compactification of $\mathbb{C}^n$ and Reeb dynamics

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Classify compactification of $\mathbb{C}^n$ (X, B) such that $X \setminus B \cong \mathbb{C}^n$ such that $b_2(X) = 1$

1. $n = 1, 2, (\mathbb{P}^n, \mathbb{P}^{n-1})$
2. for $n \geqslant 3$, $(\mathbb{P}^n, \mathbb{P}^{n-1}), (Q_n \subset \mathbb{P}^{n+1}, Q_n \cap H)$
3. $n = 3$ if D smooth (called smooth compactification), X not fano, then $(\mathbb{P}^3, \mathbb{P}^2)$
4. true for Kahler assumption.
    - proved for $n \leq 6$ by Fujita
    - short proof for $n \not\equiv 3 \mod 4$ by Li-Peternell
    - implied by $D \cong \mathbb{P}^{n-1}$ and $\leftrightarrow c_1(D) = n$