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EMA 405 – Finite Element Analysis

Homework 3 – Two-Dimensional Model for a Frame

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Overview:

In this homework question, a frame supports a floor that is subjected to a vertical load. The frame is composed of two vertical steel pipe members that support a horizontal I-beam. The task was to generate a two-dimensional model for this steel structure using BEAM 189 elements and determine the maximum vertical deflection and bending stresses in the portion BC of the frame.

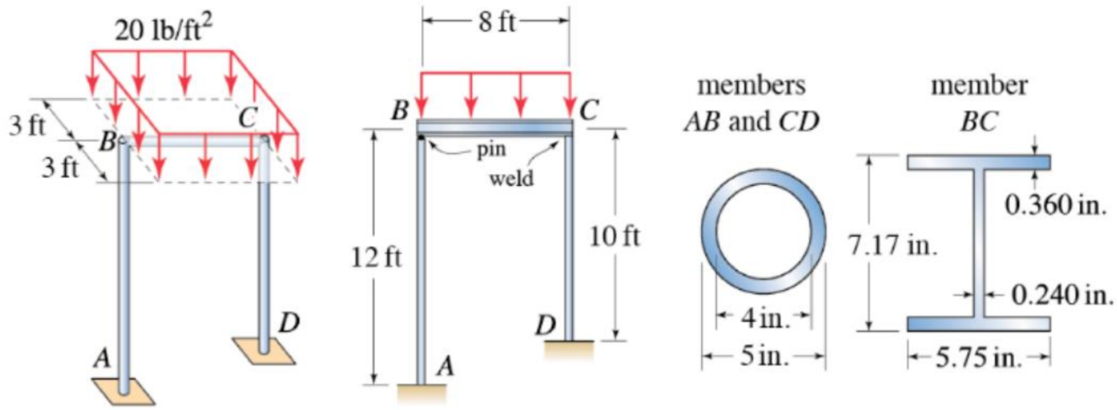
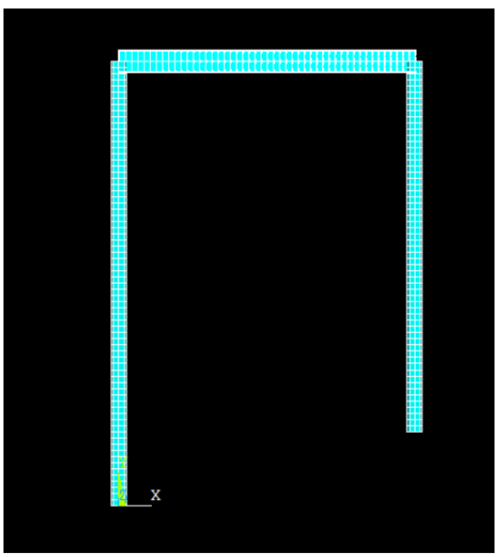
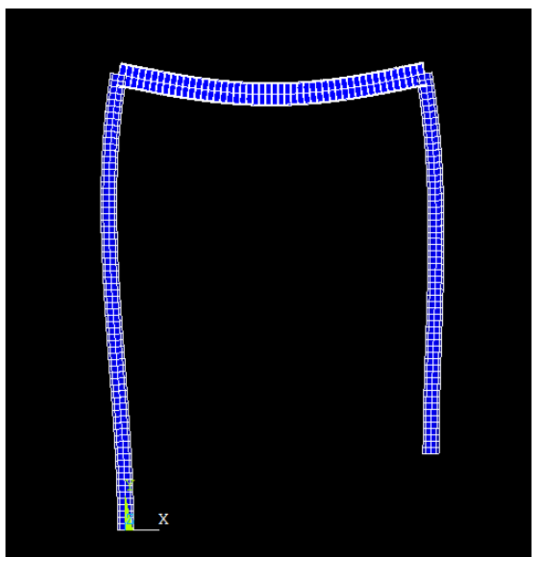


Figure 1: Frame and dimensions for members needed to solve this problem.





Figures 2 and 3: Model of the frame in Ansys and the deformed plot when the loads are applied.

Results

Maximum Vertical Deflection

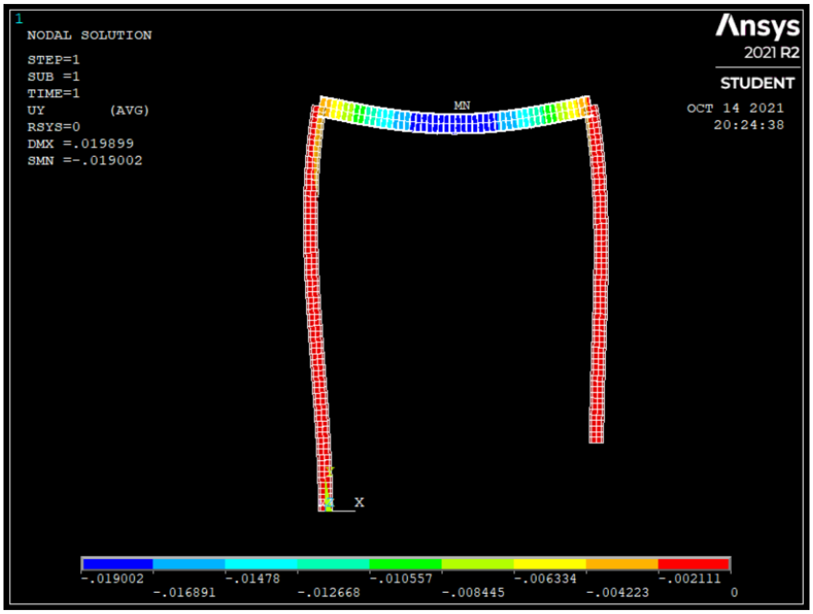


Figure 3: a contour map of deflection in the y-direction modeled in Ansys with gravity.

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313 -0.18997E-001
314 -0.19002E-001
315 -0.18989E-001
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Figure 4: Maximum nodes of deflection with gravity.

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313 -0.60355E-002
314 -0.60385E-002
315 -0.60353E-002
316 -0.60259E-002
317 -0.60104E-002
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Figure 5: Nodes with maximum deflection without gravity

For the maximum vertical deflection, it is clear from the contour map above that the maximum deflection happens in the beam section BC. The reported maximum deflections on Ansys are -0.21E-1 in. with gravity and -0.604E-2 in. without gravity. It is clear and expected that gravity would increase the deflection of the beam. The result in Ansys aligns with the analytical solution (see Appendix (8)) with a maximum deflection of -0.6E-3 in. of deflection without gravity and -0.0205 in. with gravity. These values are very close to each other, and the analytical is slightly smaller than the actual, which makes sense because the Ansys solution also takes into account the combined deflection of the pipes.

Maximum Bending Stress

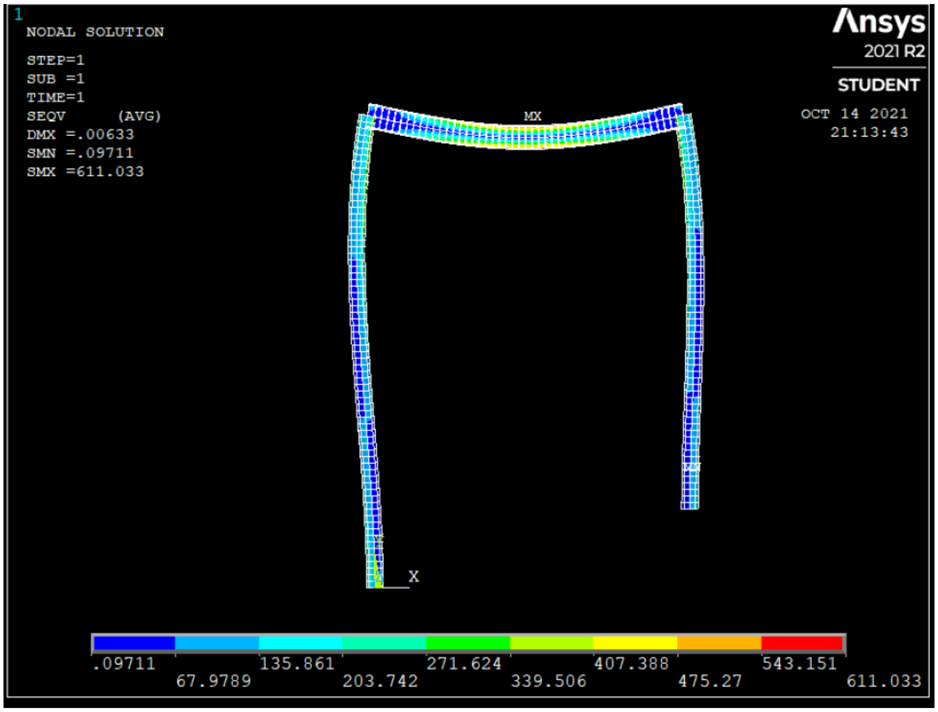


Figure 6: contour map of von Mises stresses in Ansys of the system without gravity.

Max=	601.18	0.0000	0.0000	611.03	611.03

Fig 7: Maximum von Mises stress without gravity along BC in psi.

Max=	1790.5	0.0000	0.0000	1819.8	1819.8

Fig 8: Maximum von Mises stress with gravity along BC in psi.

Without gravity, Ansys calculated that the maximum bending stress in BC was 611 psi. With gravity, Ansys calculated the maximum bending stress in BC to be 1819.8 psi. Most of this stress was concentrated in the x-direction. This is because as the beam bends, it needs to expand in the x-direction to support the movement. The actual height is not changing much compared to the expansion in the x-direction. The values are as expected, with the stress in gravity being approximately three times that without. This is expected since in Appendix (9), the calculated maximum displacement with gravity to be three times greater than without. Since Equation (7) in the appendix relates stress directly with the displacement, it was expected that the maximum stress would be three times greater with gravity.

Factor of Safety

Factor of safety refers to how much stronger a system is compared to the failure strength. In the analytical approach, we determined the critical stress that would cause the system to fail. Therefore, we can use the Equation (9) in the appendix to compare the critical stress that would cause each member to fail and compare it to the maximum stress a node experienced in Ansys with and without gravity.

	AB	AB(g)	BC	BC(g)	DC	DC(g)
$\sigma_{y/cr}$	1.264E7	1.264E7	29E3	29E3	7.53E6	7.53E6
σ_{max}	427.3	1297.9	611.0	1819.0	398.0	1188.0
F.S.	29581	9738	47.5	15.94	18919	6338

Table 1: Maximum stresses found from Ansys and corresponding factors of safety for all beams with and without gravity. With gravity noted with a (g).

The critical values for buckling and yield (σ_{cr}) were calculated by hand using Equation (6) in the appendix for buckling. The yield strength was given in the problem. The values can be found in the table above. When applying Equation (9) to these values, we find that the factors of safety are very high. None of the beams fail, even with gravity. This was expected analytically and conceptually since the applied loads are only a third of the weight of the beam itself. It is clear that a steel beam would not collapse under its own weight given these proportionally fair cross-sectional areas compared to the distance between them. However, it is noteworthy that the weight of the beams is not negligible. For example, for beam BC, the weight of the beam causes a stress approximately 1/30th of the failure strength. This would make the difference in a case where an acceptable factor of safety is 2, and the maximum stress anticipated reached the 2.03 factor of safety threshold.

Conclusion

From this assignment, the largest takeaway is how pinned and welded joints affect a model. It was also worth noting that an interesting discovery from this assignment was that the stress concentrations of the beam BC were mostly due to stress in the x-direction, instead of along the direction of displacement. The ideas of buckling and bending were reinforced, with a focus on failure criteria along with displacement analysis. Additionally, we learn how gravity plays a role in deflections and stress concentrations. With this insight, we can apply these concepts to determine if a more complicated frame or object would fail.

To check if the two vertical pipes fail due to buckling, we can model them as Euler beams. Referencing Eq. (10.30) from Mechanics of Materials by Ferdinand Beer (referenced to later as Mechanics, Beer), the critical force that causes a beam to buckle can be written as Equation (1):

$$P_{cr} = \frac{\pi^2 E I}{L_e^2} \qquad (1)$$

where P_cr is the critical load of failure, E is the Young's Modulus of the material, I is the moment of inertia of the beam, and L_e is the effective length of the beam. For a stress analysis, we can also use EQ (10.39) for Euler's analysis of allowable stress for steel columns under centric loads, which is relevant to the homework problem since we are working with steel pipes. The critical stress (σ_cr) can be written as Equation (2):

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/r)^2} \qquad (2)$$

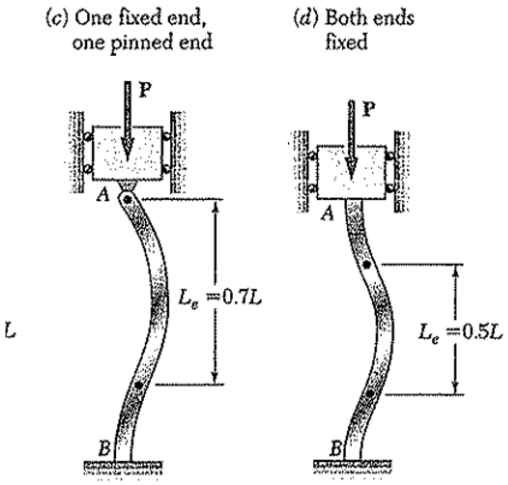
Where r represents the radius of gyration as Equation (3):

$$r_g = \sqrt{\frac{I}{A}} \qquad (3)$$

These conditions will be very handy when determining if the two vertical steel pipes fail due to buckling, and also to calculate factors of safety in this design. To calculate the radius of gyration, and the failure criteria above, we need the moment of inertia of the steel pipes, effective lengths, and their cross-sectional areas. Since the pipes are modeled with thin-walled circular cross sections, we can firstly calculate the area to be Equation (4):

$$A = \pi R^2 - \pi r^2 \qquad (4)$$

where R represents the outer radius of the cross-section and r represents the radius of the inner wall. Our beams AB and DC can then be calculated to have an area of 7.068 in^2. Furthermore, we can find the effective length of each beam referencing Fig. 10.18 from Mechanics, Beer:



Beam AB has one fixed end and one pinned end, thus its effective length Le = 0.7L. Meanwhile, beam DC has an effective length Le = 0.5L. The effective lengths of the respective beams AB and DC is calculated to be 100.8 in, 60 in.
Lastly, to find the moment of inertia of the pipes by referencing the appendix of Mechanics, Beer (A.15) to find the polar moment of inertia around their axis of rotation Equation (5):

$$I = \int_A y^2 \, dA \tag{5}$$

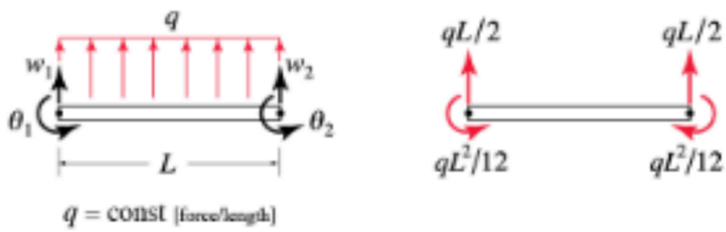
$$I = \int_0^{2\pi} \int_r^R (r \sin\theta)^2 \, r \, dr \, d\theta$$

The moment of inertia for the two beams is calculated to be 18.113 in^4.
It is worth noting that the two pipes will have different critical stresses and forces because beam AB is longer than beam DC. Additionally, pipe AB is connected to the beam with a pin, while pipe BC is welded to the beam. The stress experienced in both pipes can be modeled according to Mechanics, Beer as:

$$\sigma = \frac{P}{A} - \frac{My}{I} \tag{6}$$

The moment does not transfer on the pin, therefore, for the first beam, it can be calculated only with forces. Therefore, the analysis of pipes AB and DC are different. Referencing class handouts, the beams can equivalently be modeled similarly:

2-Node Cubic Displacement (Euler-Bernoulli) Beam



$$\sigma_{DC} = \frac{-480}{7.068} - \frac{(120)(8ft)^2}{18.113 \text{ in}^4} = -491.9 \text{ psi}$$

$$\sigma_{AB} = \frac{-480}{7.068} = -67.91 \text{ psi}$$

$$\sigma_{cr_{DC}} = \frac{\pi^2 EI}{A L_e^2} = 1.264E7 \text{ psi}$$

$$\sigma_{cr_{AB}} = \frac{\pi^2 EI}{A L_e^2} = 7.57E6 \text{ psi}$$

Therefore, the beams do not buckle.
Since the beams do not buckle, the deflections of the beams can be simply calculated using an equation given in Roarke:

$$\delta = \frac{PL}{AE} = \sigma \frac{L}{E} \qquad (7)$$

The expected analytical displacements of the pipes AB and DC are 3.26E-3 in and 1.968E-3 in respectively.

Finally, the bending in the beam can be described as a simple beam where one end has a roller, and the other end is pinned. The displacement of these beams have been characterized before. Referencing engineering toolbox, a model like this would have a maximum displacement such as Equation (8):

$$\delta_{max} = \frac{-5qL^4}{384\,EI} \qquad (8)$$

$$\delta_{max} = \frac{-5\left(10\frac{lbf}{ft}\right)(96)^4}{384 \cdot 30 \cdot 10^6 \cdot 53.41\,in^4} = \underline{-0.006902\ in}$$

The deflection with gravity included can be calculated by taking gravity as a distributed force. The distributed force of this gravity would act with a force Equation 9:

$$F = \frac{0.29\ lbf}{in^3} \cdot 5.688\ in^2 = 1.649\ lbf/in$$

$$= 19.79\ lbf/ft$$

$$\delta_{max\ gravity} = \frac{-5(10+19.79)(96)^4}{384 \cdot 30 \cdot 10^6 \cdot 53.41} = \underline{-0.0205}$$

Factor of safety is a very well known equation that can be stated as Equation (9):

$$F.S. = \frac{\sigma_{cr}}{\sigma_{allow}} \qquad (9)$$