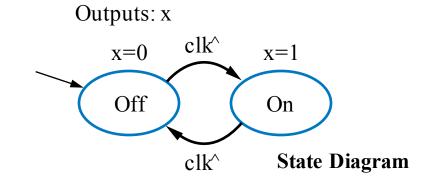
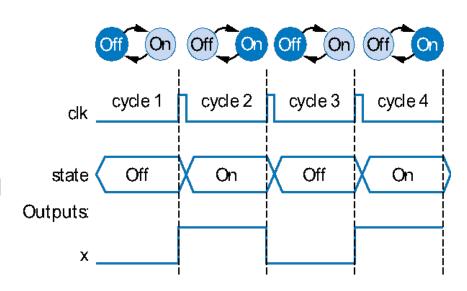
Topic 10 Finite State Machine

Describing Behavior of Sequential Circuit

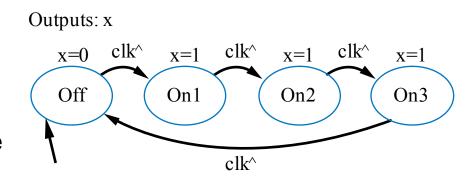
- Finite-State Machine (FSM)
 - A way to describe desired
 behavior of a sequential circuit
 - Consists of a set of states, transitions between states, and maybe inputs and outputs
 - present state: currently happening
 - next state: next to happen
 - Example: Toggle output x every clock cycle
 - Two states: "Off" and "On"
 - Corresponding outputs: x=0 or 1
 - No input
 - Transition from Off to On, or On to Off, on rising clock edge
 - Arrow with no starting state points to initial state

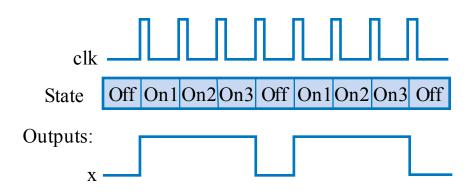




Example: Output Special Pattern

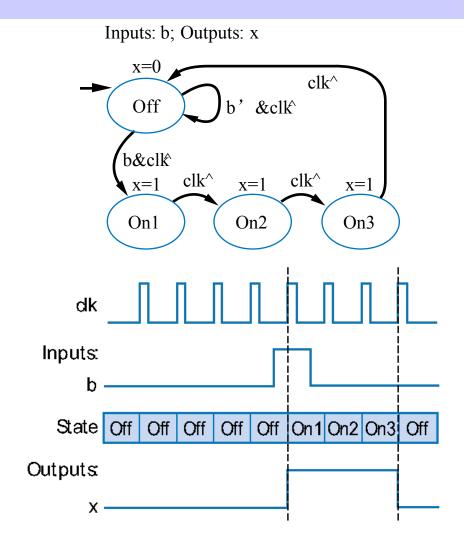
- Want a circuit to output
 - 0, 1, 1, 1, 0, 1, 1, 1, ...
 - One bit at a time
 - Each value for one clock cycle
- Can describe as FSM
 - Four states
 - Each state corresponds to an output, 0, 1, 1, 1
 - Then repeat
 - Transition on rising clock edge to next state





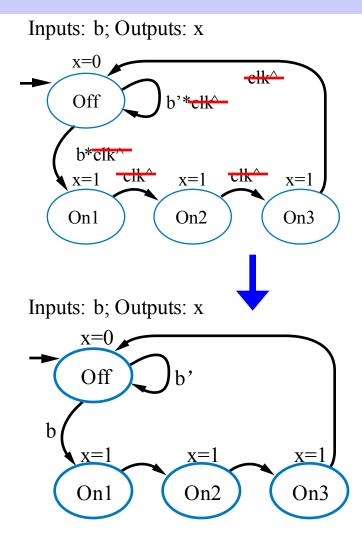
Example: FSM with Input

- b is a push button, output x to stay on for exactly 3 clock cycles no matter how long b is pushed
- Wait in "Off" state while b is 0 (b')
- When b is 1 (and rising clock edge), transition to On1
 - Sets x=1
 - On next two clock edges, transition to On2, then On3, which also set x=1
- So x=1 for three cycles after button pressed
- Potential issue: if button b stays on, what will happen?



State Diagram Simplification: Clock Implicit

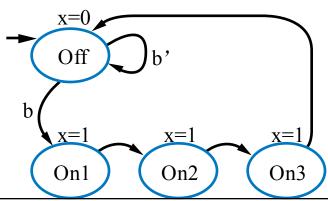
- Synchronous FSMs FSM behaviors synchronized to active edge of clock
 - Asynchronous FSMs -- less common, advanced topic
- Make implicit all state transitions are triggered by rising edge of clock
- Make implicit Unlabeled path means transition is triggered only by clock, inputs don't matter



FSM Definition

- FSM consists of
 - Set of states
 - Ex: {Off, On1, On2, On3}
 - Set of inputs, set of outputs
 - Ex: Inputs: {b}, Outputs: {x}
 - Initial state
 - Ex: "Off"
 - Set of transitions
 - Describes next states
 - Ex: Has 5 transitions
 - Set of actions (outputs)
 - Sets outputs while in states
 - Ex: x=0, x=1, x=1, and x=1

Inputs: b; Outputs: x



FSM can be represented graphically, known as state diagram

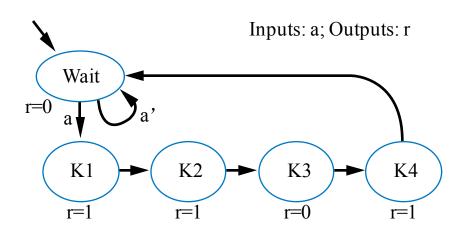
]	Inputs	;	(Dutpu	ts
	s 1	s0	b	Х	n1	n0
Off	0	0	0	0	0	0
	0	0	1	0	0	1
On1	0	1	0	1	1	0
	0	1	1	1	1	0
On2	1	0	0	1	1	1
	1	0	1	1	1	1
On3	1	1	0	1	0	0
	1	1	1	1	0	0

FSM can also be represented in tabular form, known as **state table**

Example: Secure Car Key

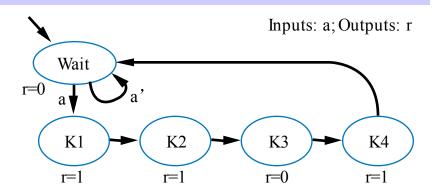
- Many new car keys contain a tiny computer chip
 - When car starts, car's computer (under engine hood) requests identifier from key
 - Key transmits identifier
 - If not, computer shuts off car
- FSM
 - Wait until computer requestsID (a=1)
 - Transmit ID (in this case, 1101)

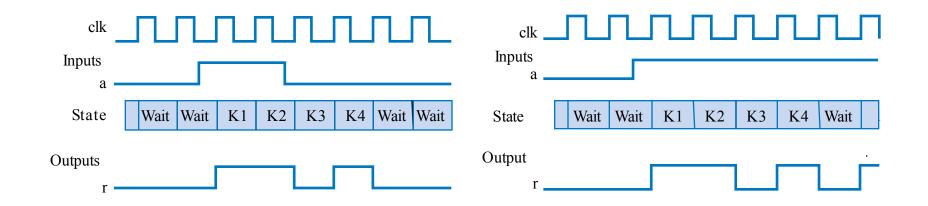




FSM Example: Secure Car Key (cont.)

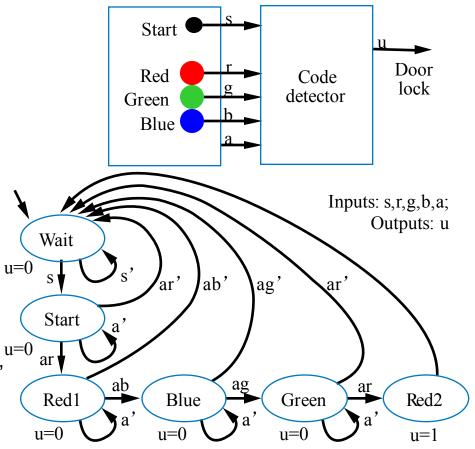
 Timing diagrams show states and output values for different input waveforms





Example: Digital Lock

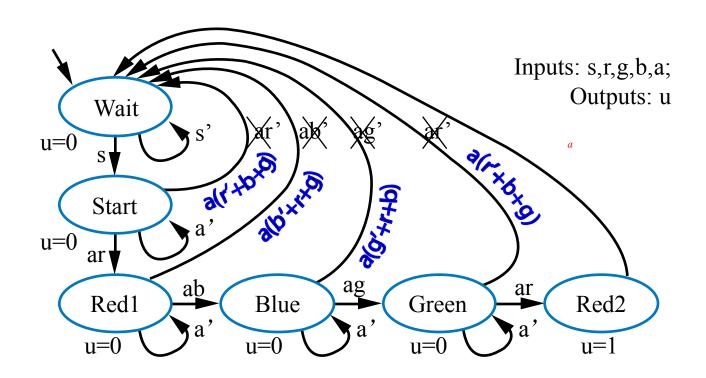
- Unlock door (u=1) only when buttons pressed in sequence:
 - start, then red, blue, green, red
- Input buttons: s, r, g, b
- Input a indicates that some color button pressed
- FSM
 - Wait for start (s=1) in "Wait"
 - Once started, go to "Start", then
 - If see red, go to "Red1"
 - Then, if see blue, go to "Blue"
 - Then, if see green, go to "Green"
 - Then, if see red, go to "Red2", and u=1
 - Wrong button at any step, return to "Wait", without opening door



Q: Can you trick this FSM to open the door, without knowing the code?

A: Yes, hold all buttons simultaneously

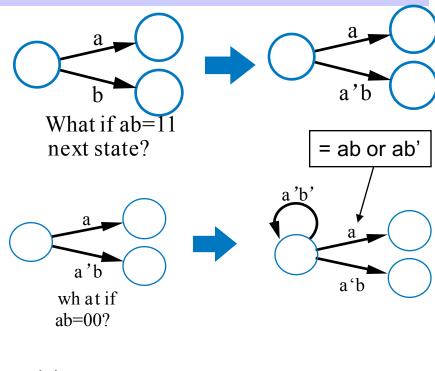
Improve FSM for Code Detector

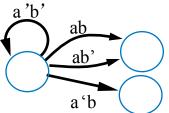


 New transition conditions detect if wrong button pressed, returns to "Wait"

Common State Transition Property

- Only one condition should be true, among all transitions leaving a state
- One condition must be true
 - For any input combination
- All conditions must be considered when leaving a state





Pitfall is Common

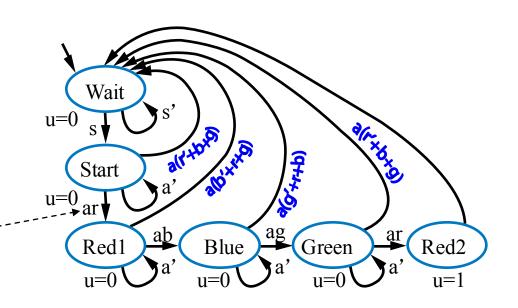
- Recall code detector FSM
 - Do the transitions obey the two required transition properties?

NO!

– How would it go wrong?

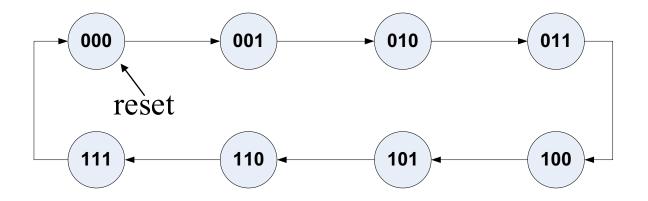
E.g. arbg = 1111 How to solve?

Answer: ar should be arb'g' (likewise for ab, ag, ar)



FSM Example: Synchronous Binary Counter

- FSM that counts binary numbers counter
- An n-bit binary counter can count in binary from 0 up to 2ⁿ-1 and repeat
- An n-bit binary counter consists of n flip-flops
- All the flip-flops are synchronized to the same clock synchronous counter
- May be implemented by different type of flip-flops
- Example: a 3-bit binary counter can count through this sequence



Synchronous Binary Counter Design

- An FSM (without external inputs)
 - State Table

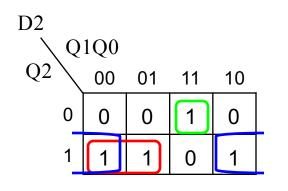
Pre	esent S	tate	1	Next Stat	е
Q2	Q1	Q0	Q2 ⁺	Q1+	Q0+
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	0

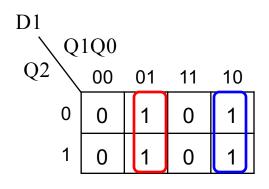
Counter Implemented with D Flip-Flop

- Use D flip flops to hold values: Q+ = D upon active edge
- Next state equations

Present State		tate	Ne	ext Sta	ate	D flip flop input		
Q2	Q1	Q0	Q2+	Q1+	Q0+	D2	D1	D0
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	0
0	1	0	0	1	1	0	1	1
0	1	1	1	0	0	1	0	0
1	0	0	1	0	1	1	0	1
1	0	1	1	1	0	1	1	0
1	1	0	1	1	1	1	1	1
1	1	1	0	0	0	0	0	0

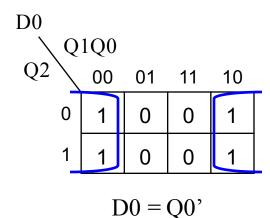
State Registers Implemented with D Flip-Flop





$$D2 = Q2Q1'+Q2Q0'+Q2'Q1Q0 D1 = Q1'Q0+Q1Q0' = Q2(Q1'+Q0')+Q2'Q1Q0 = Q1 \oplus Q0 = Q2(Q1Q0)'+Q2'(Q1Q0) = Q2 \oplus (Q1Q0)$$

$$D1 = \frac{Q1}{Q0} + \frac{Q1}{Q0}$$
$$= Q1 \oplus Q0$$



Pre	sent St	ate	D fli	p flop i	nput
Q2	Q1	Q0	D2	D1	D0
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	0

•	The input equation can be generalized as
	Dn = Qn ⊕ (Qn-1Q1Q0)

Characteristic table and equation of T-FF

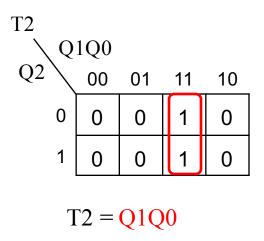
Т	T Q+		$\mathbf{O}^+ = \mathbf{T} \oplus \mathbf{O}$
0	Q	No Change	Q I U Q
1	Q'	Complement	

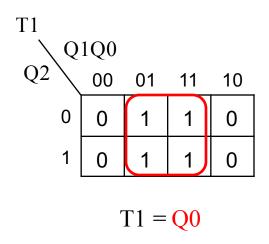
Excitation table of T-FF

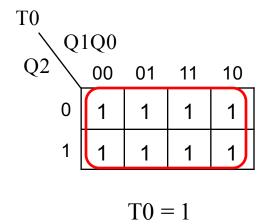
Q	Q ⁺		Т	Q	Q ⁺		T
0	0	No Change	0	Χ	Q	No Change	0
0	1	Toggle	1	X	Q'	Toggle	1
1	0	Toggle	1				
1	1	No Change	0				

Intermediate columns State Table with T input **Next State** T Input **Present State** Q2+ Q2 Q0 Q0+ Q1 Q1+ T2 T0 0 0

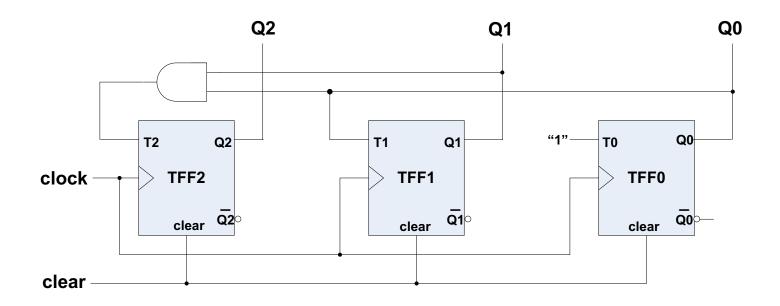
T input equations in terms of Qs can be found from the state table







3-bit binary counter by T FF



+ K'Q

Characteristic table and equation of JK-FF

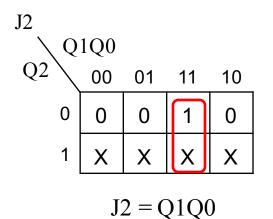
J	K	Q ⁺	Action	
0	0	Q	Hold	$Q^+ = JQ$
0	1	0	Reset	
1	0	1	Set	
1	1	Q'	Toggle	

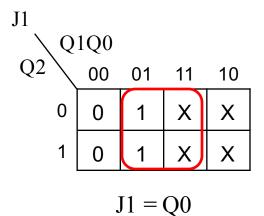
Excitation table of JK-FF

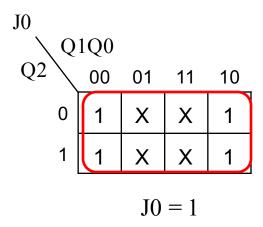
Q	Q ⁺	Action	J	K	J	K
0	0	Reset/Hold	0/0	1/0	0	X
0	1	Set/Toggle	1/1	0/1	1	X
1	0	Reset/Toggle	0/1	1/1	X	1
1	1	Set/Hold	1/0	0/0	Χ	0

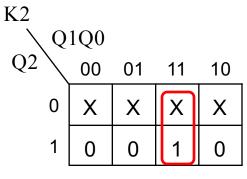
•	Sta	te Ta	ıble v	vith 、	JK in	put	/ In	term	ediat	e colu	ımns	
	Pres	sent S	State	Ne	ext Sta	ate,/			JK Ir	nputs		
	Q2	Q1	Q0	Q2	Q1	Q0	J2	K2	J1	K1	J0	K0
,	0	0	0	0	0	1_	0	X)	0	X	1	X
	0	0	1	0	1	0	0	X	[1	X)	X	1
	0	1	0	0	1	1	0	X	X	0	1	X
	0	1	1	1	0	0_	1	X	X	1.	X	1)
	1	0	0	1	0	1	X	0	0	X	1	X
	1	0	1	1	1	0	X	0	1	X	X	1
	1	1	0	1	1	1	X	0	X	0	1	X
	1	1	1	0	0	0	X	1	X	1	X	1

J-K Input equations

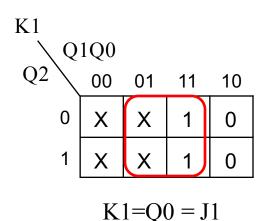


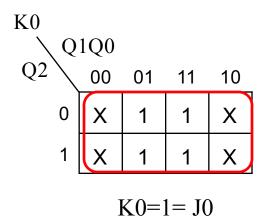




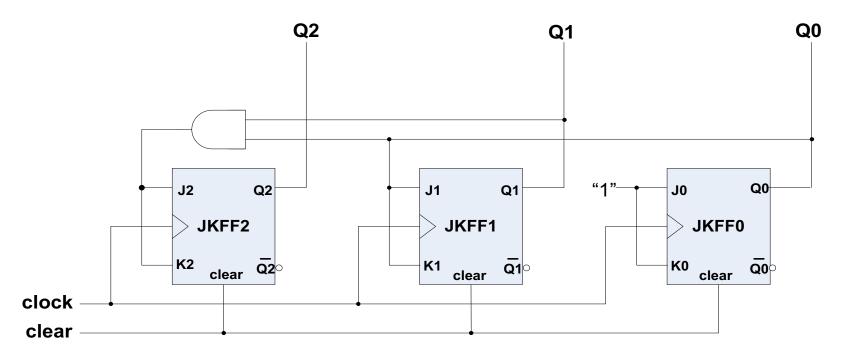






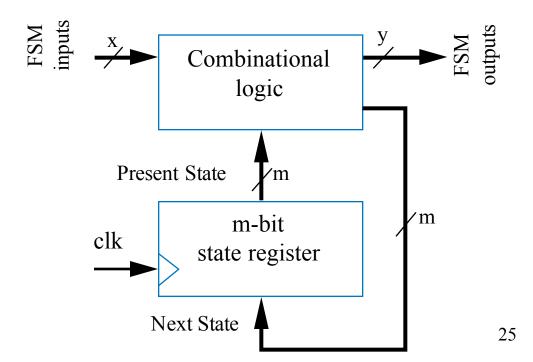


Circuit diagram



Standard FSM Architecture

- How to design sequential circuit?
 - Design as FSM
 - Use standard architecture
 - State register -- to store the present state
 - Combinational logic -- to compute outputs, and next state
 - Known as controller



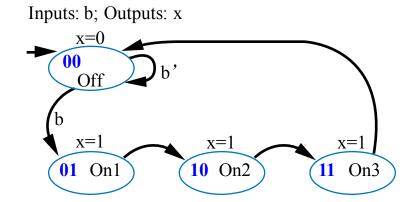
FSM (Controller) Design

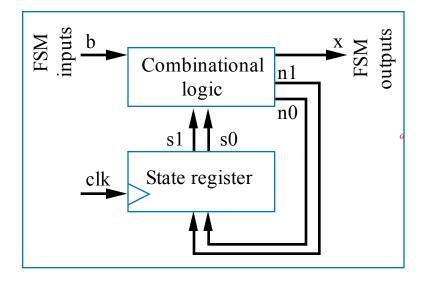
Five step FSM design process

	Step	Description
Step 1	Capture the FSM	Create an FSM that describes the desired behavior of the controller.
Step 2	Create the architecture	Create the standard architecture by using a state register of appropriate width, and combinational logic with inputs being the state register bits and the FSM inputs and outputs being the next state bits and the FSM outputs.
Step 3	Encode the states	Assign a unique binary number to each state. Each binary number representing a state is known as an <i>encoding</i> . Any encoding will do as long as each state has a unique encoding.
Step 4	Create the state table	Create a truth table for the combinational logic such that the logic will generate the correct FSM outputs and next state signals. Ordering the inputs with state bits first makes this truth table describe the state behavior, so the table is a state table.
Step 5	Implement the combinational logic	Implement the combinational logic using any method.

FSM Design Example: Push Button

- Step 1: Capture the FSM
 - Already done
- Step 2: Create architecture
 - 2-bit state register (for 4 states)
 - Input b, output x
 - Present state signals (s1, s2)
 - Next state signals (n1, n0)
- Step 3: Encode the states
 - Any encoding with unique representation for each state will work

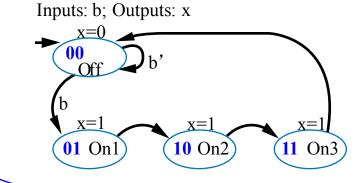


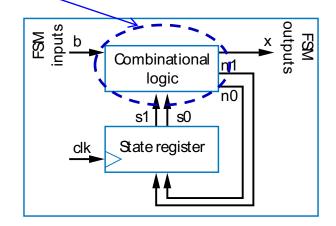


FSM Design Example: Push Button (cont.)

Step 4: Create state table

		Input	S	Outputs			
		Present state		Х	Next state		
Off	0	0 0	0 1	0 0	0 0	0	
On1	0	1 1	0 1	1 1	1 1	0	
On2	1 1	0 0	0 1	1 1	1 1	1 1	
On3	1 1	1 1	0 1	1 1	0 0	0 0	
C	ombina In	ntional puts	Com	bination Outpu	_	gic	

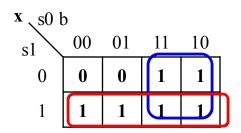




FSM Design Example: Push Button (cont.)

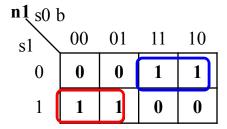
Step 5: Implement combinational logic

$$x = s1 + s0$$

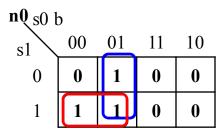


]	Inputs	,	Outputs			
	s 1	s0	b	Х	n1	n0	
Off	0	0	0	0	0	0	
	0	0	1	0	0	1	
On1	0	1	0	1	1	0	
	0	1	1	1	1	0	
On2	1	0	0	1	1	1	
	1	0	1	1	1	1	
On3	1	1	0	1	0	0	
	1	1	1	1	0	0	

$$n1 = s1's0 + s1s0'$$

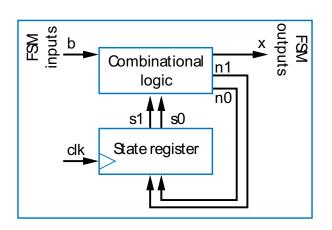


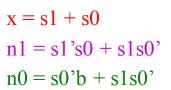
$$n0 = s0$$
'b + $s1s0$ '

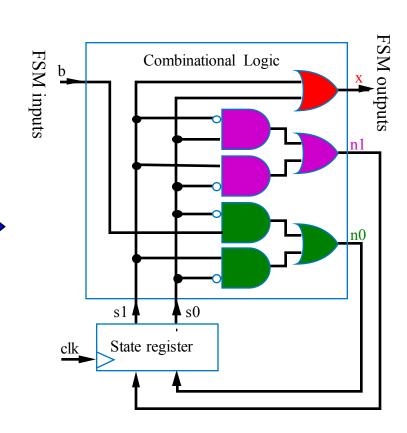


FSM Design Example: Push Button (cont.)

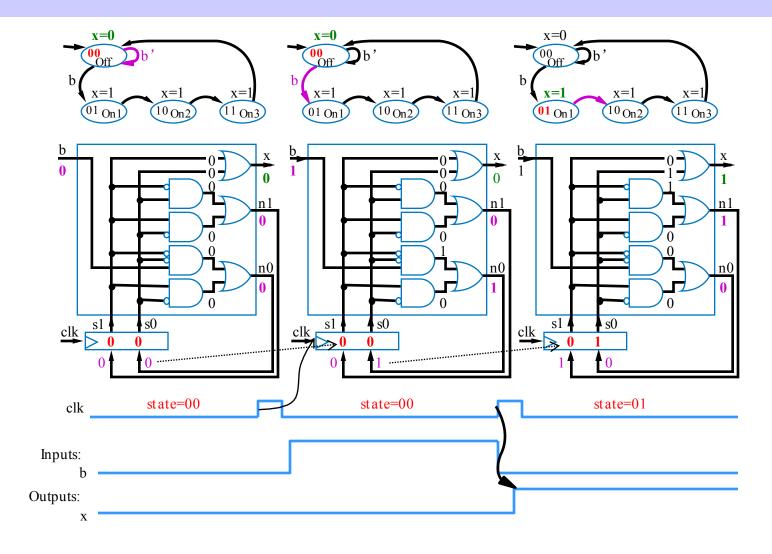
 Step 5: Implement combinational logic (cont)



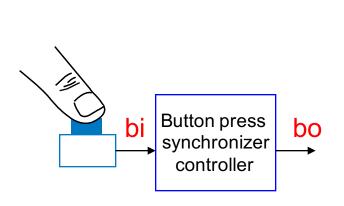


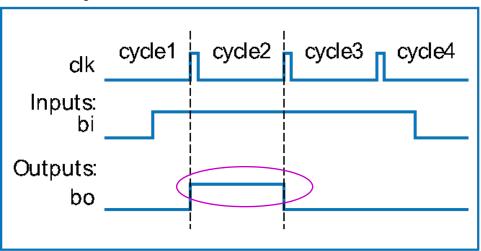


Understanding the Controller's Behavior



FSM Design Example: Button Press Synchronizer

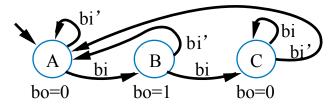




 Want simple sequential circuit that converts button press to single clock cycle duration, regardless of length of time that button actually pressed

FSM Design Example: **Button Press Synchronizer (cont.)**

FSM inputs: bi; FSM outputs: bo



Step 1: FSM

FSM nputs Combinational logic n0s1State register

Combinational logic Inputs Outputs

s1 s0 bi n1 n0 bo 0 0 0 0 0

0 0 0 1 1 0 1

0 1 0 0 0 0 0 unused

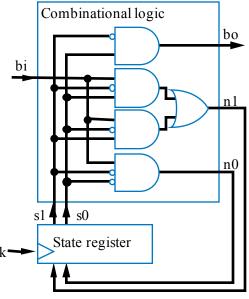
Step 4: State table

may be 'x'

Step 2: Create architecture

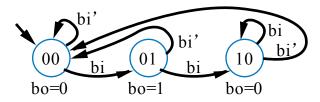
n1 = s1's0bi + s1s0bin0 = s1's0'bi

bo = s1's0bi' + s1's0bi = s1s0



Step 5: Create combinational circuit

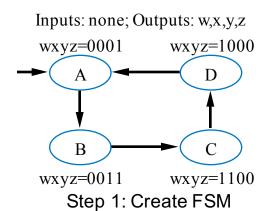
FSM inputs: bi; FSM outputs: bo

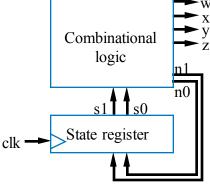


Step 3: Encode states

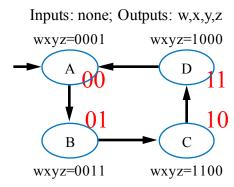
FSM Example: Sequence Generator

- Want generate sequence 0001, 0011, 1100, 1000, (repeat)
 - Each value for one clock cycle





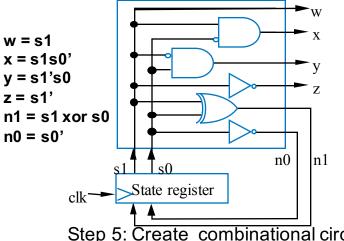
Step 2: Create architecture



Step 3: Encode states

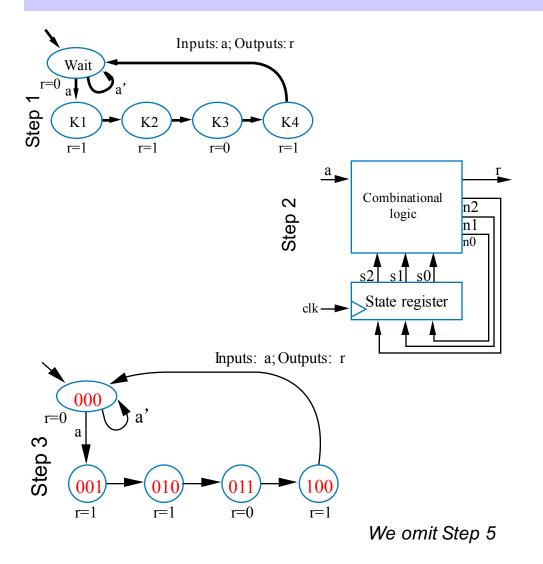
	Inp	outs	Outputs							
	s 1	s 0	W	Χ	У	Z	n1	n0		
A	0	0	0	0	0	1	0	1		
В	0	1	0	0	1	1	1	0		
C	1	0	1	1	0	0	1	1		
D	1	1	1	0	0	0	0	0		

Step 4: Create state table



Step 5: Create combinational circuit

FSM Example: Secure Car Key



	Inputs				Outputs			
	s 2	s1	s 0	а	r	n2	n1	n 0
Wait	0 0	0 0	0 0	0 1	0 0	0 0	0 0	0 1
K1	0	0 0	1 1	0 1	1 1	0 0	1 1	0 0
K2	0 0	1 1	0 0	0 1	1 1	0 0	1 1	1 1
K3	0	1 1	1 1	0 1	0 0	1 1	0	0 0
K4	1 1	0	0 0	0 1	1 1	0	0	0
Unused	1 1 1 1 1	0 0 1 1 1	1 0 0 1 1	0 1 0 1 0 1	00000	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0
Step 4 may be 'x'							35	

Simplifying Notations

FSMs

Assume unassigned output implicitly assigned 0



Sequential circuits

- Assume unconnected clock inputs connected to same external clock

