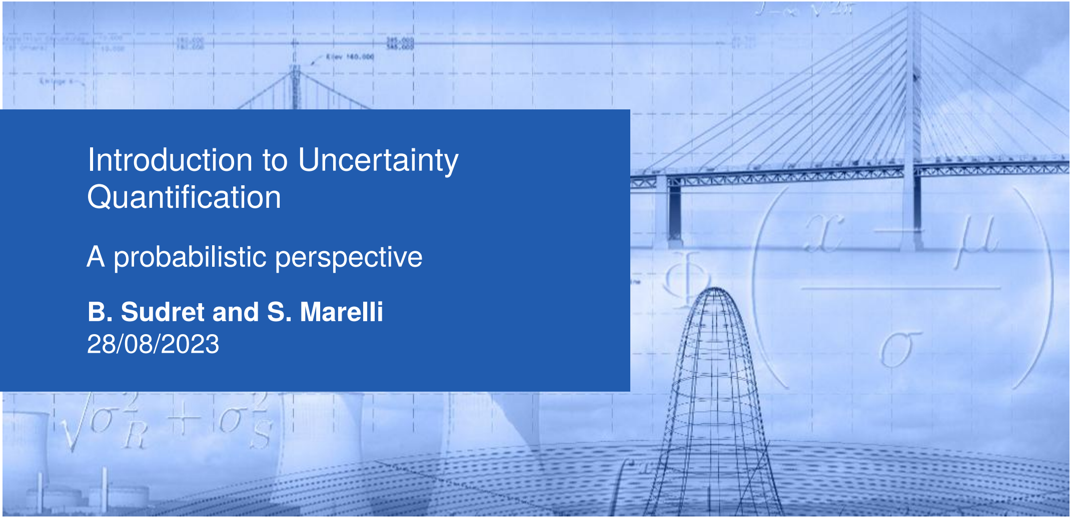


Introduction to Uncertainty Quantification

A probabilistic perspective

B. Sudret and S. Marelli

28/08/2023

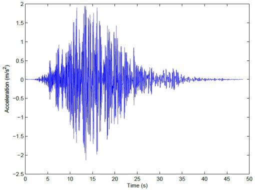
Outline

1. A simple case study: Fragility curves
2. Uncertainty quantification framework
3. Monte Carlo Simulation
4. Conclusions

Outline

1. A simple case study: Fragility curves
2. Uncertainty quantification framework
   - Aleatory vs. epistemic uncertainty
   - Common framework
3. Monte Carlo Simulation
   - Moments- and distribution analysis
   - UQ with Monte Carlo Simulation
   - Back to EXAR
4. Conclusions

Uncertainties in earthquake engineering



Earthquakes are not predictable in nature. Historical records help develop probabilistic models:

- Occurrence: # earthquakes of large magnitude (Gutenberg-Richter law)

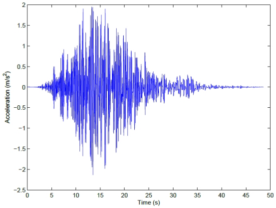
$$\log_{10} N_{\text{Mag}>M} = a - b\, M$$

- Attenuation law: link between local intensity measure (*e.g.* Peak Ground Acceleration (PGA)) and earthquake properties

$$\log \text{PGA} = c_1 + c_2\, M - c_3 \log R - c_4\, R + \varepsilon$$

where M is the magnitude, R is the source-to-site distance

Response of structures to earthquakes



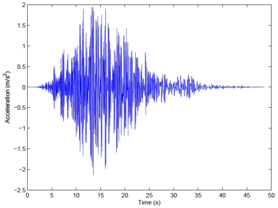
- The effect of earthquakes on structures may be assessed by finite element models, *e.g.* nonlinear transient analysis under different ground motions (accelerograms)
- Each accelerogram is characterized by its peak ground acceleration:

$$\text{PGA} = \max_{t \in [0,T]} |a(t)|$$

- Different earthquakes with the same PGA may have a different impact on the structure, *e.g.* in terms of maximal interstorey drift

Response of structures to earthquakes

- The effect of earthquakes on structures may be assessed by finite element models, *e.g.* nonlinear transient analysis under different ground motions (accelerograms)

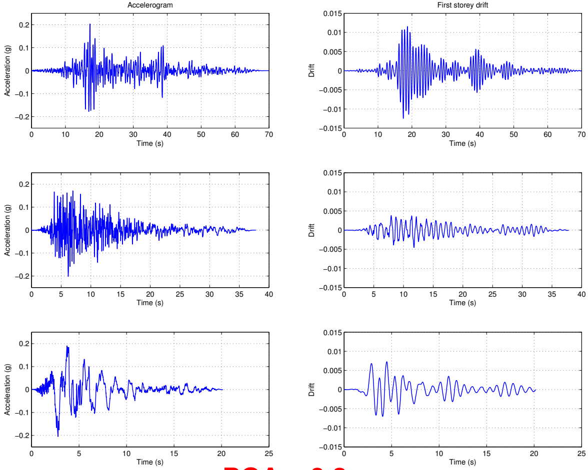


- Each accelerogram is characterized by its peak ground acceleration:

$$\text{PGA} = \max_{t \in [0,T]} |a(t)|$$

- Different earthquakes with the same PGA may have a different impact on the structure, *e.g.* in terms of maximal interstorey drift

Example: 3 accelerograms and associated maximal drift



PGA = 0.2 g

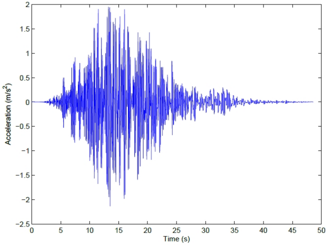
Sources of uncertainty

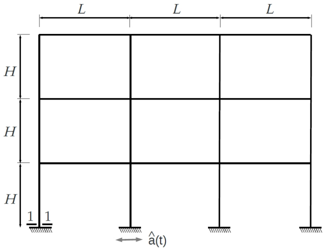
Earthquake

- Magnitude
- Duration, frequency content
- Peak ground acceleration

Structure

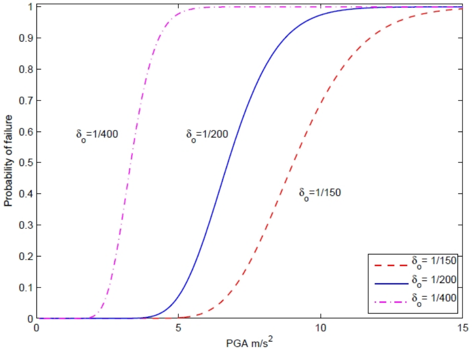
- Geometrical description (existing buildings)
- Material strength
- Ductility of the connections
- Damping





As a consequence a quake with given PGA may or may not lead to failure/collapse

Fragility curves



- An engineering demand parameter (*e.g.* the maximal interstorey drift Δ) is defined
- The vulnerability of the structure is represented by a fragility curve:

$$\text{Frag}(\text{PGA}) = \mathbb{P}\left(\Delta \geq \delta_0 | \text{PGA}\right)$$

- It is the probability of attaining some state of damage conditionally on the PGA
- Damage-related costs may be incorporated towards a global risk assessment: performance-based engineering

Outline

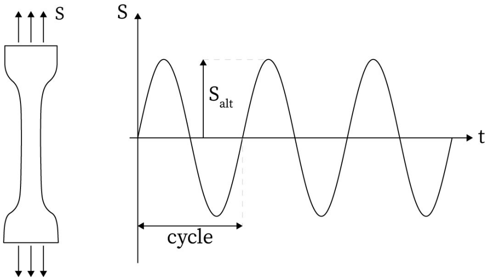
1. A simple case study: Fragility curves
2. Uncertainty quantification framework
   Aleatory vs. epistemic uncertainty
   Common framework
3. Monte Carlo Simulation
   Moments- and distribution analysis
   UQ with Monte Carlo Simulation
   Back to EXAR
4. Conclusions

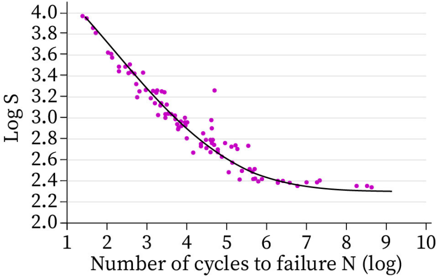
Aleatory uncertainty

Definition

The parameter under consideration *e.g.* a material property is “naturally variable” from one specimen to the other: this is called intrinsic variability

Example : the fatigue life time of a specimen





Aleatory uncertainty

Dimensions of specimens

- Size and morphology of people
- Height / width of concrete beams

Material properties

- Prefabricated concrete elements
- Glue laminated beams

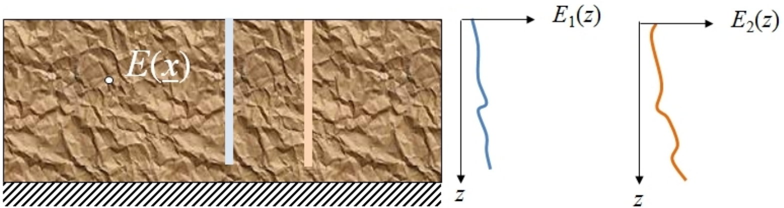
Aleatory uncertainty
Time variability

- When designing offshore structures the wave height and the wind velocity are variable in time at a given point
- From sufficiently long records this randomness may be analyzed and modelled by stochastic processes

Aleatory uncertainty
Spatial variability


Potrerillos dam, Argentina

- The soil properties (*e.g.* friction angle, cohesion) are varying in space within the soil mass
- From soil core samples the spatial variability may be represented by random fields



Epistemic uncertainty
Measurement uncertainty

- In order to select values as input of computational models, experimental data is gathered through measurements
- Any measurement device has a limited precision and resolution which is given by the provider

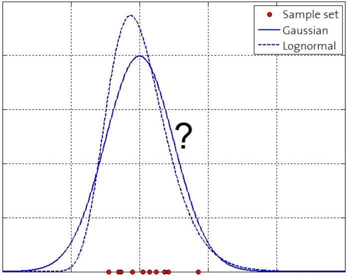
This measurement uncertainty is (theoretically) reducible *e.g.* by using a more accurate device.

Epistemic uncertainty
Measurement uncertainty

- In order to select values as input of computational models, experimental data is gathered through measurements
- Any measurement device has a limited precision and resolution which is given by the provider

This measurement uncertainty is (theoretically) reducible *e.g.* by using a more accurate device.

Epistemic uncertainty
Statistical uncertainty



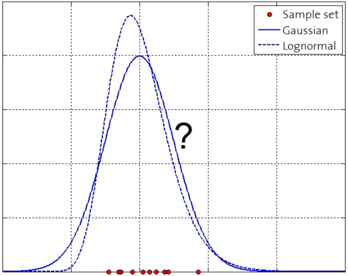
Given is a sample set of values of concrete strength

- What is the 5%-characteristic value that could be used for design ?
- What is the best fitting distribution (probability density function (PDF))?

- The number of observations is always limited: statistical uncertainty
- This uncertainty may be reduced by increasing the size of the data set

These types of uncertainty are called epistemic since they are not intrinsic but result from some lack of information / knowledge

Epistemic uncertainty
Statistical uncertainty



Given is a sample set of values of concrete strength

- What is the 5%-characteristic value that could be used for design ?
- What is the best fitting distribution (probability density function (PDF))?

- The number of observations is always limited: statistical uncertainty
- This uncertainty may be reduced by increasing the size of the data set

These types of uncertainty are called epistemic since they are not intrinsic but result from some lack of information / knowledge

Classification of uncertainties

Aleatory uncertainty

- Refers to the natural variability of some parameter
- Is irreducible: more data will exhibit more extreme values

Epistemic uncertainty

- The name comes from the Greek word $\acute{\varepsilon}\pi\iota\sigma\tau\acute{\eta}\mu\eta$ (épistemê): science / knowledge
- Refers to a lack of knowledge or information
- Is (in principle) reducible

NB:

- This classification is always related to a specific time- and space description of the system
- Both types will be modelled by probability theory hereon

Der Kiureghian, A. & Ditlevsen, O. (2009) Aleatory or epistemic? Does it matter? Structural Safety, 31, 105-112.

Classification of uncertainties

Aleatory uncertainty

- Refers to the natural variability of some parameter
- Is irreducible: more data will exhibit more extreme values

Epistemic uncertainty

- The name comes from the Greek word $\acute{\varepsilon}\pi\iota\sigma\tau\acute{\eta}\mu\eta$ (épistemê): science / knowledge
- Refers to a lack of knowledge or information
- Is (in principle) reducible

NB:

- This classification is always related to a specific time- and space description of the system
- Both types will be modelled by probability theory hereon

Der Kiureghian, A. & Ditlevsen, O. (2009) Aleatory or epistemic? Does it matter? Structural Safety, 31, 105-112.

Classification of uncertainties

Aleatory uncertainty

- Refers to the natural variability of some parameter
- Is irreducible: more data will exhibit more extreme values

Epistemic uncertainty

- The name comes from the Greek word $\acute{\varepsilon}\pi\iota\sigma\tau\acute{\eta}\mu\eta$ (épistemê): science / knowledge
- Refers to a lack of knowledge or information
- Is (in principle) reducible

NB:

- This classification is always related to a specific time- and space description of the system
- Both types will be modelled by probability theory hereon

Der Kiureghian, A. & Ditlevsen, O. (2009) Aleatory or epistemic? Does it matter? Structural Safety, 31, 105-112.

Common features of the previous examples

- A computational model that describes the physics of the problem and computes some quantities of interest which have a meaningful use for decision making
- Various sources of uncertainty in the parameters
- Possibly time- or space variability of the uncertain parameters
- A large spectrum of statistical output quantities:
  - mean, standard deviation
  - distribution, quantiles
  - probability of exceedance/failure

Global framework for uncertainty quantification

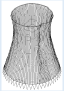
Step B
Quantification of sources of uncertainty

Step A
Model(s) of the system
Assessment criteria

Step C
Uncertainty propagation

Random variables

Computational model



Distribution
Mean, std. deviation
Probability of failure

Sudret, B. Uncertainty propagation and sensitivity analysis in mechanical models, Habilitation à diriger des recherches, 2007.

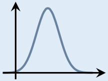
Global framework for uncertainty quantification

Step B
Quantification of sources of uncertainty

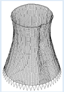
Step A
Model(s) of the system
Assessment criteria

Step C
Uncertainty propagation

Random variables



Computational model



Distribution
Mean, std. deviation
Probability of failure

Sudret, B. Uncertainty propagation and sensitivity analysis in mechanical models, Habilitation à diriger des recherches, 2007.

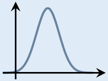
Global framework for uncertainty quantification

Step B
Quantification of sources of uncertainty

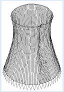
Step A
Model(s) of the system
Assessment criteria

Step C
Uncertainty propagation

Random variables



Computational model



Distribution
Mean, std. deviation
Probability of failure

Sudret, B. Uncertainty propagation and sensitivity analysis in mechanical models, Habilitation à diriger des recherches, 2007.

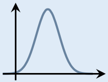
Global framework for uncertainty quantification

Step B
Quantification of sources of uncertainty

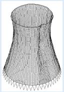
Step A
Model(s) of the system
Assessment criteria

Step C
Uncertainty propagation

Random variables



Computational model



Distribution
Mean, std. deviation
Probability of failure

Sudret, B. Uncertainty propagation and sensitivity analysis in mechanical models, Habilitation à diriger des recherches, 2007.

Step A: computational models

Vector of input parameters $\boldsymbol{x} \in \mathbb{R}^M$ — Computational model $\mathcal{M}$ — Model response $\boldsymbol{y} = \mathcal{M}(\boldsymbol{x}) \in \mathbb{R}^N$

- Geometry
- Material properties
- Loading

- Analytical formula
- Finite element model
- Computational workflow

- Displacements
- Strains, stresses
- Temperature, etc.

Step A: computational models

Vector of input parameters
$\boldsymbol{x} \in \mathbb{R}^M$

Computational model $\mathcal{M}$

Model response
$\boldsymbol{y} = \mathcal{M}(\boldsymbol{x}) \in \mathbb{R}^N$

- Geometry
- Material properties
- Loading

- Analytical formula
- Finite element model
- Computational workflow

- Displacements
- Strains, stresses
- Temperature, etc.

Step B: probabilistic models of input parameters

No data exist

- Expert judgment for selecting the input PDF's of $\boldsymbol{X}$
- Literature, data bases (*e.g.* on material properties)
- Maximum entropy principle

Input data exist

- Classical statistical inference
- Bayesian statistics to handle prior knowledge together with scarce data

Step B: probabilistic models of input parameters

No data exist

- Expert judgment for selecting the input PDF's of $\boldsymbol{X}$
- Literature, data bases (*e.g.* on material properties)
- Maximum entropy principle

Input data exist

- Classical statistical inference
- Bayesian statistics to handle prior knowledge together with scarce data

Step B: probabilistic models of input parameters

Expert judgment

Transform the available information into a relevant probabilistic model

Physical bounds

- Positive quantities shall be modelled by positive-valued random variables, *e.g.* lognormal, Gamma distributions
- Quantities obtained as maxima over time shall be modelled by extreme value distributions, *e.g.* Gumbel or Weibull distributions
- Bounded values shall be modelled by bounded distributions: Beta or truncated Gaussian distributions

Step B: probabilistic models of input parameters

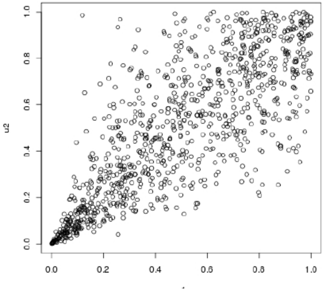
Expert judgment

Transform the available information into a relevant probabilistic model

Physical bounds

- Positive quantities shall be modelled by positive-valued random variables, *e.g.* lognormal, Gamma distributions
- Quantities obtained as maxima over time shall be modelled by extreme value distributions, *e.g.* Gumbel or Weibull distributions
- Bounded values shall be modelled by bounded distributions: Beta or truncated Gaussian distributions

Step B: probabilistic models of input parameters

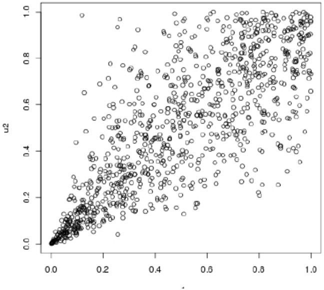


Statistical inference

- Marginal distributions of each parameter
- Correlation/dependence between parameters: copula theory

- Descriptive statistics: graphical representation of the sample set (histograms, scatterplots), numerical descriptors (mean value, standard deviation, correlation coefficients, etc.)
- Statistical inference: selection of classes of marginal probability distributions $f_X(x;\theta)$ and computation of "the best set" of parameters $\hat{\theta}$ (maximum likelihood method). Selection of a best representative dependence structure.

Step B: probabilistic models of input parameters

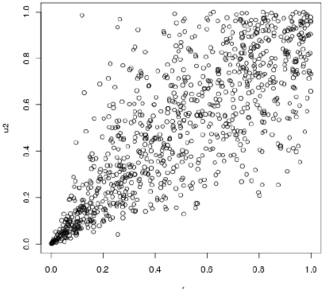


Statistical inference

- Marginal distributions of each parameter
- Correlation/dependence between parameters: copula theory

- Descriptive statistics: graphical representation of the sample set (histograms, scatterplots), numerical descriptors (mean value, standard deviation, correlation coefficients, etc.)
- Statistical inference: selection of classes of marginal probability distributions $f_X(x;\theta)$ and computation of "the best set" of parameters $\hat{\theta}$ (maximum likelihood method). Selection of a best representative dependence structure.

Step B: probabilistic models of input parameters



Statistical inference

- Marginal distributions of each parameter
- Correlation/dependence between parameters: copula theory

- Descriptive statistics: graphical representation of the sample set (histograms, scatterplots), numerical descriptors (mean value, standard deviation, correlation coefficients, etc.)
- Statistical inference: selection of classes of marginal probability distributions $f_X(x; \boldsymbol{\theta})$ and computation of "the best set" of parameters $\hat{\boldsymbol{\theta}}$ (maximum likelihood method). Selection of a best representative dependence structure.

Step C: principles of uncertainty propagation

Input parameters $\boldsymbol{x} \in \mathbb{R}^M$ — Computational model $\mathcal{M}$ — Model response $\boldsymbol{y} = \mathcal{M}(\boldsymbol{x}) \in \mathbb{R}^N$

Random variables $\boldsymbol{X}$

Computational model $\mathcal{M}$

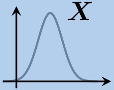
Random response $\boldsymbol{Y} = \mathcal{M}(\boldsymbol{X})$

?

Step C: principles of uncertainty propagation

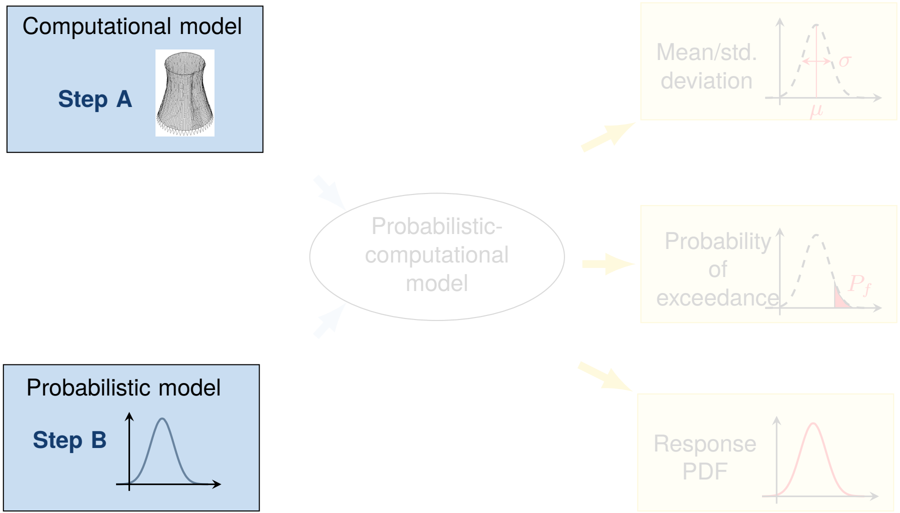
Input parameters $\boldsymbol{x} \in \mathbb{R}^M$ — Computational model $\mathcal{M}$ — Model response $\boldsymbol{y} = \mathcal{M}(\boldsymbol{x}) \in \mathbb{R}^N$

Random variables $\boldsymbol{X}$

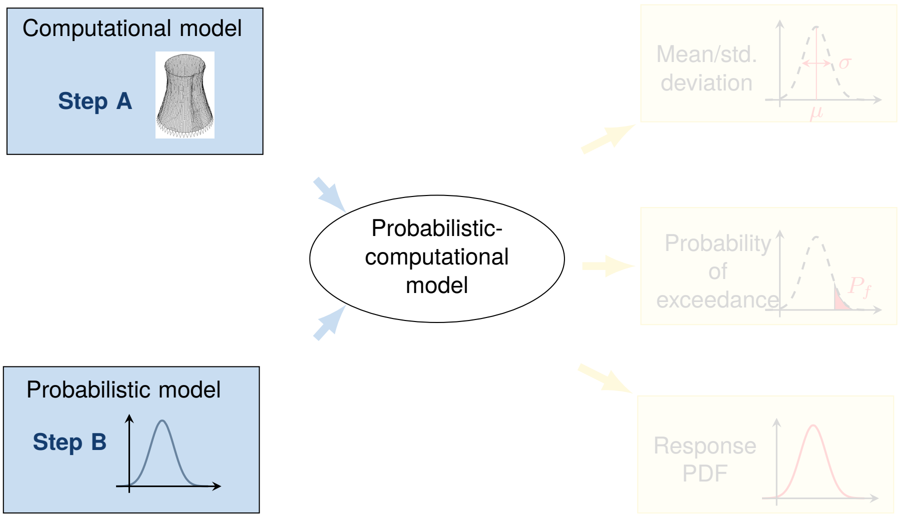


— Computational model $\mathcal{M}$ — Random response $\boldsymbol{Y} = \mathcal{M}(\boldsymbol{X})$ **?**

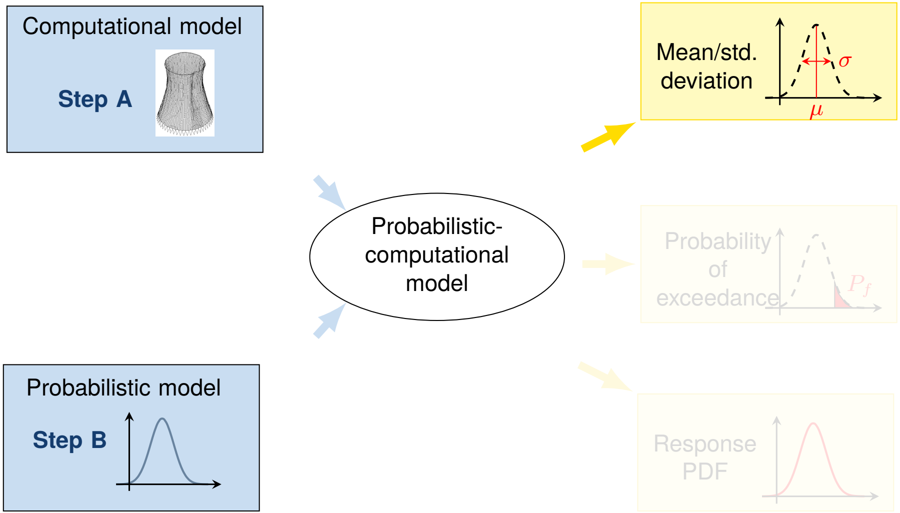
Step C: uncertainty propagation methods



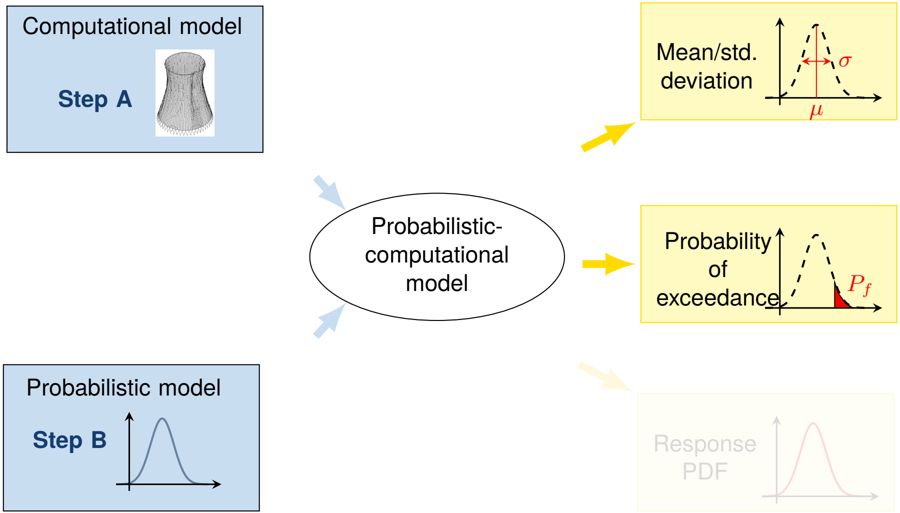
Step C: uncertainty propagation methods



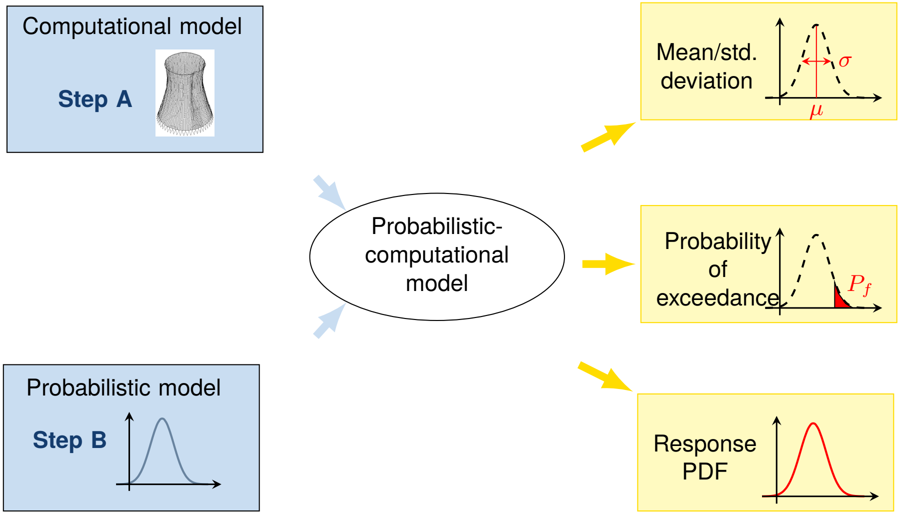
Step C: uncertainty propagation methods



Step C: uncertainty propagation methods



Step C: uncertainty propagation methods



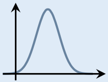
Step C’: sensitivity analysis

Step B
Quantification of sources of uncertainty

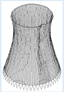
Step A
Model(s) of the system
Assessment criteria

Step C
Uncertainty propagation

Random variables



Computational model



Distribution
Mean, std. deviation
Probability of failure

Step C’
Sensitivity analysis

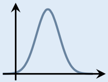
Step C’: sensitivity analysis

Step B
Quantification of sources of uncertainty

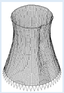
Step A
Model(s) of the system
Assessment criteria

Step C
Uncertainty propagation

Random variables



Computational model



Distribution
Mean, std. deviation
Probability of failure

Step C’
Sensitivity analysis

Step C': sensitivity analysis

Heuristics

- When considering complex models with a large number of input parameters (say 20-50), not all parameters influence the variability of the model output
- Sensitivity analysis aims at finding the "important" parameters, *i.e.* those whose variability explain the output variability
- Reducing the uncertainty on those parameters will reduce the model output uncertainty
- Unimportant parameters may be replaced by a fixed value (model simplification)

Outline

Uncertainty propagation using Monte Carlo simulation

Principle: Generate virtual variations of the engineering system using random numbers

- A sample set of input parameters $\mathcal{X} = \{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n\}$ is drawn according to the input distribution $f_{\boldsymbol{X}}$
- For each sample the quantity of interest (*e.g.* bearing capacity, displacement, global safety factor, etc.) is evaluated, say $\mathcal{Y} = \{\mathcal{M}(\boldsymbol{x}_1), \ldots, \mathcal{M}(\boldsymbol{x}_n)\}$
- The set of response quantities is used for moments-, distribution-, sensitivity or reliability analysis

Uncertainty propagation using Monte Carlo simulation

Principle: Generate virtual variations of the engineering system using random numbers

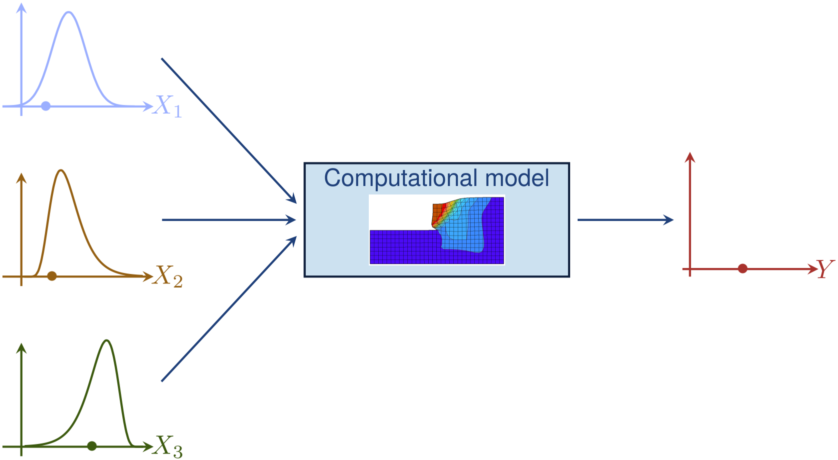
- A sample set of input parameters $\mathcal{X} = \{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n\}$ is drawn according to the input distribution $f_{\boldsymbol{X}}$
- For each sample the quantity of interest (*e.g.* bearing capacity, displacement, global safety factor, etc.) is evaluated, say $\mathcal{Y} = \{\mathcal{M}(\boldsymbol{x}_1), \ldots, \mathcal{M}(\boldsymbol{x}_n)\}$
- The set of response quantities is used for moments-, distribution-, sensitivity or reliability analysis

Uncertainty propagation using Monte Carlo simulation

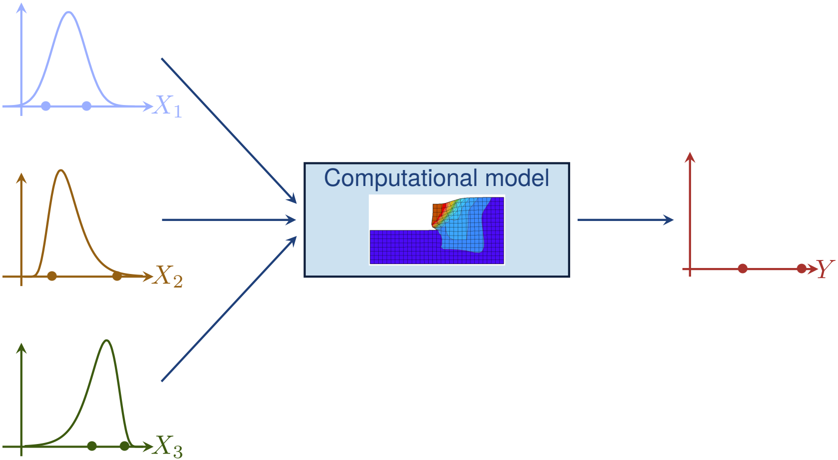
Principle: Generate virtual variations of the engineering system using random numbers

- A sample set of input parameters $\mathcal{X} = \{\boldsymbol{x}_1, \dots, \boldsymbol{x}_n\}$ is drawn according to the input distribution $f_{\boldsymbol{X}}$
- For each sample the quantity of interest (*e.g.* bearing capacity, displacement, global safety factor, etc.) is evaluated, say $\mathcal{Y} = \{\mathcal{M}(\boldsymbol{x}_1), \dots, \mathcal{M}(\boldsymbol{x}_n)\}$
- The set of response quantities is used for moments-, distribution-, sensitivity or reliability analysis

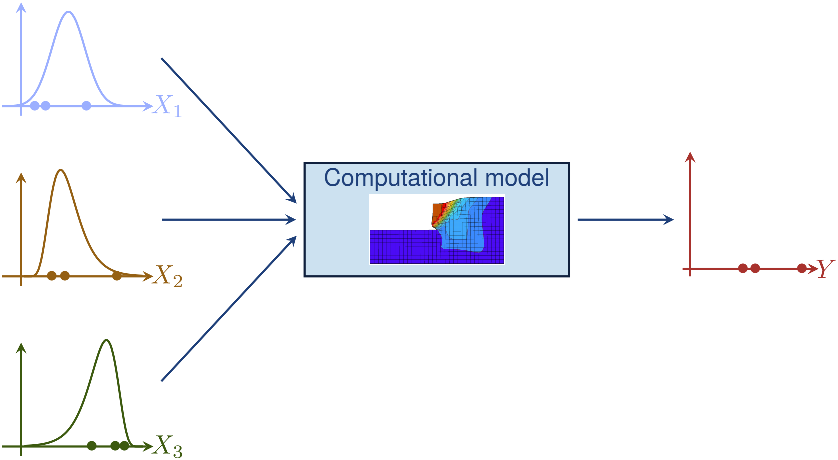
Uncertainty propagation using Monte Carlo simulation



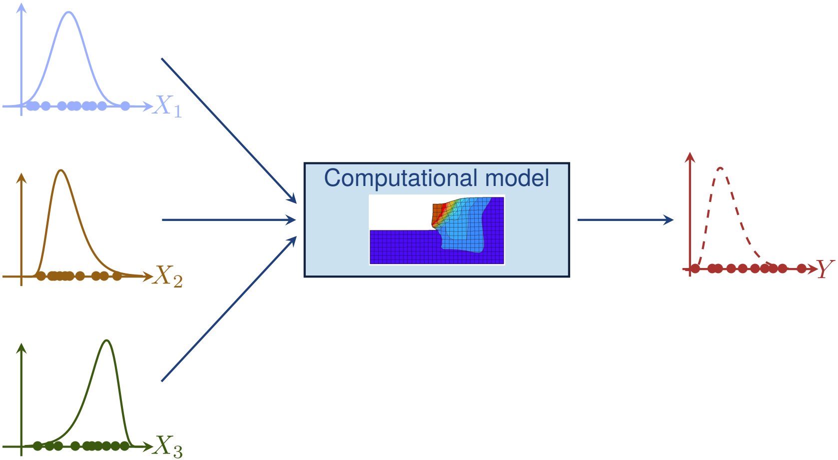
Uncertainty propagation using Monte Carlo simulation



Uncertainty propagation using Monte Carlo simulation



Uncertainty propagation using Monte Carlo simulation



Moments: mean and variance

From the Monte Carlo procedure a sample set of model response quantities is available:

$$\mathcal{Y} = \{y_i = \mathcal{M}(\boldsymbol{x}_i),\ i = 1, \ldots, N_{\mathsf{MCS}}\}$$

Estimators of the moments

- **Mean value**:

$$\hat{\mu}_Y = \frac{1}{N_{\mathsf{MCS}}} \sum_{i=1}^{N_{\mathsf{MCS}}} y_i$$

- **Variance**:

$$\widehat{\sigma_Y^2} = \frac{1}{N_{\mathsf{MCS}} - 1} \sum_{i=1}^{N_{\mathsf{MCS}}} (y_i - \hat{\mu}_Y)^2$$

- **Standard deviation / coefficient of variation**

$$\widehat{\sigma_Y} = \sqrt{\widehat{\sigma_Y^2}} \quad , \quad CV_Y = \frac{\widehat{\sigma_Y}}{\hat{\mu}_Y}$$

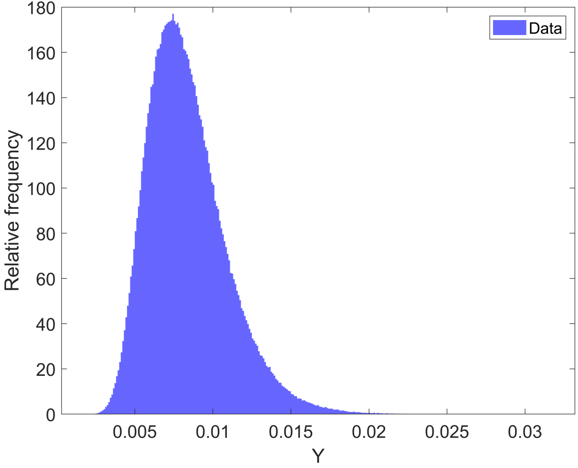
What can we do with MCS?

- Uncertainty propagation
  - Histograms and moments
  - Confidence bounds on the predictions
  - Quantiles, etc.
- Reliability analysis (rare event estimation)
  - Calculate the probability of exceedance
  - Simulate rare events
  - Identify *failure modes*
- Sensitivity analysis
  - Identify the effect of each source of uncertainty on the prediction uncertainty
  - Exclude unimportant variables from the analysis
  - Identify where to invest to reduce uncertainty

Descriptive statistics

From the response sample, different types of properties can be extracted

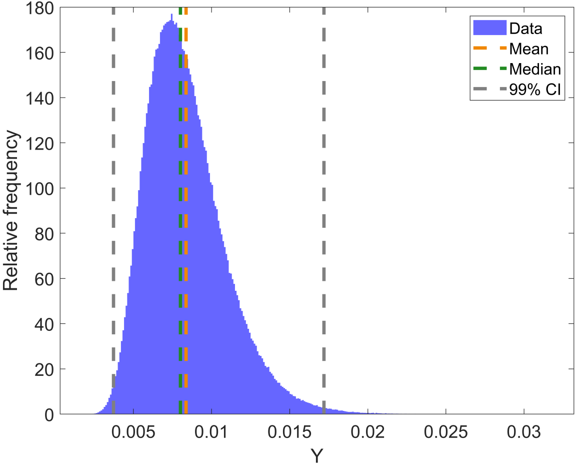
- Histogram of the response
- Empirical statistics
- PDF estimation
- Exceedance probability



Descriptive statistics

From the response sample, different types of properties can be extracted

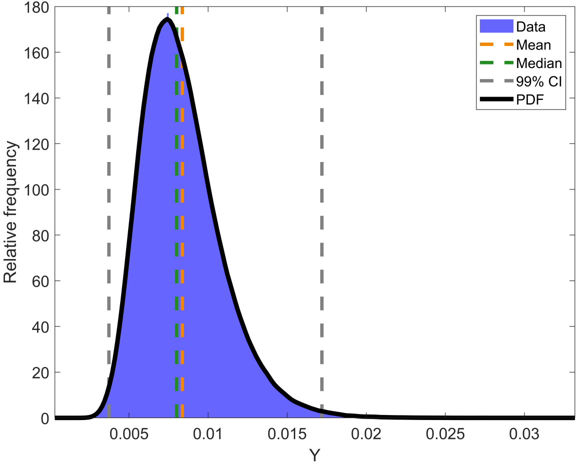
- Histogram of the response
- Empirical statistics
- PDF estimation
- Exceedance probability



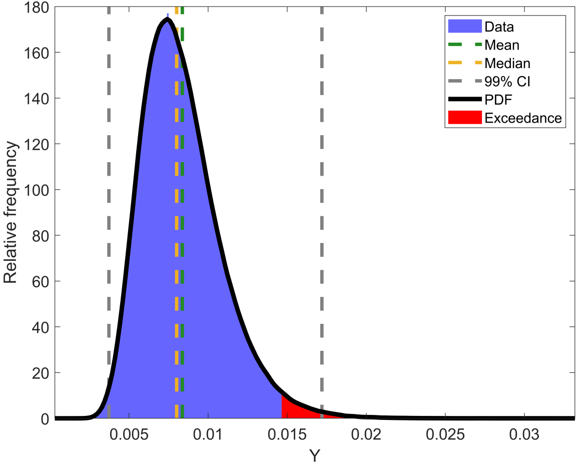
Descriptive statistics

From the response sample, different types of properties can be extracted

- Histogram of the response
- Empirical statistics
- PDF estimation
- Exceedance probability



Descriptive statistics



From the response sample, different types of properties can be extracted

- Histogram of the response
- Empirical statistics
- PDF estimation
- Exceedance probability

Caveat

Monte Carlo simulation is one of the most powerful techniques available...

However

- It can require many model runs
- Not feasible for complex engineering models
- Much more advanced tools are available:
  - **Surrogate models**: replace the computational model with a cheap but accurate copy
  - **Structural reliability analysis**: forget most of the output, only focus on exceedance
  - **AI-based learning**: mix surrogate models and machine learning to perform propagation and reliability analysis at very low costs

Caveat

Monte Carlo simulation is one of the most powerful techniques available...

However

- It can require many model runs
- Not feasible for complex engineering models
- Much more advanced tools are available:
  - **Surrogate models**: replace the computational model with a cheap but accurate copy
  - **Structural reliability analysis**: forget most of the output, only focus on exceedance
  - **AI-based learning**: mix surrogate models and machine learning to perform propagation and reliability analysis at very low costs

Caveat

Monte Carlo simulation is one of the most powerful techniques available...

However

- It can require many model runs
- Not feasible for complex engineering models
- Much more advanced tools are available:
  - **Surrogate models**: replace the computational model with a cheap but accurate copy
  - **Structural reliability analysis**: forget most of the output, only focus on exceedance
  - **AI-based learning**: mix surrogate models and machine learning to perform propagation and reliability analysis at very low costs

Caveat

Monte Carlo simulation is one of the most powerful techniques available...

However

- It can require many model runs
- Not feasible for complex engineering models
- Much more advanced tools are available:
  - **Surrogate models**: replace the computational model with a cheap but accurate copy
  - **Structural reliability analysis**: forget most of the output, only focus on exceedance
  - **AI-based learning**: mix surrogate models and machine learning to perform propagation and reliability analysis at very low costs

Back to EXAR 1: UQ approach

Full UQ framework applied

Eingabe-Parameter (Vektor) $x \in \mathbb{R}^M$	→	Berechnungsmodell $\mathcal{M}$	→	Modellantwort **Resultat aus BASEMENT** $y = \mathcal{M}(x) \in \mathbb{R}^Q$
Rauigkeitsklassen im Gesamtsystem (17 Klassen) **100 Parameter-Sätze**		Modellierung mit **BASEMENT v3** **100 Simulationen**		**Maximale Wasserspiegellagen** • Alle 2'500 m entlang der Hauptgewässer • Für sämtliche Szenarien- und Referenzpunkte

- Modellantwort aus einem kleinen Stichproben-datensatz der Eingabe-Parameter (100 Simulationen)
- Statistische Aufbereitung der Resultate (Polynomial Chaos Expansion)

Eingabe-Parameter (Vektor) $x \in \mathbb{R}^M$	→	Ersatzmodell $\widetilde{\mathcal{M}}$	→	Modellantwort **Resultat aus Ersatzmodell** $\forall x (y = \widetilde{\mathcal{M}}(x) \in \mathbb{R}^Q)$

- Die Modellantwort aus dem Ersatzmodell kann für alle möglichen Eingabeparameter innert kürzester Zeit ausgegeben werden.

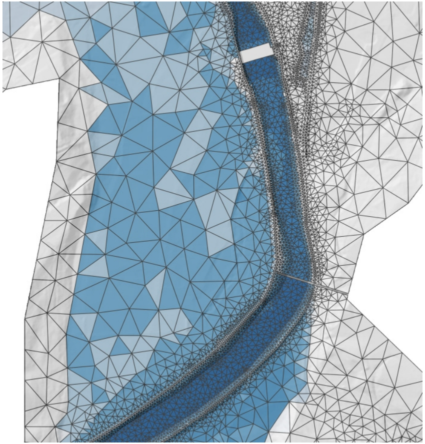
EXAR detailed report E

Pfäffli, M., Baer P., Sutter A., Irniger, A., Hunziker, R. 2021: Extremhochwasser an der Aare. Detailbericht E Projekt EXAR. Hydraulische Modellierungen. ARGE GEOTEST-HZP-IUB. Zollikofen, Aarau, Bern

Back to EXAR 2: Computational model

High resolution 3D BASEMENT flood-propagation model

- A high resolution hydraulic mesh,
- with a landslide,
- and **sharks!**
- calibrated on real data,
- that accounts or meshing errors



Back to EXAR 2: Computational model

High resolution 3D BASEMENT flood-propagation model

- A high resolution hydraulic mesh,
- with a landslide,
- and **sharks!**
- calibrated on real data,
- that accounts or meshing errors

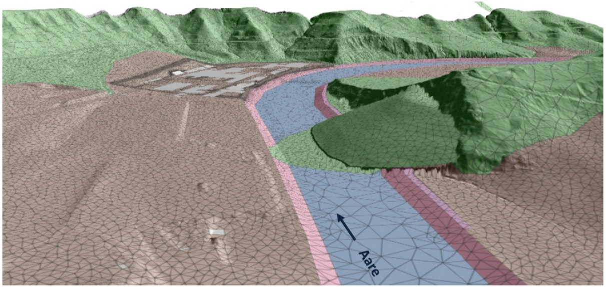


Abbildung 13: *Beispiel einer Rutschung, welche physisch in das Höhenmodell eingebaut wurde, mit darüber gelegtem Berechnungsgitter für die Gefährdungsbeurteilung. Rutschung Brättele, nahe des Beurteilungsperimeters Mühleberg.*

Back to EXAR 2: Computational model

High resolution 3D BASEMENT flood-propagation model

- A high resolution hydraulic mesh,
- with a landslide,
- and **sharks!**
- calibrated on real data,
- that accounts or meshing errors

Back to EXAR 2: Computational model

High resolution 3D BASEMENT flood-propagation model

- A high resolution hydraulic mesh,
- with a landslide,
- and **sharks!**
- calibrated on real data,
- that accounts or meshing errors

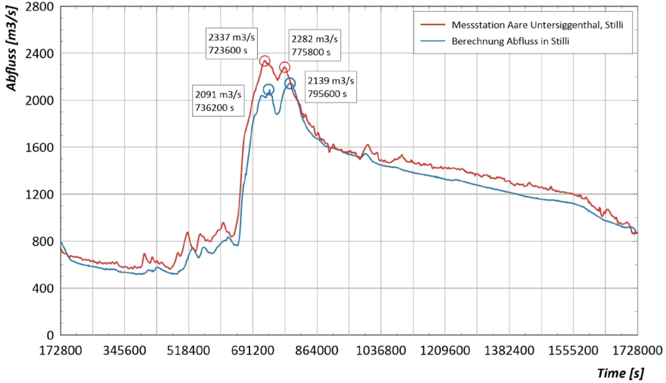


Abbildung 22: ***Ganglinienvergleich zwischen der Messstation Aare Untersiggenthal, Stilli und der Berechnung im 2D-Modell für das Hochwasser 2005.***

Back to EXAR 2: Computational model

High resolution 3D BASEMENT flood-propagation model

- A high resolution hydraulic mesh,
- with a landslide,
- and **sharks!**
- calibrated on real data,
- that accounts or meshing errors

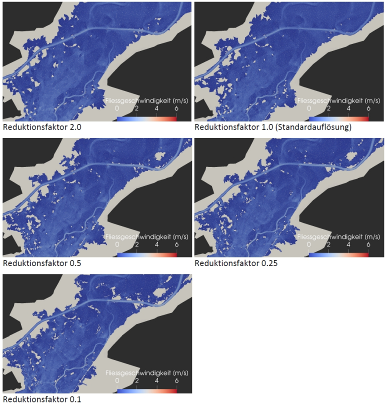
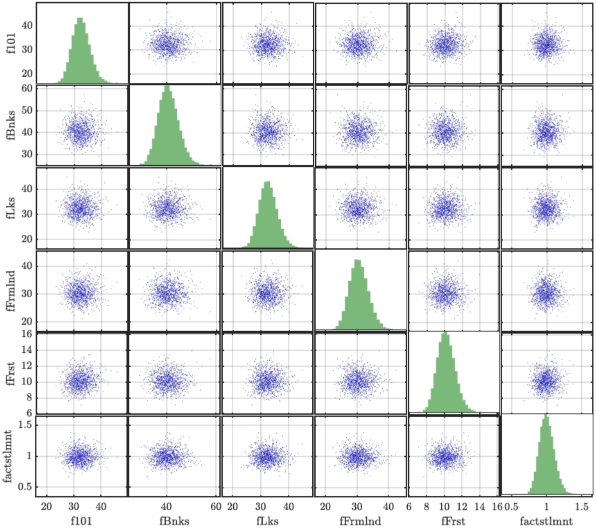


Abbildung 34: Überflutungsfläche im Bereich der Mündung der Alten Aare in die Aare bei Büren an der Aare. Dargestellt ist das hydrologische Top-Ereignis im Bereich des Spitzenabflusses für verschiedene Auflösungen der Modellgeometrie

Back to EXAR 3: Input distributions

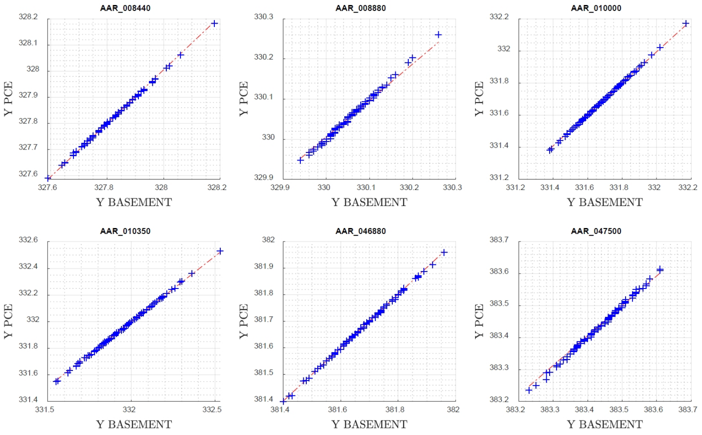


Back to EXAR 4: Surrogate models

- Limited computational budget: only $N = 100$ runs
- Surrogate model needed
- Sparse polynomial chaos expansions (PCE)
- Very good accuracy

Back to EXAR 4: Surrogate models

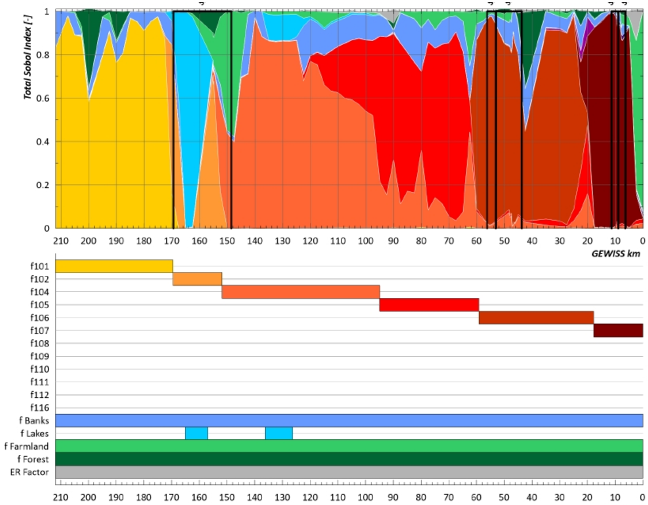
- Limited computational budget: only $N = 100$ runs
- Surrogate model needed
- Sparse polynomial chaos expansions (PCE)
- Very good accuracy



Back to EXAR 5: Sensitivity analysis

Question: which uncertainties affect the water level the most?

- Space dependent problem
- Once again surrogate model-based
- Sobol' indices as a function of coordinate along the river s
- Some variables "activate" only in specific regions



Back to EXAR 6: Prediction Confidence bounds

5%-95% bounds on the predicted water levels at different stations

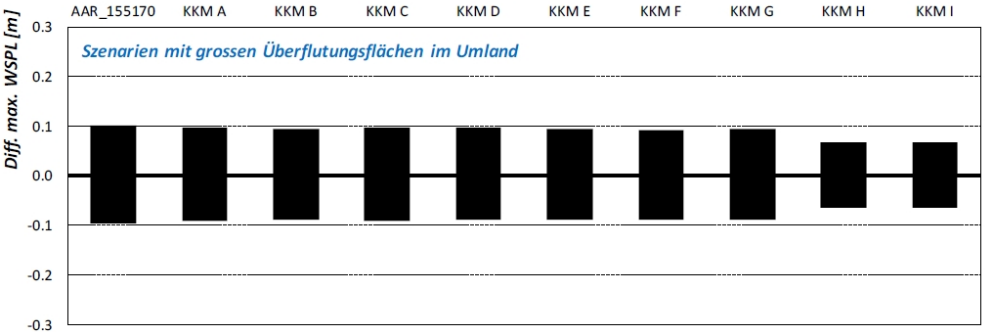


Abbildung 52: ***Hydraulische Unsicherheiten für die Szenarien- und Referenzpunkte im Beurteilungsperimeter Mühleberg für Szenarien mit grossen Überflutungsflächen.***

Back to EXAR 6: Prediction Confidence bounds

5%-95% bounds on the predicted water levels at different stations

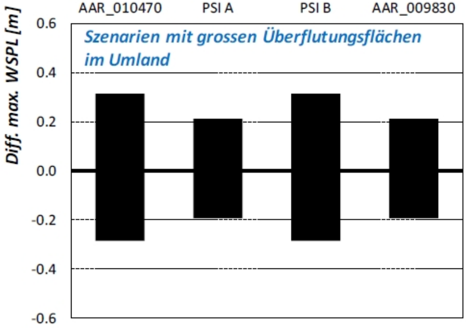


Abbildung 57: ***Hydraulische Unsicherheiten für die Szenarien- und Referenzpunkte im Beurteilungsperimeter PSI für Szenarien mit grossen Überflutungsflächen.***

Outline

1. A simple case study: Fragility curves
2. Uncertainty quantification framework
   - Aleatory vs. epistemic uncertainty
   - Common framework
3. Monte Carlo Simulation
   - Moments- and distribution analysis
   - UQ with Monte Carlo Simulation
   - Back to EXAR
4. Conclusions

Conclusions

- Computational models only provide an approximate description of real-world engineering or natural systems
- Uncertainties in the geometry, material properties, environmental conditions, etc. preclude using a single set of parameters in the analysis
- Uncertainty quantification techniques allow us to:
  - set up the problem
  - quantify the sources of uncertainty
  - and study their impact onto the model predictions and the system performance
- According to the problem under consideration (reliability analysis, robust estimations, sensitivity analysis, etc.), many different methods can be applied to propagate uncertainties

Thank you very much for your attention!

Conclusions

- Computational models only provide an approximate description of real-world engineering or natural systems
- Uncertainties in the geometry, material properties, environmental conditions, etc. preclude using a single set of parameters in the analysis
- Uncertainty quantification techniques allow us to:
  - set up the problem
  - quantify the sources of uncertainty
  - and study their impact onto the model predictions and the system performance
- According to the problem under consideration (reliability analysis, robust estimations, sensitivity analysis, etc.), many different methods can be applied to propagate uncertainties

Thank you very much for your attention!