= Put in Matlab.

We have:

$x \quad y \quad z \quad \psi \quad N$

$$\psi = q_2 + q_3 + q_4$$

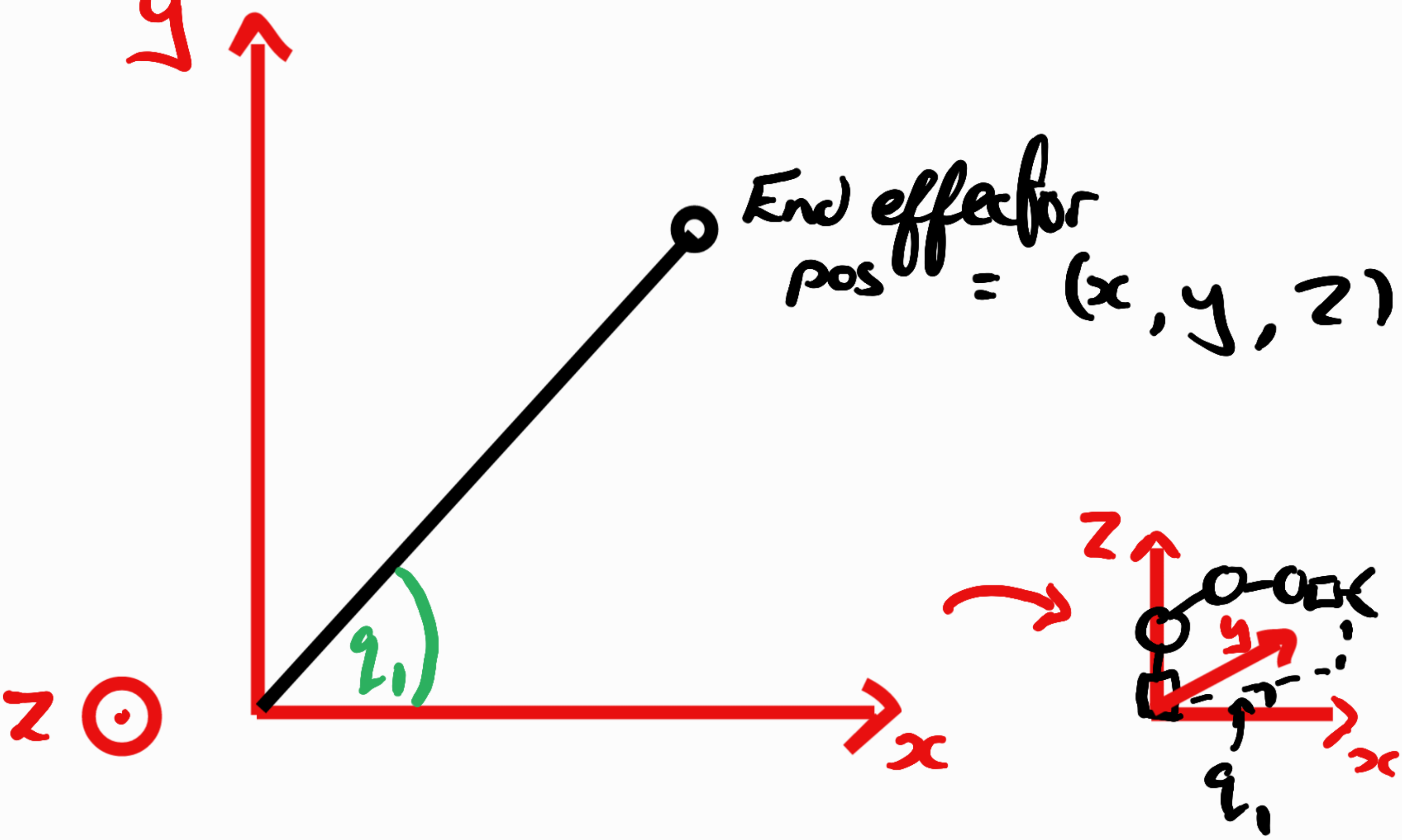
$$N = q_5$$

find:

$$q_1, q_2, q_3, q_4, q_5$$

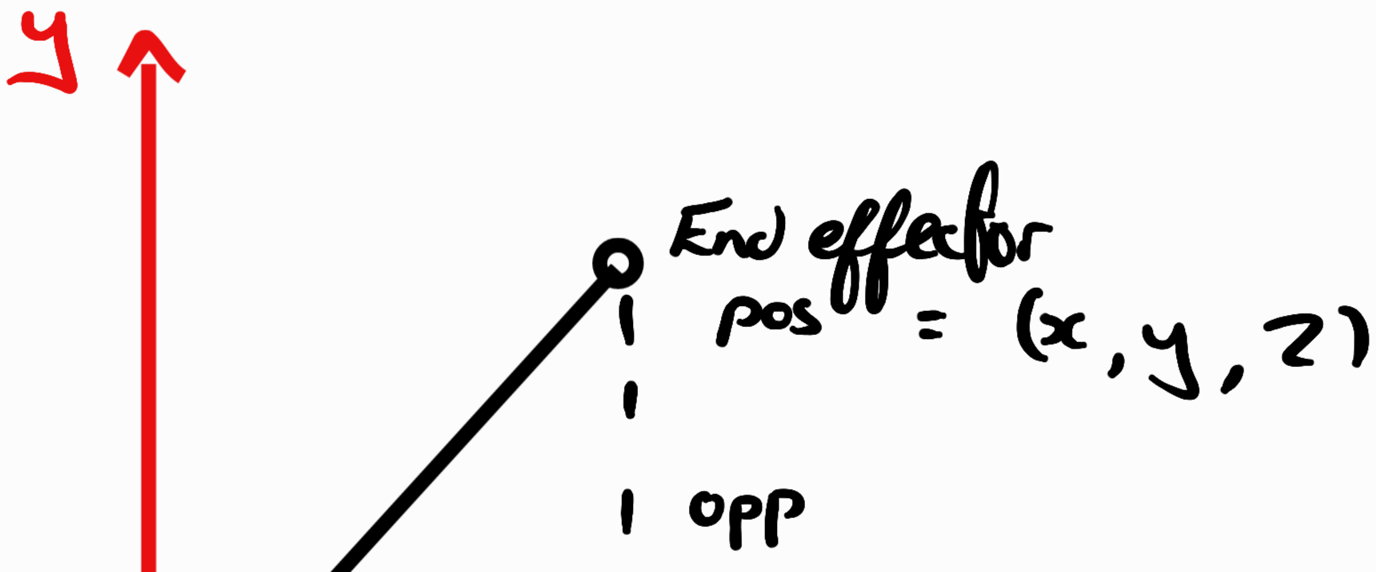
1. $\boxed{q_5 = N}$ known.

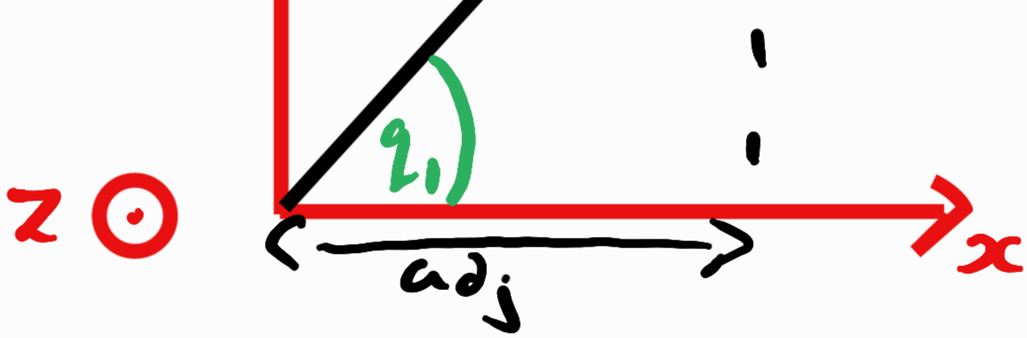
2. Find q_1 using a top down view:



In the x,y plane, end effector posit^n is:

(x, y) which is known.





clearly:

$$q_1 = \arctan\left(\frac{y}{x}\right)$$ known

or in matlab:

$$q_1 = \text{atan2}(y, x)$$

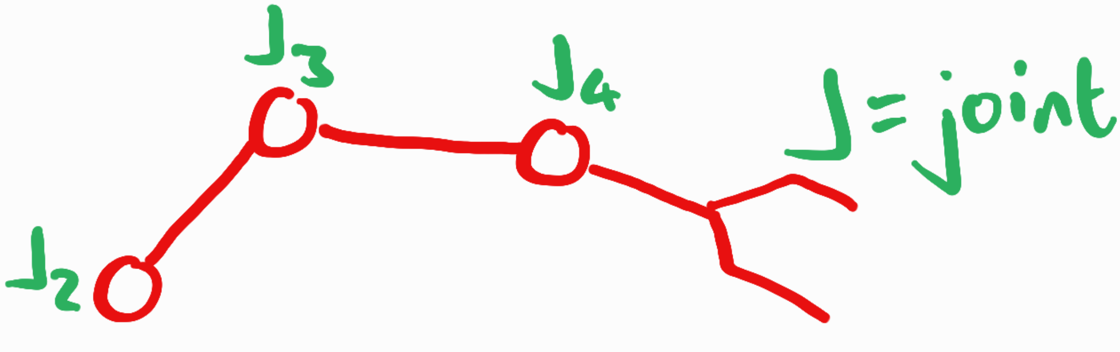
3.a Setup

Remaining are

$q_2, q_3\ q_4$

These comprise

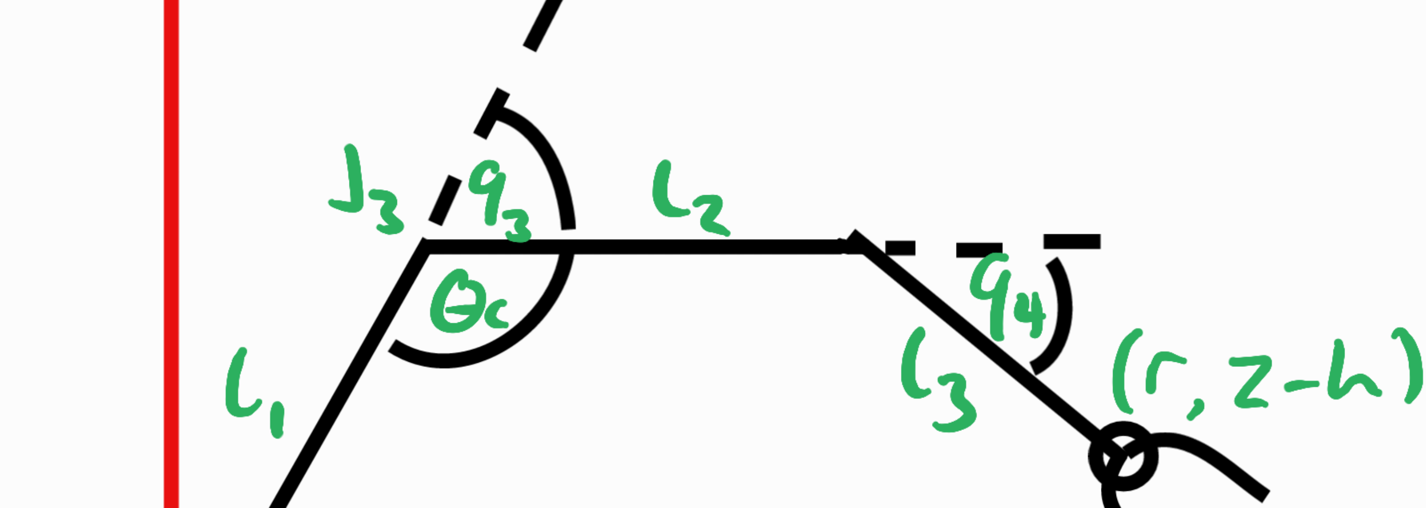
3 joints operating in the <u>same plane</u>:



We can show this in the z-r plane:

where $x = r\cos(q_1)$, $y = r\sin(q_1)$

$r^2 = x^2 + y^2$



Notes on the above diagram

- r axis is the vector of arm in the x-y plane, hence $r = \sqrt{x^2 + y^2}$

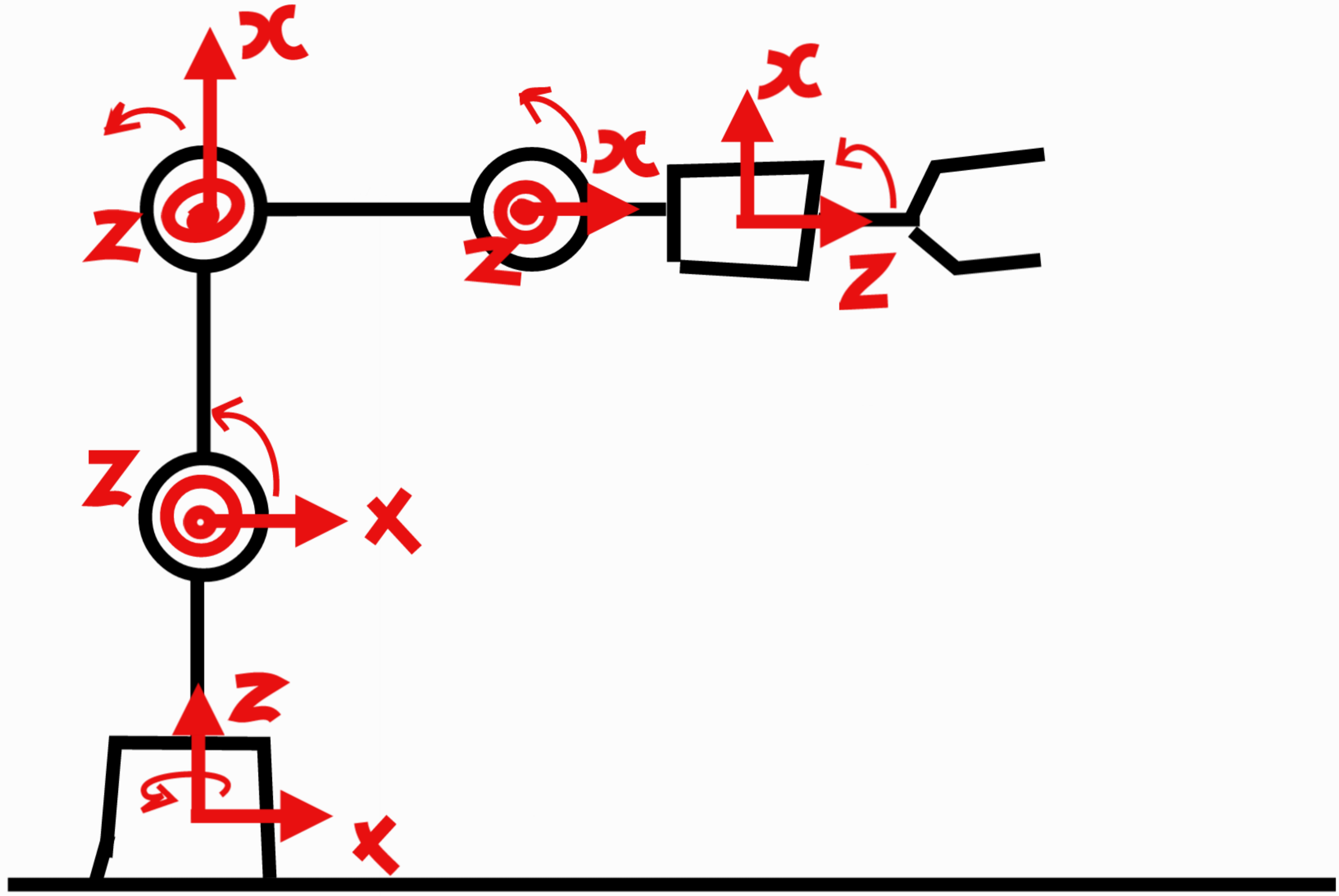
- end effector height (z) must include base height (not in diagram above, hence end effector at $z = 6$

[illegible]

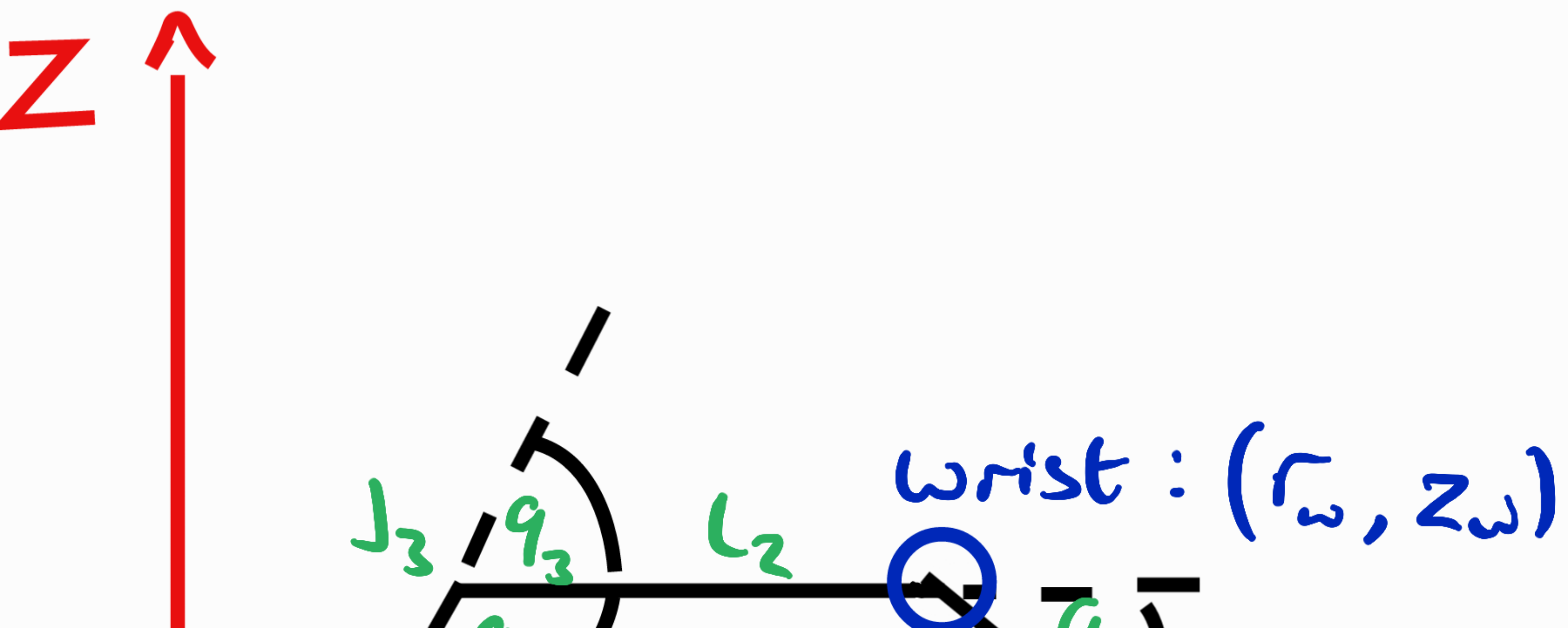
rather than z on diagram, obviously $h = d_1 =$ known

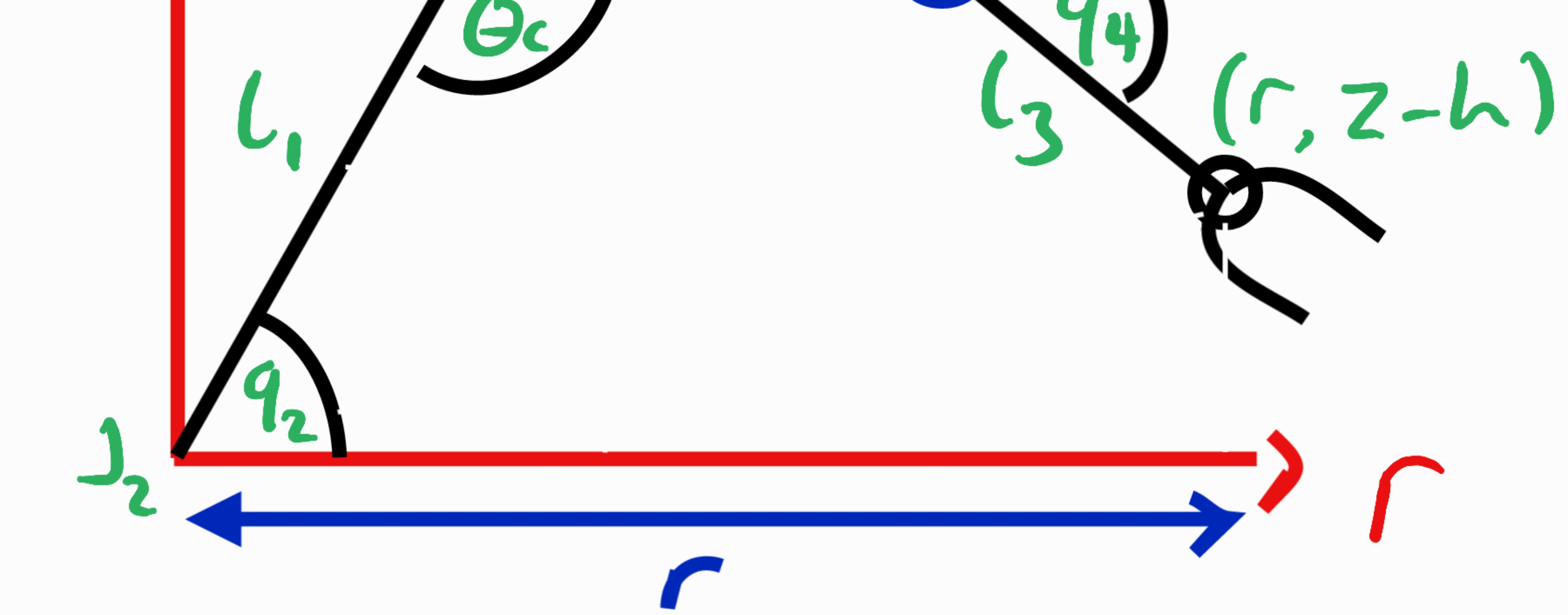
- q_5 affects only end effector orientat$^{\underline{n}}$, not posit$^{\underline{n}}$ $\therefore$ not included in diagram

- Angle zero points for $q_2\ q_3\ q_4$ are given by the DH diagram for it:



36. Finding q_2, q_3, q_4





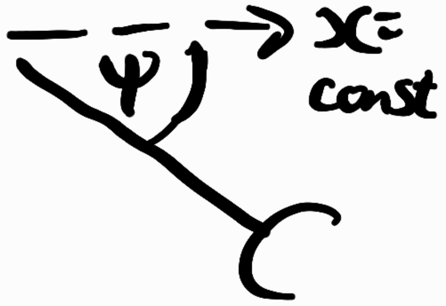
3b.i : Find wrist posit$^{\underline{N}}$.

We know value and eq$^{\underline{N}}$:

$$\Psi = q_1 + q_2 + q_3$$

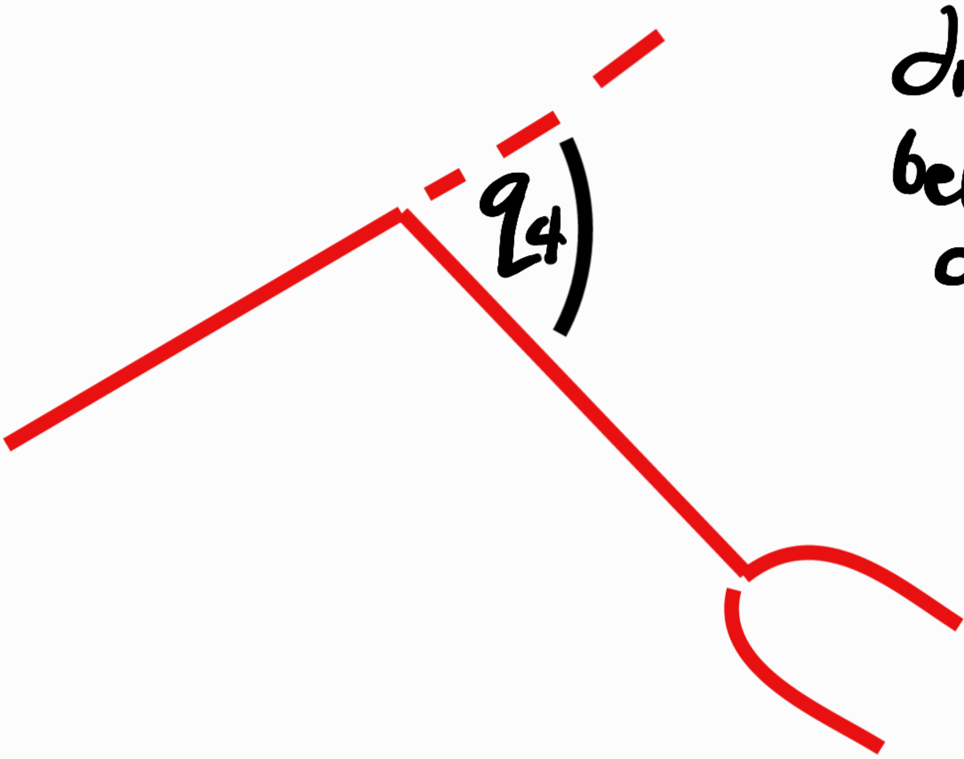
Ψ is final angle of end effector relative to the x axis

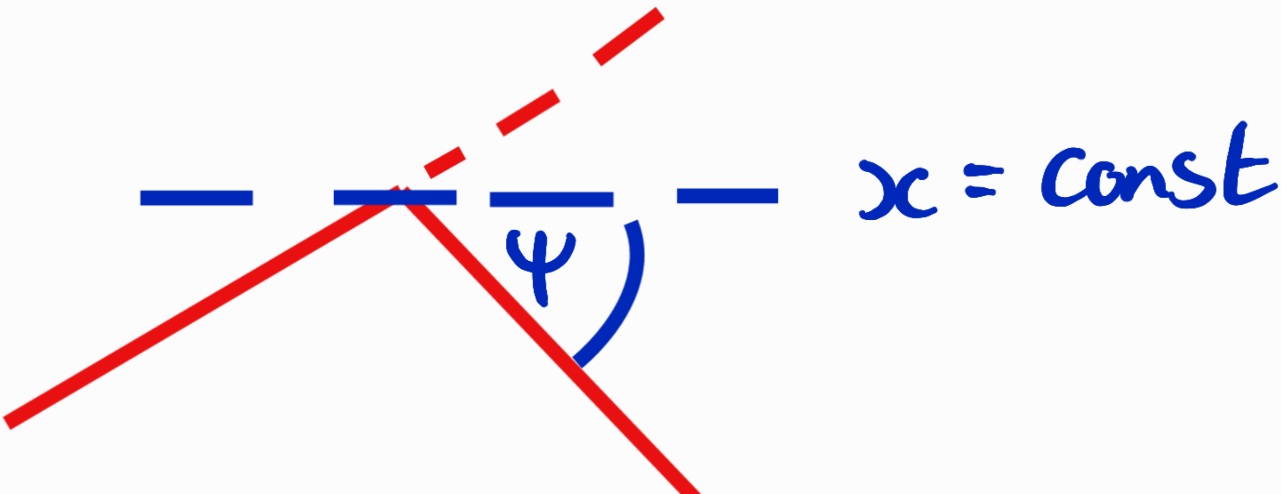
So above :



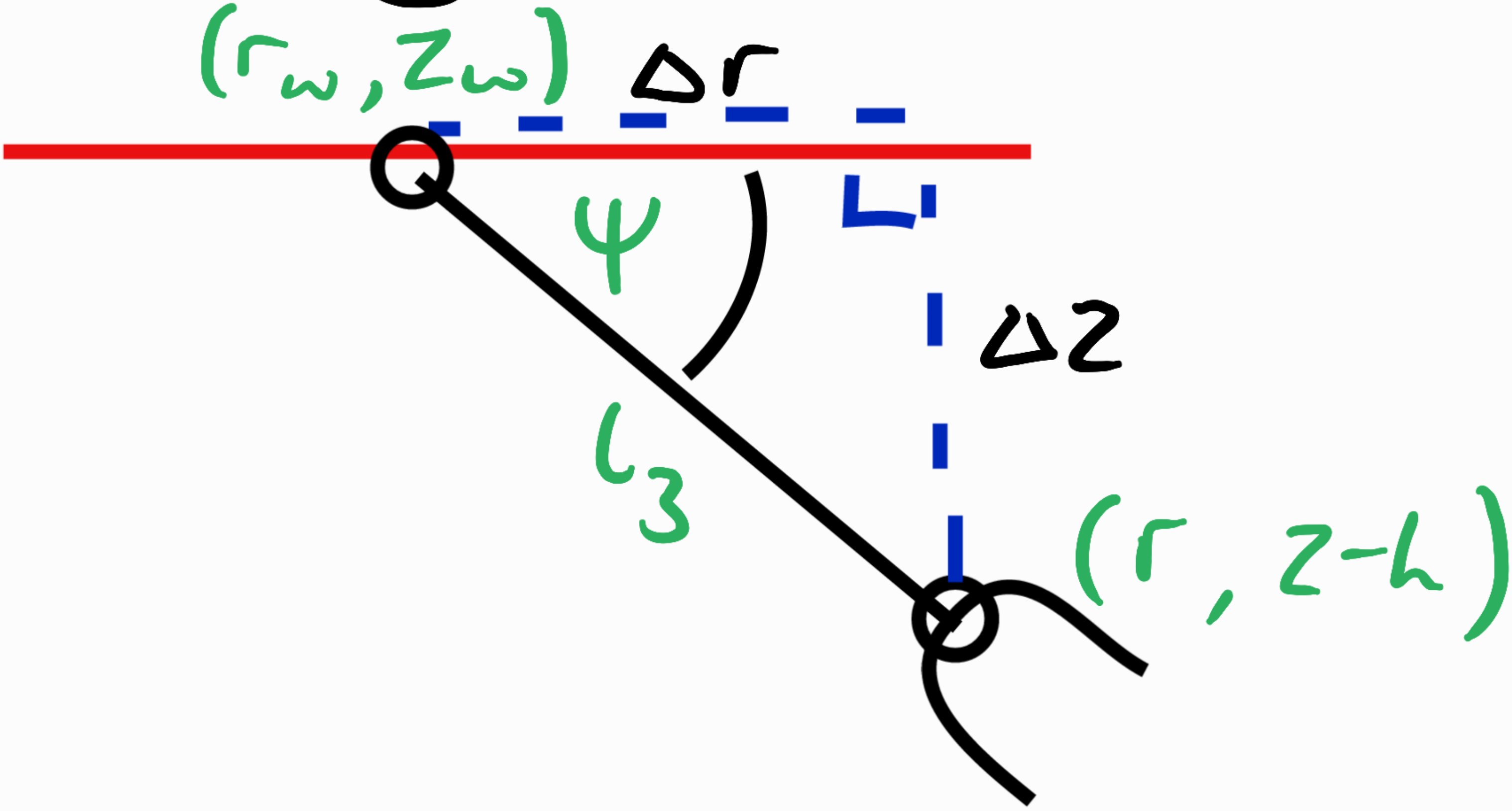
We know from wrist to end effector is length l_3

to show diff between ψ and q_4





Anyway :



We can see using vector subtraction,

$$\begin{pmatrix} r_w \\ z_w \end{pmatrix} = \begin{pmatrix} r - \Delta r \\ (z-h) - \Delta z \end{pmatrix}$$

And using trig:

$$\Delta r = l_3 \cos(\psi)$$

$$\Delta z = l_3 \sin(\psi)$$

$\therefore$ wrist posit$^{\sim}$:

$$\boxed{\begin{pmatrix} r_w \\ z_w \end{pmatrix} = \begin{pmatrix} r - l_3 \cos(\psi) \\ z - h - l_3 \sin(\psi) \end{pmatrix}}$$

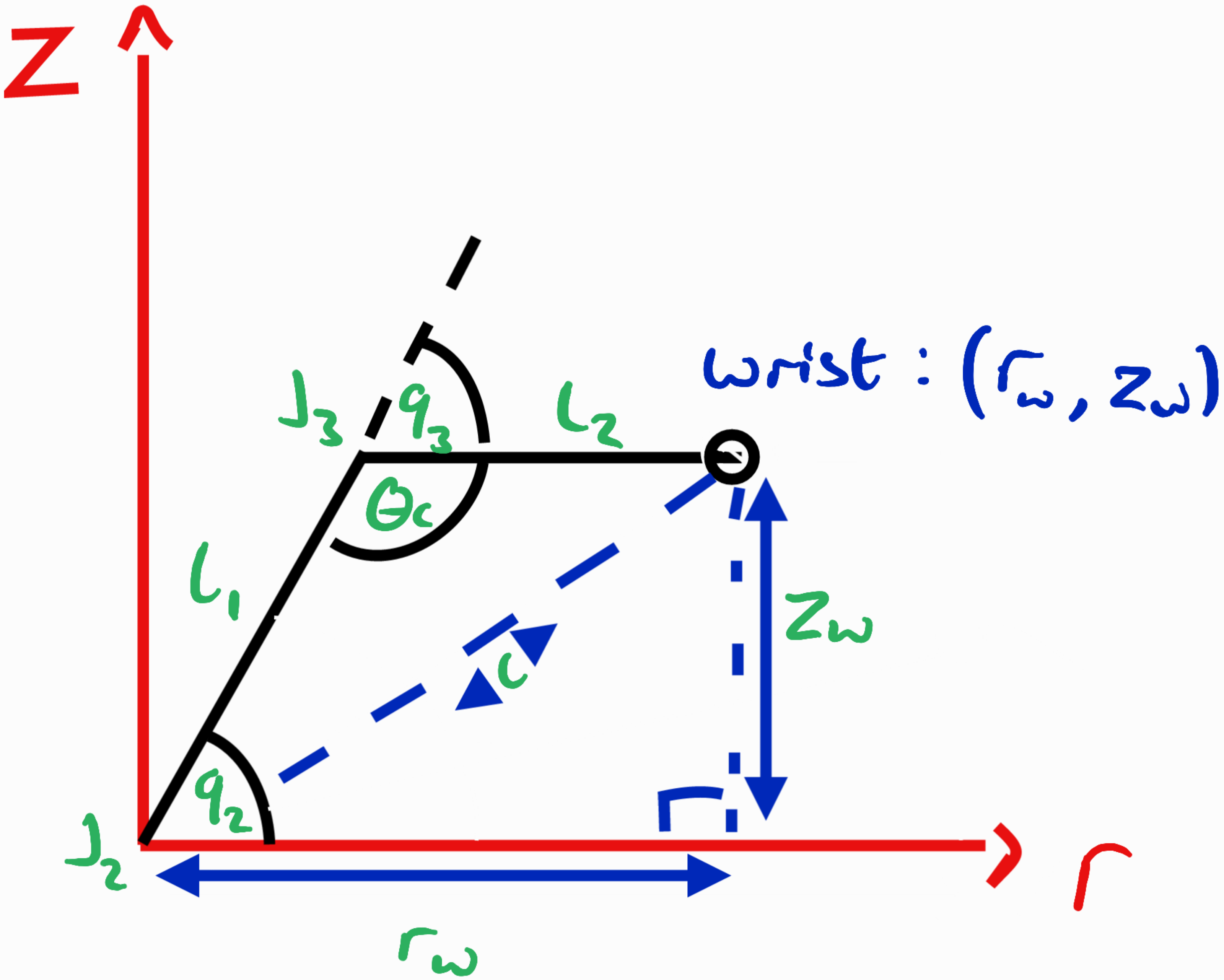
where $h = d1$

For Matlab

Remember, ψ is a known value.

3.6.ii : Find q_3

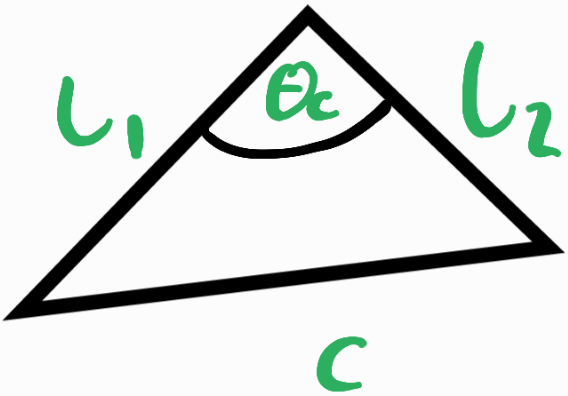
The graph can now reduce in scope:



Pythag :

$$c = \sqrt{r_w^2 + z_w^2} \quad (1)$$

And for



use law of cosines:

$$(2) \quad c = \sqrt{l_1^2 + l_2^2 - 2 l_1 l_2 \cos\theta_c}$$

(1) = (2), rearrange

for $\cos\theta_c$

(3) $\quad r_w^2 + z_w^2 - l_1^2 - l_2^2$

$$\cos(\theta_c) = -\frac{r_w^2 + z_w^2 - l_1^2 - l_2^2}{2 l_1 l_2}$$

We can also see:

$$q_3 = \pi - \theta_c$$

$$\downarrow \tan$$

$$\tan(q_3) = \tan(\pi - \theta_c)$$

$$= \tan(-\theta_c)$$

let $\cos\theta_c = D$

$$\theta_c = \arccos(D)$$

$$\therefore \tan(q_3) = \tan\left(-\arccos(D)\right)$$

trig identity:

$$\tan(\arccos(x)) = \frac{\sqrt{1-x^2}}{x}$$

$$q_3 = \arctan\left(-\frac{\sqrt{1-D^2}}{D}\right)$$

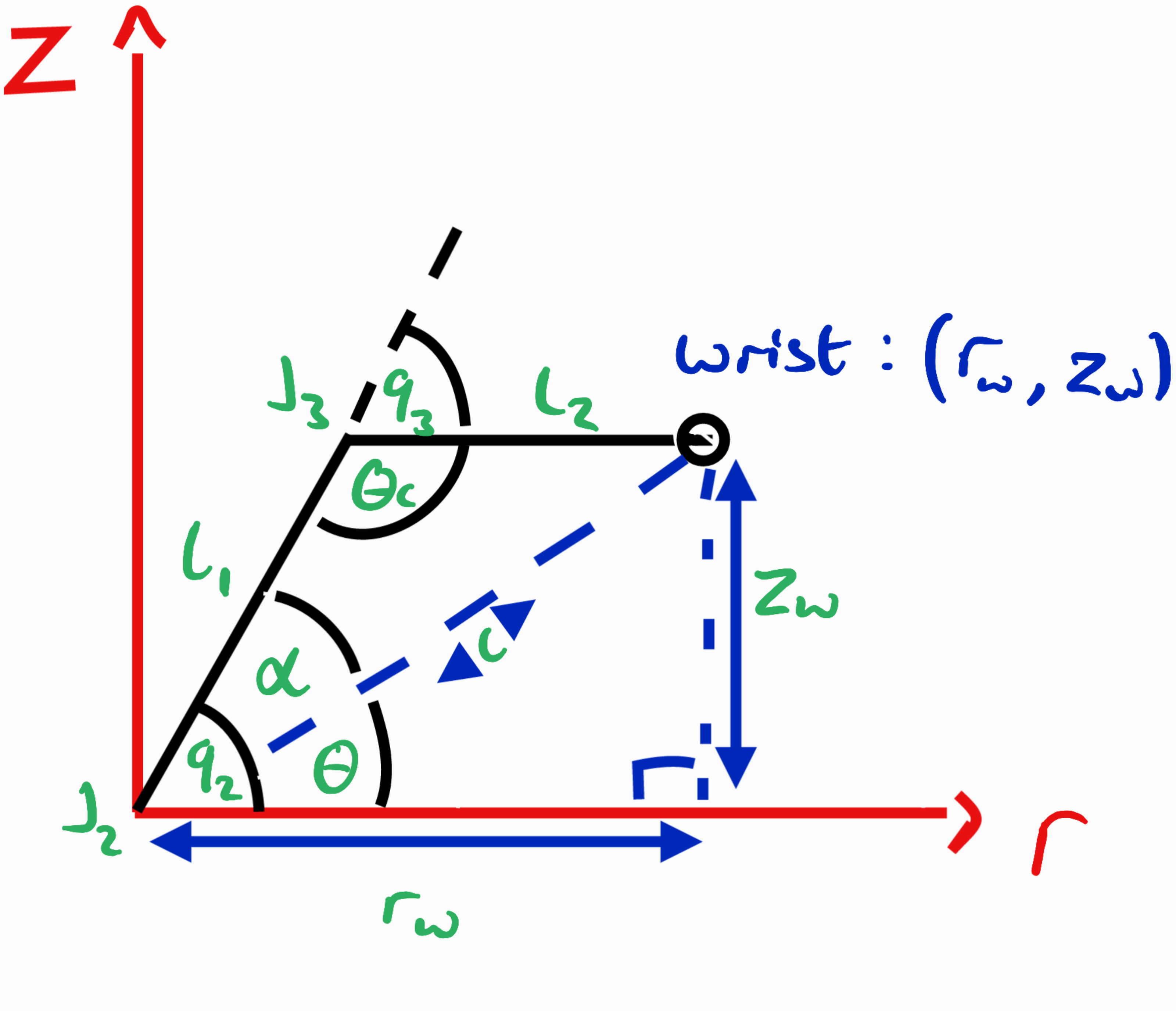
Or in matlab: two sol$^{\underline{n}}$s.

$$q_3 = \operatorname{atan2}\left(\pm\sqrt{1-D^2}, -D\right)$$

where:

$$D = -\frac{r_w^2 + z_w^2 - l_1^2 - l_2^2}{2 l_1 l_2}$$

3.6.iii : Find q_2



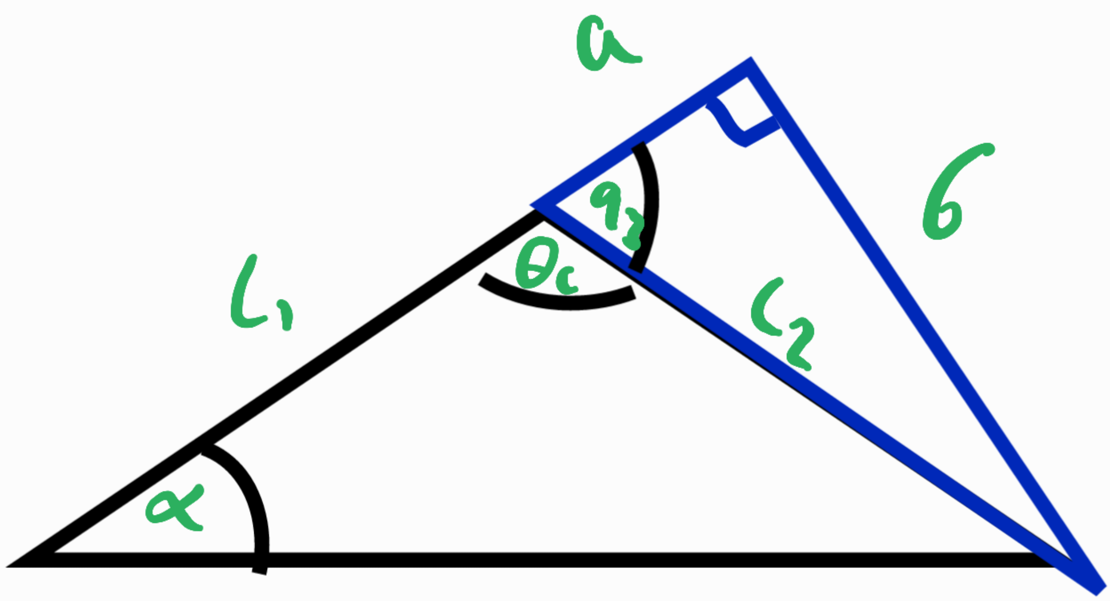
we can see :

$$q_2 = \alpha + \theta$$

clearly

$$\theta = \arctan\left(\frac{z_w}{r_w}\right) \quad (1)$$

to find α, use



clearly:

$$\alpha = \arctan\left(\frac{b}{l_1 + a}\right)$$

from blue triangle:

$$b = l_2 \sin(q_3)$$

$$a = l_2 \cos(q_3)$$

2

$$\alpha = \arctan\left(\frac{l_2 \sin(q_3)}{l_1 + l_2 \cos(q_3)}\right)$$

use 1 & 2

$$q_2 = \arctan\left(\frac{z_w}{r_w}\right) + \arctan\left(\frac{l_2 \sin(q_3)}{l_1 + l_2 \cos(q_3)}\right)$$

or in matlab:

$$\boxed{\begin{aligned} q_2 = \operatorname{atan2}(z_w, r_w) + \\ \operatorname{atan2}(l_2 \sin(q_3), l_1 + l_2 \cos(q_3)) \\ \hookrightarrow \text{2 values as 2 } q_3 \text{ values possible} \end{aligned}}$$

3.6.iii : find q_4

$$\boxed{\begin{aligned} q_4 = \psi - q_2 - q_3 \\ \hookrightarrow \text{2 values as 2 sets of } q_2, q_3 \end{aligned}}$$

4. Ensure only real

answers → eliminate imaginary answers by ensuring the imaginary component of each angle is zero:

$\boxed{\text{if } \ \mathrm{imag}(q_n) == 0:}$