EMA 405 – Finite Element Analysis

Homework 5 – Plate With Pressure Forces

Nikolaj Hindsbo

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University of Wisconsin–Madison
Hindsbo@wisc.edu

Overview:

In this homework question, the task is to model the plate pictured below in Figure 1 that is subject to axial and normal pressures. The plate is made of 409 stainless steel. The goal is to determine the stresses in the plate and use symmetry to reduce the computing power to model the plate.

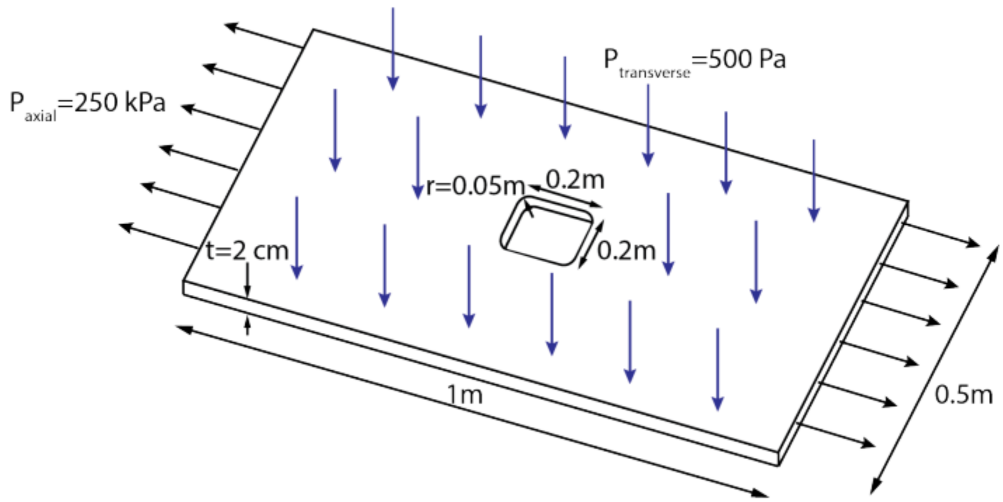

Figure 1: Plate under axial and normal pressure

Description of FEA model:

The plate was created using a Shell 281 element. According to the Ansys library, Shell281 is suitable for analyzing thin to moderately-thick shell structures. Each Shell 281 element is defined by eight nodes. It is suitable for linear, large rotation and large strain nonlinear applications, and we can tweak the amount of integration points to allow for even more accurate solving. In my solution, I used 5 points of integration to interpolate to a higher power. Comparatively, plane elements are used to model 2-D solid structures. This is usually applicable to plane strain or plane stress problems. Since this problem is neither in plane stress or plane strain, plane elements are not applicable for this problem. The thickness is a very important aspect to this problem, because the bending will cause tension on the top, and net compression in some parts of the plate on the bottom side. Plane elements are not able to model the out-of-plane thickness and differences in stress on the top and bottom. The shell has a thickness of 0.02 m, which is applicable in this case because we can model the P_axial as a line load of 250kPa*thickness = 5000 Pa*m. Also, the P_transverse can be applied on the shell area. Therefore, Shell281 is applicable for this application.

409 Stainless Steel is a heat-resistant steel that was developed for automobiles. The benefits of 409 Stainless Steel are its low cost, medium strength, and temperature corrosion resistance. According to Azo Materials, 409 Stainless Steel has a density of 7700 kg/m^3, elastic modulus of 220 GPa, and a Poisson's ratio of 0.28.

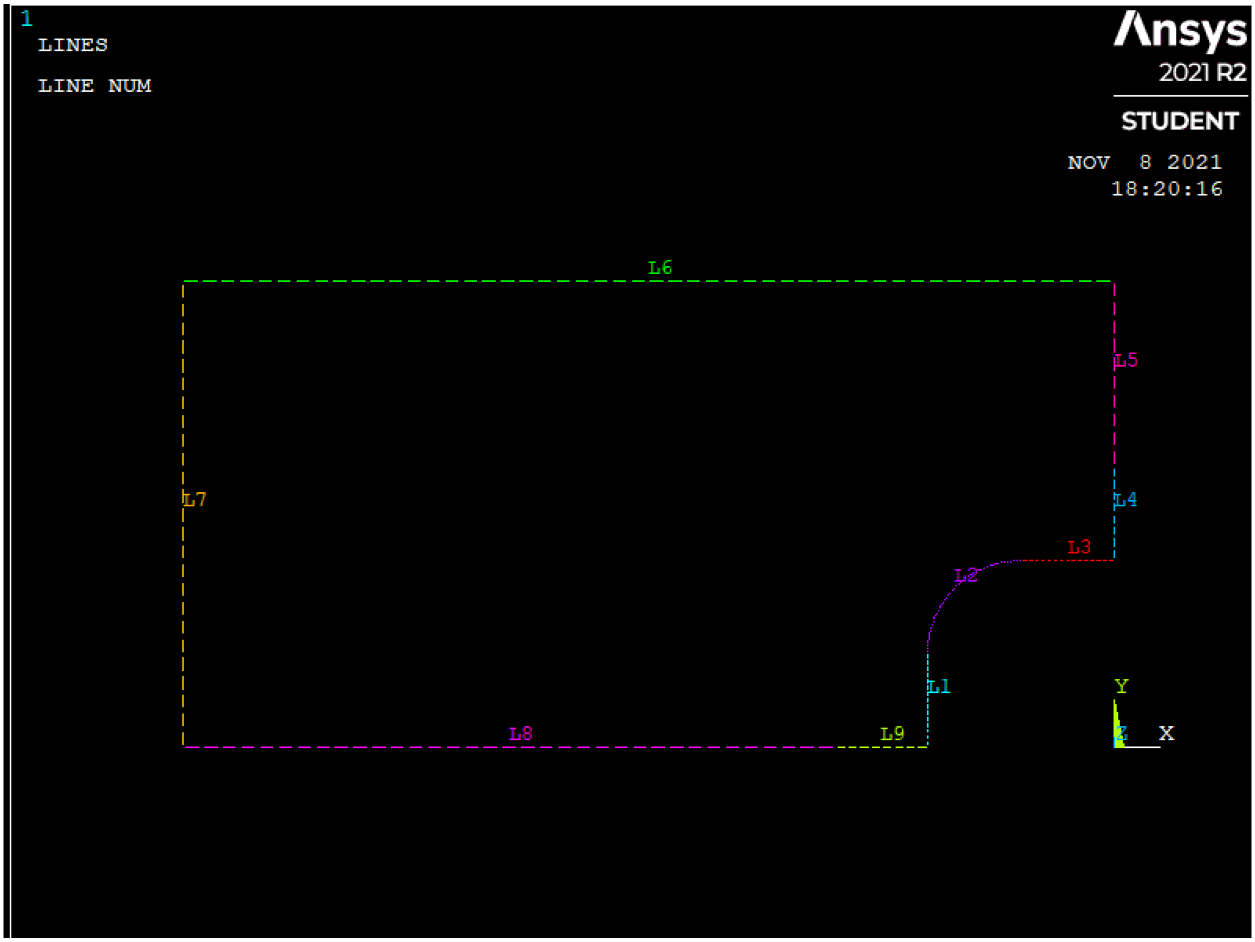

Figure 2: Model of the frame with lines.

In Ansys, we modeled the solution with symmetry. Lines 8, 9, 4, and 5 were modeled with symmetry boundary conditions which is a trick that allows Ansys to understand that this is only a quarter of the model, so we can obtain more accurate results. The line 7 was simply supported, which for a shell element means that displacement is constrained in the perpendicular direction (modeled as the Z-direction in Ansys), and only the rotation parallel to the line is allowed (ROTY). In other words, UZ = ROTX = ROTZ = 0 for line 7.

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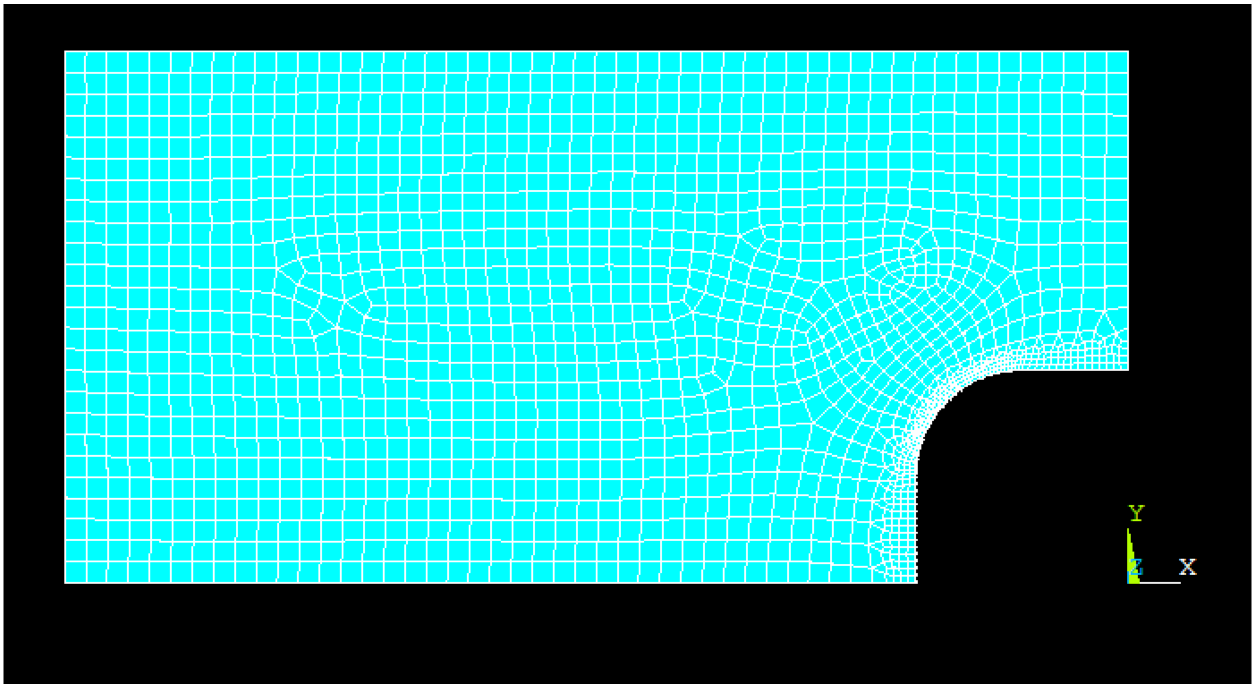


Figure 3: Elemental model of the plane

The concentration of elements towards the hole is necessary to accurately model the stress concentrations that exist on the hole. The element size was refined until the solution did not change. Larger elements led to inaccurate results because of a stress density that is shown later in the report. There were a total of 1600 elements. The highest density of elements is around the hole.

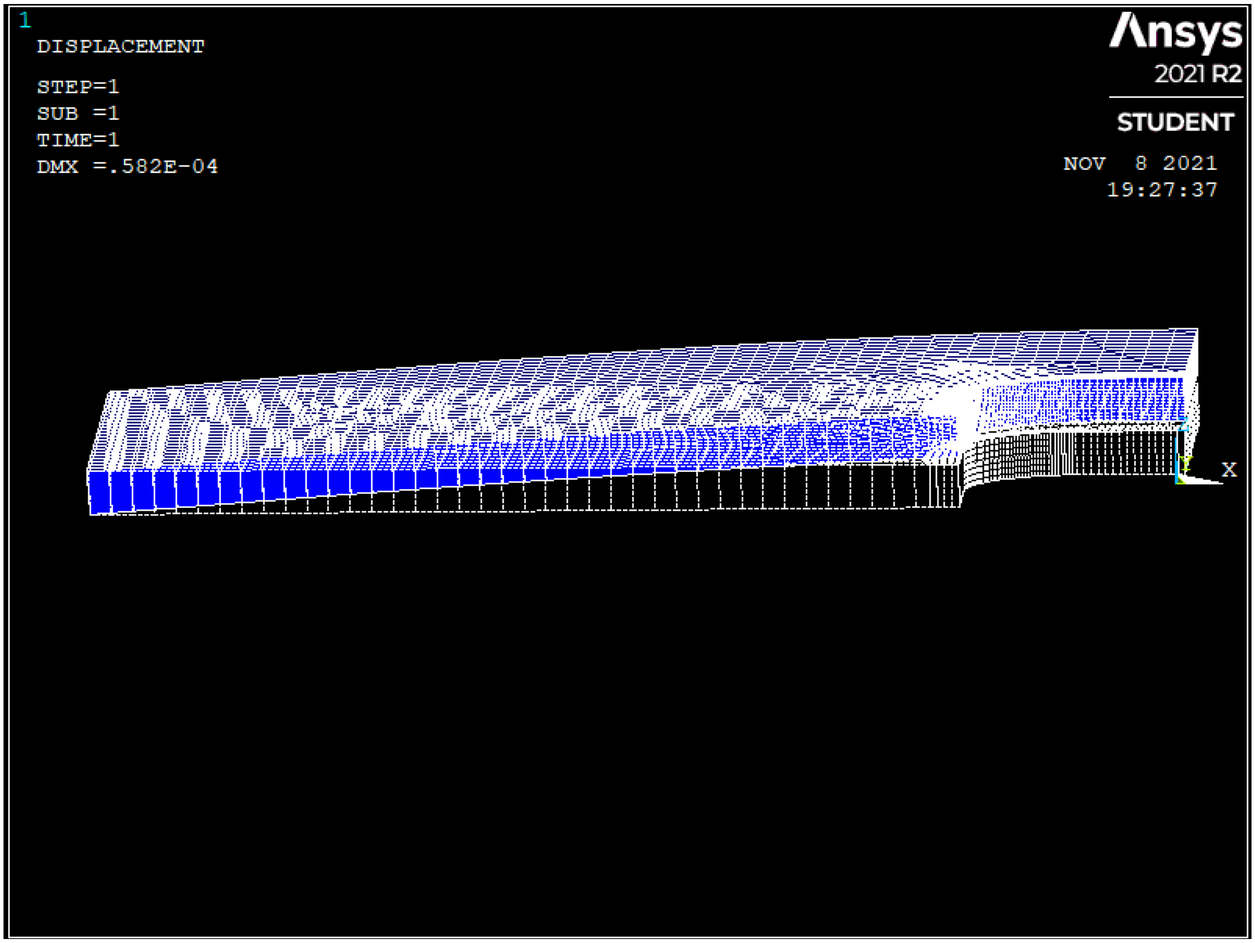


Figure 4: Plot of design deformed shape

Figure 4 is a plot of the deformed and undeformed shape. The undeformed shape is shown by the blue solid shape, and the deformed shape (represented by the white lines). It is evident that the shape bends downwards, as is expected in the analytical approach, because the bending stresses were larger than the tensile (reference lines 13, 23). The bending has a higher impact than the strain in the tensile direction, so the tensile displacement is not as obvious in the deformed shape. Refer to the page above for the boundary conditions and forces applied.

FEA Results

As a result of the applied transvers and axial pressures, the system experiences bending and tensile strains. The main stresses come from the bending and tensile stresses which are experienced primarily in the x-direction. This report will focus on the magnitude of maximum tensile and compressive normal stresses on the top, middle, and bottom surfaces of the plate. Since the plate experiences bending forces that result in tensile stresses on the top of the beam, and compressive on the bottom, we would expect that the stresses in the x-direction are different for the top, middle, and bottom of the plate. This will be expanded upon further later in the report.

Normal Stresses Analysis

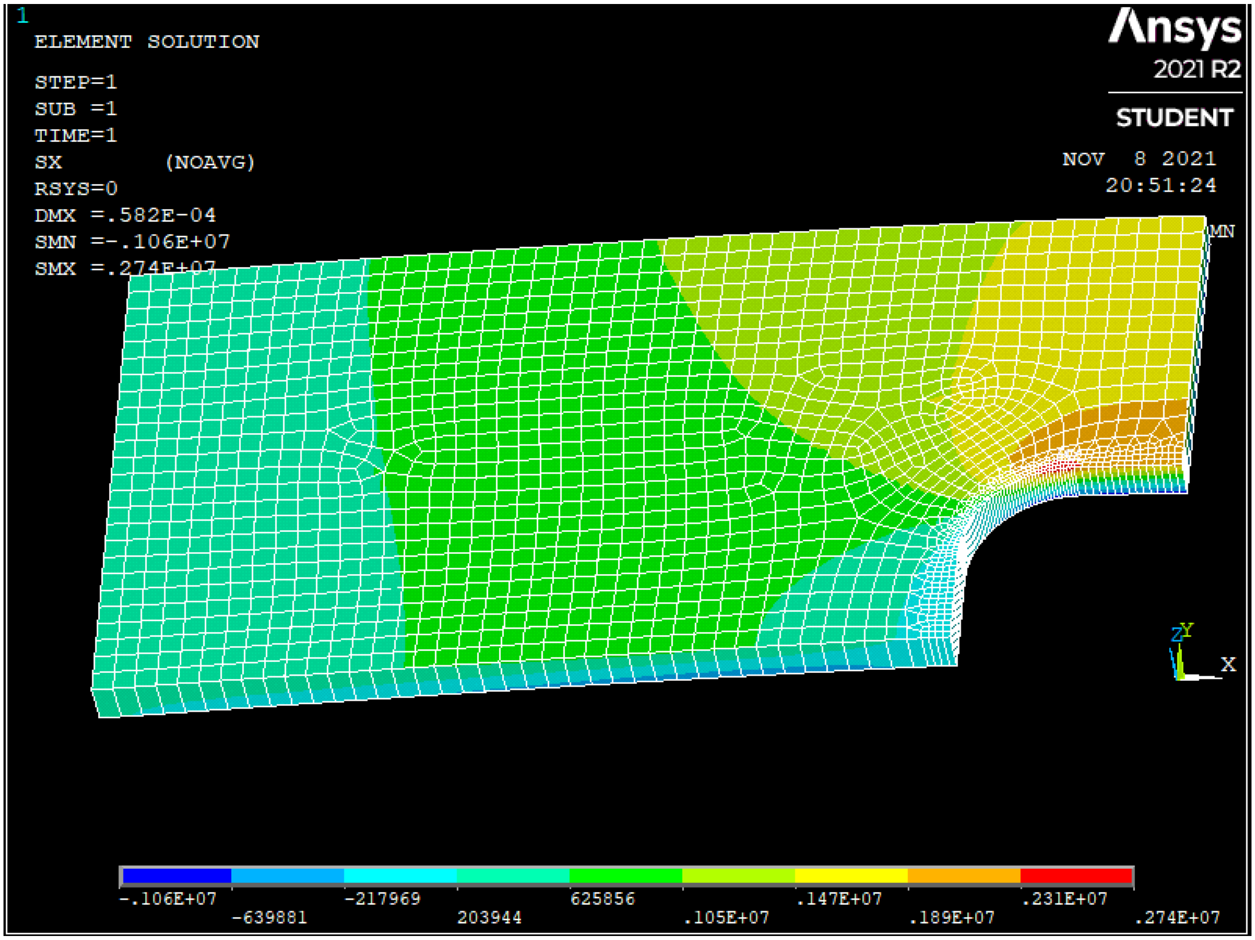


Figure 5: Contour plot of the stresses in the x-direction

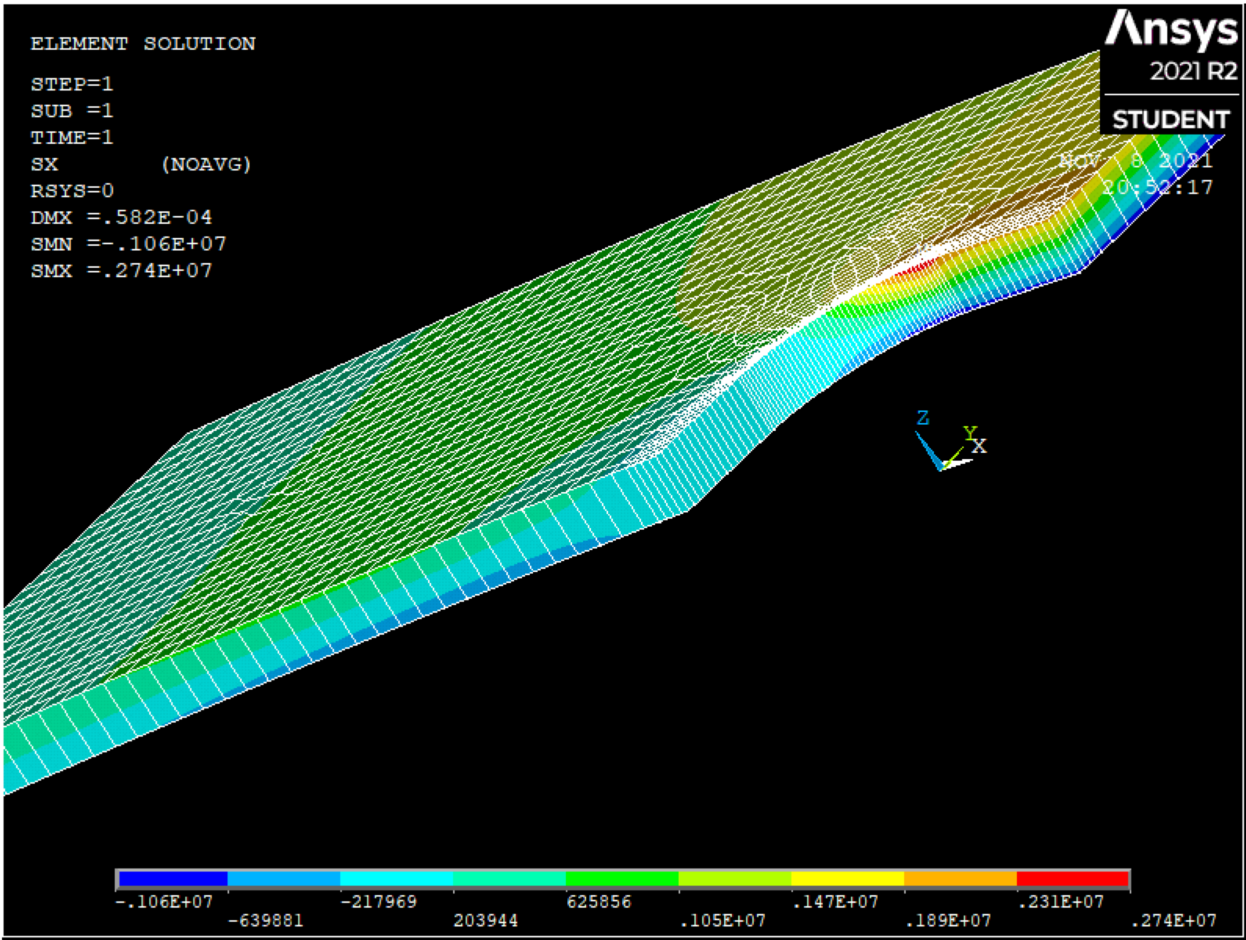


Figure 6: A zoom in on the stress concentration and around the thickness of the plate.

Evidently, there was a large stress concentration around the hole. The hole caused the stresses to be larger than the analytical would predict. In the analytical, we expected the stresses from the bending to result in a maximum stress of 0.94 MPa (see analytical line 23). Of course, the bending stresses would be tensile on the top and compressive on the bottom (see the last part of the Analytical). The axial forces, or the tensile force, therefore, would constructively add on the top, and subtract on the bottom. The expected maximum tensile stress was .74 MPa. However, our analytical solution did not account for the hole when solving for bending. A rough approximation would be to multiply the stress concentration factor, k, with the maximum bending stress found divided by two and add the maximum expected tensile load and see if this value lies is close to the solution in Ansys. In our analytical, a rough approximation for the maximum stress was 2.13E6, which is very close to the maximum stress calculated in Ansys, so there is confidence that the model is correct and our analytical was done right. Overall, the plate experiences major stress concentrations around the hole.

Top Plane Stress in the X-direction

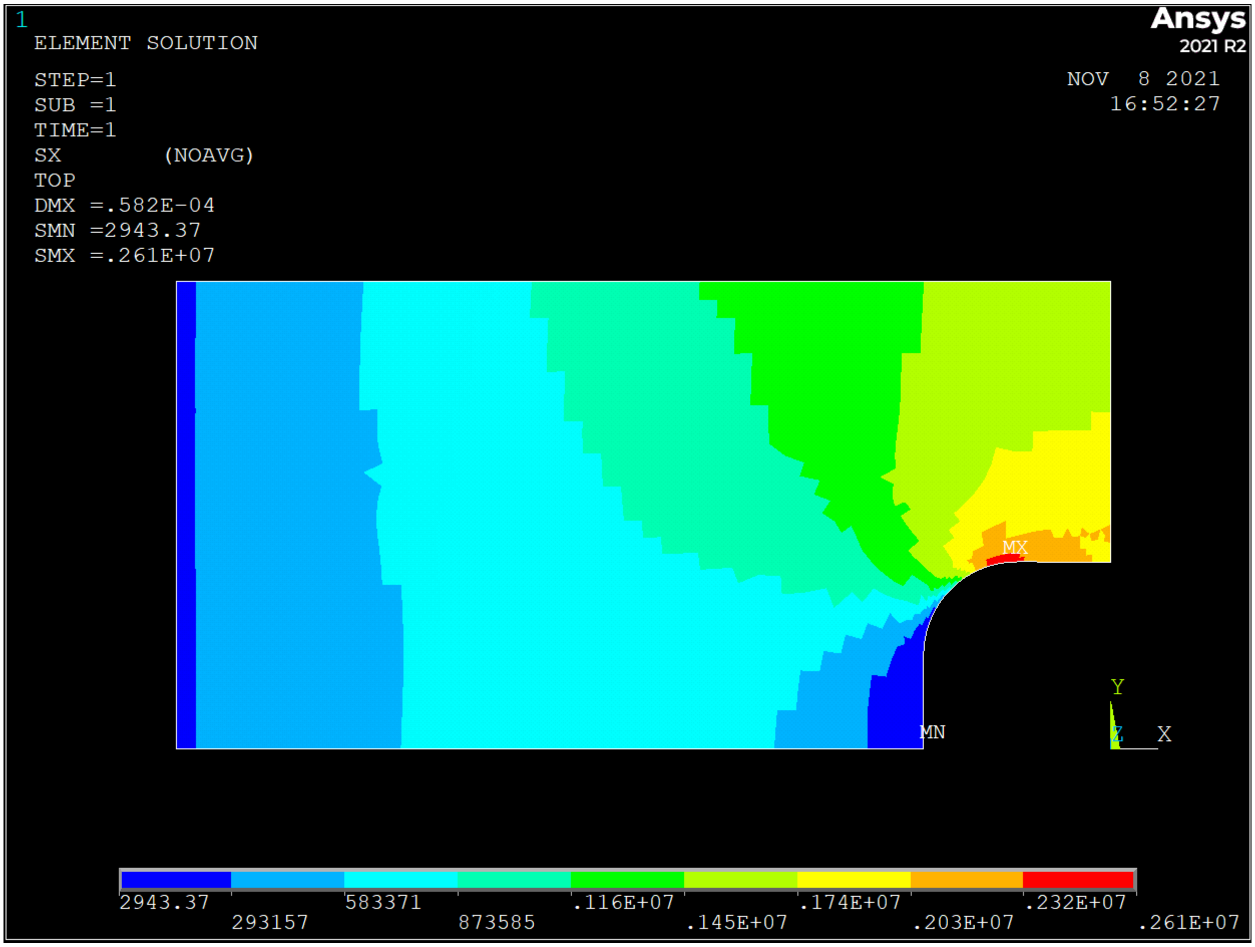


Figure 7: contour plot of the x component of stress in the top plane of the plate

Middle Plane Stress in the X-direction

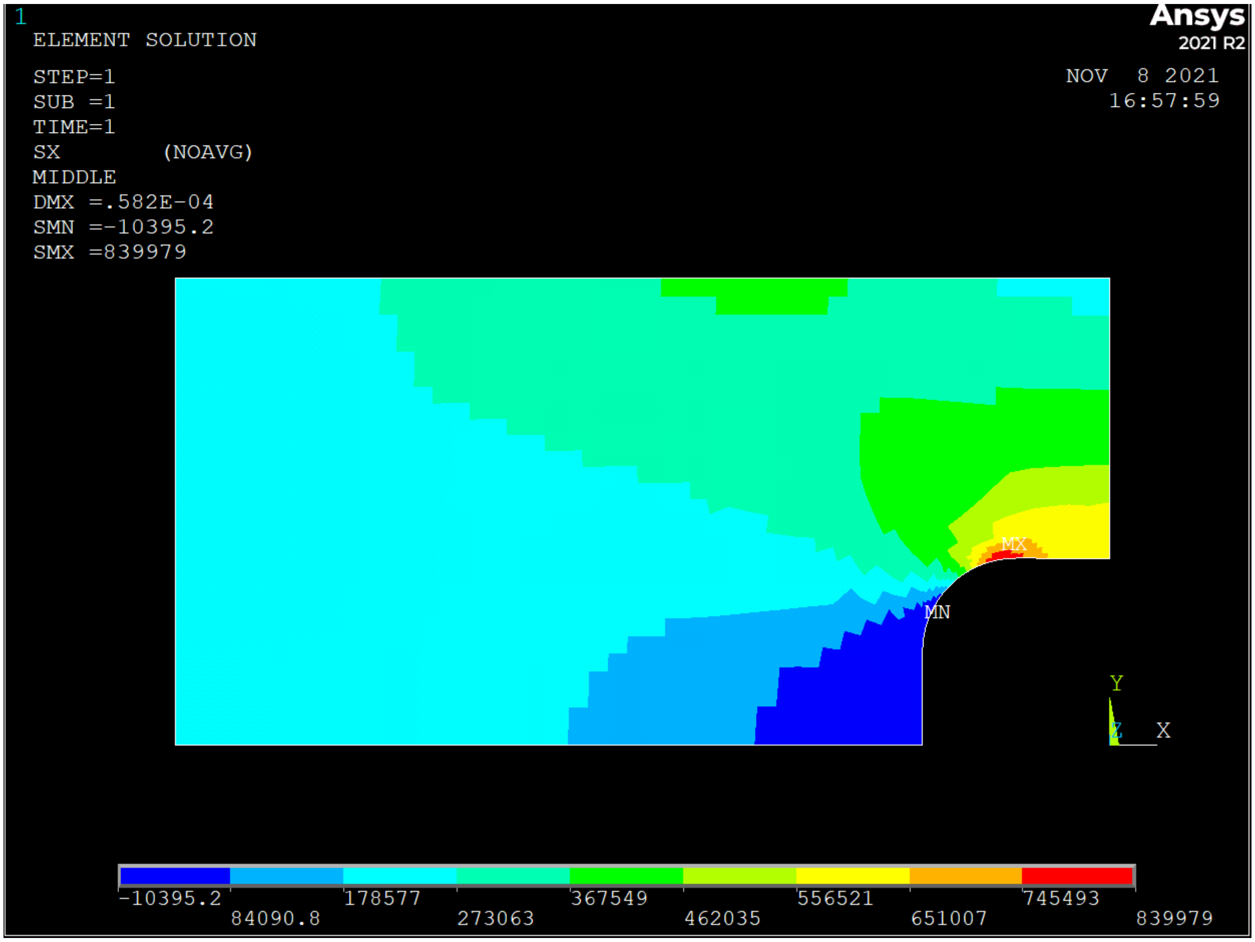


Figure 8: Contour plot of the x-component of stress for the middle plane of the plate

Bottom Plane Stress in the X-direction

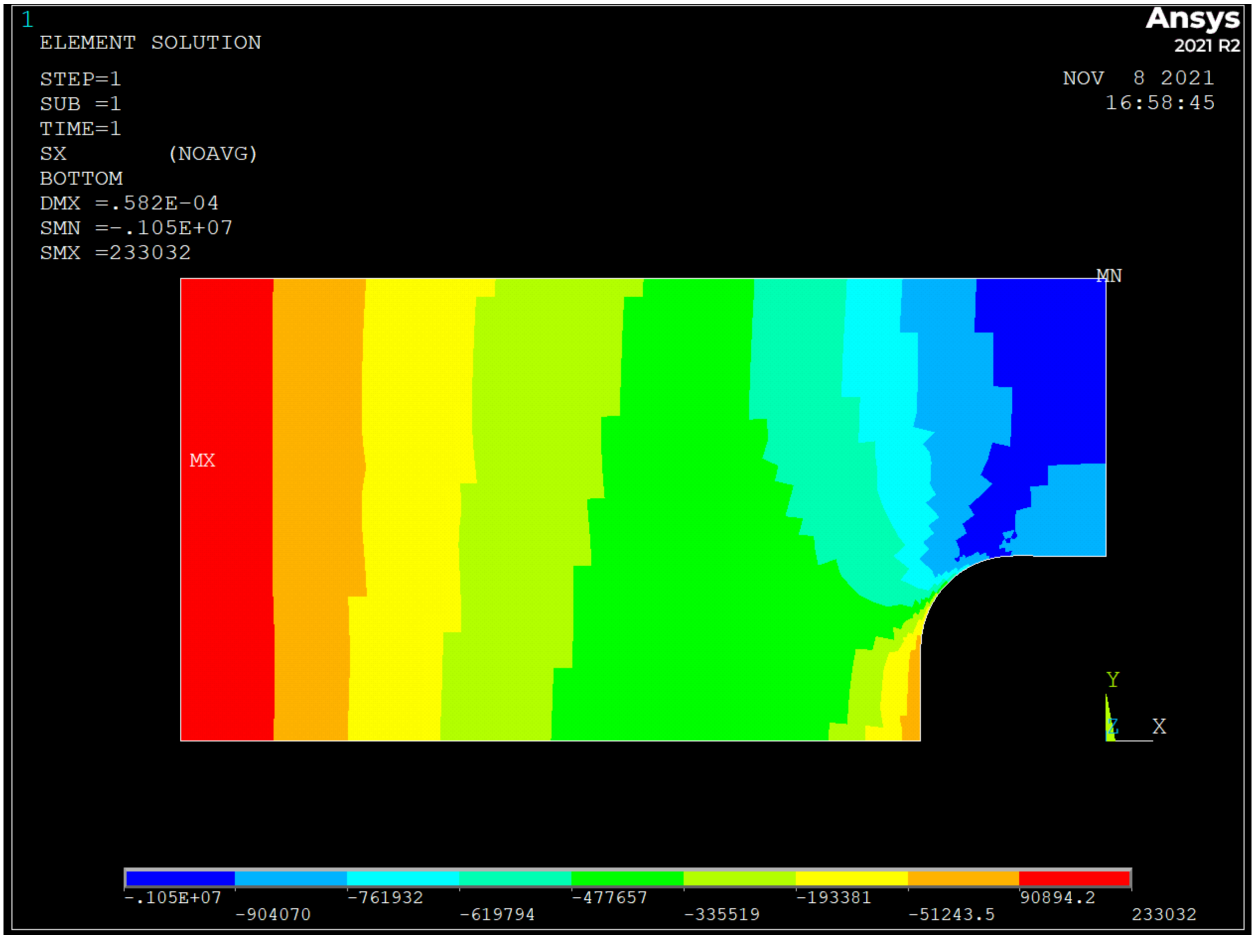


Figure 9: Contour plot of the x-component of stress in the bottom plane of the plate

Section	Element Number	Max Stress (Pa)	Element Number	Min Stress (Pa)
Top	1594	.26E7	1229	2943
Middle	1594	.839E6	1293	-10395
Bottom	1282	.23E6	1206	-0.1E7

Table 1: Maximum and Minimum stress in the x-direction and element numbers

Evidently from the contour plots in figures 7, 8, and 9, and the table of maximum and minimum stresses in the x-direction, the tensile stress in the x-direction decreases as the plane is analyzed top to bottom. That is, the bending stresses contribute to compression in the x-direction as you go down the plate. This is because the bending stress is tensile on the top, and compressive in the bottom (reference analytical). With a tensile axial load, we expect the bending stress to slowly override the tensile axial force as it goes down the plate and in areas of maximum bending (towards the stress concentration in the hole and the +x side of the plate), see Analytical diagram of moment for more. This is evident in the bottom plate, where element 1206 has a minimum stress with a large compressive stress value. The bottom plane evidently shows that the bending stresses are much larger in magnitude than the axial tensile stresses, since only the most leftward section of the plate is in tensile stress. This is the part that is closest to the tensile axial pressure (experiences a stress concentration factor as well) and experiences the least amount of bending stress. The top plane is as expected, experiencing purely tensile stresses because both the bending and axial forces contribute to the top plane having a tensile stress in the x-direction.

Conclusion

From this assignment, we further reinforce the ideas that bending stresses have tensile and compressive aspects to them. Additionally, this problem reinforced ideas of how to model problems with symmetry. A new topic of stress concentrations related to holes was introduced, and because of this the number of elements is very important because course elements led to inaccurate results that stemmed the entire body. We also worked with superposition of results (although they were approximations) in the analytical.

Reference for Table

```
MINIMUM VALUES
ELEM       1229
VALUE    2943.4

MAXIMUM VALUES
ELEM       1594
VALUE   0.26149E+007
```

Figure 10: maximum and minimum values of x-direction stress on the top plane

```
MINIMUM VALUES
ELEM       1293
VALUE   -10395.

MAXIMUM VALUES
ELEM       1594
VALUE   0.83998E+006
```

Figure 11: maximum and minimum values of x-direction stress on the middle plane

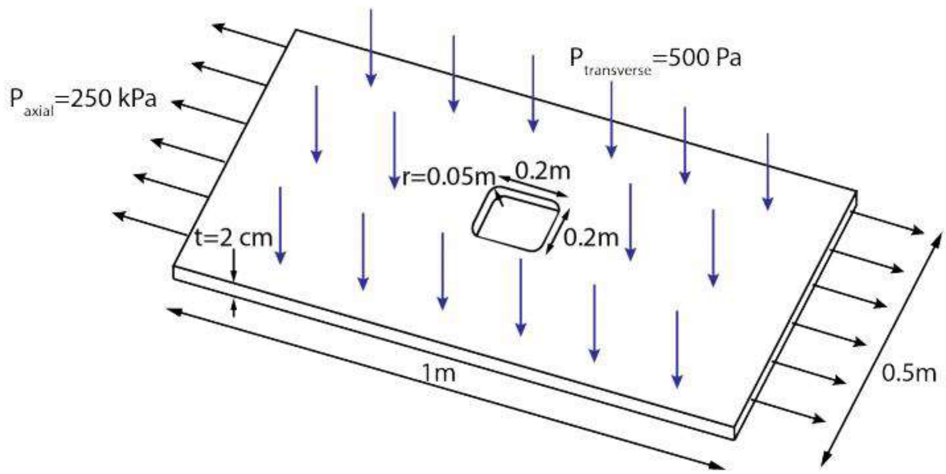
```
MINIMUM VALUES
ELEM       1206
VALUE  -0.10462E+007

MAXIMUM VALUES
ELEM       1282
VALUE   0.23303E+006
```

Figure 12: maximum and minimum values of x-direction stress on the bottom plane

Analytical Analysis

restart



The plate is simply supported on the right and left sides. For our hand calculations/analysis we need to estimate the stress from tensile pressure using the following criterias:

Type of form irregularity or stress raiser	Stress condition and manner of loading	Factor of stress concentration k for various dimensions
2. Rectangular hole with round corners in an infinite plate	12a. Elastic stress, axial tension	$\sigma_{max} = k\sigma_1$ and $k = K_1 + K_2\left(\frac{b}{a}\right) + K_3\left(\frac{b}{a}\right)^2 + K_4\left(\frac{b}{a}\right)^3$ where for $0.2 \le r/b \le 1.0$ and $0.3 \le b/a \le 1.0$ $K_1 = 14.815 - 15.774\sqrt{r/b} + 8.149r/b$ $K_2 = -11.201 - 9.750\sqrt{r/b} + 9.600r/b$ $K_3 = 0.202 + 38.622\sqrt{r/b} - 27.374r/b$ $K_4 = 3.232 - 23.002\sqrt{r/b} + 15.482r/b$

There is a stress tension of 250 kPa, which is the stress in the plate. 2b = 0.2m, 2a = 0.2m. So b = a for our hole. The rounded corners have r = 0.05 m. We can plug in information below using SI units.

$r := 0.05$

$$r := 0.05 \tag{1}$$

$b := 0.1$

$$b := 0.1 \tag{2}$$

$a := 0.1$

$$a := 0.1 \tag{3}$$

$k := K\_1 + K\_2 \cdot \left(\frac{b}{a}\right) + K\_3 \cdot \left(\frac{b}{a}\right)^2 + K\_4 \cdot \left(\frac{b}{a}\right)^3$

$$k := K\_1 + 1.000000000\, K\_2 + 1.000000000\, K\_3 + 1.000000000\, K\_4 \tag{4}$$

Check that 0.2 <= r/b <=1 and 0.3 <= b/a <= 1

$\frac{r}{b}$

$$0.5000000000 \tag{5}$$

$\frac{b}{a}$

$$1.000000000 \quad \textbf{(6)}$$

So, this is a valid model to use.

$RB := \frac{r}{b}$

$$RB := 0.5000000000 \quad \textbf{(7)}$$

$K\_1 := 14.815 - 15.774 \cdot \mathrm{sqrt}(RB) + 8.149 \cdot RB$

$$K\_1 := 7.735597630 \quad \textbf{(8)}$$

$K\_2 := -11.201 - 9.750\ \mathrm{sqrt}(RB) + 9.600 \cdot RB$

$$K\_2 := -13.29529112 \quad \textbf{(9)}$$

$K\_3 := .202 + 38.622 \cdot \mathrm{sqrt}(RB) - 27.374 \cdot RB$

$$K\_3 := 13.82487810 \quad \textbf{(10)}$$

$K\_4 := 3.232 - 23.002 \cdot \mathrm{sqrt}(RB) + 15.482 \cdot RB$

$$K\_4 := -5.291870180 \quad \textbf{(11)}$$

k

$$2.973314430 \quad \textbf{(12)}$$

So, the stress factor is 4.6193 in the center.

$sigma\_1 := 250 \cdot 10^3$

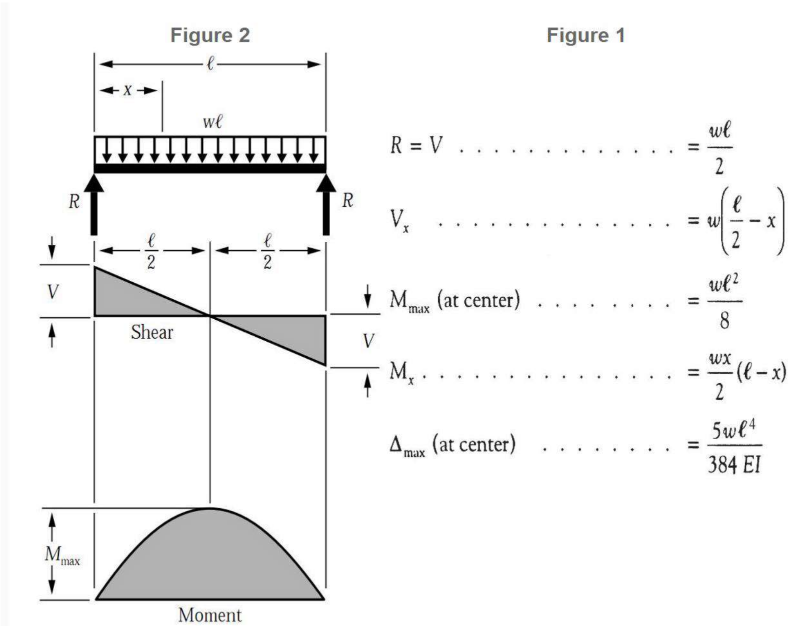
$$sigma\_1 := 250000 \quad \textbf{(13)}$$

$sigma\_max\_tensile := k \cdot sigma\_1$

$$sigma\_max\_tensile := 743328.6075 \quad \textbf{(14)}$$

The maximum stress is expected to be 1.15*10^6 Pa, or 1.15 MPa for the hole due to its tensile loading stress.

The second part of the analysis that we need to do is the bending stress. We can assume that the plate is simply supported for a beam with distributed load and ignore the hole. Referencing Roark's 9th for accuracy, I also obtained this visual for maximum deflection of a beam under



The maximum bending stress happens at the top and bottom of the beam, such that:

$$\sigma_{b.max} = \frac{Mc}{I_c}$$

Where Ic is the moment around the instantaenous center. From the two above, we know that stress is maximized where moment is maximized, which is at the center. Therefore, we can use this information.

For 409 stainless steel: (from azom)
$E := 200E9$

$$E := 2.00 \times 10^{11} \qquad \textbf{(15)}$$

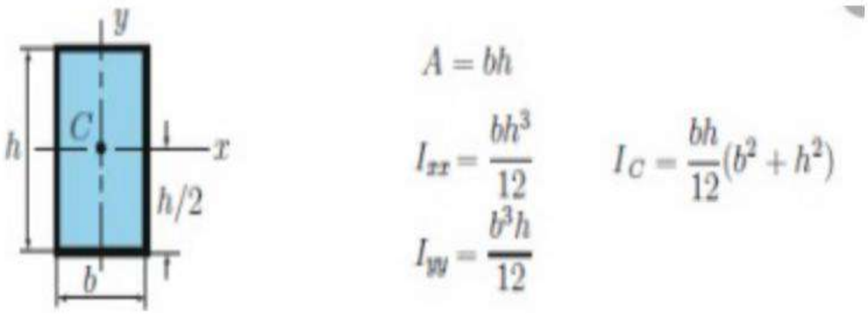
$base := 0.5$

$$base := 0.5 \qquad \textbf{(16)}$$

$H := \frac{2}{100}$

$$H := \frac{1}{50} \qquad \textbf{(17)}$$

The moment of inertia of a rectangular cross section around its instantaenous center (the x axis in the figure below) is used for the moment of inertia around its intantaneous center. Our cross section has a base of 0.5 m and a height of 0.02 m (2 cm). This can be found in Roark's 9th edition



$$I\_c := \frac{1}{12} \cdot base \cdot H^3$$

$$I\_c := 3.333333333 \times 10^{-7} \tag{18}$$

The pressure should be a line pressure, not an area pressure, so we need to multiply by the out-of-page distance (0.5 m)

$$w\_line := 500 \cdot 0.5$$

$$w\_line := 250.0 \tag{19}$$

$$l\_line := 1$$

$$l\_line := 1 \tag{20}$$

$$M\_max := \frac{w\_line \cdot l\_line^2}{8}$$

$$M\_max := 31.25000000 \tag{21}$$

The max moment is 0.0000443 N*m.

$$c := \frac{H}{2}$$

$$c := \frac{1}{100} \tag{22}$$

$$sigma\_max\_bend := \frac{M\_max \cdot c}{I\_c}$$

$$sigma\_max\_bend := 937500.0001 \tag{23}$$

The maximum bending stress is 937500 Pa. Both the tensile and the bending stresses are very close.

$$\frac{sigma\_max\_bend - sigma\_max\_tensile}{sigma\_max\_bend} \cdot 100$$

$$20.71161521 \tag{24}$$

The maximum bending stress is 20.7 % larger than the maximum tensile stress.

We would expect these stresses to add on the top. Therefore, the expected stress on the top would be

$$max\_tensile := sigma\_max\_bend + sigma\_max\_tensile$$

$$max\_tensile := 1.680828608 \times 10^{6} \tag{25}$$

We could expect this to be higher though, since our bending was calculated without the hole. An approximate solution would be to multiply by k.

$$\frac{1}{2}\, k \cdot sigma\_max\_bend + sigma\_max\_tensile$$

$$2.137069746 \times 10^{6} \tag{26}$$