Brill-Noether general curves on K3 surfaces

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Theorem 0.1 (lagassfeld1986)**.** *Let X be smooth K3 surface, H ample divisor such that all curves in $|H|$ are integral, then for a general smooth curve C in $|H|$, C is Petri general and all smooth curve C in $|H|$ are Brill-Noether general.*

Remark 0.2.   *1. The condition on $|H|$ is satisfied if $Pic(X) = \mathbb{Z}[H]$*

2. *a curve C Brill-Noether general if and only if $W_d^r(C)$ has expected dimension for all $d \geq 1, r \geq 0$ where expected dimension is $\rho(g, d, r) = g - (r+1)(g - d + r)$.*

note that if $\mathcal{L}$ on C with $h^0(\mathcal{L}) = r + 1, \deg \mathcal{L} = d$, then $\rho(C, \mathcal{L}) = g - h^0 \cdot h^1 = \rho(g, d, r)$. And

$$W_d^r(C) = \{[\mathcal{L}] \in Pic(C) : \deg \mathcal{L} = d, h^0 \geq r + 1\}$$