Overview

The greedy layer-wise approach for pre-training a deep network works by training each layer in turn. Autoencoders can be "stacked" in a greedy layer-wise fashion for pre-training (initializing) the weights of a deep network.

A stacked autoencoder is a neural network consisting of multiple layers of sparse autoencoders in which the outputs of each layer is wired to the inputs of the successive layer. Formally, consider a stacked autoencoder with n layers. Using notation from the autoencoder section, let $W^{(k,1)}, W^{(k,2)}, b^{(k,1)}, b^{(k,2)}$ denote the parameters $W^{(1)}, W^{(2)}, b^{(1)}, b^{(2)}$ for kth autoencoder. Then the encoding step for the stacked autoencoder is given by running the encoding step of each layer in forward order:

$$a^{(l)} = f(z^{(l)})$$
$$z^{(l+1)} = W^{(l,1)}a^{(l)} + b^{(l,1)}$$

The decoding step is given by running the decoding stack of each autoencoder in reverse order:

$$a^{(n+l)} = f(z^{(n+l)})$$
$$z^{(n+l+1)} = W^{(n-l,2)}a^{(n+l)} + b^{(n-l,2)}$$

The information of interest is contained within $a^{(n)}$, which is the activation of the deepest layer of hidden units. This vector gives us a representation of the input in terms of higher-order features.

The features from the stacked autoencoder can be used for classification problems by feeding $a(n)$ to a softmax classifier.

Training

A good way to obtain good parameters for a stacked autoencoder is to use greedy layer-wise training. To do this:

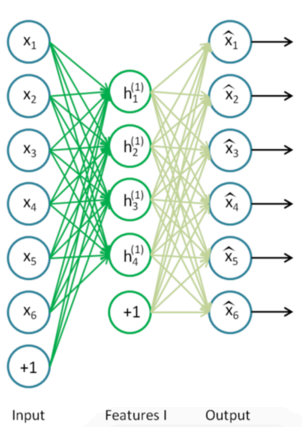
- Train the first layer on raw input to obtain parameters $W^{(1,1)}, W^{(1,2)}, b^{(1,1)}, b^{(1,2)}$.
- Use the first layer to transform the raw input into a vector consisting of activation of the hidden units, A.
- Train the second layer on this vector to obtain parameters $W^{(2,1)}, W^{(2,2)}, b^{(2,1)}, b^{(2,2)}$.
- Repeat for subsequent layers, using the output of each layer as input for the subsequent layer.

This method trains the parameters of each layer individually while freezing parameters for the remainder of the model. To produce better results, after this phase of training is complete, fine-tuning using backpropagation can be used to improve the results by tuning the parameters of all layers are changed at the same time.

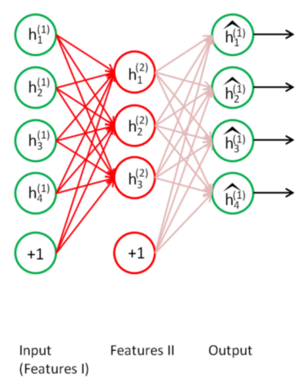
Example

To give a concrete example, suppose you wished to train a stacked autoencoder with 2 hidden layers for classification of MNIST digits.

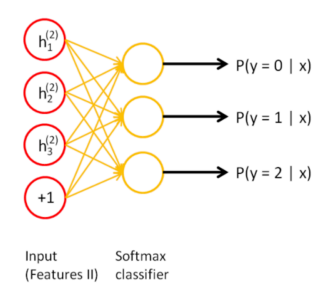
First, you would train a sparse autoencoder on the raw inputs $x^{(k)}$ to learn primary features $h^{(1)(k)}$ on the raw input.



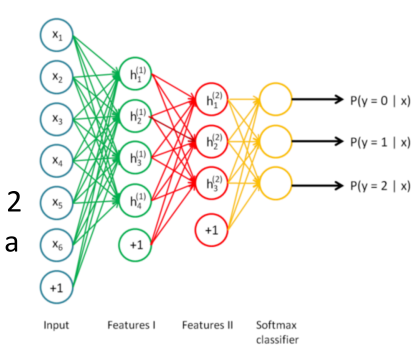
Next, you would feed the raw input into this trained sparse autoencoder, obtaining the primary feature activations $h^{(1)(k)}$ for each of the inputs $x^{(k)}$. You would then use these primary features as the "raw input" to another sparse autoencoder to learn secondary features $h^{(2)(k)}$ on these primary features.



Following this, you would feed the primary features into the second sparse autoencoder to obtain the secondary feature activations $h^{(2)(k)}$ for each of the primary features $h^{(1)(k)}$ (which correspond to the primary features of the corresponding inputs $x^{(k)}$). You would then treat these secondary features as "raw input" to a softmax classifier, training it to map secondary features to digit labels.

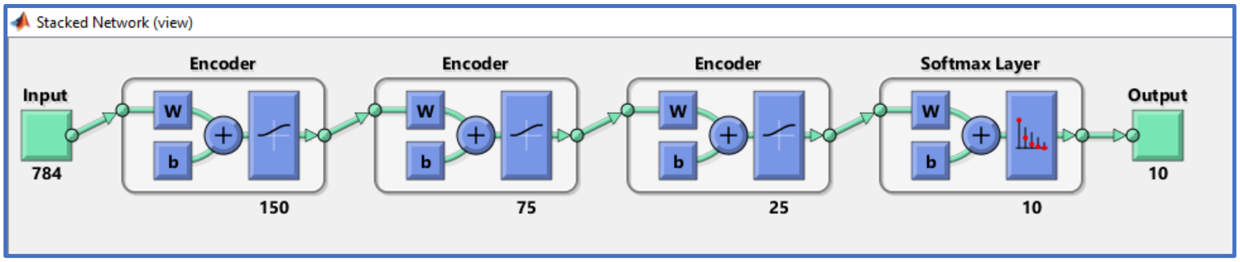


Finally, you would combine all three layers together to form a stacked autoencoder with 2 hidden layers and a final softmax classifier layer capable of classifying the MNIST digits as desired.

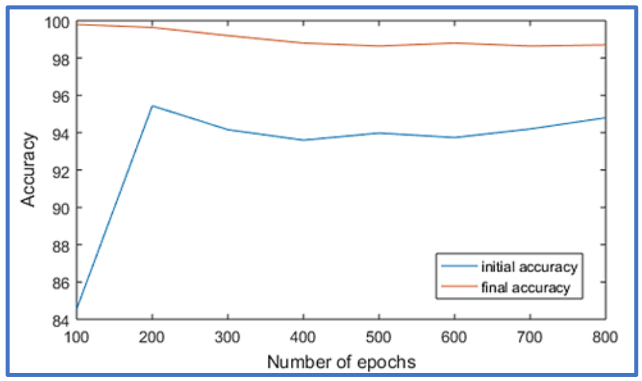


Discussion

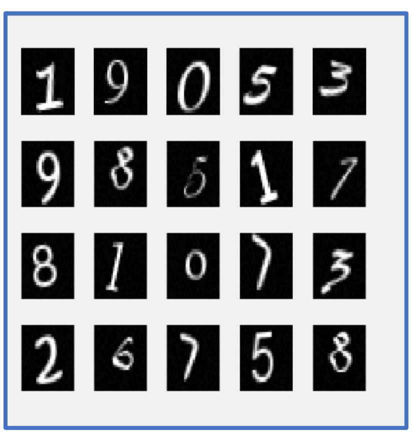
Autoencoder tends to learn features that form a good representation of its input. The first layer of a stacked autoencoder tends to learn first-order features in the raw input (such as edges in an image). The second layer of a stacked autoencoder tends to learn second-order features corresponding to patterns in the appearance of first-order features (e.g., in terms of what edges tend to occur together--for example, to form contour or corner detectors). Higher layers of the stacked autoencoder tend to learn even higher-order features.



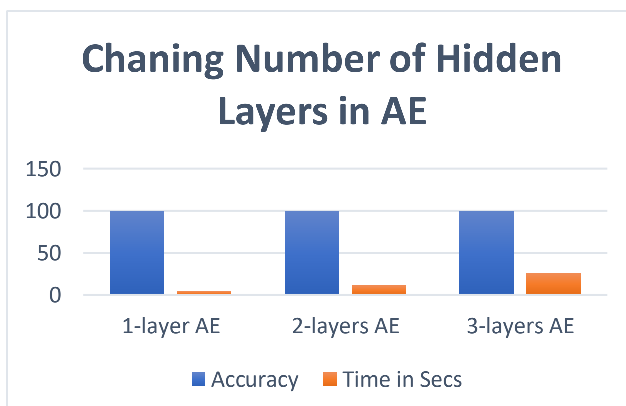
The best performance of 99.8% **is achieved at 100 epochs** even though the initial performance with same number of epochs was 84.5%. The improved performance is due to **fine-tuning**. After all the weights have been initialized, the whole encoder-decoder network is treated as a multilayer FFN and the weights are fine-tuned via backpropagation to minimize the reconstruction error. For further experiments we set epochs to 100.



Different **number of neurons in the two hidden layers** [50,25], [100,50], [150,75], [200,100] are tested. With the combination of [150, 25], [200, 50], and [200, 75] number of neurons in hidden layers, we achieve an accuracy of 99.9%.

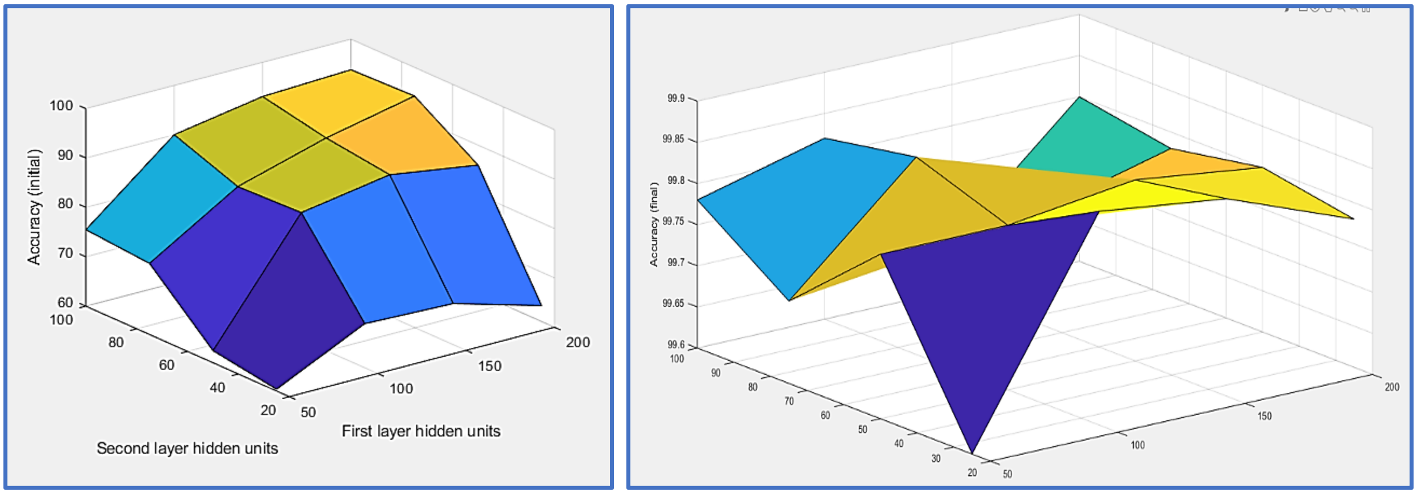


Stacked AE Digit Classification



Accuracy and number of hidden layers in SAEs

Stacked autoencoders with one [150], two [150,75] and three [150 75 25] hidden layers are tested. Accuracy and computation time of the AEs with different number of layers shows by adding more hidden layers, there isn't much difference in accuracy of the results, but the computation time increases by adding more hidden layers.



Initial Accuracy and Final Accuracy for different number of hidden neurons with in SAEs

References

[1] UFLDL Tutorial: Autoencoders*: http://ufldl.stanford.edu/tutorial/unsupervised/Autoencoders*
[2] UFLDL Tutorial: Stacked Autoencoders*: http://ufldl.stanford.edu/wiki/index.php/Stacked Autoencoders*