Steps

1. Model as a combinat$^{\underline{n}}$ of serial robots.

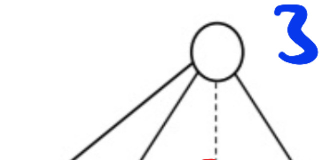
2. Each arm has different base & orientat$^{\underline{n}}$ but same EE posit$^{\underline{n}}$

3. For each arm:
   a. Perform vector chain
   b. find vector magnitude and altitude (rotat$^{\underline{n}}$) of vector.

Coursework robot:

$q_3 = a + \pi/6$

We can



$S_3 = a + 5\pi$

See, all angles we defined relative to +ve x axis

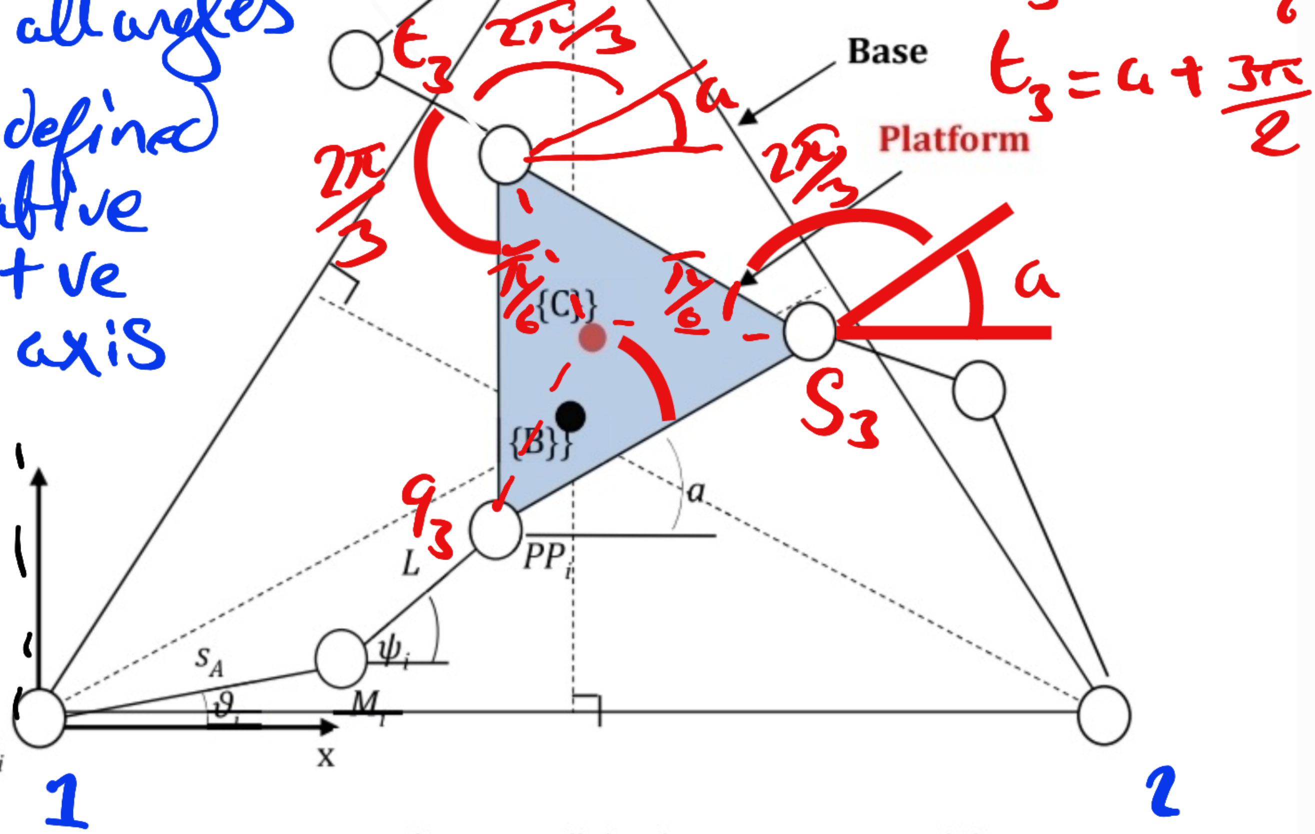
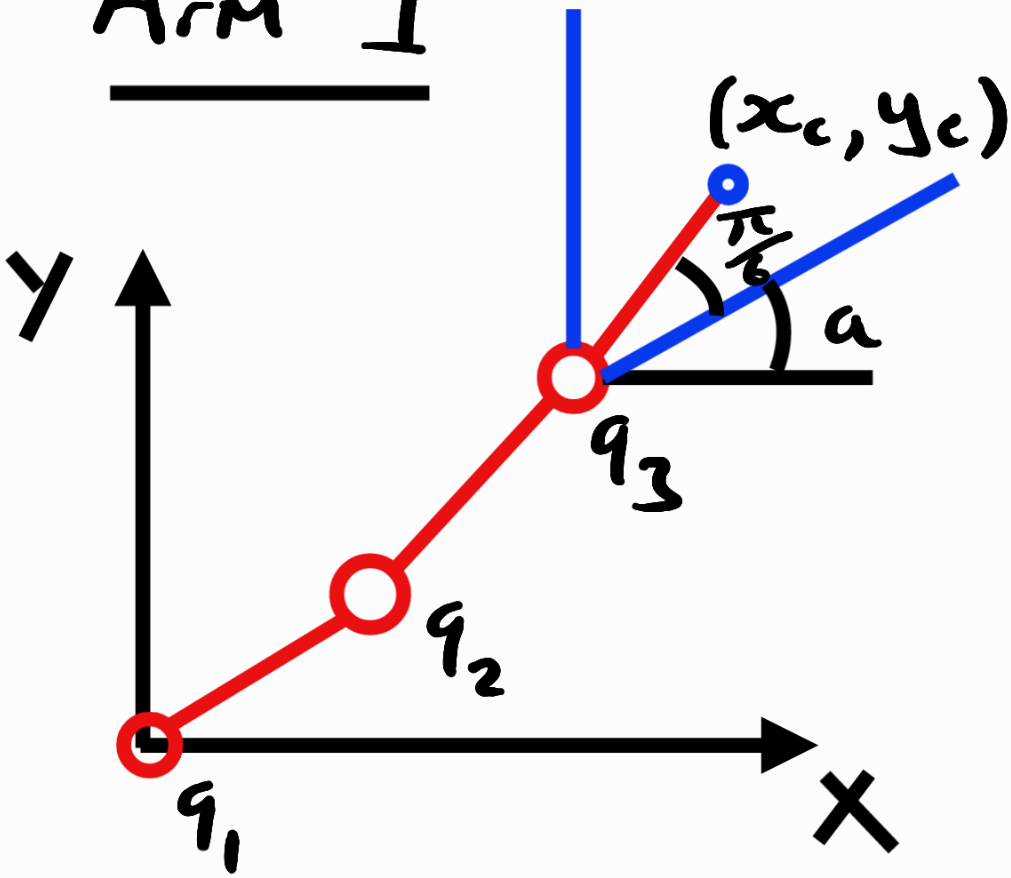


Figure 2: Planar Parallel Robot Kinematic Model

→ Arms we numbered.

Arm 1

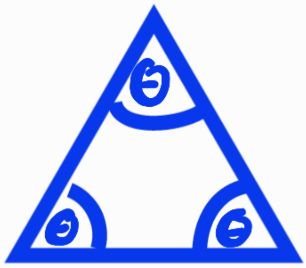


We know: $\begin{pmatrix} x_c \\ y_c \\ a \end{pmatrix}$

where:

- x_c, y_c = Centre of end effector (EE)
- a = angle between EE and zero point : x axis.
- EE is equilateral triangle

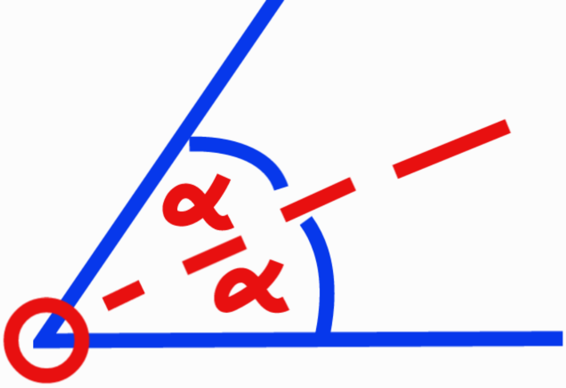
$\therefore$



$\theta = \pi/3$

So for line between joint and centre:

$\alpha = \pi/6$



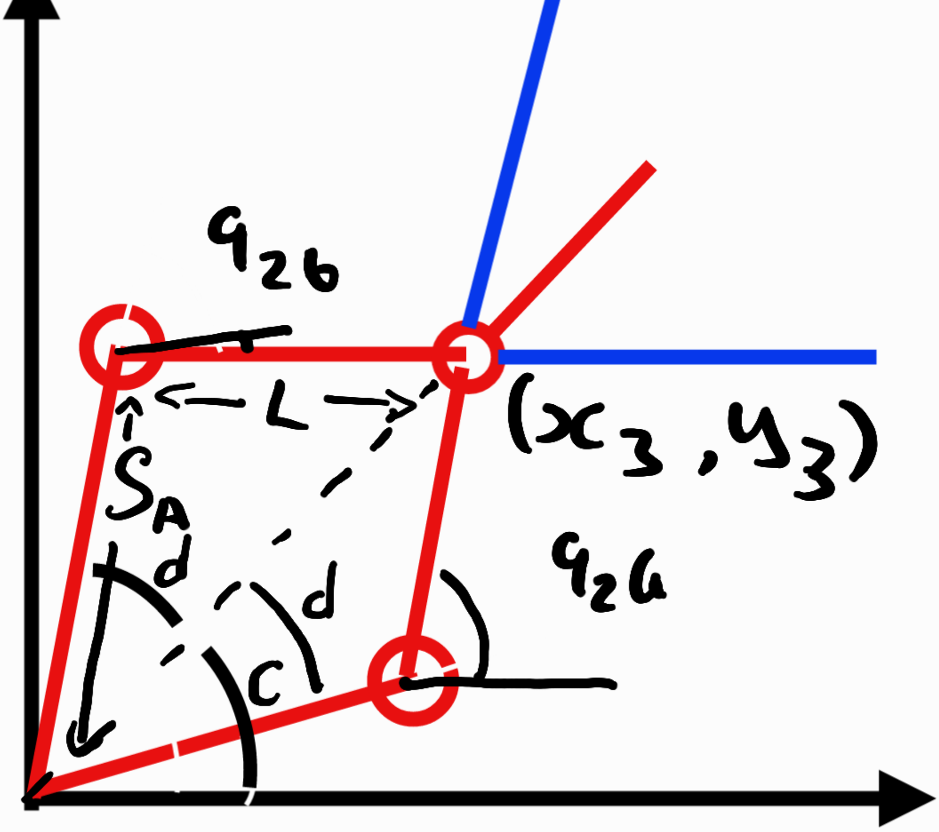
Clearly angle of final joint is:

$$\boxed{q_3 = a + \frac{\pi}{6}}$$

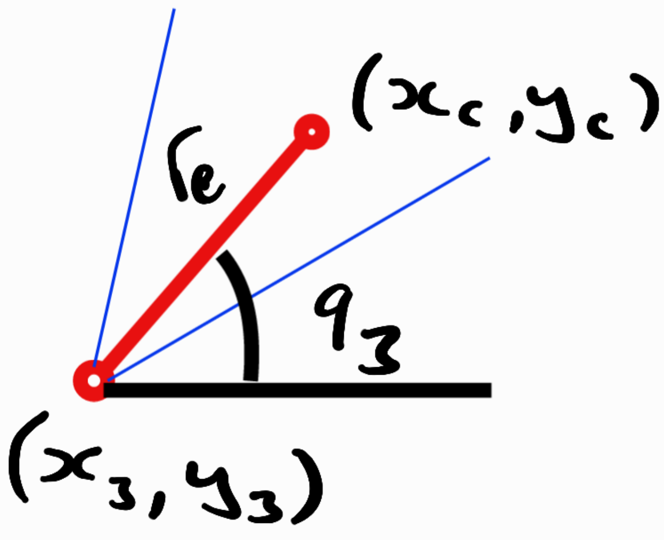
only 1 possible value.

Find q_1 & q_2:

for any $\begin{pmatrix} x_c \\ y_c \\ a \end{pmatrix}$, there are two possible q_1 & q_2 values:



Find (x_3, y_3)

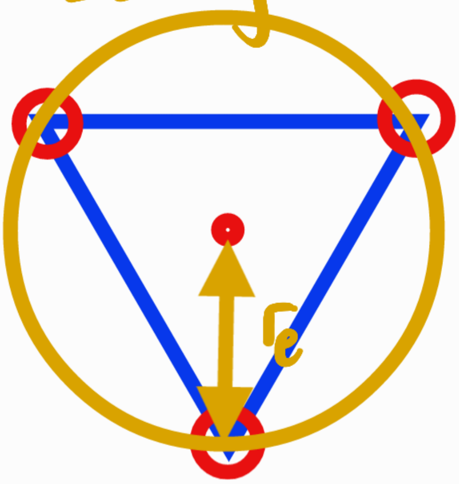


$$x_3 = x_c - r_e \cos(q_3)$$

$$y_3 = y_c - r_e \sin(q_3)$$

r_e = radius of circle formed

r_e = radius ... by 3 EE joins:



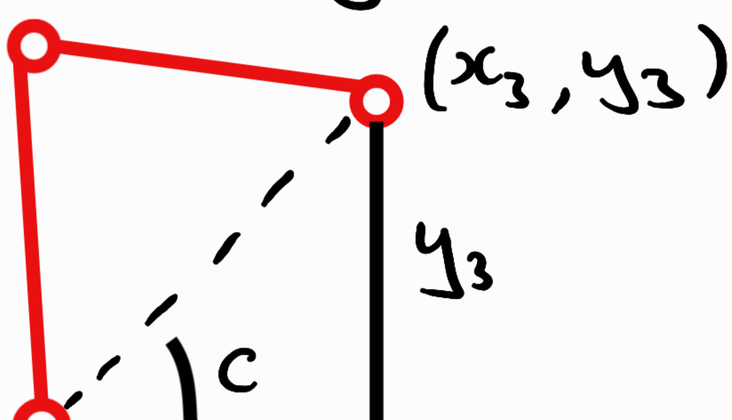
Find q_1

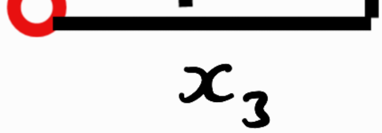
from the graph above, we can see

$$q_1 = c \pm d$$

, depending on orientation.

$\boxed{\text{Find } c}$

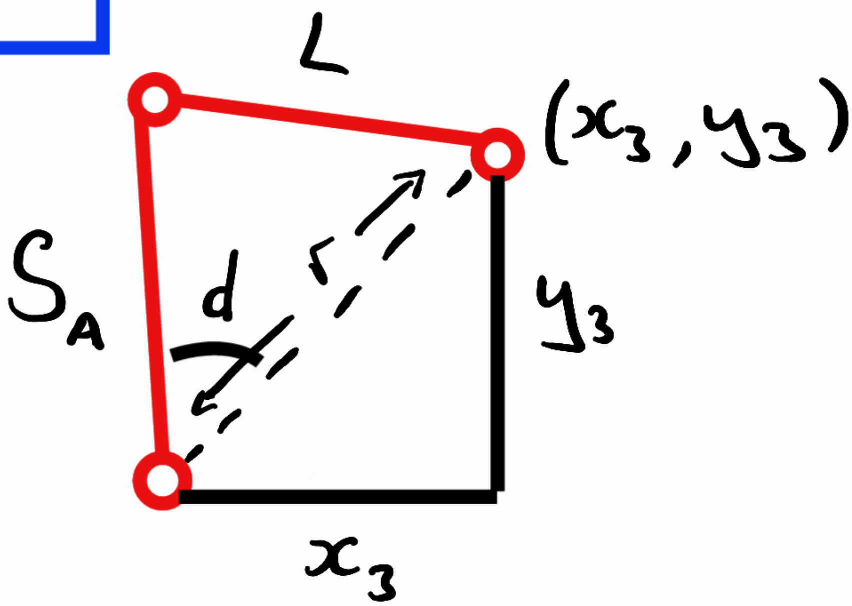




We can see then:

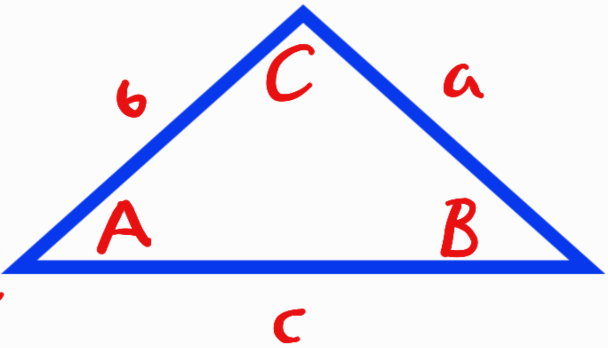
$$C = \arctan\left(\frac{y_3}{x_3}\right)$$

Find d



Pythag: $r = \sqrt{x_3^2 + y_3^2}$

Law of cosines:

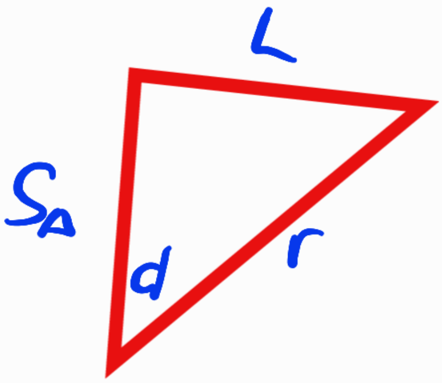


$$c^2 = a^2 + b^2 - 2ab\cos C$$

↓

$$\therefore \quad C = \arccos\left(\frac{a^2 + b^2 - c^2}{2ab}\right) \quad (1)$$

Subbing into 1 with our triangle:



$$d = \arccos\left(\frac{S_A^2 + r^2 - L^2}{2S_A r}\right)$$

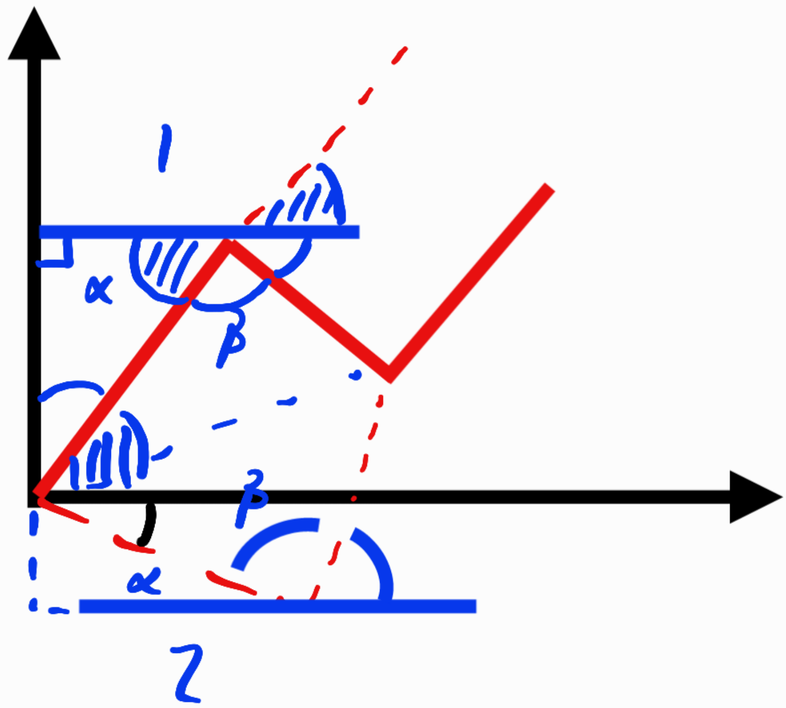
$$r = \sqrt{x_3^2 + y_3^2}$$

$$d = \arccos\left(\frac{S_A^2 + x_3^2 + y_3^2 - L^2}{2S_A\sqrt{x_3^2 + y_3^2}}\right)$$

Therefore:

$$\boxed{q = \arctan\left(\frac{y_3}{x_3}\right) \pm \arccos\left(\frac{S_A^2 + x_3^2 + y_3^2 - L^2}{2S_A\sqrt{x_3^2 + y_3^2}}\right)}$$

Find q_2

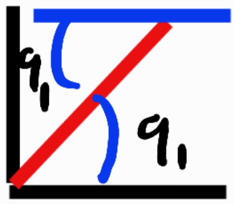


$$1: \quad q_2 = \alpha + \beta - \pi, \quad q_1 = c + d$$

$$2: \quad q_2 = \pi - \alpha - \beta, \quad q_1 = c - d$$

$$\therefore \quad q_2 = \pm(\pi - \alpha - \beta)$$

$\boxed{\alpha}$ Similar angles

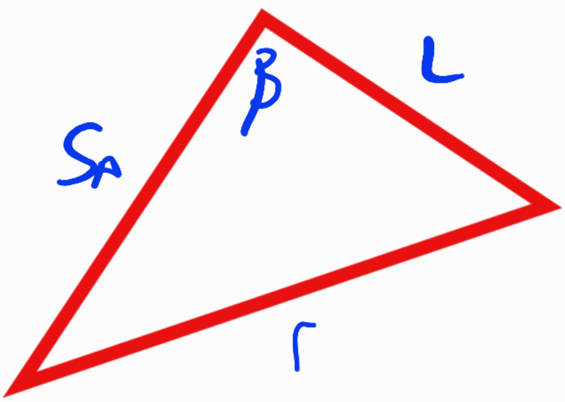


$\alpha = q_1$

We need to take
$\alpha = abs(q_1)$

to avoid messing up our q_2 eq$^{\underline{n}}$. as β will never return as -ve but q_1 can be -ve.

$\boxed{\beta}$



law of cosines:

$$C = \arccos\left(\frac{a^2 + b^2 - c^2}{2ab}\right) \quad (1)$$

Sub in: above & $r = \sqrt{x_3^2 + y_3^2}$

$$\beta = \arccos\left(\frac{S_A^2 + L^2 - x_3^2 - y_3^2}{2 S_A L}\right)$$

Therefore:

$$\boxed{\begin{gathered} q_2 = \pm\left[\pi - \operatorname{abs}(q_1) - \arccos\left(\frac{S_A^2 + L^2 - x_3^2 - y_3^2}{2 S_A L}\right)\right] \\ + \text{ for } q_1 = c - d \\ - \text{ for } q_1 = c + d \end{gathered}}$$

That's all three angles

for arm 1.

Arm 2 :

$$\boxed{\begin{aligned} \text{Base} &= (\sqrt{3}\, r_6, 0) \\ s_3 &= a + \frac{5\pi}{6} \end{aligned}}$$

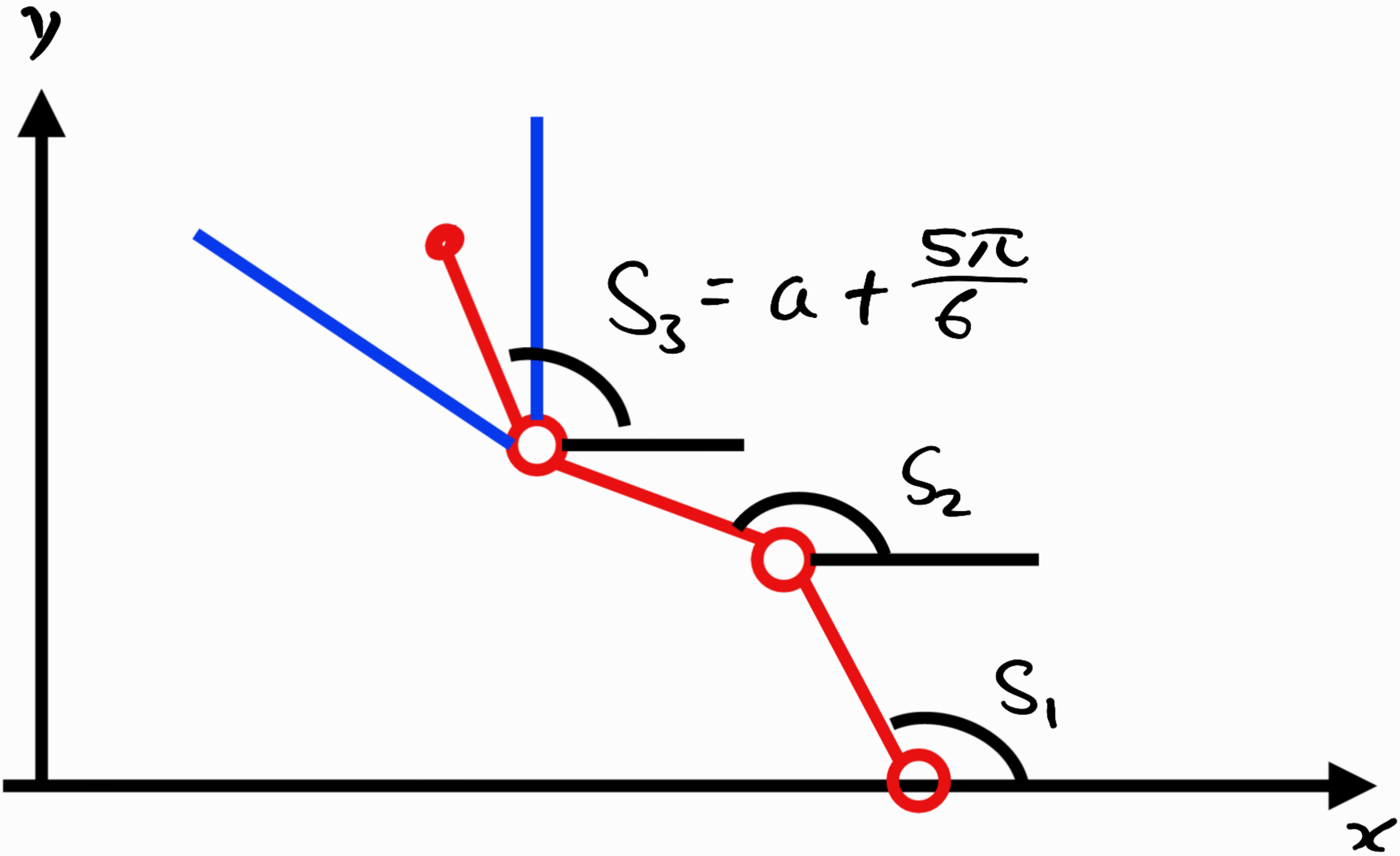
Arm 3 :

$$\boxed{\begin{aligned} \text{Base} &= \left(\frac{\sqrt{3}}{2} r_6, \frac{3}{2} r_6\right) \\ t_3 &= a + \tfrac{3}{2}\pi \end{aligned}}$$

- s_3 and t_3 derived above
- Base positions given by fact that side length of equilateral triangle inside circle of radius r_6 is

$$L_{side} = \sqrt{3}\, r_6 .$$

Arm 2

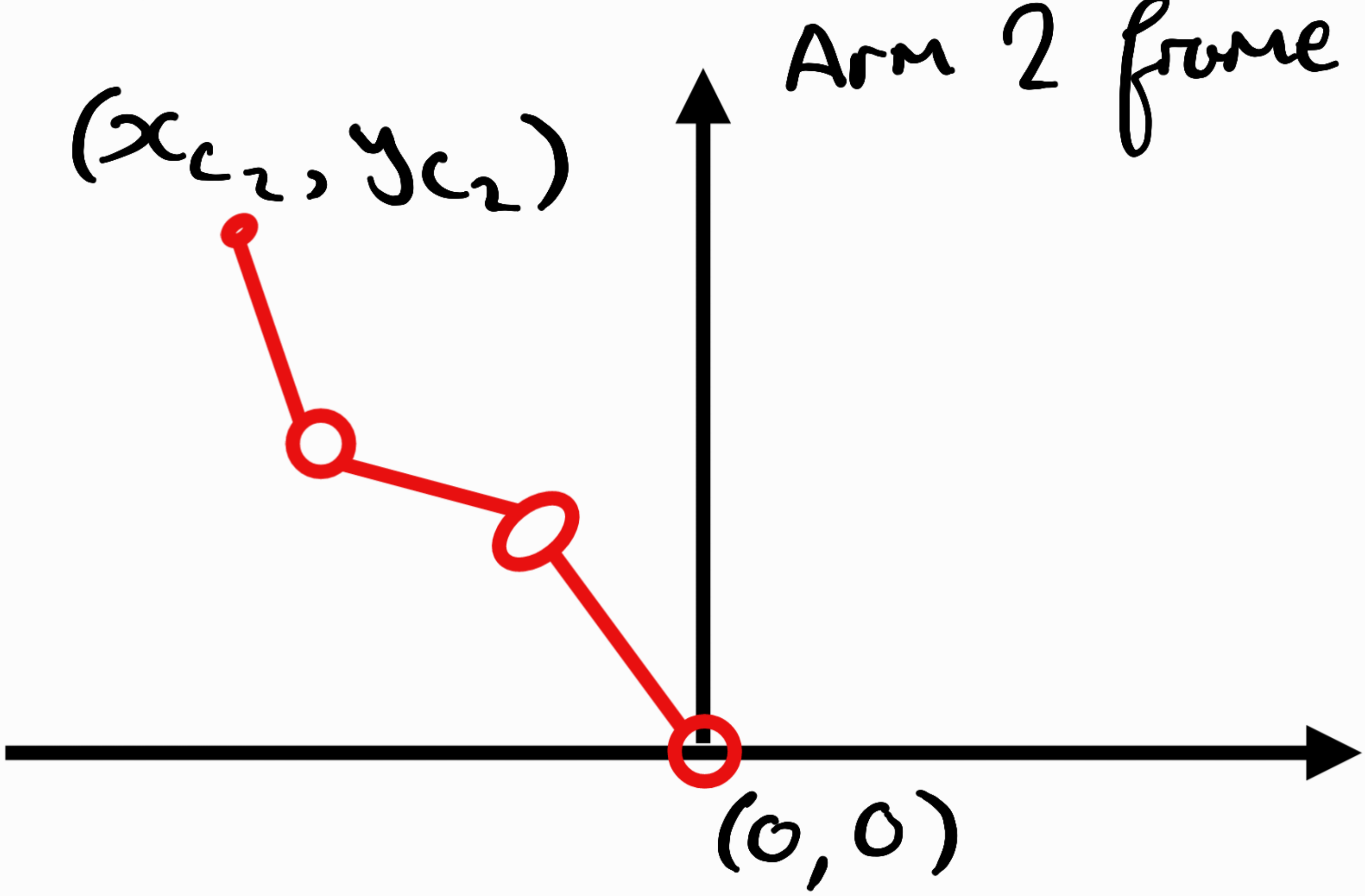


Find S_1 & S_2

These are the same as arm 1, just

apply a different base axis ∴ different end effector posit^n.

In Base axis of Arm 2:



frame 2 to frame 1

is a shift of

$$\begin{pmatrix} -\sqrt{3}\, r_b \\ 0 \end{pmatrix}$$ (no change in y)

∴

$$x_{c_2} = x_c - \sqrt{3}\, r_b$$

$$y_{c_2} = y_c$$

Apply this end effector position to same eq^ns for q_1, q_2 to find S_1, S_2

Arm 3

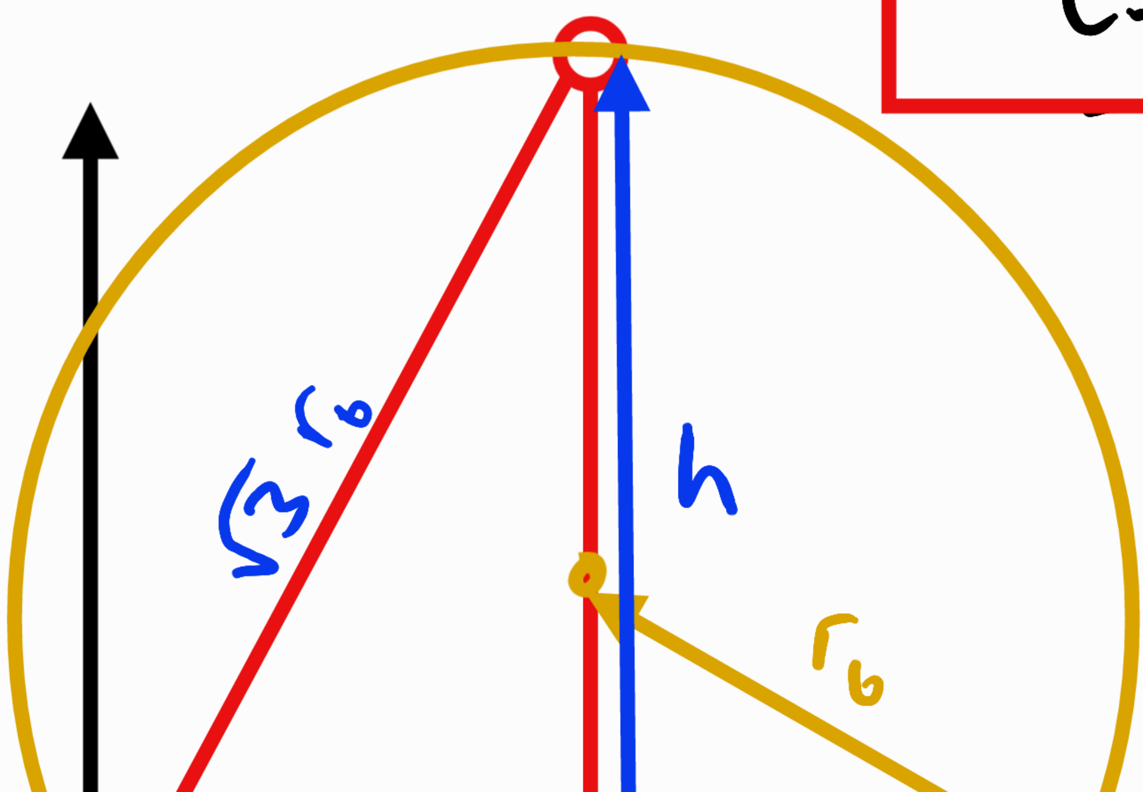
As with arm 2, find the frame shift

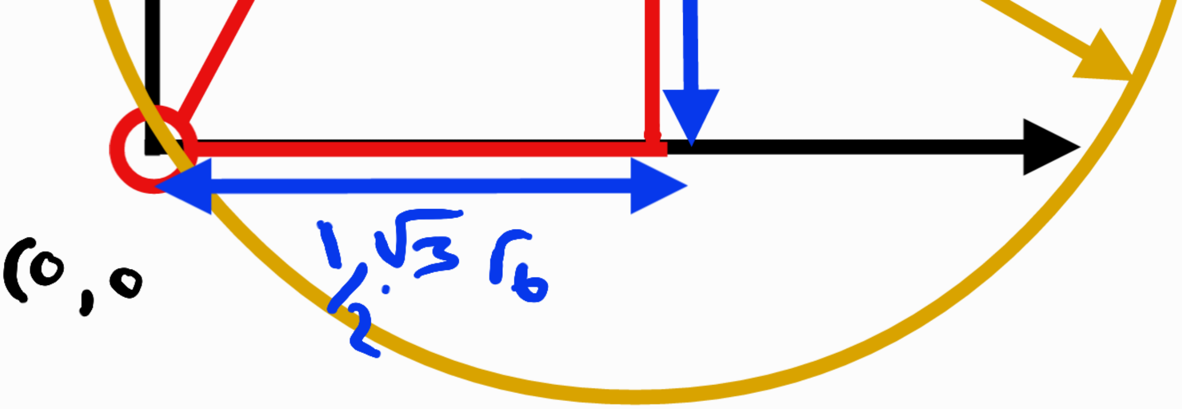
Base $= \left(\frac{\sqrt{3}}{2} r_b, \frac{3}{2} r_b\right)$

Equilateral Triangle in a circle Identity:

$L = \sqrt{3} r$

AS





$$h = \sqrt{\left(\sqrt{3}\, r_6\right)^2 - \frac{\sqrt{3}}{2} r_6{}^2}$$

$$h = \sqrt{3 r_6{}^2 - \frac{3}{4} r_6{}^2}$$

$$h = \sqrt{\frac{9}{4} r_6{}^2} = \frac{3}{2} r_6$$

Frame 3 to frame 1 is shift of

$$\begin{pmatrix} -\frac{\sqrt{3}}{2} r_6 \\ -\frac{3}{2} r_6 \end{pmatrix}$$

$\therefore$

$$x_{c_3} = x_c - \frac{\sqrt{3}}{2} r_6$$

$$y_{c_3} = y_c - \frac{3}{2} r_6$$

Apply this end effector position to same eq^n^s for q_1, q_2 to find t_1, t_2

We have now derived eq$^{\underline{n}}$s for :

$$\boxed{\begin{matrix} q_1 & q_2 & q_3 \\ s_1 & s_2 & s_3 \\ t_1 & t_2 & t_3 \end{matrix}}$$

Matlab Implementat$^{\underline{n}}$

angle 1 : manual input

angle 2&3 : make a funct$^{\underline{n}}$ which takes

- Angle 3
- x_c, y_c in base frame

and outputs:

q_1, q_2.