EMA 405 – Finite Element Analysis

Homework 7 – Nonlinear Analysis of Plastic Deformation

Nikolaj Hindsbo

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University of Wisconsin–Madison
Hindsbo@wisc.edu

Overview:

In this homework, the task is to model the thick-walled pipe that is subject to an internal pressure. We are assuming that the pipe is in plane stress, so that it is a 2D analysis. The pipe has a Young's modulus of 200E9, yield stress of 150E6, and a tangent modulus of 2E9 Pa. A Poisson's ratio of 0.3 is used.

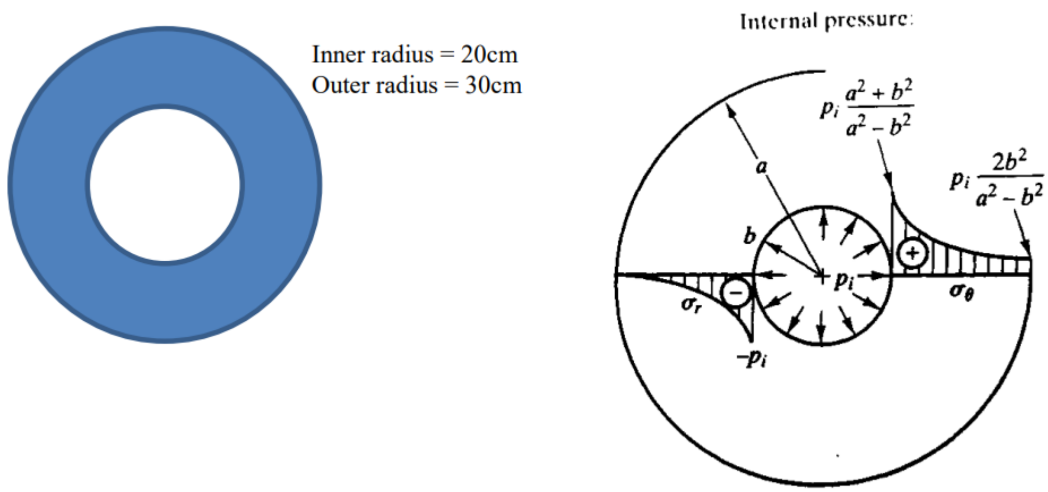


Figure 1: Thick-walled pipe problem

Description of FEA model:

The pipe was created using a 8-node quad element, which is a Plane82 element. The element was used mostly out of consideration of how the hole can be modeled with symmetry by modeling a quarter circle with symmetry boundary conditions. Additionally, it was used in consideration that this problem is assuming a 2D analysis with plane stress.

A key part of this assignment was recognizing the symmetry could be used. I modeled the top right corner of the pipe, and the horizontal and vertical lines were modeled with symmetry.

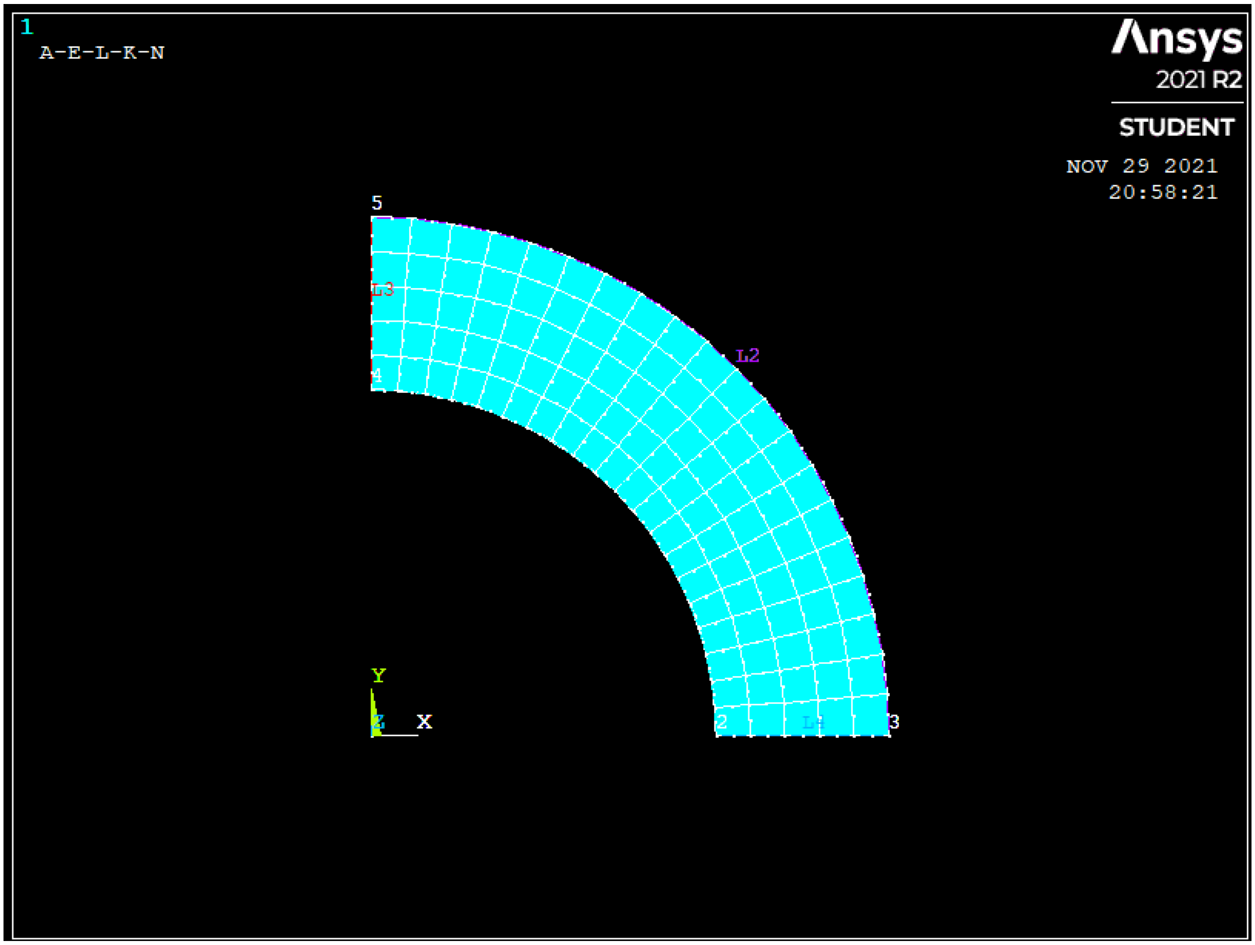


Figure 2: Model of the pipe with lines and elements shown.

The pipe was modeled with symmetry. The vertical and horizontal lines (L3, L4) were modeled with symmetry boundary conditions, but no other boundary conditions were used. When applying the pressures found in the appendix for the conditions that were asked in the assignment, the results were stored in a LS file so that each step was considered by Ansys.

Working Through Yield Conditions

The problem statement described four steps that we should consider. Firstly, we should consider when the pipe first yields (any point). Then, we should see how much of the pipe would yield when 1.2 times that pressure was applied internally. Additionally, we should find minimum pressure such that the entire area yields. Then, we should calculate the residual stress that is left in the pipe after the entire area has gone through yield. Please note that since the internal pressures were applied outward, all of the stresses are shown in tension for this report.

First Yield (Load Step 1)

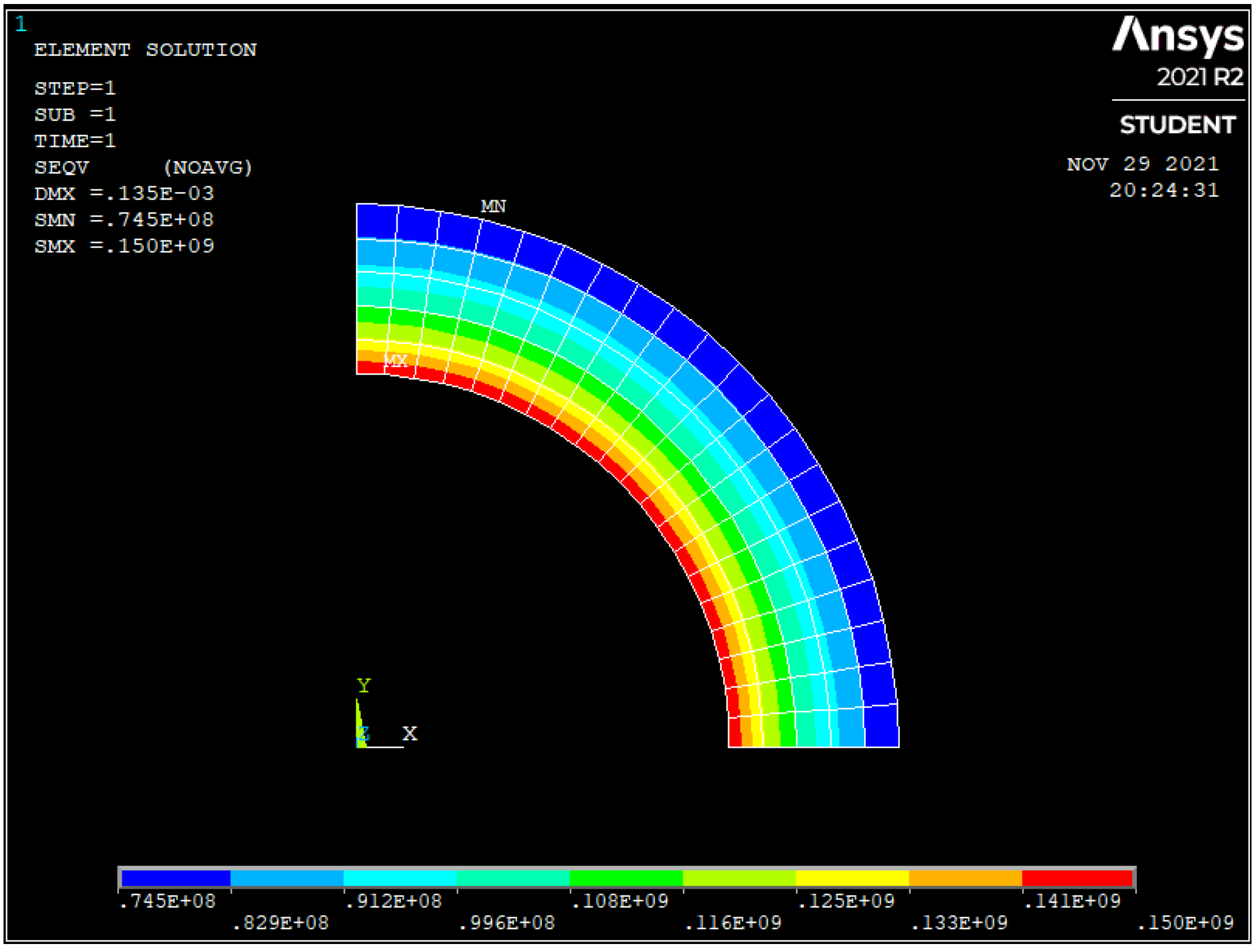


Figure 3: Contour plot of von mises stress where yielding just occurs.

The first scenario the report asked to consider was when yielding first occurs through the cross-sectional area. When an internal force is applied, the stress is maximized at the location of the hole (the inner area). Using our analytical approach in the appendix (line 17), we calculated the minimum pressure for yield to be 46 MPa. When that pressure was applied, we can see from our von mises stress contour plot in Figure (3) that the inner region of the cross-sectional area is the only one that yields, and that the maximum stress is exactly that of the yield stress that was identified in the report. Therefore, there is strong confidence that both the Ansys and analytical approach are correct in modeling the minimum yielding criteria.

1.2 Criteria (Load Step 2)

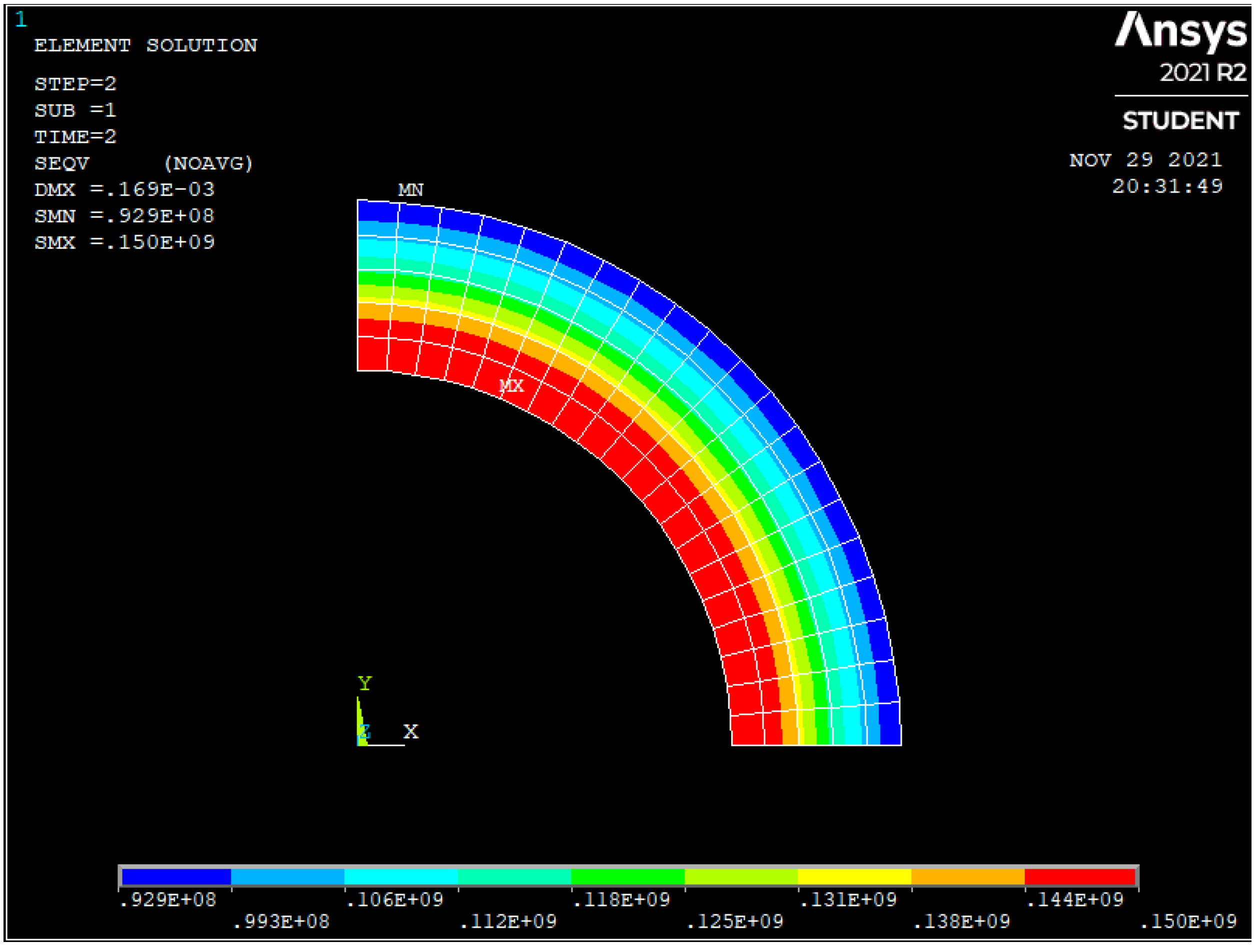


Figure 4: Contour plot of von mises stress where internal pressure is 1.2 times initial yield.

When the minimum yield was considered, the second part of the report was to identify how much of the cross-sectional area would yield when 1.2 times the minimum pressure was applied internally. In our analytical, we found that the pipe should yield until approximately r = 0.22m, which would correspond to about 1/5th of the distance between the inner and the outer radii. Based on our contour plot, our analytical is a good estimate of the Ansys solution, however a little more of the area would yield than expected. Of course, this does not mean that 1/5th of the area yields. As is continues through the radius, each cross-sectional area is larger. However, it should be approximately the region between 0.2 m and 0.22 meters that yields based on our analytical and Ansys answers, so there is strong confidence in the answer. Using hand calculations (A_outer-inner), this is approximately 16.8-20% of the cross-sectional area that has yielded.

All Area Yield (Load Step 3)

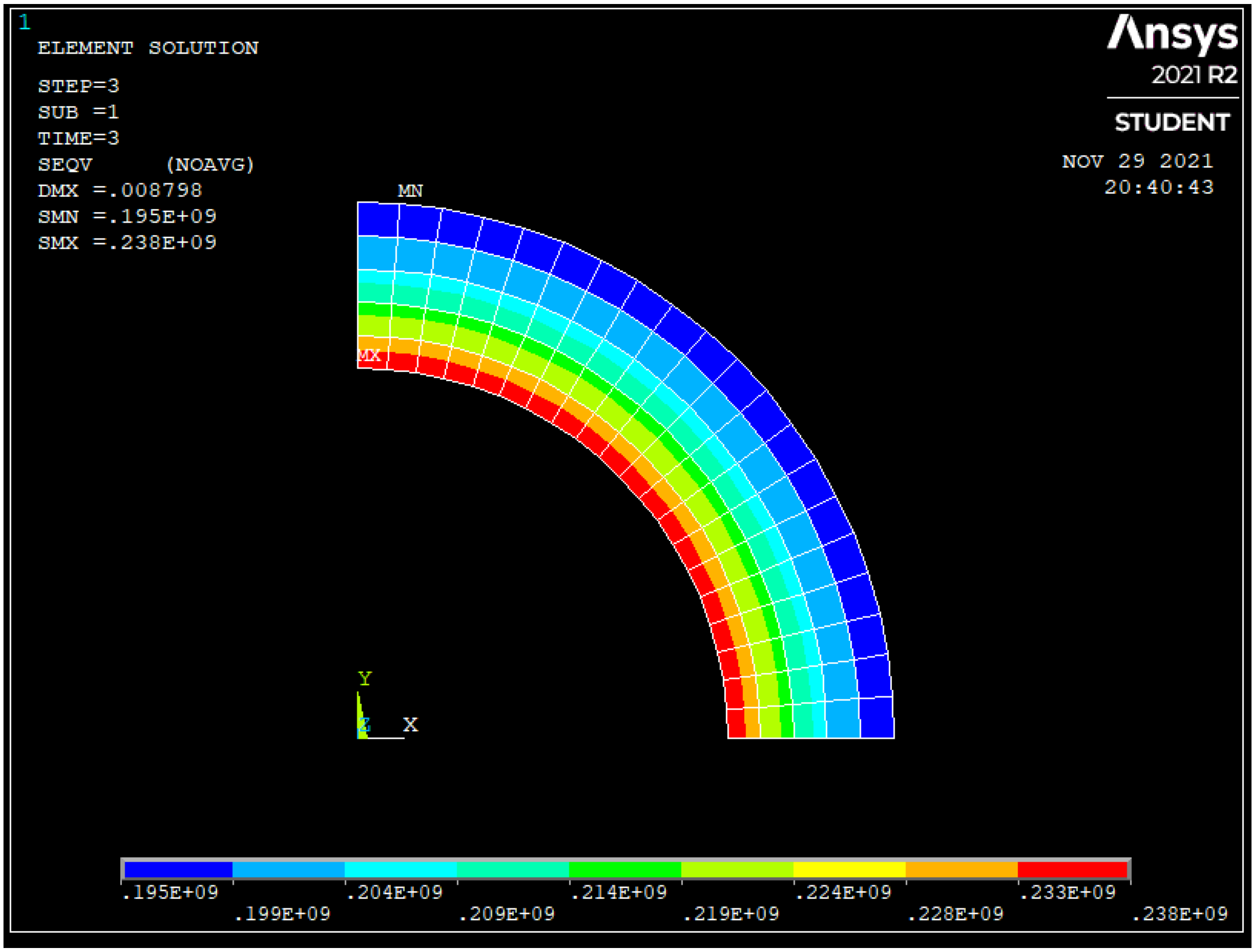


Figure 5: Contour plot of von mises stress where the entire area yields

The pressure to consider was the pressure at which the entire area yielded. Based on our analytical (line 22), we would expect that with a pressure of approximately 94 MPa, the entire area would yield. Based on our answer from Ansys, we can see that even the outmost of the area is beyond its expected yield stress. Therefore, the minimum pressure for the entire area to yield is smaller than the expected analytical. This may be due to the fact that the yielded areas do little to withstand the additional stress, so the outer regions yield faster. This could also be a result of the thick-walled pipe material behaving as a bilinear-kinematic hardening material, which is hard to model accurately analytically. An interesting result, nonetheless.

Residual Stresses (Load Step 4)

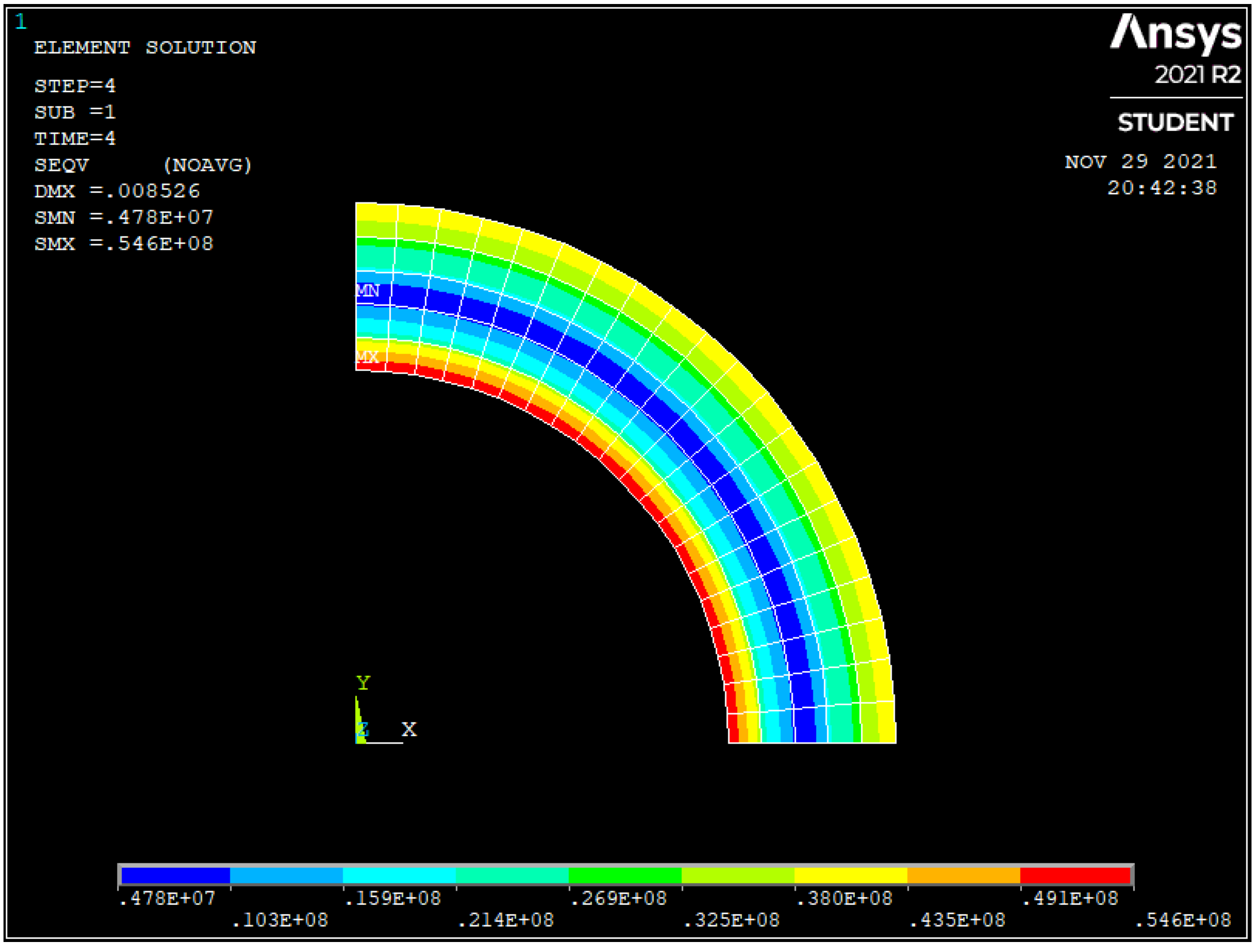


Figure 6: Contour plot of von mises stress after pressure was released.

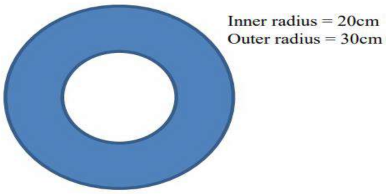
The last step of the report is to find the residual stresses left in the pipe when the pressures are released. Since the object was pushed to the point where the entire object was yielded, some of it was strained plastically. In other words, there is permanent deformation existing in the pipe's cross section. These permanent deformations leave residual stresses, which can indicate not only how the pipe was strained, but also how the material compresses after the tension was released. An analytical was not possible for this, other than the concepts. Therefore, a sanity check is in order to confirm the results. The highest residual stresses exist in the inner part of the tubes. This is expected, since the highest stresses were located along that arc. The residual stresses decreased (as expected) as we went towards the outer surface; interestingly, they started to increase across the midpoint. This is interesting since the von mises stresses did not follow this same pattern. Thus, we are lead to believe that this is either because of the way that the object is able to compress, or perhaps because there was a concentration of stress away from the midpoint of the material. Either explanation is a sanity check based on our knowledge of stress concentrations (especially shear), so there is high confidence in the Ansys solution.

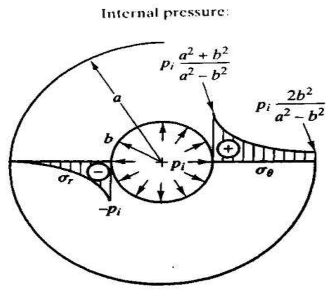
Conclusion

From this homework assignment, we learned a lot about von mises stress concentrations on pipes. We were introduced to important ideas such as plastic deformations and residual stresses. A sanity check was necessary which required thinking like an engineer based on our previous knowledge. This thick-walled pipe is a very relevant problem in the real world, and the idea that sections of the pipe can yield without the entire system yielding is a very interesting idea that was learned from this activity.

Analysis

Context:
A thick walled pipe is subjected to an internal pressure such that yielding occurs throughout the entire cross sectional area. Assume that this cross section of the pipe is in plane stress,so that we can model it using 2D analysis.
The pipe material is assumed to behave as bilinear-kinematic hardening with a yield stress of 150MPa. The initial modulus is 200GPa and the tangent modulus after yielding is 2GPa. Assume a Poisson's ratio of 0.3. Include a figure of the ANSYS stress-strain curve in your report.





Properties:

$\sigma_y := 150E6$ # *Yield stress in Pa*

$$\sigma_y := 1.50 \times 10^8 \tag{1}$$

$E_{pipe} := 200E9$ # *Young's modulus in Pa*

$$E_{pipe} := 2.00 \times 10^{11} \tag{2}$$

$E_{tangent} := 2E9$ # *Tangent modulus in Pa*

$$E_{tangent} := 2. \times 10^9 \tag{3}$$

$v := 0.3$ # *Poisson's ratio*

$$v := 0.3 \tag{4}$$

$r_{inner} := 20E-2$

$$r_{inner} := 0.20 \tag{5}$$

$r_{outer} := 30E-2$

$$r_{outer} := 0.30 \tag{6}$$

Internal stresses @ the inner and outer locations (where r = a, r = b)

$b := r_{inner}$

$$b := 0.20 \tag{7}$$

$a := r_{outer}$

$$a := 0.30 \tag{8}$$

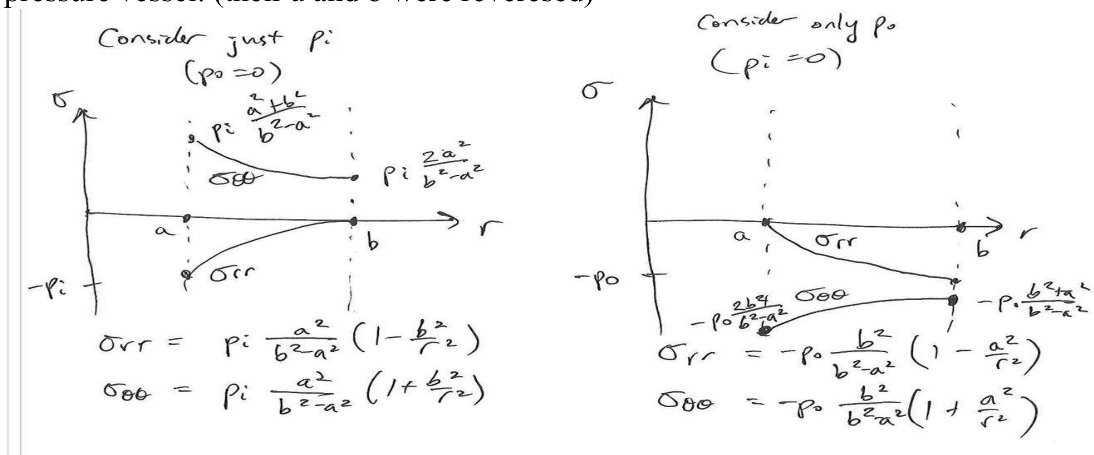
$$p_{inner} := \frac{p_i \cdot (a^2 + b^2)}{a^2 - b^2}$$

$$p_{inner} := 2.600000000\, p_i \tag{9}$$

$$p_{outer} := \frac{p_i \cdot 2\, b^2}{a^2 - b^2}$$

$$p_{outer} := 1.600000000\, p_i \tag{10}$$

This pressure is maximized at the inner portion of the thick walled pipe. We can prove this by comparing inner and outer results for what the inner pressure must be to yield. However, we are using von mises criteria, so we should use both hoop stress and radial stress. the above was just hoop stress at the inner and outer locations. Using in-class notes from adv. Mechanics of materials, which is sourced from the Intermediate Solid Mechanics by Lubarda, the following is true for hoop and radial stresses of a pressure vessel: (their a and b were revereseed)



$$\sigma_r := \frac{p_i \cdot b^2}{a^2 - b^2} \cdot \left(1 - \frac{a^2}{x^2}\right)$$

$$\sigma_r := 0.8000000000\, p_i \left(1 - \frac{0.0900}{x^2}\right) \tag{11}$$

$$\sigma_{\text{theta}} := \frac{p_i \cdot b^2}{a^2 - b^2} \cdot \left(1 + \frac{a^2}{x^2}\right)$$

$$\sigma_\theta := 0.8000000000\, p_i \left(1 + \frac{0.0900}{x^2}\right) \tag{12}$$

In the case of plane stress, the von mises expression can be simplified:

$$vm\_criteria := \left(\sigma_r - \sigma_{\text{theta}}\right)^2 + \sigma_r^{\ 2} + \sigma_{\text{theta}}^{\ \ 2} = 2 \cdot \sigma_y^{\ 2}$$

$$vm\_criteria := \left(0.8000000000\, p_i \left(1 - \frac{0.0900}{x^2}\right) - 0.8000000000\, p_i \left(1 + \frac{0.0900}{x^2}\right)\right)^2 + 0.6400000000\, p_i^2 \left(1 - \frac{0.0900}{x^2}\right)^2 + 0.6400000000\, p_i^2 \left(1 + \frac{0.0900}{x^2}\right)^2 = 4.5000 \times 10^{16} \tag{13}$$

$$max\_stress\_hoop := subs\left(x = b, \sigma_{\text{theta}}\right)$$

$$max\_stress\_hoop := 2.600000000\, p_i \tag{14}$$

$$max\_stress\_radial := subs\left(x = b, \sigma_r\right)$$

$$max\_stress\_radial := -1.000000000\, p_i \tag{15}$$

$$eq1 := subs(x = b, vm\_criteria)$$

$$eq1 := 20.72000000\, p_i^2 = 4.5000 \times 10^{16} \tag{16}$$

$p\_ans = solve(eq1, p_i)$

$$p\_ans = (4.660273245 \times 10^7, -4.660273245 \times 10^7) \tag{17}$$

Therefore, it is clear that the cylinder yields in the inner region first, with an internal pressure of 46.6 MPa.

Further questions:
* What fraction of the cylinder area yields when the pressure increase to 1.2 times the yield pressure?
--> To do this, we need to find an equation that relates the pressure along the way, and solve for what location the internal pressure is equal to the yield stress.
* At what pressure does the entire area yield
--> To do this, simply we can solve for when the outer (the lowest pressure felt) yields.
* What happens if we remove the pressure after the entire area has just yielded?
--> We need to compare the tangent modulus to see what strain it is permanantly at.

Firstly, At what pressure does the entire area yield:
We already know this from the above. 93.8 MPa is when the outer region yields, and thus the entire area yields.

Secondly:
What fraction of the cylinder area yields when the pressure increase to 1.2 times the yield pressure?

$p\_ans1 := 4.66027E7$

$$p\_ans1 := 4.66027 \times 10^7 \tag{18}$$

$eq2 := subs(p_i = 1.2 \cdot p\_ans1, vm\_criteria)$

$$eq2 := \frac{6.484994830 \times 10^{13}}{x^4} + 2.001541614 \times 10^{15} \left(1 - \frac{0.0900}{x^2}\right)^2 + 2.001541614 \times 10^{15} \left(1 + \frac{0.0900}{x^2}\right)^2 = 4.5000 \times 10^{16} \tag{19}$$

$solve(rhs(eq2) = lhs(eq2), x)$

$$0.2207051282, 0.2207051282\,I, -0.2207051282, -0.2207051282\,I \tag{20}$$

Third, what pressure does the entire area yield? that should be when the end (x = a) yields.
$eq3 := subs(x = a, vm\_criteria)$

$$eq3 := 5.120000000\, p_i^2 = 4.5000 \times 10^{16} \tag{21}$$

$p\_ans = solve(eq3, p_i)$

$$p\_ans = (9.3750000 \times 10^7, -9.3750000 \times 10^7) \tag{22}$$

So, it should fully yield when p_i = 9.37E7 Pa.
Lastly, what happen sif we remove the pressure after the entire area has just yielded?

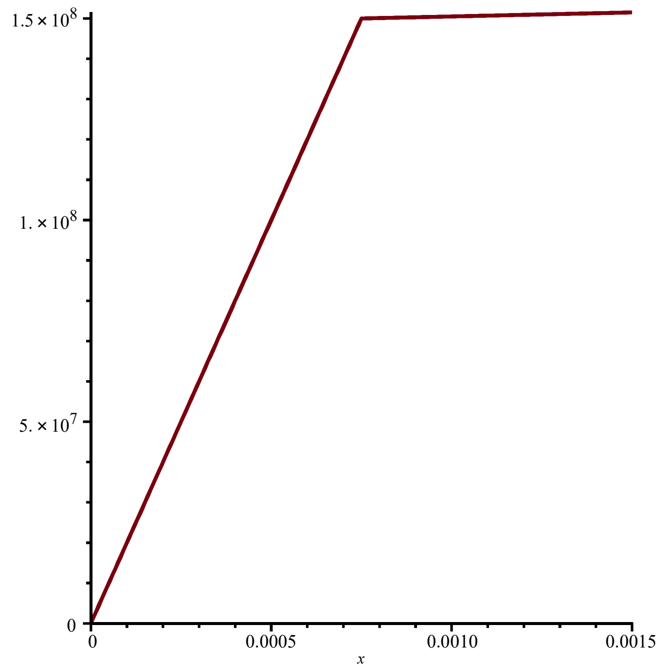
$$\epsilon_{yield} := \frac{\sigma_y}{E_{pipe}}$$

$$\epsilon_{yield} := 0.0007500000000 \tag{23}$$

$$eq\_piecewise := piecewise\left(0 \le x < \epsilon_{yield}, E_{pipe} \cdot x, \epsilon_{yield} \le x, E_{tangent} \cdot \left(x - \epsilon_{yield}\right) + \sigma_y\right)$$

$$eq\_piecewise := \begin{cases} 2.00 \times 10^{11}\, x & 0 \le x < 0.0007500000000 \\ 2. \times 10^{9}\, x + 1.485000000 \times 10^{8} & 0.0007500000000 \le x \end{cases} \tag{24}$$

$$plot\left(eq\_piecewise, x = 0 .. \epsilon_{yield} \cdot 2\right)$$



After the entire area has yielded, we would expect the pipe to be in state of residual stress. However, an analytical approach of this would require more information and a non-elastic analysis that has not been taught yet. If we knew how much the different places along the pipe had strained, we could solve for stresses by drawing back the elastic line and calculating for stress that way.