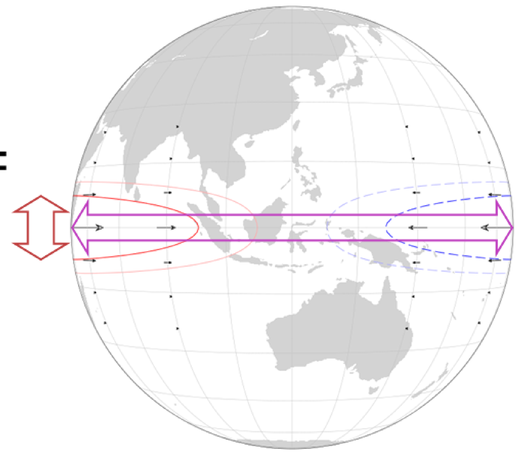
# The time scale of the MJO arises from its length scale.

The geometry of the pressure anomalies (contoured) in this schematic is all we need to derive an intraseasonal time scale.

$$\frac{\text{Period}}{\text{day}} = \frac{a^2}{\left(\sqrt{2}Y\right)^2} = \left(\frac{\text{Planetary diameter}}{\text{e-folding width of the pressure anomalies}}\right)^2$$



Given the observed ratio of the planetary radius to the meridional *e*-folding scale from 6 to 8, this scale analysis yields an oscillation period ranging from 36 to 64 days, like the observed MJO period.

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## Scale analysis for the Madden–Julian oscillation (MJO)


Hing Ong

Independent Scholar (hing5ong5@gmail.com)


### Motivation

Understanding the time scale of Madden–Julian oscillation (MJO) is the key to predict global weather patterns beyond two weeks. However, the time scale have remained mysterious as MJO theories vastly disagree on what assumptions are needed to explain the time scale. However, the geometry of the MJO is well known.

### Scale Analysis

Derive the intrinsic oscillation period for the long, narrow, and shallow pressure anomalies of the MJO from the mass continuity and momentum equations using three geometric assumptions:

**Assumption 1: Meridional narrowness**

This assumption allows us to use the following nondimensional equations.

$$\frac{YU}{XV}\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} + \frac{YW}{HV}\frac{\partial \hat{w}}{\partial \hat{z}} = 0 \qquad (1)$$

$$\frac{FaV}{2\Omega YU}\frac{\mathrm{D}\hat{v}}{\mathrm{D}\hat{t}} + \hat{y}\hat{u} + \frac{a\Phi}{2\Omega Y^2 U}\frac{\partial \hat{\varphi}}{\partial \hat{y}} = 0 \qquad (2)$$

$$\frac{FaU}{2\Omega YV}\frac{\mathrm{D}\hat{u}}{\mathrm{D}\hat{t}} - \hat{y}\hat{v} + \frac{aW}{YV}\hat{w} + \frac{a\Phi}{2\Omega YVX}\frac{\partial \hat{\varphi}}{\partial \hat{x}} = 0 \qquad (3)$$

**Assumption 2: Zonal length of planetary scale**

This assumption allows us to approximate $X = a$.

**Assumption 3: Vertical shallowness**

This assumption allows us to omit $aW/YV$.

**Flow regime selection: Zonal divergence scales on the leading order** given the easterly and westerly winds of the MJO alternating around the equator.

No further assumption is needed to deduce the highlighted terms are on the order 1. One can then derive the $V$ scale from Eq. (1), $\Phi$ scale from Eq. (2), and from Eq. (3):

$$\frac{\Omega}{F} = \frac{a^2}{2Y^2}$$

### Notation

Known variables:

$\Omega$: planetary rotation rate (2π/day); $a$: planetary radius; $Y$: meridional width scale

Unknown target variable:

$F$: oscillation angular frequency (2π/period)

Other unknown variables:

$X$: zonal length scale; $U$: zonal velocity scale; $V$: meridional velocity scale; $\Phi$; geopotential scale; $H$: depth scale; $W$: vertical velocity scale