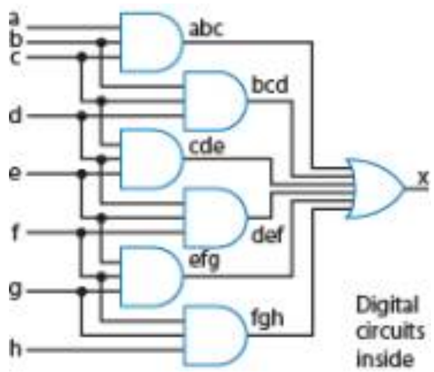
Topic 1

Introduction to Digital Design

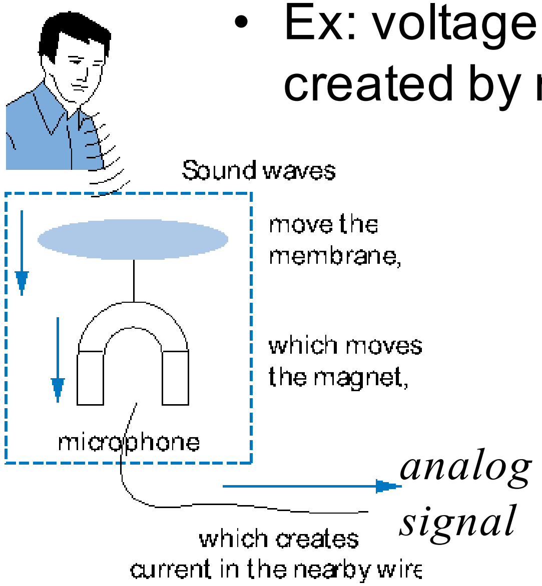
Why Study Digital Design?

- Many elements of our lives are or are to become digital
  - Computer, camera, cell phone, TV, car…
- Electronic devices are made smaller and smarter
  - Enabled by shrinking and more capable chips
  - Supported by "embedded systems"
  - Providing better quality
- Solid understanding benefits ECE engineers
  - For computer engineer – fundamental
  - For electrical engineer – many times necessary
  - Even for software engineer – confident and insightful when aware of hardware resource issues

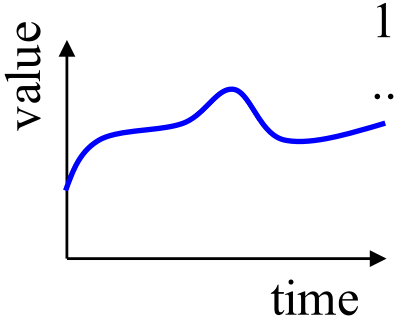


What Does "Digital" Mean?

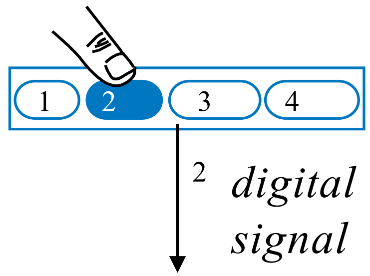
- Analog signal
  - Infinite possible values
    - Ex: voltage on a wire created by microphone

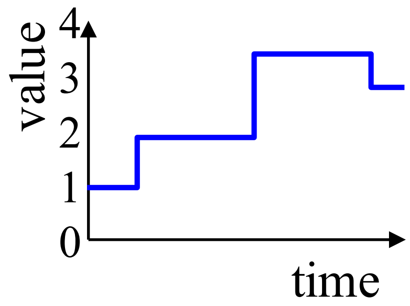


Possible values:
1.00, 1.01, 2.0000009,
... infinite possibilities



- Digital signal
  - Finite possible values
    - Ex: button pressed on a keypad

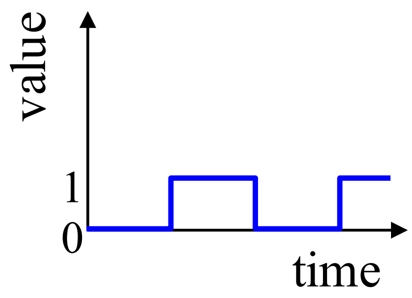




Possible values:
0, 1, 2, 3, or 4.
That's it.

Digital Signals with Only Two Values: Binary

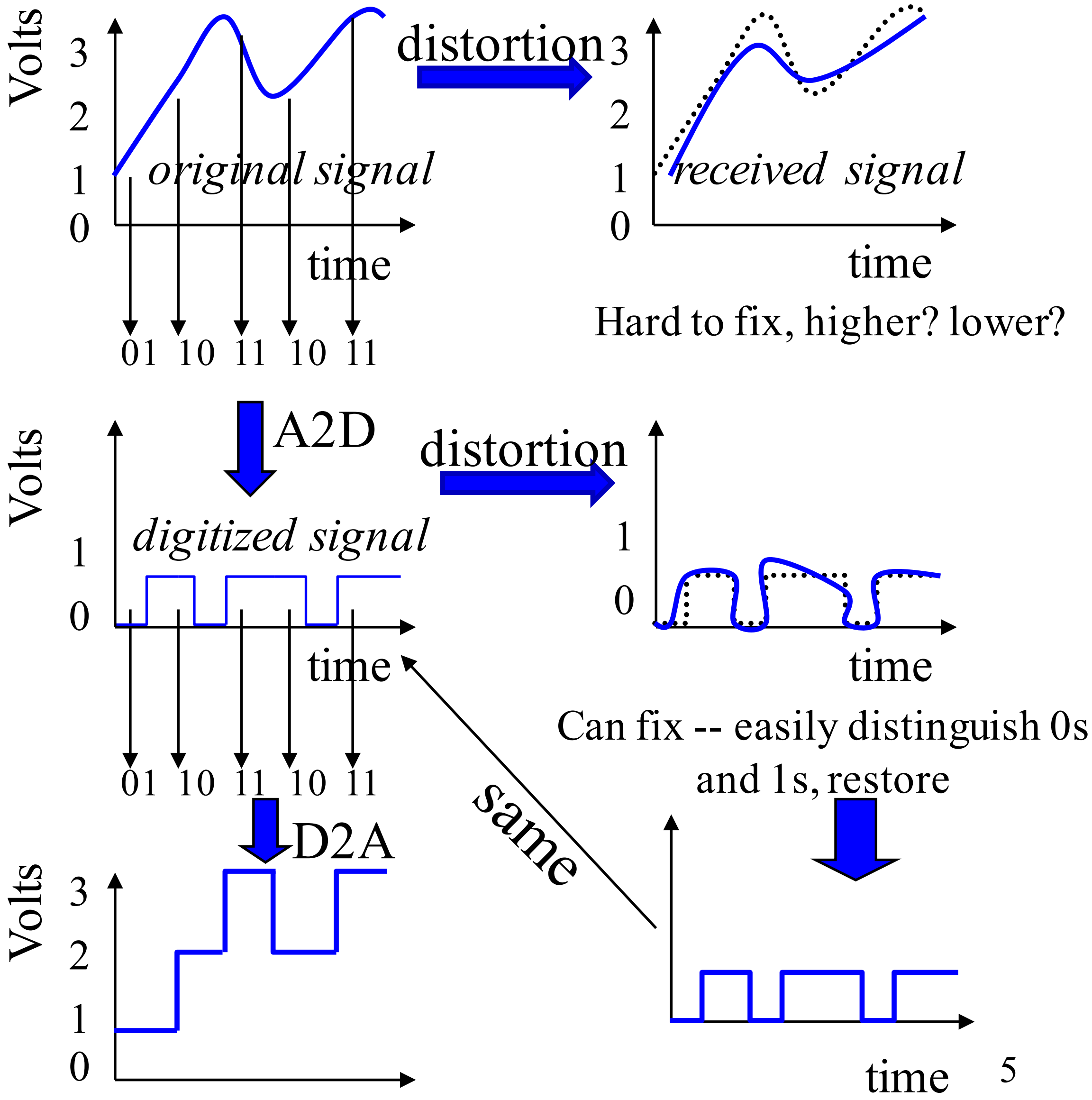
- ***Binary*** digital signal -- only *two* possible values
  - low voltage (e.g. 0V or -5V) and high voltage (e.g. 3.3V or 5V)
  - Typically represented as **0** and **1**, respectively
  - All values are represented as combinations of 0's and 1's, e.g. 1011, 11010
    - Called binary value or binary number
    - Each binary digit is a ***bit***
  - We'll only consider binary digital signals
    - Although there are other types of digital signals
  - Binary is popular because
    - Transistors, the basic digital electric component, operate using *two* voltages
    - Storing/transmitting one of *two* values is easier than three or more



From Analog to Digital – Digitization

- Analog signal (e.g., audio) may lose quality
  - Voltage levels not saved/copied/transmitted perfectly
  - Hard to recover
- Digitized version:
  - "Sample" voltage at particular rate
  - Easy to distinguish 0s from 1s, thus easy to recover
  - Increase sample rate to improve quality

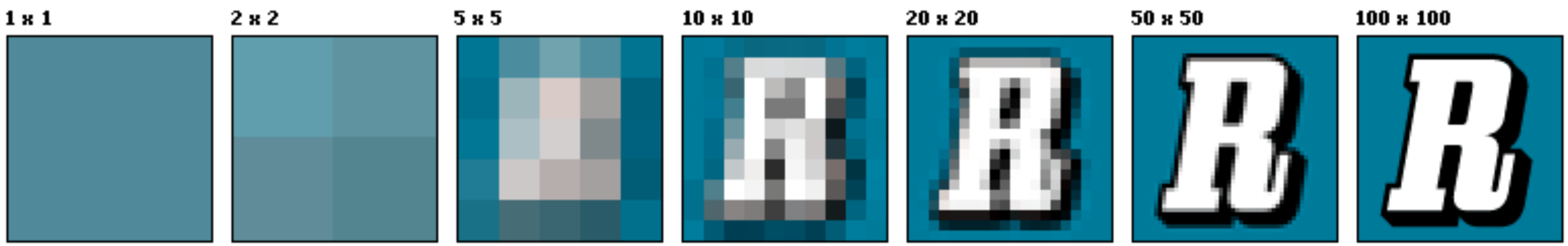
Example:
if only 4 sampled values
let binary representation be:
0 V: "00"
1 V: "01"
2 V: "10"
3 V: "11"



Resolution

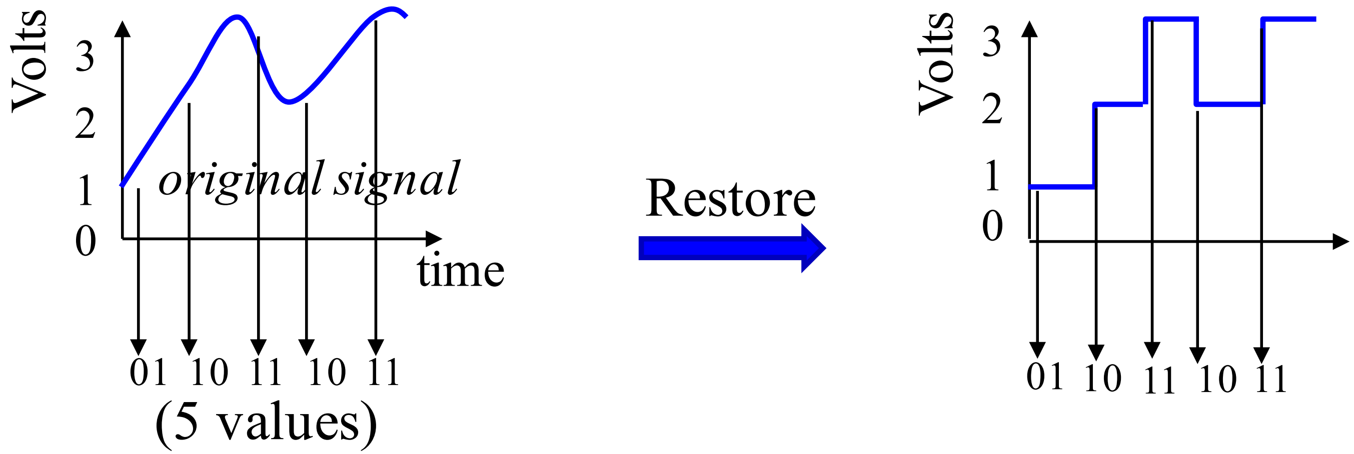
What happened?

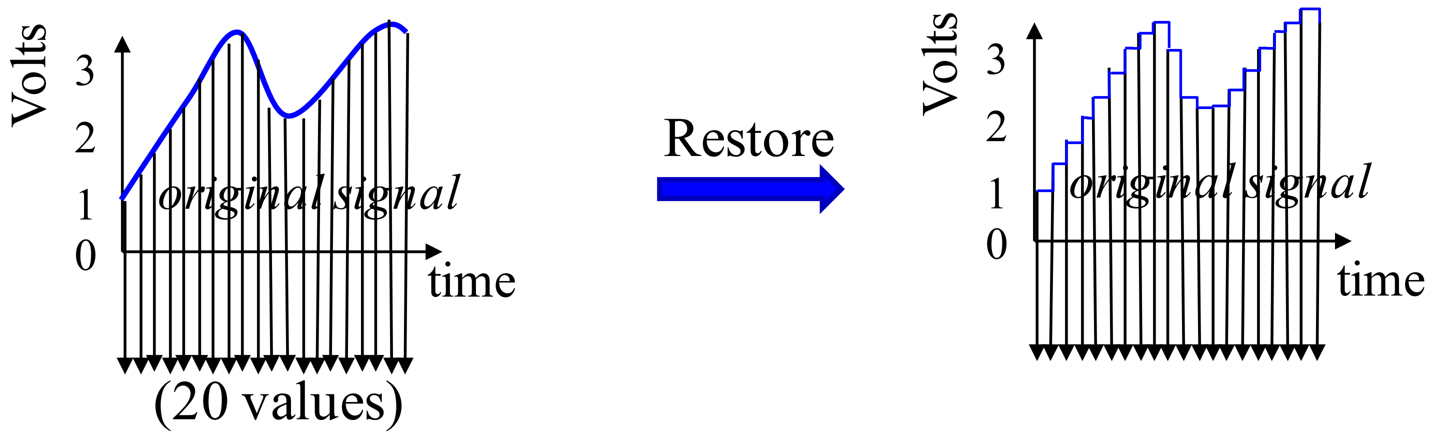
(*source: cntv.cn*)



(*source: wikipedia.org*)

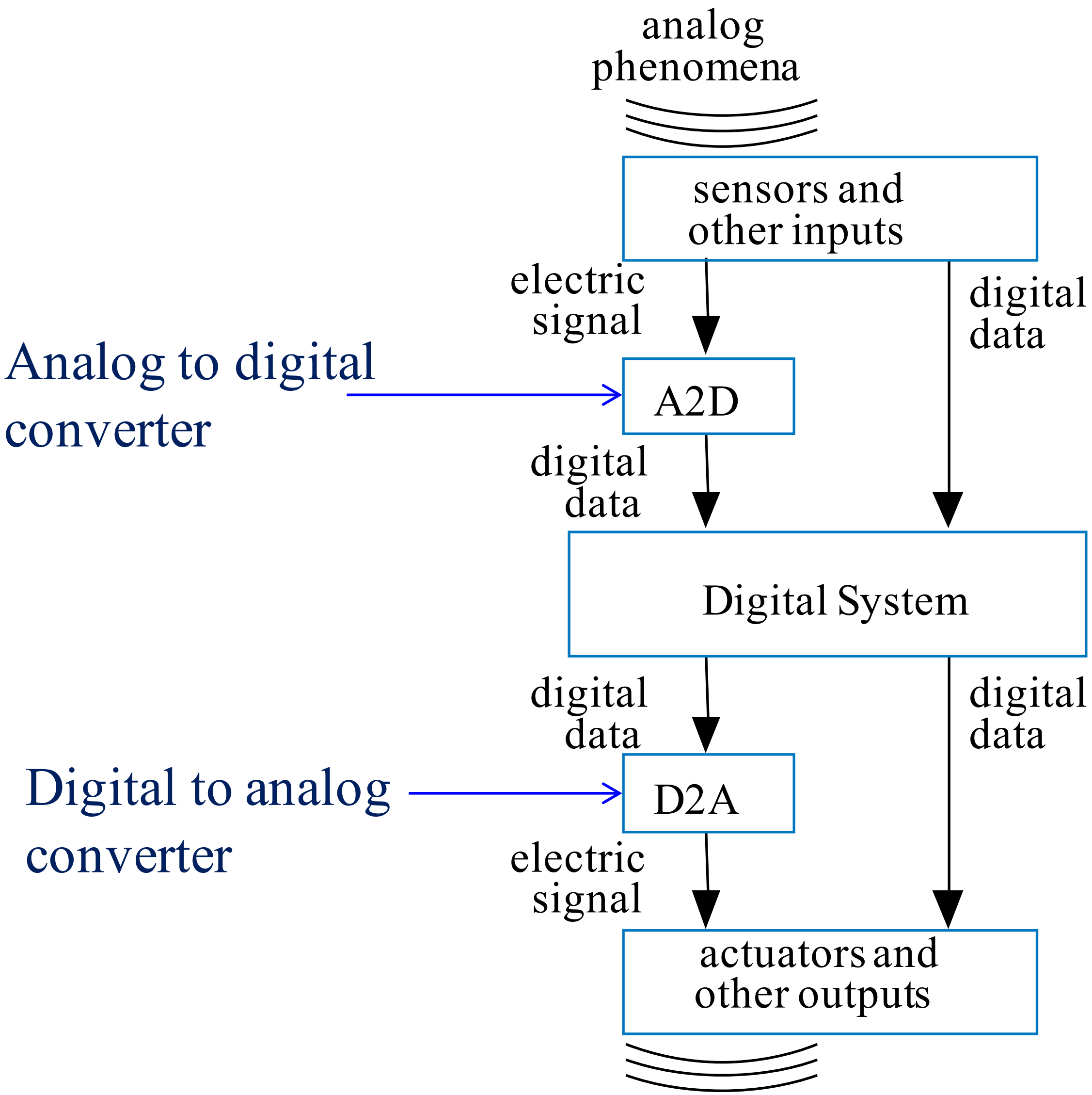
From Analog to Digital – Digitization



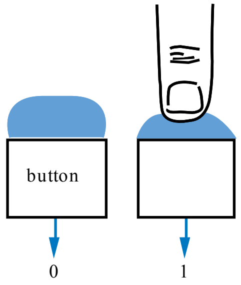


Higher resolution (sample rate)

Typical Digital System

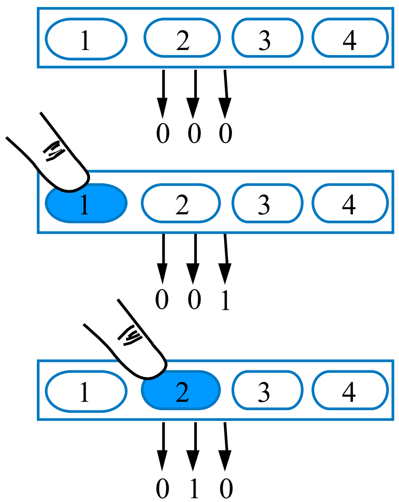


How Do We Encode Signal as Binary?



- Some signals are inherently binary
  - Example:

    Button: not pressed (0), pressed (1)



- Some signals are inherently digital
  - But not binary digital
  - Outputs finite possible values
  - Each value maybe encoded in binary with multiple bits
  - More values need more bits to represent
  - e.g., multi-button input: encode “1”=001, “2”=010, ...

How Do We Encode Numbers with Bits

- Number systems: decimal, binary, octal, hexadecimal, …
- Each position of a number is associated with a weight quantity
  - Base ten (***decimal***)

$$\frac{}{10^4}\ \frac{}{10^3}\ \frac{5}{10^2}\ \frac{2}{10^1}\ \frac{3}{10^0}$$

  - Base two (***binary***)

$$\frac{}{2^4}\ \frac{}{2^3}\ \frac{1}{2^2}\ \frac{0}{2^1}\ \frac{1}{2^0}$$

Binary System

- The Binary System is a base 2 (modulo 2) number system:
  - 2 digits: 0 or 1
- Counting beyond 1 requires additional place
- In a binary number, each position has a decimal weight in power of 2, **10011.01**

	1	0	0	1	1	.	0	1
	$\times 16$	$\times 8$	$\times 4$	$\times 2$	$\times 1$		½	¼
weight	(2^4)	(2^3)	(2^2)	(2^1)	(2^0)		(2^{-1})	(2^{-2})
position	4	3	2	1	0		-1	-2

Find Equivalent Decimal for Binary Numbers

- Example: Convert binary number 10011.01_2 to decimal

Number:	**1**	**0**	**0**	**1**	**1 .**	**0**	**1**
Position:	**4**	**3**	**2**	**1**	**0**	**-1**	**-2**
Weight:	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}

$$1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2}$$

$$= 16 + 0 + 0 + 2 + 1 + 0 + 1/4$$

$$= 19.25_{10} = 10011.01_2$$

Encode Decimal as Binary Numbers:
Subtraction Method (Easy for Humans)

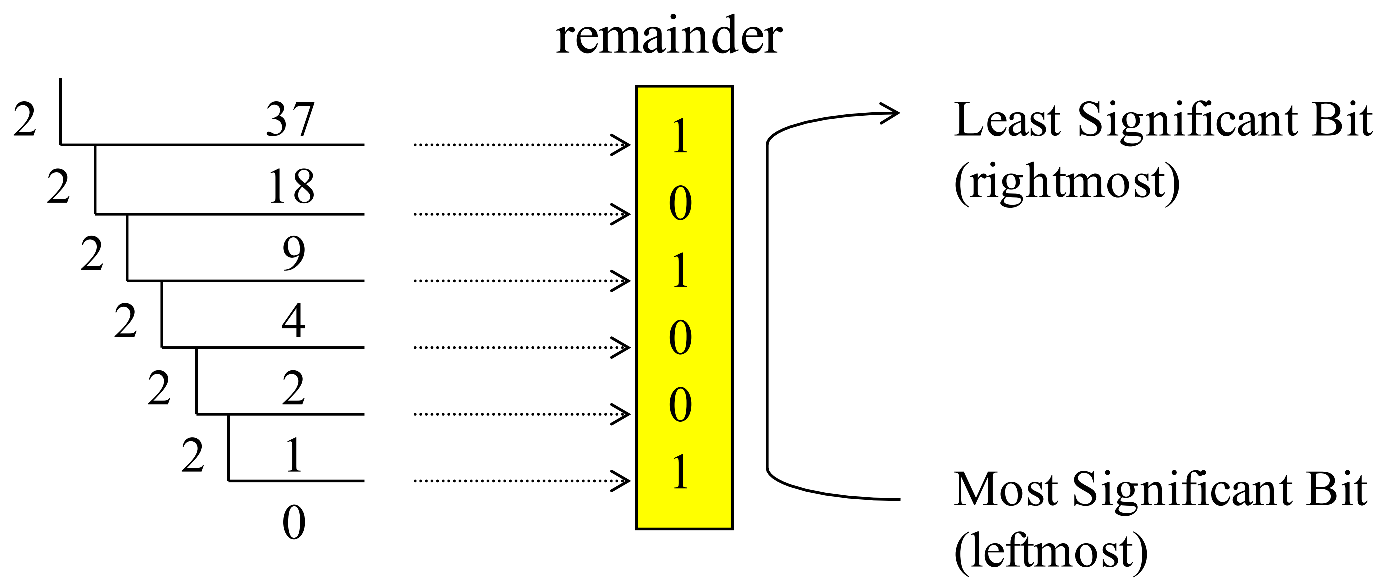
- Subtraction method
  - To make the job easier (especially for big numbers), we can just subtract a selected binary weight from the (remaining) quantity
    - Then, we have a new remaining quantity, and we start again (from the present binary position)
    - Stop when remaining quantity is 0

Remaining quantity: **12**

32	16	8	4	2	1	
___	___	___	___	___	___	
1	___	___	___	___	___	32 is too much
0	1	___	___	___	___	16 is too much
0	0	1	___	___	___	**12 – 8 = 4**
0	0	1	1	___	___	**4-4=0** DONE
0	**0**	**1**	**1**	**0**	**0**	answer

Encode Decimal in Binary Numbers:
Division Method (Good for Computers)

- Example: Convert decimal number 37 to binary
  - Repeated-division-by-base (here, base 2)



$(37)_{10} = (100101)_2$

Encode Fractional Decimal in Binary

- Example: Convert fractional part 0.715_{10} to binary
  - Repeated-multiplication-by-base (here, base 2)

$0.715 \times 2 = 1\ .430$	1	Most Significant Bit (leftmost)
$0.430 \times 2 = 0\ .860$	0	
$0.860 \times 2 = 1\ .720$	1	
$0.720 \times 2 = 1\ .440$	1	
$0.440 \times 2 = 0\ .880$	0	Least Significant Bit (rightmost)

$(0.715)_{10} \approx (0.10110\ldots\ldots)_2$

Encode Numbers with Bits

- Bigger number needs more bits to encode
  - $37_{10} = 100101_2$ (6 bits)
  - $137_{10} = 10001001_2$ (8 bits)
  - $10307_{10} = 10100001000011_2$ (14 bits)

- N bits can represent 2^N non-negative integers
  - 0, 1, 2, …, 2^N-1
  - Negative numbers will be discussed later

Encode Decimal Numbers by Binary Bits

Binary				Decimal
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7
1	0	0	0	8
1	0	0	1	9
1	0	1	0	10
1	0	1	1	11
1	1	0	0	12
1	1	0	1	13
1	1	1	0	14
1	1	1	1	15
				16
	……			……

……

Hexadecimal System

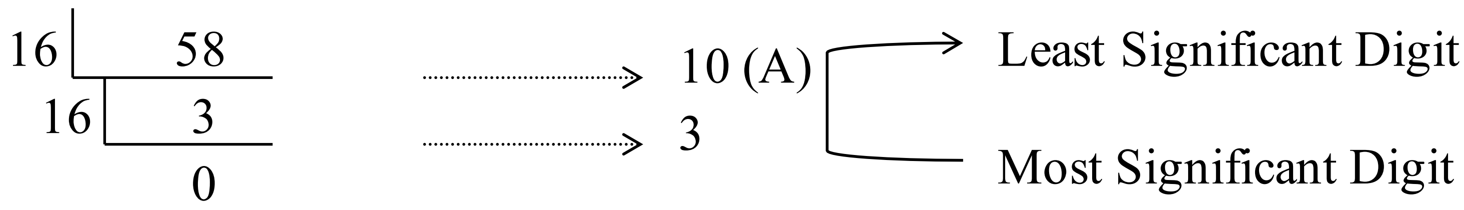
- The Hexadecimal system is a base 16 (modulo 16) number system:
  - 16 digits: 0 1 2 3 4 5 6 7 8 9 A B C D E F
- Letters A ~ F represent decimal 10 through decimal 15
- Each position has a decimal weight in power of 16, e.g. **E3A**

	E	**3**	**A**
	$\times$**256**	$\times$**16**	$\times$**1**
weight	**(16^2)**	**(16^1)**	**(16^0)**

$$(E3A)_{16} = E\times 256 + 3\times 16 + A\times 1 = 3584 + 48 + 10 = 3642$$

Encode Decimal to Hexadecimal

- Example: Convert decimal number 58 to hexadecimal
  – Repeated-division-by-base (here, base 16)



$(58)_{10} = (3A)_{16}$

Summary

Binary				Decimal	Hexaecimal
0	0	0	0	0	0
0	0	0	1	1	1
0	0	1	0	2	2
0	0	1	1	3	3
0	1	0	0	4	4
0	1	0	1	5	5
0	1	1	0	6	6
0	1	1	1	7	7
1	0	0	0	8	8
1	0	0	1	9	9
1	0	1	0	10	A
1	0	1	1	11	b
1	1	0	0	12	C
1	1	0	1	13	d
1	1	1	0	14	E
1	1	1	1	15	F

Convert Binary to Hexadecimal

- Look for groups of 4 bits starting from the LSB
- Example: Convert 11 1011 0101.11 to hexadecimal:

$$11\ 1011\ 0101.11 = (\underbrace{0011}_{3}\ \underbrace{1011}_{b}\ \underbrace{0101}_{5}.\underbrace{1100}_{c})_2$$

$$(11\ 1011\ 0101.11)_2 = (3b5.c)_{16}$$

Binary				Decimal	Hexaecimal
0	0	0	0	0	0
0	0	0	1	1	1
0	0	1	0	2	2
0	0	1	1	3	3
0	1	0	0	4	4
0	1	0	1	5	5
0	1	1	0	6	6
0	1	1	1	7	7
1	0	0	0	8	8
1	0	0	1	9	9
1	0	1	0	10	A
1	0	1	1	11	b
1	1	0	0	12	C
1	1	0	1	13	d
1	1	1	0	14	E
1	1	1	1	15	F

Convert Hexadecimal to Binary

- Each digit is converted to 4 bits in binary
- Arrange the groups of 4 bits in the same order
- Example: convert $(3F7)_{16}$ to binary:

$$(3\ F\ 7)_{16}$$

$$(\ \mathbf{0011}\quad \mathbf{1111}\quad \mathbf{0111}\ )_2$$

Binary	Decimal	Hexaecimal
0 0 0 0	0	0
0 0 0 1	1	1
0 0 1 0	2	2
0 0 1 1	3	3
0 1 0 0	4	4
0 1 0 1	5	5
0 1 1 0	6	6
0 1 1 1	7	7
1 0 0 0	8	8
1 0 0 1	9	9
1 0 1 0	10	A
1 0 1 1	11	b
1 1 0 0	12	C
1 1 0 1	13	d
1 1 1 0	14	E
1 1 1 1	15	F

- Drop the initial 0's to simplify

$$(3F7)_{16} = (11\ 1111\ 0111)_2$$

Octal System

- The Octal number system is a base 8 (modulo 8) number system:
  - 8 digits: 0 1 2 3 4 5 6 7
- Each position has a decimal weight in power of 8
- Each octal digital corresponds to a 3-bit binary number

$$(3\ 1\ 7)_8$$

$$(\ \mathbf{011}\quad \mathbf{001}\quad \mathbf{111}\ )_2$$

Convert Base-M System to Base-N System

- Decimal can always be used as the intermediate number system
- Generally, the rule "divide/multiply by the base of destination system" applies to all the number system conversions
  - Example, to convert a Hex number to base-3 number, just divide the Hex number by 3

Binary Arithmetic

- Example:

```
carry   11110111          borrow 11000010
         10110101                  10110101
       + 11010011                - 11010011
-----------------------        -------------------
        110001000                  11100010
```

Hexadecimal Arithmetic

- Example:

```
carry    111              borrow   11
         8F5A                     8F5A
       + 11BC                    - 11BC
---------------------           ---------------
         A116                     7D9E
```

How Do We Encode Text with Binary Bits

- A popular code: ASCII (American Standard Code for Information Interchange)
  - 7- (or 8-) bit encoding of each letter, number, or symbol

Symbol	Encoding
R	1010010
S	1010011
T	1010100
L	1001100
N	1001110
E	1000101
0	0110000
.	0101110
<tab>	0001001

Symbol	Encoding
r	1110010
s	1110011
t	1110100
l	1101100
n	1101110
e	1100101
9	0111001
!	0100001
<space>	0100000

- Unicode: Increasingly popular 16-bit encoding
  - Encodes characters from various world languages

Question:

What does this ASCII bit sequence represent?

1010010 1000101 1010011 1010100

R E S T

ASCII Coding Chart

Decimal	Binary	Octal	Hex	ASCII	Decimal	Binary	Octal	Hex	ASCII
64	01000000	100	40	@	96	01100000	140	60	`
65	01000001	101	41	A	97	01100001	141	61	a
66	01000010	102	42	B	98	01100010	142	62	b
67	01000011	103	43	C	99	01100011	143	63	c
68	01000100	104	44	D	100	01100100	144	64	d
69	01000101	105	45	E	101	01100101	145	65	e
70	01000110	106	46	F	102	01100110	146	66	f
71	01000111	107	47	G	103	01100111	147	67	g
72	01001000	110	48	H	104	01101000	150	68	h
73	01001001	111	49	I	105	01101001	151	69	i
74	01001010	112	4A	J	106	01101010	152	6A	j
75	01001011	113	4B	K	107	01101011	153	6B	k
76	01001100	114	4C	L	108	01101100	154	6C	l
77	01001101	115	4D	M	109	01101101	155	6D	m

⋮

Signed Binary Numbers

- To represent negative numbers
  - Cannot use minus sign: binary systems work with only two values, 0 and 1
  - The left-most bit of a binary number represents the sign of a number – sign bit
    - Sign bit 0 indicates positive numbers
    - Sign bit 1 indicates negative number

Representation of Negative Numbers

- Negative numbers are represented by
  - Sign and magnitude
  - 1's complement code
  - 2's complement code
- Sign and magnitude
  - MSB is the sign bit: 0 → positive, 1 → negative
- 1's complement representation of –N
  - Negation of every bit of N
  - Example, 1's complement representation of -3
    - N = 3 = 0011
    - -N = -3 = 1100
- 2's complement representation of –N is
  - Negation of every bit of N, then plus 1
  - Example, 2's complement representation of -3
    - N = 3 = 0011
    - -N = -3 = 1100 + 1 = 1101

Signed 2's Complement Number

- **Signed numbers are represented as 2's complement numbers in computers**
- **Recognize a signed 2's complement number**
  - Sign bit = 0, positive number, recognize as a regular binary number
    - **0**101 = +5;
  - Sign bit = 1, negative number, the magnitude of the number is obtained by 2's complement operation
    - **1**011
      - Sign: negative number
      - Magnitude: 2's complete operation of (1011) = 0100+1 = 0101 = 5
      - So 1011 = –5

Signed 2's Complement Number

- Represent following numbers in 2's complement system

  5

  7

  –6

  –2

- Recognize following 2's complement numbers

  1100

  0110

  0000

  1000

Ranges of Signed 2's Complement Number

- In general the 2's complement values range from -2^{n-1} to $2^{n-1}-1$
- For n = 4, the 2's complement values range from -8 to 7
- For n = 8, the 2's complement values range from -128 to 127
- For n = 16, the 2's complement values range from -2^{15} to $2^{15}-1$

- Overflow
  - If an n-bit 2's complement number is greater than $2^{n-1}-1$ or less than -2^{n-1}, we say there is an overflow

Detecting Overflow: Method 1

- Overflow detection logic
  - Two numbers' sign bits are the same but are different from the result's sign bit
  - If the two numbers' sign bits are different, overflow is impossible
    - Adding a positive and negative can't exceed largest magnitude positive or negative
- 4-bit example
  - overflow = a3'b3's3 + a3b3s3'

sign bits

$$\begin{array}{r} 0\ 1\ 1\ 1 \\ +\ 0\ 0\ 0\ 1 \\ \hline 1\ 0\ 0\ 0 \end{array} \qquad \begin{array}{r} 1\ 1\ 1\ 1 \\ +\ 1\ 0\ 0\ 0 \\ \hline 0\ 1\ 1\ 1 \end{array} \qquad \begin{array}{r} 1\ 0\ 0\ 0 \\ +\ 0\ 1\ 1\ 1 \\ \hline 1\ 1\ 1\ 1 \end{array}$$

overflow (a) | overflow (b) | no overflow (c)

Binary Number Subtraction

- Using two's complement representation

  A – B = A + (-B)

  = A + (two's complement of B)

  = A + invert_bits(B) + 1

- Example:

```
  11000010                       00111101
   10110101                        10110101
  - 11010011                     + 00101100
-----------------------          +          1
   11100010      same          ----------------
                                   11100010
```