Minseg Linear Dynamical Model and Feedback Control

EE 547 (PMP) Project Report
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1. Introduction to Inverted Pendulum

The objective of this project is to develop a linear state-space model of the MinSeg Test Platform (shown Figure 1), to use the model to derive a stabilizing feedback controller and to deploy the feedback controller to the Minseg. The purpose of the stabilizing feedback controller is to balance the Minseg in an upright position as shown in Figure 1.

1.1 Linear Dynamical Model of the MinSeg Robot

The MinSeg robot is an inverted pendulum that can be modeled by the nonlinear equation below.

$$\begin{bmatrix} -(I_p + m_p L^2) & m_p L\cos\alpha \\ m_p L\, r_w^2 \cos\alpha & -(I_{cm,w} + m_w r_w^2 + m_p r_w^2) \end{bmatrix} \begin{bmatrix} \ddot{\alpha} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} T_m - m_p L g \sin\alpha \\ T_m r_w + m_p L\, r_w^2 \dot{\alpha}^2 \cos\alpha \end{bmatrix}$$

The equation is a function of the pendulum's angle with the vertical axis, α; the horizontal position of the pendulum, x; and the first and second derivatives of those quantities.
The non-linear equation would typically be linearized by finding the Jacobian matrix. In this case, the equation is only non-linear due to $\cos(\alpha)$, $\sin(\alpha)$ and α'^2 terms; and those terms can be approximated by 0, α, and 0 respectively while α is small. The linearized equation around the equilibrium point of $\alpha=0, x=0$ is shown below. The term Tm is expanded in terms of inpthe ut voltage, horizontal velocity, and angular velocity. Finally, the A and B matrices of a linear state-space representation of the MinSeg robot are found and can be seen in step 1 of the Matlab code. The state is $x = [\alpha\ \alpha'\ x\ x']'$, the input is $u=V$, the C matrix is the identity matrix, and the D matrix is a zero vector. The complete A and B matrices are omitted from this report because they are large.

$$T_m = \frac{k_t}{R}V + \frac{k_t k_b}{R r_w}\dot{x} + \frac{k_t k_b}{R}\dot{\alpha},$$

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{gLm_p(i_{cmw}+(m_p+m_w)r_w^2)}{i_{cmw}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2} & -\frac{k_bk_t(i_{cmw}+r_w(m_wr_w+m_p(L+r_w)))}{R(i_{cmw}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2)} & 0 & -\frac{k_bk_t(i_{cmw}+r_w(m_wr_w+m_p(L+r_w)))}{Rr_w(i_{cmw}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2)} \\ 0 & 0 & 0 & 1 \\ \frac{gL^2m_p^2r_w^2}{i_{cmw}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2} & -\frac{k_bk_tr_w(i_p+Lm_p(L+r_w))}{R(i_{cmw}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2)} & 0 & -\frac{k_bk_t(i_p+Lm_p(L+r_w))}{R(i_{cmw}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2)} \end{bmatrix}$$

$$B=\begin{bmatrix} 0 \\ -\frac{k_t(i_{cmw}+r_w(m_wr_w+m_p(L+r_w)))}{R(i_{cmw}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2)} \\ 0 \\ -\frac{k_tr_w(i_p+Lm_p(L+r_w))}{R(i_{cmw}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2)} \end{bmatrix}$$

The equations describing the behavior of the inverted pendulum include several physical parameters of the pendulum including multiple physical dimensions, masses, and moments of inertia. The next section describes the acquisition of these parameters.

2. Coefficients and their measurements and calculation for a moment of Inertia

2.1 Moment of Inertia at the center of mass of Wheel *(Icm,w)*

The moment of inertia of the wheels was calculated by timing how long it took for the wheel to roll down a ramp. Assuming no slipping between the wheels and the ramp all the potential energy is converted to translation and rotational energy. This can be described as:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2}$$

Where:

m = rolling mass in *kg; I* = moment of inertia in $kg.m^2$; g = gravity in m/S^2; h = vertical distance travelled in *m*
v = velocity in *m/s ; r* = radius of wheel in *m*

Wheel mass was obtained using a digital scale, shown below in Picture. The total mass of both wheels and axel (m) is 37 grams.

The diameter of the wheels is measured with the help of caliper. It comes to 43.28 mm. The radius of the wheel is 43.28/2 = 21.64 mm. The radius (r)=0.02164 m.

Wheel inertia was found by setting a ramp at a certain angle, a starting height (h) of 0.0425 m, and using a timer to measure the time required to travel distance of on-ramp.

From the table shown below, it can be said that the ten runs combined give an average time of 0.716 sec for 0.281 m distance traveled, gives a velocity of 0.3924 m/s.

Table 1 Runtime for traveling same distance on a ramp

Run	Distance Travelled (m)	Time consumed (sec)
1	0.281	0.74
2	0.281	0.8

3	0.281	0.79
4	0.281	0.66
5	0.281	0.59
6	0.281	0.66
7	0.281	0.86
8	0.281	0.73
9	0.281	0.67
10	0.281	0.66

Solving the below Equation for the moment of inertia:

$$I = \frac{r^2 m(2gh - v^2)}{v^2}$$

Plugging: h= 0.04255 *m;* *V*= 0.3924 *m/s;* g=9.80665 *m/*S^2*;* r=0.0216 *m;* m=0.037 *kg*

We found moment of inertia of wheels (Icm,w) = 7.62997e-5 $kg.m^2$

2.2 Moment of Inertia at the reference point of the pendulum *(Ip)*

The moment of inertia can readily be found by allowing the MinSeg to swing while the period of oscillation is measured. Once known, the period of oscillation and the physical mass of the MinSeg can be used to estimate the inertia. Below Equation shows this relationship.

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

Where:

T = period of oscillation in *sec*
I = moment of inertia in $kg.m^2$
m = mass of pendulum in *kg*
g = gravity in *m/*S^2
h = distance from pivot to center of gravity in *m*

For that, the center of mass is required. The figure shows the nomenclature used for this calculation.

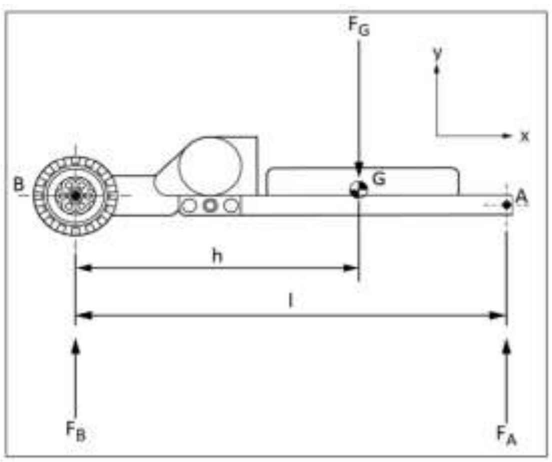


Figure 1 Minseg Center of Mass Calculation

Applying Newton's second law to this system and summing forces in the vertical (y-direction) and moments about point B results in the following system of equations:

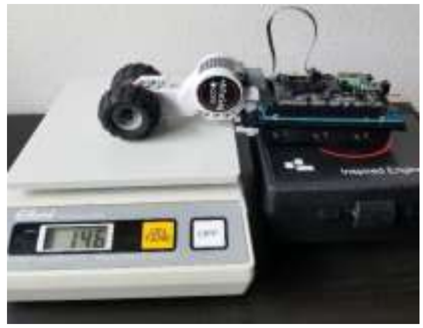
$$\sum F_y = 0 \Rightarrow \mathrm{F}_b + \mathrm{F}_a - \mathrm{F}_g = 0$$

$$\sum M_B = 0 \Rightarrow \mathrm{F}_a l - \mathrm{F}_g h = 0$$

Fb and Fg cab be found as below:

Fg can be found by placing the Minseg vertically and measuring its mass as shown in fig.
Fb can be found be placing the Minseg horizontally and measuring its mass as shown below.

F_g = 340 + 4 (wire weight) + 37 (weight of rolling mass)= 381 gm
F_b = 146 gm



Thus, the first equation can be solved directly for the unknown reaction force, Fa;
$F_a = -F_b + F_g = -146 + 381 = 235$ gm= 0.235 kg
The length (l) needs to be measured as below. It comes to l=19.5 cm = 0.195 m.

Likewise, the length of moment arm (h) can be found from the second equation as:
$h = F_a/F_g * l = 146/381*0.195 = 0.07472$ m
Below figure shows the physical arrangement for suspending the MinSeg. Two axles provided support, allowing the MinSeg to swing freely. During the test, the MinSeg was connected to Matlab so that the signal from the on-board gyro could be captured.

Below Figure shows the resulting data. It performs 28 oscillations in 20 sec. Therefore, it has a frequency of 28/20= 1.4 Hz showing a time of 0.714 seconds for one oscillation.

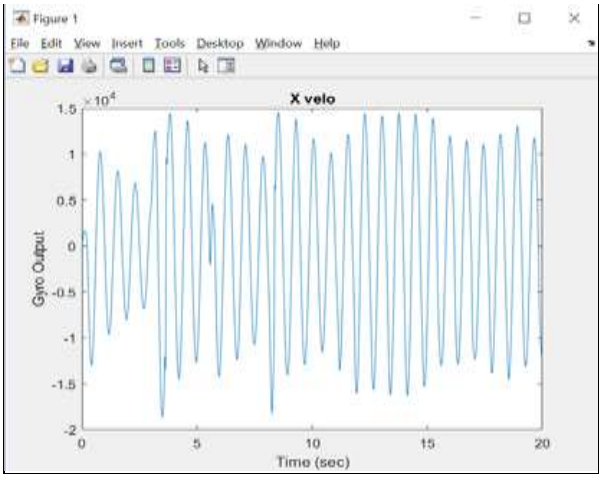


Figure 2 Oscillations of Suspended Minseg

Now, the equation can be rearranged to solve for the moment of inertia:

$$I = \frac{g\,h\,m\,T^2}{4\,\pi^2}$$

Plugging:

g = 9.80665 m/S^2; *h* = 0.07472 *m*; *m* = 0.381 *kg*; *T* = 0.714 *sec*

The moment of inertia at the reference point of the pendulum is obtained as Ip = 3.60511562e-3 $kg.m^2$

2.3 Summary of Parameters

The last 2 parameters, Kt and Kb, are constants relating the motor's torque to voltage, horizontal acceleration, angular acceleration, and voltage; they were provided in the project specification. The table below shows all of the constants needed to define the A and B matrices of the linearized system.

Table 2 List of Measured Parameters

Parameter	**Measured Quantity**
The length between wheel center and a reference point (*h*)	0.07472 *m*
Mass of pendulum (m_p)	0.381 *kg*
Moment of inertia at the reference point (I_p)	3.6051156*1e-3 $kg.m^2$
Mass of wheel (m_w)	0.037 *kg*
The radius of the wheel (r_w)	0.0216 *m*
Moment of inertia at the center of mass of wheel ($I_{cm,w}$)	7.62997*1e-5 $kg.m^2$
K_t	0.3233 *Nm/A*
K_b	*1.4953 Vs/rad*

3. Open Loop System Characteristics

The linearized system with the state space representation with A, B, C and D matrices and the eigenvalues of the system can be found in the Matlab code.

One of the poles of the system is lying on the right half plane. Therefore the system is unstable. The eigenvalues are included in the Matlab Code.

4. Stability, Controllability, Observability and its Canonical forms

The system was found unstable as some of the eigenvalues are placed on the right half of the S-Plane.
For stabilizing the system, we checked the controllability and observability of the system. The system was found to be controllable and observable from its controllability and observability matrices. Both matrices have full rank. The controllability, observability, controllable canonical form of the A and B matrices, and observable canonical form of the A and B matrices can be seen in the Matlab code. The system is controllable and observable as the rank of controllability and observability matrix is 4.

5. State Estimator

The closed-loop state-estimator was found using the A and C matrices from the state-space model of the pendulum. The A matrix of the estimator was calculated as

$$A_{estimator} = A_{system} - L * C_{system}$$

Where L is the gain of the feedback within the estimator block. The gain was calculated with Matlab place command. To set the estimator pole at [0 -935.5185 -31.3975 -30.7085] to make the system 5 to 6 times faster than the original system, the gain was found. A plot of the estimated-state variables and the actual states variables can be seen with the Matlab code. The gain L is included in the Matlab Code.

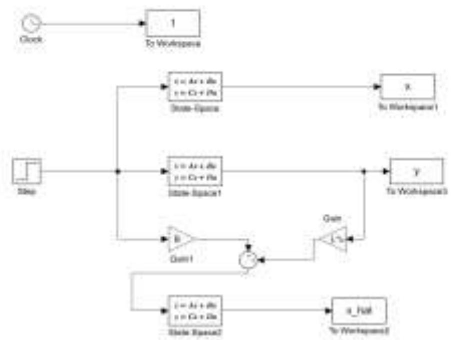


Figure 3 Simulink model of System with Estimator

From the plot of estimator states and system states, it can be seen that the estimator states perfectly follow the original system states. The system is unstable yet and therefore, the states are diverging form x-axis towards infinity.

6. Feedback Control

Now, the system is already improved to track the states perfectly with the estimator. Now, the system can be made stable by designing the feedback controller. The new A matrix can be shown with the following equation.

$$A_{controlled} = A_{system} - B_{system} * K$$

We got good results by placing poles at [-67.2, -59.2,-12,-6] and found the relevant gain K by using the place command in Matlab. A plot of the estimated-state variables, the actual states variables can be seen with the Matlab code. The value of the gain K is [-1.3989 0.4737 0 22.9307].

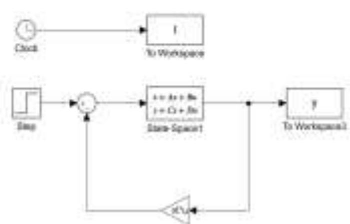


Figure 4 Simulink Model of System with Feedback

From the above plot, initially, the system output changes a lot but eventually, it stabilizes to zero. The output is now bounded, and the system is stabilized.

7. Feedback Control using State Estimator

The estimator and controller can be combined to give the complete stabilized system.

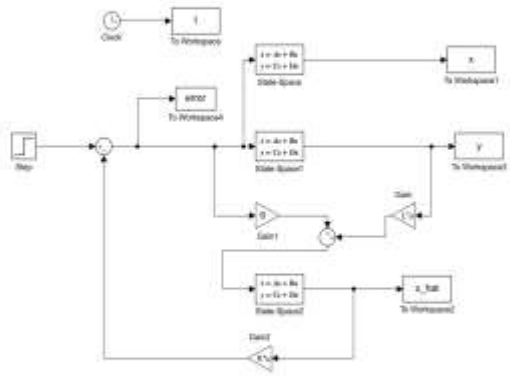


Figure 5 Simulink Model of System with Estimator and Controller

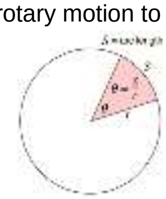
The error signal is the error in reference and the output of the system. It reduces with time and makes the system stable. In addition to this, it can be confirmed that the system has become stable as the states are bounded and stabilize to zero and estimator states perfectly follows the states of the original system.

8. Implementation of Feedback Control to Minseg

LQR feedback was used to balance the Minseg. The states were measured by sensors, and the feedback gain was determined with the LQR command in Matlab. It is extremely important to set the gain K in such a way that it balances the Minseg. Additionally, a safety switch was added to stop driving the motor if the angular position exceeded 30 degrees to prevent the motor from spinning after the Minseg had fallen over.
Steps for implementing the model of Minseg using Matlab Simulink:

8.1 Measuring the Minseg States

The rotary encoder, gyroscope, and accelerometer were used to measure the states of the Minseg system.
Horizontal Position (x): The horizontal position was found by multiplying the change in angular position of the wheel by the radius of the wheel. The change in the angular position of the wheel was obtained from the rotary encoder. Below figure demonstrates the relationship between the rotary motion to the linear motion of the wheel.



Horizontal Velocity: The horizontal acceleration was found by taking the derivative of the horizontal position with respect to time.

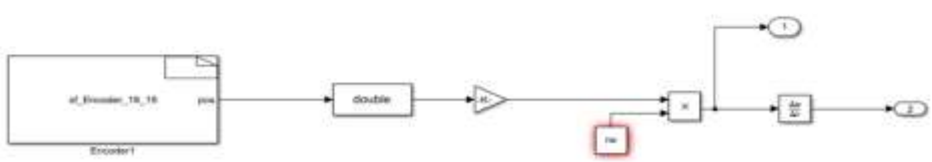
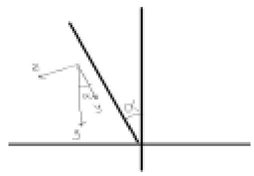


Figure 6 Sensing the position from Encoder

Angular Position (α) and Angular Velocity (α'): Multiple techniques were tried to find the angular position and angular velocity of the Minseg. The initial approach was to measure the angular velocity with the gyroscope and integrate it to find the angular position. The gyroscope, however, has a poorly defined offset in its measurement of angular velocity,

resulting in the angular position having a drift that would eventually trip the safety switch and prevent the Minseg from balancing. This motivated an alternative method of measuring the angular position.
The second method used was to measure the acceleration parallel to the axis of the Minseg using the accelerometer. The direction parallel to the Minseg is labeled y, the direction perpendicular to the Minseg is labeled z, and the acceleration of gravity is labeled g in the diagram below.



The angular position of the Minseg could be calculated as

$$\alpha = acos(y/g)$$

This calculation only gave the absolute value of the angular position and did not include information on which side of the vertical axis the Minseg was leaning to. The sign of the angular position (positive or negative) was calculated from the integral of the angular velocity acquired by the gyroscope. This puts a limitation on the system performance because, at some point, the offset in the gyroscope angular velocity will result in an integrated error large enough that the sign for the angular position derived from the accelerometer will be incorrect. This limitation has much-improved performance over using the gyroscope for the angular position, however, because it will take much longer for the error to accumulate to the point that the sign of the angular position from the accelerometer will be incorrect.

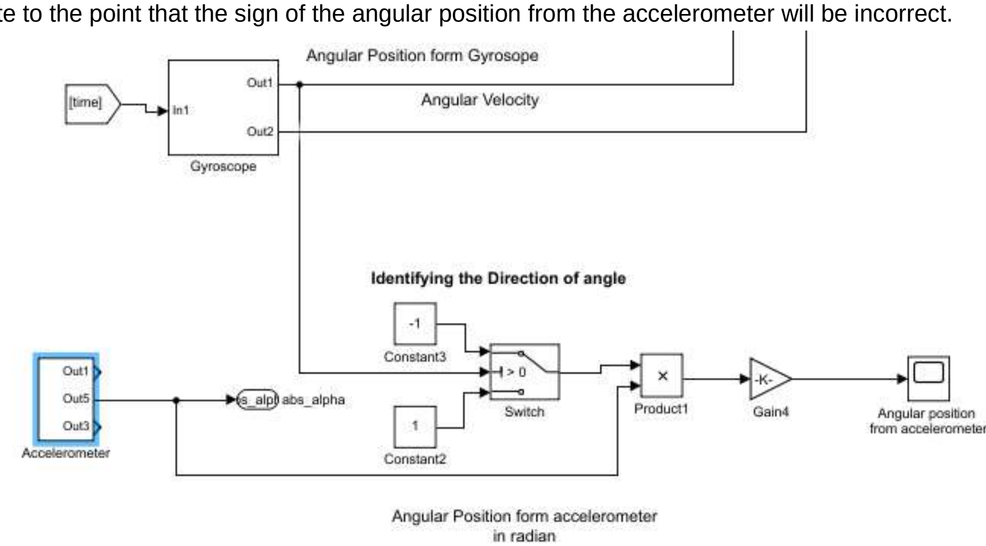


Figure 7 Getting the Sign of Angle from Gyroscope

8.2 Calibrating the States

A procedure was developed to acquire a reference for the vertical axis before the Minseg attempted to balance itself. The Minseg is held in a vertical position up against a wall while the motor is turned off and the angular position is being measured. The angular position is averaged for 5 seconds, and then a sample and hold block stores that average value. Since the measured value corresponds to an angular position of 0 degrees, it is subtracted from the measured angular position for all time after 5 seconds to calibrate any offset in the angular position measurement.

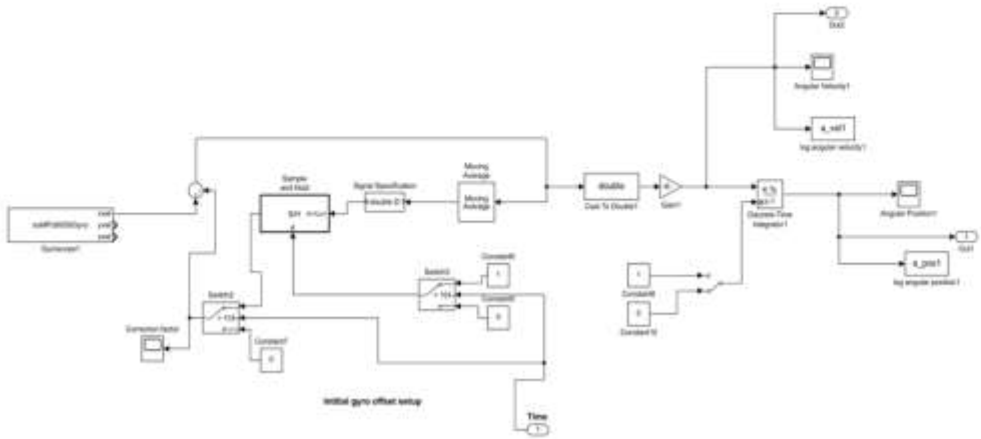


Figure 8 Calibration of Gyroscope and offset setting

8.3 Adjusting Feedback Gain

Below is the complete Simulink model. Now, the perfect value K is needed to find so that the Minseg can balance. The states are measured from sensors and are calibrated. The motor can be turned off if the angle is greater than 30 degrees. There is a motor overdrive detection Simulink model which detects whether the motor voltage is beyond its capacity or not. The perfect gyro offset can be calculated by taking the average and using it as a bias for setting the perfect gyro reference. The potentiometer can be used to fine-tune the feedback gain K. Below is the complete model for Minseg.

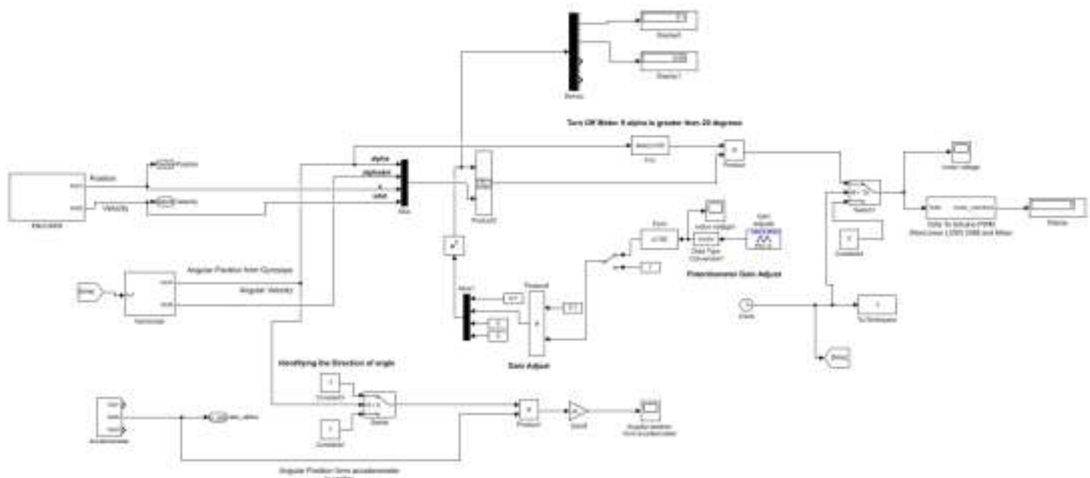


Figure 9 Complete Simulink Model for Minseg Balancing

The feedback gain was acquired with the LQR command in Matlab. The Q and R matrices were derived empirically in an effort to balance the Minseg. The angular position and angular velocity were weighted more heavily in the Q matrix because the horizontal position of the Minseg is not important as long as it has a reasonable range (<1m). The R matrix was weighted lightly because we are not concerned with energy-efficiency.
We changed the weight of diagonal of Q matrix and changed the value of the R matrix and tried different combinations to find the Gain K. The gain was too high that motor was overriding.
To reduce the motor voltage, we increased the value of R. By increasing the value of R, the system requires very less effort and thus it makes the system energy-efficient.

9. Challenges

With all bove effort with Q and R matrix, we didn't get a satisfactory response to the Minseg. One of the problems was motor overdrive. The motor was running beyond the voltage capacity. This can be controlled by reducing the value of Gain To resolve this issue, we set a model to detect the motor-overdrive condition as shown below.

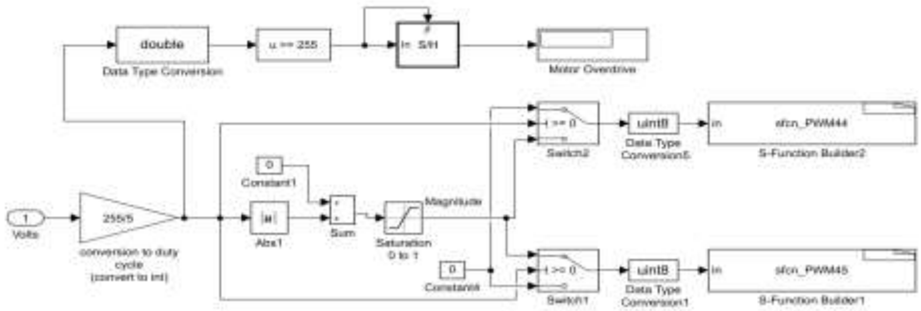


Figure 1 Simulink model of Motor Overdrive Detection

To obtained very less gain, we started using the pole placement method. We chose different poles as below, but the gain was still too high.

This value of gain was still high that it was overdriving the motor. Then we put the value of gain very low and started using the potentiometer for fine tuning the gain K. We developed a model like shown below. All the individual parameter of gain K can be changed with a potentiometer. We set the gain for alpha and alphadot and tried changing the ratio of both. We got a slightly satisfactory result with the gain value shown below.

```
K =

    0.1000    0.2950         0         0
```

Looking at the above value of gain, it can be said that only the feedback of alpha and alphadot was taken into consideration. Unfortunately, we were not able to balance the Minseg perfectly.

In addition to the above problems, we were unable to get the perfect offset for a gyro. The signal coming from gyroscope was very distorted and even though we take the moving average for the definite amount of time, there was some distortion in the signal which was creating an issue to set the reference alpha to zero perfectly.

10. Future Scope

There are still some ways to obtain the Minseg perfectly balanced.

To get rid of the noisy signal from the Gyro, a complementary filter can be used. A hybrid model of Gyroscope and accelerometer is able to get the perfect smooth signal of position.

Q and R matrix can be perfectly set to get the desired value of gain K such that it doesn't overdrive the motor.

11. Conclusion

The MinSeg is designed with a center of mass roughly half the length of the full assembly. The MinSeg Kit provides an excellent environment for implementing Linear Control theory on practical physical systems.

The team concluded that the key factor in achieving a balancing MinSeg, is an appropriate gain matrix, *K*.

Video of Minseg: https://www.youtube.com/watch?v=-a4SHM3lE_E&feature=youtu.be