Numerical Method – Homework 1

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Exercise 1

Determine the real root of

$$f(x) = 4x^3 - 6x^2 + 7x - 2.3$$

Using bisection to locate the root. Employ initial guesses of $x_l = 0$ and $x_u = 1$ and iterate until the estimated error ε_a falls below a level of $\varepsilon_a = 10\%$

Solution

We have initiate the computation with guesses of $x_l = 0$ and $x_u = 1$

So now, we need to verify that $f(x_l)f(x_u) < 0$ for confirm x_l and x_u guesses for the root such that the function changes sign over the interval

$$f(x_l)f(x_u) = f(0) * f(1) = -2.3 * 2.7 = -6.21 < 0$$

Therefore, the initial estimate of the root x_r lies at the midpoint of the interval (1)

$$x_r = \frac{0+1}{2} = 0.5$$

So now, we have $f(x_l)f(x_r) = f(0) * f(0.5) = 0.54 > 0$, so in the next iteration, we set $x_u = x_r$ (set $x_l = x_r$ when $f(x_l)f(x_r) < 0$)

So, repeate from (1) to end, with approximate percent relative error equation

$$\varepsilon_a = |\frac{x_r^{new} - x_r^{old}}{x_r^{new}}| \times 100\%$$

so, we have below table

Iteration	x_l	x_u	x_r	ε_a	$f(x_r)$	**Action**
1	0	1	0.5		0.2	$x_u = x_r$
2	0	0.5	0.25	100%	-0.8625	$x_l = x_r$
3	0.25	0.5	0.375	33.33%	-0.307812	$x_l = x_r$
4	0.375	0.5	0.4375	14.286%	-0.050977	$x_l = x_r$
5	0.4375	0.5	0.46875	6.67%	0.074877	$x_u = x_r$

Because, iterate until the estimated error ε_a falls below a level of $\varepsilon_a = 10\%$, so we can see the maximum iteration is 5 and the root of $f(x)$ is $x_r = 0.46875$

Exercise 2

Determine the real root of

$$f(x) = -13 - 20x + 19x^2 - 3x^3$$

with bisection use initial guesses of $x_l = -1$ and $x_u = 0$, and a stopping criterion of 1%

We have initiate the computation with guesses of $x_l = -1$ and x_u =0

So now, we need to verify that $f(x_l)f(x_u) < 0$ for confirm x_l and x_u guesses for the root such that the function changes sign over the interval

$$f(x_l)f(x_u) = f(-1)f(0) = 29 \times -13 = -377 < 0$$

Therefore, the initial estimate of the root x_r lies at the midpoint of the interval (1)

$$x_r = \frac{-1+0}{2} = -0.5$$

So now, we have $f(x_l)f(x_r) = f(-1)f(-0.5) = 61.625 > 0$, so in the next iteration, we set $x_u = x_r$ (set $x_l = x_r$ when $f(x_l)f(x_r) < 0$)

So, repeate from (1) to end, with approximate percent relative error equation

$$\varepsilon_a = |\frac{x_r^{new} - x_r^{old}}{x_r^{new}}| \times 100\%$$

so, we have below table

Iteration	x_l	x_u	x_r	ε_a	$f(x_r)$	Action
1	0	-1	-0.50		2.125	x_u= x_r
2	-0.5	0	-0.25	100%	-6.765625	x_l = x_r
3	-0.5	-0.25	-0.375	33.33%	-2.669922	x_l = x_r
4	-0.5	-0.375	-0.4375	14.286%	-0.362061	x_l = x_r
5	-0.5	-0.4375	-0.46875	6.67%	0.858795	x_u= x_r
6	-0.46875	-0.4375	-0.453125	3.448%	0.242733	x_l = x_r
7	-0.453125	-0.4375	-0.445313	1.7542%	-0.061068	x_l = x_r
8	-0.453125	-0.445313	-0.449219	0.869%	0.090481	

Because, iterate until the estimated error ε_a falls below a level of $\varepsilon_a = 1\%$, so we can see the maximum iteration is 8 and the root of $f(x)$ is $x_r = -0.445313$