EMA 405 – Finite Element Analysis

Homework 6 – Modal Analysis

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November 17th, 2021

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Overview:

In this homework question, task is to model the space truss pictured in Figure 1. A space truss can be used in space with various functions. The space truss is made of steel with an Elastic modulus of 200 GPa, a Poisson's ratio of 0.3, cross-sectional members of 0.01m^2, and a mass density of 7860 kg/m^3.

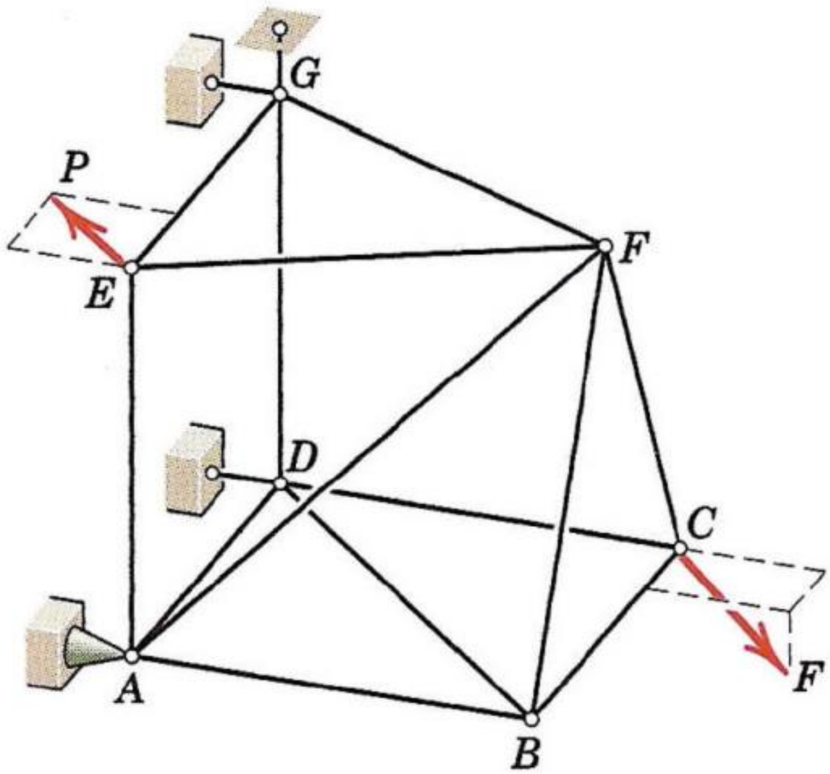


Figure 1: Space Truss model

Description of FEA model:

The space truss was creating using Beam189 elements. For each connection point only a single element was used (that is, for example the member A to B is one element). Beam189 elements were suitable for this problem because firstly, the alternative would be Link180 elements that do not account for the bending in the beams. Secondly, instead of Beam188, we used Beam189 because they have quadratic interpolation and are more suitable for problems like this that will only use one element between nodes.

I decided to use a circular cross-sectional area for the beams because that is very suitable for the application. The space truss had specified coordinates of all the points, along with stating that the points A, D, and G were fixed in all translational DOF. In the case of the space truss, we then know that the connection to the body would be at A, D, and G, so when analyzing for the rigid body modes we firstly will add constraints to A, D, and G in the rotation to try to remove these because that is the easiest product design constraint to add. All joints were originally connected as ball joints before analyzing for rigid body modes.

A key part of the assignment was identifying rigid body modes and adding constraints to the model to get rid of the rigid body modes. A rigid body mode presents itself when a free translation or body can happen without significant deformation. We want a rigid body that does not move by itself, so we will need to limit the degrees of freedom of the system until there are no rigid body modes.

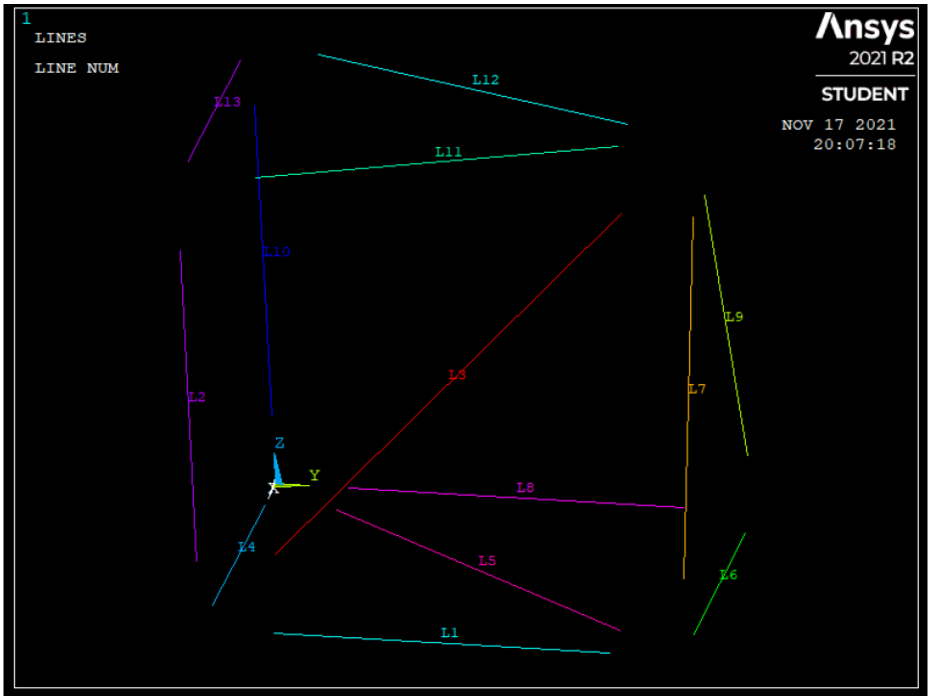


Figure 2: Model of the space truss with lines.

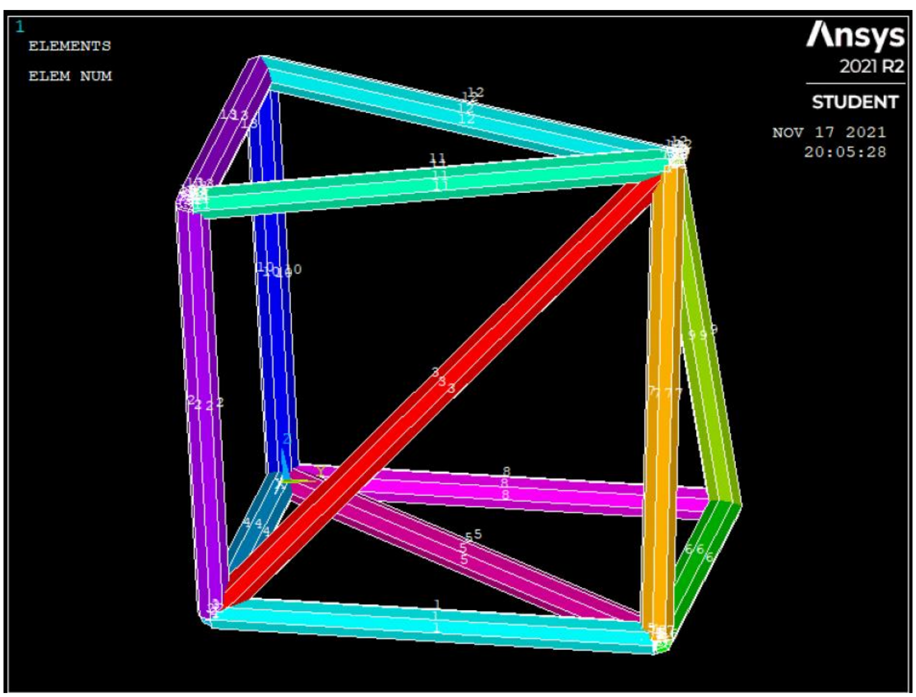


Figure 3: Elements plot of the space truss

Working Through Boundary Conditions

In Ansys, we modeled the solution with one element for each member. Originally, there were no constraints. The document will try to answer the questions about rigid bodies in transition from no constraints to the full constraints that we have found for the system along with a brief reasoning on why the constraints were taken.

***** INDEX OF DATA SETS ON RESULTS FILE *****

SET	TIME/FREQ	LOAD STEP	SUBSTEP	CUMULATIVE
1	0.0000	1	1	1
2	0.0000	1	2	2
3	0.0000	1	3	3
4	0.0000	1	4	4
5	0.0000	1	5	5
6	0.0000	1	6	6
7	0.0000	1	7	7
8	0.0000	1	8	8
9	0.39455E-05	1	9	9
10	0.70010E-05	1	10	10
11	0.96110E-05	1	11	11
12	0.96111E-05	1	12	12
13	0.11816E-04	1	13	13
14	0.11816E-04	1	14	14
15	0.12557E-04	1	15	15
16	0.13244E-04	1	16	16
17	0.15808E-04	1	17	17
18	0.34029E-04	1	18	18
19	0.44090E-04	1	19	19
20	0.52975E-04	1	20	20
21	0.54375E-04	1	21	21
22	55.624	1	22	22
23	58.541	1	23	23
24	80.702	1	24	24
25	80.903	1	25	25

Figure 4: Modal solution of natural frequencies for the first model

Figure 4 shows the modal solution of natural frequencies for the model with zero constraints. With zero constraints, there are 21 zero energy modes (interchangeably referred with rigid body modes in this document). At this point, the translations at A, D, and G were not constrained but all connections were made as ball joints.

***** INDEX OF DATA SETS ON RESULTS FILE *****

SET	TIME/FREQ	LOAD STEP	SUBSTEP	CUMULATI
1	0.0000	1	1	1
2	0.0000	1	2	2
3	0.0000	1	3	3
4	0.0000	1	4	4
5	0.0000	1	5	5
6	0.96113E-05	1	6	6
7	0.96114E-05	1	7	7
8	0.11816E-04	1	8	8
9	0.11816E-04	1	9	9
10	0.15808E-04	1	10	10
11	0.37339E-04	1	11	11
12	0.41526E-04	1	12	12
13	0.52935E-04	1	13	13
14	0.54343E-04	1	14	14
15	52.388	1	15	15
16	54.950	1	16	16
17	74.683	1	17	17

Figure 4: Modal Solution of natural frequencies for the second model

The second model was the addition of the translational constraints that were identified in the problem statement. That is, UX = UY = UZ = 0 for points A, D, and G. At this point, our model is at the stage that is specified in the problem statement. There are therefore 14 rigid body modes that must be removed before we can arrive at our final answer of the five first natural frequencies. The logical step from here would be to weld the connection points at A, D, and G since they are linked with the spaceship. Since there are 14 rigid body modes, this will not be enough to get it down to zero, but it is an important step to not just constrain randomly.

SET	TIME/FREQ	LOAD STEP	SUBSTEP	CUMULATIVE
1	0.0000	1	1	1
2	0.21155E-05	1	2	2
3	0.33903E-04	1	3	3
4	0.43947E-04	1	4	4
5	9.9194	1	5	5
6	75.944	1	6	6
7	80.776	1	7	7
8	91.725	1	8	8

Figure 5: Modal solution of natural frequencies for the third model

The third model was the addition of welding the points A, D, and G. The welds removed a considerable amount of rigid body modes. Now, there are just four rigid body modes to remove. To consider which degrees of freedom to remove, we should look at the deformed shape to see which parts of the truss are moving the most in the modal analysis. The deformed shape showed that the place to go was node F.

SET	TIME/FREQ	LOAD STEP	SUBSTEP	CUMULATIVE
1	22.765	1	1	1
2	107.71	1	2	2
3	112.88	1	3	3
4	120.28	1	4	4
5	147.60	1	5	5
6	162.39	1	6	6

Figure 6: Modal solution of natural frequencies for the last model

When all rigid body modes were removed, the first six natural frequencies can be seen in the table in Figure 6. The document will briefly summarize the boundary conditions that I added to remove all rigid body motion. The results are similar to the analytical, where an estimated 14 Hz would be the first natural frequency. Of course, the analytical was a very rough estimation.

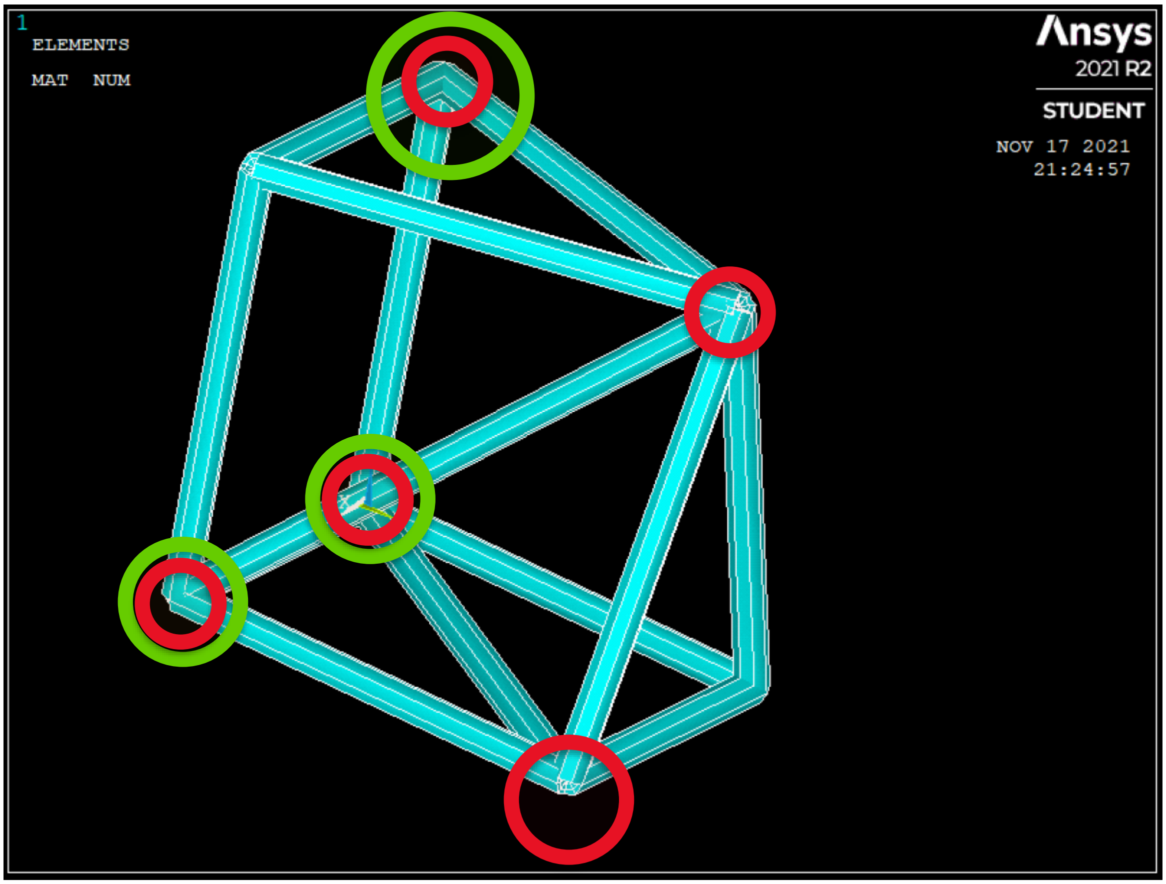


Figure 7: Ansys model with highlighted additional boundaries added

The figure above shows the additional boundaries added to the system. The red circled nodes represent joints that were welded to add constraints. The green circled nodes were joints that were locked in translation as specified in the document. These five welds and three translation restrictions were enough to remove all rigid body motion. The amount of boundary conditions needed to do this was 24, since all rotations were locked (welded), and all translations were constrained in all of these restrictions. The nodes welded were A, B, D, F, and G (ROTX = ROTY = ROTZ on the join). The nodes constrained in translation were A, D, and G (UX = UY = UZ = 0 for the nodes.

Model Shape Figures

At first, significant deformations were present in the system because of the present of zero frequency modes. As part of the assignment specifications, two of the first elastic mode shapes of the final model will be shown as well as a deformation plot of the original model with no constraints.

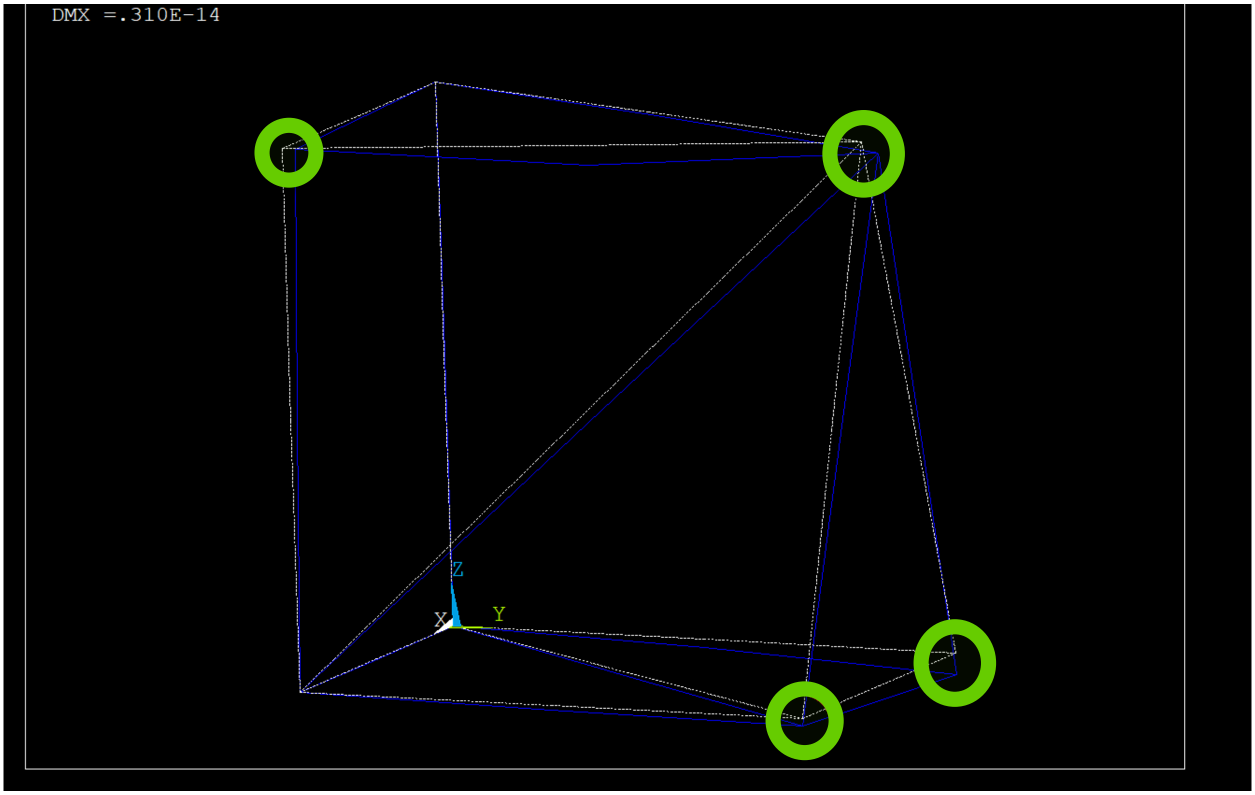


Figure 8: Undeformed and deformed plot of the original Ansys model (first natural frequency)

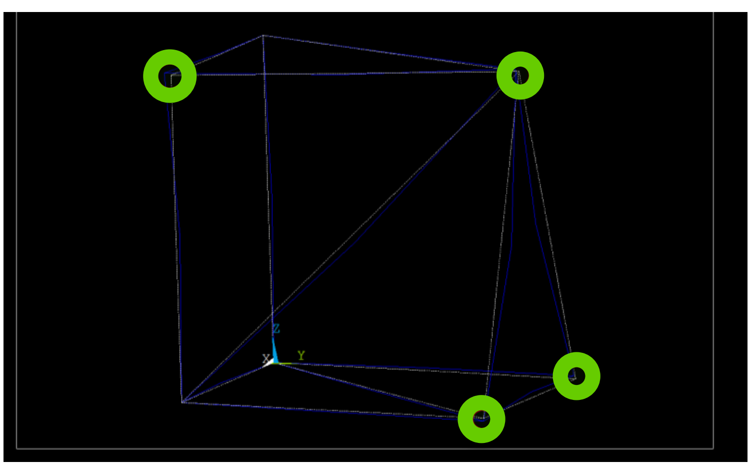


Figure 9: Undeformed and deformed plot of the original Ansys model (second natural frequency)

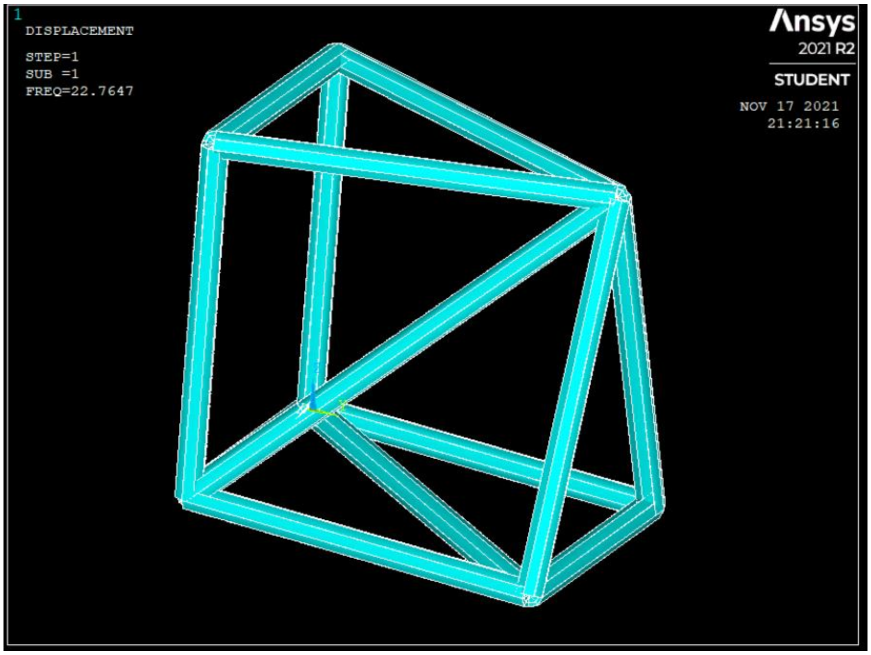


Figure 10: Undeformed and deformed plot of the final Ansys model (first natural frequency)

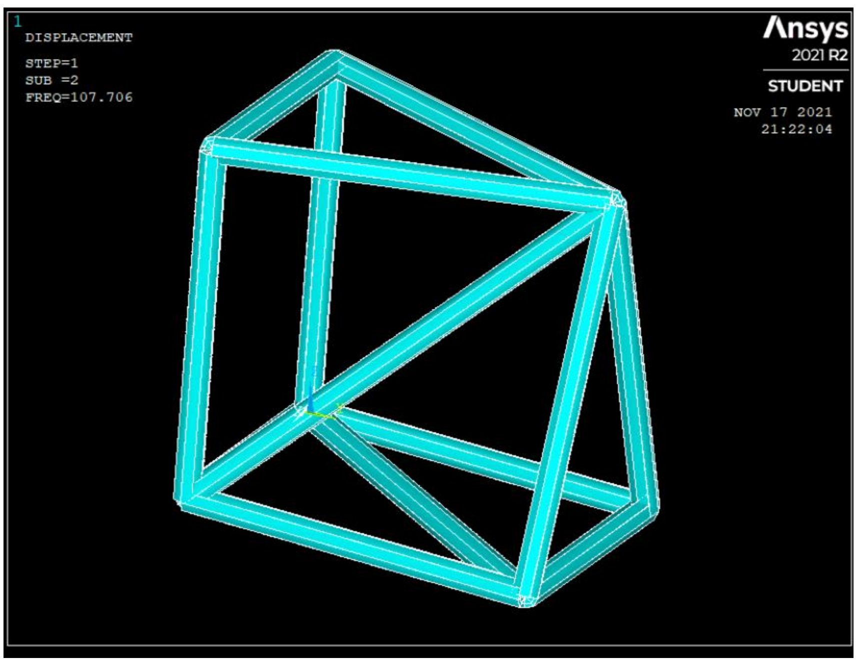


Figure 11 Undeformed and deformed plot of the final Ansys model (second natural frequency)

Clearly, when the model is restrained to have no rigid body modes, there is almost no deformation present in the system for the first two natural frequencies as compared to when the zero energy modes were present. The circled sections were the identified regions of largest deformation, which were used in the analysis of where to weld the last two points to remove all rigid body modes.

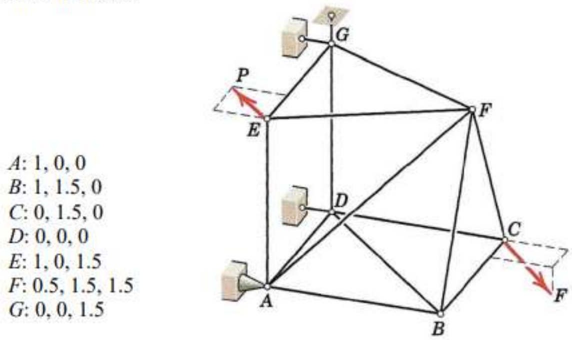
Conclusion

From this homework assignment, we learned a lot about vibrations and rigid body modes in systems. We were introduced to important ideas such as constraining degrees of freedom, analyzing model solutions, and thinking like an engineer when it comes to the design process in constraining the system.

Appendix

Problem statement - Space truss

Consider this section of a space truss, and ignore external forces P and F. Assume that points A, D, and G are fixed in all translational DOF. The coordinates of *A* through *G* are listed below in meters.



Firstly, some of the given information about the material that makes up the space truss. It is made of steel.

$E := 200\text{E}9;$ # *Elastic modulus in Pascals*

$$E := 2.00 \times 10^{11} \tag{1}$$

$A := 0.01;$ # *Cross sectional area in* m^2

$$A := 0.01 \tag{2}$$

rho $:= 7680;$ # *Mass density kg per* m^3

$$\rho := 7680 \tag{3}$$

$v := 0.3;$ # *Poisson's Ratio*

$$v := 0.3 \tag{4}$$

The first part of the problem statement asked to determine if this problem is a mechanism. You can determine if the problem is stat. determinate, indeterminate, or a mechanism by the following equations (b = # members, r = # reactions, j = # joints).

b + r = 2j - Stat. Determinate
b + r > 2j - Stat. Indeterminate
b + r < 2j - Mechanism

$b := 13$

$$b := 13 \tag{5}$$

$r := 9$

$$r := 9 \tag{6}$$

$j := 7$

$$j := 7 \tag{7}$$

$b + r > 2j$

$$14 < 22 \tag{8}$$

That would mean that this problem is Statically Indeterminate, and not a mechanism.
Firstly, we should solve for the cross-sectional area. We are going to use a circular cross section, so we should solve for the radius of the cross-section.

$A\_circ := \text{Pi} \cdot r\_circ^2$

$$A\_circ := \pi\, r\_circ^2 \tag{9}$$

$fsolve(\{A\_circ = A\}, r\_circ)[2]$

$$\{r\_circ = 0.05641895835\} \tag{10}$$

Removed the negative answer. The radius of the circle is 0.0564 meters.

Roark's Table 16.2 can help us analytically calculate modal frequencies (approximations) of the system.

$f\_n := \dfrac{C\_n \cdot r\_g}{L^2} \cdot 10^4 \cdot K\_m$

$$f\_n := \frac{4.050000000 \times 10^6\, C\_n \sqrt{\pi}\, K\_m}{L^2} \tag{11}$$

$r\_g := \text{sqrt}\left(\dfrac{Ixx}{A}\right)$

$$r\_g := 405.0000000 \sqrt{\pi} \tag{12}$$

$Ixx := \dfrac{\text{pi} \cdot r^4}{4}$

$$Ixx := \frac{6561\, \pi}{4} \tag{13}$$

f_n

$$\frac{4.050000000 \times 10^6\, C\_n \sqrt{\pi}\, K\_m}{L^2} \tag{14}$$

Using values from the book,

omega $:= subs(K\_m = 1, A = A\_circ, r = r\_circ, L = 1.5, C\_n = 31.73, f\_n)$

$$\omega := 5.711400000 \times 10^7 \sqrt{\pi} \tag{15}$$

$\dfrac{5.711 \cdot 10^7}{39^4} \cdot \text{sqrt}\left(\dfrac{1}{\text{Pi}}\right)$

$$13.92768051 \tag{16}$$

The first frequency calculated should be approximately 13.92 Hz.