EMA 405 – Finite Element Analysis

Homework 4 – Pulley-Shaft System

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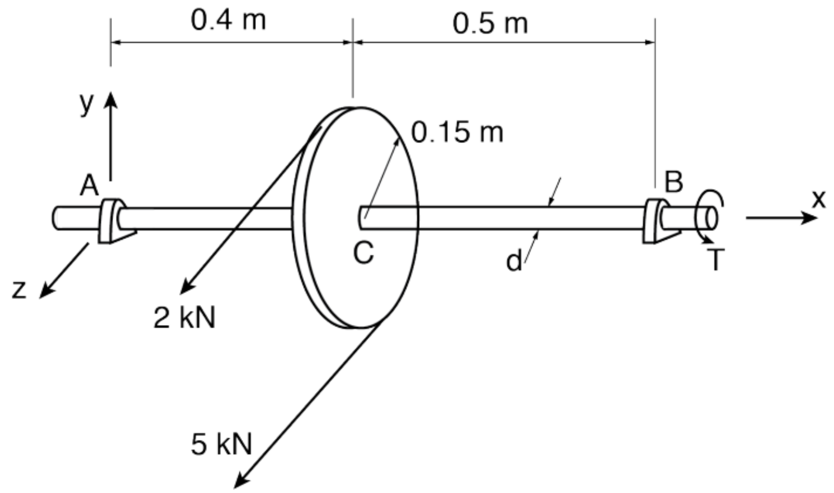
Overview:

In this homework question, the task of a new engineer from BadgerDyn Corp. is to design a solid round shaft for the pulley-shaft system below. The task of the engineer is to determine the minimum size of the shaft that will allow a maximum angle of twist of 0.26 degrees per meter of length and deflection of the beam less than 0.005 inches. The material being used is cold-rolled 1020 steel. In this case, we will do an analytical examination and test it in Ansys to help determine the appropriate diameter of the shaft to meet these specifications.



Description of FEA model:

Figure 1: Pulley-Shaft System specified for the assignment.

All beams were created using 3-noded BEAM189 elements. The origin was specified at point A throughout the report. The global coordinate system used is consistent to that in Figure (1), however each image in the Ansys report has an attached coordinate system at the origin. The meshing size controls were set to define elements with lengths equal to the width of the shaft, which was defined as the diameter of the shaft that was found to be adequate to meet the specifications detailed above. This created 11 elements with a total of 23 nodes.

For nodal degrees of freedom, key points created at points A, B, and C were defined. The nodes at A and B had constraints to model the simply supported bearing as specified. The node at A had the constraints: UX = UY = UZ = 0. The node at point B had the constraints: UX = UY = UZ = ROTX = 0. The reason for the zero rotation is the torque shown on the right side should make this a static problem that Ansys can solve. It also allows the shaft to be modeled in Ansys as simply supported, rather than fixed.

Additionally, instead of modeling the pulley system, we can apply equivalent forces and moments at the node defined as C. The equivalent forces and moments in this case would be +7000N in the z direction and -450 N*m in the x direction. This allows us to equivalently model the system to get accurate answers. The modulus of elasticity for this system was 186E9 Pa, Poisson's ratio of 0.29, and the polar moment of inertia and moment of inertia for the beam was found to be $\frac{1}{2}\pi\, r^4$ and $\frac{1}{4}\pi\, r^4$ respectively. Reference appendix equations 1, 2, 9, and 18 for the reasoning. The diameter of the shaft was found to be 0.0975 m analytically (ref eqs. 1-21 Appendix) and 0.0982 m in Ansys, so there is confidence in the answer.

Figures 2 and 3: Model of the frame in Ansys and the deformed plot when the loads are applied.

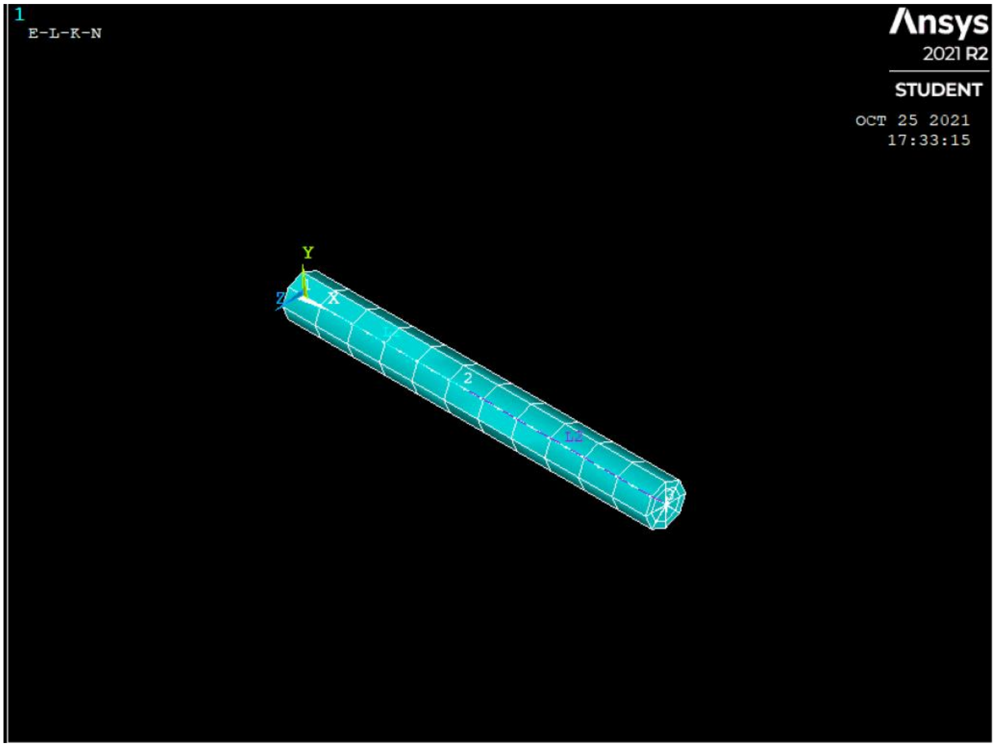


Figure 2- Design FEM with lines shown.

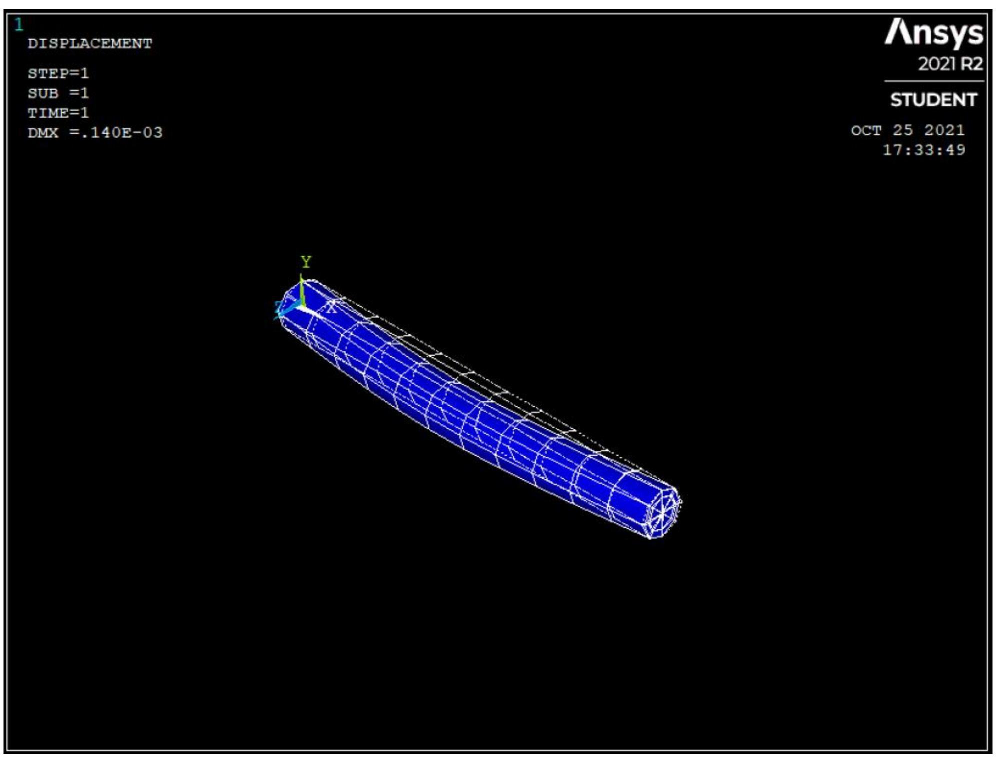


Figure 3- Plot of design deformed shape

Figure 2 shows the FEM, along with its lines that define the elements. This is to show that the lines are defined as between points A and C and C and B. Figure 3 is a plot of its undeformed shape (represented by the white lines), along with the deformed shape (represented by the blue element.) It is not evident what the exact deformations are from the image, so this report will detail the types of deformations in the shaft system with specific contour plots later. Because of an issue with graphing, the forces and boundary conditions are not present in the image. Refer to the description above on page 2 for a description of the boundary conditions and forces applied.

FEA Results

As a result of the of the applied force and moment from the pulley, the shaft experience torsion which results in rotation around the x-axis and a deflection primarily in the z-direction, with some displacements in the x and y directions that come from the nature of the simply supported ends caused by bending. Because the maximum x and y displacements were approximately ten times lower than the maximum z-direction displacement, this report will focus on the z-direction displacements and rotation around the x-axis of the beam.

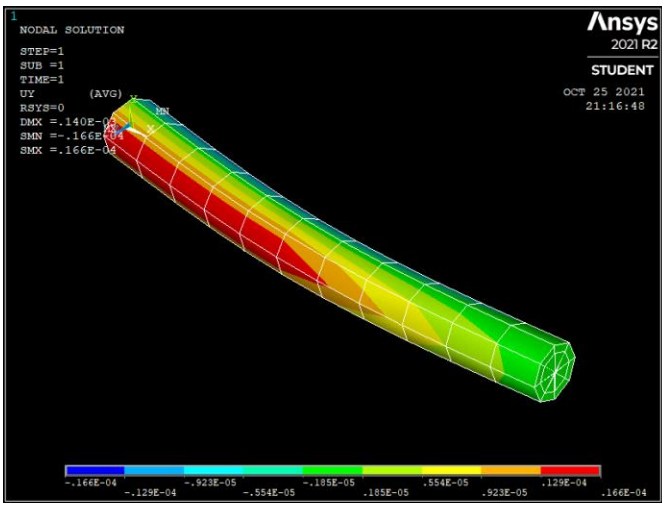


Figure 4 – Contour plot of displacement in the y-direction

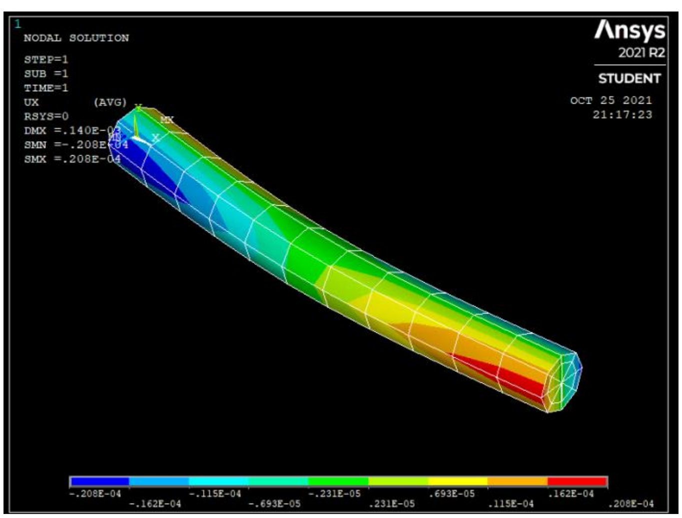


Figure 5 – Contour plot of the displacement in the x-direction

Maximum Twist

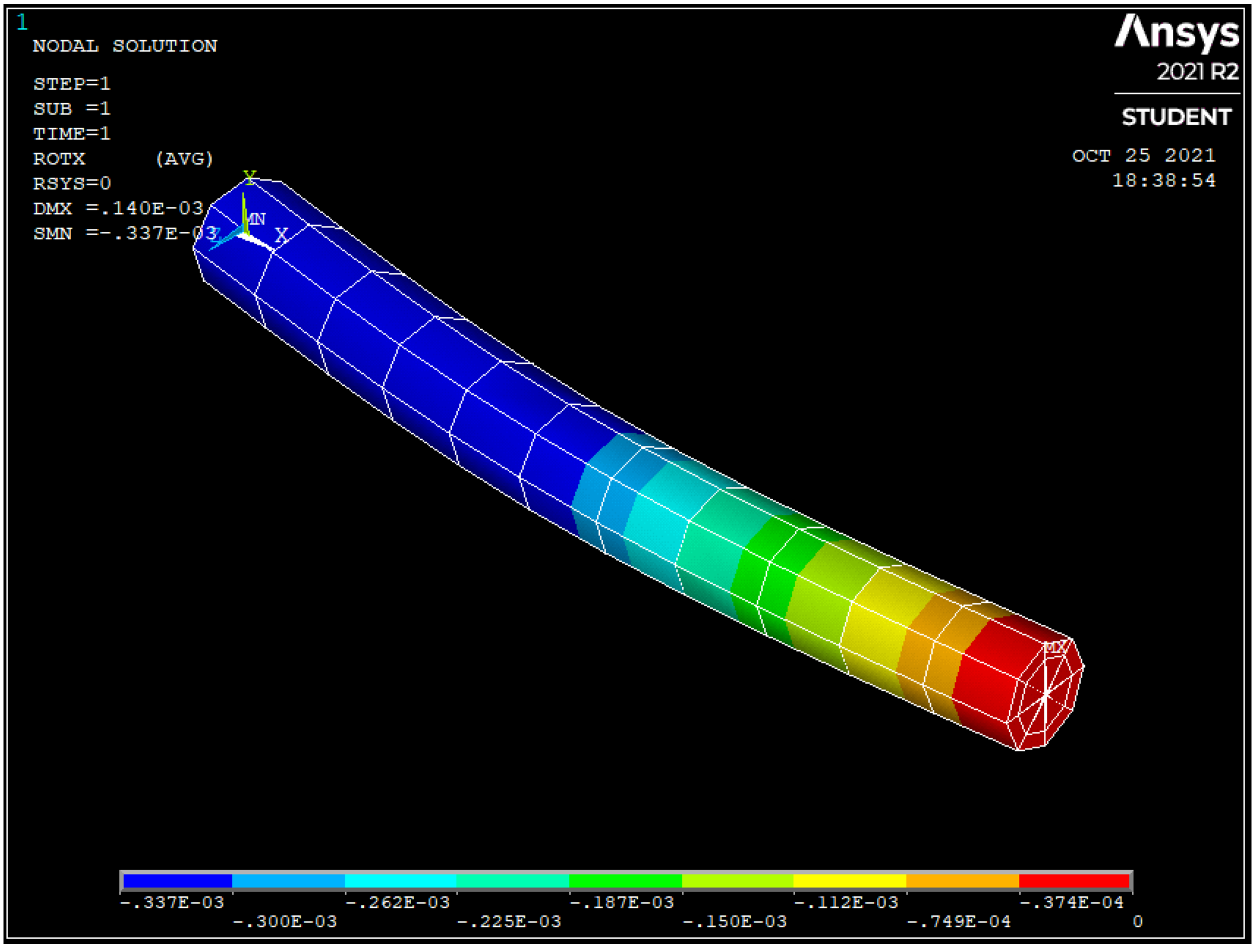


Figure 6 – Contour plot of rotation in the x-direction

The maximum twist allowed in this specification is 0.26 degrees per meter of length between B and C. Since the beam is only 0.5 meters in length, it is important to consider that does not mean that the maximum twist is 0.26 degrees, or 4.538E-3 radians. Instead, Equation 8 in the appendix shows that the maximum twist allowed in this specification is 2.27E-3 radians. Using equations 4-14 in the appendix, to meet the maximum twist specifications, the analytical answer for the minimum size of the shaft to allow for this maximum twist is 0.06 m. Therefore, the shaft must have a 0.06m or greater (refer to Equation 14 in the appendix for reasoning). However, the analytical solution also shows that the maximum bending criteria requires a larger diameter, so the Ansys design shown uses that higher radius of 0.0493 m. Because of this, the twist calculated in the software is less than if the dependent criteria would be the twist. To account for this, the expected value of twist was calculated in Equation 23 in the appendix to be 3.336E-4 radians. Evidently from Figure 6, the maximum twist happens around the point C, with a value of twist of 3.37E-4 radians. Referencing the list of nodal rotation in the x-direction (End of appendix), the maximum x-direction rotation happens at the point C, and does not twist any further than that as it approaches A. The analytical answer is consistent with the FEM which yields confidence in the answer not only that this system will be able to resist the specified rotation, but also that it is modeled correctly.

Maximum Bending

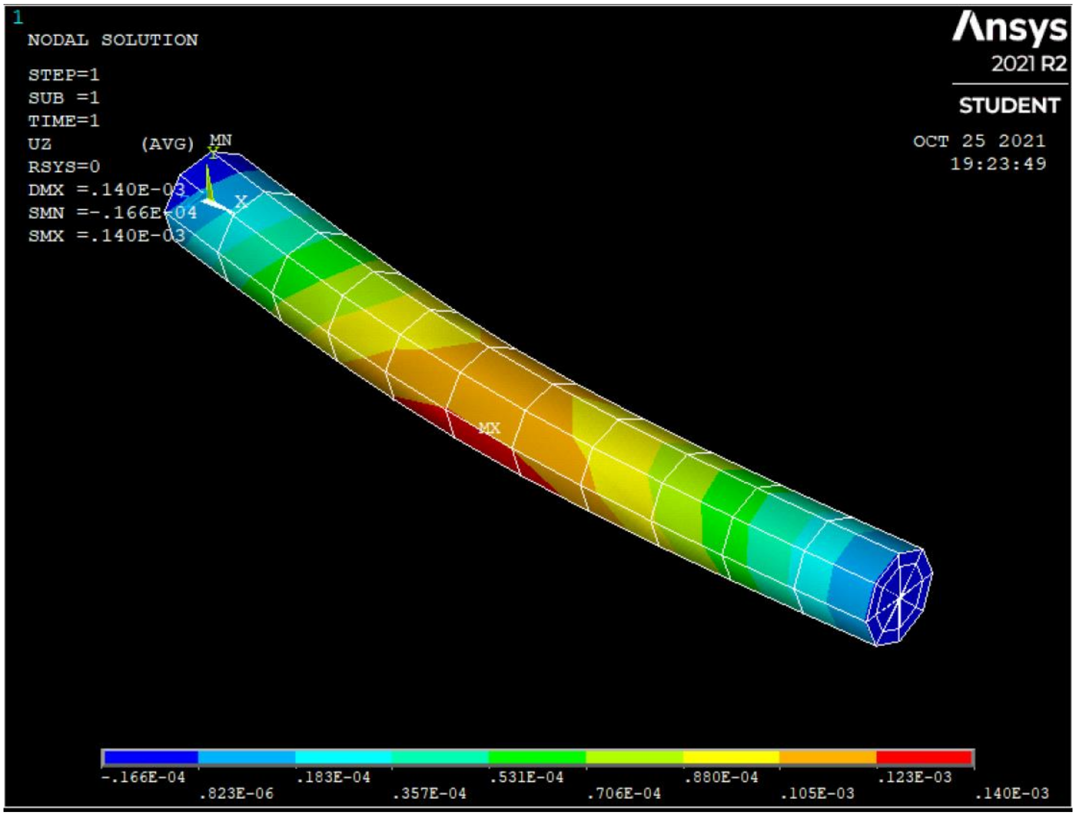


Figure 7 – Contour plot of displacement in the z-direction.

The other specification for this design was the maximum bending. In the analytical solution, we found that the shaft must have a diameter of 0.0975 m to deflect less than 0.005 inches under the loads specified (ref steps 15-22 in the Appendix). When modeling in Ansys in small increments, I found that a diameter of 0.0986 m allowed the design to deflect less than 0.005 inches (1.27E-4 m, Ansys model was made in SI units). Since these two values are very similar, this difference is most likely because of interpolation. With smaller elements, it would get closer to the analytical solution. Since these values are very similar, this should yield confidence that the minimum diameter of this shaft should be 0.0986 meters to satisfy bending requirements. The contour plot in figure 7 above shows an interesting distribution of displacement. The reasoning for this is that this distribution along the edges experiences a bending displacement and also a displacement in the z from the rotation in the x, so Ansys thinks that the points on the bottom displace more in the z-direction, because they are also rotated in the x-direction. This isn't necessarily how we define deflection in the z-direction. Therefore, the true z-displacement can be found in the list solution of z-displacement, on the nodes in the center of the shaft where torsion and rotation effect is zero. Therefore, the maximum displacement in the list is 1.244E-4 m, and happens at the node 13, which exists at x = 0.442 meters which is slightly to the right of point C (see end of the Appendix). This displacement is slightly lower than the maximum displacement allowed (1.27E-4 m). This is consistent with the expected analytical solution, which expects the maximum displacement of 1.27E-4 with the diameter of 0.0974.

Factor of Safety

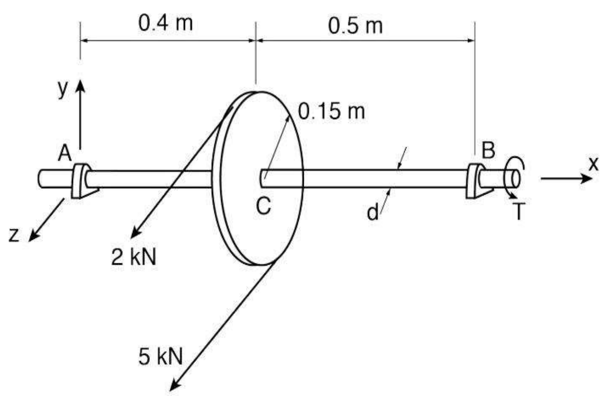
Factor of safety refers to how much stronger a system should be made compared to the failure strength. In the analytical approach, we determined the critical stress that would cause the system to fail. It would be appropriate to contact the chief engineer for their opinion on what an appropriate factor of safety would be. As an example, a blanket factor of safety of 2 would require the shaft to have a radius of 0.115 m instead (ref steps 24-27 Appendix)

Conclusion

From this assignment, the largest takeaway is designing a system, rather than modeling one. It resembles an introduction level assignment for someone new in the workspace. This assignment also is a learning experience for torsion in Ansys. Additionally, it is a learning experience for shell elements mixed with beams, and correctly defining a bearing system. With this insight, we can apply these concepts to a more complicated problem with this newfound experience. As for the delivery, my analysis concluded that in order to stay within the specifications the chief engineer laid out, the minimum diameter the shaft should be is 0.0986 meters.

Appendix

EMA 405: Homework 4



Design specifications: Maximum twist of 0.26 degrees/meter, and maximum deflection of 0.005 inches.
Using material
AISI 1020 cold-rolled steel.

Note: all units here are SI. The maple software is stubborn about units when it comes to solving, so all have been configured to SI.

AISI 1020 Steel properties: from matweb properties of 1020 colr-rolled steel:

restart

$E := 186 \cdot 10^9$ *# elastic modulus*

$$E := 186000000000 \quad \textbf{(1)}$$

$v := 0.29$ *# Poisson's ratio*

$$v := 0.29 \quad \textbf{(2)}$$

$G := \dfrac{E}{2 \cdot (1 + v)}$ *# Shear modulus*

$$G := 7.209302325 \times 10^{10} \quad \textbf{(3)}$$

Now, to answer his question about the minimum size of the shaft, we must analyze the minimum size of the shaft from specificiations from twisting and deflection. Firstly, I will analyze for the twist.

$L\_bc := 0.5$

$$L\_bc := 0.5 \quad \textbf{(4)}$$

$degrees := 0.26$

$$degrees := 0.26 \quad \textbf{(5)}$$

$radians := \dfrac{degrees \cdot \mathrm{Pi}}{180}$

$$radians := 0.004537856056 \quad \textbf{(6)}$$

$L := 0.9$

$$L := 0.9 \quad \textbf{(7)}$$

theta $:= radians \cdot L\_bc$

 # because the shaft is not 1m long, must account for that, since then it should twist less than 0.26 degrees --> radians

$$\theta := 0.002268928028 \tag{8}$$

From Roark's Stress and Strain 9th edition, further refered to as Roark's 10.1-1:

$eq1 := \text{theta} = \frac{T \cdot L\_bc}{J \cdot G}$

$$eq1 := 0.002268928028 = \frac{6.935483872 \times 10^{-12}\, T}{J} \tag{9}$$

Also from Roark's, for a solid circular cross-section J- the polar moment of inerta can be calculated:

$J := .5 \cdot \text{Pi} \cdot r^4$

$$J := 1.570796327\, r^4 \tag{10}$$

Additionally, the torque applied to the bar can be calculated using the right-hand rule: *** IN the NEGATIVE x direction ***

$T := (5 \cdot 0.15 - 2 \cdot 0.15) \cdot 10^3$

$$T := 450.00 \tag{11}$$

Then, the radius of the shaft can be calculated using this information.

$sol := evalf(solve(eq1, r), 2)$

$$sol := 0.030, 0.030\, \mathrm{I}, -0.030, -0.030\, \mathrm{I} \tag{12}$$

$answer1 := simplify(sol[1], 3)$

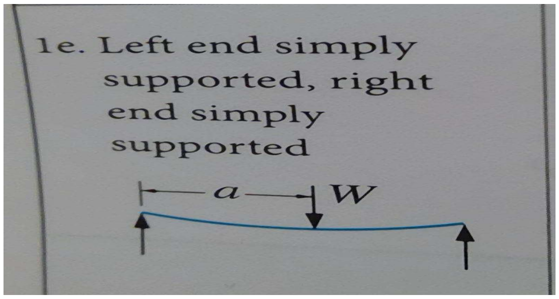
$$answer1 := 0.030 \tag{13}$$

$diameter\_twist := 2 \cdot answer1$

$$diameter\_twist := 0.060 \tag{14}$$

So, in order to be able to resist the 0.26 degrees of twist per unit length, the circular shaft must have a diameter of 0.06 meters.

Now, we should consider the minimum size of the shaft to resist the deflection.
For a shaft configured as above, we could consider the bearing system, and how it would work. In Roark's chapter 8 Table 8.1, it says that for a pinned-pinned beam with a non-centered load, the beam would have its max deflection at the centered load with a value of:



$d\_max := convert(0.005,\ 'units',\ inch,\ meter)$

$$d\_max := 0.0001270000000 \tag{15}$$

$$eq2 := d\_max = \frac{W \cdot a}{3 \cdot E \cdot I\_shaft \cdot L} \cdot \left(\frac{(L^2 - a^2)}{3} \right)^{\frac{3}{2}}$$

$$eq2 := 0.0001270000000 = \frac{1.991238550 \times 10^{-12}\, W\, a \left(0.2700000000 - \frac{a^2}{3}\right)^{3/2}}{I\_shaft} \tag{16}$$

W, the force applied, is the 5 kN + 2 kN force.

$W := (5 + 2) \cdot 10^3$

$$W := 7000 \tag{17}$$

A, the distance, is 0.4m.

$a := 0.4$

$$a := 0.4 \tag{18}$$

I is the moment of inertia of the beam, which is 2*J, whose r is unknown.

$I\_shaft := \frac{J}{2}$

$$I\_shaft := 0.7853981635\, r^4 \tag{19}$$

And the maximum allowable bending was 0.005 inches, which needs to be converted to meters.

$sol2 := solve(eq2, r)$

$$sol2 := 0.04872692086,\ 0.04872692086\, \mathrm{I},\ -0.04872692086,\ -0.04872692086\, \mathrm{I} \tag{20}$$

$soln2 := sol2[1]$

$$soln2 := 0.04872692086 \tag{21}$$

$diameter\_bending := soln2 \cdot 2$

$$diameter\_bending := 0.09745384172 \tag{22}$$

Thus, the minimum diameter for the allowable bending is 0.0974 meters for the shaft.
Since we are going to use the larger solution for the minimum shaft diameter, we should calculate what we would expect Ansys to
solve at point C, also just to double check that the twisting specification/constraint is properly met.

$subs(r = 0.0493, rhs(eq1))$

$$0.0003363424828 \tag{23}$$

Since the value for theta, or the rotation expected at C for this new beam of diamater 0.097 is less than the allowed, we also predict that
it will be properly constrained in its allowed rotation for the diamater of 0.0974. It is also the number we would expect from Ansys
to make sure our model is correct.

$$Factor\_safety := \frac{d\_max}{2}$$

$$Factor\_safety := 0.00006350000000 \tag{24}$$

$$eq3 := Factor\_safety = \frac{W \cdot a}{3 \cdot E \cdot I\_shaft \cdot L} \cdot \left(\frac{\left(L^2 - a^2\right)}{3} \right)^{\frac{3}{2}}$$

$$eq3 := 0.00006350000000 = \frac{7.159448917 \times 10^{-10}}{r^4} \tag{25}$$

$$soln\_FS := solve(eq3, r)$$

$$soln\_FS := 0.05794640098, 0.05794640098\,\mathrm{I}, -0.05794640098, -0.05794640098\,\mathrm{I} \tag{26}$$

$$soln\_FS \cdot 2$$

$$0.1158928020, 0.1158928020\,\mathrm{I}, -0.1158928020, -0.1158928020\,\mathrm{I} \tag{27}$$

Monday, October 25, 2021 9:26 PM

Nodal displacement in the z-direction in radians.

```
PRINT U    NODAL SOLUTION PER NODE

 ***** POST1 NODAL DEGREE OF FREEDOM LISTING *****

 LOAD STEP=     1  SUBSTEP=     1
  TIME=    1.0000      LOAD CASE=   0

 THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

   NODE       UZ
      1   0.0000
      2  0.12372E-003
      3  0.17141E-004
      4  0.33993E-004
      5  0.50266E-004
      6  0.65673E-004
      7  0.79924E-004
      8  0.92729E-004
      9  0.10380E-003
     10  0.11285E-003
     11  0.11959E-003
     12   0.0000
     13  0.12443E-003
     14  0.12227E-003
     15  0.11749E-003
     16  0.11036E-003
     17  0.10113E-003
     18  0.90084E-004
     19  0.77466E-004
     20  0.63541E-004
     21  0.48570E-004
     22  0.32816E-004
     23  0.16538E-004

MAXIMUM ABSOLUTE VALUES
NODE          13
VALUE   0.12443E-003
```

Nodal rotation in the x-direction measured in radians.

```
PRINT ROT  NODAL SOLUTION PER NODE

 ***** POST1 NODAL DEGREE OF FREEDOM LISTING *****

 LOAD STEP=     1  SUBSTEP=     1
  TIME=    1.0000      LOAD CASE=   0

 THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

   NODE       ROTX
      1 -0.33704E-003
      2 -0.33704E-003
      3 -0.33704E-003
      4 -0.33704E-003
      5 -0.33704E-003
      6 -0.33704E-003
      7 -0.33704E-003
      8 -0.33704E-003
      9 -0.33704E-003
     10 -0.33704E-003
     11 -0.33704E-003
     12   0.0000
     13 -0.30895E-003
     14 -0.28087E-003
     15 -0.25278E-003
     16 -0.22469E-003
     17 -0.19661E-003
     18 -0.16852E-003
     19 -0.14043E-003
     20 -0.11235E-003
     21 -0.84260E-004
     22 -0.56173E-004
     23 -0.28087E-004

MAXIMUM ABSOLUTE VALUES
NODE          1
VALUE  -0.33704E-003
```

List of all nodes and coordinates

```
LIST ALL SELECTED NODES.   DSYS=      0
SORT TABLE ON  NODE  NODE  NODE

   NODE        X              Y             Z            THXY     THYZ     THZX
      1   0.0000        0.0000        0.0000         0.00     0.00     0.00
      2  0.40000        0.0000        0.0000         0.00     0.00     0.00
      3  0.40000E-001   0.0000        0.0000         0.00     0.00     0.00
      4  0.80000E-001   0.0000        0.0000         0.00     0.00     0.00
      5  0.12000        0.0000        0.0000         0.00     0.00     0.00
      6  0.16000        0.0000        0.0000         0.00     0.00     0.00
      7  0.20000        0.0000        0.0000         0.00     0.00     0.00
      8  0.24000        0.0000        0.0000         0.00     0.00     0.00
      9  0.28000        0.0000        0.0000         0.00     0.00     0.00
     10  0.32000        0.0000        0.0000         0.00     0.00     0.00
     11  0.36000        0.0000        0.0000         0.00     0.00     0.00
     12  0.90000        0.0000        0.0000         0.00     0.00     0.00
     13  0.44167        0.0000        0.0000         0.00     0.00     0.00
     14  0.48333        0.0000        0.0000         0.00     0.00     0.00
     15  0.52500        0.0000        0.0000         0.00     0.00     0.00
     16  0.56667        0.0000        0.0000         0.00     0.00     0.00
     17  0.60833        0.0000        0.0000         0.00     0.00     0.00
     18  0.65000        0.0000        0.0000         0.00     0.00     0.00
     19  0.69167        0.0000        0.0000         0.00     0.00     0.00
     20  0.73333        0.0000        0.0000         0.00     0.00     0.00

   NODE        X              Y             Z            THXY     THYZ     THZX
     21  0.77500        0.0000        0.0000         0.00     0.00     0.00
     22  0.81667        0.0000        0.0000         0.00     0.00     0.00
     23  0.85833        0.0000        0.0000         0.00     0.00     0.00
```