

一、① 谓词： $P(x, y)$: x 有 y ;

$Q(x, y)$: y 是 x 的后继实数; $C(x, y)$: y 比 x 小

$$(\forall x)(\exists y)(Q(x, y) \wedge C(x, y) \wedge P(x, y))$$

② $P(x, y)$: x 大于 y ; $Q(x)$: x 是自然数

$$\neg(\exists x)(C(x) \wedge (\forall y)(Q(y) \Rightarrow P(x, y)))$$

③ $P(x, y)$: x 是 y 的子集合

a : 空集

$$(\forall x)P(a, x)$$

二、① $(\exists x)(\forall y)P(x, f(y))$

$$\Leftrightarrow (\exists x)(P(x, f(a)) \wedge P(x, f(b)))$$

$$\Leftrightarrow (P(a, f(a)) \vee P(b, f(a)))$$

$$\wedge (P(a, f(b)) \vee P(b, f(b)))$$

$$\Leftrightarrow (P(1, 2) \vee P(2, 2)) \wedge (P(1, 1) \vee P(2, 1))$$

$$\Leftrightarrow (1 \vee 1) \wedge (1 \vee 0)$$

$$\Leftrightarrow 1$$

所以谓词公式①的真值为真。

② $(\exists x)P(x, x) \wedge (\forall x)P(x, f(x))$

$$\Leftrightarrow (\exists x)P(x, x) \wedge (\forall y)P(y, f(y))$$

$$\Leftrightarrow (P(a, a) \vee P(b, b)) \wedge (P(a, f(a)) \wedge P(b, f(b)))$$

$$\Leftrightarrow (P(1, 1) \vee P(2, 2)) \wedge P(1, 2) \wedge P(2, 1)$$

$$\Leftrightarrow (1 \vee 1) \wedge 1 \wedge 0$$

$$\Leftrightarrow 0$$

所以谓词公式②的真值为假

三、① $(\forall x)(\forall y)P(x, y) \rightarrow (\exists x)A(x)$

$$\Leftrightarrow (\forall x)(P(x, a) \wedge P(x, b)) \rightarrow (A(a) \vee A(b))$$

$$\begin{aligned} &\Leftrightarrow \neg(P(a,a) \wedge P(b,a) \wedge P(a,b) \wedge P(b,b)) \\ &\quad \vee A(a) \vee A(b) \end{aligned}$$

$$\begin{aligned} &\Leftrightarrow \neg P(a,a) \vee \neg P(b,a) \vee \neg P(a,b) \vee \neg P(b,b) \\ &\quad \vee A(a) \vee A(b) . \end{aligned}$$

$$\textcircled{2} (\forall x)(A(x) \rightarrow B(x))$$

$$\Leftrightarrow (\forall x)(\neg A(x) \vee B(x))$$

$$\Leftrightarrow (\forall x)\neg A(x) \vee (\forall x)B(x)$$

$$\Leftrightarrow (\forall x)\neg A(x) \vee (\forall y)B(y)$$

$$\Leftrightarrow (\neg A(a) \wedge \neg A(b)) \vee (B(a) \wedge B(b))$$

$$\text{四. ① 证: } (\exists x)A(x) \wedge \neg(\forall x)\neg B(x)$$

$$\Leftrightarrow (\exists x)A(x) \wedge \neg(\forall y)\neg B(y)$$

$$\Leftrightarrow (\exists x)A(x) \wedge (\exists y)B(y)$$

$$\Leftrightarrow (\exists x)(\exists y)(A(x) \wedge B(y))$$

$$\textcircled{2} (\forall x)A(x) \rightarrow (\exists x)B(x)$$

$$\Leftrightarrow \neg(\forall x)A(x) \vee (\exists x)B(x)$$

$$\Leftrightarrow (\exists x)\neg A(x) \vee (\exists x)B(x)$$

$$\Leftrightarrow (\exists x)(\neg A(x) \vee B(x))$$

$$\text{五. ① } (\exists x)(\forall y)(A(x) \rightarrow B(y)) \vee (\exists y)(\exists x)P(x,y)$$

$$\Leftrightarrow (\exists x)(\forall y)(\neg A(x) \vee B(y)) \vee (\exists v)(\exists u)P(u,v)$$

$$\Leftrightarrow (\exists x)(\forall y)(\exists u)(\exists v)(\neg A(x) \vee B(y) \vee P(u,v))$$

六. 命题符号化为:

$M(x)$: x 是人

$P(x)$: x 怕困难

$Q(x)$: x 获得成功

$R(x)$: x 失败过

前提: $(\exists x)(M(x) \wedge (P(x) \rightarrow \neg Q(x)))$,
 $(\forall x)(M(x) \rightarrow P(x) \vee R(x))$,
 $(\exists x)(M(x) \wedge \neg R(x))$.

结论: $(\exists x)(M(x) \wedge \neg P(x))$

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|---|-----------|
| ① $(\exists x)(M(x) \wedge \neg R(x))$ | 前提引入 |
| ② $M(c) \wedge \neg R(c)$ | ① ES 规则 |
| ③ $M(c)$ | ② 化简式 |
| ④ $\neg R(c)$ | ② 化简式 |
| ⑤ $(\forall x)(M(x) \rightarrow P(x) \vee R(x))$ | 前提引入 |
| ⑥ $M(c) \rightarrow P(c) \vee R(c)$ | ⑤ US 规则 |
| ⑦ $P(c) \vee R(c)$ | ③ ⑥ 假言推理 |
| ⑧ $P(c)$ | ④ ⑦ 析取三段论 |
| ⑨ $(\exists x)(M(x) \wedge (P(x) \rightarrow \neg Q(x)))$ | 前提引入 |
| ⑩ $M(c) \wedge (P(c) \rightarrow \neg Q(c))$ | ⑨ ES 规则 |
| ⑪ $P(c) \rightarrow \neg Q(c)$ | ⑩ 简化式 |
| ⑫ $\neg Q(c)$ | ⑧ ⑪ 假言推理 |
| ⑬ $M(c) \wedge \neg Q(c)$ | ③ ⑫ 合取引入 |
| ⑭ $(\exists x)(M(x) \wedge \neg Q(x))$ | ⑬ EG 规则 |