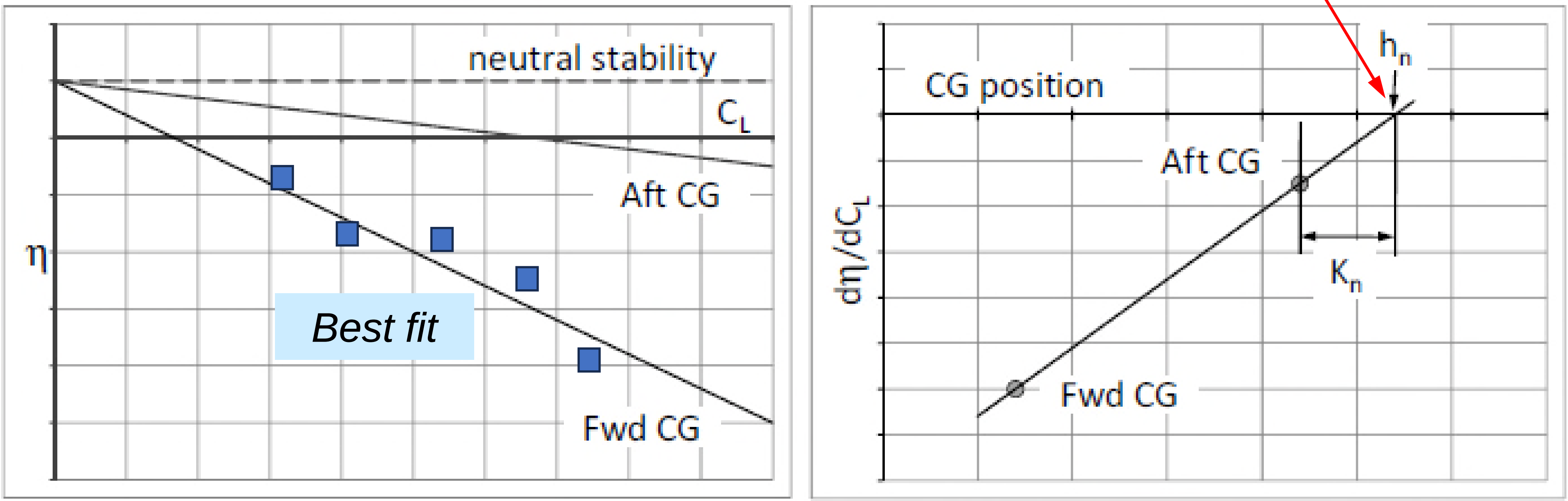
Accident report for G-CHNL - flap deployment moved the neutral point, making a marginally stable aircraft very unstable.

AAIB, Aircraft Accident report 2/2000, *Report on the accident to Fokker F27-600 Friendship, G-CHNL Near Guernsey Airport*, Channel islands on 12 Jan 1999 (2000)

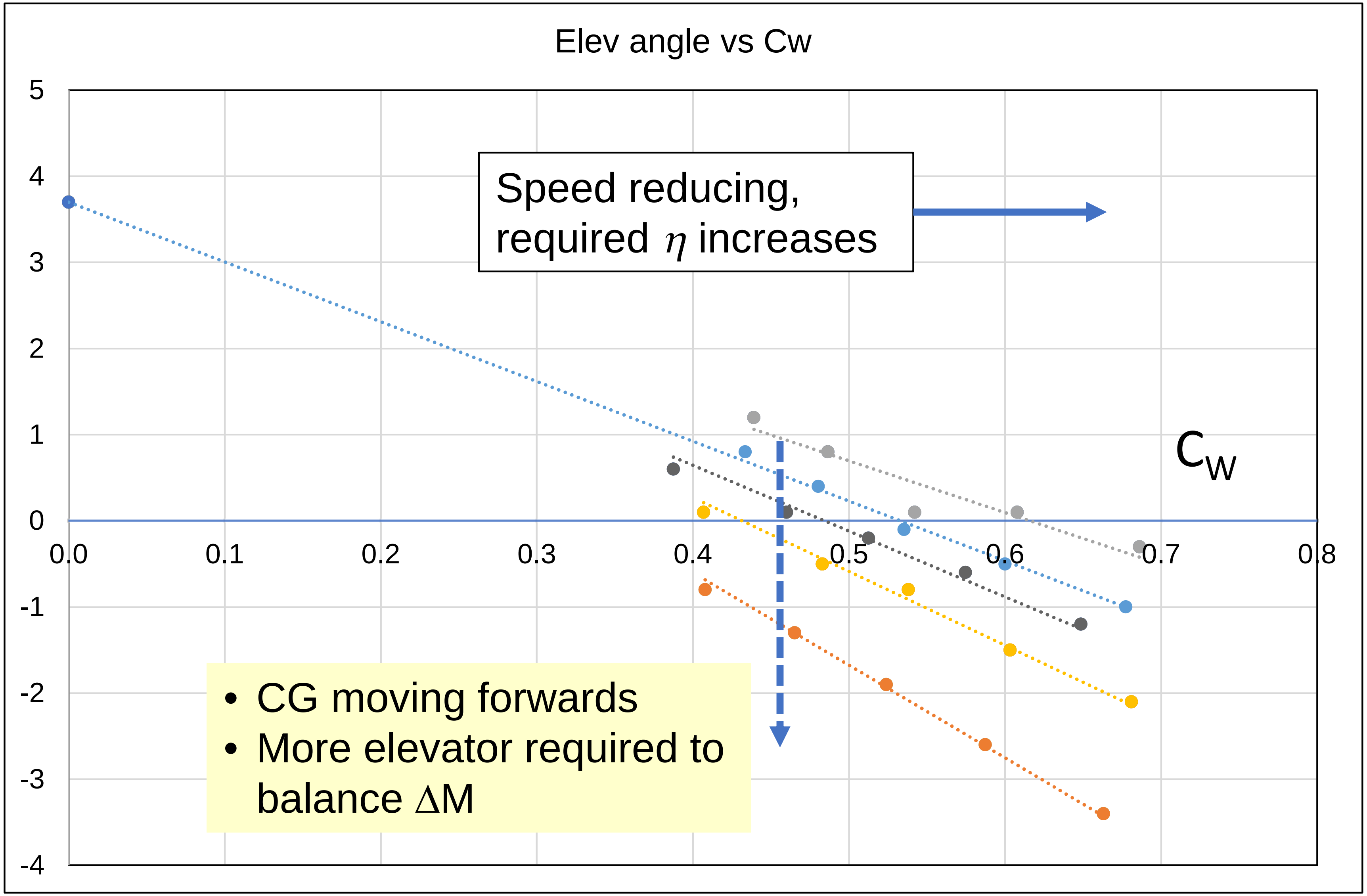
Determining the Neutral Point (Stick fixed)

- *Fly aircraft at a range of C_L (or α)*
- *Measure **elevator angle** to balance ΔM*
- *Plot slope for a range of CG*

Stick Fixed Neutral Point



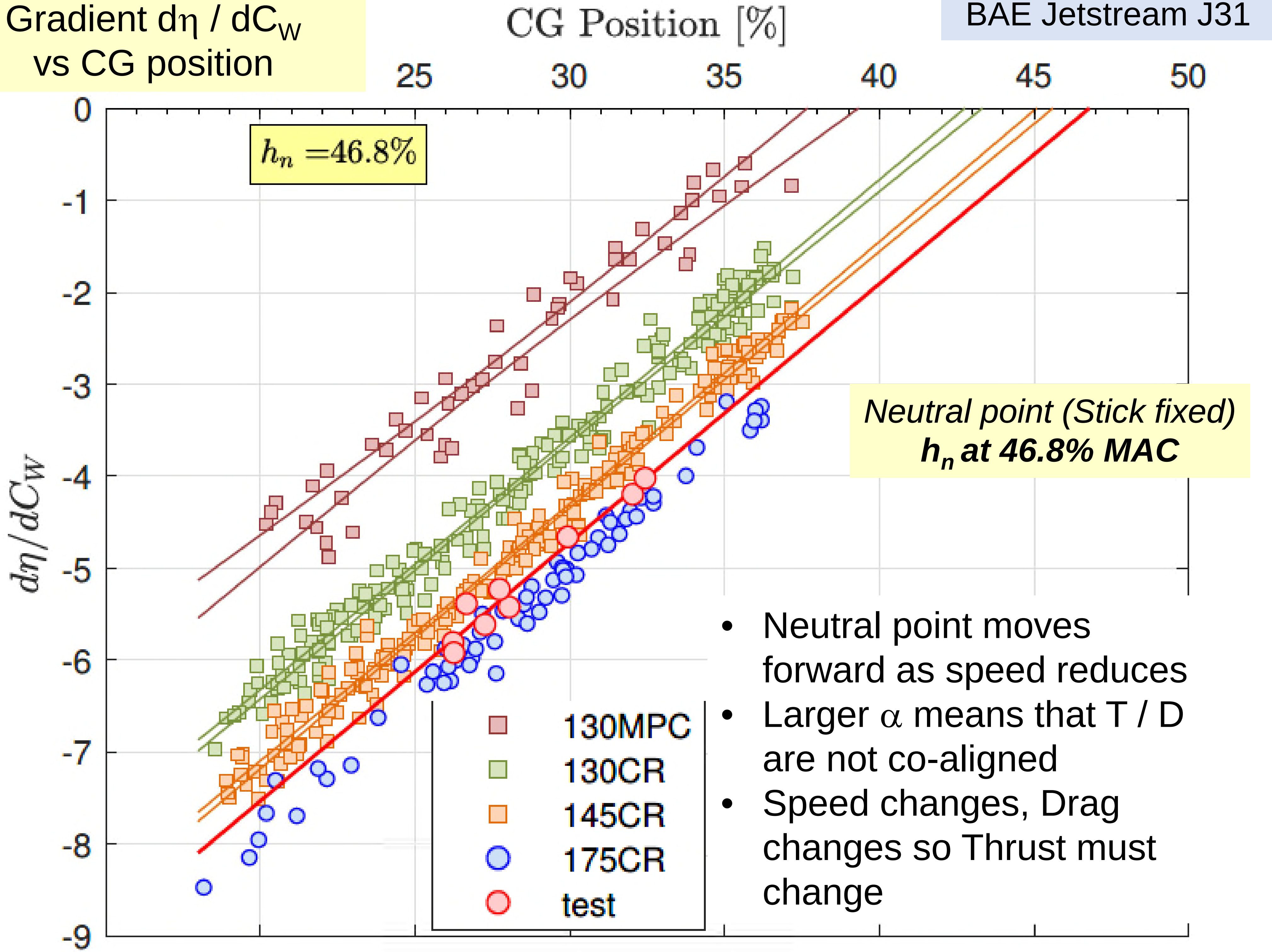
Determining the Neutral Point (Stick fixed)
Typical Saab 340B data



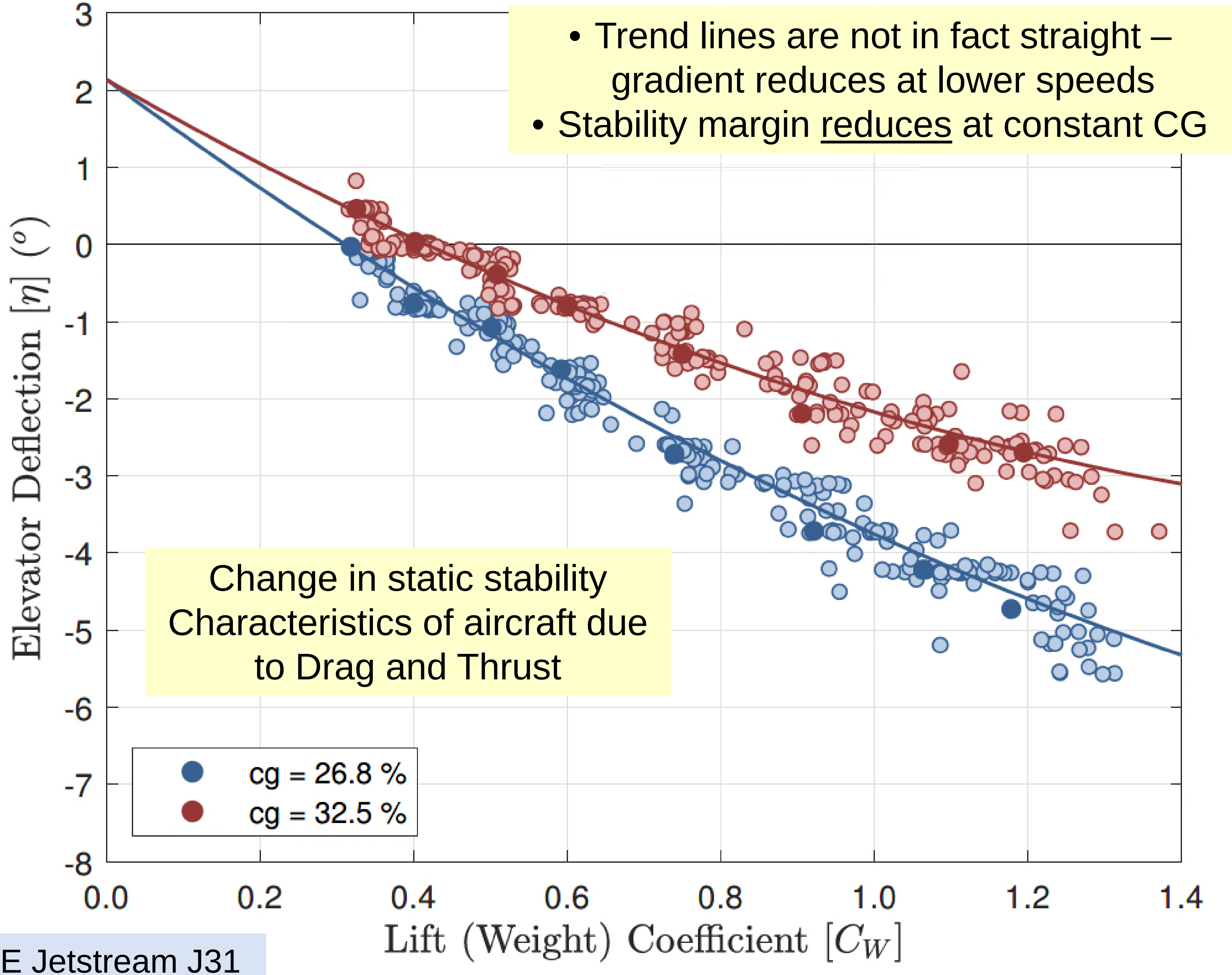
Estimated Stick fixed neutral point h_n = 48%

Gradient dη / dC_W vs CG position

BAE Jetstream J31



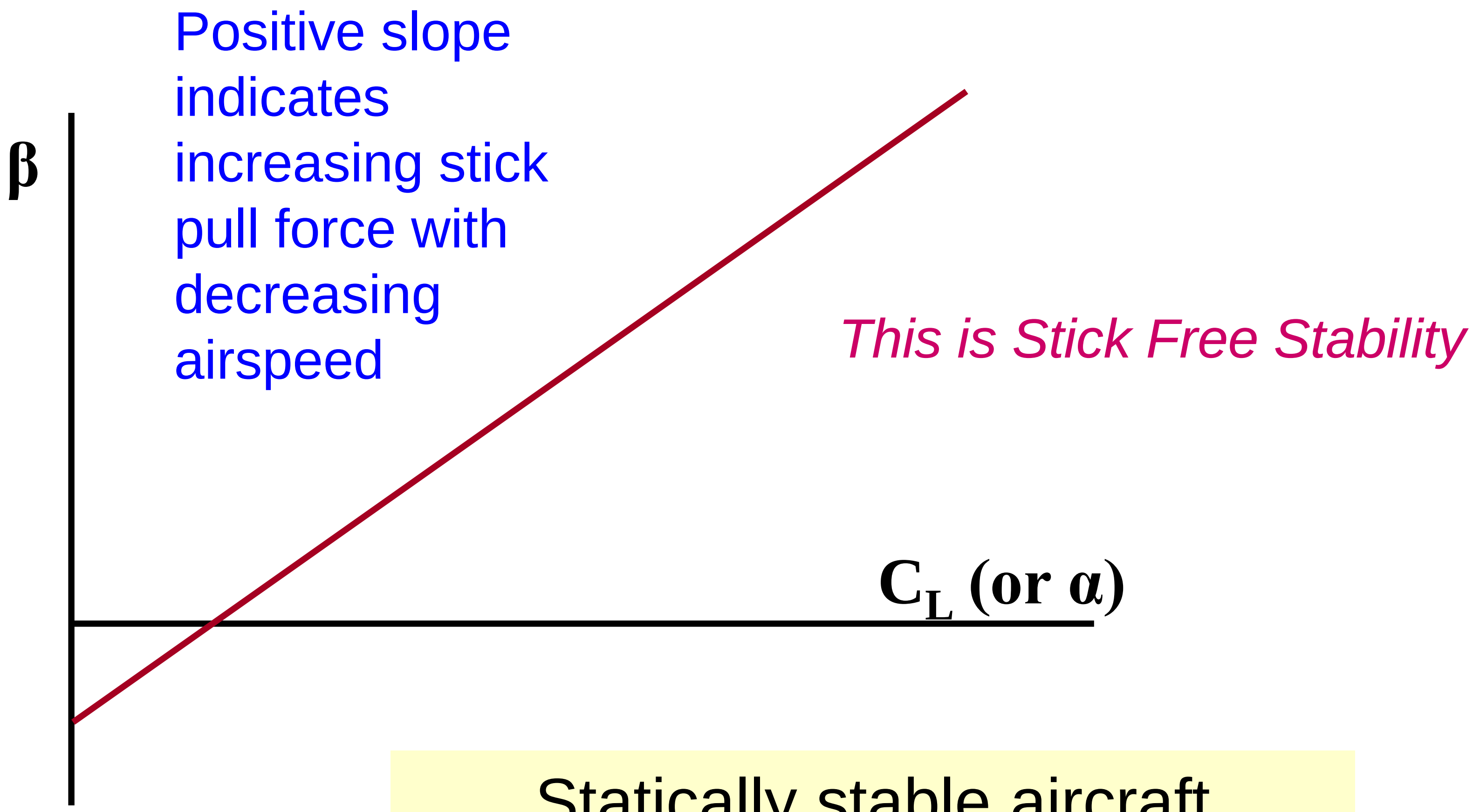
- Neutral point moves forward as speed reduces
- Larger α means that T / D are not co-aligned
- Speed changes, Drag changes so Thrust must change



BAE Jetstream J31

Longitudinal Static Stability (Stick-free)

Using the trim tab for the measurements yields:

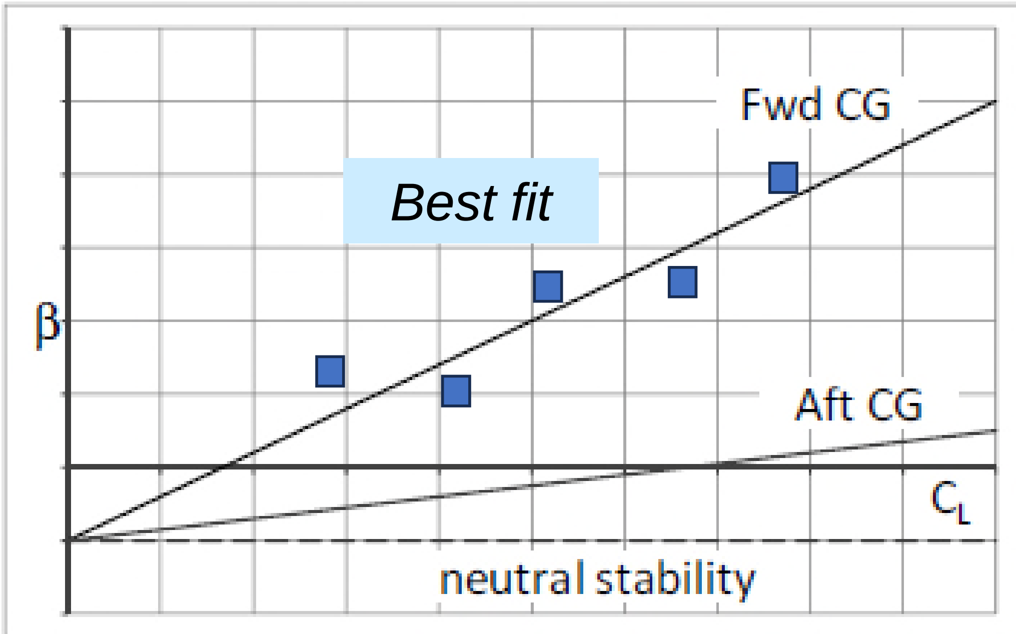


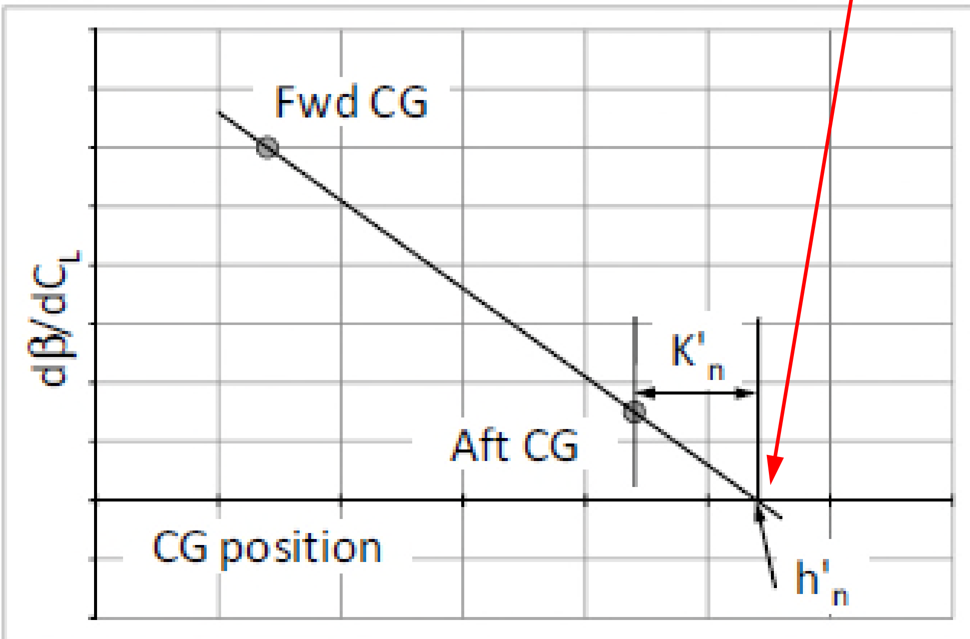
Statically stable aircraft indicated by positive slope

Determining the Neutral Point (Stick Free)

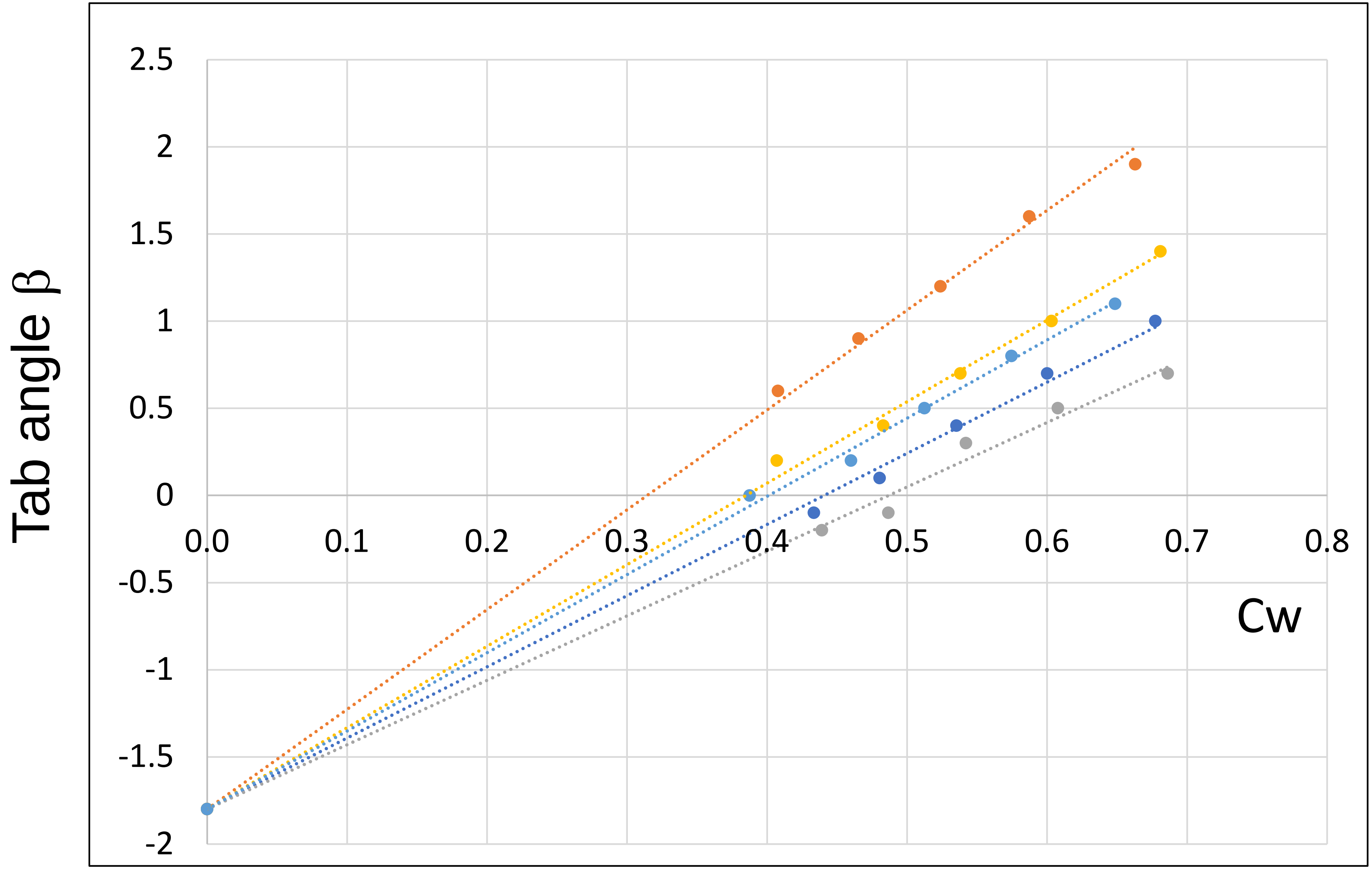
- *Fly aircraft at a range of C_L (or α)*
- *Measure **tab angle** to balance hinge moment from elevator for each ΔM*
- *Plot slope for a range of CG*

Stick Free Neutral Point

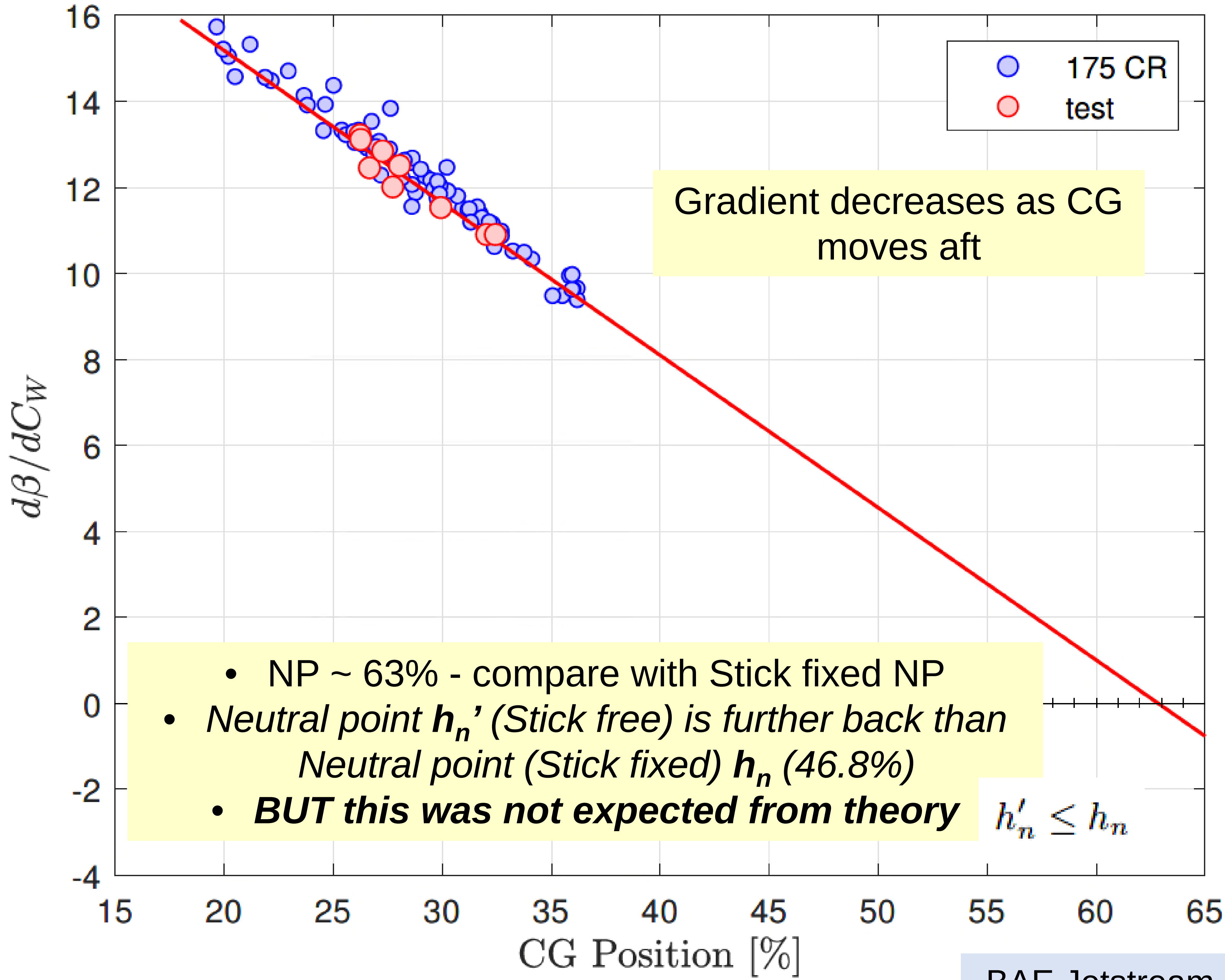




Determining the Neutral Point (Stick free)
Typical Saab 340B data



Estimated Stick free neutral point h'_n = 57%



Gradient decreases as CG moves aft

- NP ~ 63% - compare with Stick fixed NP
- *Neutral point* ***h_n'*** *(Stick free) is further back than Neutral point (Stick fixed)* ***h_n*** *(46.8%)*
- ***BUT this was not expected from theory***

$$h'_n \leq h_n$$

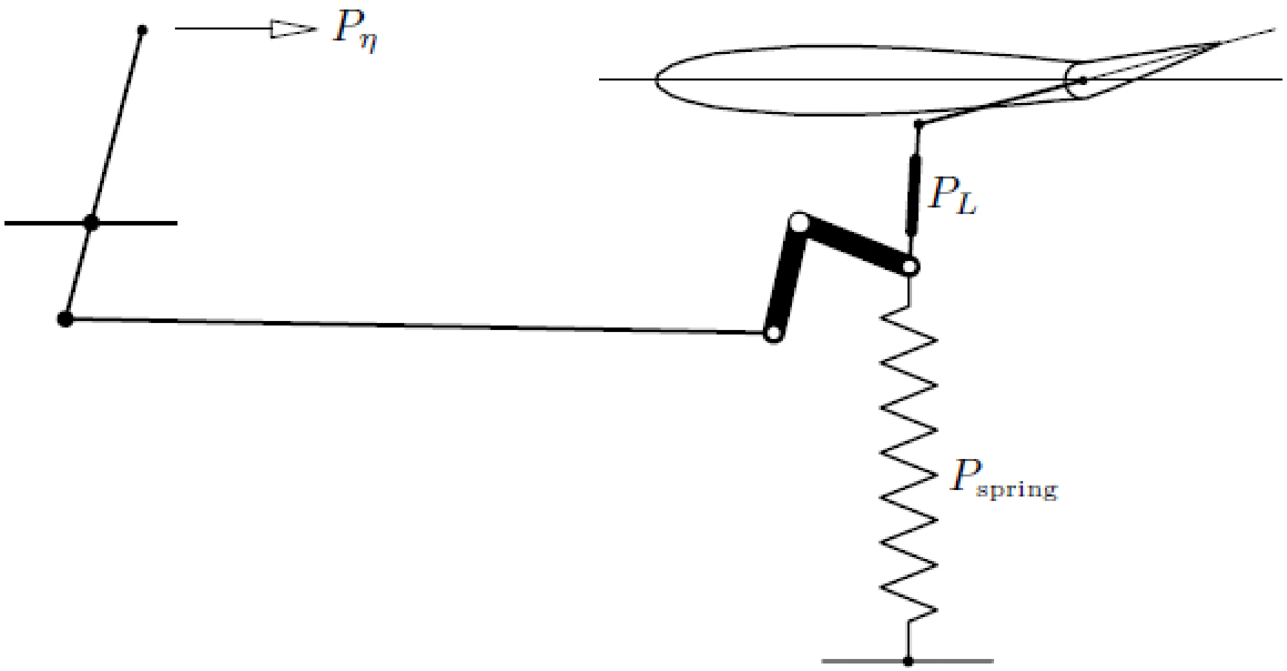
BAE Jetstream J31

Neutral point(s)

- Usually the two neutral points will occur at different CG locations
- This arises from the aerodynamic balancing of the elevator control and the lost ‘force’ due to friction in the control circuit.
- Thus typically the neutral point for Stick free is further forward than that for Stick fixed:

$$h'_n \leq h_n$$

Elevator Control Circuit – Example

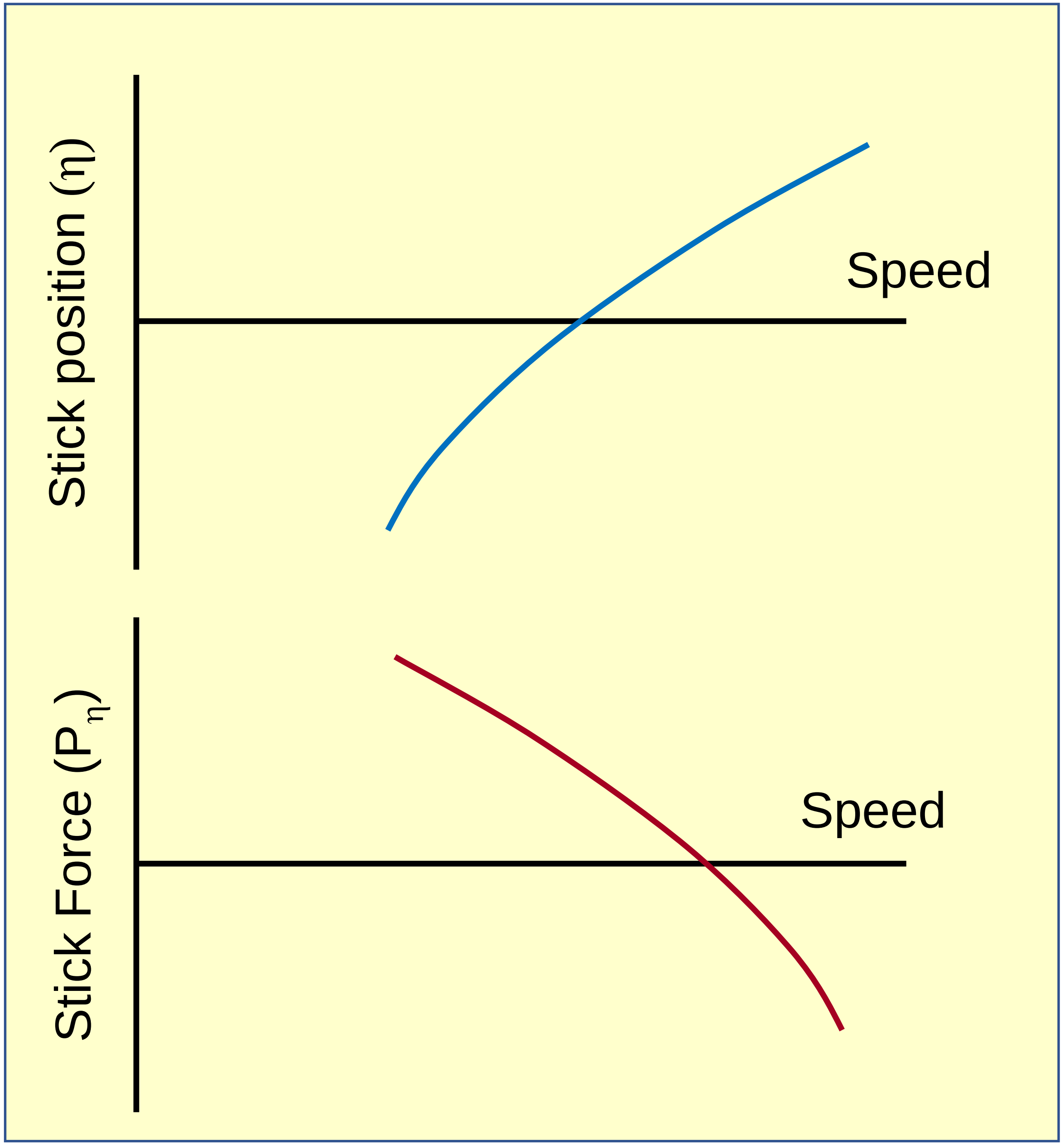


- *Variation of tab angle β with C_L is equivalent to variation of Stick force with speed*
- CS25 and CS23 give requirement - see **CS 25.173(c)**:

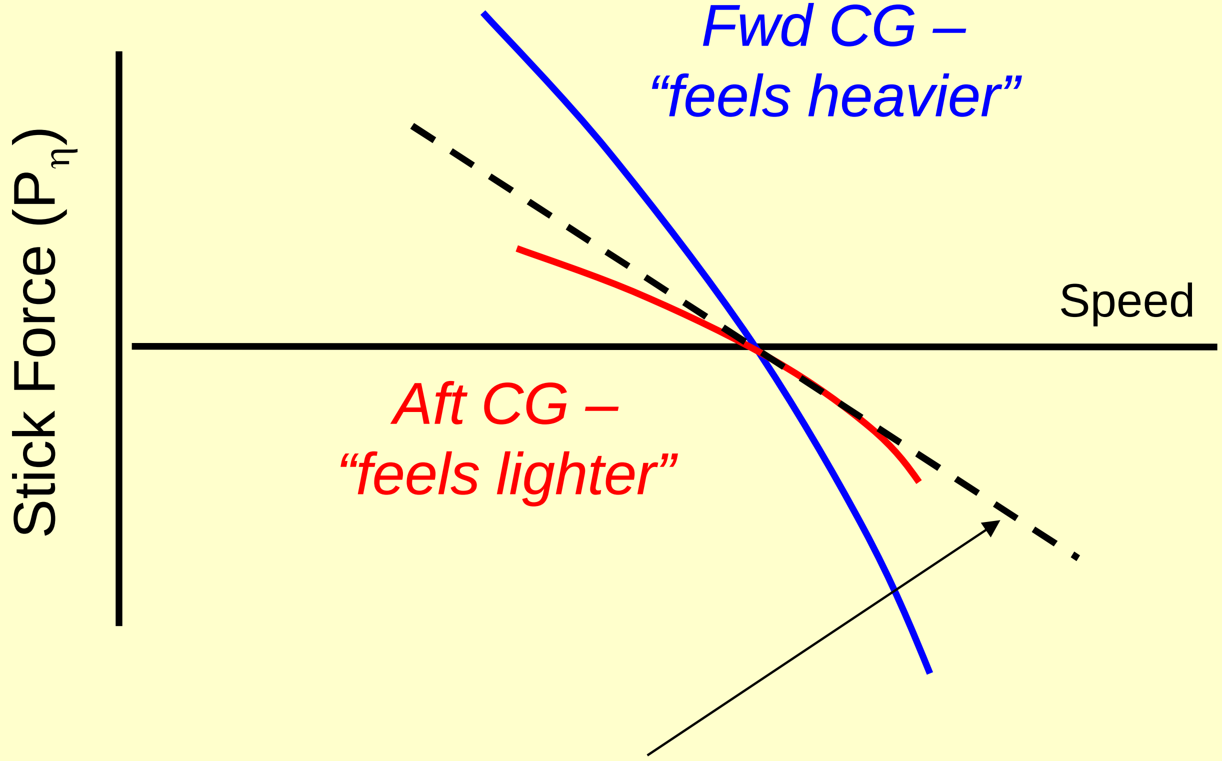
(c) The average gradient of the stable slope of the stick force versus speed curve may not be less than 1 pound for each 6 kts, [7.4 N/10 kts]..."

Pilot interpretation of Static Stability:
Control inputs produce a change of speed

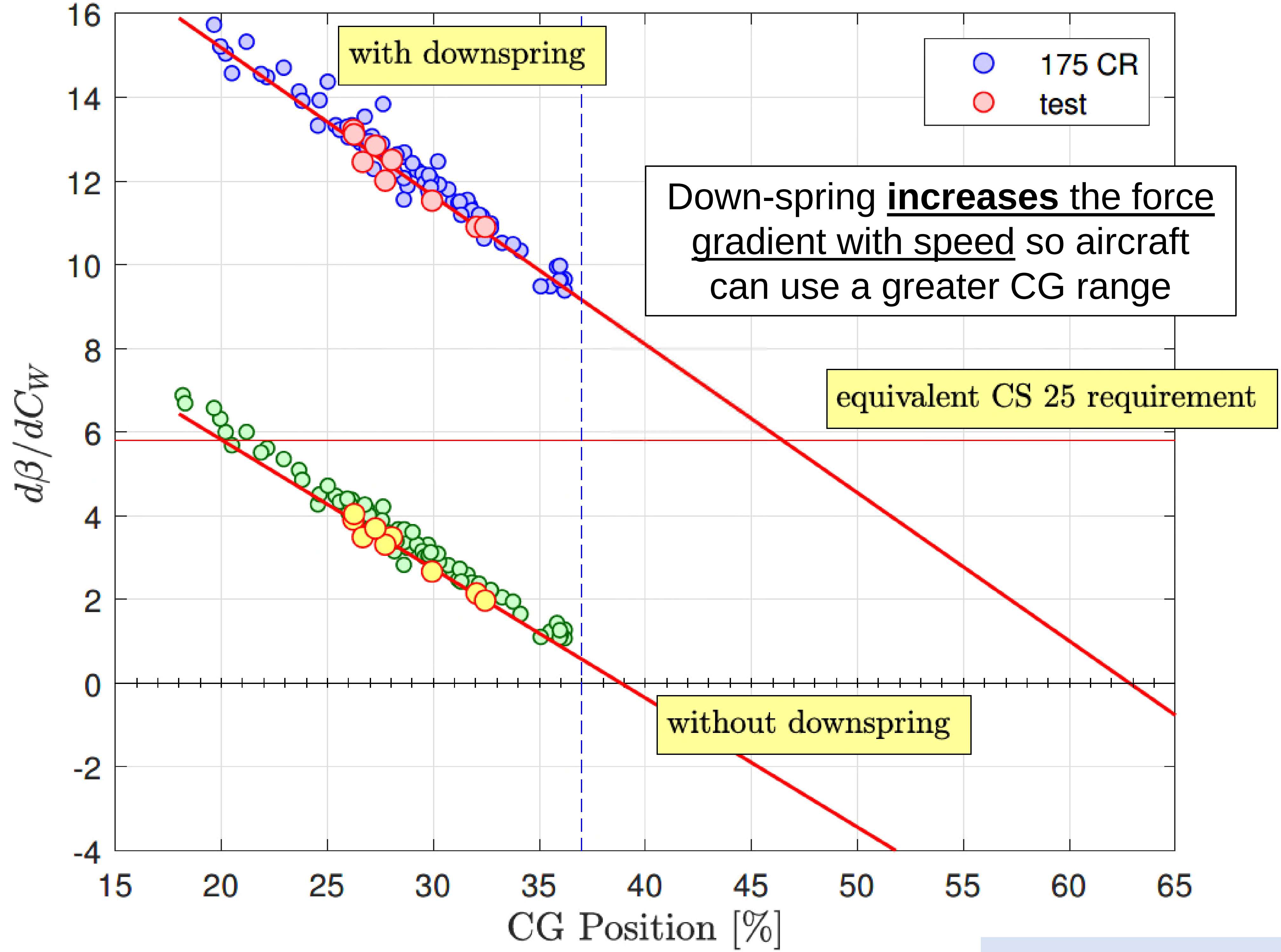
Forces are in natural sense:
Push to increase speed
Pull to decrease speed



Pilot interpretation of Static Stability: ***Control inputs produce a change of speed***



CS-25 (JAR) requirements:
Average slope shall not be less than 1lb/6 knots (7.4N/10 knots)



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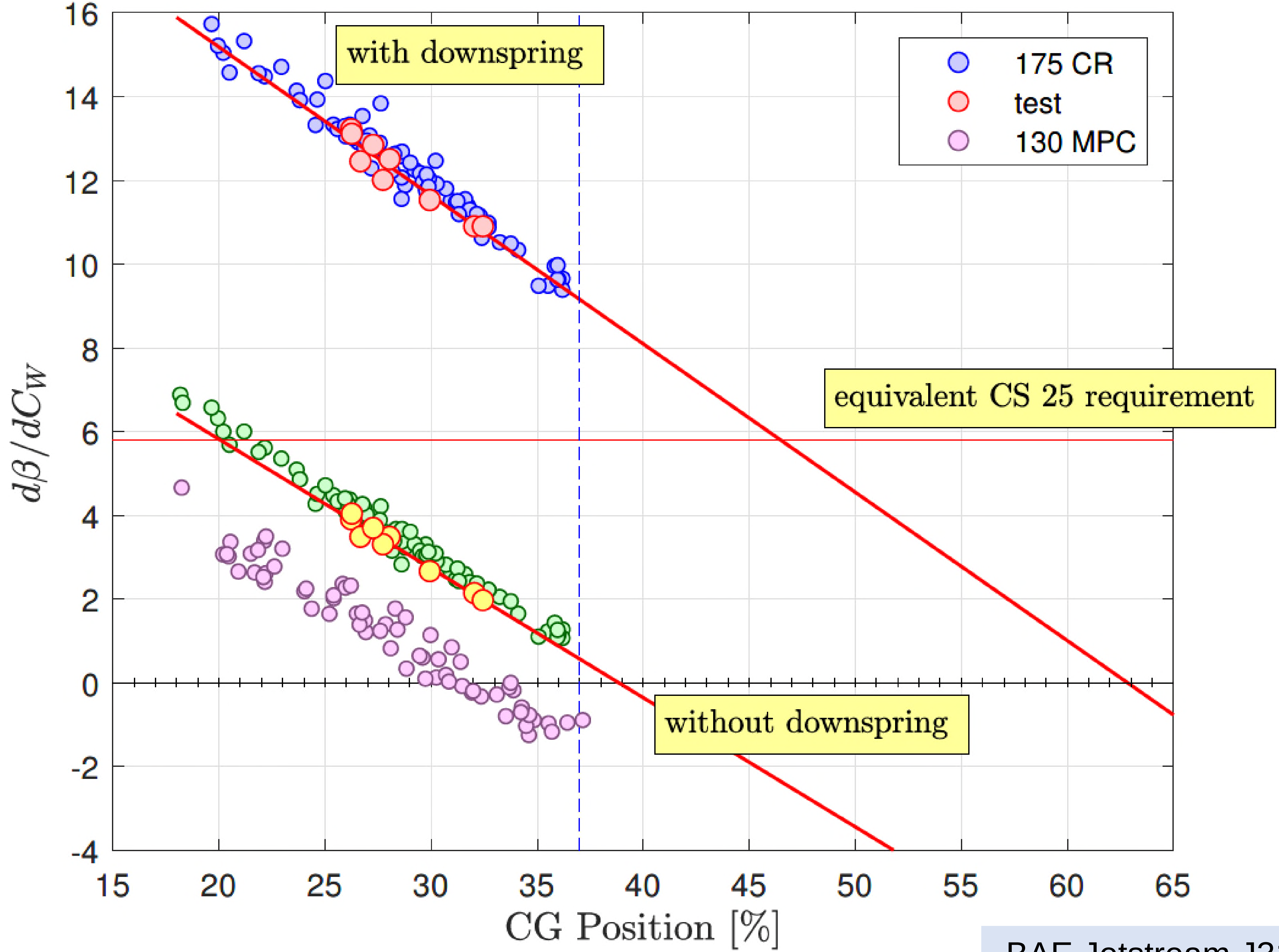
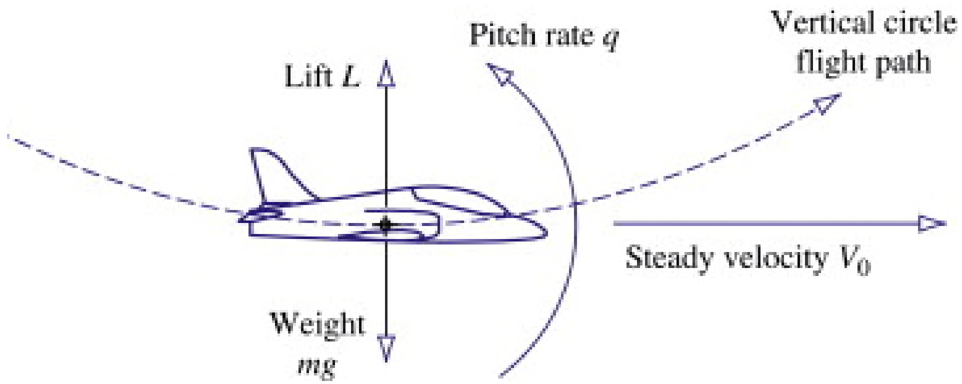


BAE Jetstream J31

3 Manoeuvre stability [*Longitudinal*]

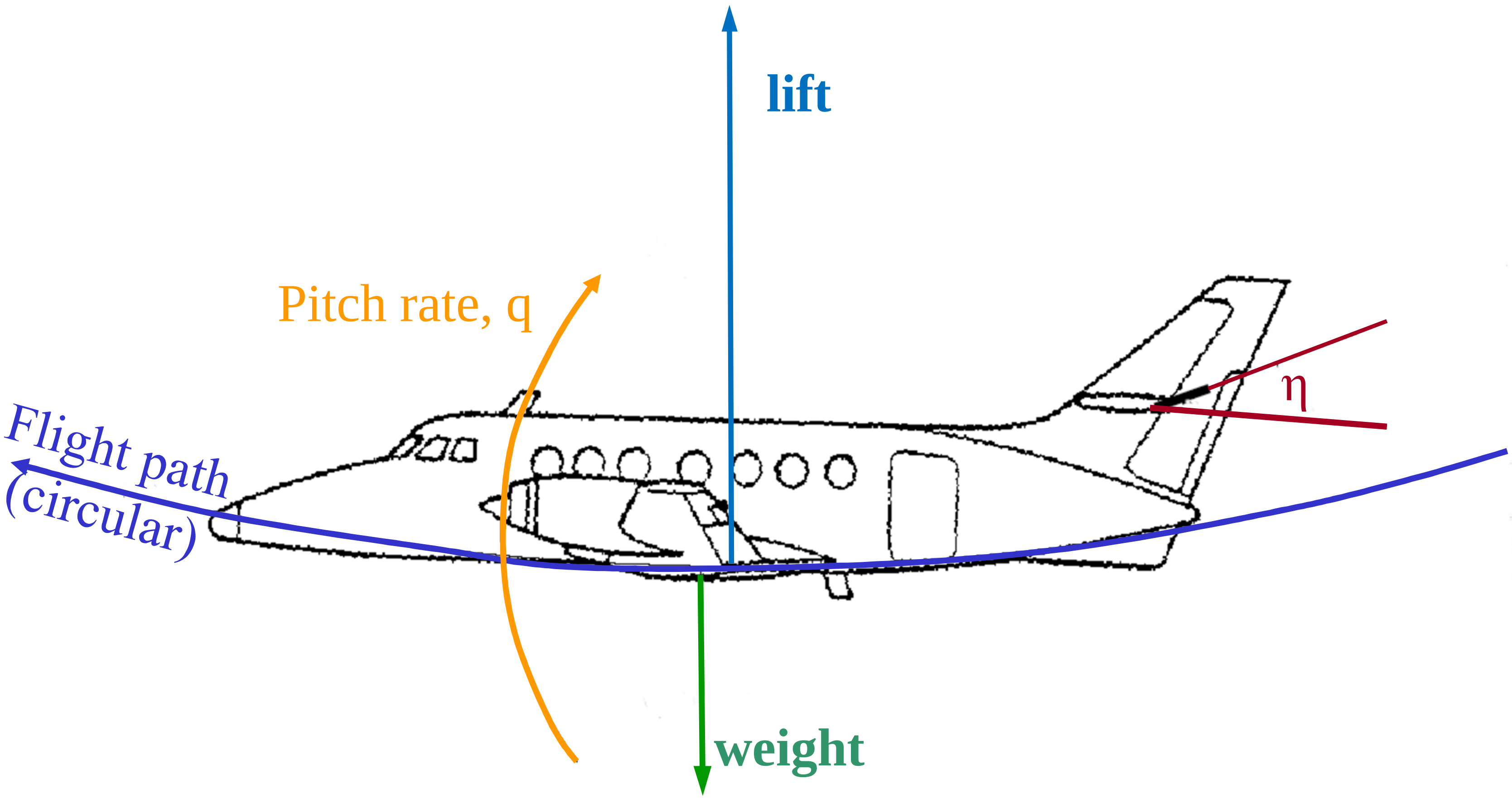
Based on a steady manoeuvre - aircraft is subject to a steady normal acceleration in response to a pitch control input



- Important to achieve the right level of stability
  - ➢ Too much stability would mean large control displacements needed to manoeuvre
  - ➢ Too little manoeuvre stability could lead to inputs that may overstress the airframe - excessive acceleration.
- May use inside or outside loop or steady banked turn (up to 60 degree bank angle gives up to 2g)

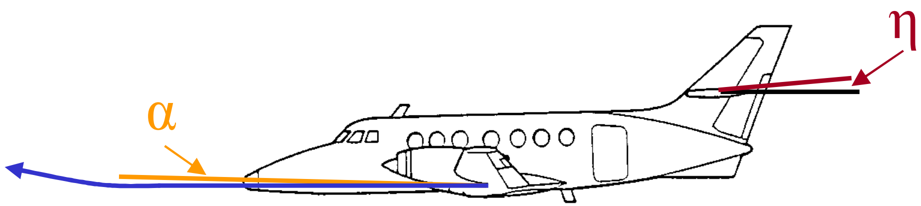
[Cook, 2013]

Manoeuvre stability



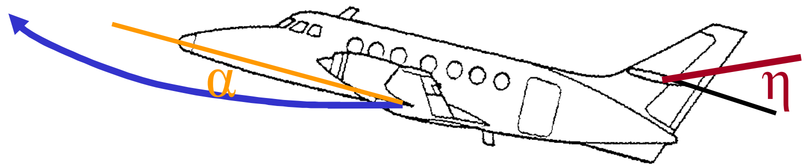
In the steady manoeuvre, the aerodynamic pitching moment
*(from increase in AoA **and** from pitch rate)*
is balanced by elevator moment.

*Manoeuvre takes place at **Constant Speed***



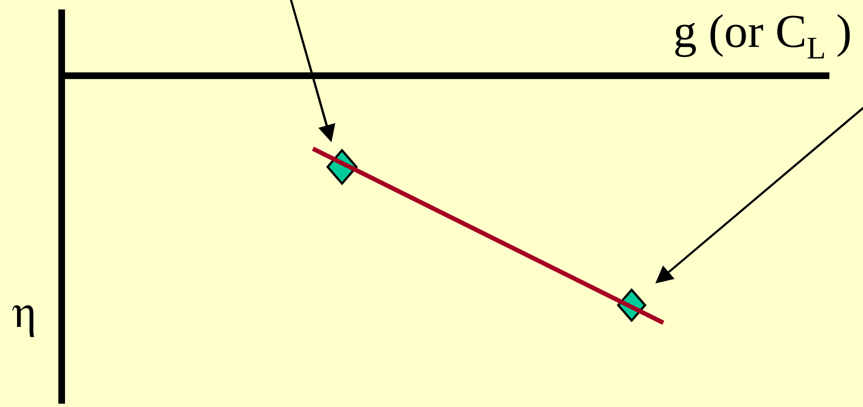
Small η, low q

Therefore low α and low "g" (C_L)



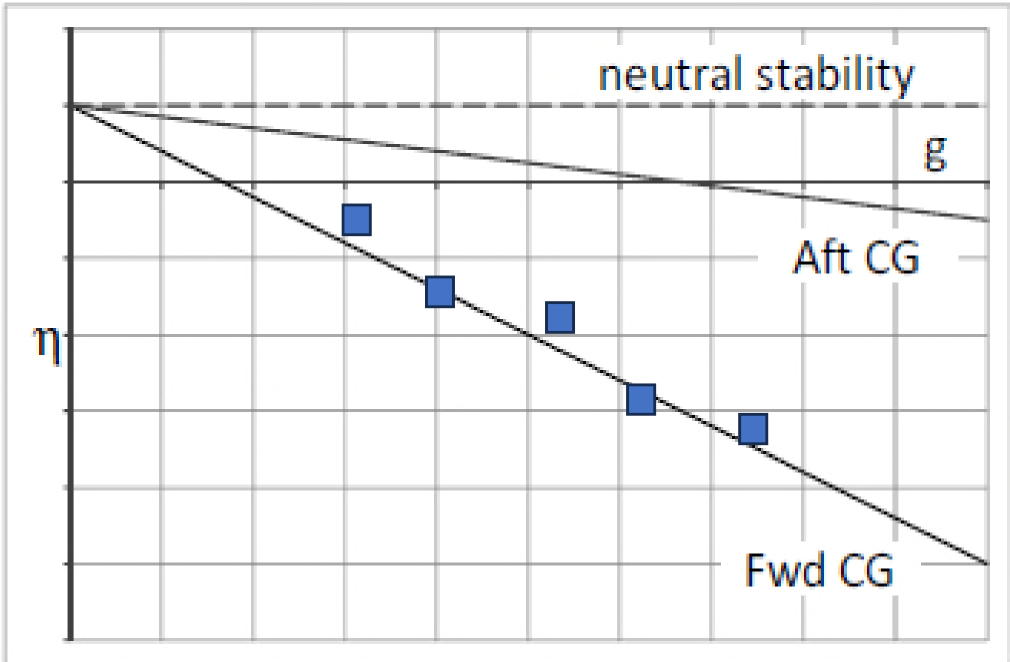
Large η, high q

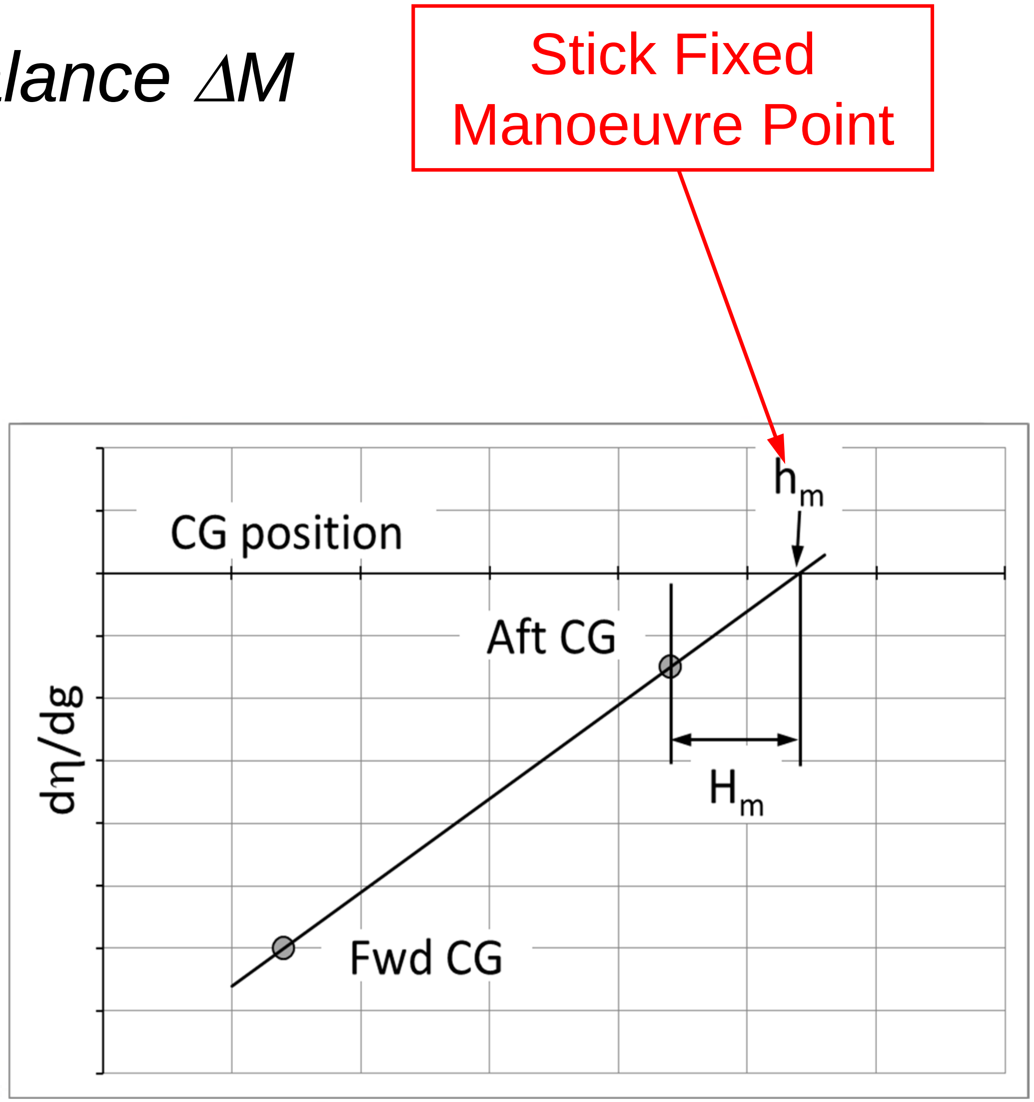
Therefore high α and high "g" (C_L)



Determining the Manoeuvre Point (Stick Fixed)

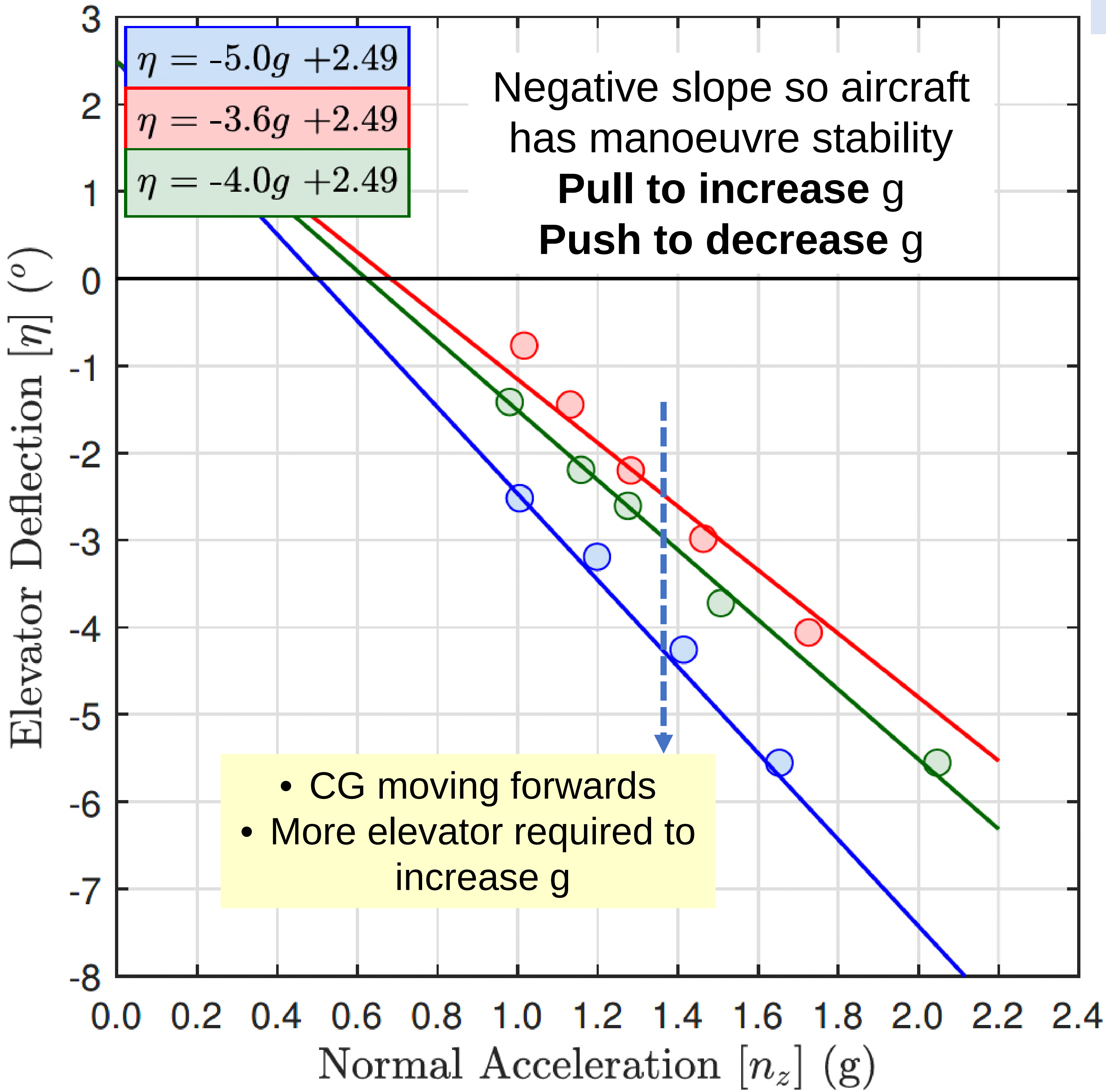
- *Fly aircraft at a range of g*
- *Measure elevator angle to balance ΔM*
- *Plot slope for a range of CG*

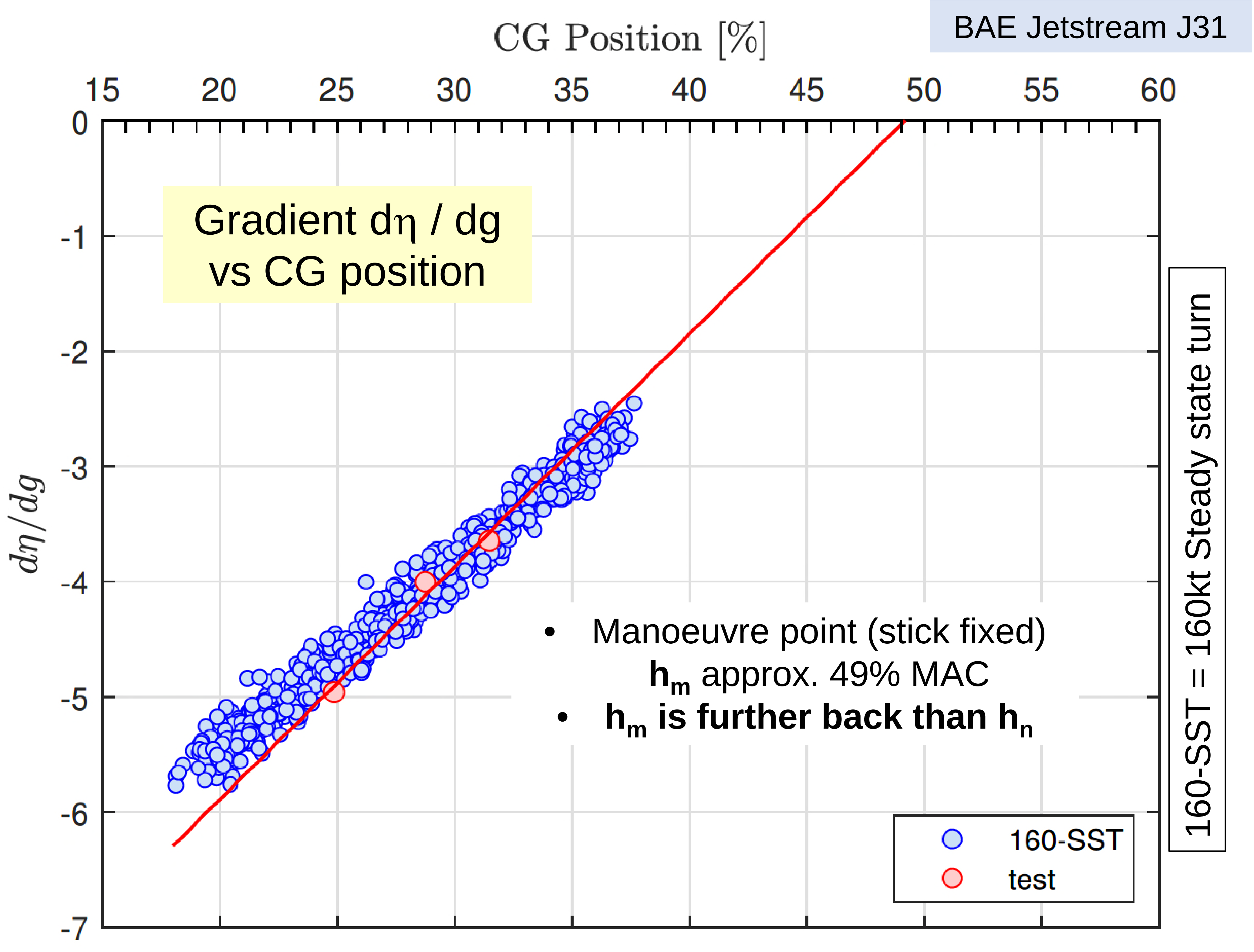




Elevator deflection η vs n_z

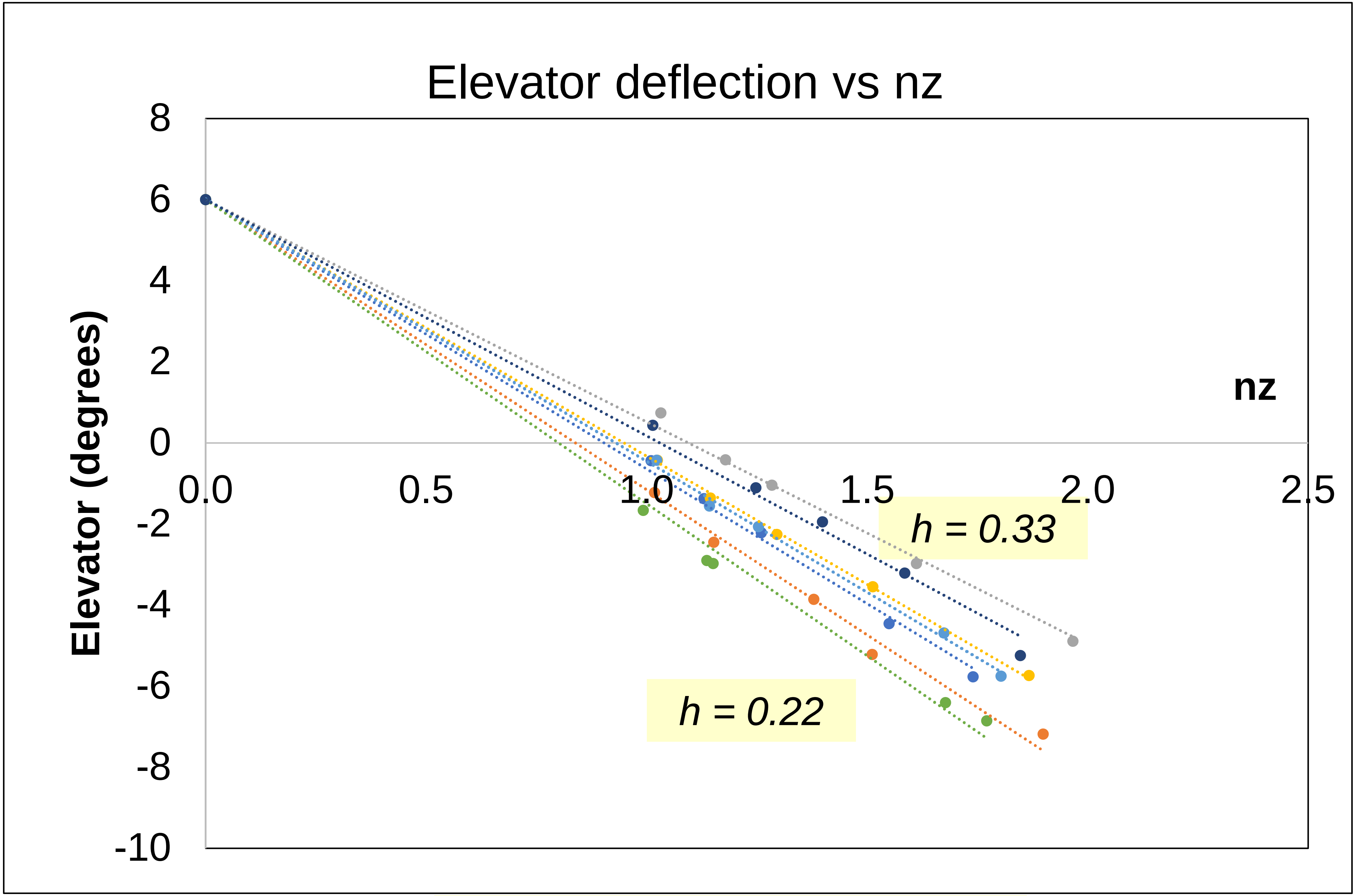
BAE Jetstream J31





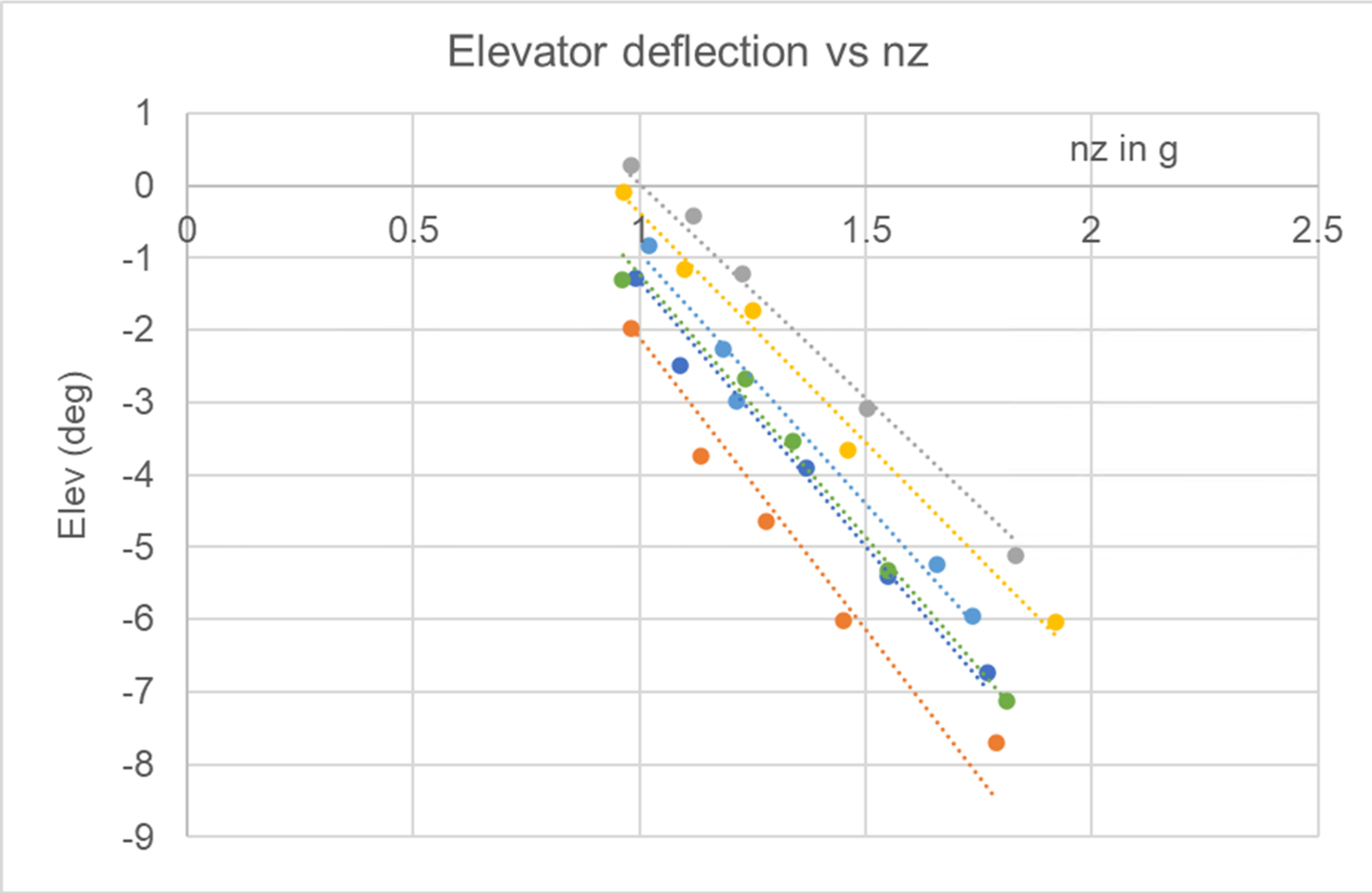
BAE Jetstream J31
CG Position [%]
15
20
25
30
35
40
45
50
55
60
0
-1
-2
-3
-4
-5
-6
-7
dη/dg
Gradient dη / dg
vs CG position
• Manoeuvre point (stick fixed)
hm approx. 49% MAC
• hm is further back than hn
160-SST
test
160-SST = 160kt Steady state turn

Manoeuvre Point (Stick fixed) - Saab data 2022



Estimated Manoeuvre point ~ 59%

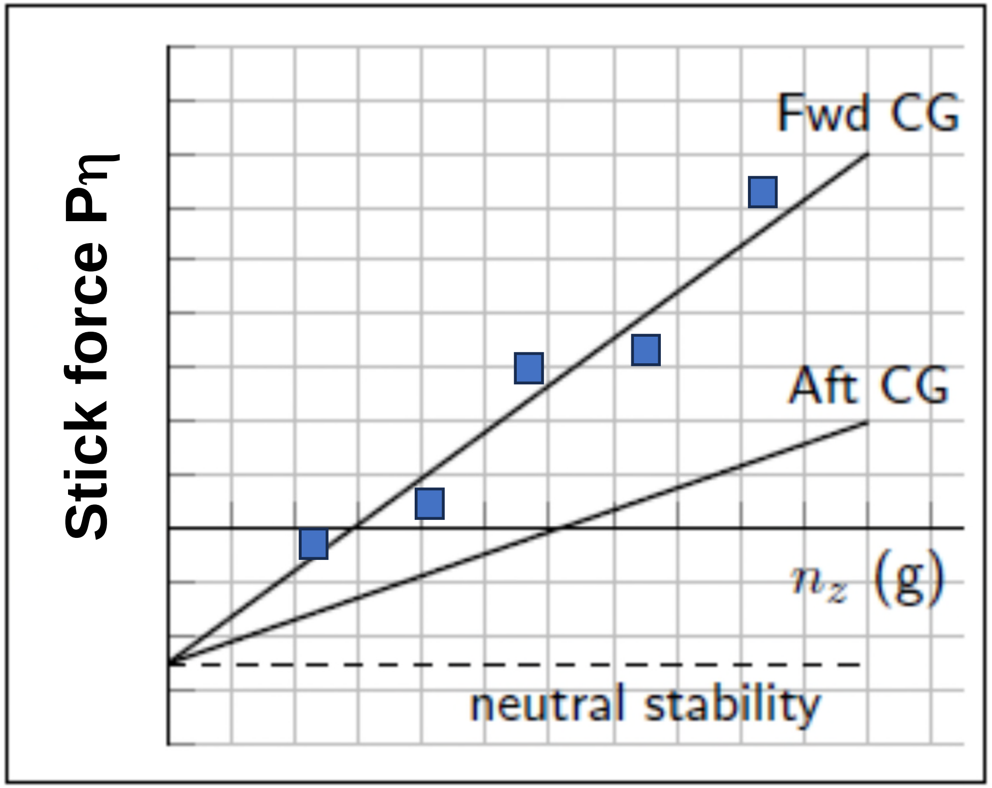
Manoeuvre Point (Stick fixed)
Saab 2022-23

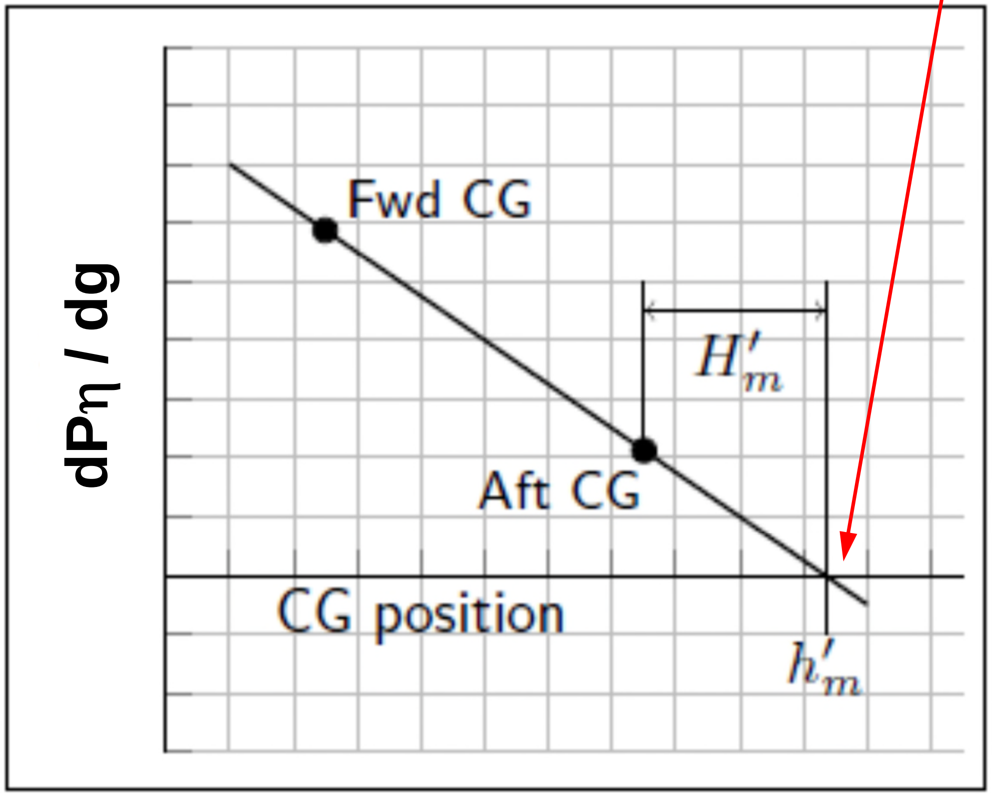


Determining the Manoeuvre Point
"Stick Free"

- *Fly aircraft at a range of g*
- *Measure Stick force to balance ΔM*
- *Plot slope for a range of CG*

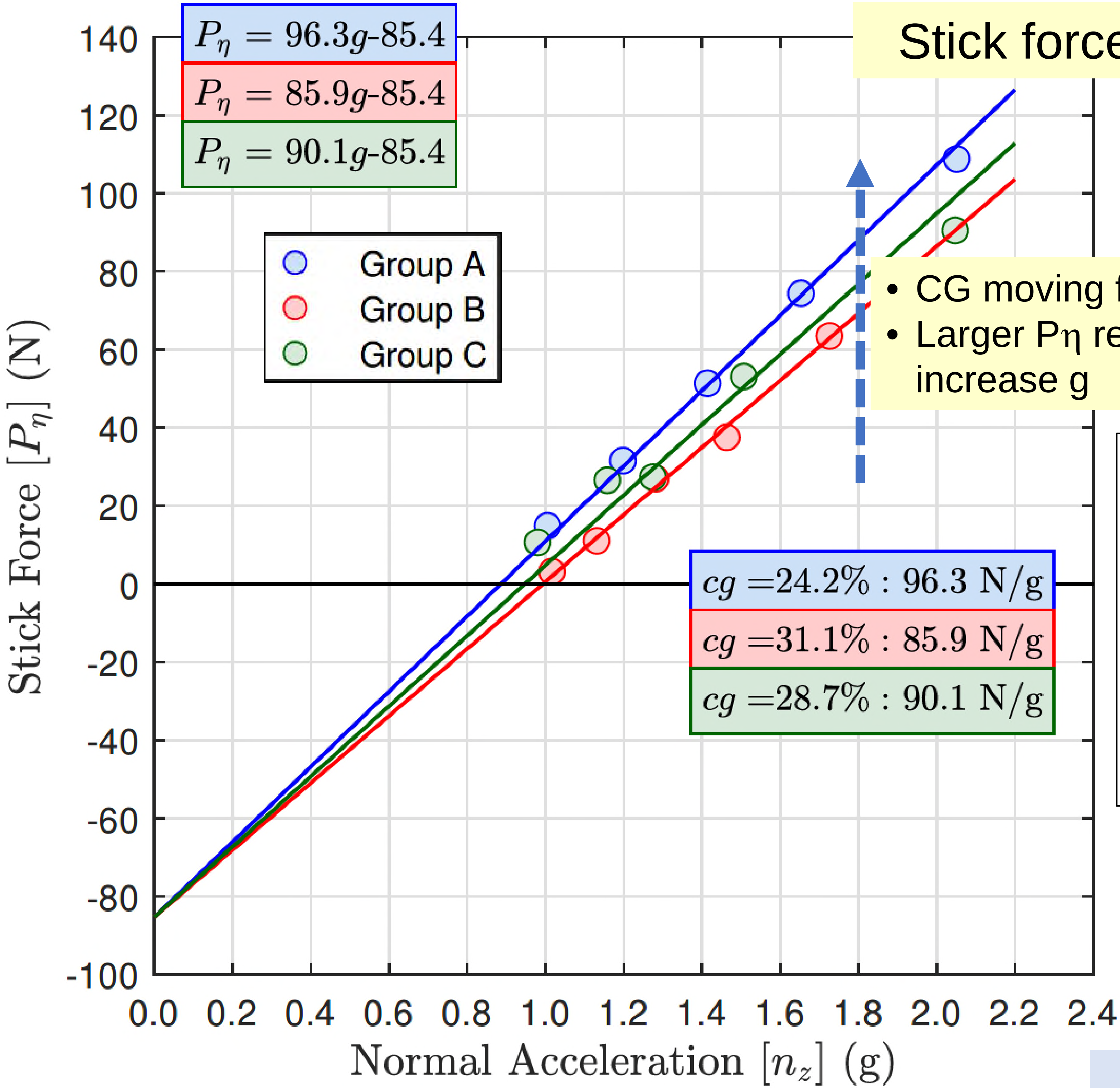
"Stick Free"
Manoeuvre Point





Stick Force per g is a very important parameter

Stick force Pη vs n_z



- CG moving forwards
- Larger Pη required to increase g

- Pilot can independently set elevator trim tab
- Tab is moved to datum condition at **1g**

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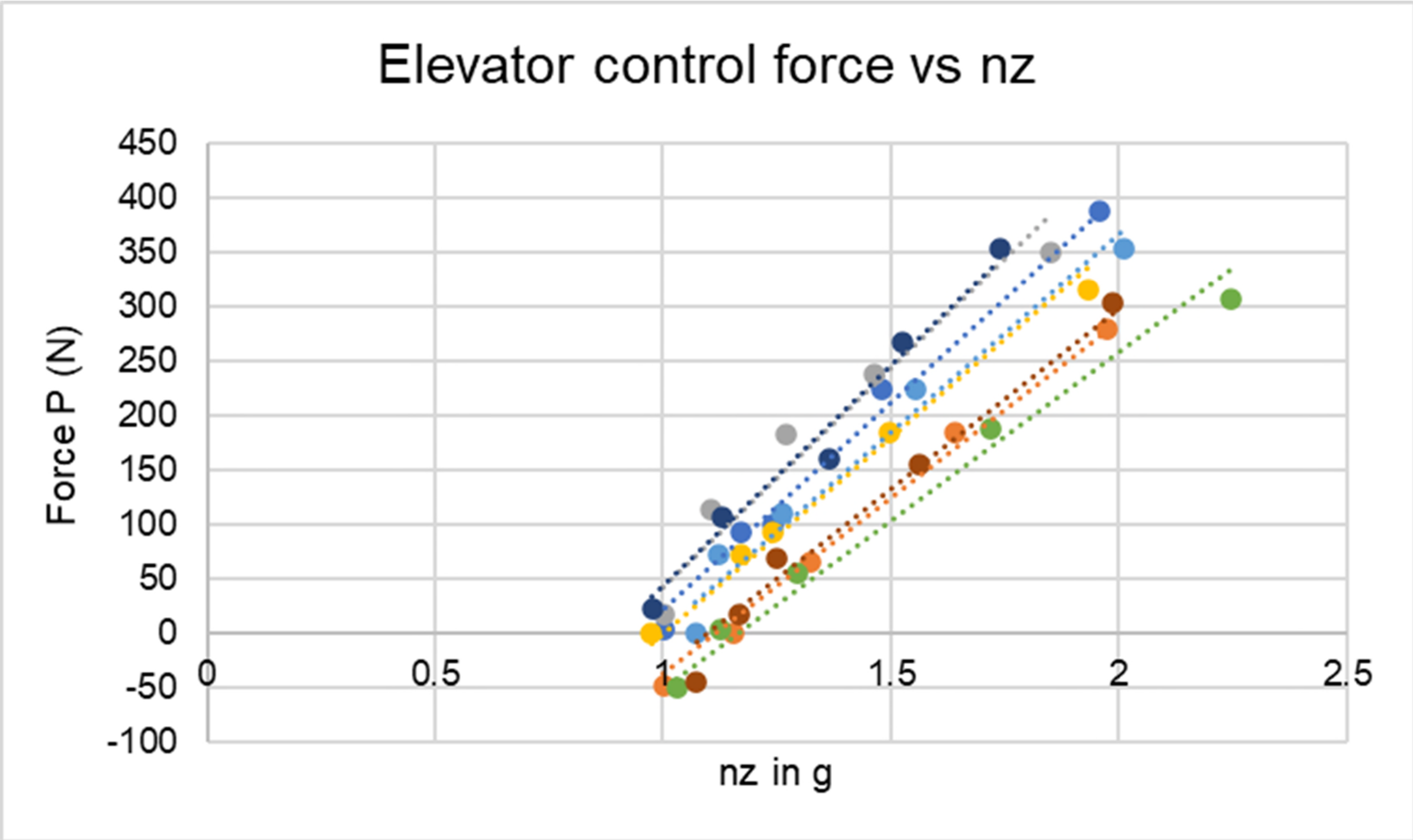
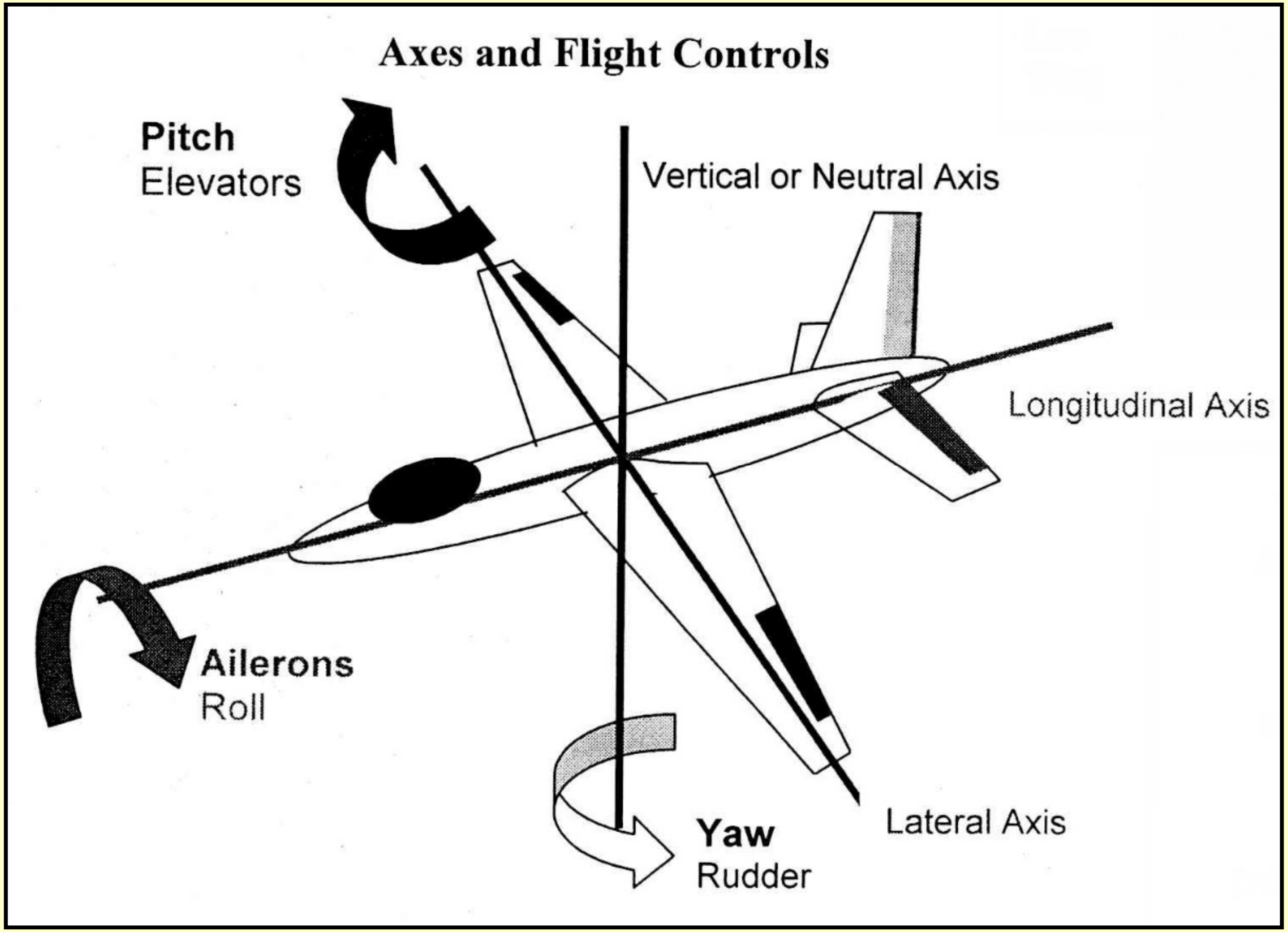
Plot of Stick force vs g - Saab data



4. Dynamic Stability



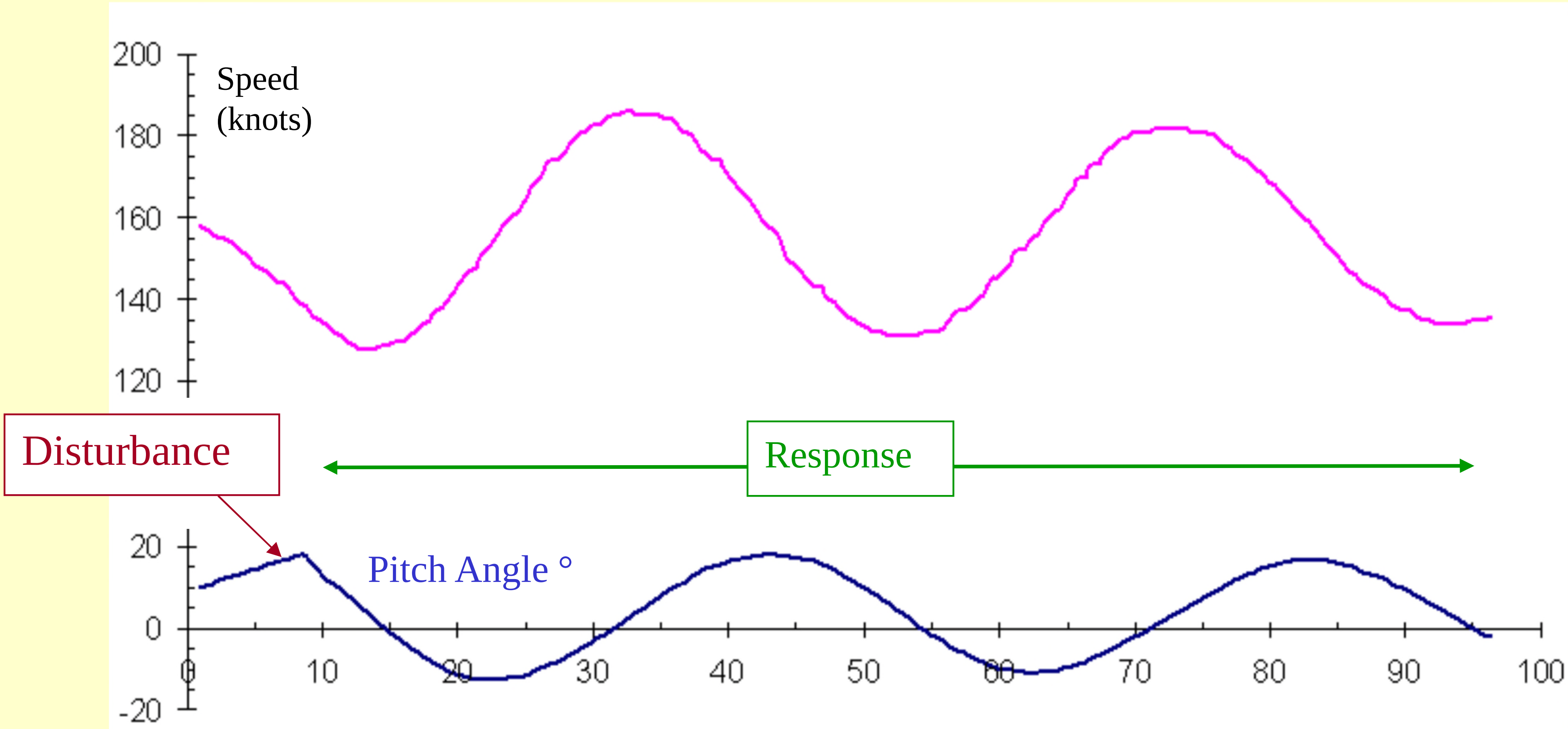
…the transient motion involved as the aircraft attempts to recover to equilibrium following a disturbance

Longitudinal Modes of Motion

I. Phugoid

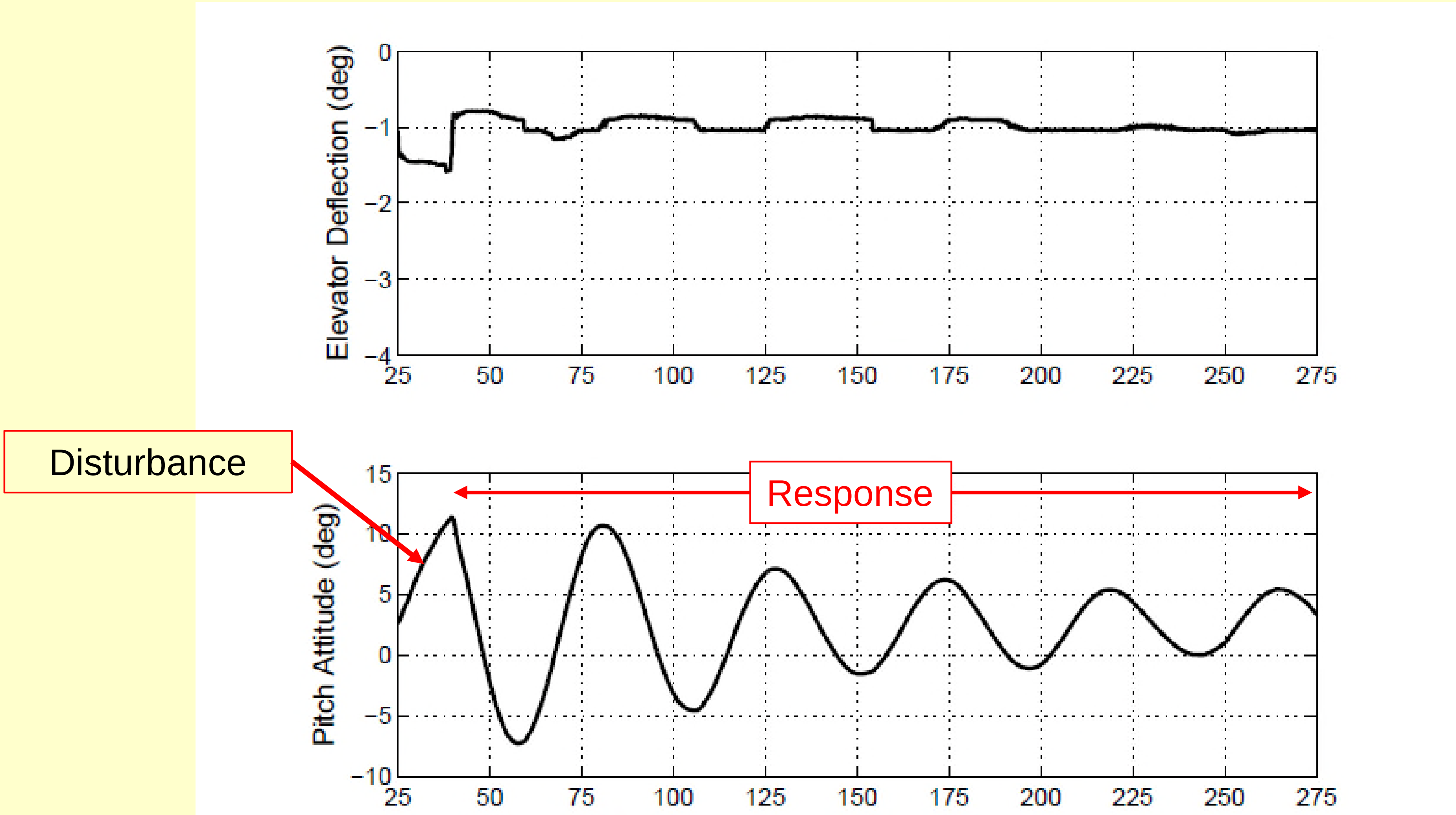
Phugoid Oscillation

- an interaction between **pitch angle** and **speed change**



Phugoid

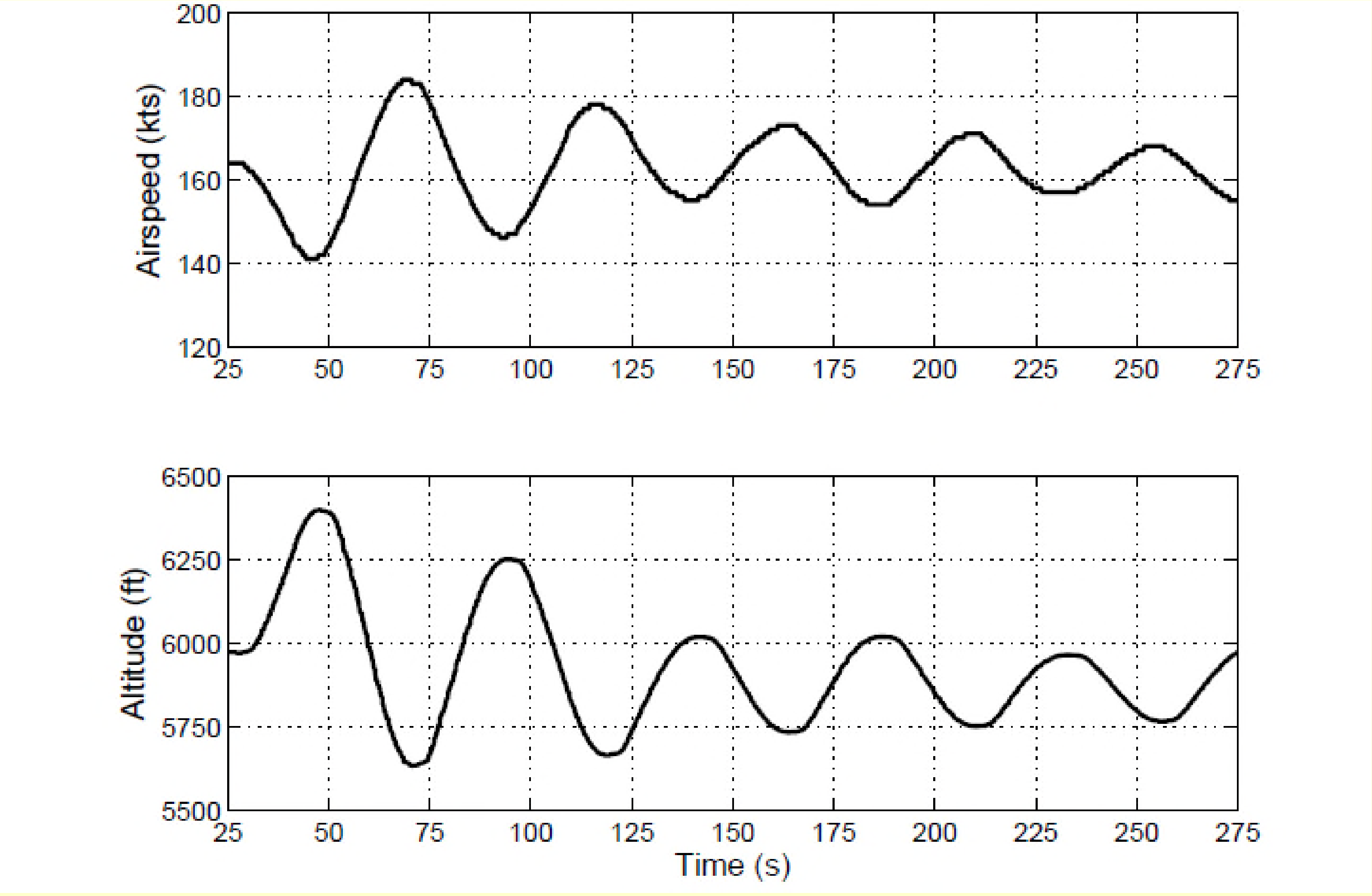
- Angle of attack is constant during the mode
- However, elevator may make small movements – mode can become unstable



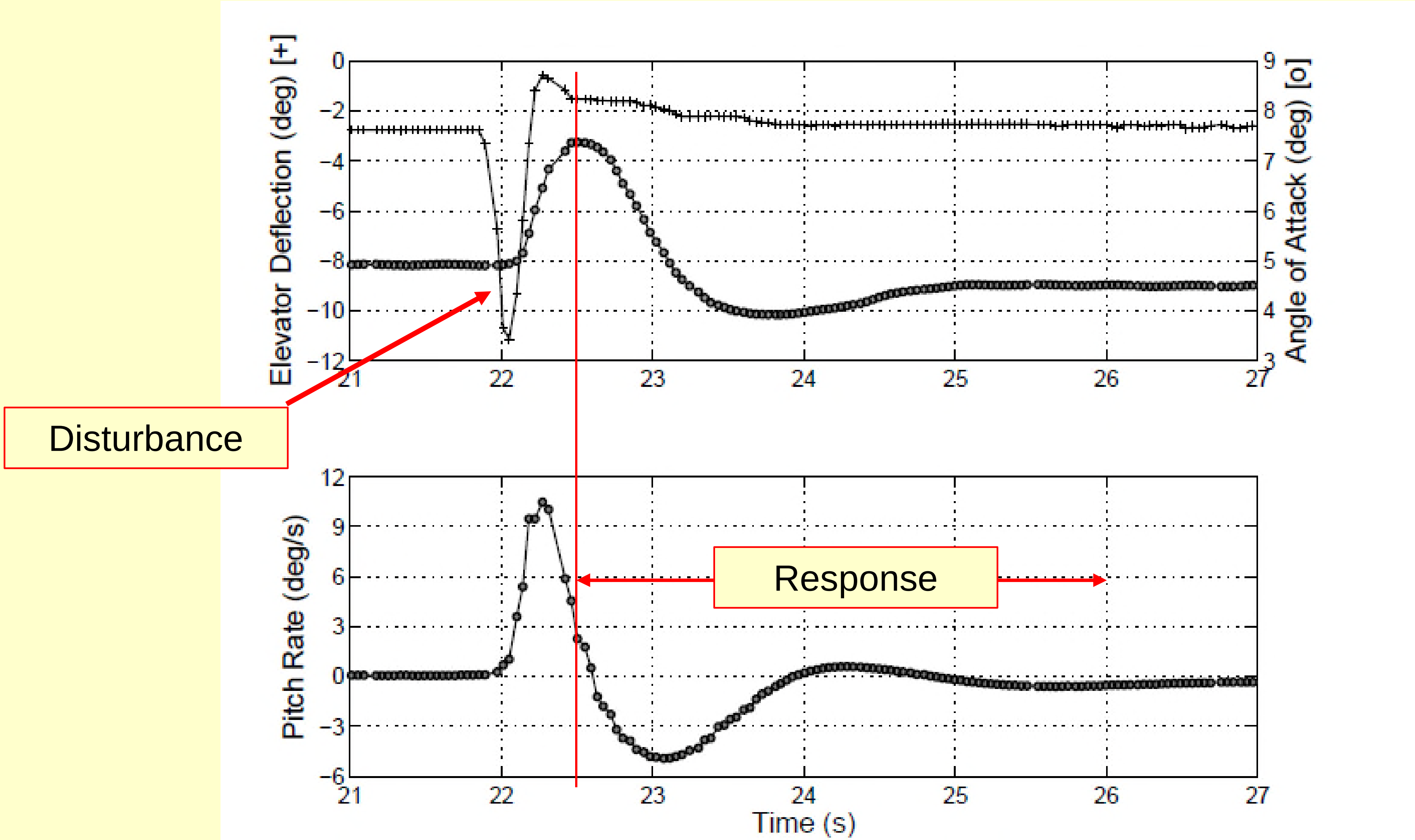
Phugoid

- Since α is constant during the mode; C_L is constant and so is $C_D = C_{D0} + kC_L^2$

Hence speed excursions cause increase and decrease in Lift and Drag

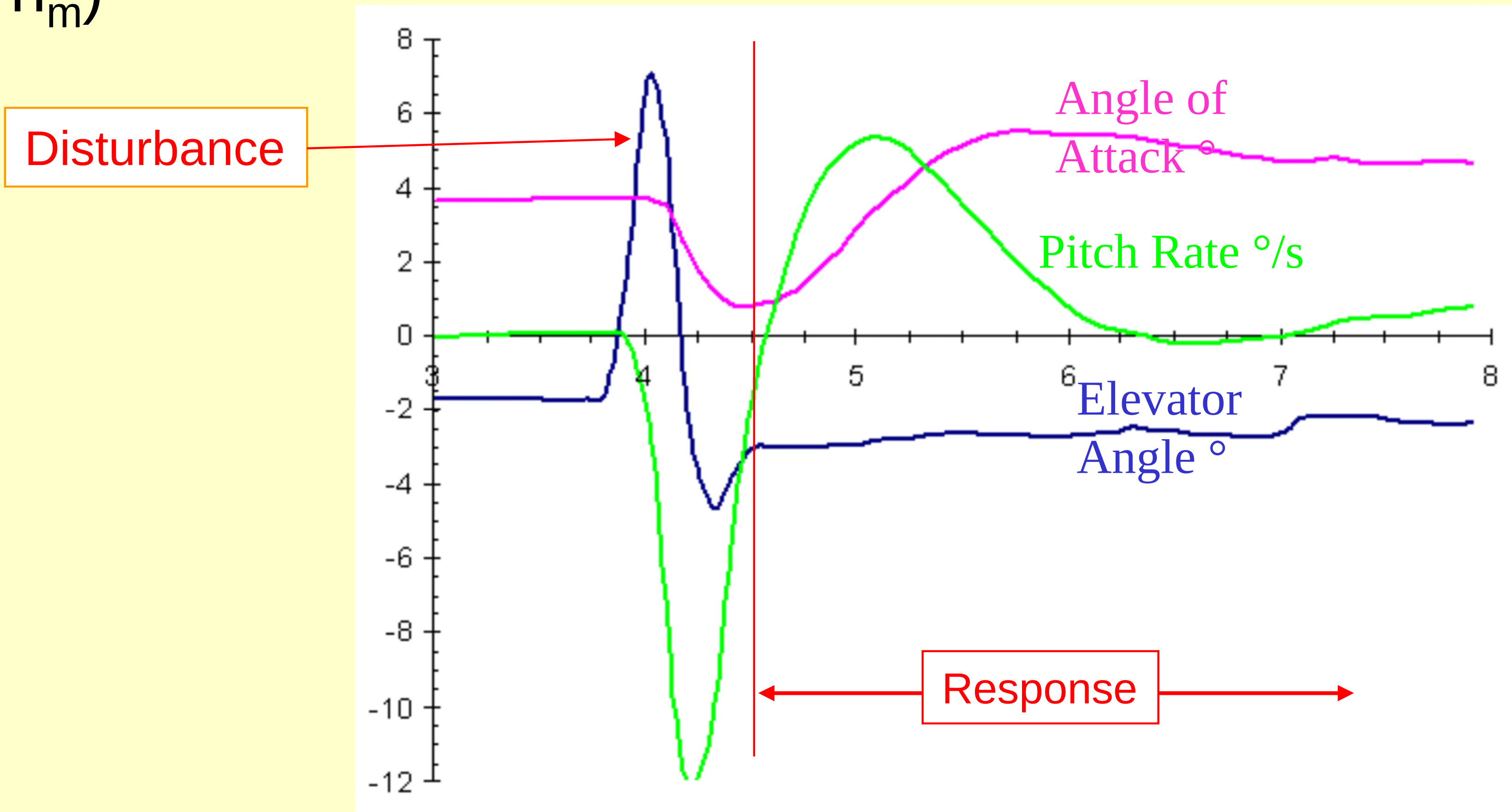


II. Short Period Pitching Oscillation

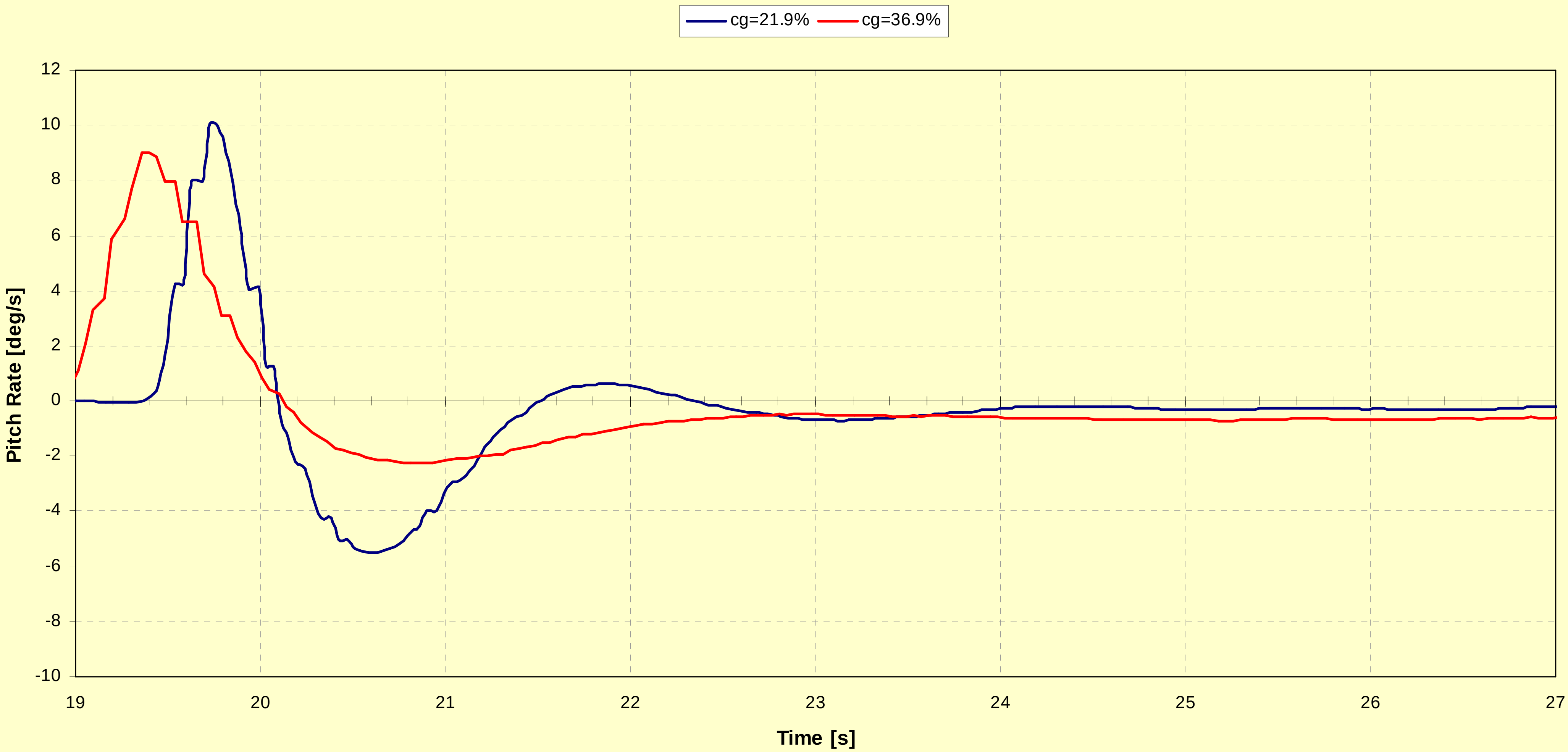


Short Period Pitching Oscillation

- Interaction between **pitch rate** and **angle of attack**
- This mode is critical for aircraft handling qualities
- Response of affected by Manoeuvre margin K_m (distance CG to h_m)



Effect of CG Position of Short Period Dynamics



CG = 21.9% (Fwd) – High manoeuvre margin K_m

CG = 36.9% (Rear) – Low manoeuvre margin K_m

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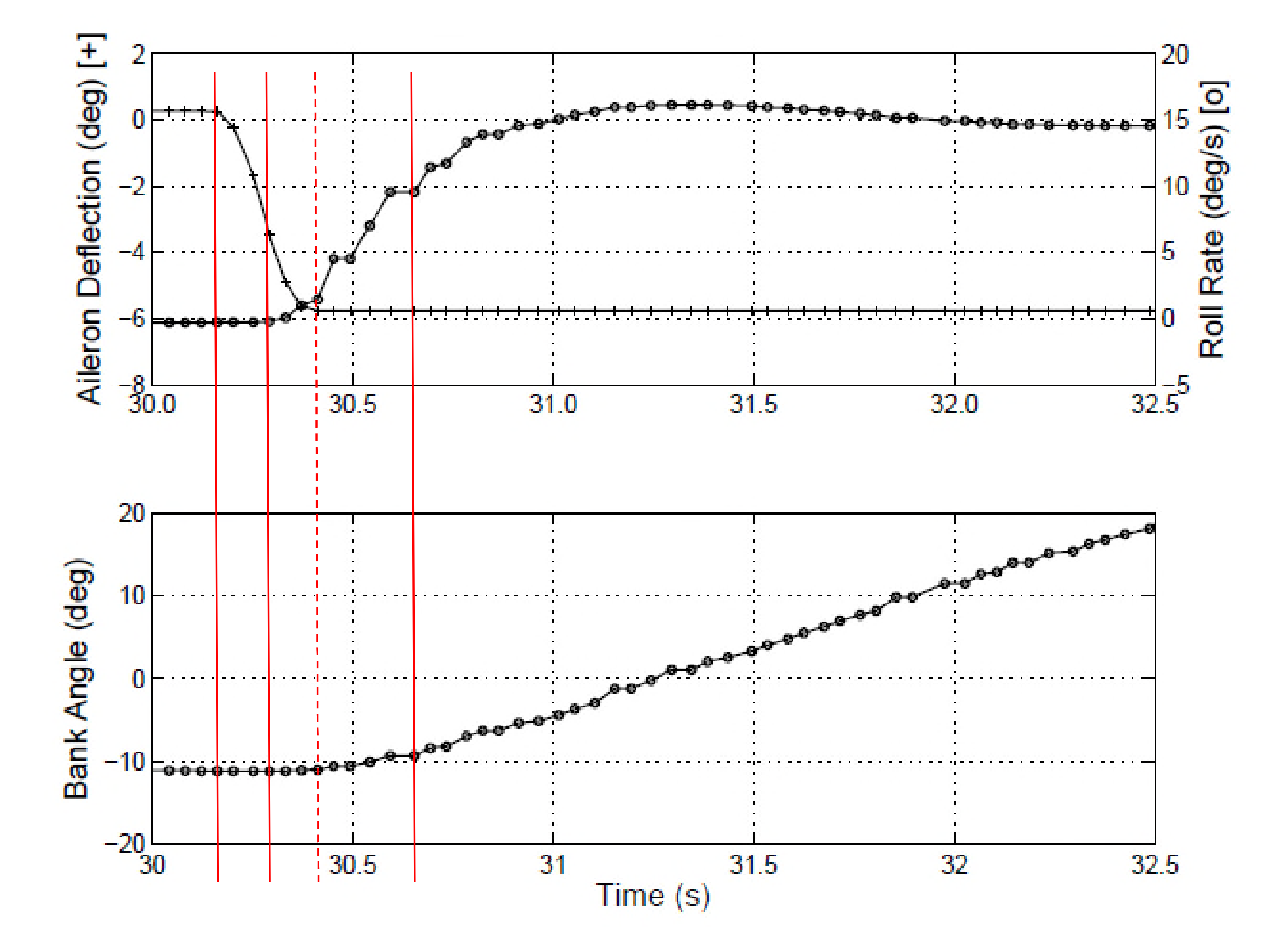
Dynamic Stability

Lateral/Directional Modes of Motion

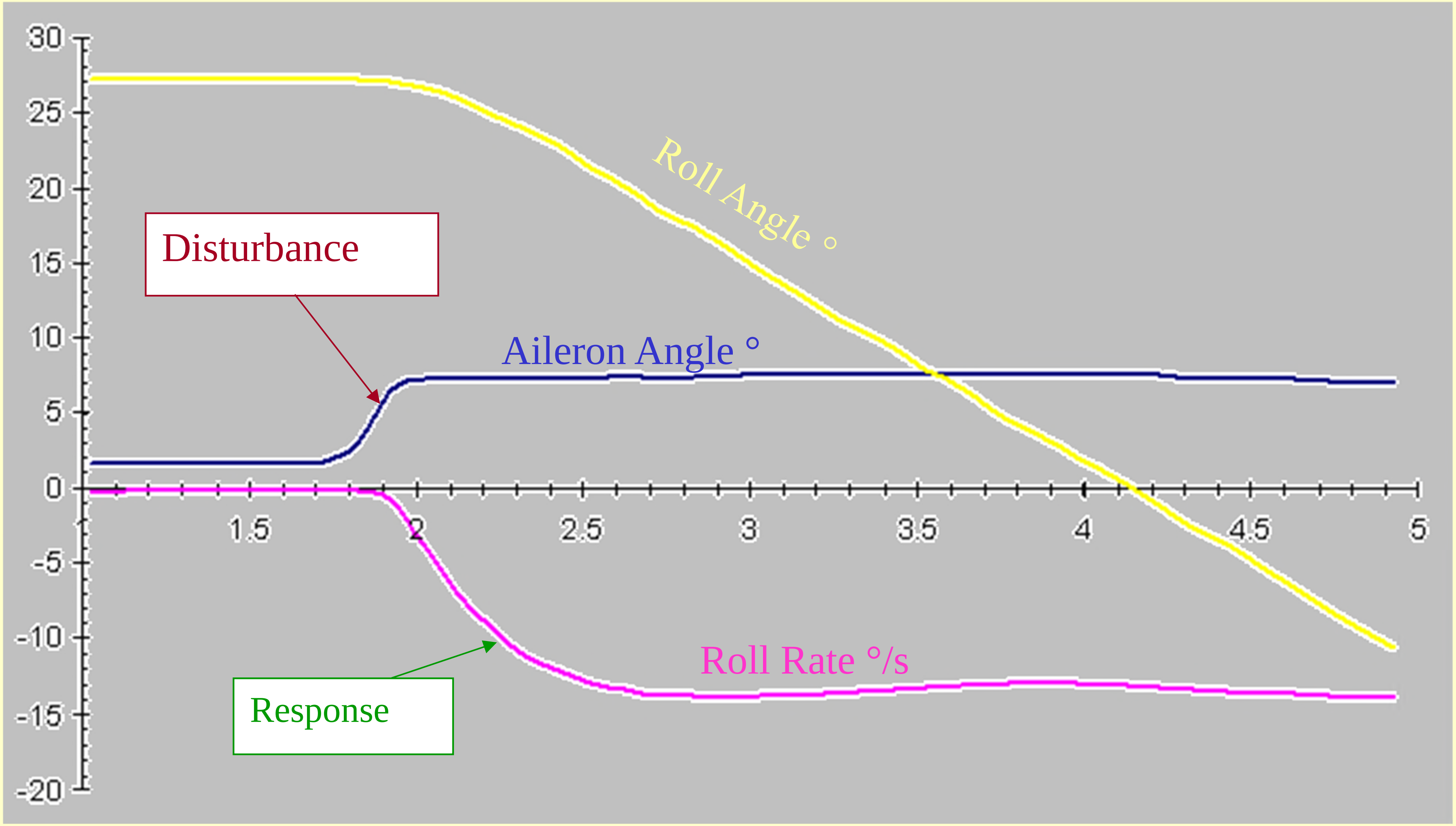
III. Roll (Subsidence) Mode



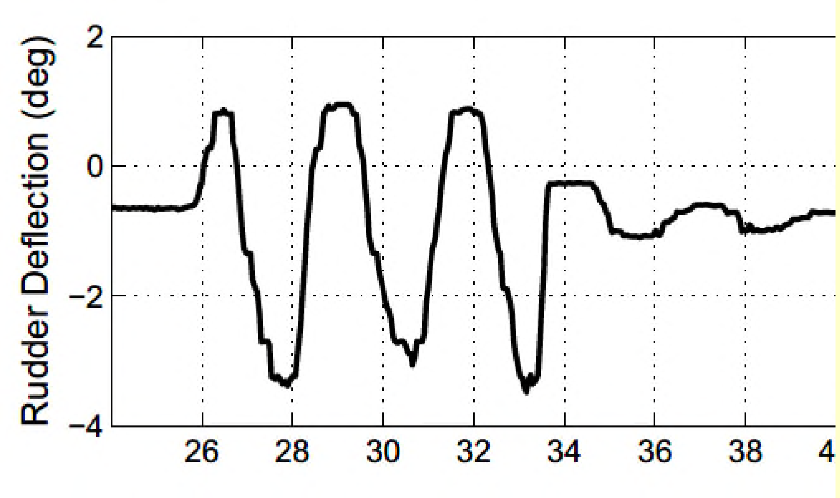
- First order response
- At *Time constant* T, Roll rate = 63.2% of steady value

Roll (Subsidence) Mode

Dynamic mode in which **roll rate** stabilises

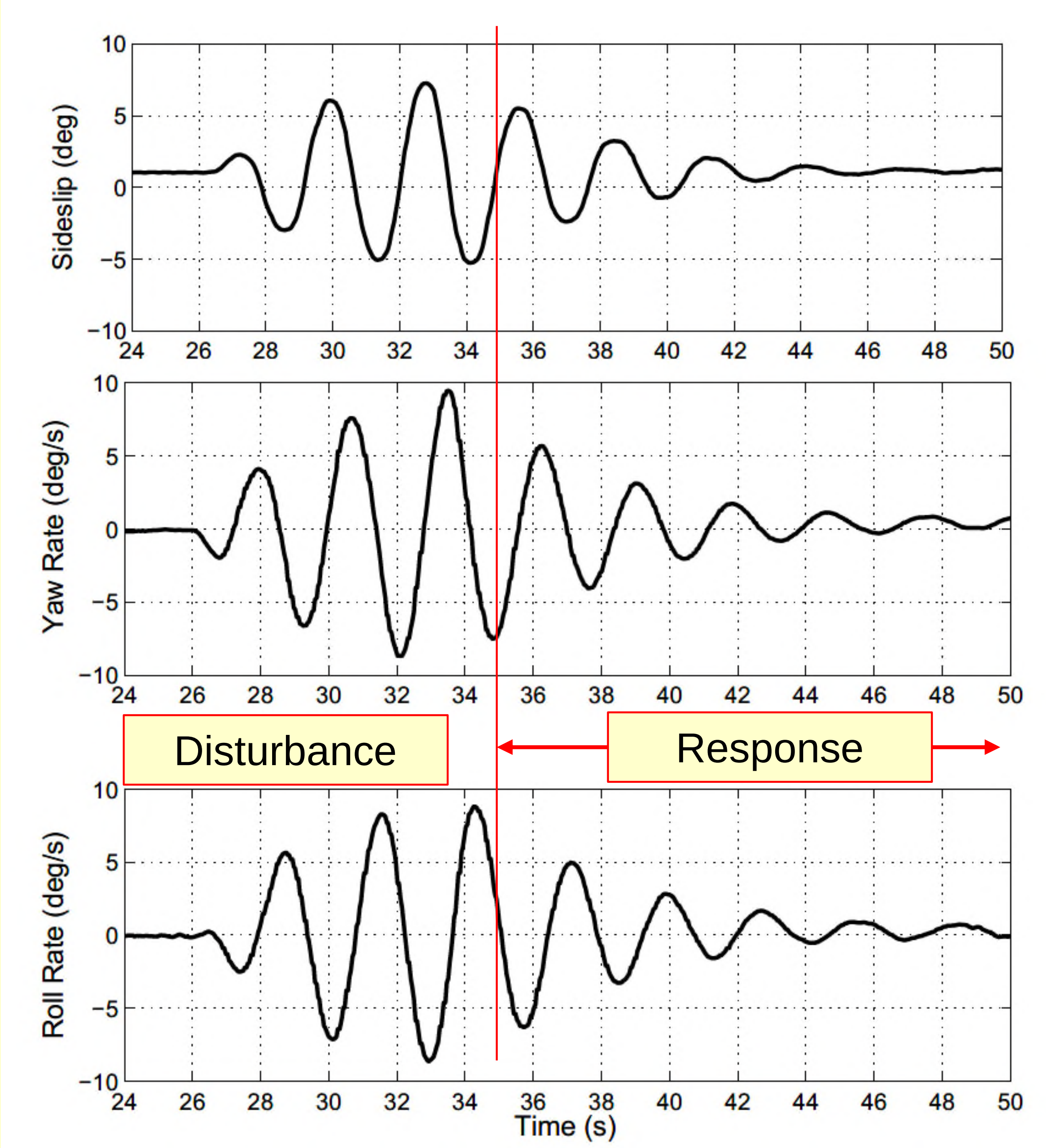


IV. Dutch Roll Oscillation



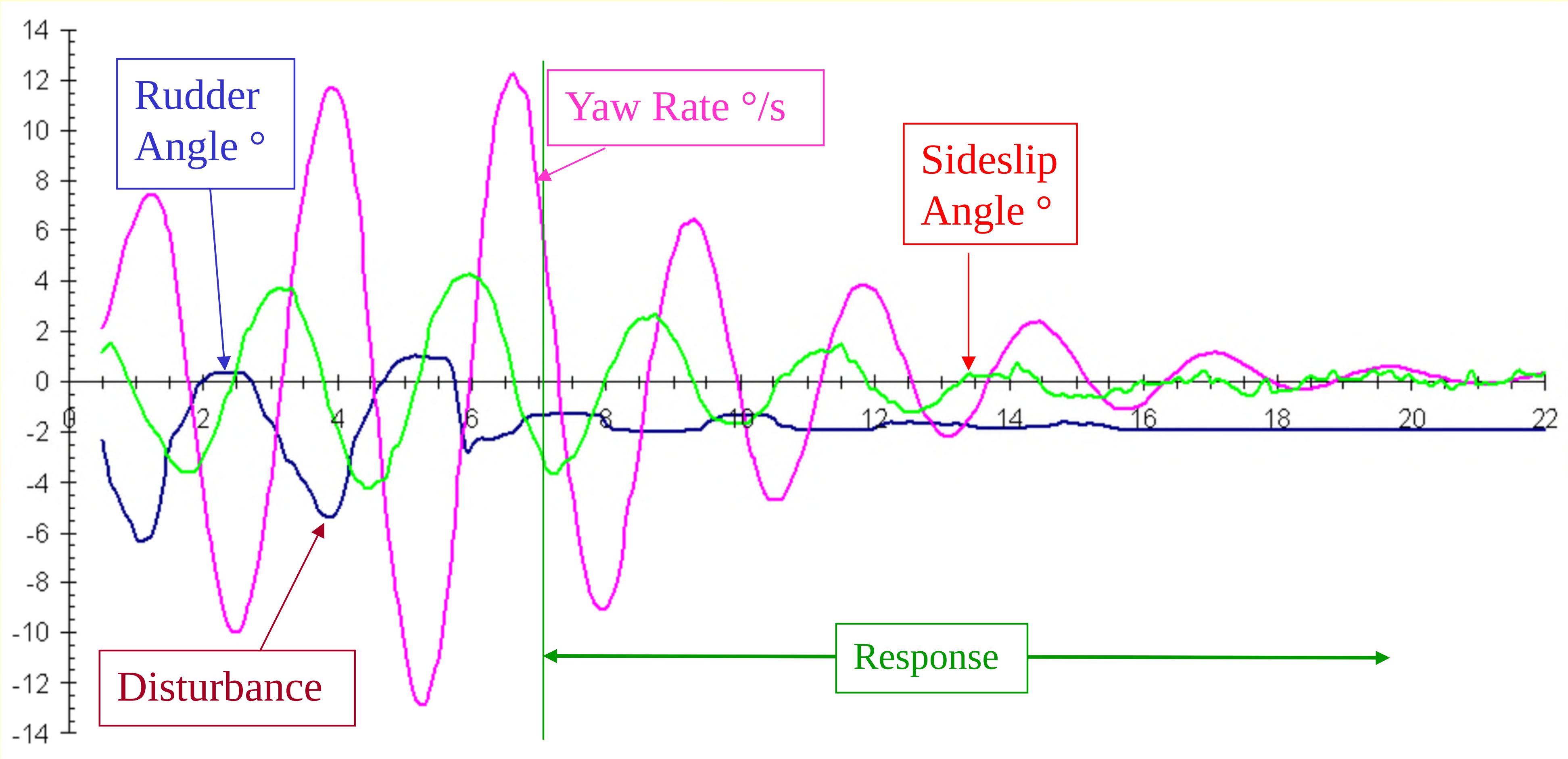
Rudder input

Lateral stability (dihedral effect) opposes sideslip so both yawing and rolling motion occur

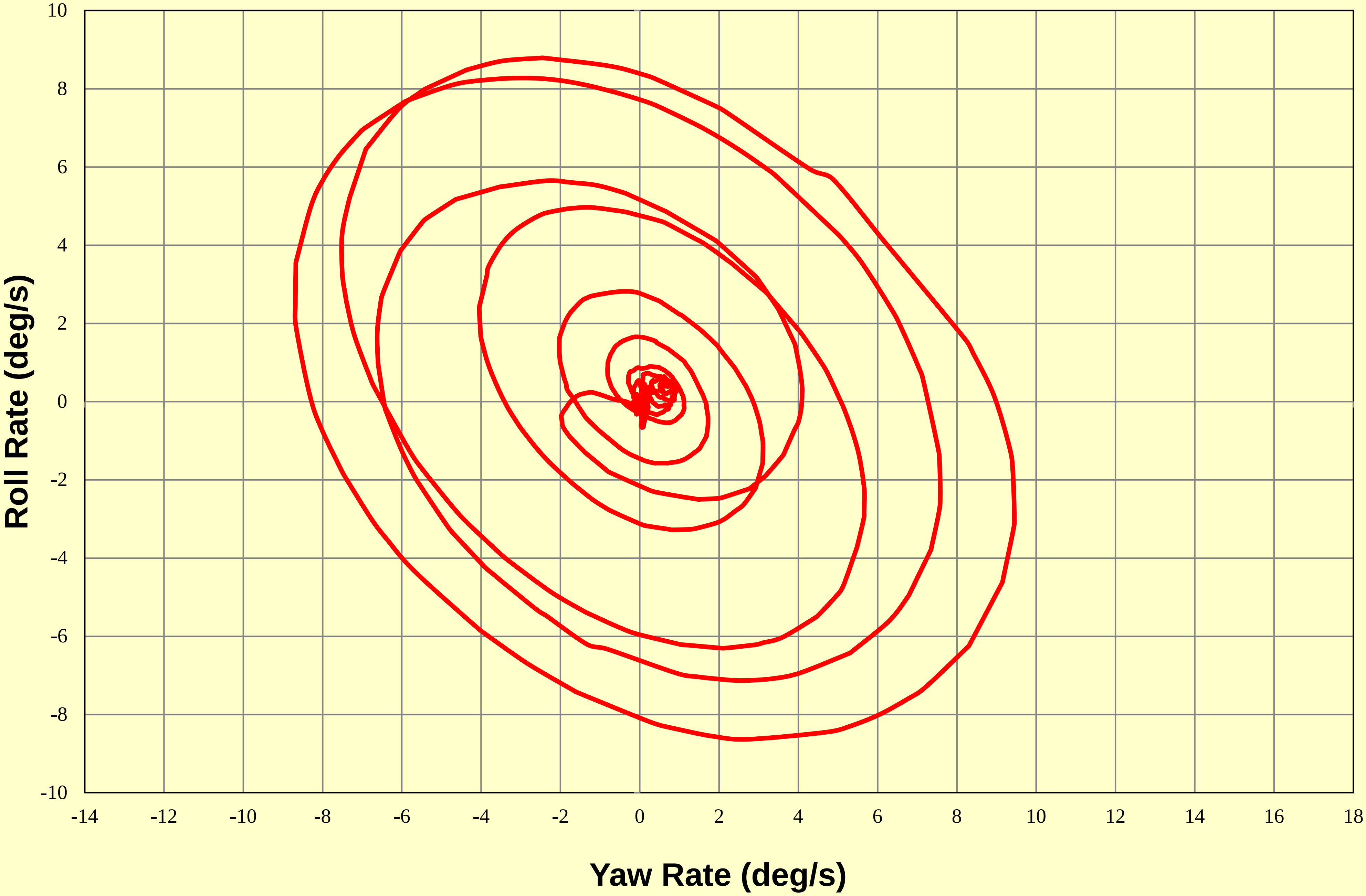


Dutch Roll Oscillation

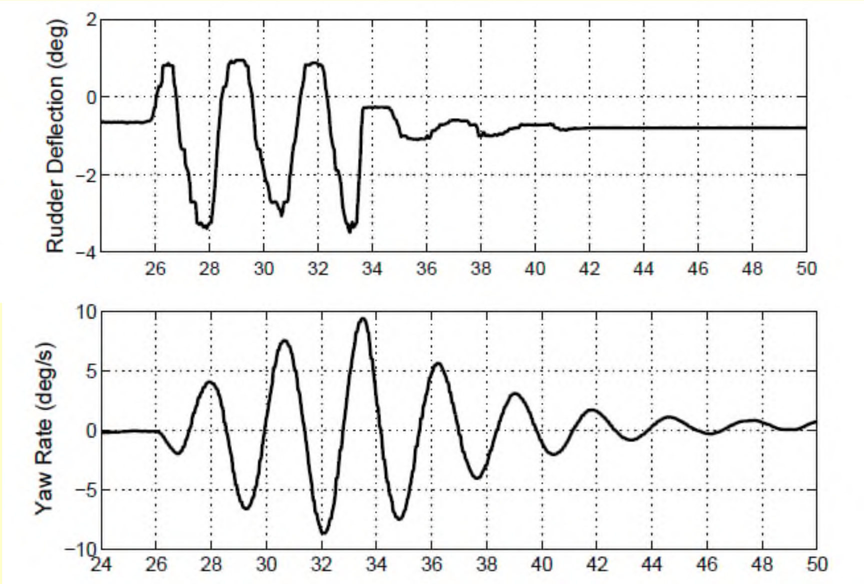
- Time history shows interaction between **yaw rate** and **sideslip angle**
- Mode is stable but damping is low

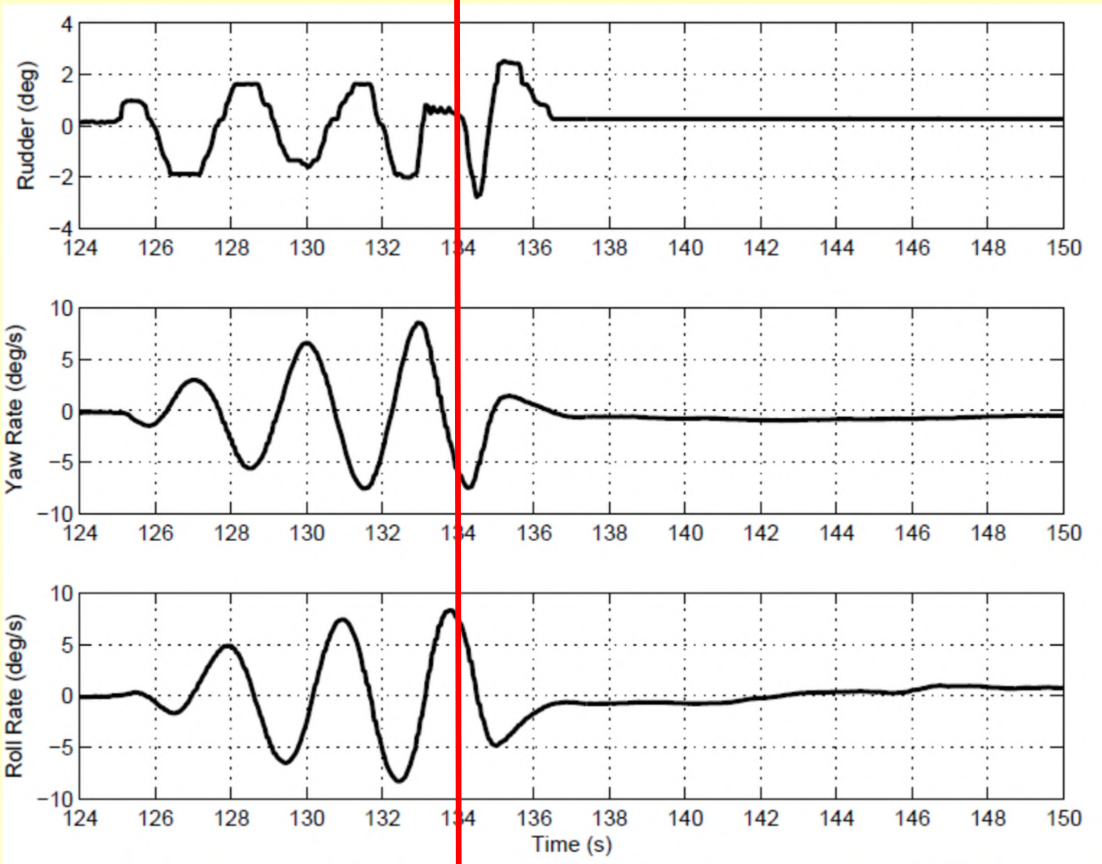


Dutch Roll Oscillation



Dutch Roll Oscillation





Yaw Damper OFF

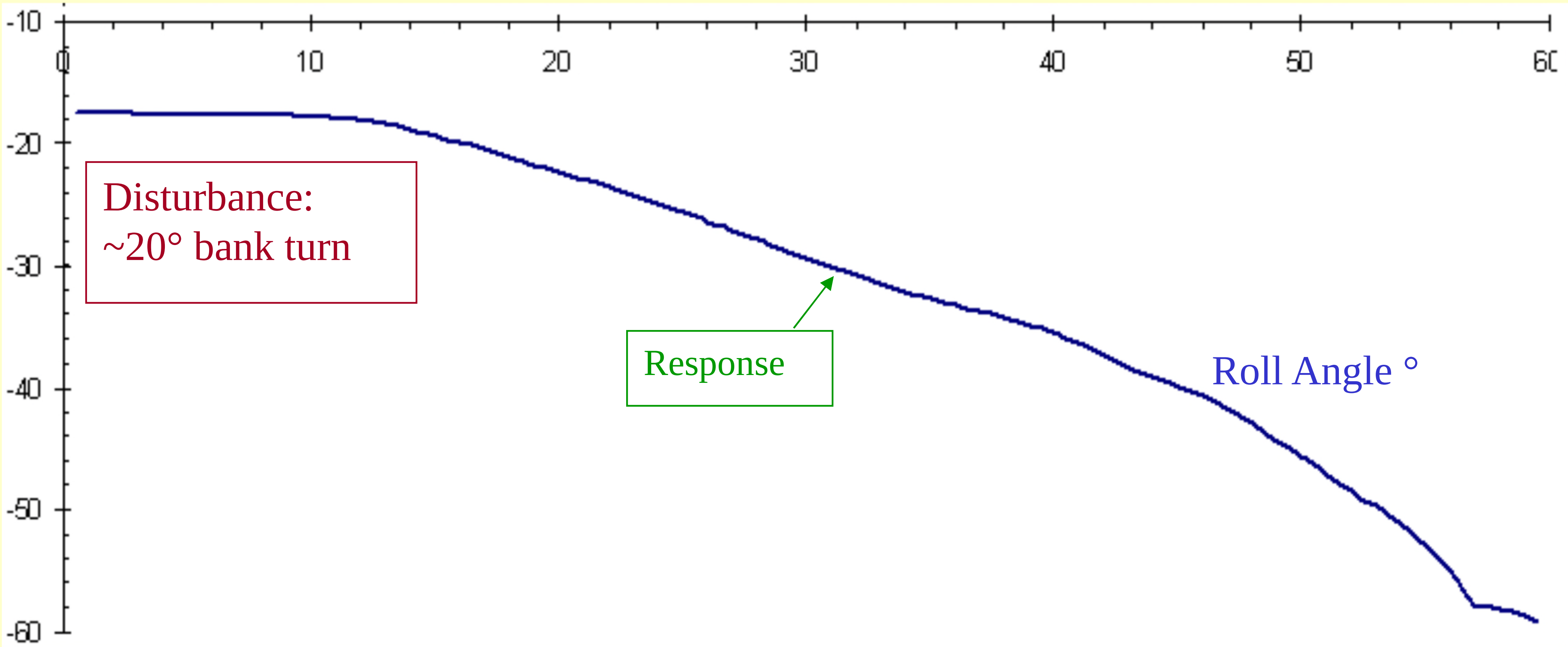
Yaw Damper ON

www.youtube.com/watch?v=Zmjam1evDD4

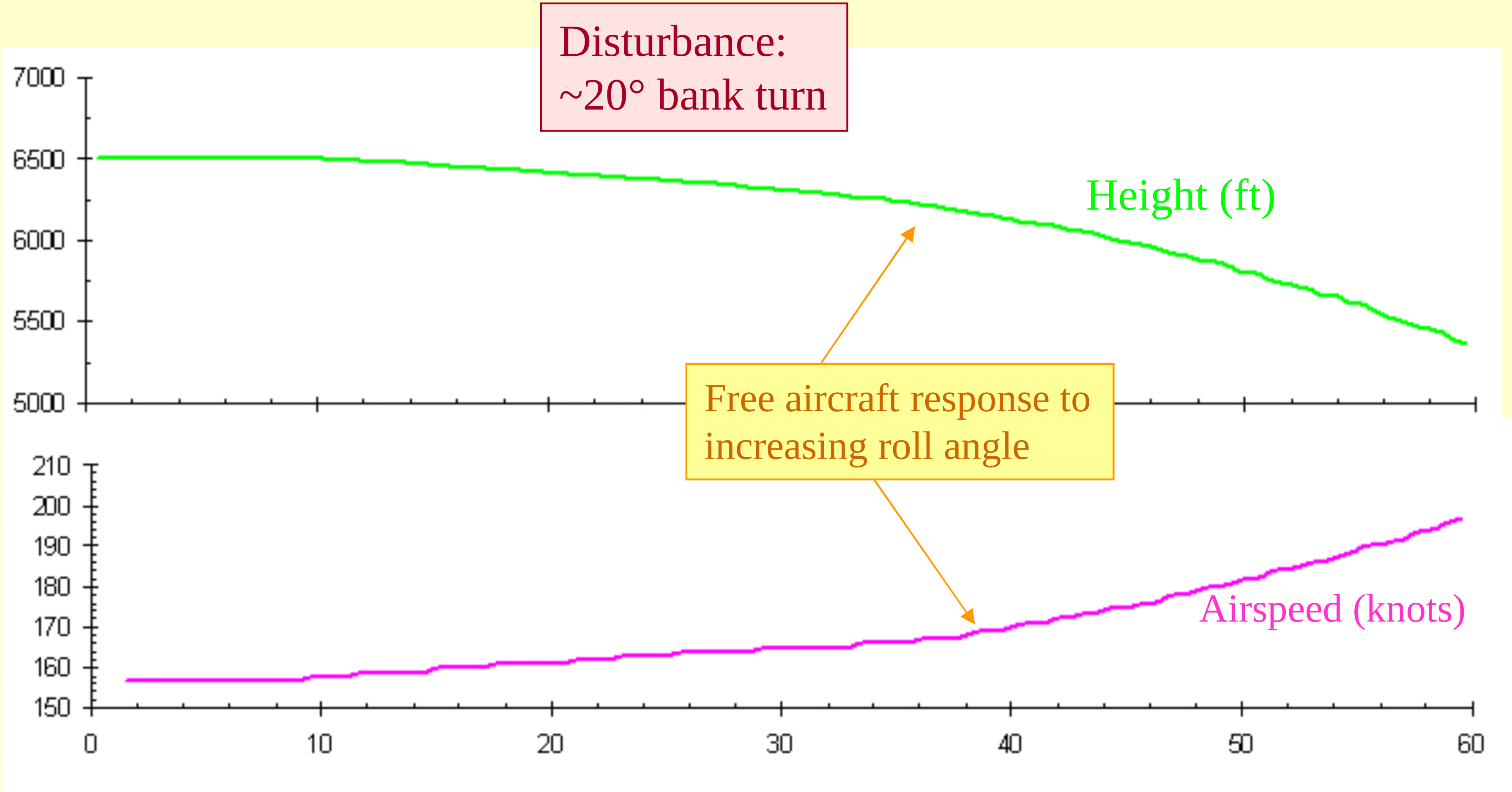
V. Spiral Mode

***Spiral Mode -** the way in which **roll angle** changes*

- Mode behaviour dependent on relative balance between lateral/directional stability

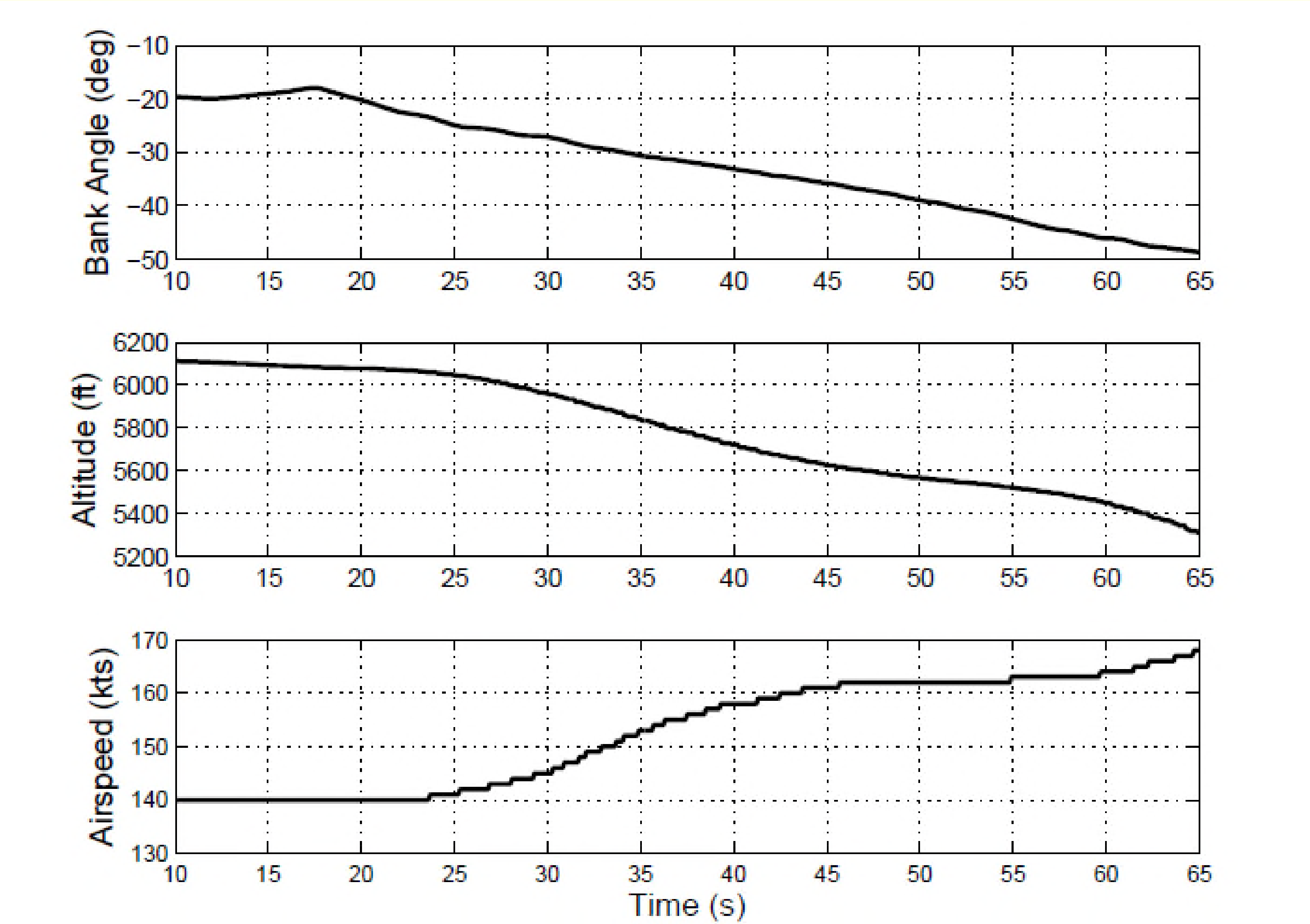


Spiral Mode



Disturbance:
~20° bank turn
Height (ft)
Free aircraft response to
increasing roll angle
Airspeed (knots)
7000
6500
6000
5500
5000
210
200
190
180
170
160
150
0
10
20
30
40
50
60

Spiral Mode



Bank Angle (deg)
−10
−20
−30
−40
−50
Altitude (ft)
6200
6000
5800
5600
5400
5200
Airspeed (kts)
170
160
150
140
130
10 15 20 25 30 35 40 45 50 55 60 65
Time (s)

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