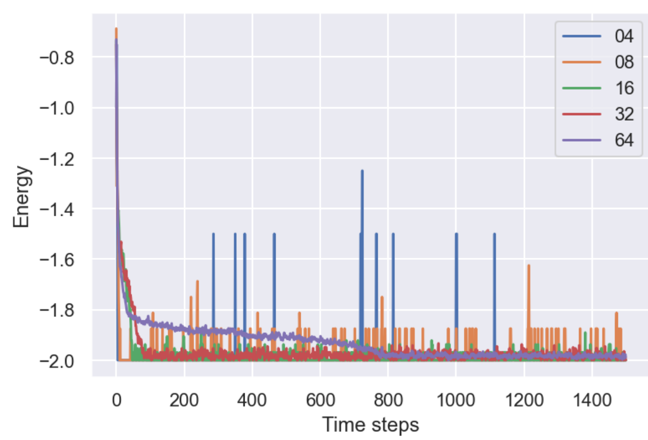
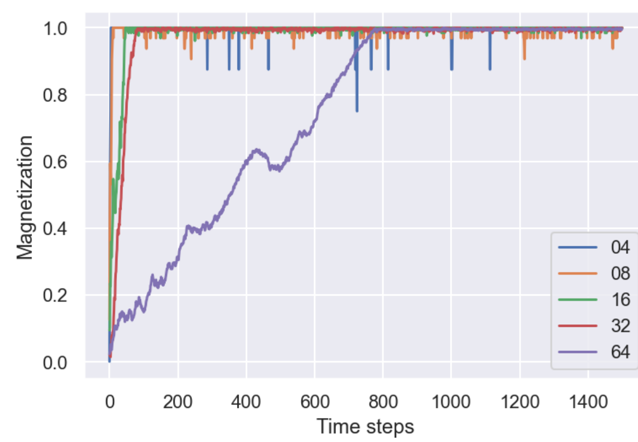
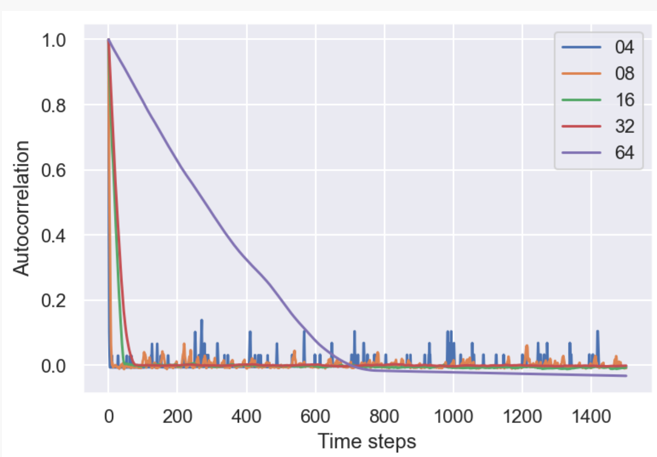
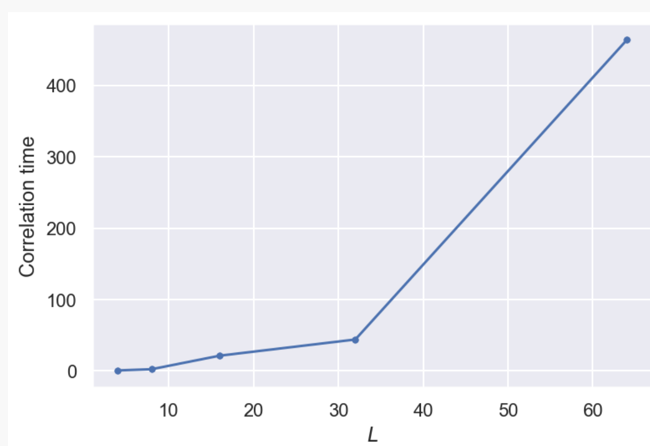
Ising model

Simulation setup

We considered lattice size $L \in \{4, 8, 16, 32, 64\}$. For each lattice, the model is thermalized and then passed through 10^5 Monte-Carlo steps. The Metropolis algorithm is used for this exercise.

Thermalization and correlation time

For different values of L, the thermalization is done at $\beta = 0.84$, as shown in the figure below.. The correlation plots confirms the thermalizaion, from which we can compute the correaltion time given in the table.

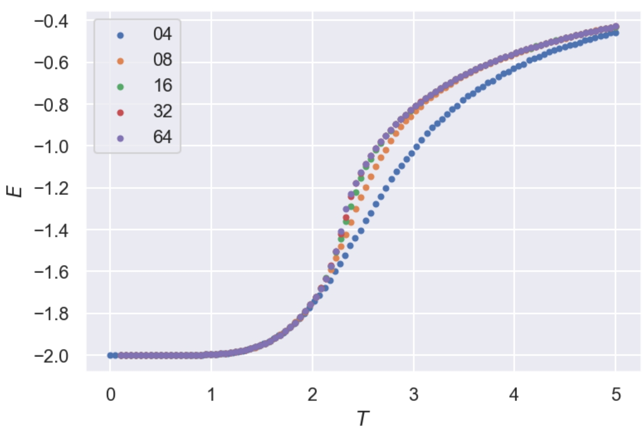
	
	

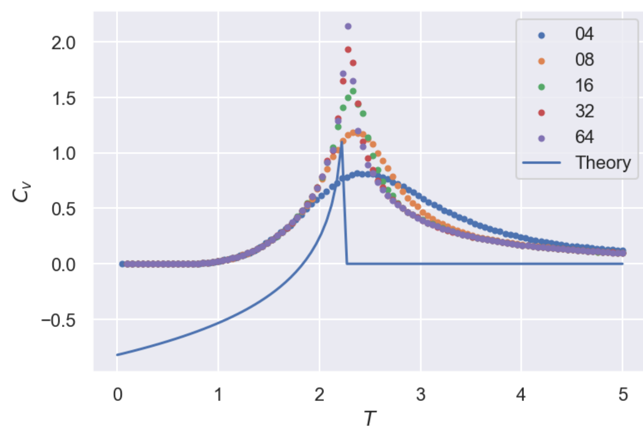
The correlation time is computed using the exponential approximation, $\tau_G = \rho(1)/(1-\rho(1))$. From the above plot, we can see that it increases with the lattice size.

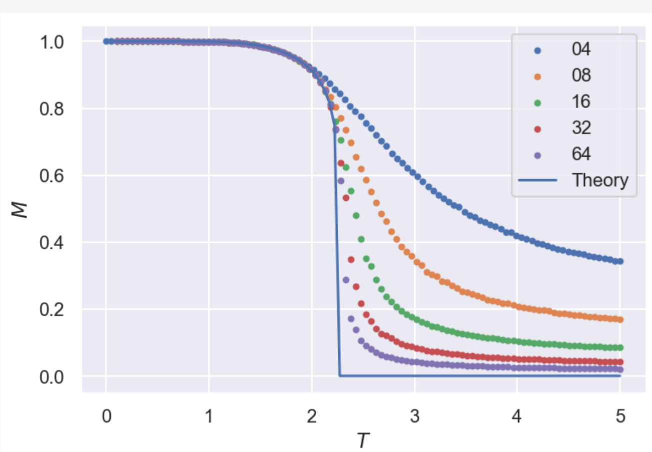
L	τ_G
4	0.634
8	2.407
16	21.297
32	43.992
64	463.587

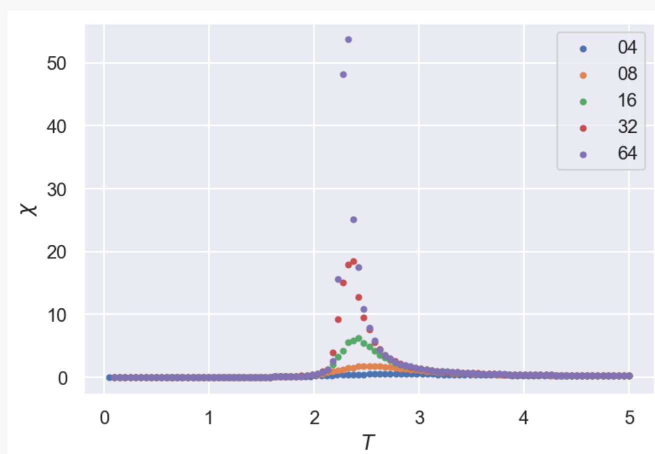
Expected results

The results are plotted for all lattice sizes as below. These plots indicates that the simulated criticla temperature reaches the therotical one as the size of the lattice gets bigger.



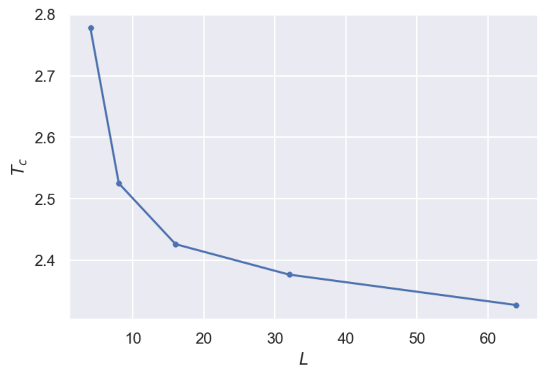


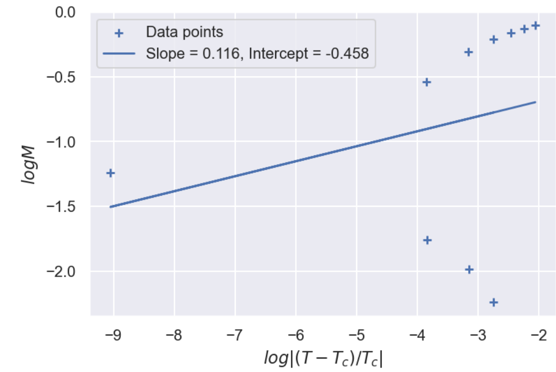


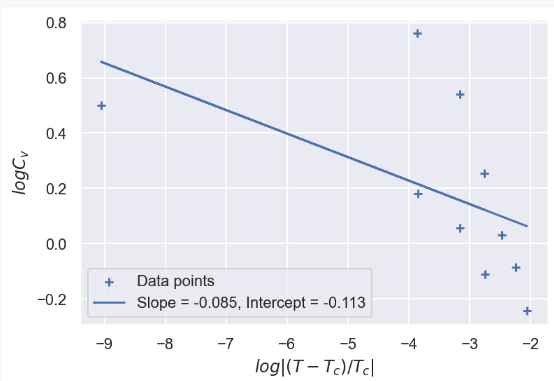


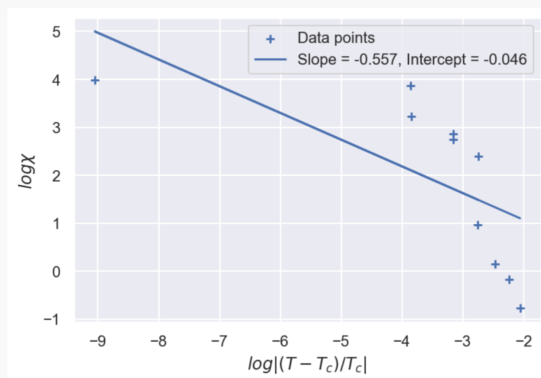
Critical exponents

First, we note that the critical temperature varies with the size of the lattice as can be seen from the table and figure.





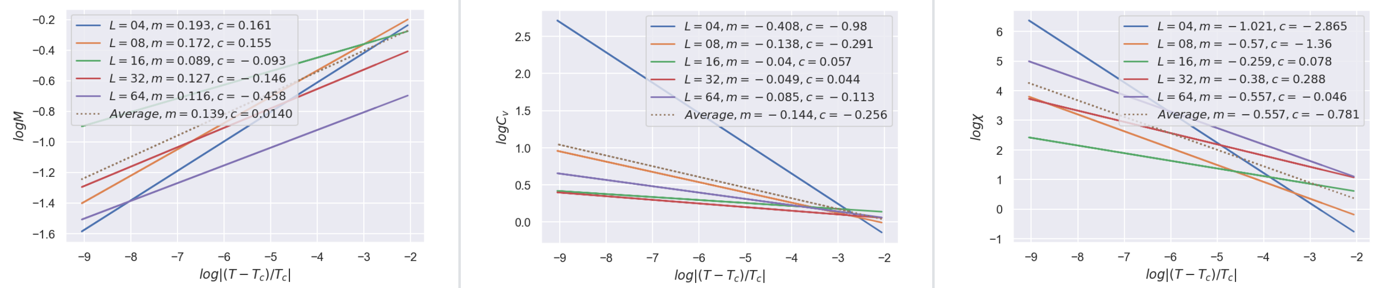




L	Tc
4	2.778
8	2.525
16	2.426
32	2.377
64	2.327

We computed critical exponent for all lattice sizes near the critical region ftom $T = 2.0$ to $T = 2.5$. The results are shown in the abe plots. For illustration, we have also provided the result for $L = 64$ with data points and fit.

For all lattice sizes, the plots are given below with averages.



The average value of critical exponents are then found to be,

$$\beta \approx 0.139, \quad \alpha \approx 0.144, \quad \gamma \approx 0.557, \tag{5}$$

while for the large lattice size $L = 64$,

$$\beta \approx 0.116, \quad \alpha \approx 0.085, \quad \gamma \approx 0.557. \tag{6}$$

Compared with the the actual values, only β and α seems to agree.

$$\beta = 1/8, \quad \alpha = 0, \quad \gamma = 7/4, \tag{7}$$

Correlation length

The correlation length was found as following. We first inverted the equation,

$$\Gamma_{ij} =< (\sigma_i - < \sigma >)(\sigma_j - < \sigma >) >\approx exp[-|i-j|/\zeta(\tau)], \tag{8}$$

to get $\zeta(\tau)$. The correlation function is the simulated for $\tau \in (0.937, 0.729, 0.52, 0.311, 0.103, 0.106, 0.314, 0.523, 0.731, 0.94)$ for the lattice of size 64. The results are shown in the figure below, that gives $\nu = 0.423$ that is far from the theoretical value 1.