DESI 3PCF Tutorial

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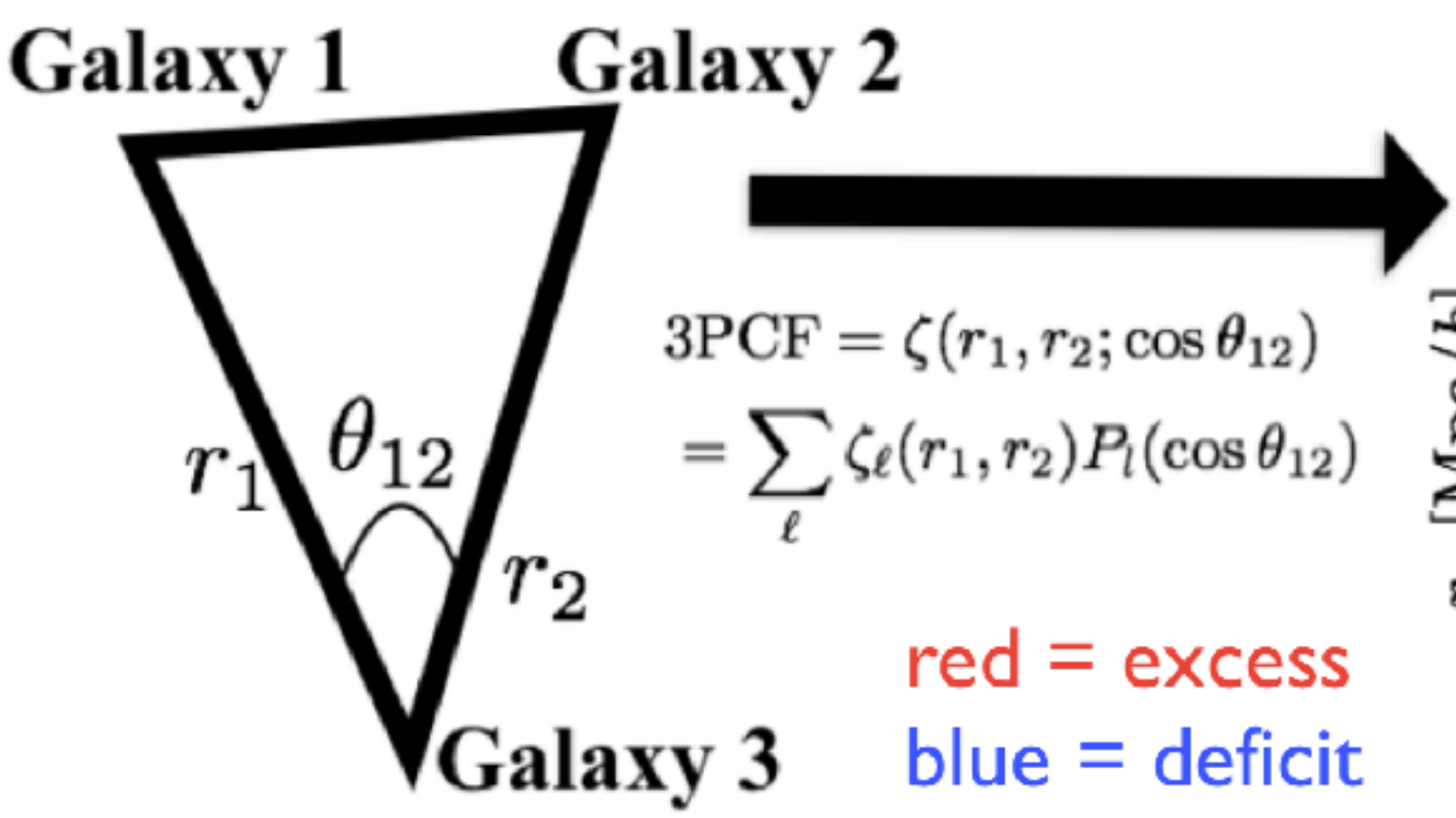
THE 3-POINT CORRELATION FUNCTION

Leading-order statistic sensitive to information beyond the Gaussian Random Field

Galaxy clustering has a 3PCF

Can we use it to sharpen the BAO standard ruler?

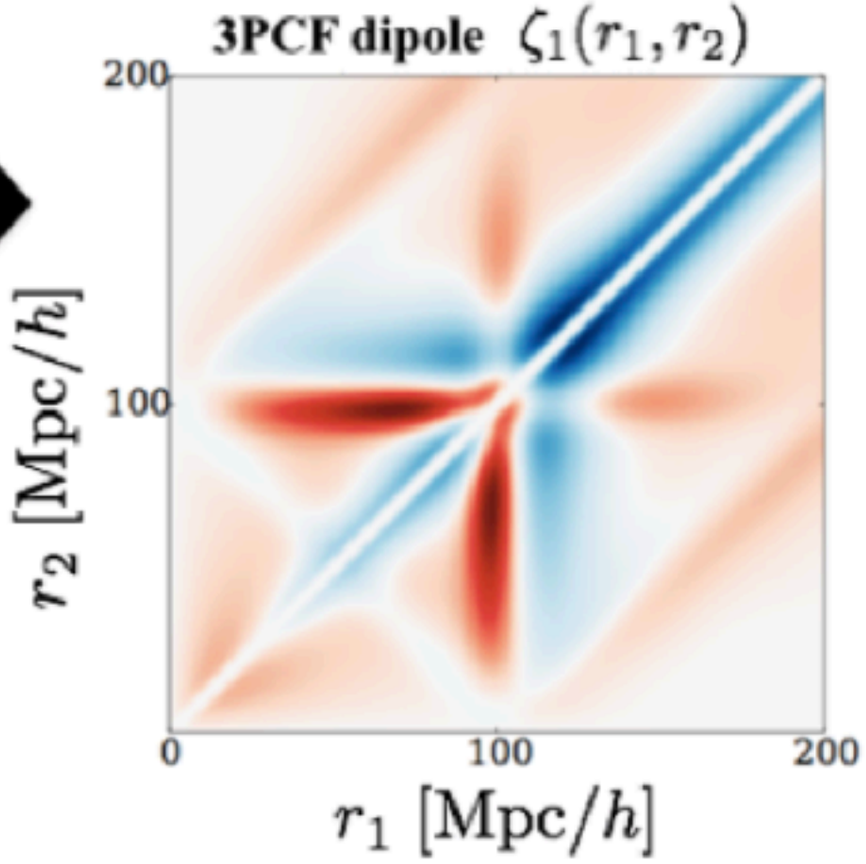
A NEW BASIS: MULTIPOLES



$$\text{3PCF} = \zeta(r_1, r_2; \cos\theta_{12}) = \sum_\ell \zeta_\ell(r_1, r_2) P_l(\cos\theta_{12})$$

red = excess

blue = deficit



Color shows # of triangles with given side lengths; angle dependence is projected onto Legendre polynomial P_ℓ

Proposed by Szapudi 2004 but not used

Developed by myself and collaborators

Slepian & Eisenstein 2016, MNRASL 455, 1

Slepian & Eisenstein 2015, MNRAS 454, 4

The 3PCF is computationally expensive

if you have 10 friends, choosing two to take to lunch is 10 X 9 = 90

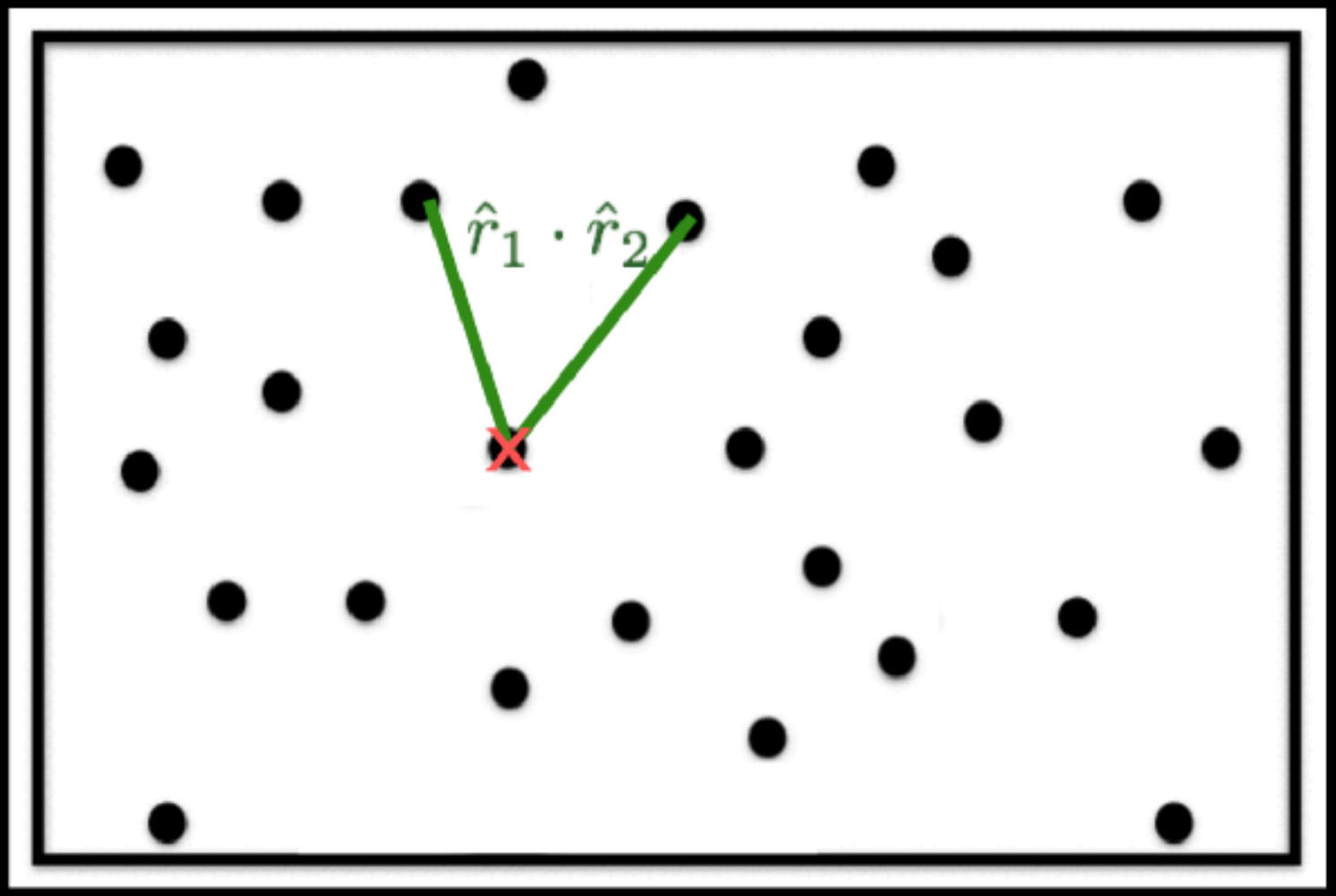
Choosing 3 is
10 X 9 X 8 = 720
—you are double-booked!

For large N, 3PCF scales as N^3 vs. 2PCF's N^2

THE N² 3PCF ALGORITHM

$$\bar{\hat{\zeta}}_l(r_1, r_2; \vec{s}) = \frac{2l+1}{(4\pi)^2} \int d\Omega_1 d\Omega_2 \delta(\vec{s}) \bar{\delta}(r_1; \hat{r}_1; \vec{s}) \bar{\delta}(r_2; \hat{r}_2; \vec{s}) P_l(\hat{r}_1 \cdot \hat{r}_2)$$

Compute about a particular galaxy given by **s** first, then average over all galaxies



looks like order N^2 about **each** galaxy

so overall order N^3

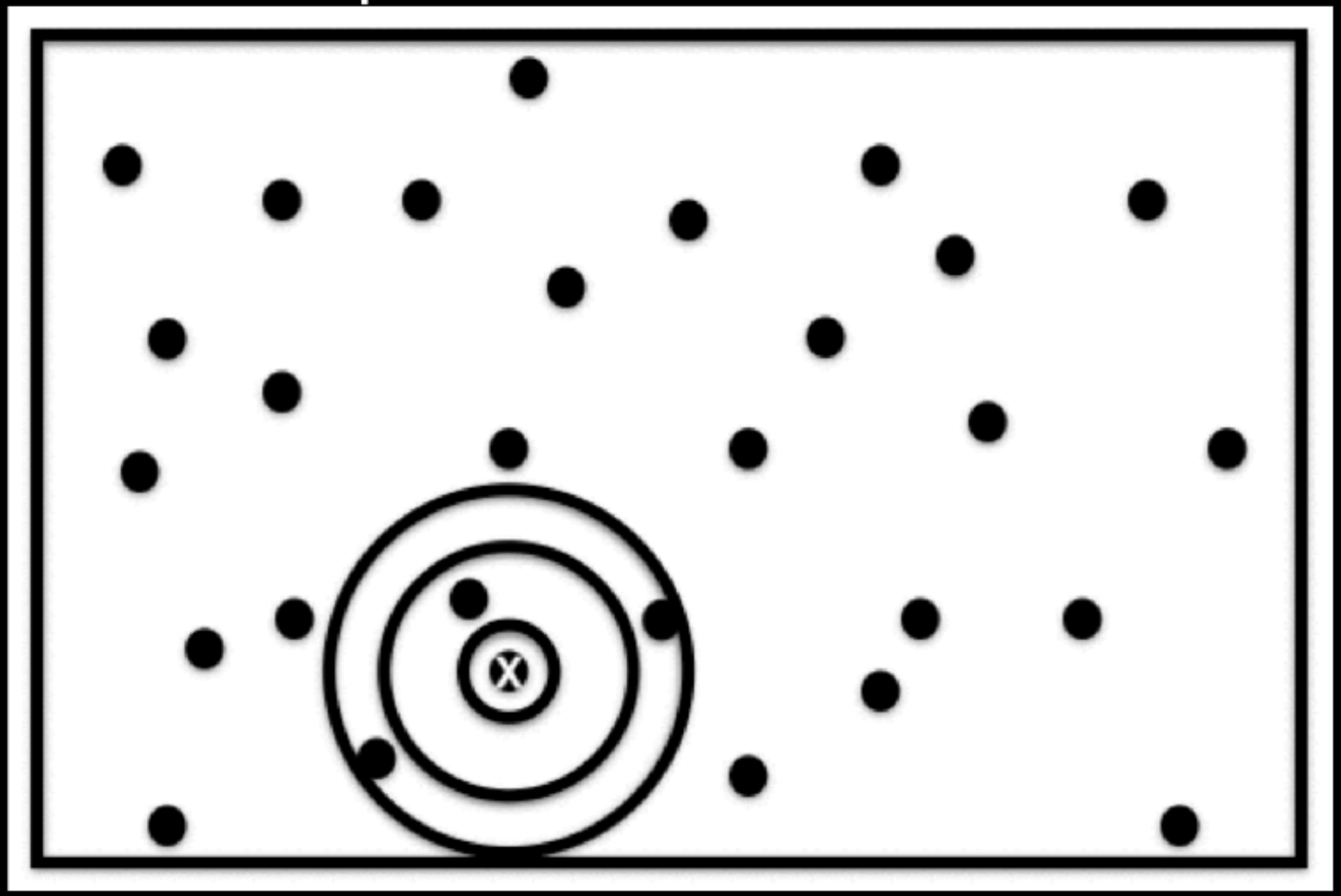
Slepian & Eisenstein 2015b

SPHERICAL HARMONIC ADDITION THEOREM ALLOWS FACTORIZATION

$$P_l(\hat{r}_1 \cdot \hat{r}_2) = \frac{4\pi}{2l+1} \sum_{m=-l}^{l} Y_{lm}(\hat{r}_1) Y^*_{lm}(\hat{r}_2)$$

Assemble 3PCF multipole moments from spherical harmonic coefficients, which depend only on direction to one galaxy each

Around each galaxy, compute a_{lm} in spherical shells/radial bins



$$a_{lm}(r;\vec{s}) = \sum_{\text{gals } j \text{ in bin}} Y^*_{lm}(\hat{r}_j)$$

NOW ORDER N ABOUT EACH GALAXY, OVERALL N^2

Slepian & Eisenstein 2016, MNRAS 454, 4

SPEED OF THE ALGORITHM

500X FASTER THAN A TRIPLET COUNT FOR 700,000 GALAXY TEST CASE

ONLY 6X SLOWER THAN COMPUTING A 2-POINT FUNCTION

Details in SE15b, SE16a

Contact/References/Links

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References

Algorithm paper:

https://arxiv.org/abs/1506.02040

FT-based version:

https://arxiv.org/abs/1506.04746

And further description of implementation of the FT version:

https://arxiv.org/abs/1711.09907

A few application papers

To BOSS data:

https://arxiv.org/abs/1512.02231

https://arxiv.org/abs/1607.06097

To MHD turbulence:

https://arxiv.org/abs/1711.09907