AERO40003 Computing & Numerical Methods 1

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2023-24 Coursework brief (Part 1: MATLAB)

The aim of this coursework is for you to write a few more complex programs than what you typically have encountered so far in tutorials. Unlike most tutorial questions, you will have to come up with the structure of the program yourself. This is what programming will involve for the rest of your career after all.

The following brief presents the background information and instructions necessary for you to complete this part of the coursework. You must submit all the files (the `.m` files for these scripts/functions and figures) through this submission box in a `.zip` file.

The coursework will be marked based on completeness, readability, elegance, consistency, correctness and obviously on whether the programs run!

Rotors on Mars - A tribute to Ingenuity

It is hard to imagine that anyone will not know about Ingenuity, but for anyone who has been living under a rock, Ingenuity is a rotorcraft technology demonstrator that NASA/JPL used to successfully demonstrate flight on Mars. Ingenuity was launched with the rover Perseverance in July 2020 and landed on Mars in February 2021, stowed in the belly of Perseverance. On April 19, 2021, in flew for the first time, making history as the first flight on another planet.

Initially conceived as a technology demonstrator with up to 5 flights planned (a singe controlled flight would have been considered a success), it eventually went on to fly a total of 72 times, higher, farther and faster than anyone could have ever imagined. After 3 years of successful flights, its blades got damaged on January 2024, marking the end of one of NASA's most successful and exciting missions.

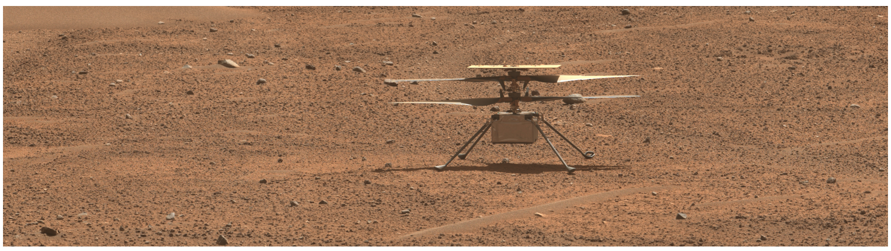

Figure 1: Ingenuity as seen by Perseverance, taken on August 2^{nd}, 2023 (courtesy of NASA/JPL).

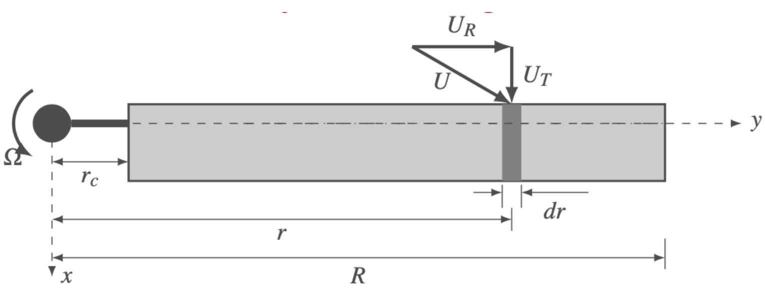

Figure 2: Blade element, top view of blade [1].

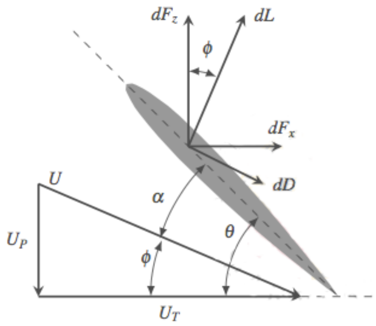

Figure 3: Sectional view of blade element [1].

1 Theoretical Background

1.1 Blade Element Theory

For this project, we will use a very simplified approach to the Blade Element Theory, limited to hover conditions only. This section will not show the full derivation of the method, but simply provide some background and the equations that you need to implement the method described in the implementation section below. The derivation in this section is extracted from [1].

Blade Element Theory (BET) is a method that assumes that each blade 2D section produces aerodynamic forces and moments that can then be integrated along the span of the blade and averaging over a rotor revolution to obtain the resultant forces and moments produced by the rotor. Despite its limitations, it offers a good starting point to evaluate the effect of the blade properties, such as the aerofoil characteristics, the blade twist or the planform shape, on the rotor performance.

Consider a 2D section of the rotor blade, a distance y from the centre of rotation, as shown in Figure 2. In hover we can ignore the radial component, which simplifies the problem. The sectional view of the blade element is shown in Figure 3. Any given section a distance y from the centre of rotation sees a flow velocity U:

$$U = \sqrt{U_T^2 + U_P^2} \tag{1}$$

In hover, U_T is due to the blade rotation, so $U_T = \Omega y$. The perpendicular component $U_P = V_c + v_i$, but in hover the climb velocity $V_c = 0$ so it is simply the induced velocity v_i. The angle that the velocity U forms with the plane of rotation is called the induced angle of attack and it is given by

$$\phi = \tan^{-1}\left(\frac{U_P}{U_T}\right) \tag{2}$$

The actual angle of attack seen by the blade section is therefore the blade pitch θ, which is formed by the local twist angle and the blade collective angle, minus the induced angle of attack, that is

$$\alpha = \theta - \phi = \theta_{twist} + \theta_{collective} - \phi \tag{3}$$

With the angle of attack known, we can now obtain the sectional lift and drag coefficients, which are provided in tables for several values of the Mach number ($M = U/a$, where a is the speed of sound). With the coefficients extracted, the sectional lift and drag per units span are obtained:

$$dL = \frac{1}{2}\rho U^2 c C_l dy \tag{4}$$

$$dD = \frac{1}{2}\rho U^2 c C_d dy \tag{5}$$

Resolving the lift and drag forces in the X and Z directions, and accounting for the contribution of all the blades, we then obtain the the differential thrust and power at this blade section:

$$dT = N_b(dL\cos\phi - dD\sin\phi) \tag{6}$$

$$dP = N_b(dL\sin\phi + dD\cos\phi)\Omega y \tag{7}$$

It is usually more convenient to work in non-dimensional terms. The thrust coefficient is defined as:

$$dC_T = \frac{N_b dT}{\rho A(\Omega R)^2} \tag{8}$$

where A is the rotor area, πR^2. After applying some small angle approximations and re-arranging, the thrust coefficient can also be expressed as:

$$dC_T = \frac{1}{2}\left(\frac{N_b c}{\pi R}\right) C_l r^2 dr \tag{9}$$

where $r = y/R$ and $dr = dy/R$.

The power coefficient is defined as:

$$dC_P = \frac{N_b dP}{\rho A(\Omega R)^3} \tag{10}$$

Again, with small angle assumptions and other simplifications it can also be written as:

$$dC_P = \frac{1}{2}\left(\frac{N_b c}{\pi R}\right)(\phi C_l + C_d)r^3 dr \tag{11}$$

Now these so far are the differential coefficients for a blade element of width dy (or dr in non-dimensional terms) located a distance y (or r) from the centre of rotation. For the total thrust and power, these values need to be integrated along the span. A simple rectangular integration will suffice, therefore:

$$C_T = \sum_{n=1}^{N} dC_{T_n} \tag{12}$$

$$C_P = \sum_{n=1}^{N} dC_{P_n} \tag{13}$$

Calculating the induced velocity v_i is usually one of the most complicated parts of BET. For the purpose of this project, we will assume that the induced velocities are uniform over the rotor and use the momentum theory solution to determine its value, that is

$$v_i = \sqrt{\frac{T}{2\rho A}} \tag{14}$$

So far, the equations described above have referred to a single rotor. For a coaxial helicopter with two equal and counter-rotating rotors, some adjustments need to be made. As there are two rotors, we assume each carries a fraction of the weight of the helicopter. Splitting the weight in half would lead to unequal power requirements on each rotor due to the fact that the lower rotor operates in the downwash of the upper rotor. To be perfectly accurate, one would model the the downwash of the upper rotor in the inner half of the lower rotor. But for simplicity, since we are assuming constant induced velocities, we will use the results from [2] to determine the fraction of the weight that each rotor carries, and calculate the induced velocity for that rotor accordingly. Therefore, the thrust of the upper rotor $T_u = 0.5897W$ and the thrust of the lower rotor is $T_l = 0.4103W$. As the values in [2] are derived with Momentum Theory rather than BET, the torque is not perfectly balanced, but it is close enough for the purpose of this exercise.

Finally, we define a parameter to measure the efficiency of a rotor in hover, the Figure of Merit (FM). The FM is defined as the ratio of the ideal power required by the rotor to the actual power, which includes induced and profile losses. The FM for a coaxial rotor [2] is defined as:

$$FM = \frac{\kappa_{int}\frac{C_{T_l}^{3/2}}{\sqrt{2}}\left[\left(\frac{C_{T_u}}{C_{T_l}}\right)^{3/2} + 1\right]}{\kappa_{int}\kappa\frac{C_{T_l}^{3/2}}{\sqrt{2}}\left[\left(\frac{C_{T_u}}{C_{T_l}}\right)^{3/2} + 1\right] + C_{P_p}} \tag{15}$$

where C_{T_u} and C_{T_l} are the thrust coefficients of the upper and lower rotor respectively, and C_{P_p} is the profile power coefficient, which is the drag contribution to the profile power, that is

$$dC_{P_p} = \frac{1}{2}\left(\frac{N_b c}{\pi R}\right) C_d r^3 dr \tag{16}$$

κ is the induced power factor and you can use 1.15. κ_{int} is the coaxial interference factor, which you can take as 1.2657 [2] .

2 The Mars Helicopter

The Mars Helicopter is a small coaxial helicopter, with two rotors, each spinning in one direction (to balance the torque). Each rotor has two blades, with a radius of 0.6 m. The rotors spin at 2600 rpm.

The Mars Helicopter is extremely light, with a mass of just 1.8 kg, to be able to fly in the extremely thin Martian atmosphere, which has approximately 1% of the density on the surface of Earth. The range of Reynolds numbers encountered by the rotor blades is very low, in the range of 10000 to 25000, and a custom aerofoil had to be developed, the cl5605 [4].

All the data you need is provided in several files that accompany this brief. You are given the following data/files:

planetdata.mat
: The planet environment data (density, gravity, etc); this contains more than one planet, so use the data for Mars for this problem.

MH_data.mat
: Some parameters describing the Mars Helicopter [4].

MH_chord.csv
: The Mars Helicopter non-dimensinalised chord distribution [3].

MH_twist.csv
: The Mars Helicopter twist distribution [3].

MH_cldata.csv
: The lift coefficient for the cl5605 aerofoil used by the Mars Helicopter, as a function of angle of attack, for different Mach numbers [3].

MH_cddata.csv
: The drag coefficient for the cl5605 aerofoil used by the Mars Helicopter, as a function of angle of attack, for different Mach numbers [3].

3 Objective

Your task is to write a script to calculate the C_T, C_P and Figure of Merit FM of the Martian helicopter rotor in hover using BET with different twist and chord distributions.

In the first instance, you will use the actual twist and chord distribution of the Mars Helicopter, as provided in the data files.

Then you will repeat the exercise, but using a simplified bilinear twist and chord distribution, given with three points as shown in Tables 1 and 2.

Finally, you will do a sensitivity study to explore the effect of changing some of these parameters on the FM. One at a time, you will change a given value in a range $\pm$ 10% around the values given in Tables 1 and 2. You will explore the sensitivity of the FM to the following 4 parameters (leaving all others at their values in the baseline case):

1. The location of the mid point radial location in the chord distribution, at station B1.
2. The value of the chord at Station B1.
3. The location of the mid point radial location in the twist distribution, at station B2.
4. The value of the twist at Station B2.

Station	Radial location (y/R)	Nondimensional chord, c/R
A	0.09	0.05
B1	0.34	0.2
C	1	0.07

Table 1: Simplified bilinear chord distribution. The chord varies linearly between Stations A and B1 and between stations B1 and C, with different slopes.

Station	Radial location (y/R)	Local twist angle, deg
A	0.09	16
B2	0.2	18
C	1	0

Table 2: Simplified bilinear twist distribution. The twist varies linearly between Stations A and B2 and between stations B2 and C, with different slopes.

4 Implementation

Please follow this steps intended to guide you in implementing the method described above. Remember to work on `.m` files. Your main file should drive the process and call all the necessary functions.

1. You must first load all the data programmatically (with code commands to read/parse it as needed). You are welcome to read the data with the method of your choice (*fread/fscanf*, *importdata*, etc) as long as you treat the datatypes consistently and your data is in one of the data types we have seen in this module (numeric arrays, cell arrays, structures). Be mindful of the type of file and its format when choosing how to load it. You can use a different approach for each type of file as you find suitable.

2. Add the lift and drag coefficient data to the helicopter structure (*veh*) by creating fields to store them.

3. Discretise the blade into N equidistant points and store into an array of radial positions. Do not start at 0, as the blade has a cutout near the root, instead assume the blade starts at $0.09R$. You can choose the number of points, but at least use 20 (finer discretisations are more accurate, but also computationally more expensive).

4. Interpolate the chord and twist values at each of the radial stations defined above based on the current chord/twist case (actual MH, baseline bilinear case or other modified bilinear). You can use MATLAB *interp1* function for this. Add fields to the structure containing the helicopter data (*veh*) for the radial stations, twist and chord values and store the values you have obtained.

5. With the updated (*veh*) structure and the environment structure containing the Mars data, call your BET function (see below) to calculate the thrust coefficient, C_T, pressure coefficient C_P and Figure of Merit FM.

6. Repeat the previous two steps with each twist and chord distribution (the baseline bilinear case and the four sensitivity studies).

7. Plot the required output. Format your figures and label them appropriately. Save them programmatically. You must provide the following plots:

   - The lift coefficient vs. angle of attack, for different values of M in the same figure.
   - The drag coefficient vs. angle of attack, for different values of M in the same figure.
   - The original chord distribution and the baseline bilinear chord distribution, in the same figure.
   - The original twist distribution and the baseline bilinear twist distribution, in the same figure.
   - The FM vs. the four variables in the sensitivity study, each in a subplot of the same figure. Include the FM of the actual MH and the baseline bilinear cases for reference.
   - The C_P vs. the four variables in the sensitivity study, each in a subplot of the same figure. Include the C_P of the actual MH and the baseline bilinear cases for reference.

Your function should take the vehicle and environment structures as input ((*veh* and *env*, all the data should be contained within the two structures in additional fields as needed) and use BET to calculate and return as output the thrust coefficient, the power coefficient and the figure of merit. You may follow the method described below:

1. Calculate the induced velocity for the upper rotor using the appropriate thrust.
2. Iterate over all the radial stations. At each station:
   - Calculate the local induced angle, the angle of attack and Mach number.
   - Interpolate the lift and drag coefficients for the current angle of attack and Mach number. You can use MATLAB's *interp1* function. You can pick the nearest value of the Mach number instead of double interpolating. For any values of the angle of attack outside of the range, use the nearest value (do not extrapolate).
   - Calculate the current element's thrust and power coefficients.
   - Calculate the total thrust and power coefficients for this rotor.
3. Repeat the previous steps with the lower rotor, noting that the induced velocity calculation carries a different weight fraction for the thrust.
4. Add the total thrust and power coefficients for the coaxial system.
5. Calculate the Figure of Merit.

5 What to submit

You should submit the following items in a **folder** called `CW_Pt1_username`, replacing username with your own, and zip the folder before uploading:

- The main script to run the entire simulation - you may name it as you wish but let it be clear it's the main file (i.e. CWmain.m) .
- The hover coaxial BET function and any other functions you have created.
- The data files given to you (so the code can run directly on the folder with everything it needs).
- Your saved plots.

6 Formatting guidelines

As you have more time for this project, it is expected that you will put some care into presenting both your code and plots neatly and professionally. Please refer to the mark scheme to make sure your code complies with the expectations we will be looking when marking. Correctness is indeed important.

But other criteria such as readability or elegance are also important.

For this exercise, it is recommended that you work on an `.m` file. Save the file with some meaningful name. Parts of your code can be provided in functions, which should also be named meaningfully so the markers can identify them. Functions might make your code more elegant and readable, but only one is required.

Additionally, the following criteria are expected for the formatting of the graphs:

- Label your axes and add titles to your plots.
- Where necessary, add a legend.
- Make the lines and markers in your plots of thickness 2.
- Adjust the font size in the plots so that they are easy to read.

7 Final tips

- Be careful with your units! Specially be mindful when angles are in radians or degrees.
- Also be mindful when data is in meters or non-dimensionalised with the radius, both in the data or in equations.

References

[1] J Gordon Leishman. *Principles of helicopter aerodynamics*. Cambridge University Press Cambridge, Cambridge, 2nd ed edition, 2006.

[2] J Gordon Leishman and Monica Syal. Figure of Merit Definition for Coaxial Rotors. *Journal of the American Helicopter Society*, 53(3):290–300, 2008.

[3] Witold J.F. Koning, Wayne Johnson, and Brian G. Allan. Generation of mars helicopter rotor model for comprehensive analyses. *Proceedings of the AHS International Technical Meeting on Aeromechanics Design for Transformative Vertical Flight 2018*, 2018.

[4] Shannah Withrow, Wayne Johnson, Larry A. Young, Witold Koning, Winnie Kuang, Carlos Malpica, J. Balaram, and Theodore Tzanetos. Mars Science Helicopter Conceptual Design. Technical Report NASA/TM—2020–220485, NASA, 2020.