WEEK 4: 2021-06-08, 2021-06-10

[AM]: Atiyah-Macdonald, Introduction to commutative algebra.

Exercise 1. Let k be a field, $R := k[X, Y, Z]$ be the polynomial ring. Set $\mathfrak{a} := \langle XY, X - YZ\rangle$, and

$$\mathfrak{q}_1 = \langle X, Z\rangle, \quad \mathfrak{q}_2 = \langle Y^2, X - YZ\rangle.$$

Show that $\mathfrak{a} = \mathfrak{q}_1 \cap \mathfrak{q}_2$, and that this is a minimal primary decomposition.

Exercise 2. Let A be a noetherian ring, M an A-module, N a submodule of M and $x \in A$. Prove that if $x \notin \mathfrak{p}$ for any $\mathfrak{p} \in \mathrm{Ass}(M/N)$, then $xM \cap N = xN$.

Exercise 3. [AM] Page 92, Exercise 3.

Exercise 4. [AM] Page 92, Exercise 6.

Exercise 5. [AM] Page 72, Exercise 28.