Topic 4

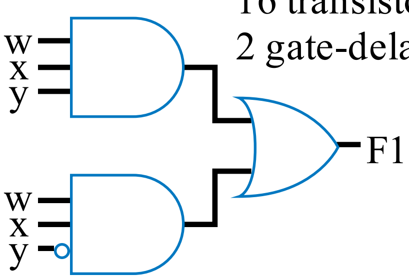
Logic Optimization

Simplification and Optimization

- **How can we build better circuits?**
- **Let's consider two important design criteria**
  - ***Delay* –** the time from inputs changing to new correct stable output
  - ***Size* –** the number of transistors
  - For quick estimation, assume
    - Every gate has delay of "1 gate-delay"
    - Every gate *input* requires 2 transistors
    - Ignore inverters for simplicity

Transforming F1 to F2 represents an ***optimization***: Better in all criteria of interest

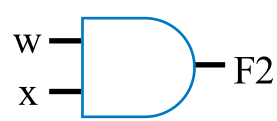
16 transistors
2 gate-delays



F1 = wxy + wxy'

(a)

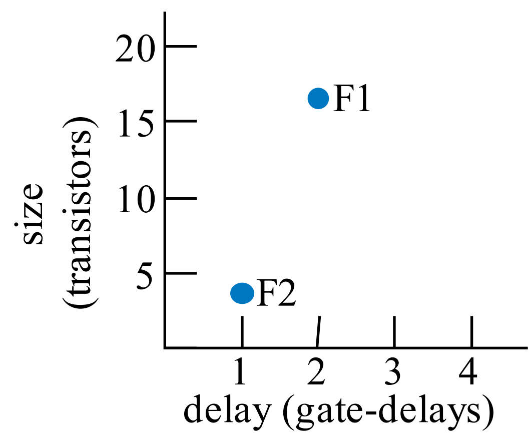
4 transistors
1 gate-delay



F2 = wx

(b)

= wx(y+y') = wx



(c)

Logic Optimization

- **Two-level size optimization using algebraic methods**
  – Goal: circuit with only two levels (AND-OR network), with minimum transistors
  – Sum-of-products yields two levels
    - F = abc + abc' is sum-of-products
    - G = w(xy + z) is not

Example

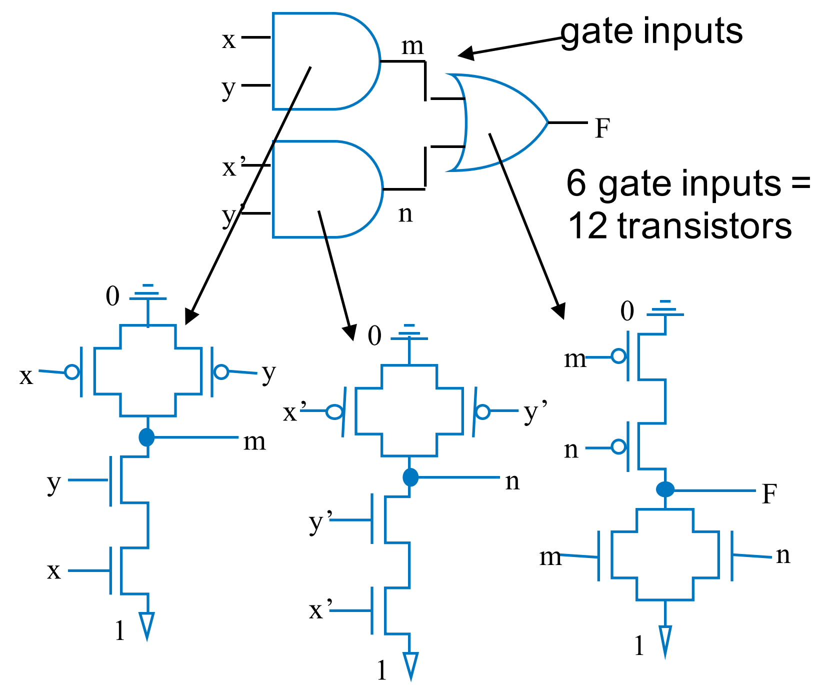
F = xyz + xyz' + x'y'z' + x'y'z

F = xy(z + z') + x'y'(z + z')

F = xy*1 + x'y'*1

F = xy + x'y'

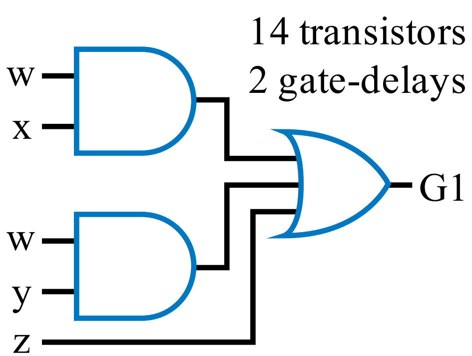
4 literals + 2 terms = 6 gate inputs



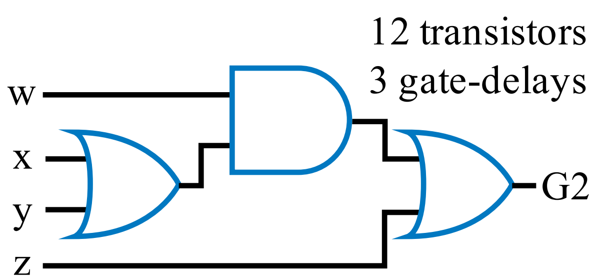
Logic Optimization

- **Multi-level optimization**
  - Improves some, but worsens other

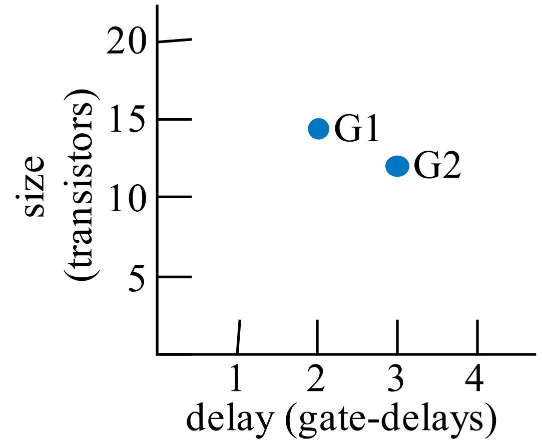
Transforming G1 to G2 represents a ***tradeoff***: Some criteria better, others worse



G1 = wx + wy + z

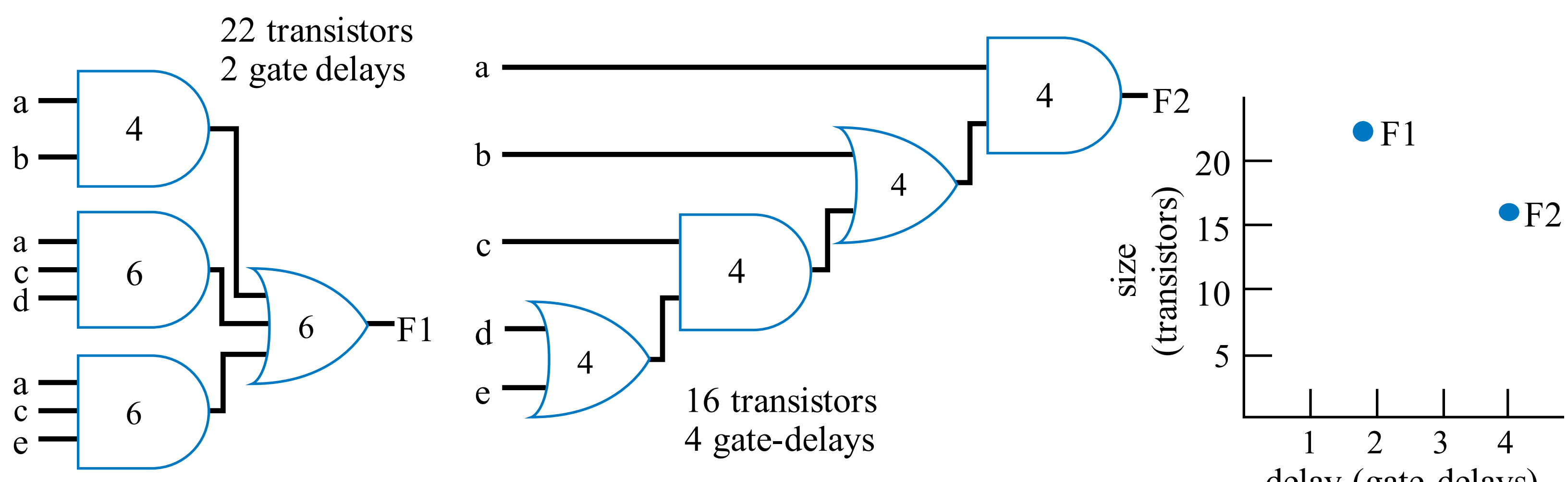


G2 = w(x+y) + z



Performance/Size Tradeoffs

- **Delay & Size tradeoff**
  - We don't always need the speed of two level logic
  - Multiple levels may yield fewer gates
  - Example
    - F1 = ab + acd + ace → F2 = ab + ac(d + e) = a(b + c(d + e))
    - General technique: Factor out literals



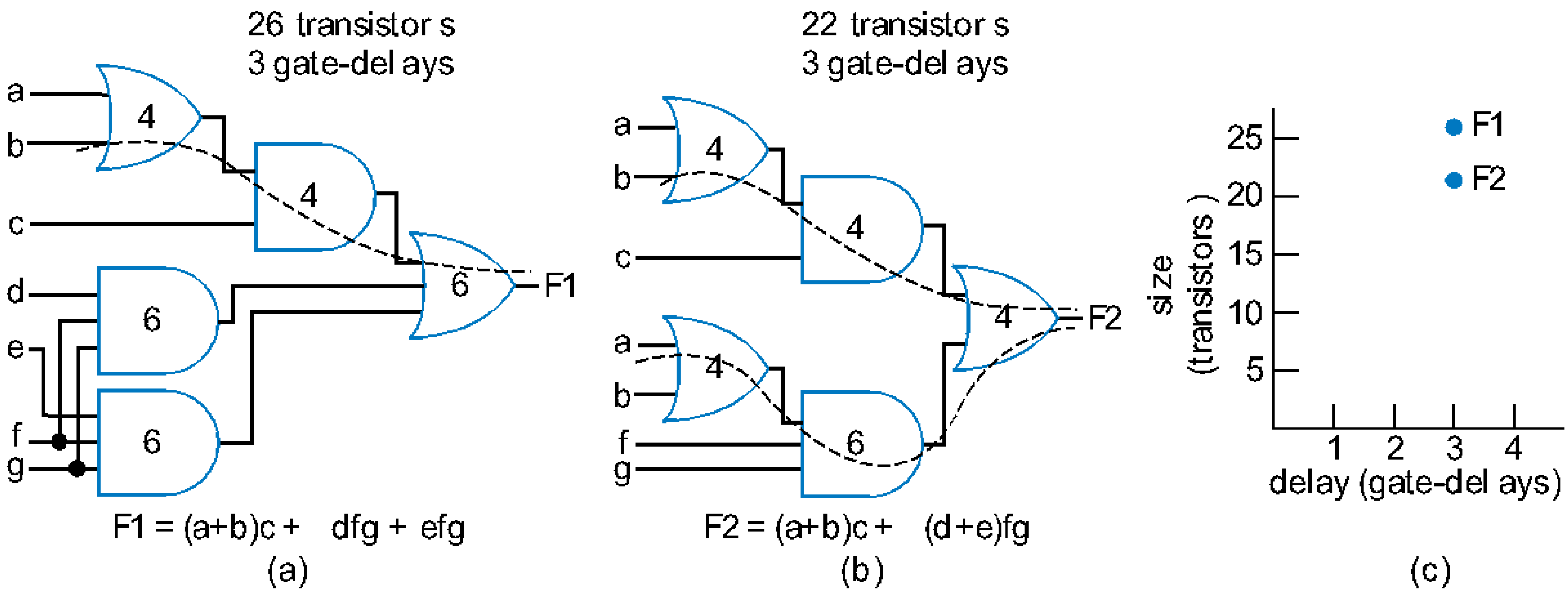
F1 = ab + acd + ace
(a)

F2 = a(b+c(d+e))
(b)

(c)

Critical Path

- **Critical path: longest delay path from an input to output**
- **Optimization**
  - Reduce delay by shortening length of critical path
  - Reduce size by using multiple levels on non-critical paths
    - But may make non-critical path become critical path

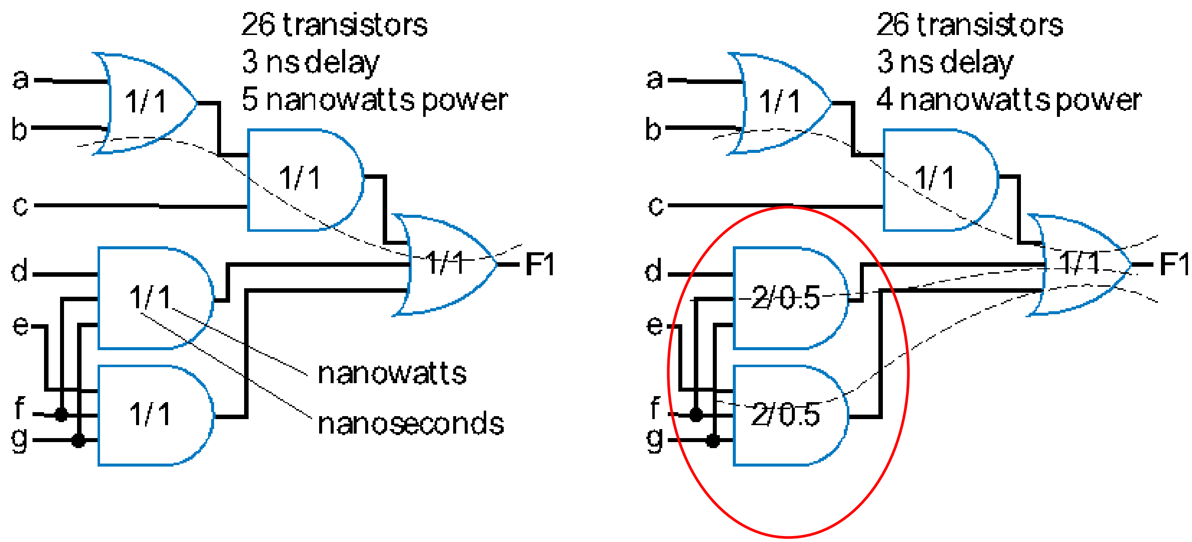


Power Optimization

- **Power is another important design criteria**
  – Measured in Watts (energy/second)
    - Rate at which energy is consumed
- **Increasingly important as more transistors on a chip**
  – Power not scaling down at same rate as size
    - cooling is difficult
  – CMOS technology: Switching a wire from 0 to 1 consumes power (known as ***dynamic power***)
    - $P = k * CV^2 f$
      – k: constant; C: capacitance of wires; V: voltage; f: switching frequency
    - Power reduction methods
      – Reduce voltage: But slower, and there's a limit
      – What else?

Using Low-Power Gates on Non-Critical Paths

- **Another method: Use low-power gates**
  - Multiple versions of gates may exist
    - Fast/high-power, and slow/low-power, versions
  - Use slow/low-power gates on non-critical paths
    - Reduces power, without increasing delay



Logic Optimization

- **optimization using other techniques**
  - Karnaugh-map (later)
  - Quine-McCluskey (not in this class)

Karnaugh Map (K-map) Technique

- **A graphical technique used to *simplify* a logic equation**
- **A way to show the *relationship* between the logic inputs and corresponding output**
  - Like truth table
- **Much cleaner and more *procedural* than algebraic simplification by theorems of Boolean algebra**
- **Theoretically, it can be used for any number of input variables,**
  - BUT is only practical for less than six, we will limit our discussion to logic equations with *five or less* variables

Building a K-map

- **K-map can be filled up directly from a truth table**
  - Each minterm corresponds to a cell in the K-map
- **K-map cells are labeled so that both horizontal and vertical movement differ only in one variable**

F (X \ Y)	**Y’**	**Y**
X’	m0	m1
X	m2	m3

F (X \ YZ)	**Y’Z’**	**Y’Z**	**YZ**	**YZ’**
X’	m0	m1	m3	m2
X	m4	m5	m7	m6

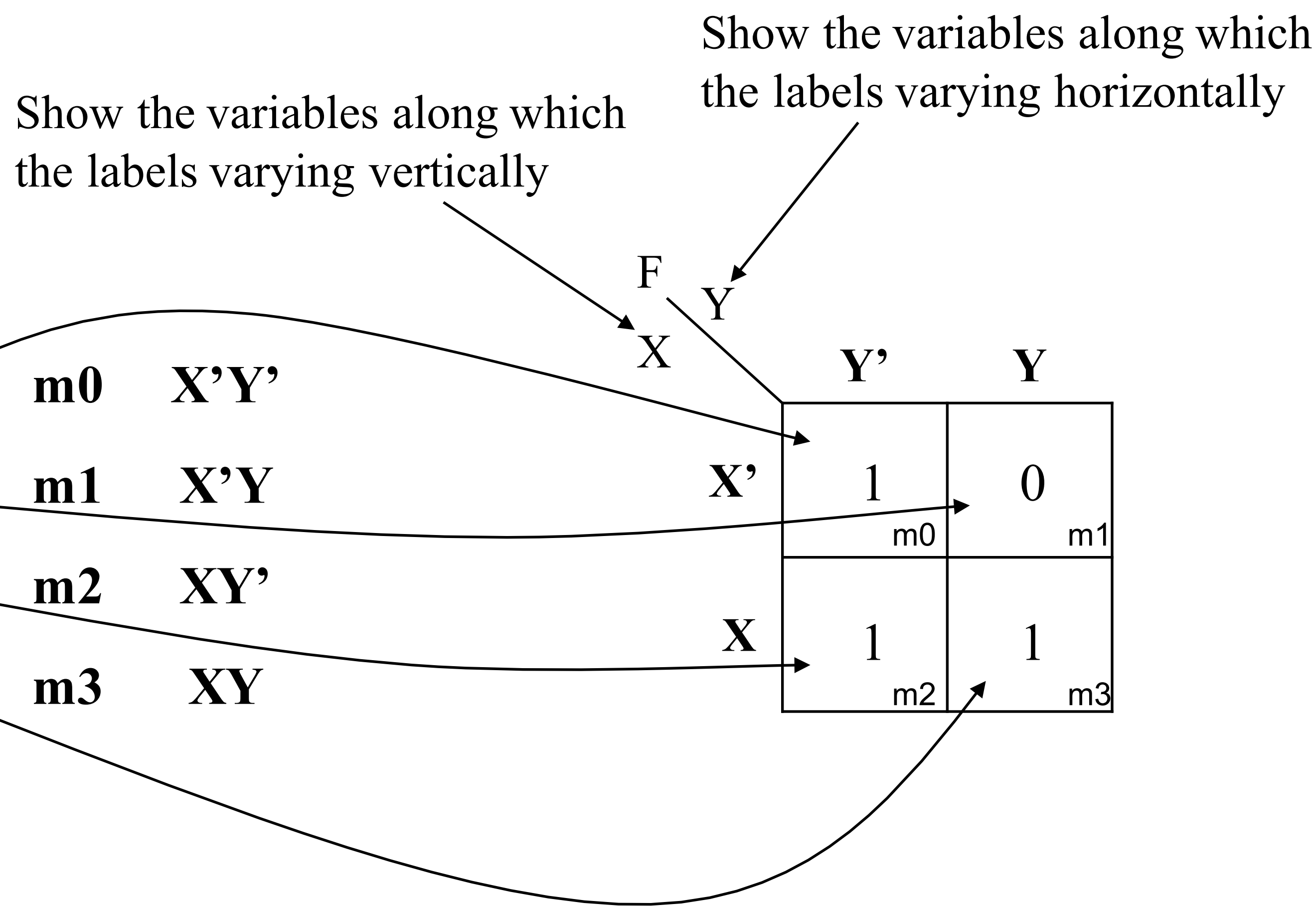
F (WX \ YZ)	**Y’Z’**	**Y’Z**	**YZ**	**YZ’**
W’X’	m0	m1	m3	m2
W’X	m4	m5	m7	m6
WX	m12	m13	m15	m14
WX’	m8	m9	m11	m10

- **Since the adjacent cells differ in only one variable, they can be grouped to create simpler terms in the sum-of-product expression.**

Two-Variable K-map

- **There are four minterms – 2 by 2 square map**

X	Y	F		
0	0	1	m0	X’Y’
0	1	0	m1	X’Y
1	0	1	m2	XY’
1	1	1	m3	XY



Three-Variable K-map

- **There are $2^3 = 8$ minterms – 2 by 4 rectangular map**

Show the variables along which the labels varying vertically

Show the variables along which the labels varying horizontally

X	Y	Z	F		
0	0	0	1	m0	X'Y'Z'
0	0	1	0	m1	X'Y'Z
0	1	0	1	m2	X'YZ'
0	1	1	1	m3	X'YZ
1	0	0	0	m4	XY'Z'
1	0	1	0	m5	XY'Z
1	1	0	0	m6	XYZ'
1	1	1	1	m7	XYZ

X \ YZ	Y'Z'	Y'Z	YZ	YZ'
X'	1 m0	0 m1	1 m3	1 m2
X	0 m4	0 m5	1 m7	0 m6

Four-Variable K-map

- **There are 2^4=16 minterms – 4 by 4 square map**

W	X	Y	Z	F		
0	0	0	0	1	m0	W'X'Y'Z'
0	0	0	1	0	m1	W'X'Y'Z
0	0	1	0	1	m2	W'X'YZ'
0	0	1	1	1	m3	W'X'YZ
0	1	0	0	0	m4	W'XY'Z'
0	1	0	1	0	m5	W'XY'Z
0	1	1	0	0	m6	W'XYZ'
0	1	1	1	1	m7	W'XYZ
1	0	0	0	1	M8	WX'Y'Z'
1	0	0	1	1	m9	WX'Y'Z
1	0	1	0	0	M10	WX'YZ'
1	0	1	1	0	m11	WX'YZ
1	1	0	0	1	m12	WXY'Z'
1	1	0	1	0	m13	WXY'Z
1	1	1	0	0	m14	WXYZ'
1	1	1	1	1	m15	WXYZ

Show the variables along which the labels varying vertically or horizontally

WX \ YZ	Y'Z'	Y'Z	YZ	YZ'
W'X'	1 m0	0 m1	1 m3	1 m2
W'X	0 m4	0 m5	1 m7	0 m6
WX	1 m12	0 m13	1 m15	0 m14
WX'	1 m8	1 m9	0 m11	0 m10

Label the Rows and Columns by 0 and 1

F: X \ Y	Y' 0	Y 1
X' 0	1 m0	0 m1
X 1	1 m2	1 m3

F: X \ YZ	Y'Z' 00	Y'Z 01	YZ 11	YZ' 10
X' 0	1 m0	0 m1	1 m3	1 m2
X 1	0 m4	0 m5	1 m7	0 m6

F: WX \ YZ	Y'Z' 00	Y'Z 01	YZ 11	YZ' 10
W'X' 00	1 m0	0 m1	1 m3	1 m2
W'X 01	0 m4	0 m5	1 m7	0 m6
WX 11	1 m12	0 m13	1 m15	0 m14
WX' 10	1 m8	1 m9	0 m11	0 m10

0 represents the primed form
1 represents the unprimed form

Simplify – Grouping and Canceling

- **Group is in shape of rectangle or square**
- **Group the adjacent 1's until all the 1's are grouped**

F (X \ Y)	0	1
0	1	0
1	0	0

X'Y'

No adjacent 1's, the minterm cannot be further simplified:
F = X'Y'

F (X \ Y)	0	1
0	1	1
1	0	0

X'

Two adjacent 1's:

$$
\begin{aligned}
F &= X'Y' + X'Y \\
&= X'(Y' + Y) \\
&= X' \cdot 1 \\
&= X'
\end{aligned}
$$

If both primed and unprimed forms of a letter appear in the same group, the letter can be canceled

A group corresponds to a Sum-of-Minterm expression

Grouping and Canceling

- **No zeros in the group**

F X\YZ	00	01	11	10
0	0	1	1	0
1	0	0	1	0

F X\YZ	00	01	11	10
0	0	1	1	0
1	0	0	1	0

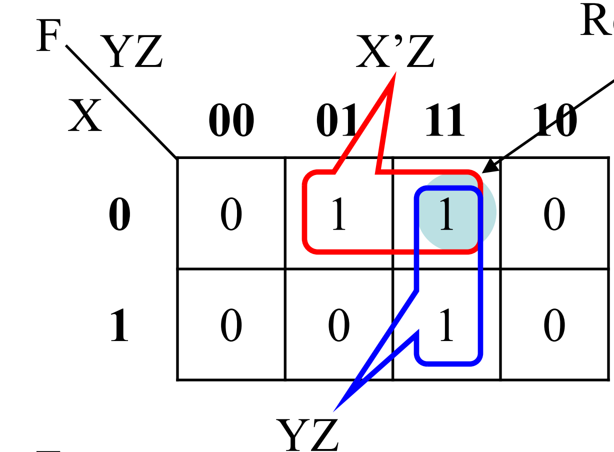
- **The number of 1's in the group should be 2^N, N = 0, 1, 2, …**

F X\YZ	00	01	11	10
0	1	1	1	0
1	0	0	0	0

F X\YZ	00	01	11	10
0	1	1	1	0
1	0	0	0	0

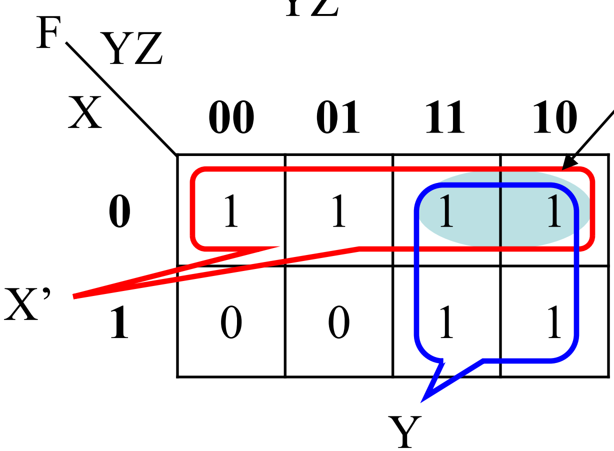
Grouping and Canceling

- **Group as many adjacent 1's as possible**



Redundant term

$$F = X'Y'Z + X'YZ + XYZ$$
$$= (X'Y'Z + X'YZ) + (X'YZ + XYZ)$$
$$= X'Z\,(Y' + Y) + YZ\,(X' + X)$$
$$= X'Z + YZ$$



Redundant terms

$$F = X'Y'Z' + X'Y'Z + X'YZ + X'YZ' + XYZ + XYZ'$$
$$= (X'Y'Z' + X'Y'Z + \cancel{X'YZ + X'YZ'}) + (X'YZ + X'YZ' + XYZ + XYZ')$$
$$= (X'Y' + X'Y) + (X'Y + XY)$$
$$= X' + Y$$

Grouping and Canceling

- **Group as many adjacent 1's as possible**

F: WX \ YZ	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	0	1	0
10	1	0	0	0

W', because
horizontally, both Y and Y' appear, Y cancels
both Z and Z' are included, Z cancels;
vertically, both X and X' appear, X cancels.
That leaves the 0 for W – primed W.

XYZ, because
only W and W' appear, W cancels.
The remaining term 111 implies XYZ

Y'Z', because
both W and W' appear, and both X and X' appear,
So W and X cancel.
That leaves the 00 for YZ – primed Y and primed Z.

Grouping and Canceling

- **Edges wrap around**

F (X \ YZ)	00	01	11	10
0	1	0	0	1
1	1	0	0	1

F = Z’

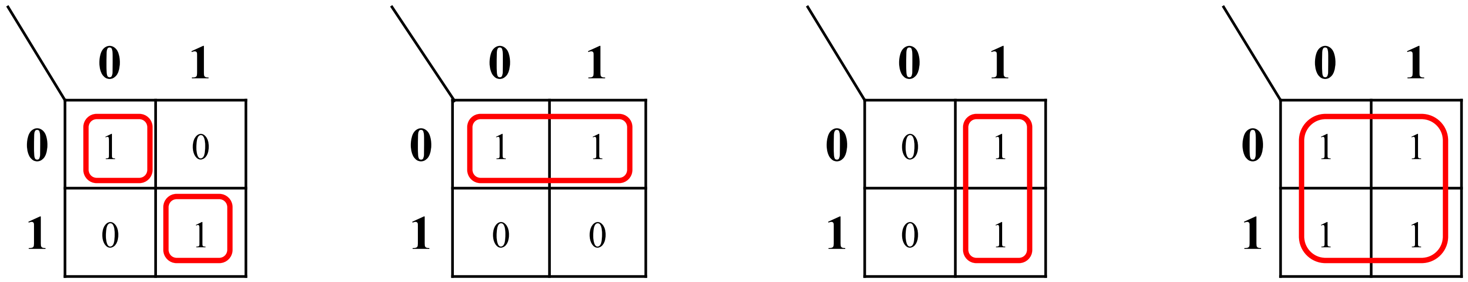
F (WX \ YZ)	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	0	0	0	0
10	1	0	0	1

F = W’Z’ + X’Z’

Grouping and Canceling

- **Summary**
  - Group is in shape of rectangle or square
  - Group the adjacent 1's until all the 1's are grouped
  - The number of 1's in the group should be 2^N, N = 0, 1, 2, …
  - Collect as many 1's as possible in the same group
  - No zeros in the group
  - Edges wrap around
  - If both primed and unprimed forms of a letter appear in a same group, the letter cancels
  - The simplified result will be a sum-of-product form; the number of the product terms is decided by the number of the groups

Group Patterns of 2-Variable Map



- **Summary**
  – A group of one cell represents a minterm, giving a term of two literals
  – A group of two cells represents a term of one literal
  – A group of all the four cells gives a logic 1

Group Patterns of 3-Variable Map

	00	01	11	10
0	1	0	1	0
1	0	1	0	1

	00	01	11	10
0	1	1	1	0
1	0	0	1	1

	00	01	11	10
0	1	1	0	1
1	1	1	1	

	00	01	11	10
0	1	1	1	1
1	1	0	0	1

	00	01	11	10
0	1	1	1	1
1	1	1	1	0

	00	01	11	10
0	1	1	1	1
1	1	1	1	1

Group Patterns of 3-Variable Map

- **Summary**
  - A group of one cell represents a minterm, giving a term of three literals
  - A group of two cells represents a term of two literals
  - A group of four cells represents a term of one literal
  - A group of all the eight cells gives a logic 1

Group Patterns of 4-Variable Map

	00	01	11	10
00	1	1	1	0
01	0	1	0	1
11	0	0	1	0
10	0	0	0	0

	00	01	11	10
00	0	1	0	1
01	1	0	0	1
11	0	0	0	0
10	0	1	0	1

	00	01	11	10
00	1	1	1	1
01	1	0	0	0
11	1	1	1	1
10	1	0	0	0

	00	01	11	10
00	1	1	1	0
01	1	1	1	0
11	1	1	0	0
10	0	0	0	0

	00	01	11	10
00	1	1	0	1
01	1	0	0	1
11	0	0	0	0
10	1	1	0	1

	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	0	1	1	0
10	0	1	1	0

	00	01	11	10
00	1	1	1	1
01	1	0	0	1
11	1	0	0	1
10	1	1	1	1

	00	01	11	10
00	1	1	1	0
01	1	1	1	0
11	1	1	1	0
10	1	1	1	0

	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	0	1	0
10	1	0	0	0

	00	01	11	10
00	1	0	1	1
01	1	0	0	1
11	1	0	1	0
10	1	0	0	0

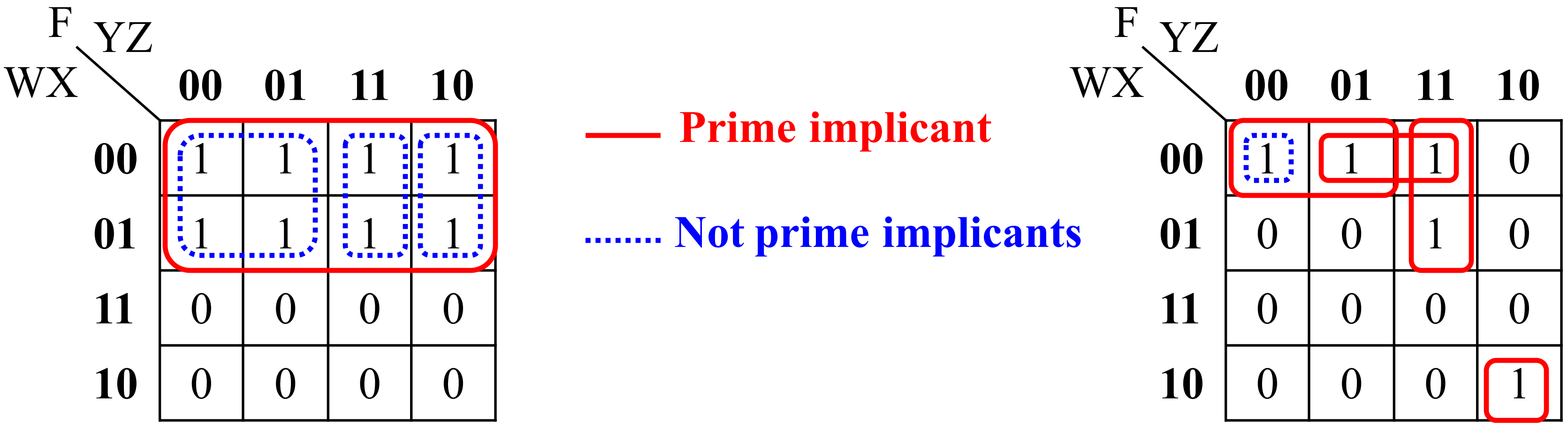
	00	01	11	10
00	0	1	0	1
01	0	1	0	1
11	1	0	1	0
10	1	0	0	1

Group Patterns of 4-Variable Map

- **Summary**
  - A group of one cell represents a minterm, giving a term of four literals
  - A group of two cells represents a term of three literals
  - A group of four cells represents a term of two literals
  - A group of eight cells represents a term of one literal
  - A group of all the sixteen cells gives a logic 1
- **The more the number of cells in one group, the less the number of literals that group represents, hence cheaper to implement using logic gates**

Prime Implicants

- Implicant: is a product term
- A **prime implicant (PI)** is a group that cannot be entirely contained by another implicant



Essential Prime Implicants

- A **prime implicant (PI)** is **essential** if a a cell is covered **ONLY** by that PI
- The **essential PIs** can be found by
  - looking at each cell marked as 1 and not covered by any other essential PI
  - and checking the number of PIs that cover it

F WX \ YZ	**00**	**01**	**11**	**10**
00	1	0	1	1
01	0	1	1	0
11	0	1	1	0
10	1	1	1	1

Essential Prime Implicants

- **Check each cell marked as 1, only if it has not been covered by an essential PI**

F: WX \ YZ	00	01	11	10
00	***1***	0	1	1
01	0	1	1	0
11	0	1	1	0
10	1	1	1	1

Essential PI: X'Z'

F: WX \ YZ	00	01	11	10
00	1	0	***1***	1
01	0	1	1	0
11	0	1	1	0
10	1	1	1	1

No essential PIs found

Essential Prime Implicants

F: WX \ YZ	00	01	11	10
00	1	0	1	1
01	0	*1*	1	0
11	0	1	1	0
10	1	1	1	1

Essential PI: XZ

F: WX \ YZ	00	01	11	10
00	1	0	1	1
01	0	1	1	0
11	0	1	1	0
10	1	*1*	1	1

No essential PIs found

Essential Prime Implicants

F (WX \ YZ)	00	01	11	10
00	1	0	1	1
01	0	1	1	0
11	0	1	1	0
10	1	1	*1*	1

No essential PIs found

Essential Prime Implicants

F (WX \ YZ)	00	01	11	10
00	1	0	**1**	1
01	0	1	1	0
11	0	1	1	0
10	1	**1**	**1**	1

Essential PIs

F (WX \ YZ)	00	01	11	10
00	1	0	**1**	1
01	0	1	1	0
11	0	1	1	0
10	1	**1**	**1**	1

Non essential PIs

- **Essential PIs have to be used in the simplified equation**
- **Cells not covered by essential PIs can be represented by any PIs covering them**

F = X'Z' + XZ + WX'(or WZ) + X'Y(or YZ)

Product-of-Sum Simplification – An Alternate Method

- **Redraw the K-map for F' by switching 1's and 0's**

W	X	Y	Z	F	F'
0	0	0	0	1	0
0	0	0	1	1	0
0	0	1	0	1	0
0	0	1	1	1	0
0	1	0	0	0	1
0	1	0	1	0	1
0	1	1	0	0	1
0	1	1	1	1	0
1	0	0	0	1	0
1	0	0	1	1	0
1	0	1	0	0	1
1	0	1	1	0	1
1	1	0	0	1	0
1	1	0	1	0	1
1	1	1	0	0	1
1	1	1	1	0	1

F

WX \ YZ	**00**	**01**	**11**	**10**
00	1	1	1	1
01	0	0	1	0
11	1	0	0	0
10	1	1	0	0

F'

WX \ YZ	**00**	**01**	**11**	**10**
00	0	0	0	0
01	1	1	0	1
11	0	1	1	1
10	0	0	1	1

Product-of-Sum Simplification – An Alternate Method

- **Two forms of the same truth table**

F: WX \ YZ	00	01	11	10
00	1	1	1	1
01	0	0	1	0
11	1	0	0	0
10	1	1	0	0

Sum-of-Product form:

$F = \underline{W'X'} + \underline{X'Y'} + \underline{WY'Z'} + \underline{W'YZ}$

F': WX \ YZ	00	01	11	10
00	0	0	0	0
01	1	1	0	1
11	0	1	1	1
10	0	0	1	1

$F' = \underline{WY} + XY'Z + W'XZ'$

Product-of-Sum form:

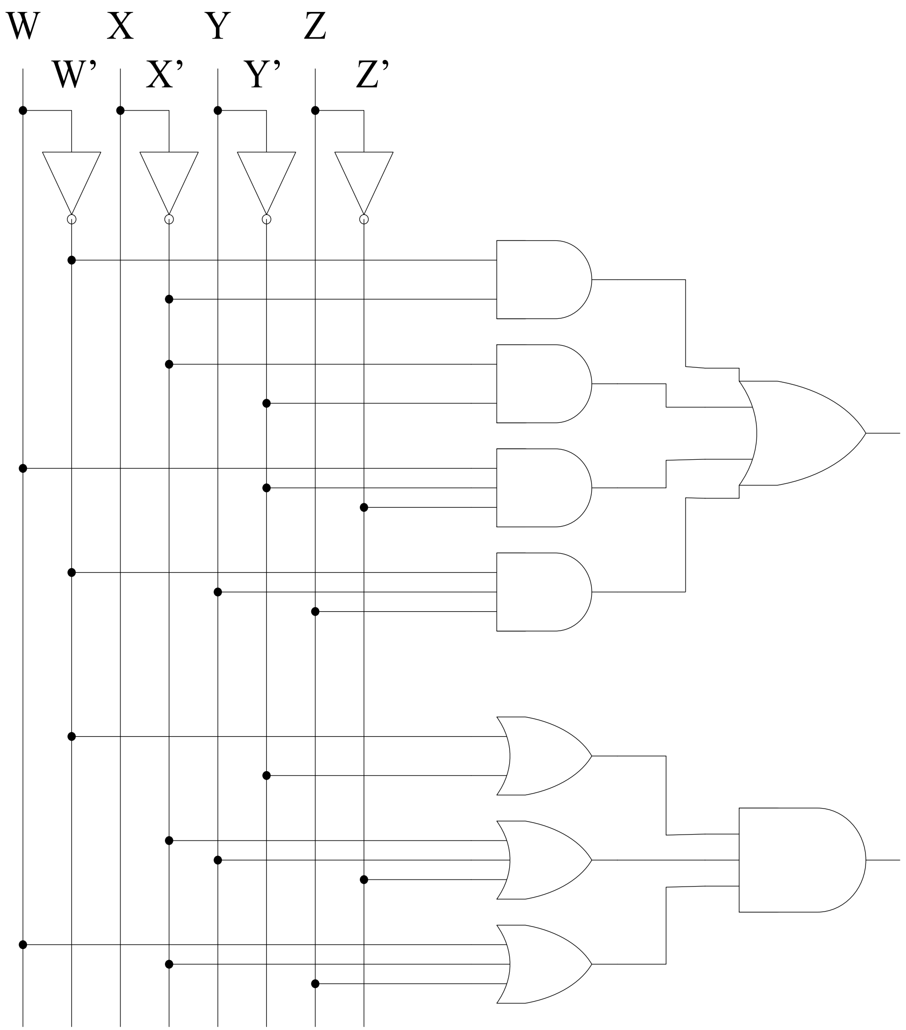
$F = (F')'$ (**DeMorgan's Law**)

$= (WY + XY'Z + W'XZ')'$

$= (WY)'\,(XY'Z)'\,(W'XZ')'$

$= (W'+Y')(X'+Y+Z')(W+X'+Z)$

Product-of-Sum Simplification – An Alternate Method



F = W'X' + X'Y' + WY'Z' + W'YZ

F = (W'+Y')(X'+Y+Z')(W+X'+Z)

Simpler

Simplify Any Standard Sum-of-Product Form

Method 1: fill out the table directly

- **F = A'C + A'BD + AB'C + BCD**

A	B	C	D	F	
0	0	0	0	0	**m0**
0	0	0	1	0	**m1**
0	0	1	0	1	**m2**
0	0	1	1	1	**m3**
0	1	0	0	0	**m4**
0	1	0	1	1	**m5**
0	1	1	0	1	**m6**
0	1	1	1	1	**m7**
1	0	0	0	0	**m8**
1	0	0	1	0	**m9**
1	0	1	0	1	**m10**
1	0	1	1	1	**m11**
1	1	0	0	0	**m12**
1	1	0	1	0	**m13**
1	1	1	0	0	**m14**
1	1	1	1	1	**m15**

F

AB \ CD	00	01	11	10
00	0	0	1	1
01	0	1	1	1
11	0	0	1	0
10	0	0	1	1

Simplify Any Standard Sum-of-Product Form

- **F = A’C + A’BD + AB’C + BCD**
  - **Method 2: convert any form of equation to sum-of-minterm**
    - AND with sum of the primed and unprimed forms of the missing literal, one at a time until all the missing literals are considered
    - Remove the duplicated minterms

F = A’C + A’BD + AB’C + BCD

= A’C (B+B’) + A’BD (C+C’) + AB’C (D+D’) + BCD (A+A’)

= A’BC + A’B’C + A’BCD + A’BC’D + AB’CD + AB’CD’ + ABCD + A’BCD

= A’BC (D+D’) + A’B’C (D+D’) + A’BCD + A’BC’D + AB’CD + AB’CD’+ABCD+A’BCD

= A’BCD + A’BCD’ + A’B’CD + A’B’CD’ + A’BCD + A’BC’D + AB’CD + AB’CD’+ABCD+A’BCD

$= \Sigma\ m(7, 6, 3, 2, 7, 5, 11, 10, 15, 7)$

$= \Sigma\ m(2, 3, 5, 6, 7, 10, 11, 15)$

Simplify Any Standard Sum-of-Product Form

- F = A'C + A'BD + AB'C + BCD

A	B	C	D	F	
0	0	0	0	0	m0
0	0	0	1	0	m1
0	0	1	0	1	*m2*
0	0	1	1	1	*m3*
0	1	0	0	0	m4
0	1	0	1	1	*m5*
0	1	1	0	1	*m6*
0	1	1	1	1	*m7*
1	0	0	0	0	m8
1	0	0	1	0	m9
1	0	1	0	1	*m10*
1	0	1	1	1	*m11*
1	1	0	0	0	m12
1	1	0	1	0	m13
1	1	1	0	0	m14
1	1	1	1	1	*m15*

F, AB \ CD	00	01	11	10
00	0	0	1	1
01	0	1	1	1
11	0	0	1	0
10	0	0	1	1

After simplification:

F = A'C + A'BD + B'C + CD

(Essential PI?)

Don’t Care Conditions

- The possible input combinations might not be all valid or not for consideration for a device
  - Hence we don’t care what the corresponding outputs are under those conditions
  - Called **don’t care** conditions
  - Mark the corresponding outputs by **X**

A	B	C	D	F
0	*0*	*0*	*0*	*X*
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	*0*	*0*	*1*	*X*
1	0	1	0	1
1	0	1	1	1
1	*1*	*0*	*0*	*X*
1	*1*	*0*	*1*	*X*
1	*1*	*1*	*0*	*X*
1	1	1	1	1

Don't Care Conditions

- By employing **don't care** conditions, logic equations can be further simplified
- Example:

  F (A, B, C, D) = $\Sigma m(2, 3, 5, 6, 7, 10, 11, 15)$
  $+ \Sigma d(0, 9, 12, 13, 14)$

  - Fill out the K-map with 1's and X's
  - Each "X" can be either 0 or 1 depending upon the needs of simplification
  - Not all X's have to be considered
- Apply the same grouping and canceling rules

  before using 'X': F = A'C + A'BD + B'C + CD

  after: F = C + BD (Essential PI?)

F

AB \ CD	**00**	**01**	**11**	**10**
00	X (0)	0 (1)	1 (3)	1 (2)
01	0 (4)	1 (5)	1 (7)	1 (6)
11	X (12)	1 (13)	1 (15)	1 (14)
10	0 (8)	X (9)	1 (11)	1 (10)

Dealing with Five Variables

E	A	B	C	D	F
0	0	0	0	0	X
0	0	0	0	1	0
0	0	0	1	0	1
0	0	0	1	1	1
0	0	1	0	0	0
0	0	1	0	1	1
0	0	1	1	0	1
0	0	1	1	1	1
0	1	0	0	0	0
0	1	0	0	1	X
0	1	0	1	0	1
0	1	0	1	1	1
0	1	1	0	0	X
0	1	1	0	1	X
0	1	1	1	0	X
0	1	1	1	1	1

E	A	B	C	D	F
1	0	0	0	0	1
1	0	0	0	1	1
1	0	0	1	0	0
1	0	0	1	1	X
1	0	1	0	0	X
1	0	1	0	1	1
1	0	1	1	0	0
1	0	1	1	1	0
1	1	0	0	0	1
1	1	0	0	1	X
1	1	0	1	0	1
1	1	0	1	1	0
1	1	1	0	0	1
1	1	1	0	1	1
1	1	1	1	0	X
1	1	1	1	1	0

E'

F AB \ CD	00	01	11	10
00	X $_{0}$	0 $_{1}$	1 $_{3}$	1 $_{2}$
01	0 $_{4}$	1 $_{5}$	1 $_{7}$	1 $_{6}$
11	X $_{12}$	X $_{13}$	1 $_{15}$	X $_{14}$
10	0 $_{8}$	X $_{9}$	1 $_{11}$	1 $_{10}$

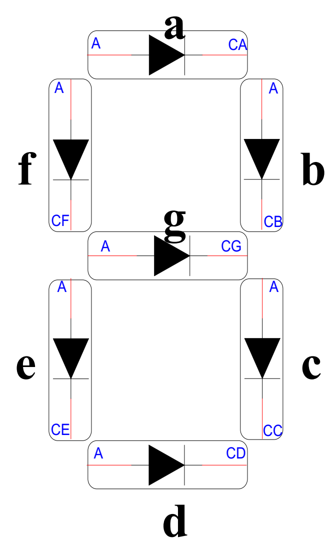
E

F AB \ CD	00	01	11	10
00	1 $_{0}$	1 $_{1}$	X $_{3}$	0 $_{2}$
01	X $_{4}$	1 $_{5}$	0 $_{7}$	0 $_{6}$
11	1 $_{12}$	1 $_{13}$	0 $_{15}$	X $_{14}$
10	1 $_{8}$	X $_{9}$	1 $_{11}$	0 $_{10}$

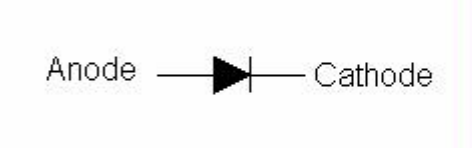
F = E'(C+BD) + E(C'+B'D)

Seven-Segment Display

- A Seven-Segment Display (SSD) consists of seven Light-Emitting Diodes (LEDs)
- The seven segments of a digit are labeled a, b, c, d, e, f, and g by convention
- Each LED has one anode (+) and one cathode (-)
- Connecting anode to higher-level voltage (logic 1) and cathode to lower-level voltage (logic 0) turns ON an LED
- Each LED can be turned on independently in a seven-segment display

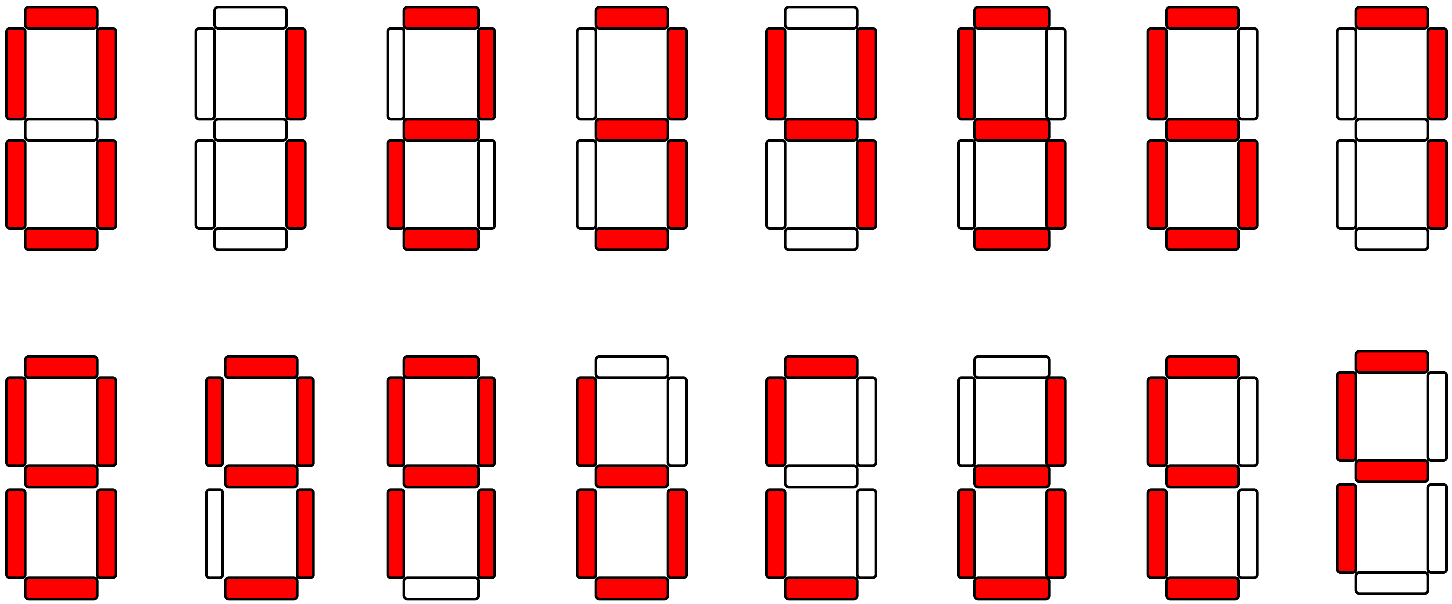


Seven-Segment Display



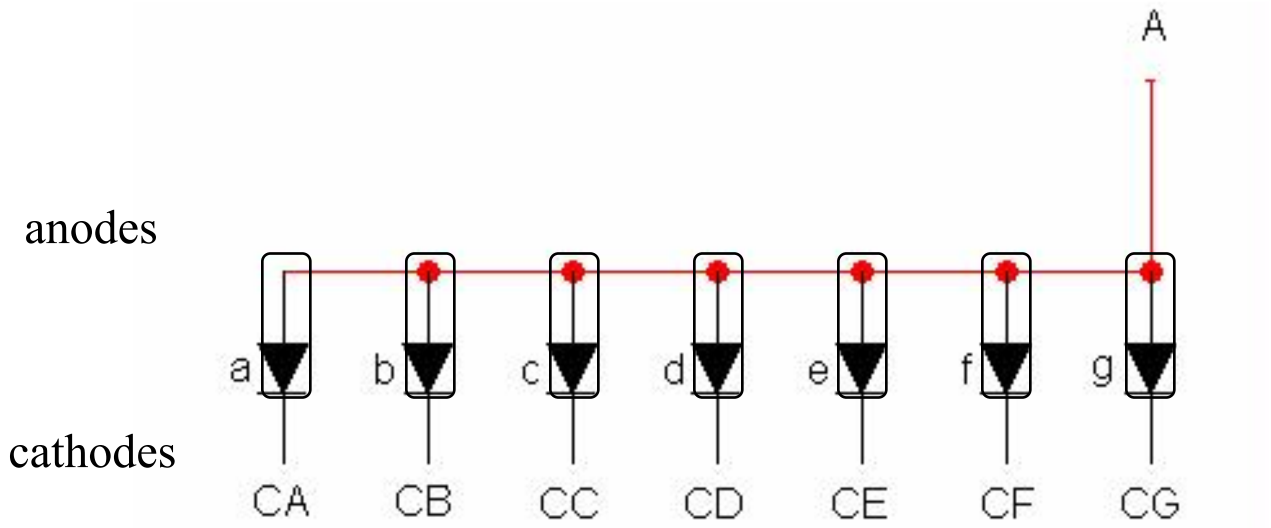
One LED

Seven-Segment Display

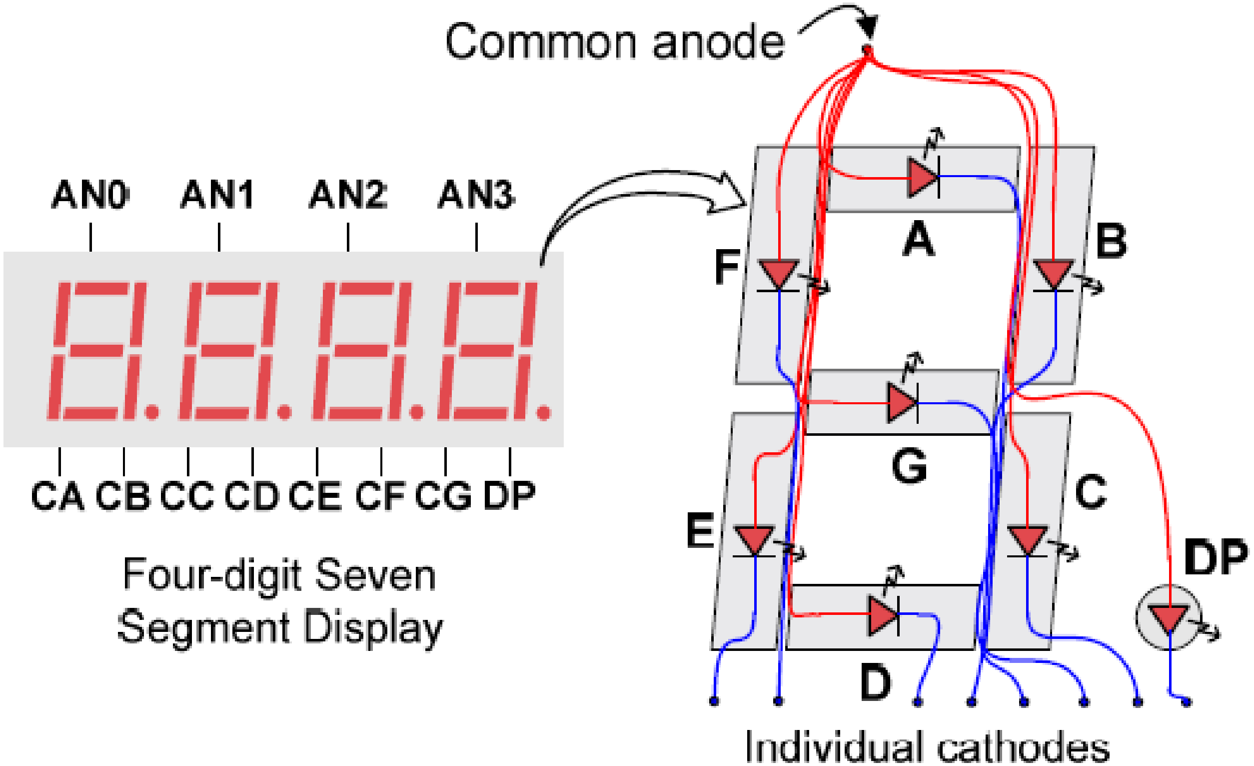


Common Anode Seven-Segment Display

- In order to reduce the number of variables needed to address the terminals of each segment (LED), either the anodes or cathodes of the seven LEDs are tied to one pin. The display will then be called **common anode** or **common cathode** seven-segment display
- Common anode SSD
  - The anodes of the seven LEDs are tied together
  - The anode of each LED is labeled as A, while the cathodes are labeled as CA, CB, CC, CD, CE, CF, and CG for segments a to g, respectively.

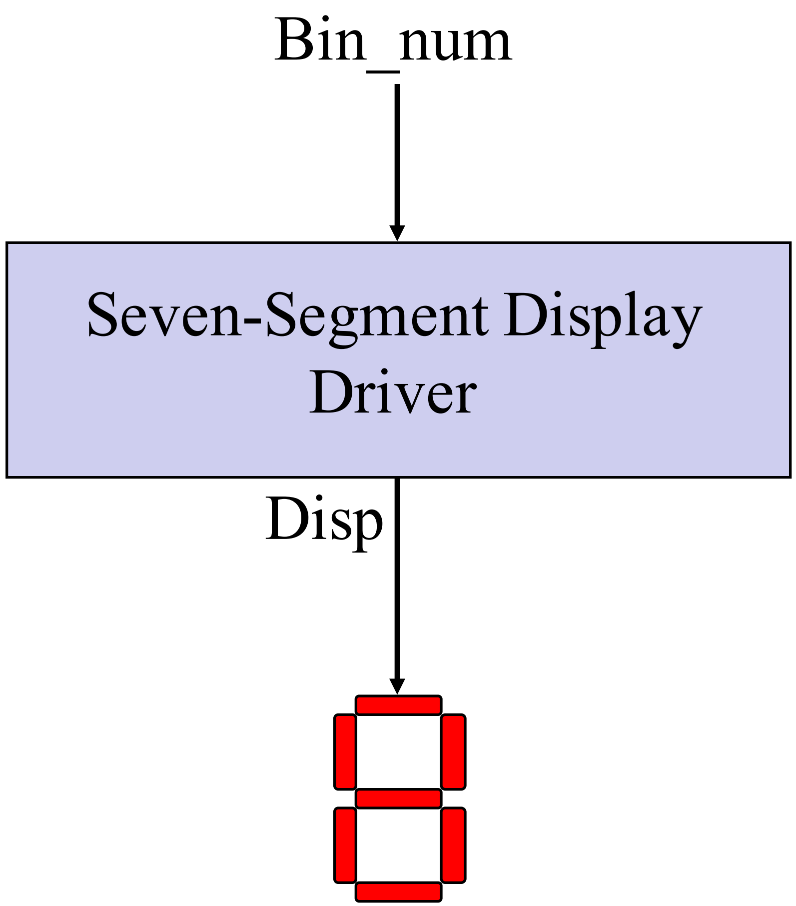


Seven Segment Displays on Nexys2

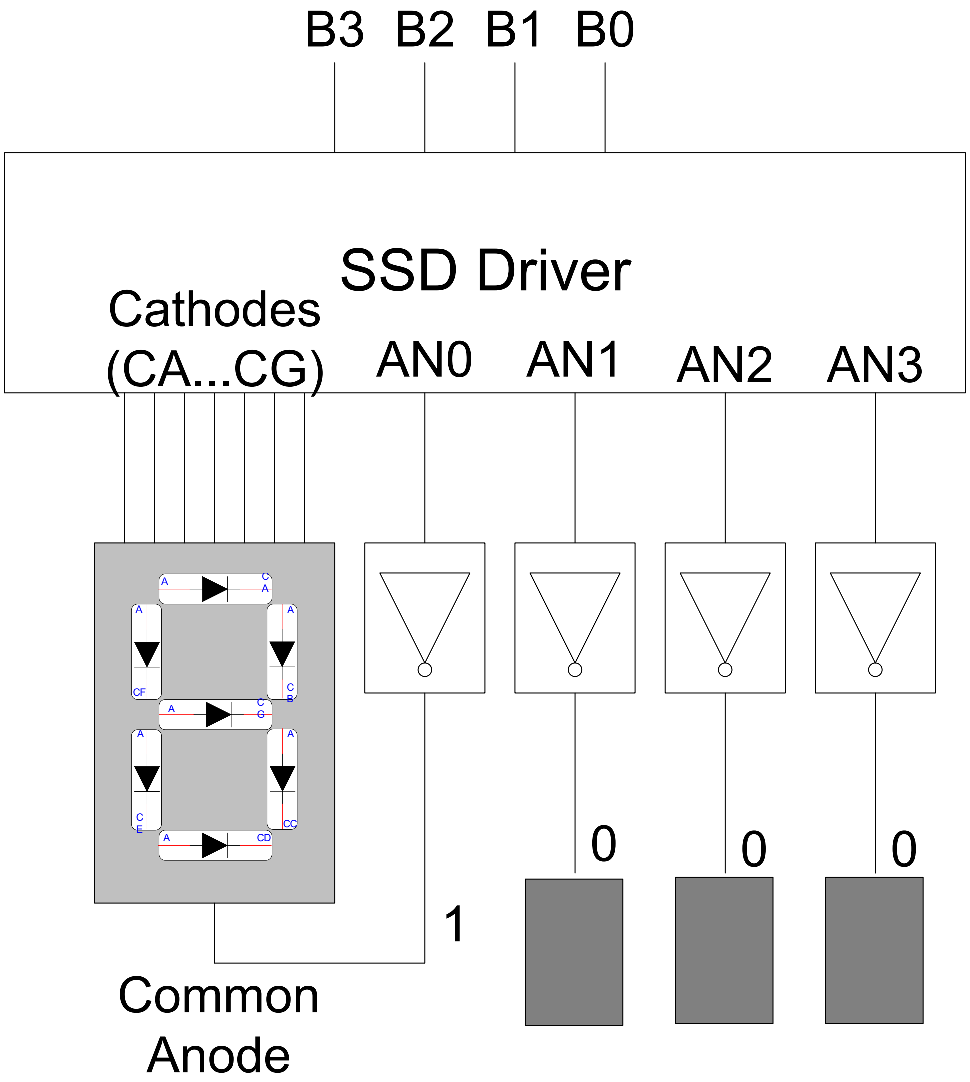


SSD Driver

- A circuit that provides control signals to SSD



SSD Driver for Nexys2

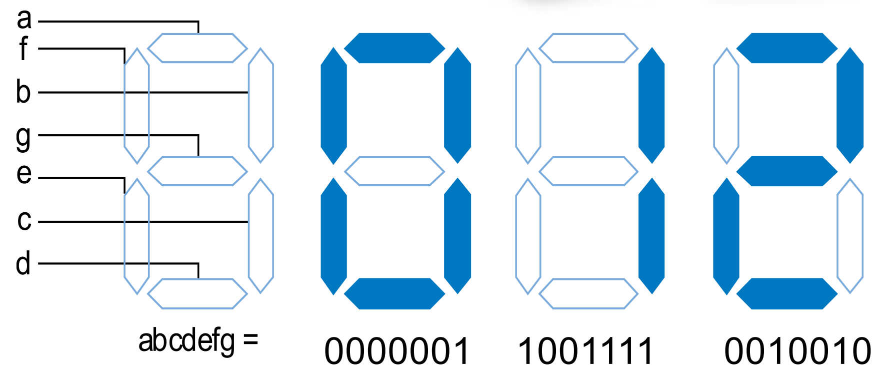


Control of Common Anode SSD

- **The illumination of a common anode seven-segment display is primarily controlled by the common anode and secondarily by the seven cathodes.**
  - Common anode of a SSD should be connected to “1” for displaying
  - On when cathode = 0, off when cathode = 1
  - If the anode is connected to “0”, then the seven-segment display is turned off, cathodes are ignored.

Multiple-Output Example:
BCD to 7-Segment Converter

W	X	Y	Z	a	b	c	d	e	f	g
0	0	0	0	0	0	0	0	0	0	1
0	0	0	1	1	0	0	1	1	1	1
0	0	1	0	0	0	1	0	0	1	0
0	0	1	1	0						
0	1	0	0	1						
0	1	0	1	0						
0	1	1	0	0						
0	1	1	1	0						
1	0	0	0	0						
1	0	0	1	0						
1	0	1	0	0						
1	0	1	1	1						
1	1	0	0	0						
1	1	0	1	1						
1	1	1	0	0						
1	1	1	1	0						



$a = \Sigma_m(1, 4, 11, 13)$

$b = \ldots$

$c = \ldots$

……