EMA 405 – Finite Element Analysis

Homework 8 – Shrink-Fit Thermal Analysis

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Overview:

In this homework, the model is to task a shrink fit problem, where two circular rings are assembled by shrink fit. The outer ring is at a significantly higher temperature (100 C, 20 C) than the inner ring when they fit together perfectly, and the model needs to cool until they are at equilibrium (defined as 20 degrees C). Using FEA, I determined the final minimum inside radius of the assembly. There will be some interference, which is modeled as perfectly bonded (no slip).

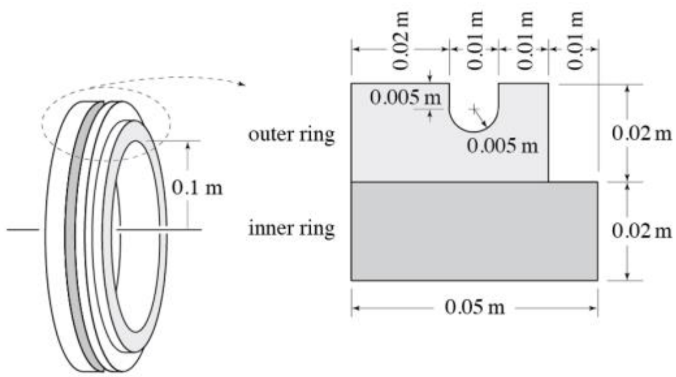

Figure 1: Shrink-fit problem

Description of FEA model and boundary conditions:

The two circular rings can be modeled in APDL using a Thermal 8-node Quad 77 element. This is a thermal element. When setting up the quad element, make sure to set an elementary behavior of axisymmetric so that we can model the rings as a 2D element, where Ansys will know that it is actually just a cross-section.

To ensure that the ring does not slip, you need to strategically add frictionless supports to ensure there is no internal movement in the ring.

Some key properties that were required to model this ring in Ansys were: A Young's modulus of 200 GPa, a Poisson's ratio of 0.3, thermal expansion coefficient of 12E-6/degree C, and a thermal conductivity value of 40 W/(m*degree C).

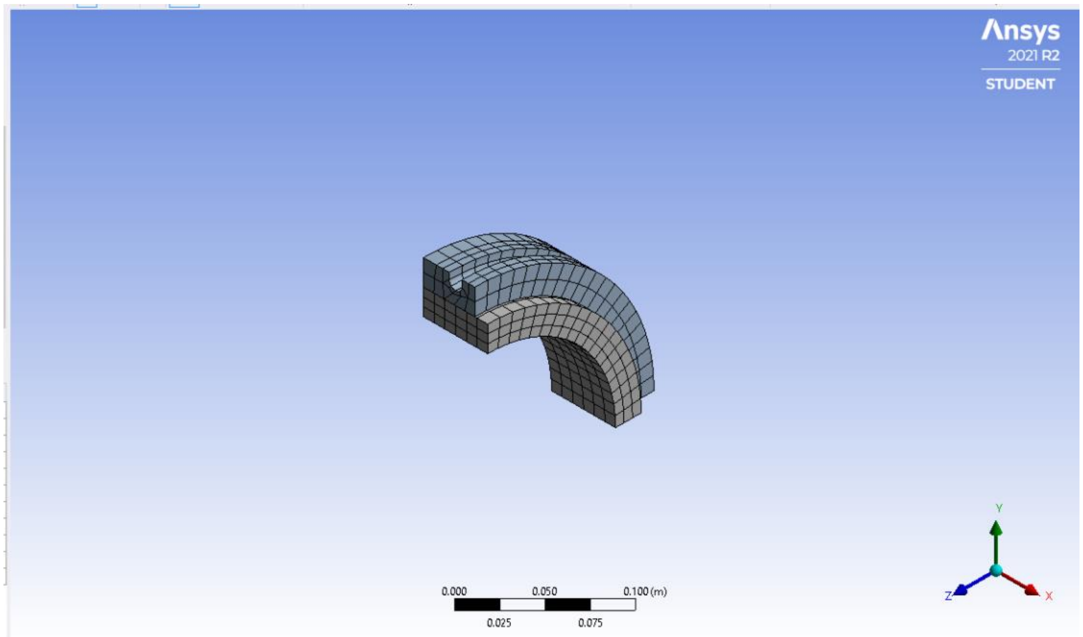

Figure 2: Workbench model of the ring with elements shown

The model can be made in SolidWorks. Make sure that there are two separate materials to be modeled. The reference temperatures for each of the materials should be set according to the problem statement (100 degrees for the outer, 20 for the inner). You can model each as Structural Steel. This ring was modeled with symmetry.

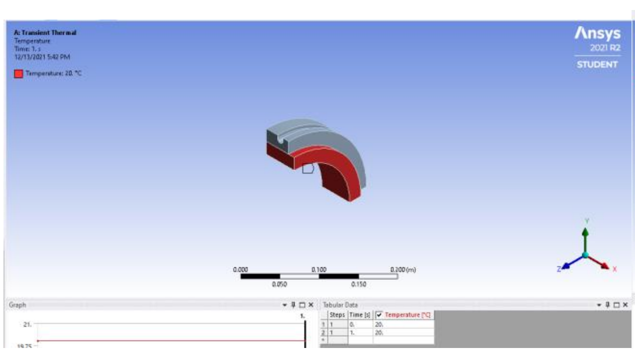

Figure 2: Inner ring temperature steps

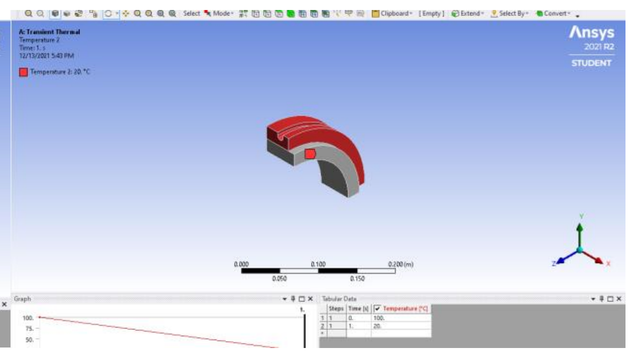

Figure 3: Outer ring temperature steps

To analyze the transient-thermal step of the problem, we need to set two reference steps. At first, the model has a temperature of 20 degrees for the inner ring, and 100 degrees Celsius for the outer ring. The second step should have the model with a temperature of 20 degrees Celsius for both ring parts.

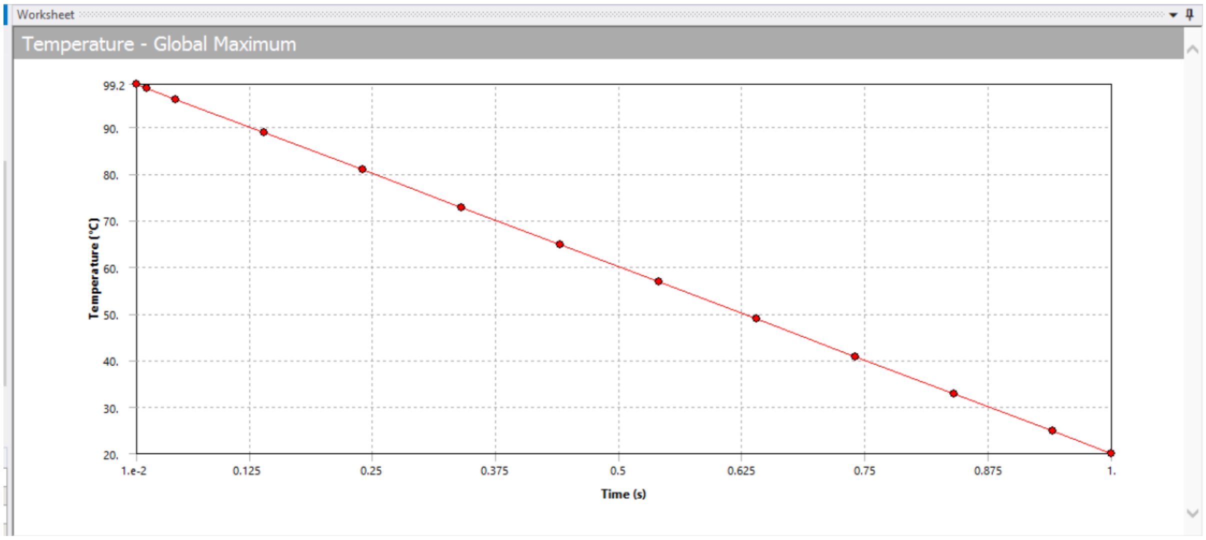

Figure 4: Global maximum temperature versus time

As seen in the temperature steps solution, our model has a global maximum of 100 degrees and converges when the entire model is at 20 degrees Celsius.

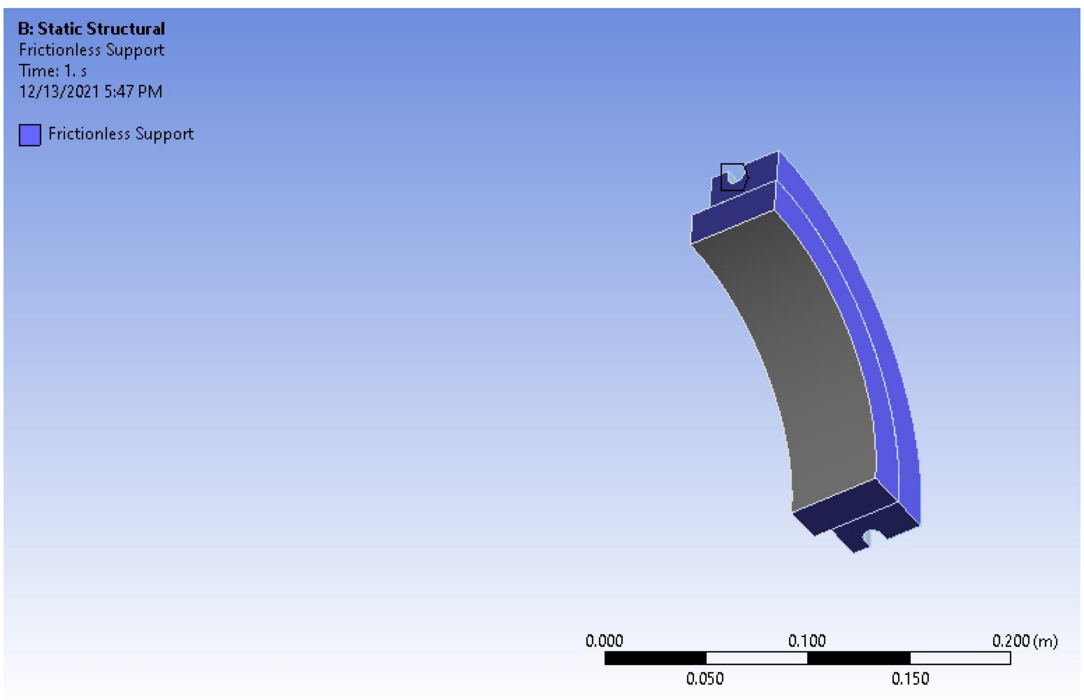

Figure 5: Frictionless Support boundaries

To ensure both that the inner ring and outer ring do not slip, and that the model is accurately modeled with symmetry, I added a frictionless support on six of the faces.

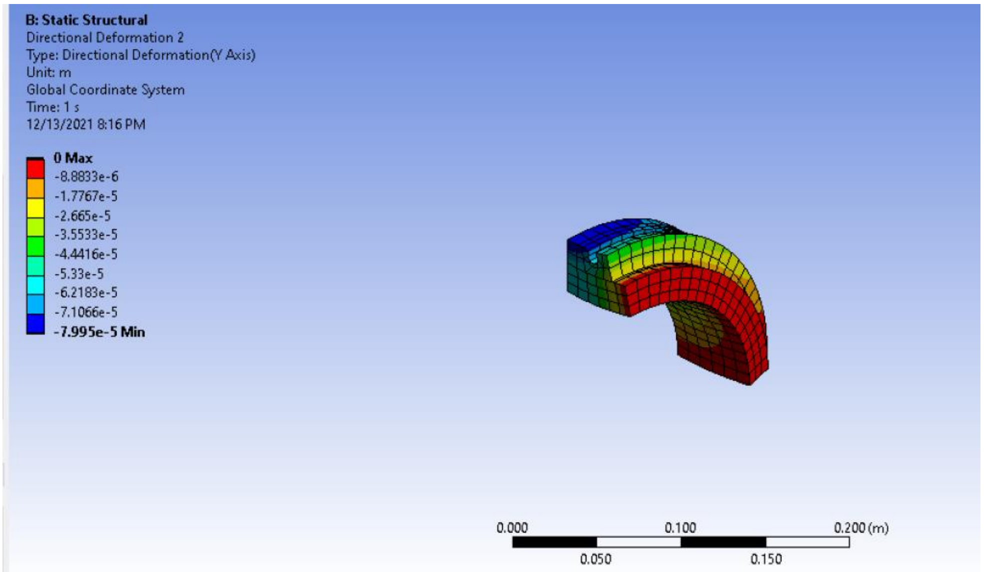

Figure 6: Directional deformation in the y-direction

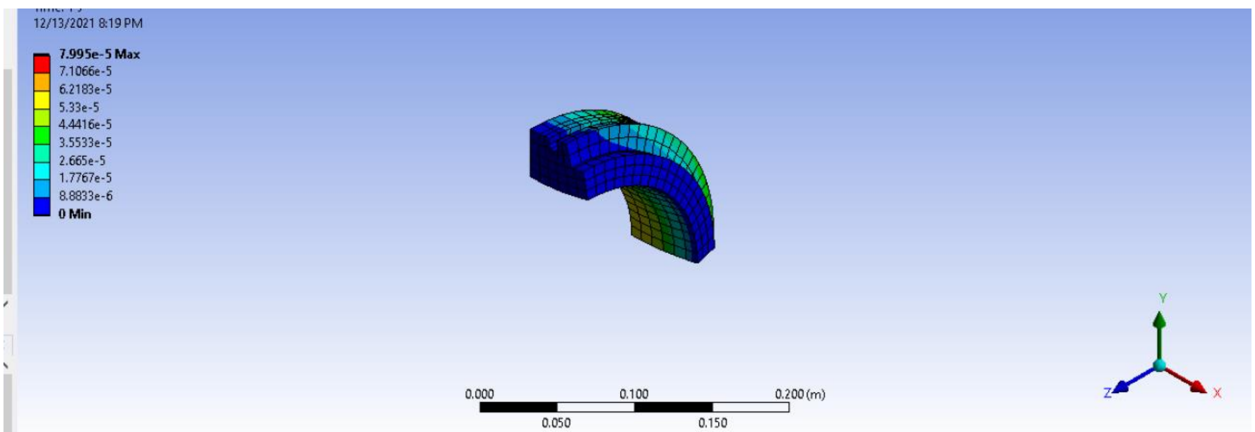

Figure 7: Direction deformation in the z-direction.

Figures 6 and 7 are important to understand if the total deformation (radial, which helps us identify the minimum inner radius) are in tension or compression. That is, if our ring was expanded or contracted. Evidently, the inner ring was compressed, and looking along the inner radius, we can expect the ring to radially be displaced inwards. This is evident since the contour plots of y-and z-directions were mirrored to each other and positive or negative with their respective axis (a negative y-direction is tension, positive z-direction is tension), so we are confident that the model was accurately shrink-fit since it was forced in compression with mirrored x and y-directions, such that the deformation can be modeled radially.

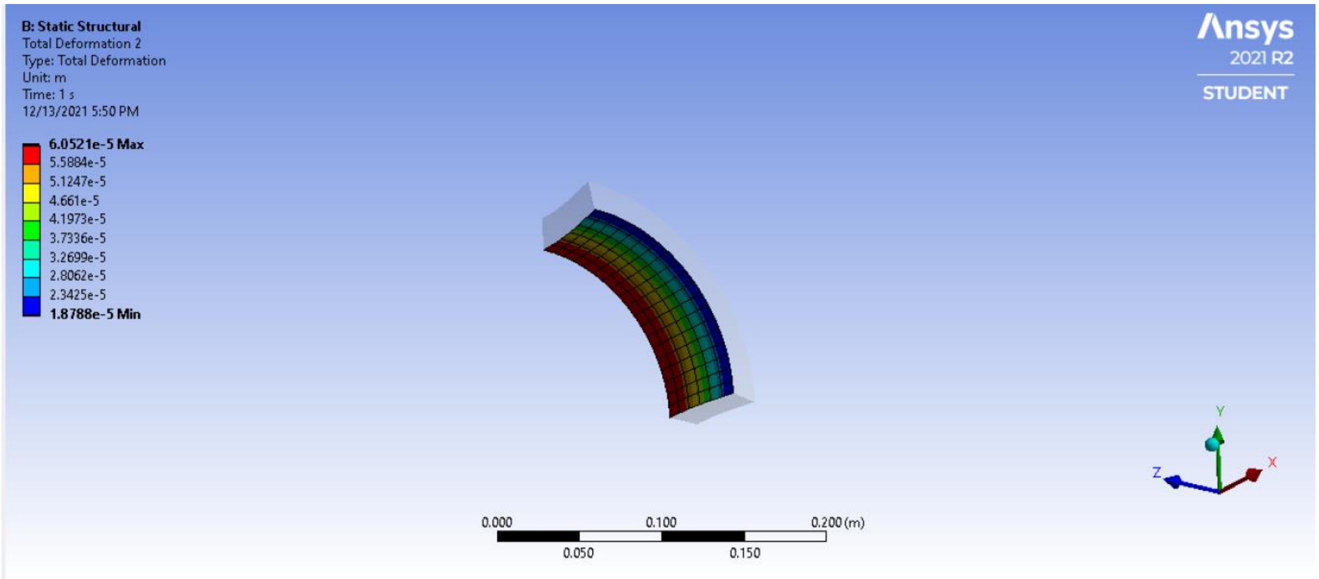

Figure 6: Total deformation on the inner ring plane

When the transient thermal solution was solved, we can import those solutions into a static-structural analysis when both objects are in thermal equilibrium. The above contour plot is a contour plot of the radial deformation on the inner plane of the inner ring. What we can see is that the part of the model where the inner ring is connected to the outer ring has a higher deformation of the "loose" end. Along the midpoint, we see a deformation of approximately 4E-5 meters. This is a vector sum, and as explained in the previous step, we are expecting this deformation to be inwards (that is, the inner ring was compressed). There is also confidence in our answer since the deformation of 4E-5 meters is very close to the analytical approach, which found an approximate deformation of 9.6E-5 meters in the analytical (line 1-12). The analytical is far from accurate, we would expect the real answer to be smaller, and our answer of 4E-5 is smaller than the analytical, and not off by a large amount, so I have confidence in this answer for deformation of the inner ring. Therefore, we would expect our final inner ring to have a minimum radius of 0.1-6.05E-5 meters = 0.09994 meters. This is practically the same as the analytical, which expected a radius 0.099904 meters.

Conclusion

From this homework assignment, we learned about thermal expansion and compression in real-world examples. The shrink fit problem is a very applicable problem to the real world because many of our real-world objects are fit this way. We learned how to model objects in Ansys that are subject to thermal change and map those to static-structural deformation. From the analytical approach, I learned how to map static indeterminacy to account for loss of deformation in resistance of stresses. I would also conclude that the final minimum inner radius of the inner ring after cooling is 0.09994 meters.

Analytical

$E := 200E9$

$$E := 2.00 \times 10^{11} \quad \textbf{(1)}$$

alpha := 12E−6

$$\alpha := 0.000012 \quad \textbf{(2)}$$

$delta\_T := -80$

$$delta\_T := -80 \quad \textbf{(3)}$$

$L := 0.02$

$$L := 0.02 \quad \textbf{(4)}$$

delta := $L \cdot \text{alpha} \cdot delta\_T$

$$\delta := -0.00001920 \quad \textbf{(5)}$$

$radius_{final} := L + \text{delta}$

$$radius_{final} := 0.01998080 \quad \textbf{(6)}$$

However, we need to consider the stresses inside of the ring have changes this ideal case. It is a static determinate problem because the inner ring will have stresses that cause less deformation.

$R_i := 0.10$

$$R_i := 0.10 \quad \textbf{(7)}$$

$inner\_strain := \frac{x}{R_i}$

$$inner\_strain := 10.00000000\, x \quad \textbf{(8)}$$

$R_o := 0.12$

$$R_o := 0.12 \quad \textbf{(9)}$$

$outer\_strain := \frac{x}{R_o}$

$$outer\_strain := 8.333333333\, x \quad \textbf{(10)}$$

$eq1 := E \cdot inner\_strain = E \cdot \left(outer\_strain + \frac{\text{delta}}{R_o} \right)$

$$eq1 := 2.000000000 \times 10^{12}\, x = 1.666666667 \times 10^{12}\, x - 3.200000000 \times 10^{7} \quad \textbf{(11)}$$

$solve(eq1, x)$

$$-0.00009600000010 \quad \textbf{(12)}$$

We would expect the radius to deform approximately 9.6E-5 meters. However, I would expect this to be significantly smaller

$0.1 + \%$

$$0.09990400000 \quad \textbf{(13)}$$