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Three-axes predictive control of autopilot for missile

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Abstract
Purpose – The purpose of this paper is to present the autopilot design for the missile under various disturbances.
Design/methodology/approach – In this study, model predictive control (MPC) method has been used for autopilot design for each axis. The aim of autopilot is that to keep the roll angle value around the zero degree and to track pitch/yaw acceleration commands. This three-axes control methodology also takes into consideration the interaction between pitch, yaw and roll motions.
Findings – The purpose of using MPC method for three-axes of the autopilot is to decrease the control effort and to make the close-loop system insensitive against modeling uncertainties and stochastic effects.
Originality/value – This study shows that the missile is able to reach to the desired target with good robustness, low control effort and little miss-distance under disturbances such as aerodynamic uncertainties, thrust misalignment and gust affect by using this alternative control method.

Keywords Simulation, Autopilot design, Guidance algorithm, Missile dynamic, Model predictive control, Three-axes control

Paper type Research paper

Nomenclature

A	=	Reference area, m^2;
a_n, a_y	=	Normal and yaw accelerations of the missile, m/s^2;
cg	=	center-of-gravity;
d	=	Diameter of the missile, m;
C_m, C_n, C_l	=	Aerodynamic moment coefficients;
C_x, C_y, C_z	=	Aerodynamic force coefficients;
EOM	=	Equation of motion;
F_x, F_y, F_z	=	Aerodynamic forces, N;
G_x, G_y, G_z	=	Gravity forces, N;
H_p	=	Prediction horizon;
H_u	=	Control horizon;
$I_{x,y,z}$, $J_{xy,xz,zy}$	=	Moment of inertia of the missile, $kg.m^2$;
l_{ref}	=	Reference length, m;
LOS	=	Line-of-sight angle;
M, N, L	=	Aerodynamic moments, Nm;
MPC	=	Model predictive control;
m	=	Mass of missile, kg;
p, q, r	=	Roll, pitch and yaw rate, rad/s;
x_b, y_b, z_b	=	Body coordinate system;
x_{ap}, y_{ap}, z_{ap}	=	Autopilot coordinate system;
t_0, t_f	=	Initial and final time of the simulation;
T_x, T_y, T_z	=	Propulsion forces, N;
u, v, w	=	Components of the velocity, m/s;
Q	=	Dynamic pressure, Pa;
V	=	Speed of missile, m/s;
α, β	=	Angle of attack and sideslip angle, rad;
ϕ, θ, ψ	=	Roll, pitch and yaw angles of missile, rad;
$\delta_{a/e/r}$	=	Control surface deflections (elevator, rudder and aileron), rad;
$\delta_{a/e/r\_com}$	=	CAS commands, rad;
$\underline{\hat{x}}$	=	Prediction state variables.

Introduction

Control algorithms of a missile can be divided to two different sections – guidance and autopilot algorithms. The guidance algorithm generates appropriate guidance commands for the autopilot algorithm. The autopilot algorithm gives the direction the missile to track the guidance commands. The aerospace industry mostly uses three-loop control technique (Defu *et al.*, 2009; Devaud *et al.*, 2001), and gain-scheduling method is also applied to missile (Stilwell, 2001; Wu *et al.*, 2002). Robust control design formulation is proposed for autopilot design problem (Lin *et al.*, 1995; Nam *et al.*, 2001). Robust roll autopilot based on the extended state observer technique is proposed (Talole *et al.*, 2011). Additionally, missile autopilot

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design is studied by using sliding mode control (Huang *et al.*, 1993; Shima *et al.*, 2006), and observer-based control for skid-to-turn missile is presented (Chwa and Choi, 2004; Das *et al.*, 2005). Receding horizon and predictive control methodologies have been applied successfully to autopilot design (Kim and Kwon, 1997; Kim *et al.*, 2000; Upreti and Talole, 2004; Tang and Cai, 2012). Additionally, constrained model predictive control (MPC) method has been applied to spacecraft (Kolmanovsky *et al.*, 2014)

In this paper, the aim of autopilot is that to keep the roll angle value around the zero degree and to track guidance commands on pitch and yaw axes. The aerodynamic coefficients of a missile were reached by using Missile DATCOM program (Ada, 2011). The nonlinear missile dynamic was linearized around the trim conditions to design autopilot algorithm by using MPC method. Three-axes MPC methodology has been applied for various scenarios unlike the relevant literature in this field. These scenarios include model uncertainties and some environment disturbances such as aerodynamic uncertainties and thrust misalignment and gust effects. The aim of autopilot is that to keep the roll angle value around the zero degree and to track the guidance acceleration commands. Three-axes control methodology also takes into consideration the interaction between pitch, yaw and roll motions. Thus, the performance of autopilot improves. The autopilot calculates control surface deflection commands to guide the missile to the target. To do this, the autopilot uses some information from the navigation algorithm and sensors. The nonlinear simulation including the missile dynamic model set up for certain scenarios were formed by using MATLAB/SIMULINK. The nonlinear simulation includes navigation model, guidance and autopilot algorithm designing for more than one trim condition.

Proposed autopilot models were used in the nonlinear simulation (nonlinear missile dynamic model, navigation algorithm and linear controllers), and their performances were checked for different scenarios. The results show that the missile is able to reach to the desired outputs with good robustness, low control effort and little miss distance which are significant issues in terms of design of missile systems.

Model of the missile

The mathematical model of the missile was obtained assuming a rigid body, constant mass and inertia of the missile.

Dynamical model of the missile

The equations of motion (EOM) are generated by Newton's second law and Euler moment equations [equations (1) and (2)] (McLean, 1990; Blakelock, 1991; Etkin and Reid, 1996):

$$\vec{F} = \begin{bmatrix} F_x + T_x + G_x \\ F_y + T_y + G_y \\ F_z + T_z + G_z \end{bmatrix} = \begin{bmatrix} m(\dot{u} + wq - vr) \\ m(\dot{v} + ur - wp) \\ m(\dot{w} + vp - uq) \end{bmatrix} \quad (1)$$

$$\overrightarrow{M} = \begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} \dot{p}I_x - \dot{r}J_{xz} + qr(I_z - I_y) - pqJ_{xz} \\ \dot{q}I_y + pr(I_x - I_z) + (p^2 - r^2)J_{xz} \\ \dot{r}I_z - \dot{p}J_{xz} + pq(I_y - I_x) - qrJ_{xz} \end{bmatrix} \quad (2)$$

To linearize the EOM, gravity and thrust terms are neglected. Assuming that roll rate is equal to zero ($p = 0$ rad/s) and the missile is symmetric at XY and XZ planes, J_{xz}, J_{xy} are equal to zero. The linear missile equations (f_x, f_y and f_z signify linear forces, and l, m and n signify linear moments) are given below (McLean, 1990; Blakelock, 1991; Etkin and Reid, 1996):

$$f_x = m[\dot{u} + wq] \quad (3)$$

$$f_y = m[\dot{v} + ur] \quad (4)$$

$$f_z = m[\dot{w} - uq] \quad (5)$$

$$l = \dot{p} \cdot I_x \quad (6)$$

$$m = \dot{q} \cdot I_y \quad (7)$$

$$n = \dot{r} \cdot I_z \quad (8)$$

Aerodynamic model of the missile

Aerodinamic forces and moments can be written as follows in terms of coefficients:

$$f_x = Q \cdot A \cdot C_x \quad (9)$$

$$f_y = Q \cdot A \cdot C_y \quad (10)$$

$$f_z = Q \cdot A \cdot C_z \quad (11)$$

$$l = Q \cdot A \cdot l_{ref} \cdot C_l \quad (12)$$

$$m = Q \cdot A \cdot l_{ref} \cdot C_m \quad (13)$$

$$n = Q \cdot A \cdot l_{ref} \cdot C_n \quad (14)$$

The missile aerodynamic coefficients were calculated by using Missile DATCOM program (The USAF Stability and Control DATCOM, 1999) for the flight regime (Ada, 2011). The flight conditions were predicted; Mach number is changing from 0.1 to 0.8; attack (α)/sideslip (β) angles and control surface deflections are between −10 and +10 degrees. Aerodynamic coefficients graphs are shown in Figures 1-4.

Figure 1 Axial force coefficient

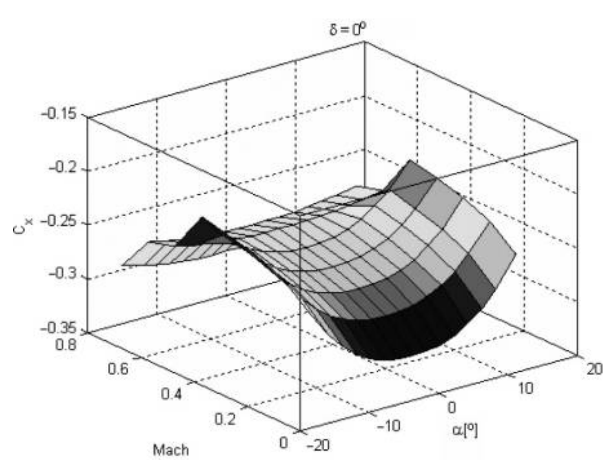


Figure 2 Roll moment coefficient

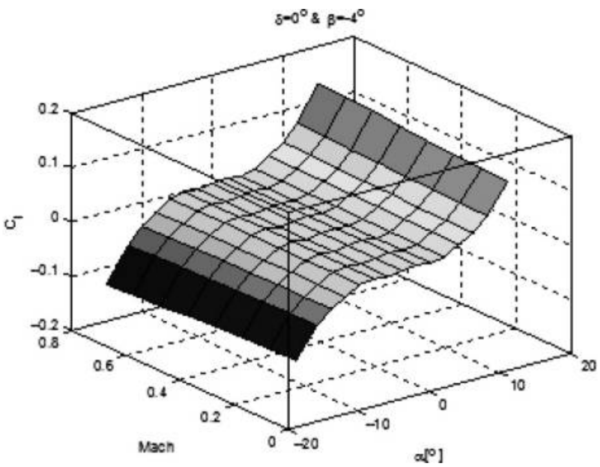


Figure 3 Normal force coefficient

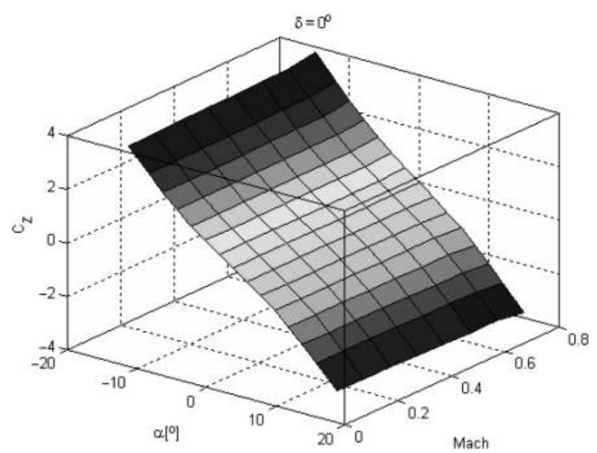
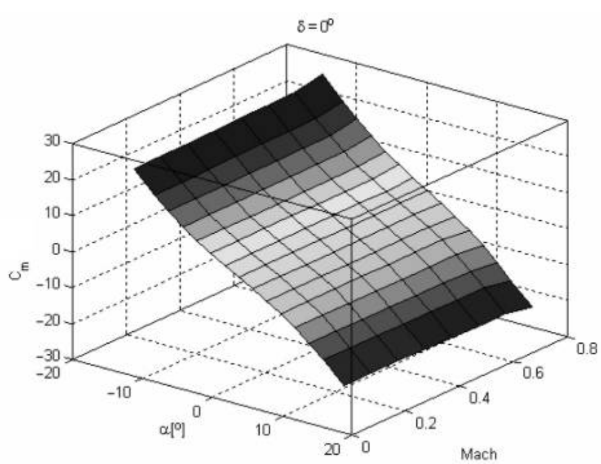


Figure 4 Pitch moment coefficient



Autopilot design

The stability algorithm of missiles, accomplishing the guidance commands along a path, is called "autopilot". To do that, autopilot calculates needed control surface deflection angles (elevator, rudder and aileron) by using sensor and navigation outputs. In this study, the missile is controlled at three axes with three different controllers – pitch, yaw and roll autopilot algorithms. The pitch and yaw autopilot algorithms are the same because the missile is symmetric at XY and XZ surfaces. It is assumed that the missile has a skid-to-turn maneuvering movement. To do skid-to-turn movement, roll stabilization is needed because the roll angle is affected by attack and side-slip angles like a disturbance effect. The roll autopilot tries to cope with this effect and keep to the roll angle of missile around to zero. Consequently, by using controllers for three axes in this study, the control methodology also takes into consideration the interaction between pitch, yaw and roll motions.

Autopilot coordinate system

Autopilot axes are defined as follows (Ada, 2011) (Figures 5 and 6).

Where $\{\}_{ap}$ is autopilot axis system; $\{\}_b$ is body axis system and cg denotes center-of-gravity of the missile.

Pitch/yaw axis system model

Pitch axis equations, which are functions of attack angle, pitch rate and elevator angle, can be written as partial derivative form:

$$F_z = Q \cdot A \cdot \left(C_{z\alpha} \cdot \alpha + C_{z\delta} \cdot \delta_e + C_{zq} \cdot \frac{d}{2V} \cdot q \right) \quad (15)$$

Figure 5 Pitch autopilot axis system

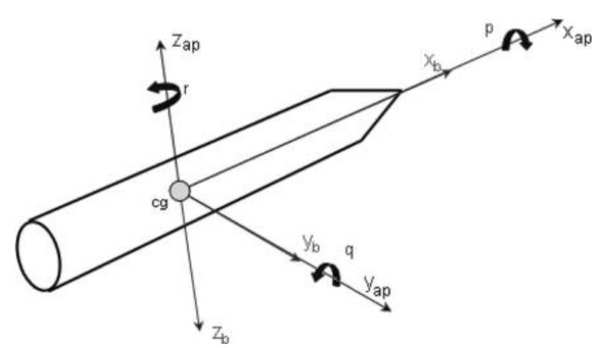
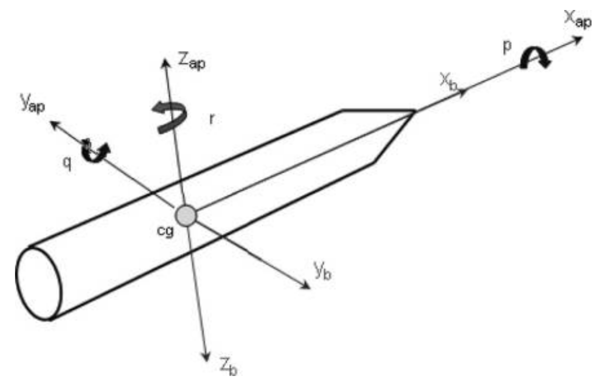


Figure 6 Yaw autopilot axis system



$$M = Q \cdot A \cdot l_{ref} \cdot \left(C_{m\alpha} \cdot \alpha + C_{m\delta} \cdot \delta_e + C_{mq} \cdot \frac{d}{2V} \cdot q \right) \quad (16)$$

The state-space model of the pitch axes is given as follows (McLean, 1990; Blakelock, 1991; Etkin and Reid, 1996):

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \left(\frac{Q \cdot A}{m \cdot V} C_{z\alpha} \right) & 1 + \left(\frac{Q \cdot A}{m \cdot V} \cdot \frac{d}{2V} C_{zq} \right) \\ \left(\frac{Q \cdot A}{I_y} \cdot l_{ref} \cdot C_{m\alpha} \right) & \left(\frac{Q \cdot A}{I_y} \cdot \frac{d}{2V} \cdot l_{ref} \cdot C_{mq} \right) \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} \left(\frac{Q \cdot A}{m \cdot V} C_{z\delta} \right) \\ \left(\frac{Q \cdot A}{I_y} \cdot l_{ref} \cdot C_{m\delta} \right) \end{bmatrix} \delta_e \quad (17)$$

According to autopilot coordinate system, system pitch acceleration is in the opposite direction according to body axis system. Therefore:

$$a_z = \left[\left(\frac{Q \cdot A}{m} C_{z\alpha} \right) \;\; \left(\frac{Q \cdot A}{m} \cdot \frac{d}{2V} C_{zq} \right) \right] \begin{bmatrix} \alpha \\ q \end{bmatrix} + \left[\left(\frac{Q \cdot A}{m} C_{z\delta} \right) \right] \delta_e, \quad a_n = -a_z \quad (18)$$

The yaw model of the missile can be obtained by using pitch model according to yaw autopilot coordinate system:

$$\begin{array}{ll} C_{z\alpha} \rightarrow C_{y\beta} & C_{m\alpha} \rightarrow C_{n\beta} \\ C_{z\delta} \rightarrow C_{y\delta} & C_{m\delta} \rightarrow C_{n\delta} \\ C_{zq} \rightarrow C_{yr} & C_{mq} \rightarrow C_{nr} \end{array}$$

The state-space model of the yaw axis is given as follows (McLean, 1990; Blakelock, 1991; Etkin and Reid, 1996):

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \left(\frac{Q \cdot A}{m \cdot V} C_{y\beta} \right) & 1 - \left(\frac{Q \cdot A}{m \cdot V} \cdot \frac{d}{2V} C_{yr} \right) \\ \left(\frac{Q \cdot A}{I_z} \cdot l_{ref} \cdot C_{n\beta} \right) & \left(\frac{Q \cdot A}{I_z} \cdot \frac{d}{2V} \cdot l_{ref} \cdot C_{nr} \right) \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} \left(\frac{Q \cdot A}{m \cdot V} C_{y\delta} \right) \\ \left(\frac{Q \cdot A}{I_z} \cdot l_{ref} \cdot C_{n\delta} \right) \end{bmatrix} \delta_r \quad (19)$$

$$a_y = \left[\left(\frac{Q \cdot A}{m} C_{y\beta} \right) \;\; \left(\frac{Q \cdot A}{m} \cdot \frac{d}{2V} C_{nr} \right) \right] \begin{bmatrix} \alpha \\ q \end{bmatrix} + \left[\left(\frac{Q \cdot A}{m} C_{y\delta} \right) \right] \delta_r \quad (20)$$

Roll axis system model

To do roll stabilization of the missile, roll model is needed. Roll moment equation is functions of roll rate and aileron angle. Then this equation can be written in partial derivative form [equation (21)] as follows:

$$L = Q \cdot A \cdot l_{ref} \cdot \left(C_{l\delta} \cdot \delta_a + C_{lp} \cdot \frac{d}{2V} \cdot p \right) \quad (21)$$

The state-space model of the yaw axis is given as follows (McLean, 1990; Blakelock, 1991; Etkin and Reid, 1996; Ada, 2011):

$$\begin{bmatrix} \dot{\phi} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & \left(\frac{Q \cdot A}{I_x} \cdot \frac{d}{2V} l_{ref} \cdot C_{lp} \right) \end{bmatrix} \begin{bmatrix} \phi \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ \left(\frac{Q \cdot A}{I_x} \cdot l_{ref} \cdot C_{l\delta} \right) \end{bmatrix} \delta_a \quad (22)$$

$$\phi = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \phi \\ p \end{bmatrix} + [0]\delta_a \quad (23)$$

Autopilot design points

Autopilot must be designed more than one trim point for flight performance. Trim points are chosen considering flight parameters, which are dynamic pressure, center-of-gravity, velocity and mass of missile. Dynamic pressure is the dominant sub-function of aerodynamic forces and moments. Dynamic pressure depends on velocity of missile and density of atmosphere which changes with altitude of missile. Additionally, changing of the center-of-gravity and mass can be neglected. Therefore, design points of controller are chosen dependent on altitude and velocity of the missile as follows (Ada, 2011):

- Mach number: 0.1-0.2-0.3-0.4-0.5-0.6-0.7-0.8
- Altitude [m]: 1,000

Model predictive control

The aim of MPC is to calculate the system input, u. The controller is explicitly a function of the system model that can be modified in real-time, and quadratic cost function is solved online at each sampling instance over a shifted prediction horizon to calculate the optimum control moves.

State variables and system output desired at the time are predicted by using discrete-time state-space model of the system. To use discrete-time system model, state variables of the system and system output can be estimated at the future steps. At k sample time, equation which represents state variables' vector equality at desired time step is given in equations (24) and (25) (Camacho and Bordons, 1999; Maciejowski, 2002; Tang and Cai, 2012; Ada and Kural, 2014). Calculated discrete-time system model whose sample time was chosen as 5 ms was used to design controllers:

$$\begin{aligned} \hat{x}(k+1|k) &= A \cdot x(k) + B \cdot \hat{u}(k|k) \hat{x}(k+2|k) \\ &= A \cdot \hat{x}(k+1) + B \cdot \hat{u}(k+1|k) \\ &= A^2 \cdot x(k) + A \cdot B \cdot \hat{u}(k|k) + B \cdot \\ &\quad \hat{u}(k+1|k) \; \vdots \; \hat{x}(k+H_p|k) \\ &= A \cdot \hat{x}(k+H_p-1) + B \cdot \hat{u}(k+H_p-1|k) \\ &= A^{H_p} \cdot x(k) + A^{H_p-1} \cdot B \cdot \hat{u}(k|k) + \cdots \\ &\quad + B \cdot \hat{u}(k+H_p-1|k) \end{aligned} \quad (24)$$

where H_u signifies control horizon, and H_p signifies prediction horizon. Equation (24) can be written in the matrix form like equation (25):

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$$\begin{bmatrix} \hat{x}(k+1|k) \\ \vdots \\ \hat{x}(k+H_u|k) \\ \hat{x}(k+H_u+1|k) \\ \vdots \\ \hat{x}(k+H_p|k) \end{bmatrix} = \underbrace{\begin{bmatrix} A \\ \vdots \\ A^{H_u} \\ A^{H_u+1} \\ \vdots \\ A^{H_p} \end{bmatrix} \cdot x(k) + \begin{bmatrix} A \\ \vdots \\ \sum_{i=0}^{H_u-1} A^i B \\ \sum_{i=0}^{H_u} A^i B \\ \vdots \\ \sum_{i=0}^{H_p-1} A^i B \end{bmatrix} \cdot u(k-1)}_{PAST} + \underbrace{\begin{bmatrix} B & \dots & 0 \\ AB+B & \dots & 0 \\ \vdots & \ddots & \vdots \\ \sum_{i=0}^{H_u-1} A^i B & \dots & B \\ \vdots & \vdots & \vdots \\ \sum_{i=0}^{H_p-1} A^i B & \dots & \sum_{i=0}^{H_p-H} A^i B \end{bmatrix} \cdot \begin{bmatrix} \Delta\hat{u}(k|k) \\ \vdots \\ \Delta\hat{u}(k+H_u-1|k) \end{bmatrix}}_{FUTURE} \quad (25)$$

At desired time step, system outputs can be calculated as equation (26):

$$\begin{aligned} \hat{y}(k+1|k) &= C_y \cdot \hat{x}(k+1|k) \\ \hat{y}(k+2|k) &= C_y \cdot \hat{x}(k+2|k) \\ &\vdots \\ \hat{y}(k+H_p|k) &= C_y \cdot \hat{x}(k+H_p|k) \end{aligned} \quad (26)$$

System error vector for the future is given in equation (27):

$$\varepsilon(k) = T(k) - \psi \cdot x(k) - \Upsilon \cdot u(k-1) \quad (27)$$

where $T(k)$ is the reference path vector [equation (30)], and ψ and Υ can be defined as follows [equations (28) and (29)]:

$$\Psi(k) = \begin{bmatrix} C_z & 0 & \dots & 0 \\ 0 & C_z & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & C_z \end{bmatrix} \cdot \begin{bmatrix} A \\ \vdots \\ A^{H_u} \\ A^{H_u+1} \\ \vdots \\ A^{H_p} \end{bmatrix} \quad (28)$$

$$\Upsilon(k) = \begin{bmatrix} C_z & 0 & \dots & 0 \\ 0 & C_z & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & C_z \end{bmatrix} \cdot \begin{bmatrix} A \\ \vdots \\ \sum_{i=0}^{H_u-1} A^i B \\ \sum_{i=0}^{H_u} A^i B \\ \vdots \\ \sum_{i=0}^{H_p-1} A^i B \end{bmatrix} \quad (29)$$

$$T(k) = \begin{bmatrix} r(k+1) \\ \vdots \\ r(k+H_p) \end{bmatrix} \quad (30)$$

Cost function of the system can be defined as given equation [equation (31)]:

$$\mathcal{J}(k) = \|Y(k) - T(k)\|_Q^2 + \|\Delta U(k)\|_R^2 \quad (31)$$

where:

$$Y(k) = \begin{bmatrix} y(k+1|k) \\ \vdots \\ y(k+H_P|k) \end{bmatrix} \quad (32)$$

By rearranging the cost function given in equation (31), the following equation can be obtained [equation (33)]:

Table I Pitch/yaw MPC autopilot design

Mach	Prediction horizon (H_p)	Control horizon (H_u)	System input weighting factor (R)	System output weighting factor (Q)
0.1	300	10	9	0.01
0.2	150	10	13	0.0084
0.3	95	10	14	0.008
0.4	83	10	15.5	0.0077
0.5	78	10	18	0.0074
0.6	60	10	20.3	0.007
0.7	50	10	22.6	0.007
0.8	175	10	25	0.0066

Table II Roll MPC autopilot design

Mach	Prediction horizon (H_p)	Control horizon (H_u)	System input weighting factor (R)	System output weighting factor (Q)
0.1	400	2	0.82	0.03
0.2	350	2	1.26	0.076
0.3	250	2	2.28	0.08
0.4	225	2	4.7	0.09
0.5	200	2	6.2	0.095
0.6	150	2	7.7	0.1
0.7	150	2	10	0.13
0.8	150	2	12.6	0.13

Figure 7 Velocity vectors of missile and the target at the firing time

$$\begin{aligned}\mathcal{J}(k) &= \|\Theta \cdot \Delta U(k) - \varepsilon(k)\|_Q^2 + \|\Delta U(k)\|_R^2 \\ &= [\Delta U(k)^T \cdot \Theta^T - \varepsilon(k)^T] \cdot Q \cdot [\Theta \cdot \Delta U(k) - \varepsilon(k)] \\ &\quad + \Delta U(k)^T \cdot R \cdot \Delta U(k) \\ &= \varepsilon(k)^T \cdot Q \cdot \varepsilon(k) - 2\Delta U(k)^T \cdot \Theta^T \cdot Q \cdot \varepsilon(k) \\ &\quad + \Delta U(k)^T [\Theta^T \cdot Q \cdot \Theta + R] \Delta U(k) \end{aligned} \tag{33}$$

The system input is calculated by minimizing the cost function to follow the reference path with minimum effort [equation (34)]:

$$\frac{\partial \mathcal{J}(k)}{\partial \Delta U(k)} = 0 \rightarrow \Delta U(k)_{opt} = [\Theta^T \cdot Q \cdot \Theta + R]^{-1} \cdot \Theta^T \cdot Q \cdot \varepsilon(k) \tag{34}$$

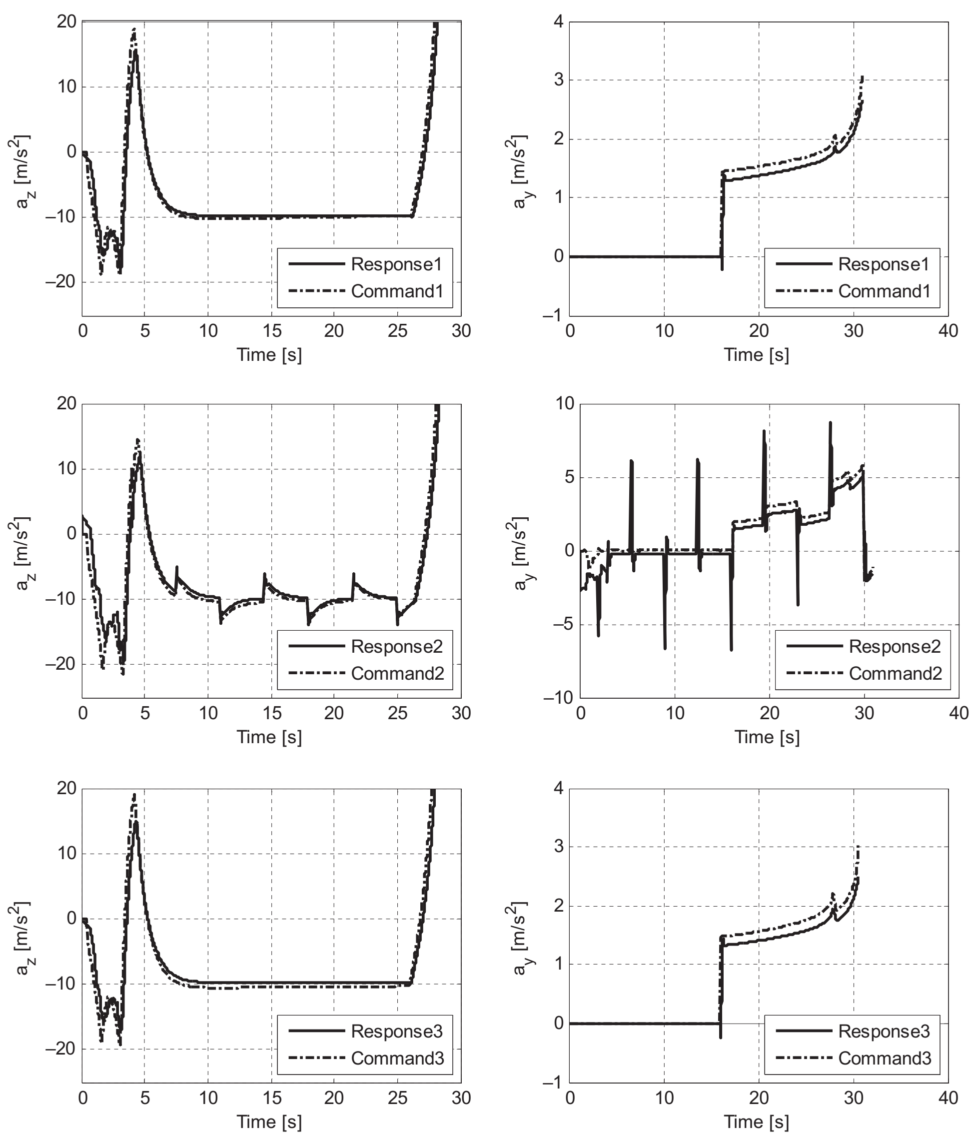
First element of obtained input vector which is optimal control input is applied to the system. Selections of tuning parameters of the pitch autopilot are given in Tables I and II. Horizons and weighting factors of the MPC were chosen according to system performance constraints (Ada, 2011).

Guidance algorithm

Guidance algorithm consists of midcourse and terminal phases. The midcourse guidance starts with the fire command. In this phase, the missile climbs the reference altitude, maintains this altitude to satisfy the ideal impact angle and tracks the target in the yaw channel. When the line-of-sight (LOS) angle exceeds the reference LOS angle, terminal guidance phase starts (Erer and Merttopcuoglu, 2010). During this phase, acceleration commands are computed according to proportional guidance law (Zarchan, 1994; Shneydor, 1998; Yanushevsky, 2008):

$$\vec{a}_{com} = N \cdot \vec{\dot{\lambda}} \times \vec{V}_{missile} \tag{35}$$

Figure 8 Pitch and yaw acceleration responses for different scenarios



where is the velocity vector of the missile; N is the effective navigation ratio; $\vec{\dot{\lambda}}$ is the LOS rate. $\vec{\dot{\lambda}}$ can be calculated from the target and missile positions ($\vec{P}_{target}$, $\vec{P}_{missile}$) as follows:

$$\vec{r} = \vec{P}_{target} - \vec{P}_{missile} \rightarrow \vec{\dot{\lambda}} = \frac{\vec{r} \times \dot{\vec{r}}}{|\vec{r}|^2} \quad (36)$$

Simulation model

The six degree-of-freedom (DOF) nonlinear missile model is developed in Matlab-SIMULINK environment. First, linear autopilots are designed by using the linear model of the missile, and, then, they are integrated to the nonlinear missile model (Ada and Kural, 2014). After the missile locks to the target, nearly 16 s of the flight, it starts to maneuver in yaw axis. Additionally, target moves with an angle of 90° with respect to the missile at time 0 which is shown in Figure 7. Target speed is assumed as 5 m/s.

Autopilot performance is observed by running nonlinear simulations for three different scenarios. In the first two of these scenarios, disturbance effects are taken into account, and the nominal case is also considered. Disturbances and uncertainties used are defined below.

Thrust misalignment and gust disturbances

Yaw and pitch axes are affected by −2° and 2° thrust misalignments, respectively. Thrust misalignment causes pitch and yaw rates during the active phase. Additionally, 0-3 m/s gust in pitch channel and 0-5 m/s gust in yaw channel are added. Because gust effects impact the velocity vector of the missile, system outputs are changed, whereas system model is not affected.

Aerodynamic uncertainties

To all aerodynamic moment and force coefficients, 5 per cent uncertainties are added. Because the missile dynamics are changed, the autopilot must be robust enough to compensate these differences.

Simulation results

Three nonlinear missile simulations are conducted. First simulation is the nominal case (Simulation 1), and the others are the simulations in which thrust misalignment and gust effects (Simulation 2) and aerodynamic uncertainties (Simulation 3) are considered. System responses and commands of each simulation are shown between Figures 8-12. Yaw and pitch acceleration graphs are shown in Figure 8. Despite the uncertainties in the system model, guidance commands are well-tracked, and the target is hit as desired.

Roll angle graphs are shown in Figure 9. Roll autopilot keeps the roll attitude around the 0°, despite of the aerodynamic uncertainties and disturbance effects.

The control surface deflection angles are given in Figure 10. Elevator, rudder and aileron angles are less than 10° for all cases; hence, the aerodynamic model constraints are satisfied.

The flight performances of each simulation are summarized in Table III. The impact angle is the missile pitch attitude at the hit moment. The distance between the target and the impact coordinate of the missile is called the miss distance. The performance outputs, which are impact angle, miss distance and control surface deflection efforts, are calculated to the nonlinear simulation by using EOM given in equations (1) and (2). The formula of the

Figure 9 Roll angle responses for different scenarios

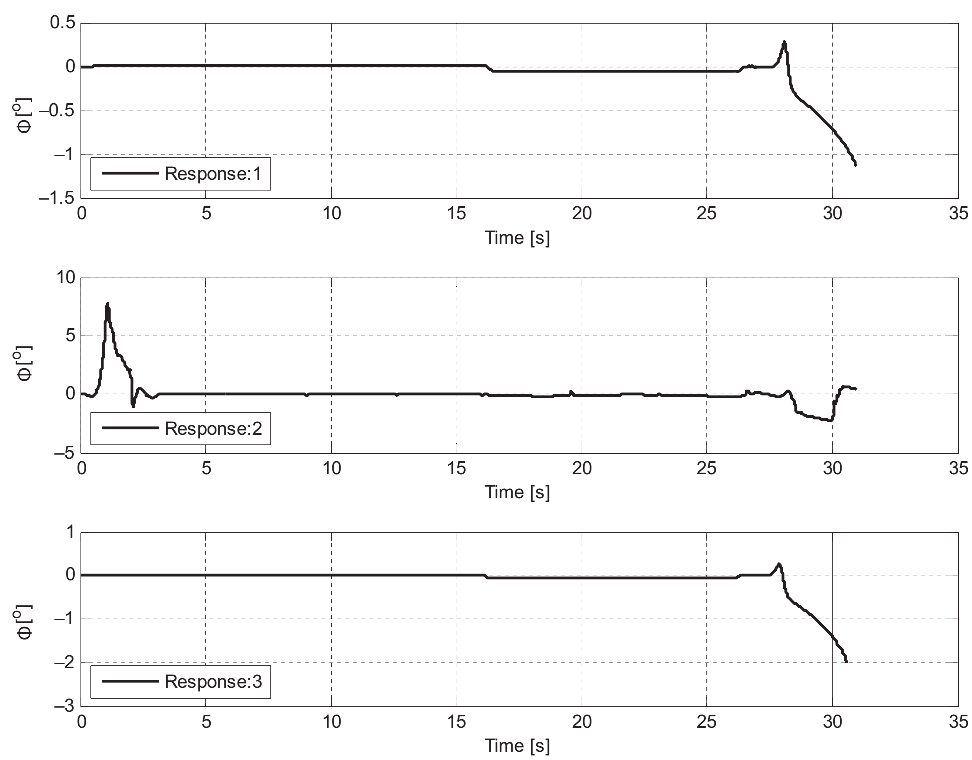


Figure 10 Surface deflections (elevator, rudder and aileron) of first scenario

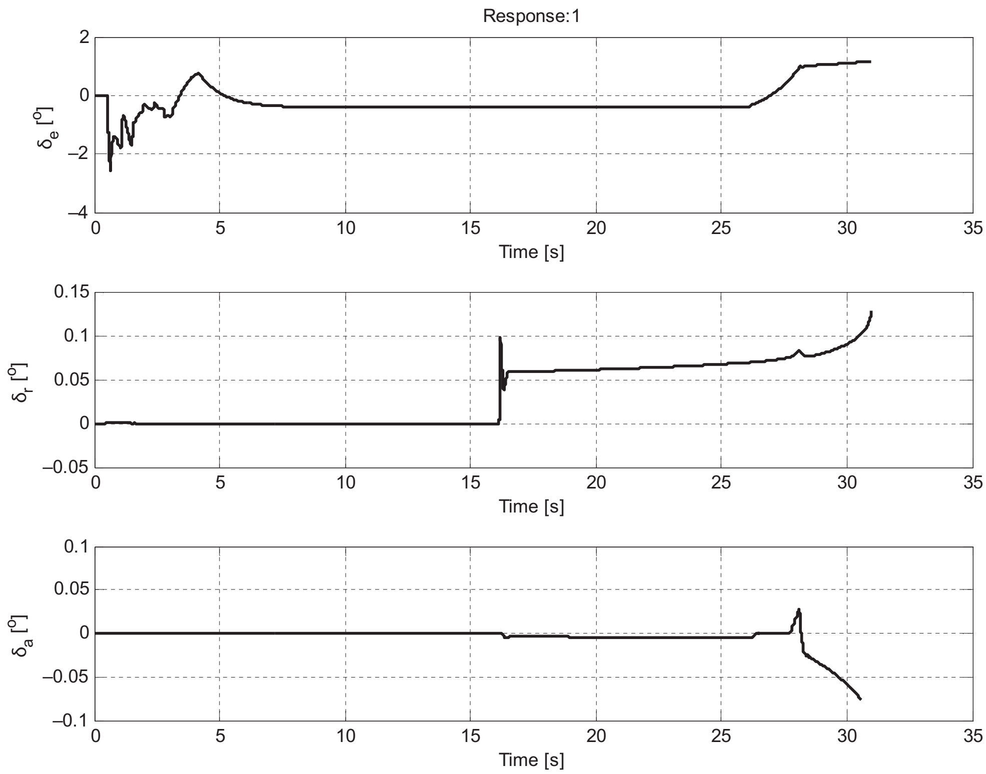


Figure 11 Surface deflections (elevator, rudder and aileron) of second scenario

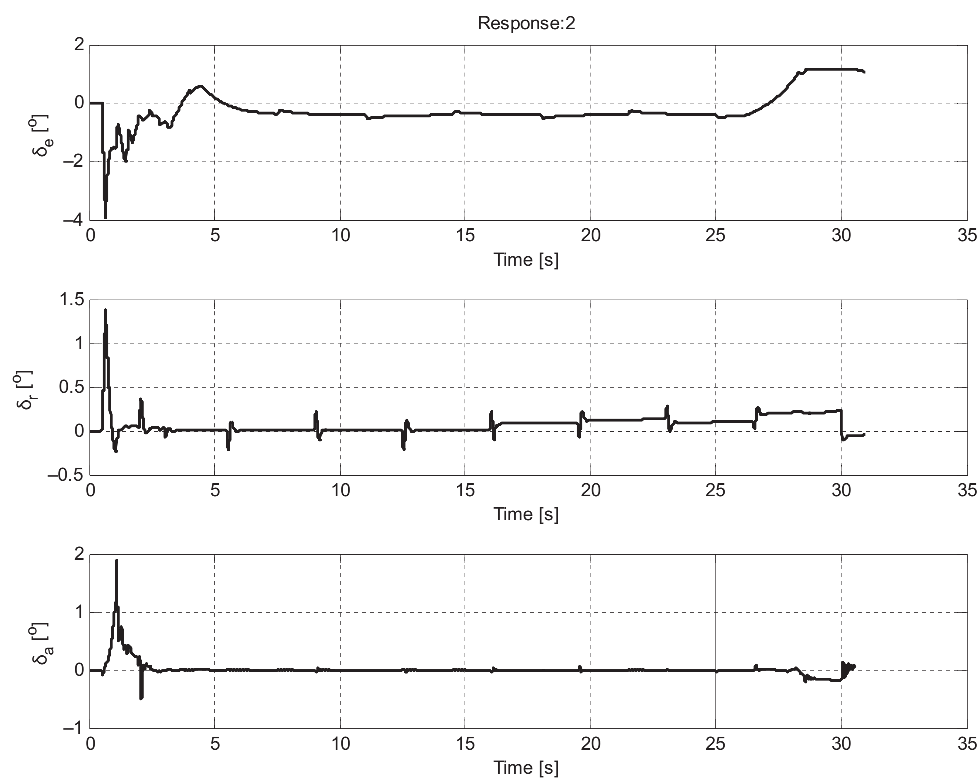


Figure 12 Surface deflections (elevator, rudder and aileron) of third scenario

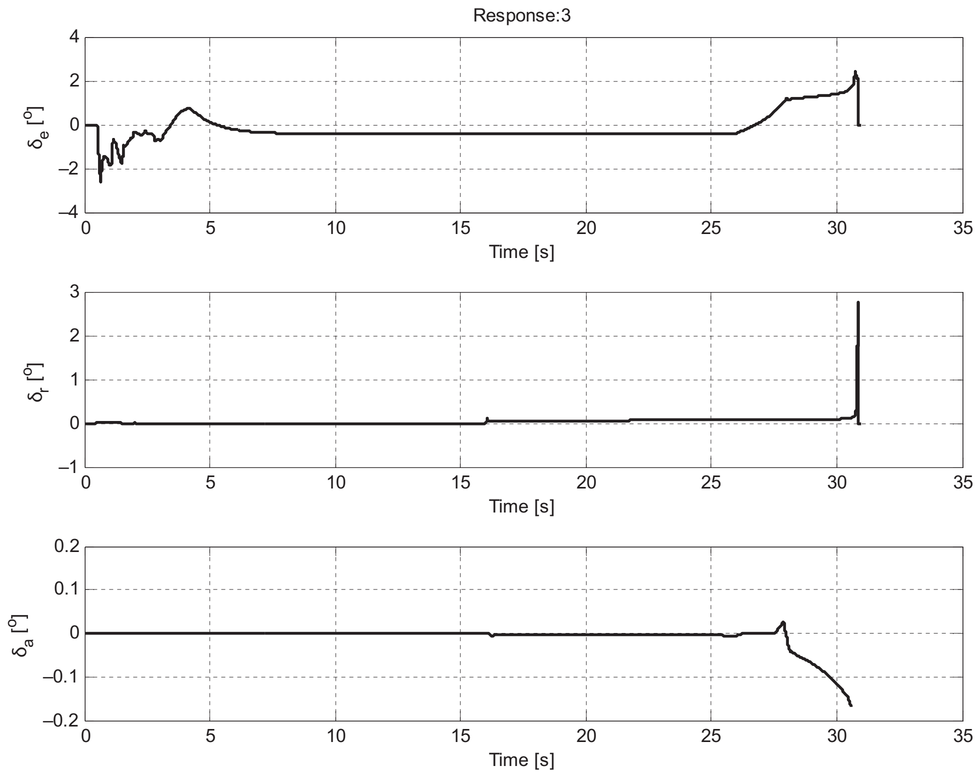


Table III Simulations performances

Run	Impact angle in degrees	Miss distance (m)	Elevator control effort in degrees	Rudder control effort in degrees	Aileron control effort in degrees
Nominal case (Simulation 1)	32.6	0.24	14.9	1.09	0.21
Thrust misalignment and gust disturbances (Simulation 2)	30.83	0.10	15.72	2.41	1.16
Aerodynamic uncertainties (Simulation 3)	31.53	0.11	14.30	1.18	0.14

control effort (Schultz and Melsa, 1967; Maiti *et al.*, 2008) is given below:

$$CE = \int_{t_0}^{t_f} |\delta_{e/r/a}| \cdot dt \tag{37}$$

Conclusion

This paper presents the autopilot designs with MPC method in three axes. MPC methodology is applied for different scenarios including aerodynamic uncertainties, thrust misalignment and gust effects. In addition, coupling between pitch, yaw and roll axes are also considered in this control methodology. MPC method is quite robust for modeling uncertainties and disturbances and needs less control efforts when compared to other methodologies. The main advantage of MPC is that the controller is explicitly a function of the model that can be modified in real-time, and it allows the current time range to be optimized while keeping future time range in account. This is achieved by optimizing a finite time-horizon but only implementing the current time range. MPC has the ability to anticipate future events and can provide control actions accordingly.

In the future, only one constrained multivariable MPC method, by limiting the angle of attack, side-slip angle and control surface deflections at all flight conditions, can be applied to autopilot to develop the performance of autopilot for the application in aerospace industry.

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